

Quasi-two-body decays

$B_{(s)} \rightarrow P[\rho, \rho'(1450), \rho''(1700)] \rightarrow P\pi\pi$

in the perturbative QCD approach

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Outline

- Motivation
- Framework
- Results
- Summary

Motivation (experiment)

Large amounts of data from CDF, D0, Belle, *BABAR*, LHCb *etc.* Collaborations.

***BABAR*:** *prd 70-092001, prd 72-072003, prd 78-052005, prd 79-072006 ...*

Belle: *prd 75-012006, prl 96-251803, prd 79-072004, prd 78-072006 ...*

LHCb: *prd 86-052006, prd 90-012003, prd 90-112004, prl 112-011801 ...*

Motivation (theory)

QCDF

- Hai-Yang Cheng, C.-H. Chen, C.-K. Chua, C. Q. Geng, Y. K. Hsiao, A. Soni, Y. Li, ...
- Xin-Heng Guo, Ya-Dong Yang, Zhen-Hua Zhang, Gang Lü, Jia-Qi Lei, ...
- Hossein Mehraban, Mahboobeh Sayahi, ...
- A. Furman, B. El-Bennich, R. Kaminski, L. Lesniak, B. Loiseau
- Thomas Mannel, Susanne Kränkl, Javier Virto

Symmetry

- Xiao-Gang He, Guan-Nan Li, Dong Xu
- $SU(3)$ and Isospin Breaking Effects on $B \rightarrow PPP$ Amplitudes

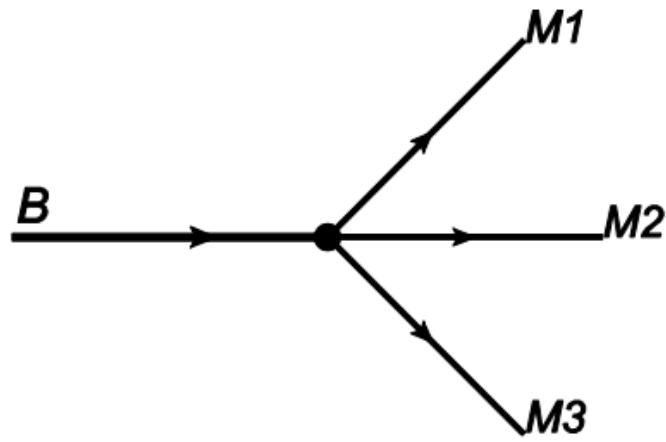
- J.L.Rosner, B. Bhattacharyaa,M. Gronaub

U-spin \rightarrow CP asymmetries in three-body B^\pm decays to charged pions and kaons

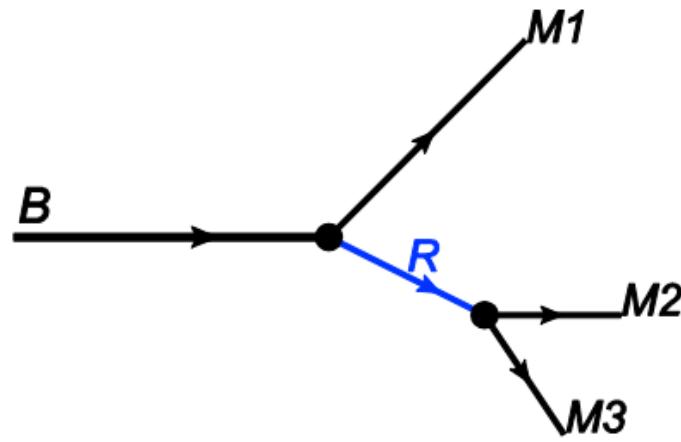
PQCD

- Hsiang-nan Li, Wen-Fei Wang, W. Wang, C.-D. Lü, Z.-J. Xiao, H.-C. Hu, C.-H. Chen, ...

Framework



$1 \rightarrow 3$ decay mode



$1 \rightarrow 2 \rightarrow 3$ decay mode

Framework

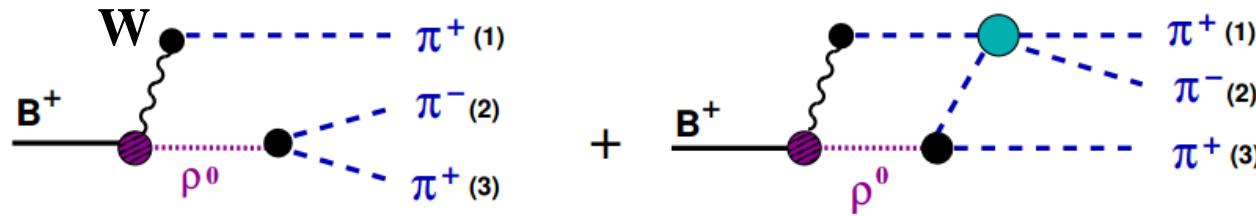
Final state interaction on $B^+ \rightarrow \pi^- \pi^+ \pi^+$

I. Bediaga and P. C. Magalhães

*Centro Brasileiro de Pesquisas Físicas - CBPF - Rio de Janeiro, RJ, Brasil**

(Dated: January 1, 2016)

