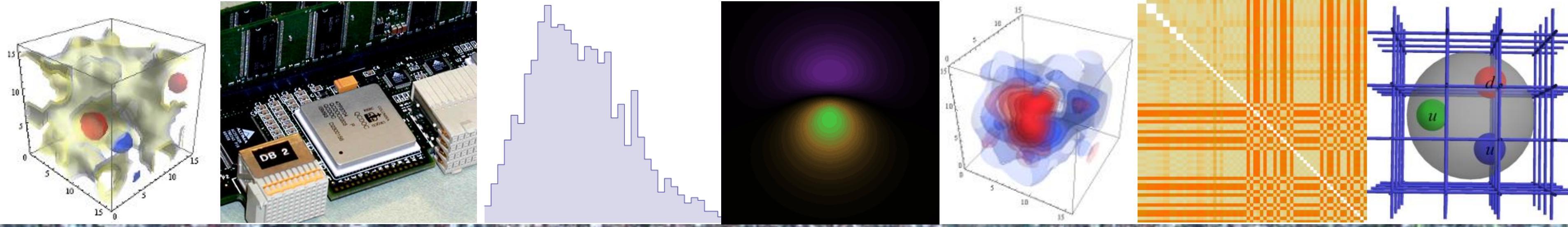


Parton Distribution Functions From Lattice QCD

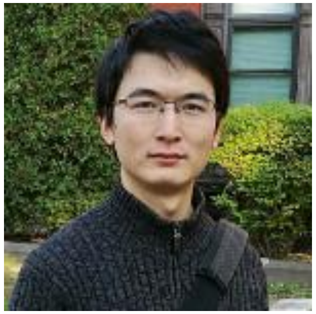
Huey-Wen Lin

Lattice Parton Physics Project (LP3)

https://www.pa.msu.edu/~hwlin/LP3/

HWL
(MSU)

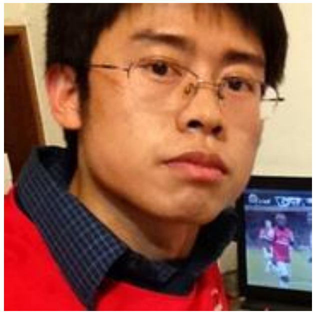
Xiangdong Ji
(UMD)



Luchang Jin
(BNL)

Peng Sun
(MSU)

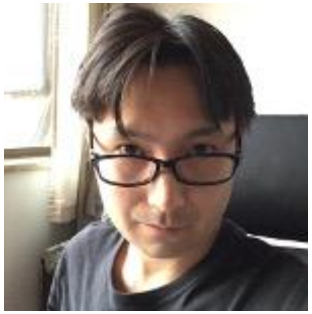
Yi-Bo Yang
(MSU)



Yong Zhao
(MIT)

International collaborators

Jiunn-Wei Chen
(NTU)



Tomomi Ishikawa
(SJTU)

Jian-Hui Zhang
(Regensburg)

Based on the work done in 1706.01295 (LP3) and ongoing work

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

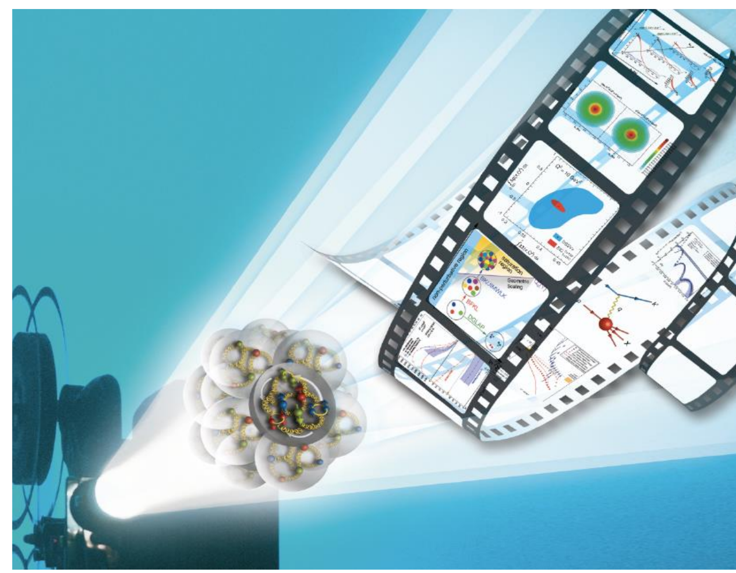
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the *sea* *quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?*

EIC White Paper, 1212.1701



Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

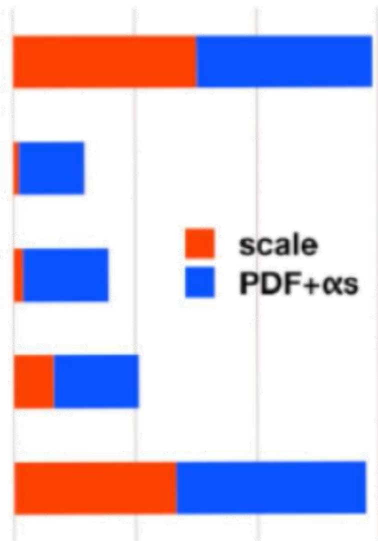
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

§ Important inputs to discern new physics at LHC

- Currently dominate errors in Higgs production

HIGGS PRODUCTION

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
	VBF	1.56 pb	2.9%
NNLO QCD +NLO EW	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%



(J. Campbell, HCP2012)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

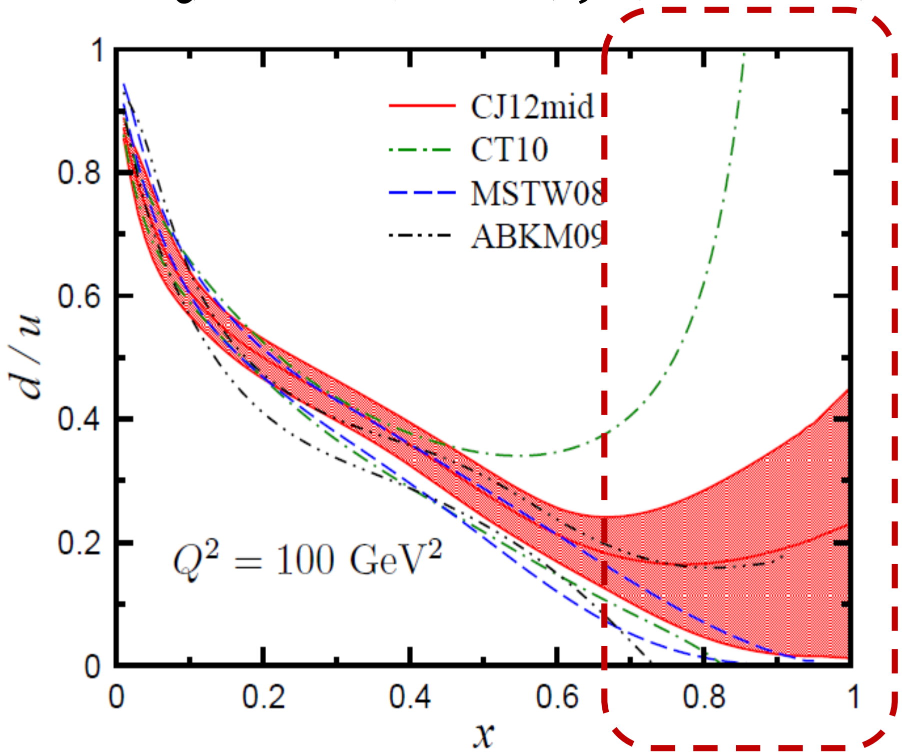
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

