New contribution to Single-spin Asymmetry

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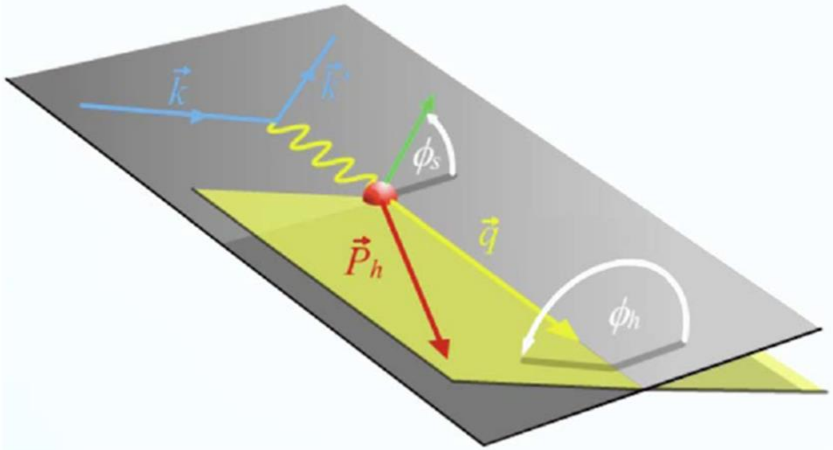
Presented at PKU

July 15, 2017

Process with polarization

- Semi-inclusive deeply inelastic scattering off polarized proton

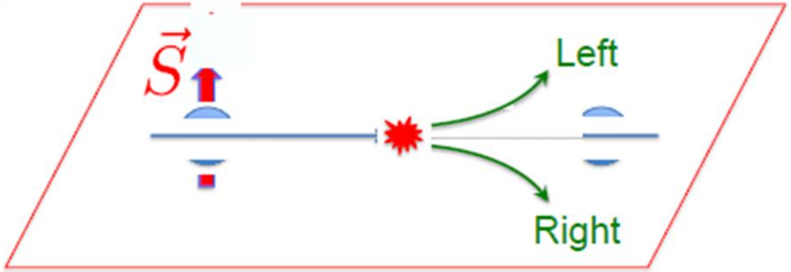


$$\ell + p^{\uparrow} \rightarrow \ell' + \pi(p_T) + X$$

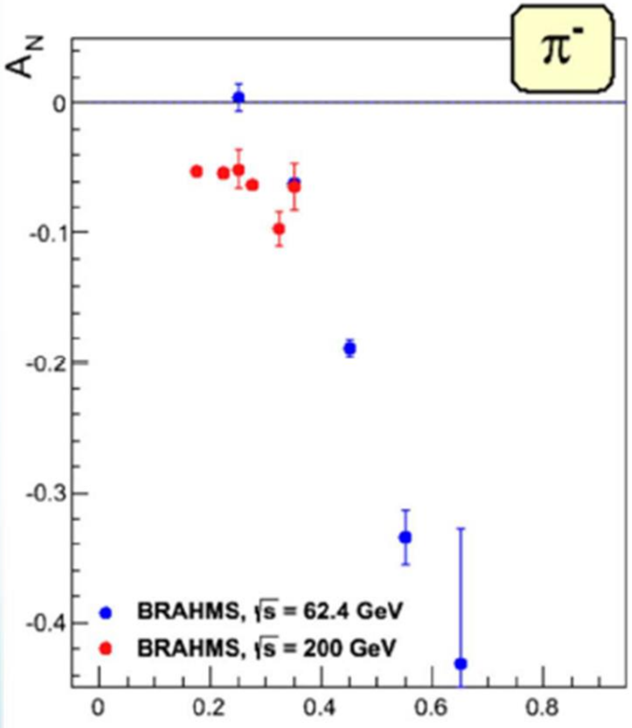
- Set $\phi_S = 90^o, \phi_h < \phi_S \; (\phi_h > \phi_S)$, produced hadron moves to left (right)

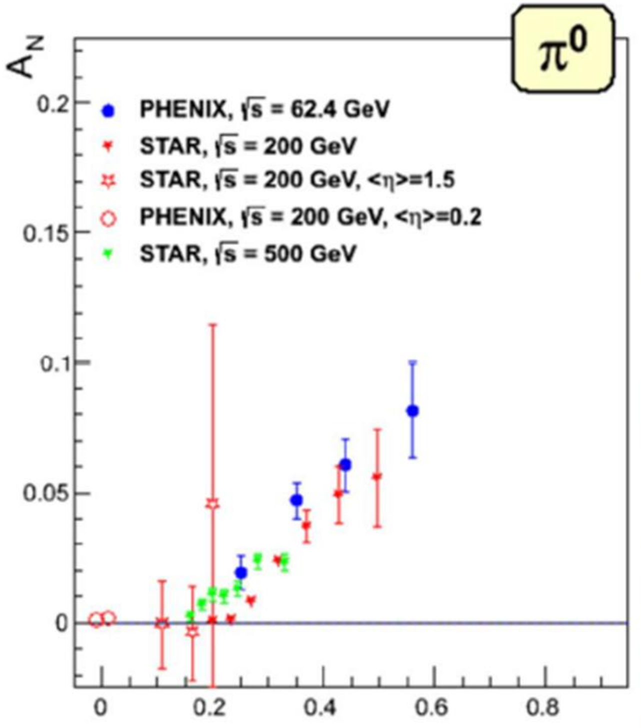
Single transverse spin asymmetry (SSA)

- Consider a transversely polarized proton scatter off an unpolarized proton or electron



$$A_N \equiv \frac{L-R}{L+R} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$





$x_F \sim 2P_z/\sqrt{s}$ scaled longitudinal momentum

Mechanism

- There exists correlation proportional to

$$\varepsilon_{\mu\nu\rho\lambda} S_T^{\mu} p_{hT}^{\nu} \cdots$$

- To generate such term in Feynman diagram, need

$$tr\left[\gamma_5 \not{S}_T \not{p}_{hT} \cdots\right] = i\varepsilon_{\mu\nu\rho\lambda} S_T^{\mu} p_{hT}^{\nu} \cdots$$

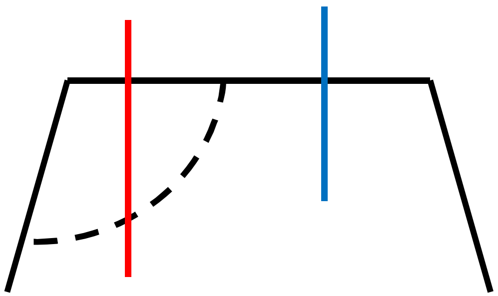
- Projector for polarized proton $(\not{p} + m)\gamma_5 \not{S}_T$
- Projector for produced hadron $\not{p}_h + m_h$
- But need strong phase to make cross section real

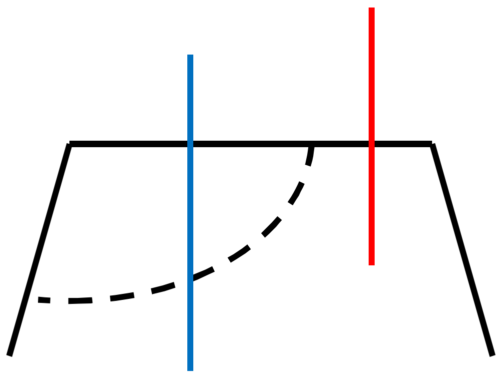
Where is phase?