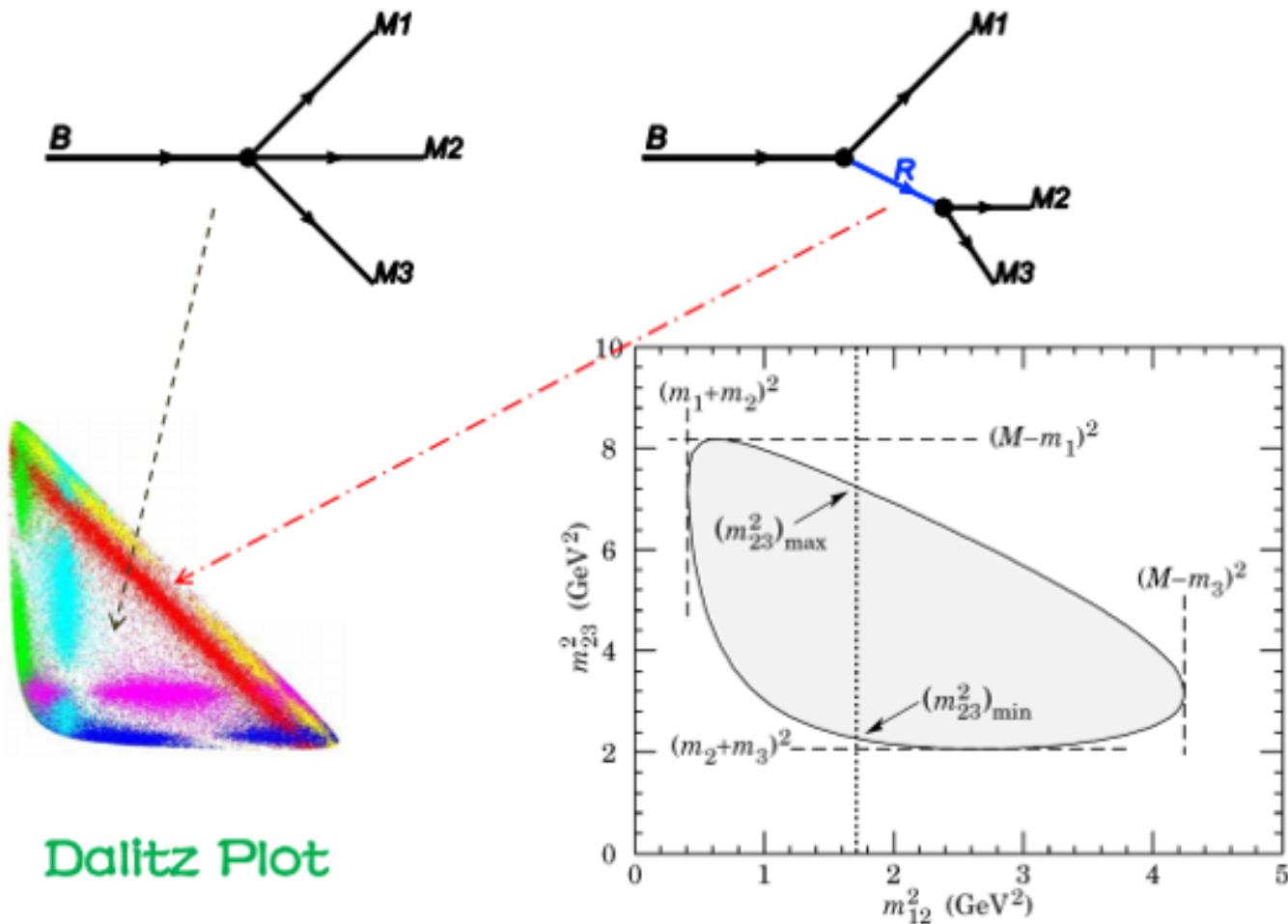
arXiv:1512.09284



Quasi-two-body mode

1→2→3 decay mode

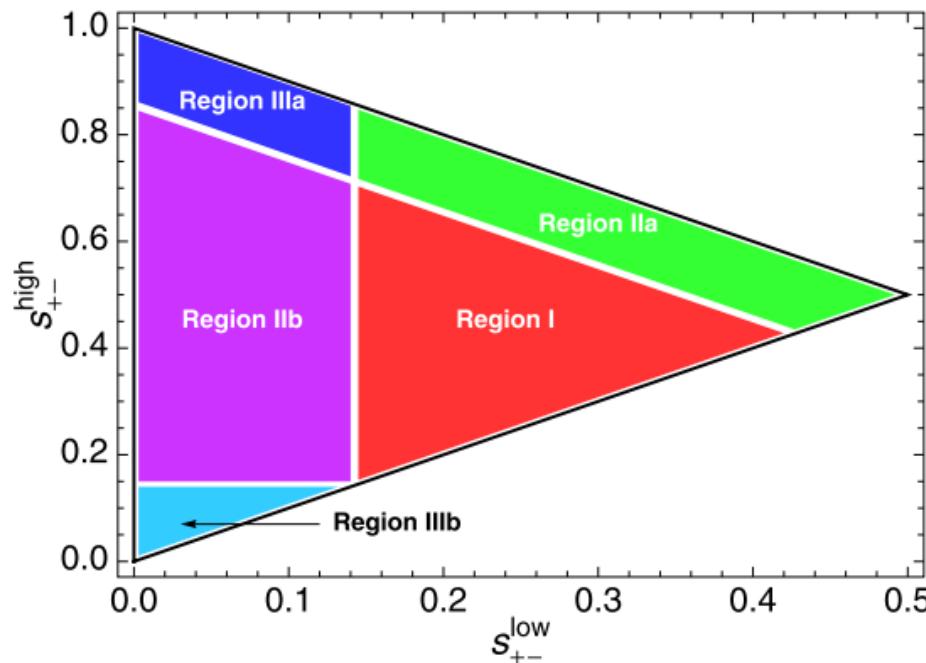
Framework



Framework

Thomas Mannel, Susanne Kränkl, Javier Virto

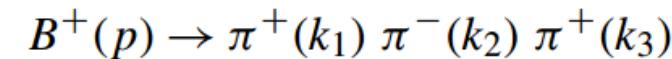
Nuclear Physics B 899 (2015) 247



Region I: $s_{++} \sim s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/3$

Region IIa: $s_{++} \sim 0, \quad s_{+-}^{\text{low}} \sim s_{+-}^{\text{high}} \sim 1/2$

Region IIIa: $s_{++} \sim s_{+-}^{\text{low}} \sim 0, \quad s_{+-}^{\text{high}} \sim 1$



$$s_{ij} \equiv \frac{(k_i + k_j)^2}{m_B^2} = \frac{2k_i \cdot k_j}{m_B^2} \quad (i \neq j).$$

$$s_{12} \equiv s_{+-}^{\text{low}}, \quad s_{13} \equiv s_{++} \quad \text{and} \quad s_{23} \equiv s_{+-}^{\text{high}}$$

$$s_{12} + s_{13} + s_{23} = 1 \quad \text{and} \quad 0 \leq s_{ij} \leq 1.$$

Region IIb: $s_{+-}^{\text{low}} \sim 0, \quad s_{++} \sim s_{+-}^{\text{high}} \sim 1/2$

Region IIIb: $s_{+-}^{\text{high}} \sim s_{+-}^{\text{low}} \sim 0, \quad s_{++} \sim 1$

Framework



Available online at www.sciencedirect.com



PHYSICS LETTERS B

Physics Letters B 561 (2003) 258–265

www.elsevier.com/locate/npe

Three-body nonleptonic B decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li

Abstract

We develop perturbative QCD formalism for three-body nonleptonic B meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing **two-meson distribution amplitudes**. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

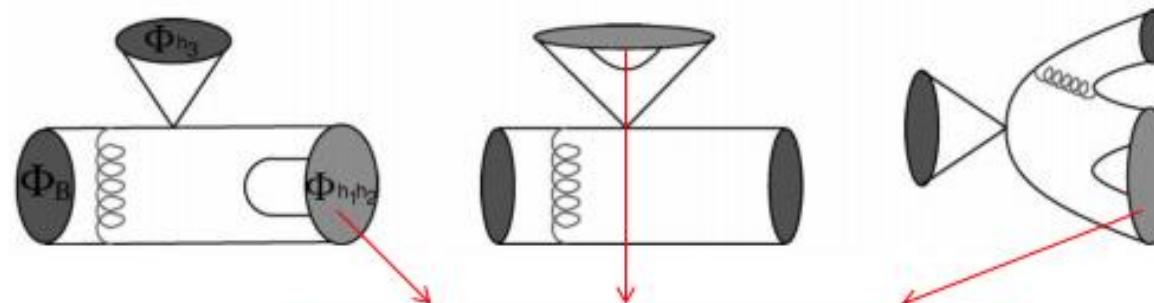
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Framework

Physics Letters B 561 (2003) 258–265

Three-body nonleptonic B decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li



two-meson distribution amplitudes

TMDA

$$B \rightarrow h_1 h_2 h_3$$

$$\Rightarrow \mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}$$

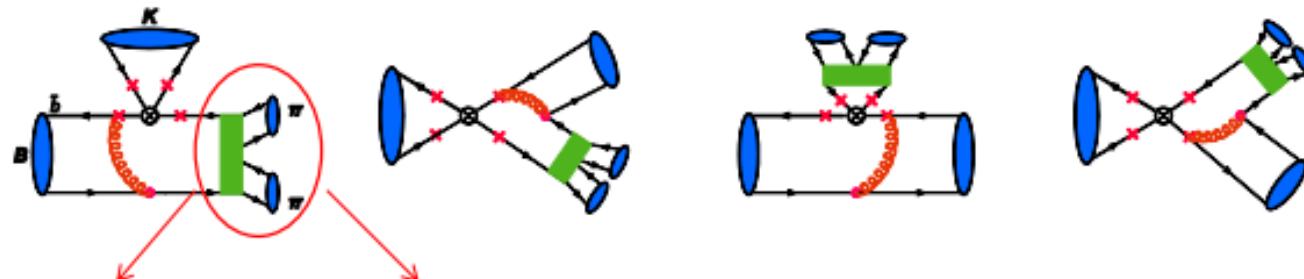


Quasi-two-body decays $B \rightarrow K\rho \rightarrow K\pi\pi$ in perturbative QCD approach



Wen-Fei Wang ^{a,b}, Hsiang-nan Li ^{a,*}

$$B \rightarrow K\rho^0 \rightarrow K\pi^+\pi^-$$



$\rho \rightarrow \pi\pi \sim 100\%$

resonant state distribution amplitude

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|p_1|}{M^2} d\Omega \quad \sim \quad d\Gamma = \frac{1}{(2\pi)^5} \frac{1}{16M^2} |\mathcal{M}|^2 |p_1^*| |p_3| dm_{12} d\Omega_1^* d\Omega_3$$



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Physics Letters B

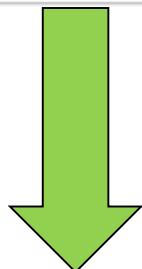
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Quasi-two-body decays $B \rightarrow K\rho \rightarrow K\pi\pi$ in perturbative QCD approach