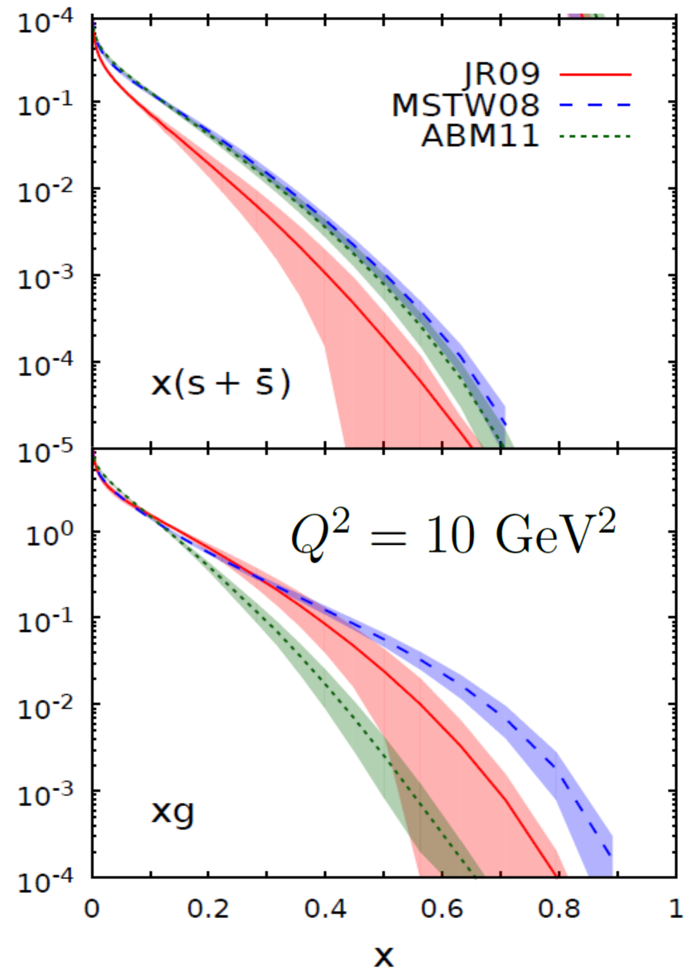
Global Analysis

§ Discrepancies appear when data is scarce

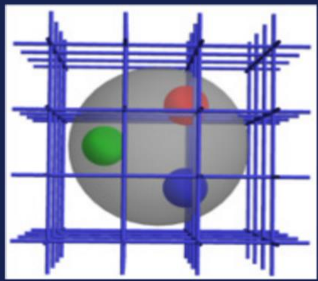
§ Many groups have tackled the analysis

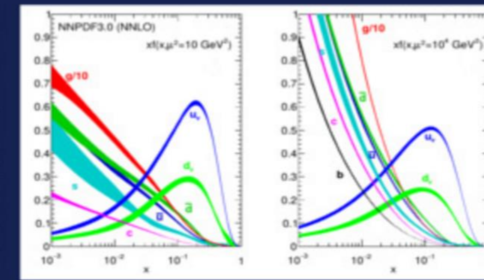
- CTEQ, MSTW, ABM, JR, NNPDF, etc.





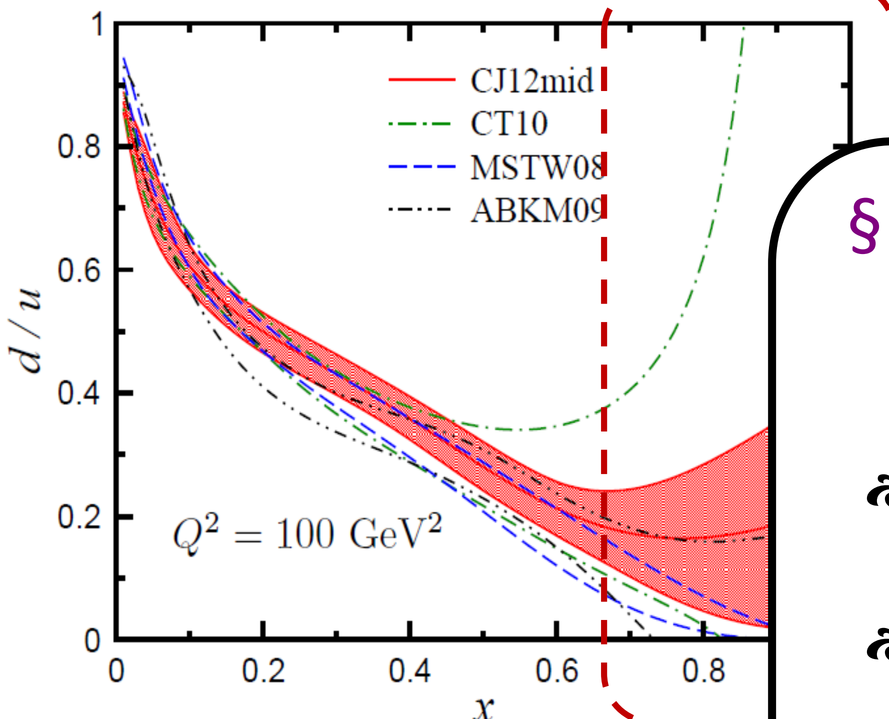
Jimenez-Delgado, Melnitchouk, Owens, J.Phys. G40 (2013) 09310

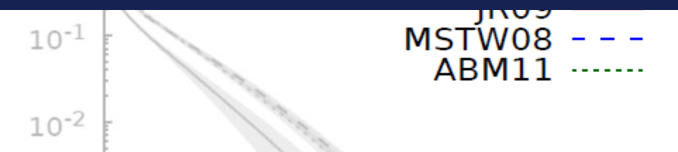




Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK





Jimenez-Delgado, Melnitchouk, O
J.Phys. G40 (2013) 09310

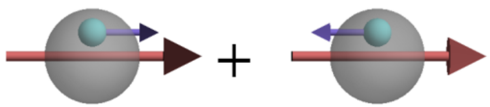
- A first joint workshop with global-fitting community to address key LQCD inputs
  - http://www.physics.ox.ac.uk/confs/PDFlattice2017
  - Whitepaper will study the needed precision of lattice PDFs in the large-x region

What can we do on the lattice?