- Phase comes from on-shell internal particles

$$\frac{1}{k^2+i\varepsilon}=\frac{P}{k^2}-i\pi\delta(k^2)$$

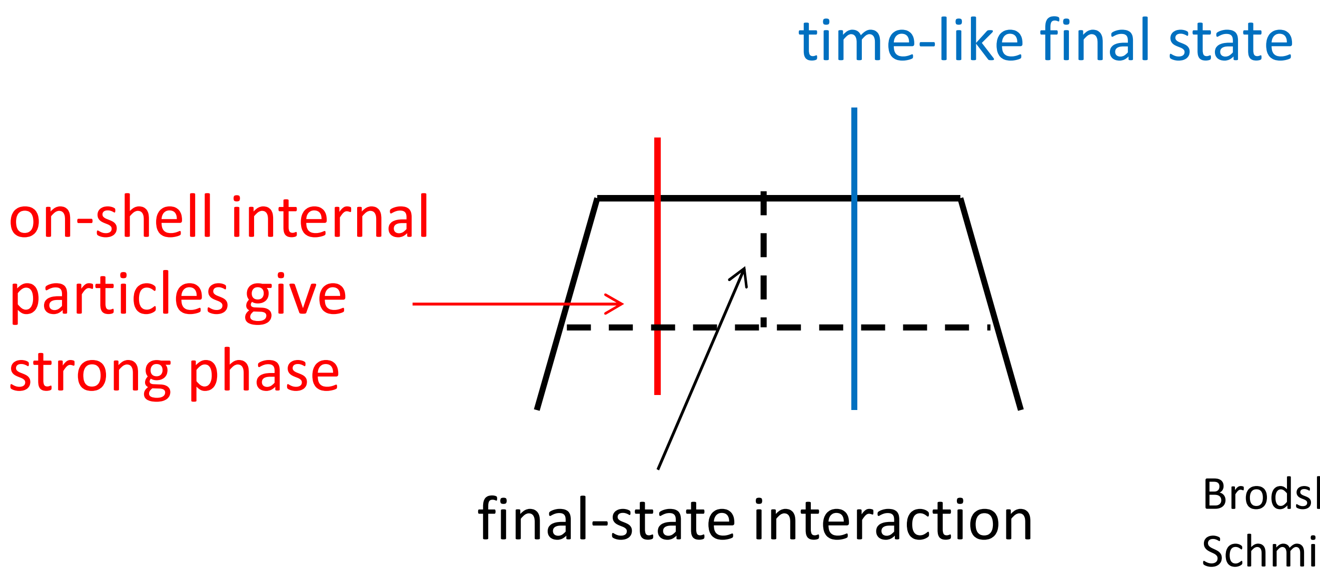
- Need time-like final states with FSI
- No strong phase at LO and one loop





Phase at two loops

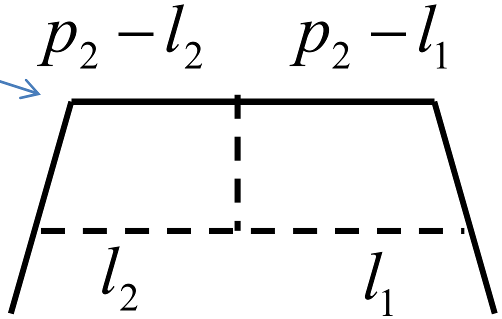
- Need two final-state particles with one gluon exchange (FSI) between them
- Nonvanishing phase appears at two loops, and comes from box diagram



Brodsky, Hwang, Schmidt 2002

Kinematics for phase

$$q = p_2 - p_1, \quad p_2 = (p_2^+, p_2^-, 0_T) \qquad p_1^+, p_2^- >> p_2^+ >> \Lambda_{QCD}$$



$$l_{1T} \leftrightarrow p_{hT}$$

$$p_2^2 > 0$$

time-like

$$l_1^- = l_{1T}^2/(2l_1^+)$$

$$p_1 = (p_1^+, 0, 0_T)$$

$$l_1^+ = \frac{1}{2}\left(p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-}l_{1T}^2}\right)$$

$$l_2^- = l_{2T}^2/(2l_2^+)$$

$$l_2^+ = \frac{1}{2}\left(p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-}l_{2T}^2}\right)$$

Collinear to initial state

- Picking up plus signs, gluons collimate to polarized proton

$$l_{1,2}^{+} \sim O(p_2^{+}) >> l_{1T,2T} >> l_{1,2}^{-}$$

$$p_1 - l_2 \approx p_1^{+} - p_2^{+}$$

collinear

$$p_2 - l_1 \approx p_2 - l_2 \approx p_2^{-}$$

- Phase goes into Sivers function
- FSI gluon is soft

Sivers function

Sivers 1990

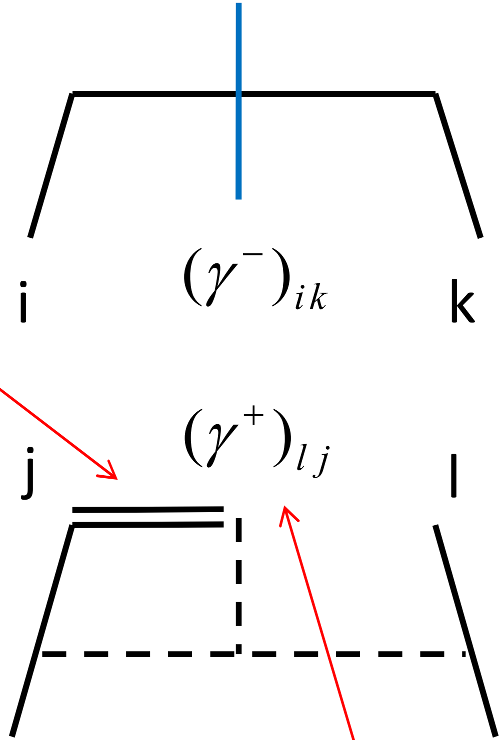
- Eikonalize outgoing quark and insert Fierz identity