Wen-Fei Wang^{a,b}, Hsiang-nan Li^{a,*}



PHYSICAL REVIEW D 95, 056008 (2017)

Quasi-two-body decays $B_{(s)} \rightarrow P\rho \rightarrow P\pi\pi$ in the perturbative QCD approach

Ya Li,^{1,†} Ai-Jun Ma,^{1,‡} Wen-Fei Wang,^{2,*} and Zhen-Jun Xiao^{1,3,§}

$$\Phi_{\pi\pi}^P = \frac{1}{\sqrt{2N_c}} [\not{p}\Phi_{v\nu=-}^{I=1}(z, \zeta, \omega^2) + \omega\Phi_s^{I=1}(z, \zeta, \omega^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{w(2\zeta - 1)}\Phi_{t\nu=+}^{I=1}(z, \zeta, \omega^2)],$$

$$\Phi_{v\nu=-}^{I=1} = \phi_0 = \frac{3F_\pi(s)}{\sqrt{2N_c}} z(1-z) \left[1 + a_{2\rho}^3 \frac{3}{2} (5(1-2z)^2 - 1) \right],$$

$$\Phi_s^{I=1} = \phi_s = \frac{3F_s(s)}{2\sqrt{2N_c}} (1-2z) \left[1 + a_{2\rho}^s (10z^2 - 10z + 1) \right],$$

$$\Phi_{t\nu=+}^{I=1} = \phi_t = \frac{3F_t(s)}{2\sqrt{2N_c}} (1-2z)^2 \left[1 + a_{2\rho}^t \frac{3}{2} (5(1-2z)^2 - 1) \right],$$

→ $a_{2\rho}^0 = 0.30, a_{2\rho}^s = 0.70, a_{2\rho}^t = -0.40.$

Phys. Rev. D 86, 032013 (2012) (BABAR)

$$F_\pi(s) = \frac{\text{GS}_\rho(s, m_\rho, \Gamma_\rho) \frac{1+c_\omega \text{BW}_\omega(s, m_\omega, \Gamma_\omega)}{1+c_\omega} + \sum c_i \text{GS}_i(s, m_i, \Gamma_i)}{1 + \sum c_i}$$

$$F_{s,t}(w^2) \approx (f_\rho^T/f_\rho) F_\pi(w^2).$$

G. Gounaris, J.J. Sakurai
 PRL21-244(1968)

$$\text{GS}_\rho(s, m_\rho, \Gamma_\rho) = \frac{m_\rho^2 [1 + d(m_\rho) \Gamma_\rho / m_\rho]}{m_\rho^2 - s + f(s, m_\rho, \Gamma_\rho) - i m_\rho \Gamma(s, m_\rho, \Gamma_\rho)},$$

Results

TABLE I: CP averaged branching ratios and direct CP -violating asymmetries of $B_{(s)} \rightarrow K(\rho \rightarrow) \pi\pi$ decays calculated in PQCD approach together with experimental data [91]

Modes		Quasi-two-body results	Experiment
$B^+ \rightarrow K^+(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$4.04^{+0.75}_{-0.58}(\omega_B)^{+0.24}_{-0.20}(a_{2\rho}^t)^{+0.27}_{-0.25}(a_{2\rho}^s)^{+0.22}_{-0.21}(a_{2\rho}^0)$	3.70 ± 0.50
	$\mathcal{A}_{CP}(\%)$	$50.7^{+3.8}_{-2.6}(\omega_B)^{+3.3}_{-4.7}(a_{2\rho}^t)^{+0.0}_{-0.7}(a_{2\rho}^s)^{+0.9}_{-1.5}(a_{2\rho}^0)$	37.0 ± 10.0
$B^0 \rightarrow K^+(\rho^- \rightarrow) \pi^- \pi^0$	$\mathcal{B}(10^{-6})$	$8.17^{+1.93}_{-1.39}(\omega_B)^{+0.36}_{-0.31}(a_{2\rho}^t)^{+0.46}_{-0.51}(a_{2\rho}^s) \pm 0.43(a_{2\rho}^0)$	7.00 ± 0.90
	$\mathcal{A}_{CP}(\%)$	$39.7^{+2.6}_{-0.6}(\omega_B)^{+5.1}_{-5.4}(a_{2\rho}^t)^{+0.5}_{-0.0}(a_{2\rho}^s)^{+1.0}_{-0.9}(a_{2\rho}^0)$	20.0 ± 11.0
$B_s^0 \rightarrow K^-(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$19.68^{+7.63}_{-5.18}(\omega_{B_s}) \pm 0.01(a_{2\rho}^t) \pm 0.01(a_{2\rho}^s)^{+0.05}_{-0.06}(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$21.8^{+3.7}_{-3.4}(\omega_{B_s}) \pm 0.3(a_{2\rho}^t) \pm 0.2(a_{2\rho}^s) \pm 1.2(a_{2\rho}^0)$	—
$B^+ \rightarrow K^0(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$8.13^{+1.82}_{-1.23}(\omega_B) \pm 0.87(a_{2\rho}^t)^{+0.44}_{-0.43}(a_{2\rho}^s)^{+0.36}_{-0.39}(a_{2\rho}^0)$	8.00 ± 1.50
	$\mathcal{A}_{CP}(\%)$	$13.8^{+3.1}_{-2.9}(\omega_B)^{+2.2}_{-1.9}(a_{2\rho}^t)^{+0.2}_{-0.0}(a_{2\rho}^s)^{+0.2}_{-0.3}(a_{2\rho}^0)$	-12.0 ± 17.0
$B^0 \rightarrow K^0(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$4.39^{+1.12}_{-0.81}(\omega_B) \pm 0.38(a_{2\rho}^t)^{+0.21}_{-0.22}(a_{2\rho}^s)^{+0.19}_{-0.16}(a_{2\rho}^0)$	4.70 ± 0.60
	$\mathcal{A}_{CP}(\%)$	$8.1^{+0.1}_{-0.2}(\omega_B)^{+0.8}_{-0.3}(a_{2\rho}^t)^{+0.8}_{-0.6}(a_{2\rho}^s) \pm 0.0(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \bar{K}^0(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.21^{+0.05}_{-0.01}(\omega_{B_s})^{+0.01}_{-0.00}(a_{2\rho}^t)^{+0.01}_{-0.00}(a_{2\rho}^s)^{+0.03}_{-0.01}(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$63.7^{+13.1}_{-15.2}(\omega_{B_s})^{+5.7}_{-7.0}(a_{2\rho}^t)^{+3.1}_{-4.0}(a_{2\rho}^s)^{+1.5}_{-2.0}(a_{2\rho}^0)$	—

Results

TABLE II: CP averaged branching ratios and direct CP -violating asymmetries of $B_{(s)} \rightarrow \pi(\rho \rightarrow) \pi\pi$ decays calculated in PQCD approach together with experimental data [91]