PDFs on the Lattice

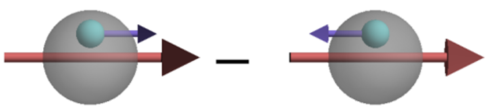
- Lattice calculations rely on operator product expansion, only provide moments

most well known



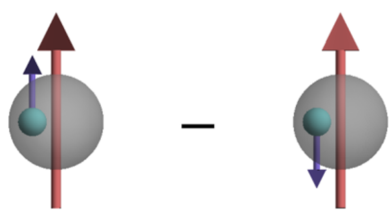
Quark density/unpolarized

$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$



Helicity
longitudinally polarized

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$



Transversity
transversely polarized

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

very poorly known

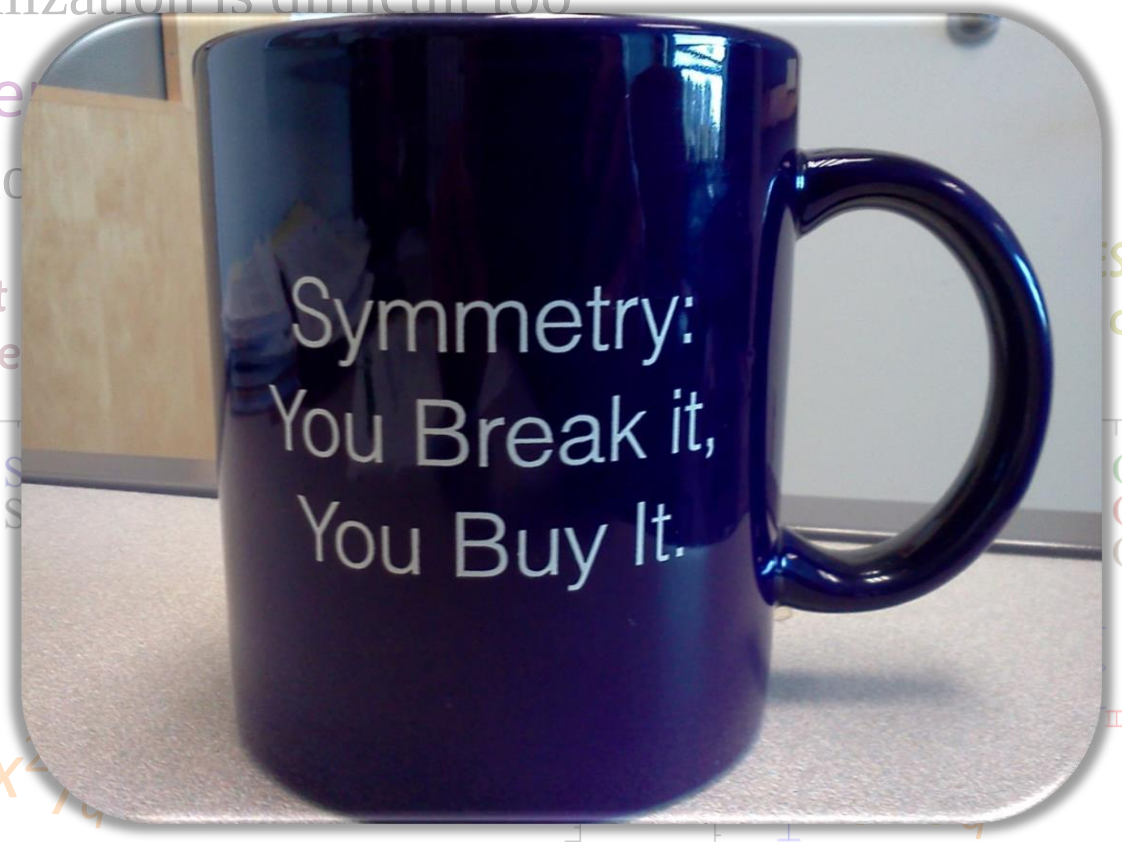
- True distribution can only be recovered with all moments

Problem with Moments

- For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- Relative e
  - Calculatic

Dolgov et
Göckeler e

SAM):
clover

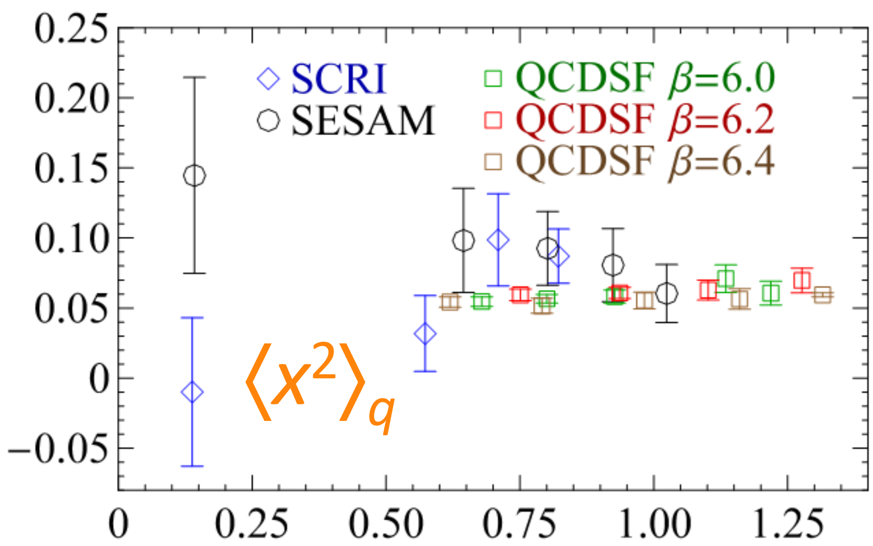


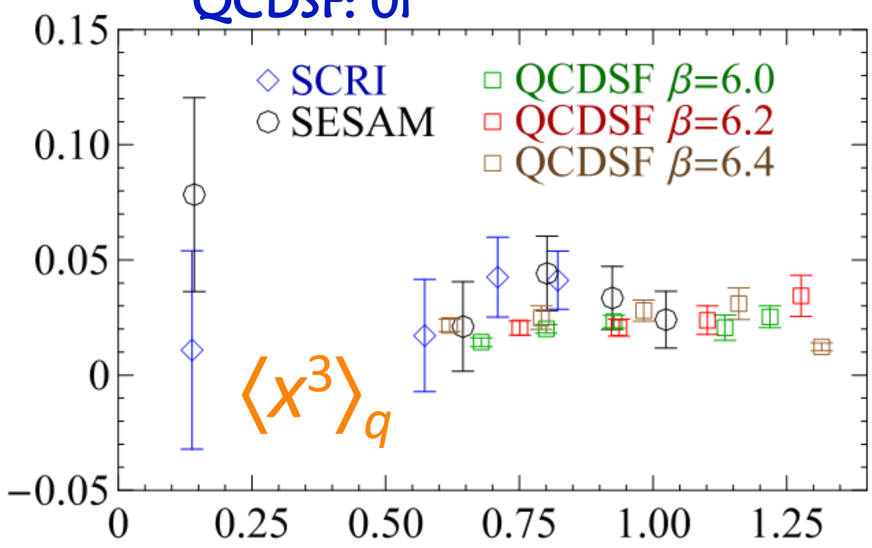
Problem with Moments

- For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- Relative error grows in higher moments
  - Calculation would be costly and difficult

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover
QCDSF: 0f





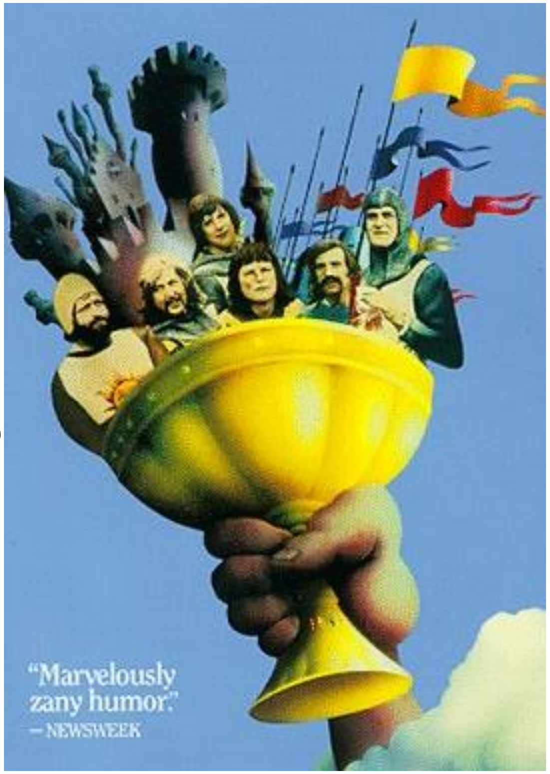
PDFs on the Lattice

Long existing obstacle!