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma^{\alpha})_{ik}(\gamma_{\alpha})_{lj}$$

$$+ \frac{1}{4}(\gamma^5\gamma^{\alpha})_{ik}(\gamma_{\alpha}\gamma^5)_{lj} + \frac{1}{4}(\gamma^5)_{ik}(\gamma^5)_{lj}$$

$$+ \frac{1}{8}(\gamma^5\sigma^{\alpha\beta})_{ik}(\sigma_{\alpha\beta}\gamma^5)_{lj}$$

give dominant (twist-2) contribution



pick up $\not{p} = p^+\gamma^-$

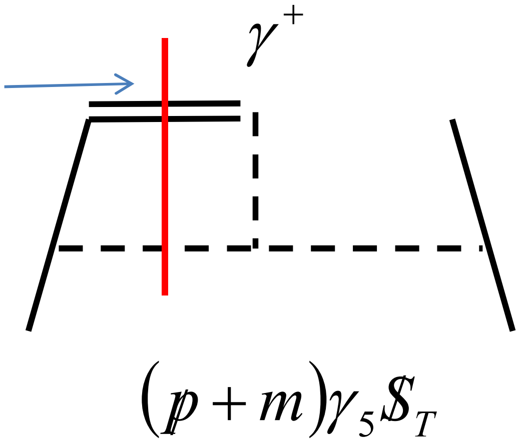
Parton transverse momentum

- Sivers function demands inclusion of parton transverse momentum

compensated by phase here

$$tr\left[\gamma_5 \not{S}_T \not{k}_T \gamma^+ \gamma^- \cdots\right] = i\varepsilon_{\mu\nu+-} S_T^\mu k_T^\nu \cdots$$

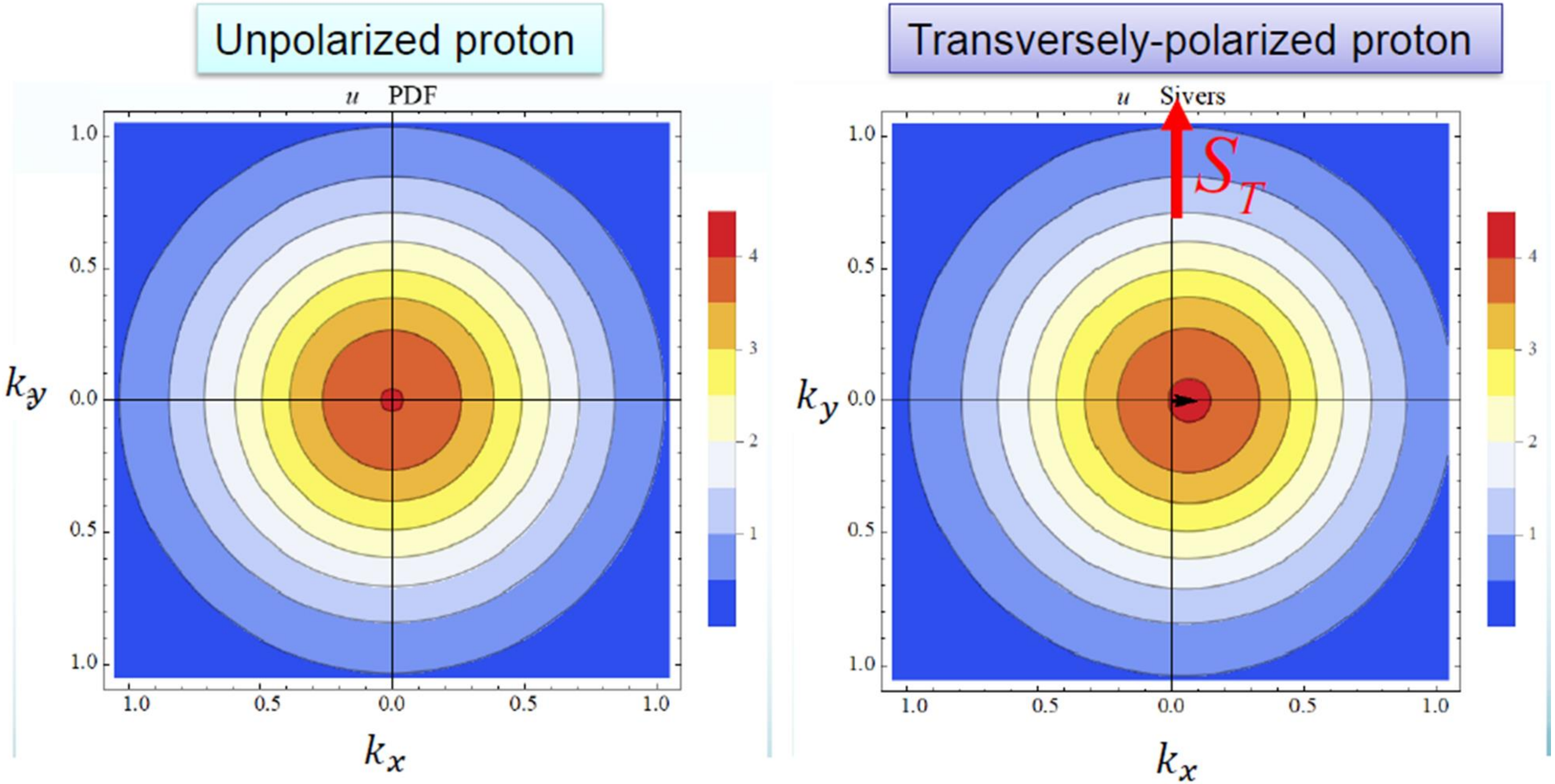
$l_{1T,2T}$



- This correlation determines preferred direction of k_T for polarized proton , which then propagates into p_h