Modes		Quasi-two-body results	Experiment
$B^+ \rightarrow \pi^+(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$8.84^{+1.48}_{-1.24}(\omega_B)^{+0.12}_{-0.13}(a_{2\rho}^t)^{+1.17}_{-1.11}(a_{2\rho}^s)^{+0.25}_{-0.26}(a_{2\rho}^0)$	8.30 ± 1.20
	$\mathcal{A}_{CP}(\%)$	$-27.5^{+2.3}_{-3.1}(\omega_B)^{+0.9}_{-1.0}(a_{2\rho}^t) \pm 1.4(a_{2\rho}^s) \pm 0.9(a_{2\rho}^0)$	$18.0^{+9.0}_{-17.0}$
$B^0 \rightarrow \pi^+(\rho^- \rightarrow) \pi^- \pi^0$	$\mathcal{B}(10^{-6})$	$7.85^{+2.60}_{-1.82}(\omega_B)^{+1.77}_{-1.58}(a_{2\rho}^t)^{+0.94}_{-0.91}(a_{2\rho}^s)^{+0.26}_{-0.25}(a_{2\rho}^0)$	23.00 ± 2.30 ^a
	$\mathcal{A}_{CP}(\%)$	$-31.4^{+3.4}_{-3.3}(\omega_B)^{+3.2}_{-4.0}(a_{2\rho}^t)^{+1.1}_{-1.6}(a_{2\rho}^s)^{+0.9}_{-0.7}(a_{2\rho}^0)$	-8.0 ± 8.0
$B^0 \rightarrow \pi^-(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$18.78^{+6.92}_{-4.80}(\omega_B)^{+0.56}_{-0.55}(a_{2\rho}^t)^{+0.20}_{-0.21}(a_{2\rho}^s) \pm 0.01(a_{2\rho}^0)$	23.00 ± 2.30 ^a
	$\mathcal{A}_{CP}(\%)$	$8.2^{+1.9}_{-1.5}(\omega_B) \pm 0.3(a_{2\rho}^t)^{+0.2}_{-0.1}(a_{2\rho}^s)^{+0.6}_{-0.5}(a_{2\rho}^0)$	13.0 ± 6.0
$B_s^0 \rightarrow \pi^+(\rho^- \rightarrow) \pi^- \pi^0$	$\mathcal{B}(10^{-6})$	$0.38 \pm 0.05(\omega_{B_s}) \pm 0.01(a_{2\rho}^t)^{+0.00}_{-0.01}(a_{2\rho}^s)^{+0.02}_{-0.03}(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$-4.9^{+0.0}_{-1.7}(\omega_{B_s})^{+1.3}_{-4.4}(a_{2\rho}^t)^{+0.0}_{-2.5}(a_{2\rho}^s)^{+0.6}_{-1.5}(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \pi^-(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$0.41 \pm 0.05(\omega_{B_s})^{+0.00}_{-0.02}(a_{2\rho}^t) \pm 0.01(a_{2\rho}^s)^{+0.02}_{-0.03}(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$-36.7^{+0.0}_{-2.5}(\omega_{B_s})^{+2.8}_{-5.4}(a_{2\rho}^t)^{+0.1}_{-0.3}(a_{2\rho}^s)^{+0.0}_{-0.3}(a_{2\rho}^0)$	—
$B^+ \rightarrow \pi^0(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$5.53^{+2.65}_{-1.79}(\omega_B)^{+0.76}_{-0.71}(a_{2\rho}^t)^{+0.49}_{-0.47}(a_{2\rho}^s)^{+0.00}_{-0.02}(a_{2\rho}^0)$	10.90 ± 1.40
	$\mathcal{A}_{CP}(\%)$	$34.9^{+7.3}_{-6.9}(\omega_B)^{+1.6}_{-2.1}(a_{2\rho}^t)^{+1.6}_{-1.7}(a_{2\rho}^s)^{+1.9}_{-1.8}(a_{2\rho}^0)$	2.0 ± 11.0
$B^0 \rightarrow \pi^0(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.11^{+0.06}_{-0.03}(\omega_B)^{+0.02}_{-0.00}(a_{2\rho}^t)^{+0.01}_{-0.00}(a_{2\rho}^s)^{+0.01}_{-0.00}(a_{2\rho}^0)$	2.00 ± 0.50
	$\mathcal{A}_{CP}(\%)$	$-14.2^{+17.1}_{-4.3}(\omega_B)^{+3.6}_{-0.7}(a_{2\rho}^t)^{+11.3}_{-9.2}(a_{2\rho}^s)^{+2.8}_{-0.0}(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \pi^0(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.35^{+0.06}_{-0.05}(\omega_{B_s}) \pm 0.01(a_{2\rho}^t) \pm 0.00(a_{2\rho}^s) \pm 0.03(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$-24.6^{+2.8}_{-0.0}(\omega_{B_s})^{+1.9}_{-0.0}(a_{2\rho}^t)^{+0.0}_{-1.6}(a_{2\rho}^s)^{+0.0}_{-2.6}(a_{2\rho}^0)$	—

Puzzle: CP violation in $B^+ \rightarrow \rho^0 \pi^+$

PHYSICAL REVIEW D **94**, 094015 (2016)

Direct CP violation in charmless three-body decays of B mesons

Hai-Yang Cheng,¹ Chun-Kiang Chua,² and Zhi-Qing Zhang³

Indeed, LHCb has measured asymmetries in $B^- \rightarrow \pi^+ \pi^- \pi^-$ in four distinct regions dominated by the ρ [8]: I: $0.47 < m(\pi^+ \pi^-)_{\text{low}} < 0.77$ GeV, $\cos \theta > 0$, II: $0.77 < m(\pi^+ \pi^-)_{\text{low}} < 0.92$ GeV, $\cos \theta > 0$, III: $0.47 < m(\pi^+ \pi^-)_{\text{low}} < 0.77$ GeV, $\cos \theta < 0$, and IV: $0.77 < m(\pi^+ \pi^-)_{\text{low}} < 0.92$ GeV, $\cos \theta < 0$. It is seen that \mathcal{A}_{CP} changes sign at $m(\pi^+ \pi^-) \sim m_\rho$. Summing over the regions I-IV yields CP asymmetry consistent with zero with slightly positive central value
Hence, the LHCb data imply positive CP violation induced by the ρ and f_0 resonances.

BABAR and LHCb measurements for this quantity, however, prefer a positive CP asymmetry in the $m(\pi^+ \pi^-)$ region peaked at m_ρ .

The theoretical predictions based on the QCDF, PQCD and SCET all give a negative CP asymmetry of order -0.20 for $B^+ \rightarrow \rho^0 \pi^+$.

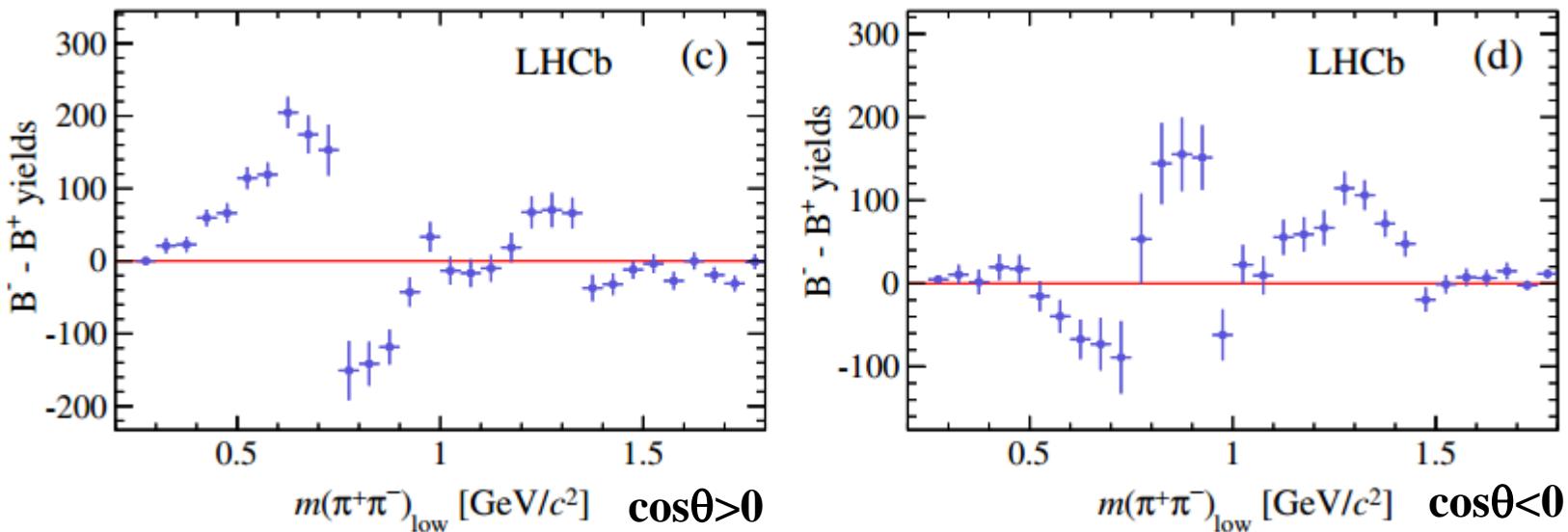
Measurements of CP violation in the three-body phase space of charmless B^\pm decays

R. Aaij *et al.*^{*}

(LHCb Collaboration)

(Received 25 August 2014; published 11 December 2014)

LHCb has measured CP asymmetries in regions dominated by vector resonances



I: $0.47 < m(\pi^+\pi^-)_{\text{low}} < 0.77 \text{ GeV}$, $\cos\theta > 0$,

II: $0.77 < m(\pi^+\pi^-)_{\text{low}} < 0.92 \text{ GeV}$, $\cos\theta > 0$,

III: $0.47 < m(\pi^+\pi^-)_{\text{low}} < 0.77 \text{ GeV}$, $\cos\theta < 0$,

IV: $0.77 < m(\pi^+\pi^-)_{\text{low}} < 0.92 \text{ GeV}$, $\cos\theta < 0$.