§ Holy grail of structure calculations

§ Applies to many structure quantities:

Generalized parton distributions (GPDs),
Transverse-momentum distributions (TMD),
Meson distribution amplitudes, …



§ A few ideas try to solve this problem

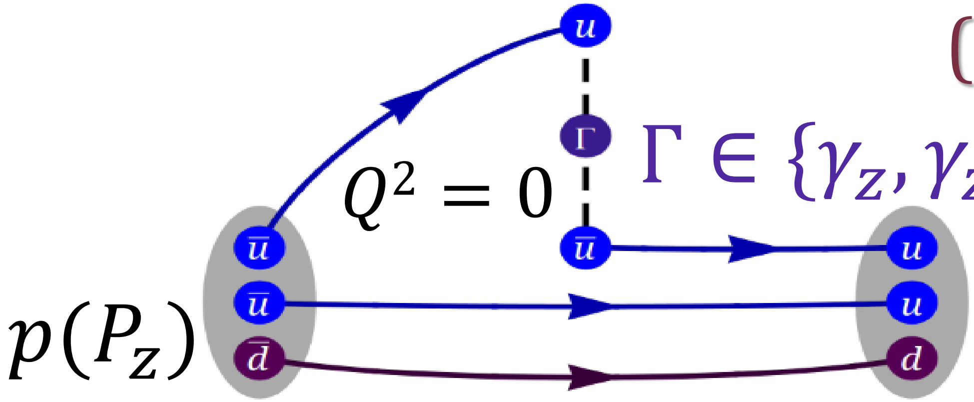
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- OPE without OPE (QCDSF, hep-lat/9809171, … 1004.2100)
- Fictitious heavy quarks (Detmold et al. hep-lat/0507007)
- Smeared lattice operators (Davoudi et al. 1204.4146)

Looking forward to more developments here

A New Direction

Large-Momentum Effective Theory for PDFs

1) Calculate nucleon matrix elements on the lattice (z dependence)



2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\, \Gamma \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

3) Recover true distribution（take $P_z \to \infty$ limit)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(\alpha_s) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

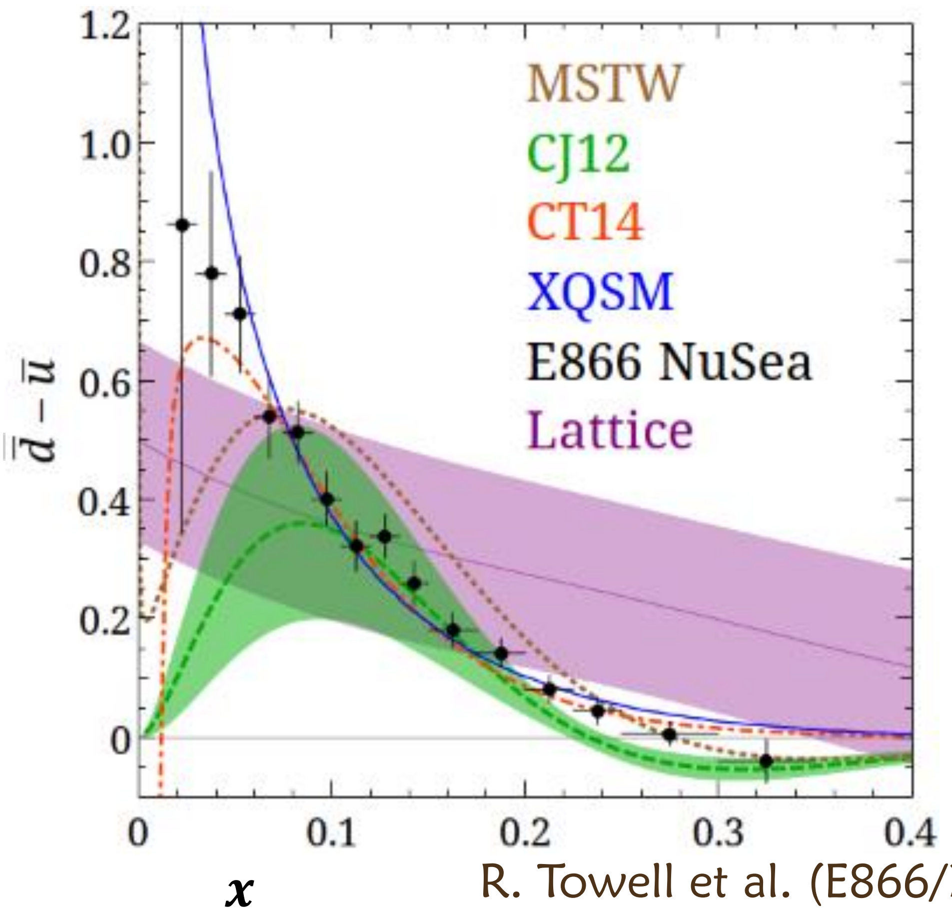
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Sea Flavor Asymmetry

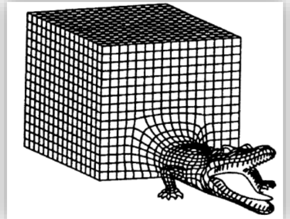
§ First time in LQCD history to study antiquark distribution!