Spin-transverse-momentum correlation

$$f_{q/p\uparrow}(x,k_T,\overrightarrow{S_T}) = f_{q/p}(x,k_T) - \frac{1}{M} f_{1T}^{\perp q}(x,k_T)\overrightarrow{S_T}\cdot(\hat{p}_h \times k_T)$$



produced hadron tends to move to right

Collinear to final state

- Picking up minus signs, gluons collimate to produced hadron

$$l_{1,2}^{-} \sim O(p_2^{-}) >> l_{1T,2T} >> l_{1,2}^{+}$$

collinear

$$p_2 - l_1 \sim O(p_2^{-}), \quad p_2 - l_2 \sim O(p_2^{-})$$

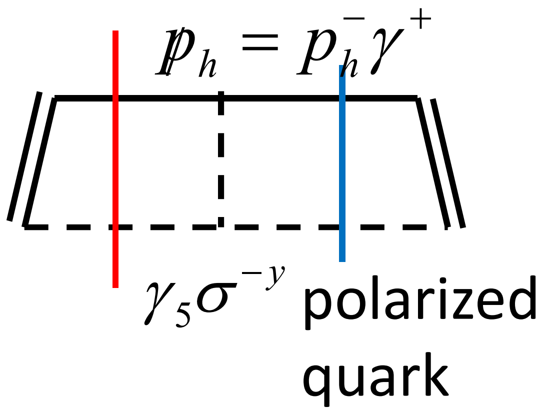
$p_1 - l_2$ highly off-shell

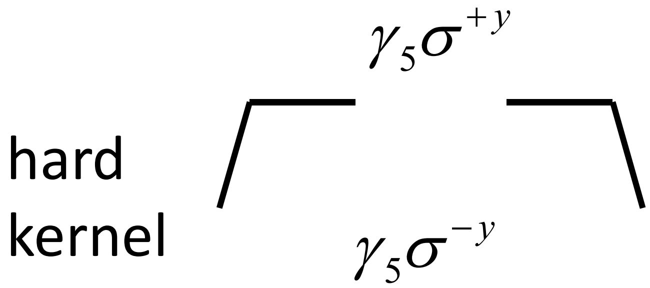
- Phase goes into Collins fragmentation function

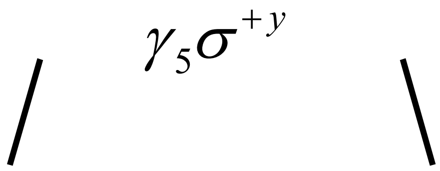
Collins 1993

Collins function

- Eikonalize incoming quark and insert Fierz identity
- $\gamma_5\sigma^{-y}$ dominates
- Collins function demands inclusion of parton k_T
- LO hard kernel demands projector for initial state





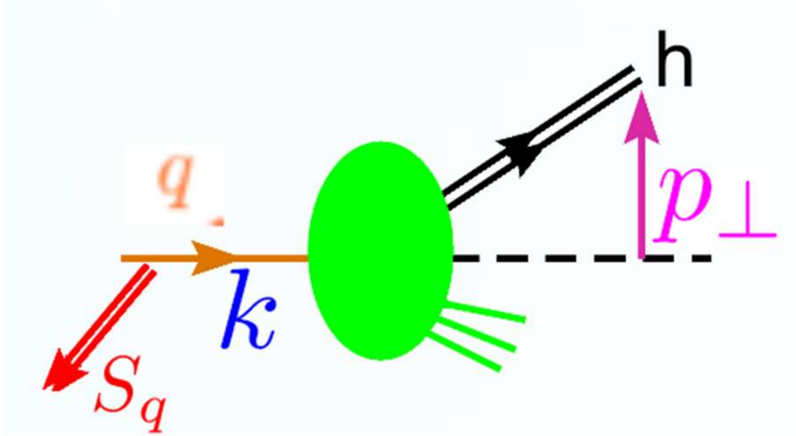


- Transversity distribution defined

$$(\not{p}+m)\gamma_5\not{S}_T$$

Mechanism

- Transversity distribution for polarized proton determines preferred direction of quark spin
- This polarized quark scattered into final state
- Collins function then determines direction of produced hadron preferred by polarized quark fragmentation
- Without preferred direction of quark spin from initial state, Collins function cannot work



Twist-2 TMDs

see J.P. Chen's talk

Sivers function

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} f_{1T}^{\perp},$$

$$\Phi^{[\gamma^+\gamma_5]} = S_L\, g_{1L} - \frac{p_T \cdot S_T}{M}\, g_{1T}\ ,$$

$$\Phi^{[i\sigma^{\alpha+}\gamma_5]} = S_T^{\alpha} h_1 + S_L\, \frac{p_T^{\alpha}}{M}\, h_{1L}^{\perp} - \frac{p_T^{\alpha} p_T^{\rho} - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, h_{1T}^{\perp} - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M} h_1^{\perp}$$

transversity function

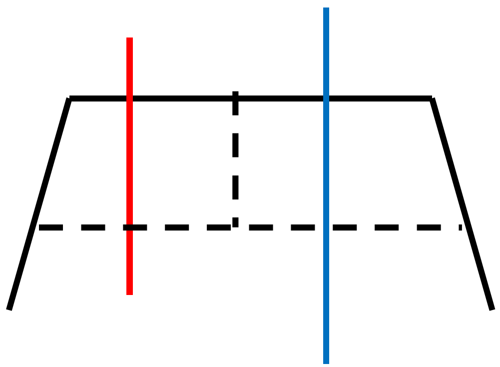
in the case of FFs, it is Collins function

Boer, Mulders 1997
Goeke, Meta, Schlegel 2005
Bacchetta et al., 2007

Phase in hard kernel

- For other sign combinations, or arbitrary transverse momenta
- phase appears in hard kernel

$H^{(2)} =$



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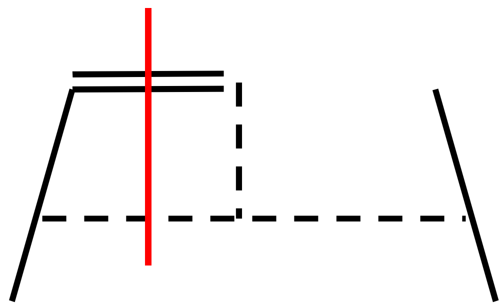


- How to extract this phase?
- Use $\gamma_5 \gamma^{\perp}$
- A new contribution to SSA

Twist-3 TMDs

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L\, e_L - \frac{p_T\cdot S_T}{M}\, e_T\right], \qquad \Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma}\, p_{T\rho}\, S_{T\sigma}}{M}\, e_T^{\perp}\right]$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\left[-\,\epsilon_T^{\alpha\rho}\, S_{T\rho}\, f_T - S_L\,\frac{\epsilon_T^{\alpha\rho}\, p_{T\rho}}{M}\, f_L^{\perp} - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\,\epsilon_{T\rho\sigma}\, S_T^\sigma\, f_T^{\perp} + \frac{p_T^\alpha}{M} f^{\perp}\right]$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\left[S_T^\alpha\, g_T + S_L\,\frac{p_T^\alpha}{M}\, g_L^{\perp} - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, g_T^{\perp} - \frac{\epsilon_T^{\alpha\rho}\, p_{T\rho}}{M}\, g^{\perp}\right]$$

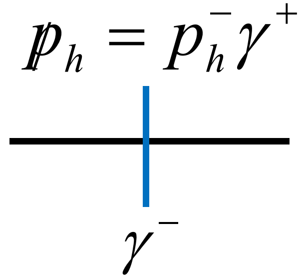
$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta}{M}\, h_T^{\perp} - \epsilon_T^{\alpha\beta}\, h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L\, h_L - \frac{p_T\cdot S_T}{M}\, h_T\right],$$

