

$B \rightarrow \pi\pi$ transition form factors in LCSR

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Outline

Background and Significance

Overview for QCD Light-cone Sum Rule

$B \rightarrow \pi\pi$ transition form factors in LCSR

With B meson DAs

With dipion DAs

Conclusion and Prospect

[S. Cheng, A. Khodjamirian and J. Virto, JHEP05(2017)157]

[S. Cheng, A. Khodjamirian and J. Virto, Phys.Rev.D96(2017)051901(R)]

[C. Hambrock and A. Khodjamirian, Nucl.Phys.B905(2016)373]

Background

- ▶ SM works beautifully, but an effective theory valid up to some scale;
- ▶ BSM are needed to explained the large C/P matter antimatter asymmetry;
 - † Direct search for new particles;
 - † Comparison of precise measurements with prediction from SM.
 - † **Precision study:**
 - beta decay, (1983, UA1,UA2, W boson);
 - kaon physics, (1974, BNL/SLAC, charm quark; 1977, Fermilab E288, beauty quark);
 - 1987, ARGUS, large $\Delta m_{B_d, B_s}$ (1994, CDF/D0, top quark);
 - $A_{CP}(t, f)$ in B decays (irreducible phase in CKM paradigm);
 - ...

Significance

Interests [S. Faller, T. F, A. K, T. M and Danny v Dyk, Phys.Rev.D89(2014)014015]

- ▶ Quark level: $b \rightarrow u$ is induced by tree-level W -exchange in SM, weighted by $|V_{ub}|$ (rare decay).
- ▶ Tensions of $|V_{ub}|$: [BABAR, Phys.Rev.D95(2017)072001, talks in FPCP2017, Prague]
CKM data analysis is 15% smaller than inclusive B decays;
 $B \rightarrow \pi l \bar{\nu}_l$, is 2σ smaller than, $B \rightarrow X_u l \bar{\nu}_l$;
⇒ Unitarity of CKM or NP?
- ▶ Traditional $B \rightarrow \rho$ form factors, single ρ meson with narrow width:
width effect ? nonresonance background?
- ▶ We need enlarge the set of exclusive processes ($B \rightarrow \pi\pi l \bar{\nu}_l / B_{l4}$).

Phenomenological Applications

- ▶ A competitive determination of the $|V_{ub}|$ with accurate FFs;
- ▶ FCNC $B \rightarrow \pi\pi l^+ l^-$, $B \rightarrow \pi\pi\pi$.

- ▶ **QCD sum rules approach:** twofold way of treating correlation function.
- ▶ Mutually versions for different types of hadronic matrix elements:
LCSRs(hadronic form factors), 2pSRs(decay constants).

LCSRs

- ▶ Non-local **correlation function**, putting the external meson on-shell, expanding on LC to get LCDAs;
- ▶ **Quark level:** evaluated with OPE **LCDAs**,
Hadron level: with intermediate interpolating, convolution of two decoupled quark current by dispersion relation;
- ▶ **Quark-hadron duality:** equate hadron dispersion integral to the OPE calculation (threshold s_0);
- ▶ **Borel transformation:** mitigate the harassment of ultraviolet subtraction scheme from the OPE side & suppress the contributions from higher excited and continuum states from the hadron aspect (M^2).

LCSR_s for $\bar{B}^0 \rightarrow \pi^+ \pi^0$ transition FFs

Some highlights:

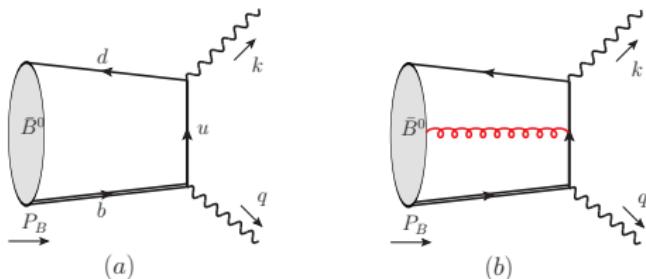
- For definiteness and conciseness, only the transition $\bar{B}^0 \rightarrow \pi^+ \pi^0$ was concentrated, **iso-vector dipion** final state,
FFs with iso-scalar dipion can be further studied.
- How much of the dominant intermediate ρ contribution to the FFs,
accurate interpretation (10%) of the $B \rightarrow \pi\pi/\bar{\nu}_l$ measurements.
- ◆ We are now at LO for preliminary attempts, NLO corrections from soft gluon, hard gluon exchanging correction will be added.
- ◆ Our predictions are effected/constrained by hadronic inputs:
dipion DAs (twist, strong phase) / B meson DAs (λ_B).

With B meson DAs

With B meson DAs

Correlation Function: [A. Khodjamirian, T. Mannel and Nils Offen, Phys.Rev.D75(2014)054013]

$$\begin{aligned} F_{\mu\nu}(k, q) &= i \int d^4x e^{ik \cdot x} \langle 0 | T\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0)\gamma_\nu(1 - \gamma_5)b(0)\} | \bar{B}^0(q+k) \rangle \\ &\downarrow \quad \text{Lorentz decomposition} \\ &\equiv \varepsilon_{\mu\nu\rho\sigma} q^\rho k^\sigma F_{(\varepsilon)}(k^2, q^2) + ig_{\mu\nu} F_{(g)}(k^2, q^2) + iq_\mu k_\nu F_{(qk)}(k^2, q^2) \\ &+ ik_\mu k_\nu F_{(kk)}(k^2, q^2) + iq_\mu q_\nu F_{(qq)}(k^2, q^2) + ik_\mu q_\nu F_{(kq)}(k^2, q^2) \quad (1) \end{aligned}$$