- $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**

HWL et al. 1402.1462



$$\bar{q}(x) = -q(-x)$$



Lost resolution in small-x region

Future improvement: larger lattice volume

$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

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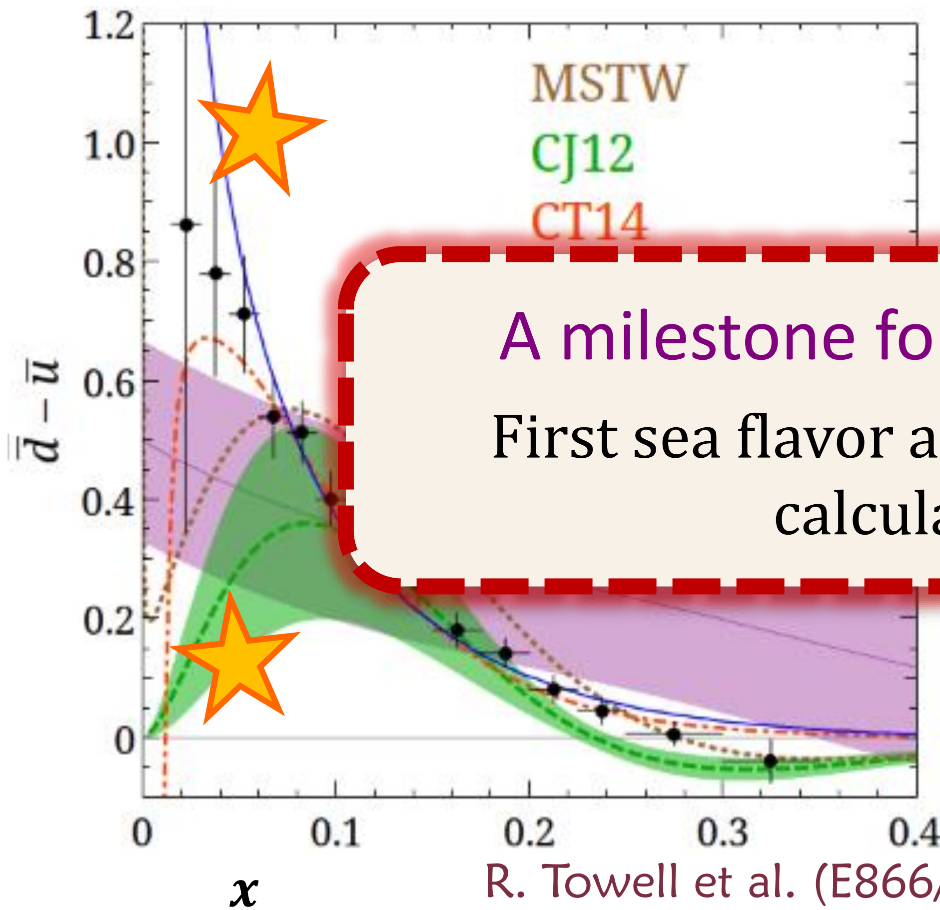
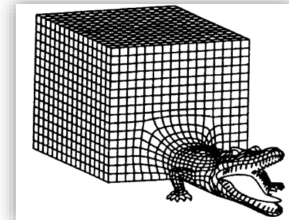


$$\bar{q}(x) = -q(-x)$$



Lost resolution in small-x re...

...ent:

...ume

$\approx -0.16(7)$

A milestone for lattice QCD!

First sea flavor asymmetry ever calculated!

Experiment	x range	$\int_0^1[\ldots - u(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
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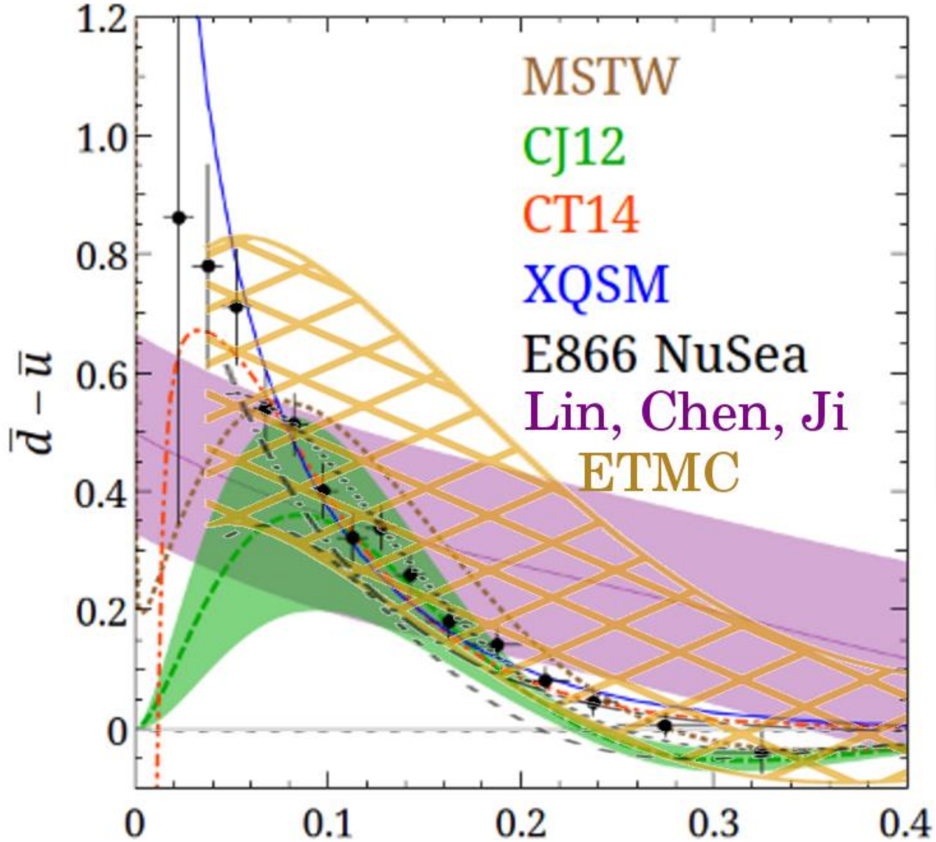
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**

HWL et al 1402.1462



Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Similar results repeated by ETMC, at $M_\pi \approx 373$ MeV

ETMC, 1504.07455

(7)

Experiment	x range	$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
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R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Missing Ingredient: Renormalization

Recent progress:

1705.00246, 1705.11193, 1706.00265, 1706.01295, 1706.08962 …

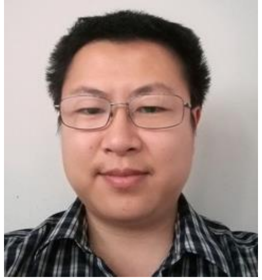
Renormalization

§ Long-link operator

$$O_\Gamma(z) = \bar{\psi}(z)\Gamma W_z(z,0)\psi(0)$$

§ Vector operator mixing with scalar ones T. Ishikawa, this conference

$$\begin{pmatrix} O^R_{\gamma_z}(z) \\ O^R_{\mathbb{I}}(z) \end{pmatrix} = \begin{pmatrix} Z_{VV}(z) & Z_{VS}(z) \\ Z_{SV}(z) & Z_{SS}(z) \end{pmatrix} \begin{pmatrix} O_{\gamma_z}(z) \\ O_{\mathbb{I}}(z) \end{pmatrix}$$



Yi-Bo Yang (MSU)

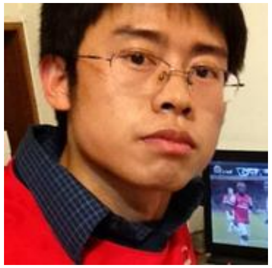
§ RI/MOM renormalization scheme 1706.01295 (LP3)