A_{CP} changes sign at
 $m(\pi^+\pi^-) \sim m_\rho$

- Interference between ρ and $f_0(500)$

PHYSICAL REVIEW D **87**, 076007 (2013)

CP violation in $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ in the region with low invariant mass of one $\pi^+ \pi^-$ pair

Zhen-Hua Zhang,^{1,*} Xin-Heng Guo,^{2,†} and Ya-Dong Yang^{1,‡}

- The fraction of tree and penguin contributions varys across the phase space

PHYSICAL REVIEW D **88**, 114014 (2013)

Branching fractions and direct CP violation in charmless three-body decays of B mesons

Hai-Yang Cheng¹ and Chun-Khiang Chua²

Results

TABLE III: CP averaged branching ratios and direct CP -violating asymmetries of $B_{(s)} \rightarrow \eta^{(\prime)}(\rho \rightarrow) \pi\pi$ decays calculated in PQCD approach together with experimental data [91]

Modes		Quasi-two-body results	Experiment
$B^+ \rightarrow \eta(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$6.74^{+2.04}_{-1.50}(\omega_B)^{+0.29}_{-0.27}(a_{2\rho}^t)^{+0.10}_{-0.09}(a_{2\rho}^s)^{+0.02}_{-0.01}(a_{2\rho}^0)$	7.00 ± 2.90
	$\mathcal{A}_{CP}(\%)$	$-0.3^{+0.2}_{-0.0}(\omega_B)^{+0.3}_{-0.2}(a_{2\rho}^t)^{+0.0}_{-0.1}(a_{2\rho}^s) \pm 0.0(a_{2\rho}^0)$	11.0 ± 11.0
$B^0 \rightarrow \eta(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.17^{+0.03}_{-0.02}(\omega_B)^{+0.03}_{-0.02}(a_{2\rho}^t)^{+0.01}_{-0.00}(a_{2\rho}^s)^{+0.02}_{-0.00}(a_{2\rho}^0)$	< 1.5
	$\mathcal{A}_{CP}(\%)$	$16.3^{+3.3}_{-1.6}(\omega_B)^{+9.1}_{-7.2}(a_{2\rho}^t)^{+0.0}_{-1.9}(a_{2\rho}^s)^{+0.0}_{-1.8}(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \eta(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.10^{+0.04}_{-0.02}(\omega_{B_s}) \pm 0.00(a_{2\rho}^t) \pm 0.00(a_{2\rho}^s) \pm 0.00(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$19.2^{+0.1}_{-0.2}(\omega_{B_s})^{+0.0}_{-0.4}(a_{2\rho}^t)^{+0.2}_{-0.4}(a_{2\rho}^s)^{+1.5}_{-1.7}(a_{2\rho}^0)$	—
$B^+ \rightarrow \eta'(\rho^+ \rightarrow) \pi^+ \pi^0$	$\mathcal{B}(10^{-6})$	$4.56^{+1.44}_{-1.02}(\omega_B)^{+0.16}_{-0.13}(a_{2\rho}^t)^{+0.04}_{-0.03}(a_{2\rho}^s)^{+0.02}_{-0.01}(a_{2\rho}^0)$	9.70 ± 2.20
	$\mathcal{A}_{CP}(\%)$	$21.0^{+1.7}_{-1.9}(\omega_B) \pm 1.6(a_{2\rho}^t) \pm 0.2(a_{2\rho}^s)^{+0.3}_{-0.2}(a_{2\rho}^0)$	26.0 ± 17.0
$B^0 \rightarrow \eta'(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.17^{+0.05}_{-0.04}(\omega_B)^{+0.01}_{-0.00}(a_{2\rho}^t) \pm 0.01(a_{2\rho}^s) \pm 0.01(a_{2\rho}^0)$	< 1.3
	$\mathcal{A}_{CP}(\%)$	$12.8^{+0.0}_{-1.2}(\omega_B)^{+21.1}_{-23.6}(a_{2\rho}^t)^{+8.3}_{-7.3}(a_{2\rho}^s)^{+0.1}_{-0.8}(a_{2\rho}^0)$	—
$B_s^0 \rightarrow \eta'(\rho^0 \rightarrow) \pi^+ \pi^-$	$\mathcal{B}(10^{-6})$	$0.23^{+0.08}_{-0.06}(\omega_{B_s})^{+0.00}_{-0.01}(a_{2\rho}^t) \pm 0.00(a_{2\rho}^s)^{+0.00}_{-0.01}(a_{2\rho}^0)$	—
	$\mathcal{A}_{CP}(\%)$	$37.9^{+0.3}_{-0.5}(\omega_{B_s}) \pm 0.2(a_{2\rho}^t) \pm 0.3(a_{2\rho}^s) \pm 0.2(a_{2\rho}^0)$	—

Results

$$F_\pi(s) = \frac{\text{GS}_\rho(s, m_\rho, \Gamma_\rho) \frac{1+c_\omega \text{BW}_\omega(s, m_\omega, \Gamma_\omega)}{1+c_\omega} + \Sigma c_i \text{GS}_i(s, m_i, \Gamma_i)}{1 + \Sigma c_i}$$

$$i = \rho', \rho'', \rho'''(2254)$$

$$\mathcal{B}(B_{(s)} \rightarrow P(\rho' \rightarrow) \pi\pi) = \mathcal{B}(B_{(s)} \rightarrow P\rho') \cdot \mathcal{B}(\rho' \rightarrow \pi\pi)$$

Results

$$\Gamma_{\rho' \rightarrow \pi\pi} = \frac{g_{\rho'\pi\pi}^2}{6\pi} \frac{|\vec{p}_\pi(m_{\rho'}^2)|^3}{m_{\rho'}^2}$$

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$$\mathcal{B}(\rho' \rightarrow \pi\pi) = 10.04^{+5.23\%}_{-2.61\%}, \quad \mathcal{B}(\rho'' \rightarrow \pi\pi) = 8.11^{+2.22\%}_{-1.47\%}$$