Boer, Mulders 1997
Goeke, Meta, and Schlegel 2005
Bacchetta et al., 2007

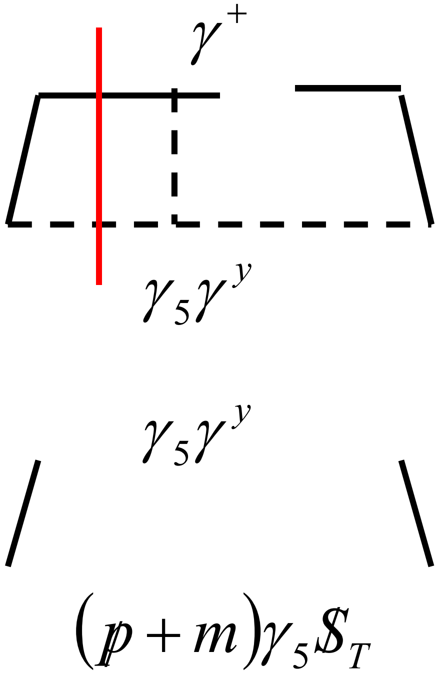
Factorization of new contribution

- Unpolarized twist-2 fragmentation function



- Polarized quark scattered into preferred direction of produced hadron

$$tr\left[\gamma_5 \gamma^y \not{p}_{hT} \gamma^+ \gamma^- \cdots\right] = i\varepsilon_{yx+-} p_{hT}^x \cdots$$



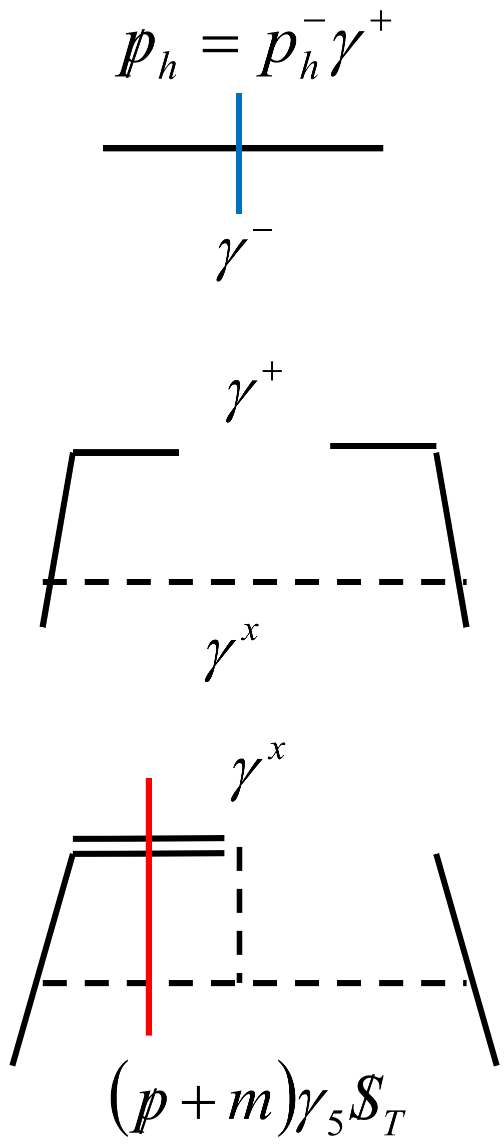
- 2-parton twist-3 TMD g_T defined for polarized proton

Lesson learned

- Both Sivers and Collins functions contribute starting from LO hard kernel
- If allowed to go to higher orders of hard kernel, other projectors can be used
- Though higher orders, COMPASS data Q ~ few GeV, hard kernel effect may be sizable
- Hard kernel is process-dependent
- Rich phenomenology!

At 3 loops

- At 3 loops, we can have 2-loop TMD for polarized proton and 1-loop hard kernel
- In addition to Sivers function, can use γ^x to extract phase in initial state in this case
- 2-parton twist-3 TMD f_T defined
- Another new contribution



Twist-3 TMDs

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L\, e_L - \frac{p_T\cdot S_T}{M}\, e_T\right], \qquad \Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M}\, e_T^\perp\right]$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\left[-\,\epsilon_T^{\alpha\rho} S_{T\rho}\, f_T - S_L\,\frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, f_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\,p_T^2\, g_T^{\alpha\rho}}{M^2}\,\epsilon_{T\rho\sigma} S_T^\sigma\, f_T^\perp + \frac{p_T^\alpha}{M} f^\perp\right]$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\left[S_T^\alpha\, g_T + S_L\,\frac{p_T^\alpha}{M}\, g_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\,p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, g_T^\perp - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, g^\perp\right]$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta}{M}\, h_T^\perp - \epsilon_T^{\alpha\beta}\, h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L\, h_L - \frac{p_T\cdot S_T}{M}\, h_T\right],$$

2-parton twist-3

- A contribution to SSA at 2-parton twist-3, with 1-loop hard kernel

$$d\sigma = f_T \otimes H^{(1)}_{\gamma^x, \gamma^+} \otimes D_1$$

- f_T proportional to S_T^y, 1-loop hard kernel to k_T^x
- k_T^x flows into produced hadron, giving rise to p_h^x with probability described by twist-2 FF D_1
- Compared to Sivers and Collins functions, this S_T^y - p_h^x correlation is perturbative

3-parton twist-3

- New contribution is of the same order as 3-parton twist-3 (Qiu-Sterman) one in, for example, Drell-Yan

$$
\begin{aligned}
&\phi^{(3)}_{i/A}(x,x')\otimes\phi_{j/B}(y)\otimes D_{h/c}(z)\otimes H^{(A)}_{ij\to c}(x,x',y,z)\\
&+\phi_{i/A}(x)\otimes\phi^{(3)}_{j/B}(y,y')\otimes D_{h/c}(z)\otimes H^{(B)}_{ij\to c}(x,y,y',z)\\
&+\phi_{i/A}(x)\otimes\phi_{j/B}(y)\otimes D^{(3)}_{h/c}(z,z')\otimes H^{(C)}_{ij\to c}(x,y,z,z')
\end{aligned}
$$

- New contribution deserves study

Qiu, Sterman 1991
Kouvaris, Qiu, Vogelsang, Yuan 2006
Yuan, Zhou 2008
Kang, Qiu, Vogelsang, Yuan 2008

Questions to you

- Is my observation correct?
- Is SSA more complicated than we though?
- If yes, how to do subleading analysis?
- What’s impact on extraction of Sivers, Collins,…?
- Is sign flip between SIDIS and DY exact?
- Can twist-3 effect be revealed by precise measurement of SSA?
- Is sign mismatch really serious?
- Other spin-dependent observables?