- $Z^{-1} =$

$$\frac{1}{12e^{-ip_z z}} \begin{pmatrix} \mathrm{Tr}\left[\tilde{\Gamma}\Lambda(p,z,\gamma_z)\right] & \mathrm{Tr}\left[\tilde{\Gamma}\Lambda(p,z,\mathbb{I})\right] \\ \mathrm{Tr}[\Lambda(p,z,\gamma_z)] & \mathrm{Tr}[\Lambda(p,z,\mathbb{I})] \end{pmatrix}_{p^2=\mu_R^2,\, p_z=P_z}$$

$$\Lambda(p,z,\Gamma) = S(p)^{-1}\left\langle \sum_w S^\dagger(p,w+zn)\Gamma W_z(w+zn)S(p,w) \right\rangle S(p)^{-1}$$

projected with $\tilde{\Gamma} = \not{p}/p_z$

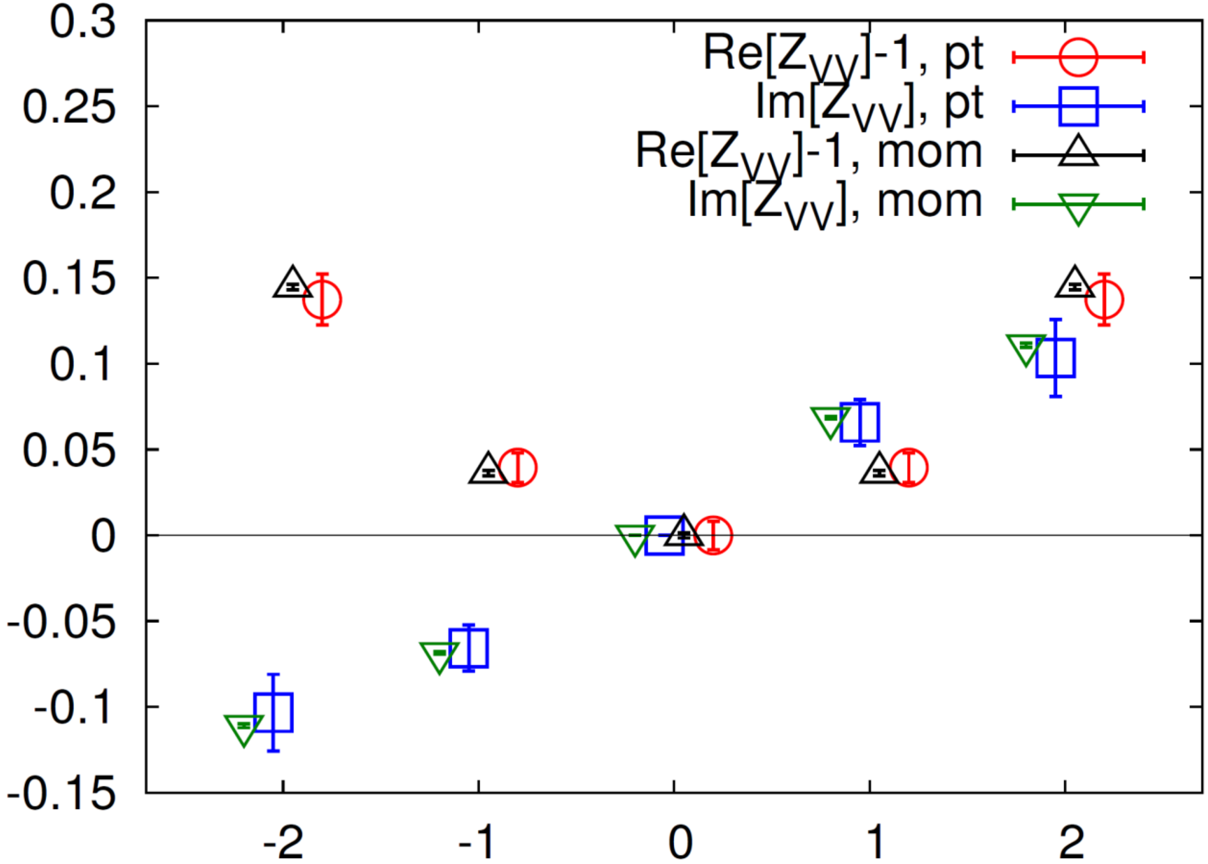


Yong Zhao (MIT)

- Test case: $a \approx 0.12$ fm, $M_\pi \approx 310$ MeV, clover/HISQ

§ RI/MOM renormalization scheme

Momentum source vs point source for $|z| \le 2$



$M_\pi \approx$ **310 MeV**

$a \approx$ **0.12 fm**

$p_z = 6\pi/L_s$

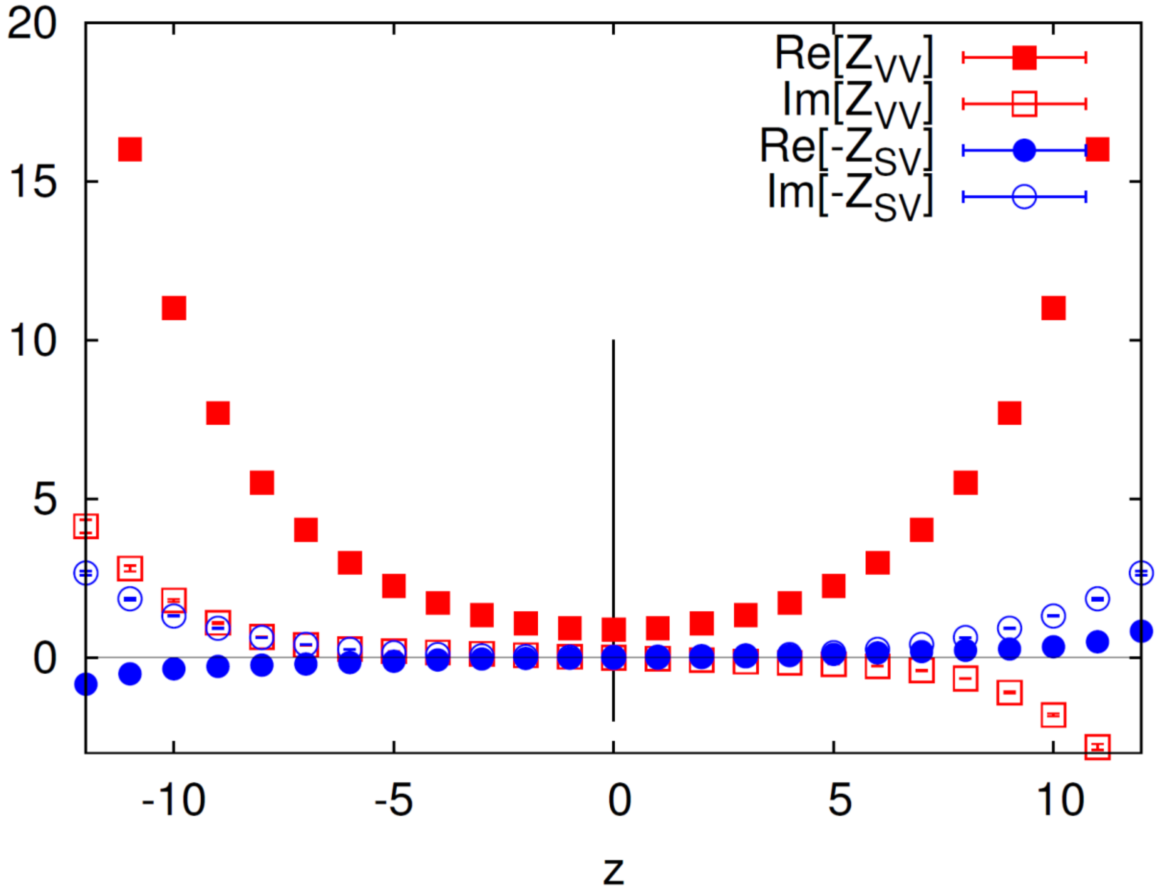
$\mu_R^2 = p^2$

$= 5.74\ \text{GeV}^2$

Plot by Yi-Bo Yang; 1706.01295 (LP3)

Renormalization

§ RI/MOM renormalization scheme



$M_\pi \approx$ **310 MeV**
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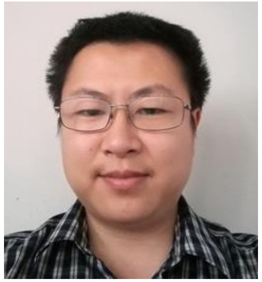
$p_z = 6\pi/L_s$
$\mu_R^2 = p^2$
$= 5.74\ \text{GeV}^2$