Decay Modes	Quasi-two-body \mathcal{B} (in 10^{-7})	Two-body \mathcal{B} (in 10^{-6})	\mathcal{A}_{CP}
$B^+ \rightarrow K^+(\rho'^0 \rightarrow) \pi^+ \pi^-$	$4.66^{+1.05+0.44+0.59+0.11}_{-0.79-0.42-0.50-0.12}$	$4.64^{+1.28}_{-1.03}$	$0.39^{+0.05+0.02+0.00+0.00}_{-0.03-0.03-0.01-0.01}$
$B^0 \rightarrow K^+(\rho'^- \rightarrow) \pi^- \pi^0$	$8.88^{+2.66+0.74+0.98+0.26}_{-1.54-0.59-0.84-0.21}$	$8.84^{+2.93}_{-1.86}$	$0.35^{+0.02+0.04+0.00+0.00}_{-0.01-0.04-0.00-0.00}$
$B_s^0 \rightarrow K^-(\rho'^+ \rightarrow) \pi^+ \pi^0$	$13.84^{+5.31+0.04+0.02+0.04}_{-3.59-0.03-0.02-0.04}$	$13.78^{+5.29}_{-3.58}$	$0.25^{+0.04+0.01+0.00+0.01}_{-0.04-0.00-0.00-0.01}$
$B^+ \rightarrow K^0(\rho'^+ \rightarrow) \pi^+ \pi^0$	$10.64^{+2.89+1.63+1.02+0.24}_{-2.16-1.55-0.95-0.22}$	$10.60^{+3.47}_{-2.82}$	$0.13^{+0.03+0.02+0.01+0.00}_{-0.02-0.01-0.00-0.00}$
$B^0 \rightarrow K^0(\rho'^0 \rightarrow) \pi^+ \pi^-$	$5.31^{+1.61+0.64+0.41+0.11}_{-1.15-0.60-0.39-0.10}$	$5.29^{+1.78}_{-1.35}$	$0.10^{+0.00+0.01+0.01+0.00}_{-0.00-0.00-0.00-0.00}$
$B_s^0 \rightarrow \bar{K}^0(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.22^{+0.03+0.01+0.00+0.02}_{-0.02-0.01-0.01-0.02}$	$0.22^{+0.04}_{-0.03}$	$0.20^{+0.15+0.11+0.05+0.00}_{-0.11-0.10-0.05-0.00}$
$B^+ \rightarrow \pi^+(\rho'^0 \rightarrow) \pi^+ \pi^-$	$8.15^{+0.00+0.05+1.44+0.22}_{-0.13-0.05-1.30-0.22}$	$8.11^{+1.45}_{-1.32}$	$-0.29^{+0.00+0.02+0.03+0.01}_{-0.00-0.01-0.03-0.00}$
$B^0 \rightarrow \pi^+(\rho'^- \rightarrow) \pi^- \pi^0$	$5.32^{+0.84+1.58+0.87+0.13}_{-0.79-1.21-0.84-0.14}$	$5.30^{+1.99}_{-1.67}$	$-0.37^{+0.00+0.06+0.02+0.01}_{-0.01-0.07-0.02-0.00}$
$B^0 \rightarrow \pi^-(\rho'^+ \rightarrow) \pi^+ \pi^0$	$12.34^{+4.73+0.66+0.25+0.03}_{-3.30-0.64-0.24-0.02}$	$12.29^{+4.76}_{-3.36}$	$0.11^{+0.02+0.00+0.00+0.00}_{-0.02-0.01-0.01-0.01}$
$B_s^0 \rightarrow \pi^+(\rho'^- \rightarrow) \pi^- \pi^0$	$0.19^{+0.02+0.07+0.01+0.02}_{-0.01-0.03-0.00-0.01}$	$0.19^{+0.08}_{-0.03}$	$0.01^{+0.00+0.00+0.01+0.02}_{-0.07-0.08-0.01-0.02}$
$B_s^0 \rightarrow \pi^-(\rho'^+ \rightarrow) \pi^+ \pi^0$	$0.29^{+0.01+0.00+0.02+0.01}_{-0.03-0.01-0.02-0.01}$	$0.29^{+0.02}_{-0.04}$	$-0.28^{+0.01+0.04+0.01+0.02}_{-0.01-0.08-0.02-0.01}$
$B^+ \rightarrow \pi^0(\rho'^+ \rightarrow) \pi^+ \pi^0$	$1.94^{+1.50+0.56+0.35+0.00}_{-0.80-0.40-0.29-0.01}$	$1.93^{+1.63}_{-0.94}$	$0.24^{+0.02+0.04+0.02+0.02}_{-0.06-0.08-0.02-0.02}$
$B^0 \rightarrow \pi^0(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.26^{+0.11+0.04+0.02+0.01}_{-0.08-0.02-0.02-0.01}$	$0.26^{+0.12}_{-0.09}$	$-0.54^{+0.05+0.07+0.05+0.00}_{-0.04-0.04-0.05-0.01}$
$B_s^0 \rightarrow \pi^0(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.15^{+0.03+0.02+0.00+0.01}_{-0.02-0.01-0.00-0.01}$	$0.15^{+0.04}_{-0.02}$	$-0.30^{+0.00+0.10+0.07+0.02}_{-0.08-0.15-0.00-0.00}$
$B^+ \rightarrow \eta(\rho'^+ \rightarrow) \pi^+ \pi^0$	$4.41^{+1.55+0.29+0.09+0.01}_{-1.08-0.27-0.09-0.00}$	$4.39^{+1.57}_{-1.11}$	$0.02^{+0.01+0.00+0.00+0.00}_{-0.02-0.00-0.01-0.00}$
$B^0 \rightarrow \eta(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.14^{+0.02+0.02+0.01+0.00}_{-0.02-0.02-0.01-0.01}$	$0.14^{+0.02}_{-0.03}$	$-0.13^{+0.01+0.06+0.01+0.02}_{-0.03-0.03-0.01-0.02}$
$B_s^0 \rightarrow \eta(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.08^{+0.03+0.00+0.00+0.00}_{-0.02-0.00-0.00-0.00}$	$0.08^{+0.03}_{-0.02}$	$0.37^{+0.00+0.00+0.00+0.01}_{-0.01-0.01-0.01-0.01}$
$B^+ \rightarrow \eta'(\rho'^+ \rightarrow) \pi^+ \pi^0$	$3.21^{+1.09+0.17+0.02+0.00}_{-0.77-0.15-0.02-0.01}$	$3.20^{+1.10}_{-0.78}$	$0.46^{+0.05+0.03+0.02+0.00}_{-0.03-0.02-0.01-0.00}$
$B^0 \rightarrow \eta'(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.22^{+0.07+0.02+0.01+0.01}_{-0.04-0.00-0.01-0.00}$	$0.22^{+0.07}_{-0.04}$	$0.20^{+0.10+0.27+0.08+0.01}_{-0.11-0.30-0.07-0.00}$
$B_s^0 \rightarrow \eta'(\rho'^0 \rightarrow) \pi^+ \pi^-$	$0.17^{+0.06+0.00+0.00+0.00}_{-0.05-0.01-0.00-0.00}$	$0.17^{+0.06}_{-0.05}$	$0.54^{+0.01+0.01+0.01+0.00}_{-0.00-0.00-0.00-0.00}$