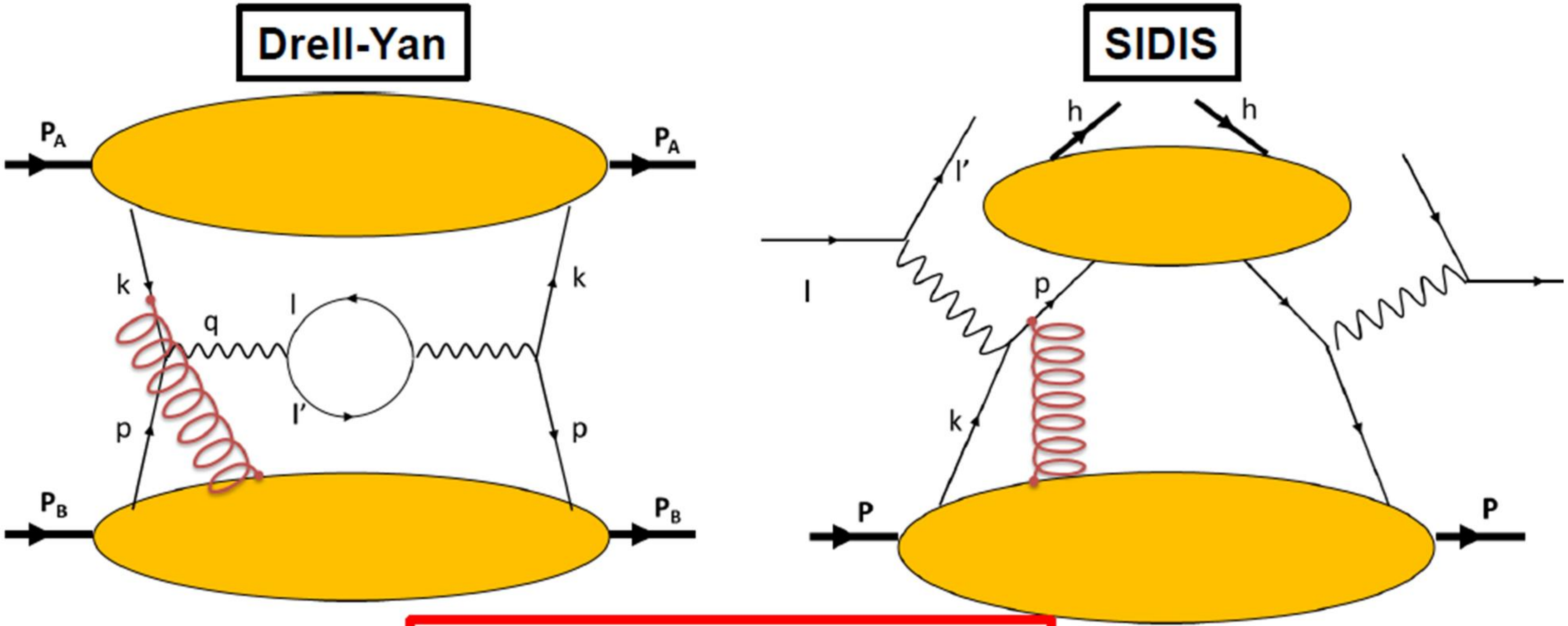
Up to twist-3 NLO

- Up to 2-parton twist-3, 1-loop in hard kernel, SSA is given by

$$
\begin{aligned}
d\sigma \;=\; & f_{1T}^{\perp} \otimes H^{(0)}_{\gamma^-,\gamma^+} \otimes D_1 + h_1 \otimes H^{(0)}_{\gamma_5\sigma^{x-},\gamma_5\sigma^{x+}} \otimes H_1^{\perp} \\
& + f_{1T}^{\perp} \otimes H^{(1)}_{\gamma^-,\gamma^y} \otimes D^{\perp} + g_{1T} \otimes H^{(1)}_{\gamma_5\gamma^-,\gamma_5\gamma^x} \otimes G^{\perp} \\
& + e_T \otimes H^{(1)}_{\gamma_5,\gamma_5\sigma^{x+}} \otimes H_1^{\perp} + f_T \otimes H^{(1)}_{\gamma^y,\gamma^+} \otimes D_1 \\
& + h_T^{\perp} \otimes H^{(1)}_{\gamma_5\sigma^{xy},\gamma_5\sigma^{x+}} \otimes H_1^{\perp} + h_T \otimes H^{(1)}_{\gamma_5\sigma^{-+},\gamma_5\sigma^{x+}} \otimes H_1^{\perp}
\end{aligned}
$$

- More terms up to NNLO

Sign change of Sivers function

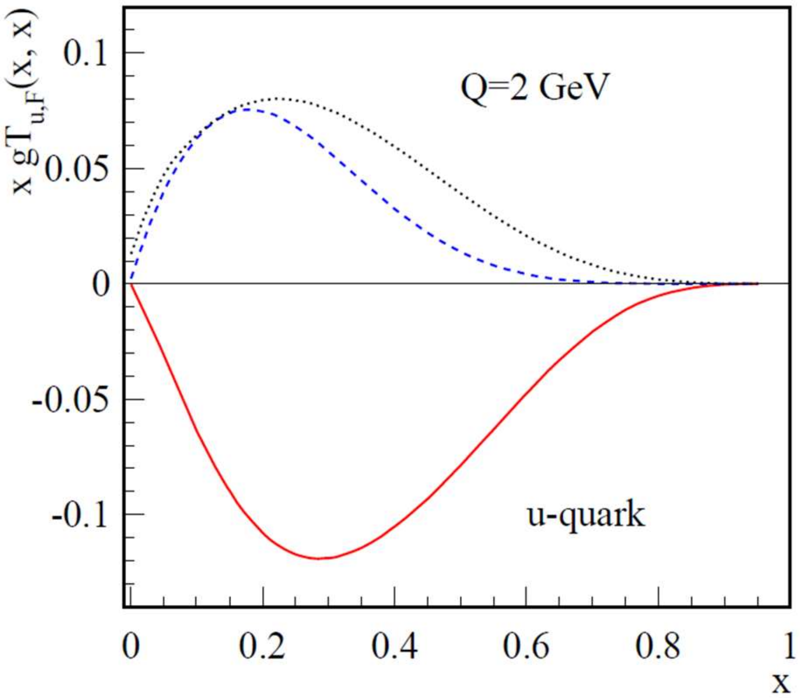


$$\boxed{\mathrm{Sivers}\,|_{DY} = -1 * \mathrm{Sivers}\,|_{SIDIS}}$$

Sign-mismatch problem

- No sign flip was seen in $p^{\uparrow}p \to \pi + X$

correlation function for polarized proton



expectation from SIDIS data under sign flip

result extracted from data

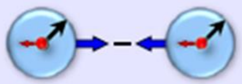
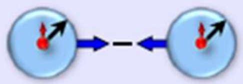
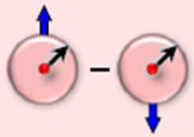
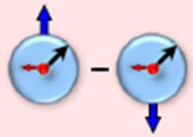
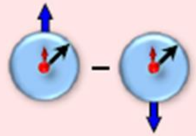
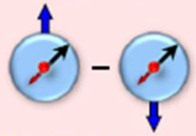
- Now there are other twist-3 contributions…