Yi-Bo Yang
(MSU)

$$h_R = Z_{VV} h_{\gamma_z} + Z_{SV} h_{\mathbb{I}}$$

Plot by Yi-Bo Yang; 1706.01295 (LP3)

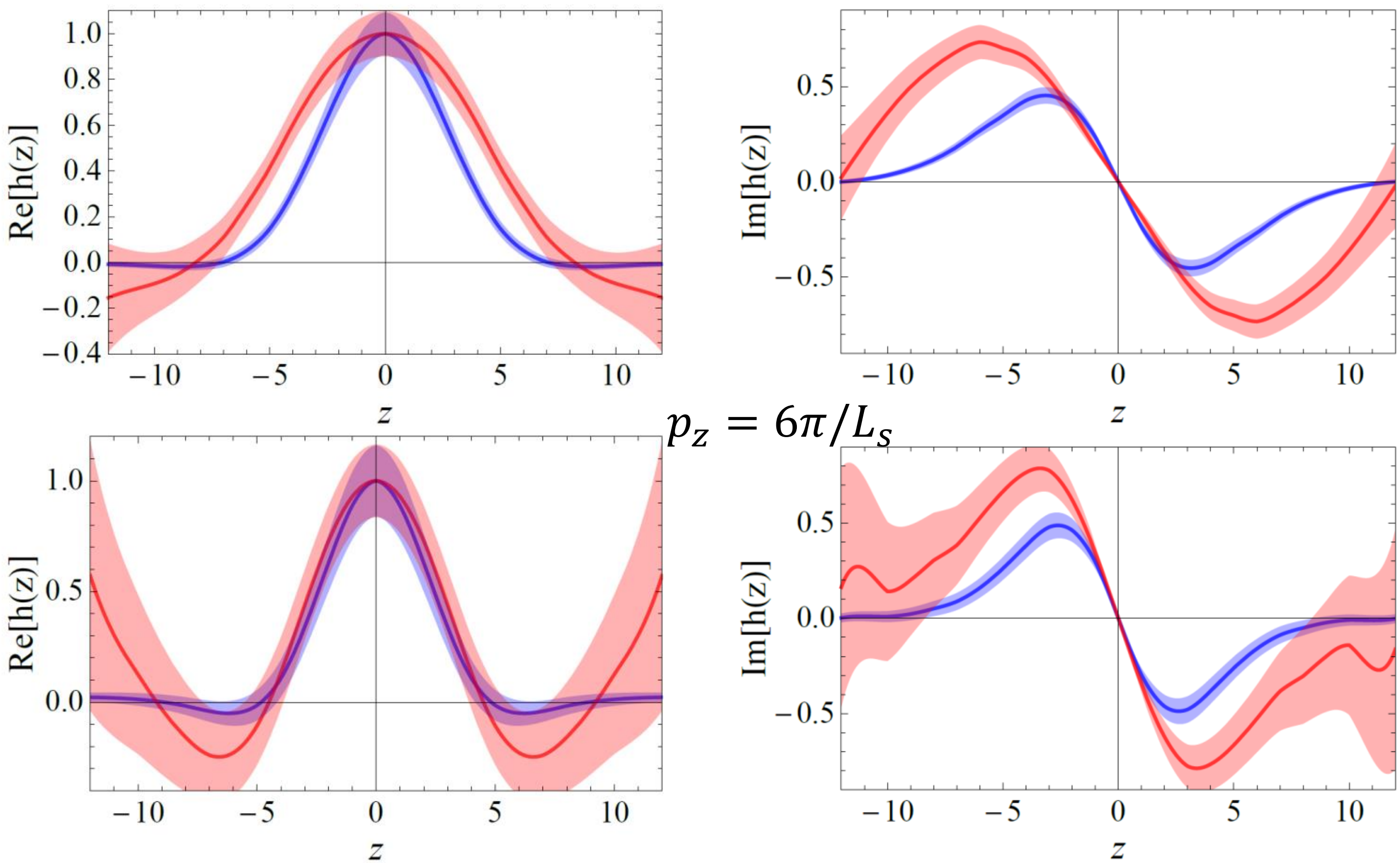
§ Effect on nucleon matrix elements as function of *z*

- $h_R \approx Z_{VV} h_{\gamma_z}$

$M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm**

$p_z = 4\pi/L_s$

$p_z = 6\pi/L_s$



Jian-Hui Zhang
(Regensburg)

Plot by Jianhui Zhang; 1706.01295 (LP3)

Summary & Outlook

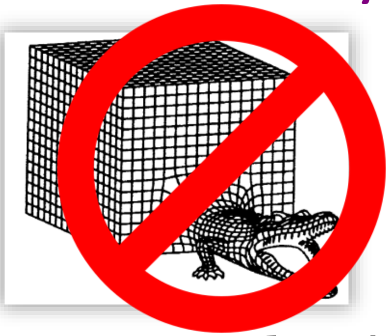
Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full *x*-distribution

- Most importantly, this can be done with today's computer
- First lattice approach to study sea asymmetry
- First look into PDA 1702.00008

§ Moving on to remove the systematics of earlier study

- Working on **renormalization**,
  statistics (all-mode averaging?),
  larger momentum boost,
  finer lattice-spacing ensembles, …



- Larger P_z with smaller *a* may reduce issues associated w/ larger *z*

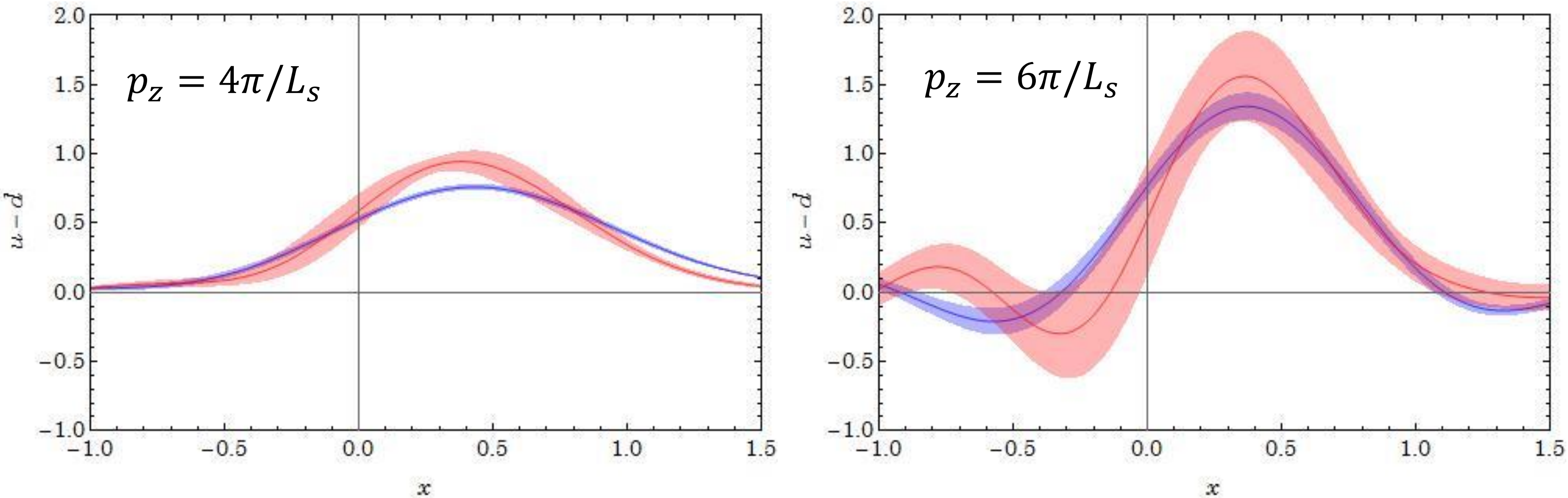
Backup Slides

Renormalization

§ Effect on quasi-PDFs

$$\tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_z z}\tilde{h}_R(z, P_z, \mu_R)$$