Decay Modes	Quasi-two-body \mathcal{B} (in 10^{-7})	Two-body \mathcal{B} (in 10^{-6})	\mathcal{A}_{CP}
$B^+ \rightarrow K^+(\rho''^0 \rightarrow) \pi^+ \pi^-$	$2.53^{+0.69+0.29+0.35+0.07}_{-0.52-0.27-0.31-0.05}$	$3.12^{+1.02}_{-0.82}$	$0.33^{+0.06+0.01+0.00+0.00}_{-0.07-0.01-0.00-0.01}$
$B^0 \rightarrow K^+(\rho''^- \rightarrow) \pi^- \pi^0$	$4.80^{+1.51+0.51+0.56+0.12}_{-1.08-0.40-0.52-0.09}$	$5.92^{+2.09}_{-1.56}$	$0.29^{+0.02+0.05+0.01+0.01}_{-0.05-0.04-0.01-0.00}$
$B_s^0 \rightarrow K^-(\rho''^+ \rightarrow) \pi^+ \pi^0$	$6.52^{+2.49+0.02+0.01+0.02}_{-1.69-0.02-0.01-0.01}$	$8.03^{+3.07}_{-2.08}$	$0.26^{+0.04+0.00+0.00+0.01}_{-0.04-0.01-0.00-0.01}$
$B^+ \rightarrow K^0(\rho''^+ \rightarrow) \pi^+ \pi^0$	$6.20^{+1.90+1.04+0.73+0.13}_{-1.43-0.95-0.65-0.11}$	$7.64^{+2.82}_{-2.27}$	$0.14^{+0.03+0.01+0.00+0.00}_{-0.04-0.03-0.01-0.01}$
$B^0 \rightarrow K^0(\rho''^0 \rightarrow) \pi^+ \pi^-$	$2.98^{+1.02+0.40+0.28+0.07}_{-0.74-0.37-0.27-0.06}$	$3.67^{+1.40}_{-1.08}$	$0.09^{+0.01+0.01+0.01+0.01}_{-0.00-0.00-0.00-0.00}$
$B_s^0 \rightarrow \bar{K}^0(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.11^{+0.01+0.01+0.00+0.01}_{-0.01-0.00-0.00-0.01}$	0.14 ± 0.02	$0.01^{+0.16+0.13+0.06+0.01}_{-0.10-0.11-0.05-0.00}$
$B^+ \rightarrow \pi^+(\rho''^0 \rightarrow) \pi^+ \pi^-$	$2.81^{+0.28+0.02+0.56+0.09}_{-0.40-0.03-0.52-0.09}$	$3.46^{+0.78}_{-0.82}$	$-0.35^{+0.03+0.01+0.02+0.00}_{-0.01-0.02-0.05-0.01}$
$B^0 \rightarrow \pi^+(\rho''^- \rightarrow) \pi^- \pi^0$	$1.28^{+0.13+0.41+0.25+0.03}_{-0.09-0.11-0.17-0.03}$	$1.58^{+0.61}_{-0.28}$	$-0.51^{+0.01+0.08+0.02+0.01}_{-0.02-0.00-0.02-0.01}$
$B^0 \rightarrow \pi^-(\rho''^+ \rightarrow) \pi^+ \pi^0$	$5.61^{+2.16+0.38+0.14+0.02}_{-1.50-0.34-0.13-0.01}$	$6.92^{+2.71}_{-1.90}$	$0.11^{+0.03+0.01+0.01+0.01}_{-0.02-0.00-0.00-0.00}$
$B_s^0 \rightarrow \pi^+(\rho''^- \rightarrow) \pi^- \pi^0$	$0.08^{+0.01+0.04+0.00+0.01}_{-0.01-0.02-0.00-0.01}$	$0.10^{+0.05}_{-0.03}$	$-0.03^{+0.05+0.08+0.03+0.02}_{-0.01-0.03-0.03-0.00}$
$B_s^0 \rightarrow \pi^-(\rho''^+ \rightarrow) \pi^+ \pi^0$	$0.16^{+0.02+0.01+0.02+0.01}_{-0.01-0.01-0.01-0.00}$	$0.20^{+0.04}_{-0.02}$	$-0.24^{+0.02+0.08+0.02+0.02}_{-0.00-0.08-0.00-0.01}$
$B^+ \rightarrow \pi^0(\rho''^+ \rightarrow) \pi^+ \pi^0$	$0.67^{+0.60+0.27+0.15+0.00}_{-0.29-0.15-0.13-0.00}$	$0.83^{+0.83}_{-0.43}$	$0.18^{+0.00+0.07+0.05+0.02}_{-0.04-0.18-0.07-0.02}$
$B^0 \rightarrow \pi^0(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.14^{+0.04+0.03+0.01+0.00}_{-0.03-0.02-0.01-0.00}$	$0.17^{+0.06}_{-0.05}$	$-0.53^{+0.00+0.08+0.04+0.00}_{-0.02-0.03-0.06-0.01}$
$B_s^0 \rightarrow \pi^0(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.06^{+0.01+0.02+0.00+0.01}_{-0.01-0.01-0.00-0.00}$	$0.07^{+0.03}_{-0.02}$	$-0.35^{+0.00+0.19+0.01+0.02}_{-0.02-0.14-0.00-0.02}$
$B^+ \rightarrow \eta(\rho''^+ \rightarrow) \pi^+ \pi^0$	$2.11^{+0.77+0.17+0.05+0.00}_{-0.53-0.14-0.04-0.00}$	$2.60^{+0.97}_{-0.68}$	$0.02^{+0.01+0.00+0.00+0.00}_{-0.01-0.00-0.00-0.00}$
$B^0 \rightarrow \eta(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.08^{+0.02+0.02+0.00+0.00}_{-0.02-0.01-0.00-0.00}$	0.10 ± 0.03	$-0.32^{+0.04+0.03+0.05+0.01}_{-0.01-0.00-0.05-0.00}$
$B_s^0 \rightarrow \eta(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.04^{+0.01+0.00+0.00+0.00}_{-0.01-0.00-0.00-0.00}$	0.05 ± 0.01	$0.44^{+0.01+0.01+0.00+0.00}_{-0.02-0.01-0.01-0.01}$
$B^+ \rightarrow \eta'(\rho''^+ \rightarrow) \pi^+ \pi^0$	$1.49^{+0.52+0.09+0.01+0.01}_{-0.38-0.08-0.01-0.00}$	$1.84^{+0.65}_{-0.48}$	$0.50^{+0.00+0.03+0.02+0.00}_{-0.01-0.03-0.02-0.00}$
$B^0 \rightarrow \eta'(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.11^{+0.02+0.01+0.00+0.00}_{-0.02-0.01-0.01-0.00}$	0.14 ± 0.03	$0.31^{+0.10+0.29+0.08+0.01}_{-0.12-0.34-0.07-0.00}$
$B_s^0 \rightarrow \eta'(\rho''^0 \rightarrow) \pi^+ \pi^-$	$0.08^{+0.02+0.00+0.00+0.00}_{-0.02-0.00-0.00-0.00}$	0.10 ± 0.02	$0.60^{+0.01+0.00+0.00+0.00}_{-0.01-0.00-0.00-0.00}$

Results

$$B^+ \rightarrow \pi^+ \rho'^0 \rightarrow \pi^+ \pi^+ \pi^-$$

$$\mathcal{B} = (8.15^{+1.46}_{-1.33}) \times 10^{-7} \quad \mathcal{A}_{CP} = -0.29^{+0.04}_{-0.03} \quad (\textbf{PQCD})$$

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$$(1.4^{+0.6}_{-0.9}) \times 10^{-6} \quad (-6 \pm 28 \pm 20^{+12}_{-35})\%$$

Results

Phys. Rev. D 86, 032013 (2012) (BABAR)

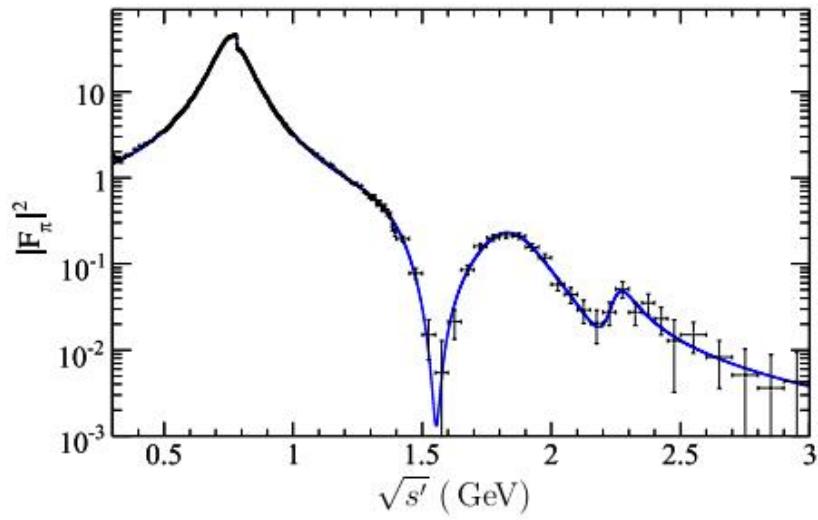
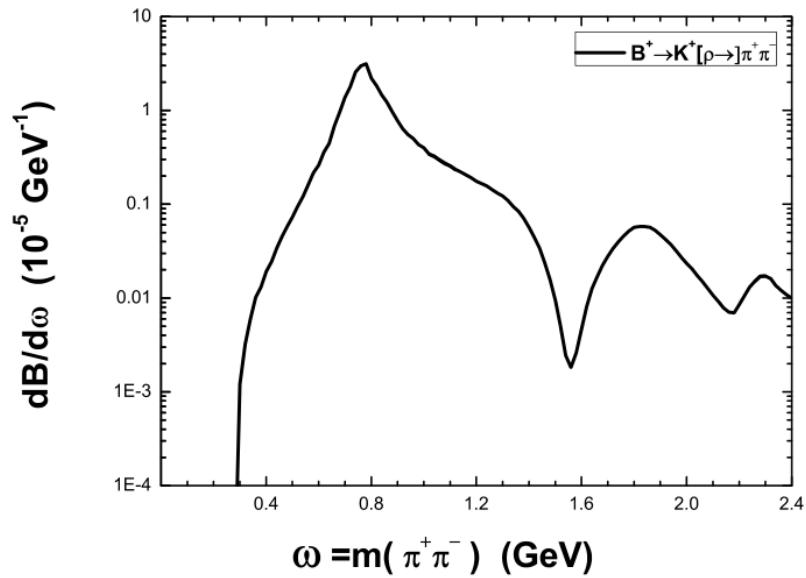