Kang, Qiu, Vogelsang, Yuan 2011

Back-up slides

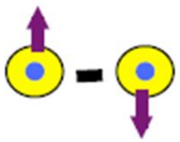
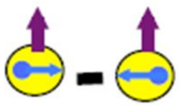
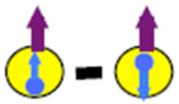
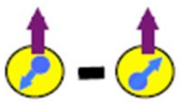
Twist-2 TMDs

- spin of the nucleon
- spin of the quark
- k_T of the quark

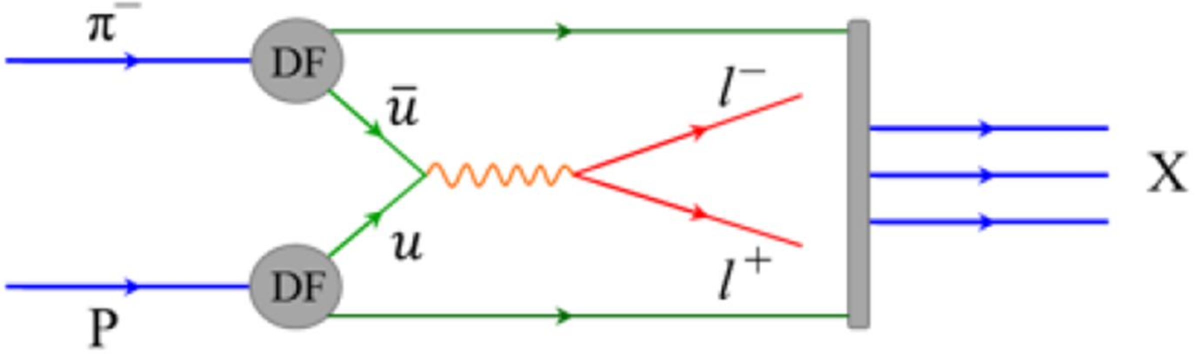
Nucleon \ Quark	U	L	T
U	 number density $f_1^q(x, k_T^2)$		 Boer-Mulders $h_1^{q\perp}(x, k_T^2)$
L		 Helicity $g_1^q(x, k_T^2)$	 worm-gear L $h_{1L}^{q\perp}(x, k_T^2)$
T	 Sivers $f_{1T}^{q\perp}(x, k_T^2)$	 Kotzinian-Mulders worm-gear T $g_{1T}^{q\perp}(x, k_T^2)$	 Transversity $h_1^q(x, k_T^2)$  Pretzelosity $h_{1T}^{q\perp}(x, k_T^2)$

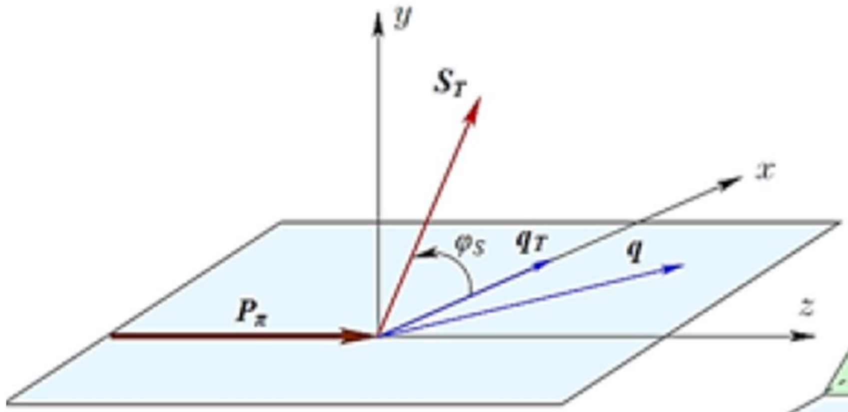
kT can be integrated out in transverse distribution

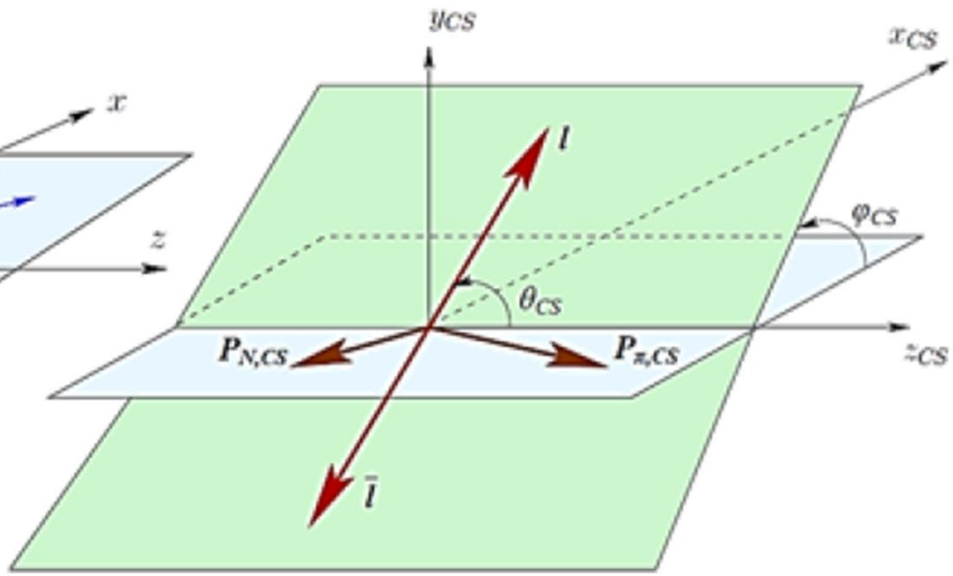
Twist-3 TMDs

	$e(x,k_\perp)$, $f^\perp(x,k_\perp)$	number density
	$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp)$, $f_T^{\perp 2}(x,k_\perp)$	Sivers function
	$e_L(x,k_\perp)$, $g_L^\perp(x,k_\perp)$	helicity distribution
	$e_T(x,k_\perp)$, $g_T(x,k_\perp)$, $g_T^\perp(x,k_\perp)$	Worm gear: trans-helicity
	$h(x,k_\perp)$	Boer-Mulders function
	$h_T^\perp(x,k_\perp)$	transversity distribution
	$h_T(x,k_\perp)$	pretzelocity
	$h_L(x,k_\perp)$	Worm gear: longi-transversity
	$f_L^\perp(x,k_\perp)$	
	$g^\perp(x,k_\perp)$	