$M_\pi \approx$ **310 MeV,** $a \approx$ **0.12 fm**



Plot by Jianhui Zhang

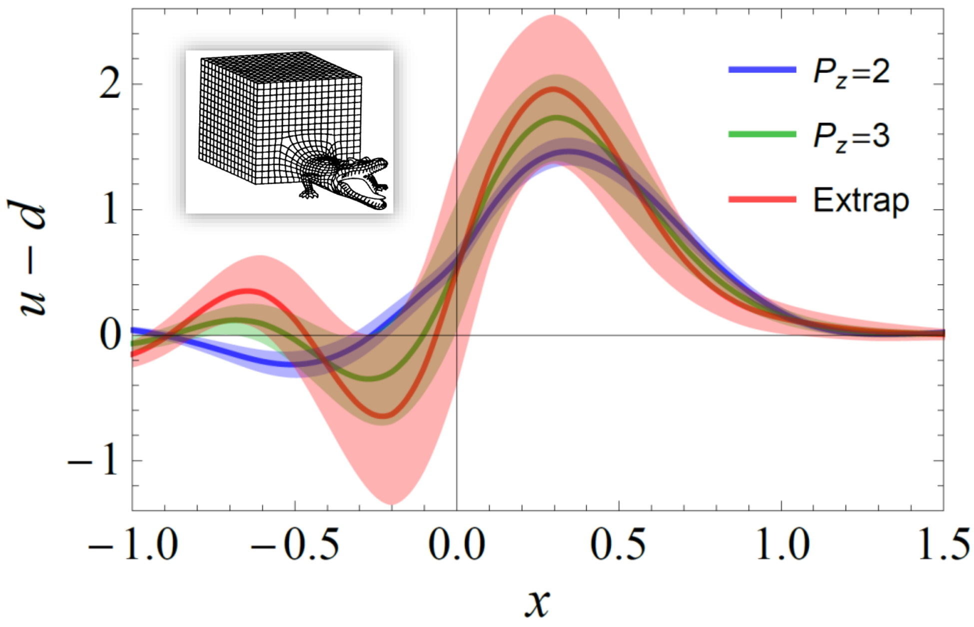
Renormalization

§ Effect on quasi-PDFs

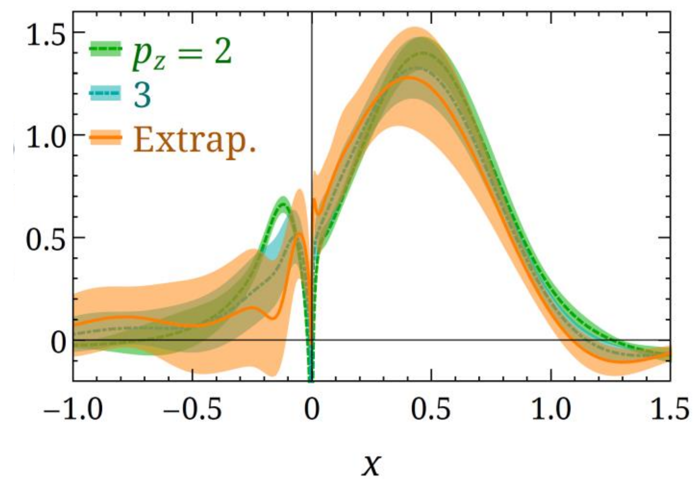
$$\tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_z z}\tilde{h}_R(z, P_z, \mu_R)$$

$+O(M_N^n/P_z^n)$
$+ O(\alpha_s)$ error
+RI/MOM to $\overline{\text{MS}}$ matching (Zhao)

$a \approx 0.12$ fm, $M_\pi \approx 310$ MeV



Earlier work (ignore renormalization)



1603.06664 (LP3)

Plot by Jianhui Zhang; 1706.01295 (LP3)

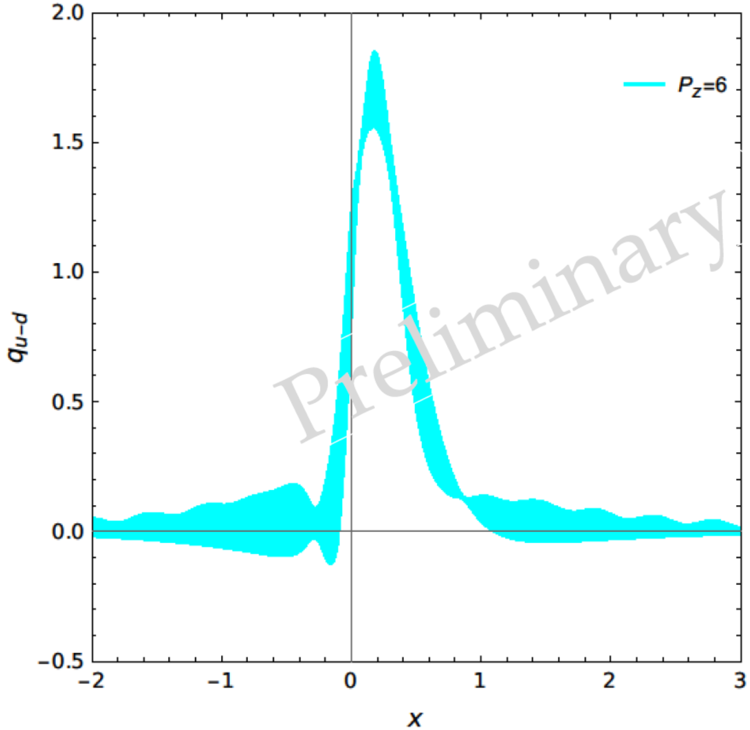
Power Divergence

§ Improved quasi-quark distribution

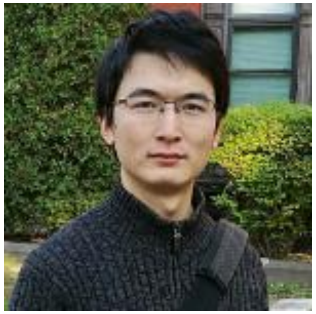
- $\tilde{q}_{\text{imp}}(x,\Lambda,p_z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{izk_z - \delta m|z|} \langle p|\bar{\psi}(0,0_\perp,z)\gamma_z L(z,0)\psi(0)|p\rangle$

§ Wilson-line renormalization to remove power divergence

- $a \approx 0.09$ fm, $L \approx 6$ fm, $M_\pi \approx 130$ MeV, clover/HISQ



Jian-Hui Zhang



Luchang Jin