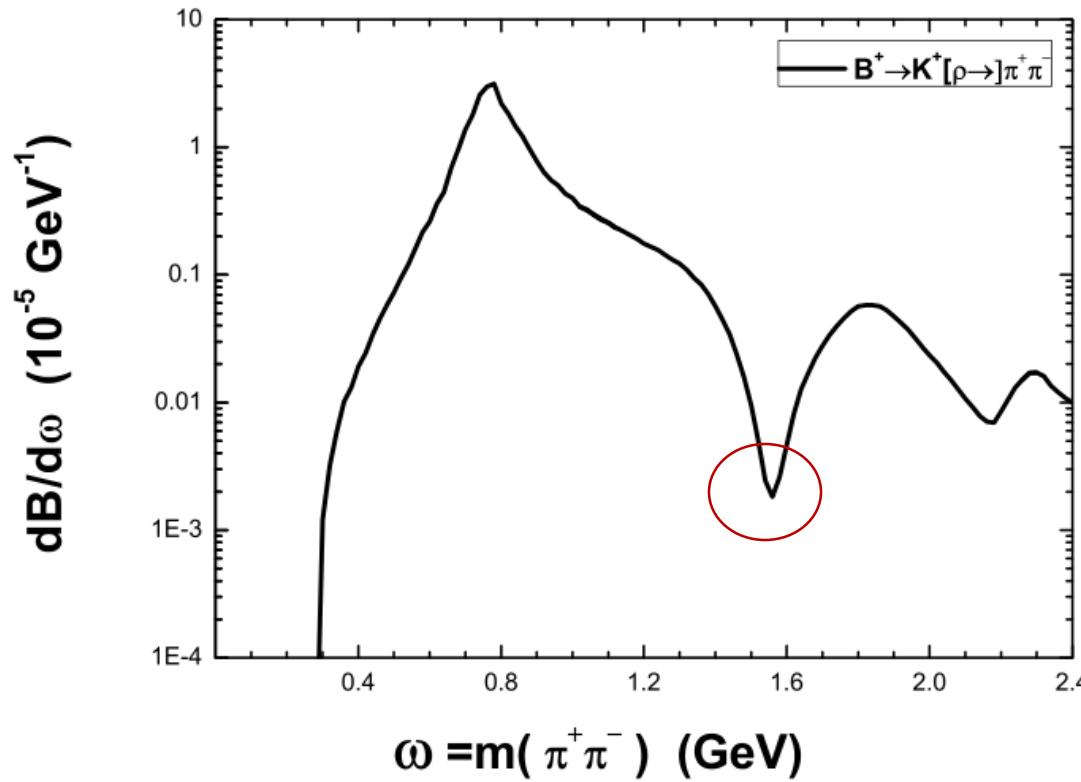
FIG. 45 (color online). The pion form factor-squared measured by BABAR as a function of $\sqrt{s'}$ from 0.3 to 3 GeV and the VDM fit described in the text.

Phys. Rev. D 96, 036014 (2017)



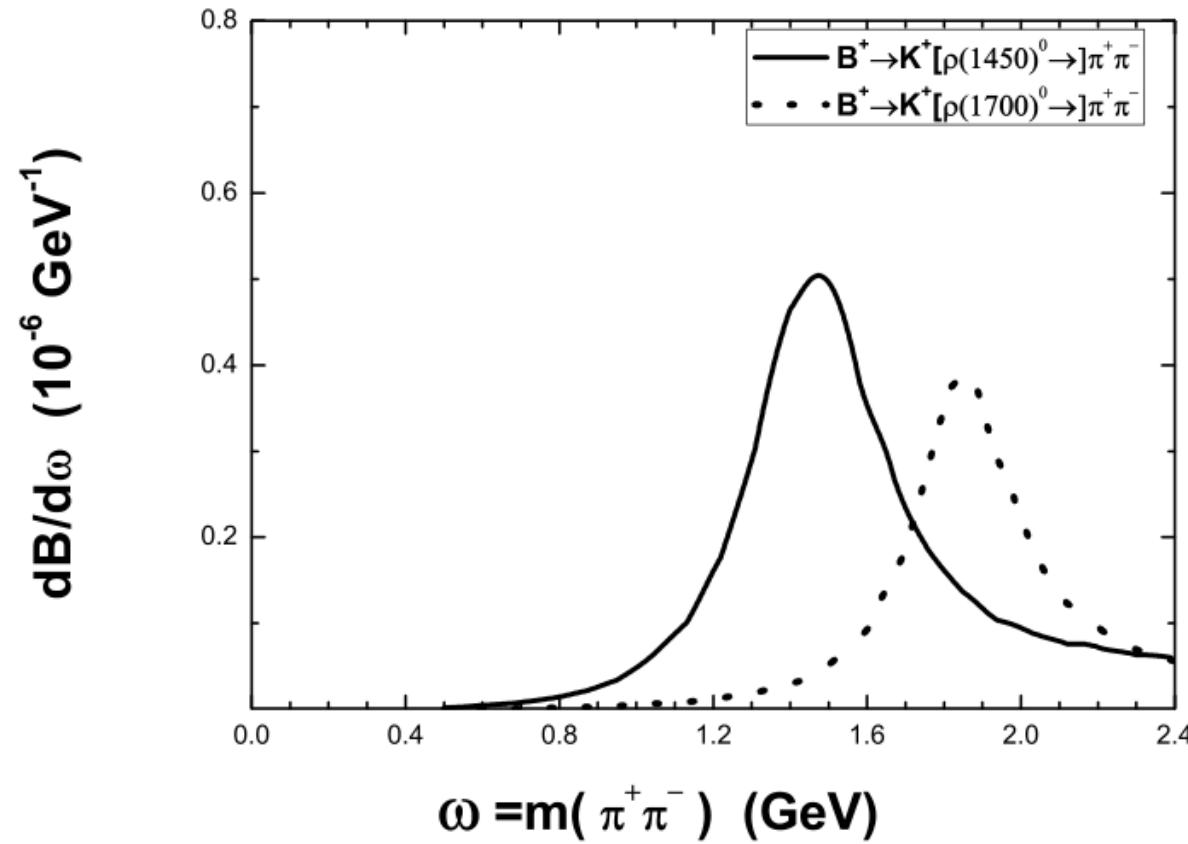
The summation of the contributions from $\rho(770)$, ρ' , ρ'' and ρ''' for the differential branching ratios of the $B^+ \rightarrow K^+ \rho \rightarrow K^+ \pi^+ \pi^-$ decays

Results



$$\begin{aligned}\mathcal{B}(B^+ \rightarrow K^+(\rho'^0 \rightarrow) \pi^+ \pi^-) &= 4.66^{+1.10}_{-1.05} \times 10^{-7}, \\ \mathcal{B}(B^+ \rightarrow K^+(\rho''^0 \rightarrow) \pi^+ \pi^-) &= 2.53^{+0.90}_{-0.67} \times 10^{-7}, \\ \text{interference term} &\approx -4.55 \times 10^{-7},\end{aligned}$$

Results



The comparison of the differential branching distributions for $B^+ \rightarrow K^+ \rho' \rightarrow K^+ \pi^+ \pi^-$ and $B^+ \rightarrow K^+ \rho'' \rightarrow K^+ \pi^+ \pi^-$

Summary

- Framework for the 3-body (quasi-two-body) hadronic B meson decays in PQCD approach
- Results of quasi-two-body decays $B_{(s)} \rightarrow P\rho$, $P\rho'(1450)$, $P\rho''(1700) \rightarrow P\pi\pi$

Thank you