Polarized Drell-Yan



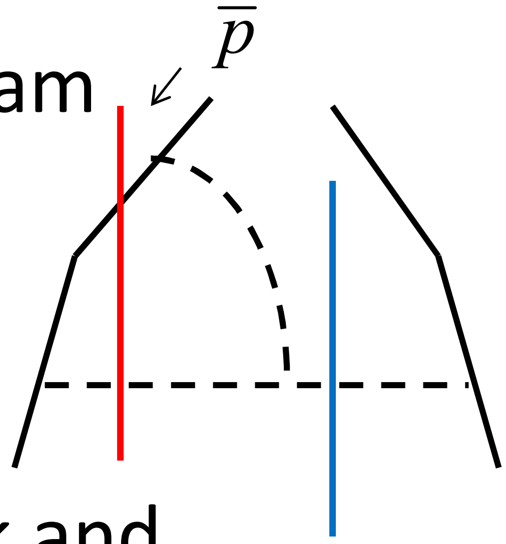




$\varphi_S > 0,\quad \gamma^*$ moves to left

$\varphi_S < 0,\quad \gamma^*$ moves to right

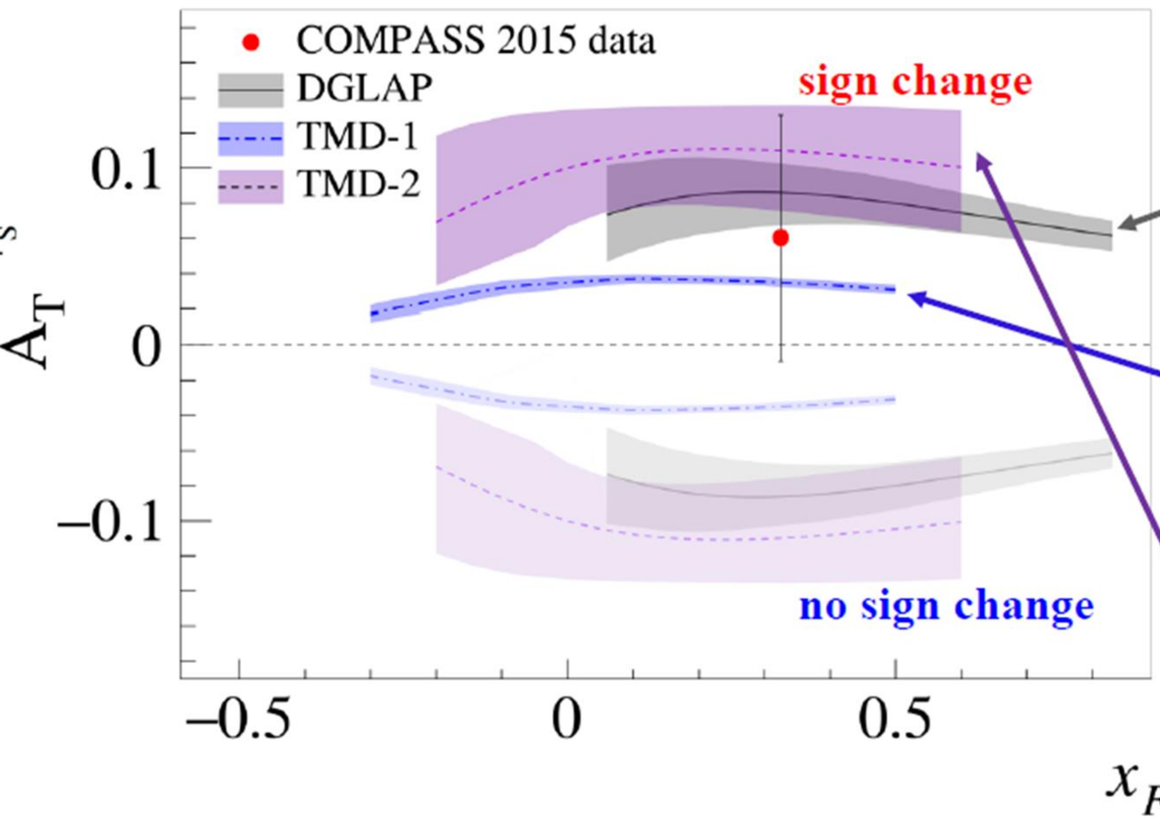
Strong phase in DY



- Strong phase appears in box diagram
- As gluons collimate to polarized proton, strong phase goes into Sivers function
- Eikonalize another incoming quark and insert Fierz identity
- Only difference from SIDIS is the sign of antiquark in DY

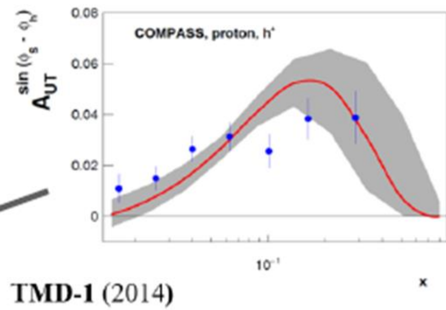
$$\mathrm{Im}\frac{\not{p}_h - \not{l}}{(p_h - l)^2 + i\varepsilon} \propto -\delta(l^+) \qquad \mathrm{Im}\frac{-(\not{\bar{p}} + \not{l})}{(\bar{p} + l)^2 + i\varepsilon} \propto \delta(l^+)$$

Sivers Asymmetry in Drell-Yan: Hint of Sign Change!

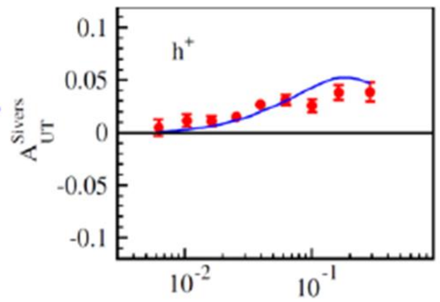


arXiv:1704.00488 [hep-ex]

DGLAP (2016)
M. Anselmino et al., **arXiv:1612.06413**



TMD-1 (2014)
M. G. Echevarria et al. **PRD89,074013**



TMD-2 (2013)
P. Sun, F. Yuan, **PRD88, 114012**