With B meson DAs

Processing

- $(q + k)^2 = m_B^2$, When $\Lambda_{QCD}^2 \ll q^2 \ll m_B^2$, $|k^2| \gg \Lambda_{QCD}^2$,
large virtuality of intermediate u quark,
OPE is applicable with DAs defined in HQET ($k \cdot x \sim 0$),
 q^2 is chosen from 0 to 10GeV².
- At LO, free propagator (u quark) + "Soft" gluon correction,
heavy-light bilocal $q\bar{q}$ and $qG\bar{q}$ matrix (two- and three-particle DAs).
- In parallel, employ the hadronic dispersion relation in the k^2 respecting to
the $\bar{d}\gamma_\mu u$ current interpolation.
- Take the invariant amplitude $F_\varepsilon(k^2, q^2)$ for example to outline.

OPE Calculation

$$F_{(\varepsilon)}^{\text{OPE}}(k^2, q^2) = f_B m_B \int_0^1 d\sigma \frac{\phi_+^B(\sigma m_B)}{(1-\sigma)(s-k^2)} + \dots \quad (2)$$

$$s = s(\sigma, q^2) = \sigma m_B^2 - \sigma q^2 / \bar{\sigma}, \quad \bar{\sigma} \equiv 1 - \sigma \quad (3)$$

- σ : the momentum carried by the light quark in B .
Ellipsis : subleading 3-particle DA contributions (**soft gluon correction**),
- Dispersion integral formula in k^2 for $F_{(\varepsilon)}^{\text{OPE}}(k^2, q^2)$

$$F_{(\varepsilon)}^{\text{OPE}}(k^2, q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^\infty ds \frac{\text{Im} F_{(\varepsilon)}^{\text{OPE}}(s, q^2)}{s - k^2} \quad (4)$$

$$\frac{1}{\pi} \text{Im} F_{(\varepsilon)}^{\text{OPE}}(s, q^2) = f_B m_B \left[\left(\frac{d\sigma}{ds} \right) \frac{\phi_+^B(\sigma m_B)}{(1-\sigma)} \right]_{\sigma(s)} + \dots \quad (5)$$

Hadronic dispersion relation

$$F_{(\varepsilon)}(k^2, q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im} F_{(\varepsilon)}(s, q^2)}{s - k^2} \quad (6)$$

$$\begin{aligned} 2 \text{Im} F_{\mu\nu}(k, q) &= \int d\tau_{2\pi} \langle 0 | \bar{d} \gamma_\mu u | \pi^+(k_1) \pi^0(k_2) \rangle \\ &\quad \langle \pi^+(k_1) \pi^0(k_2) | \bar{u} \gamma_\nu (1 - \gamma_5) b | \bar{B}^0(q + k) \rangle + \dots \end{aligned} \quad (7)$$

- Inserting the complete set of state with quantum number of the $\bar{d} \gamma_\mu u$ current (**unitarity relation**).
- Ellipsis : interpolation with other states with higher thresholds,
 4π , $K\bar{K}$, etc

Hadronic dispersion relation

$$\langle \pi^+(k_1) \pi^0(k_2) | \bar{u} \gamma_\mu d | 0 \rangle = -\sqrt{2}(k_1 - k_2)_\mu F_\pi(k^2), \quad (8)$$

$$\begin{aligned} i \langle \pi^+(k_1) \pi^0(k_2) | \bar{u} \gamma_\nu (1 - \gamma_5) b | \bar{B}^0(p) \rangle \\ = F_\perp(k^2, q^2, q \cdot \bar{k}) \frac{2}{\sqrt{k^2} \sqrt{\lambda}} i \epsilon_{\nu \alpha \beta \gamma} q^\alpha k^\beta \bar{k}^\gamma + \dots \end{aligned} \quad (9)$$

- $k = k_1 + k_2$, $\bar{k} = k_1 - k_2$; Normalization $F_\pi^{\text{em}}(0) = 1$;
In the isospin symmetry limit $F_\pi(k^2) = F_\pi^{\text{em}}(k^2)$,
- Källén function:
 $\lambda \equiv \lambda(m_B^2, q^2, k^2) = m_B^4 + q^4 + k^4 - 2(m_B^2 q^2 + m_B^2 k^2 + q^2 k^2)$
- $q \cdot \bar{k} = \frac{1}{2}\sqrt{\lambda} \beta_\pi(k^2) \cos \theta_\pi$ with $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$,
- θ_π : the angle between the 3-momenta of the neutral pion and the B -meson in the dipion rest frame,
- Ellipsis: other FFs with different kinematic definition.

Hadronic dispersion relation

Legendre expansion of $F_i(k^2, q^2, q \cdot \bar{k}), i = \perp, \parallel, t, 0$

$$\begin{aligned} F_{0,t}(k^2, q^2, q \cdot \bar{k}) &= \sqrt{3} F_{0,t}^{(\ell=1)}(k^2, q^2) P_1^{(0)}(\cos \theta_\pi) + \dots, \\ F_{\perp,\parallel}(k^2, q^2, q \cdot \bar{k}) &= \sqrt{3} F_{\perp,\parallel}^{(\ell=1)}(k^2, q^2) \frac{P_1^{(1)}(\cos \theta_\pi)}{\sin \theta_\pi} + \dots, \end{aligned} \quad (10)$$

- (Associated) Legendre polynomials:
 $P_1^{(0)}(\cos \theta_\pi) = \cos \theta_\pi$ and $P_1^{(1)}(\cos \theta_\pi) = -\sin \theta_\pi$.
- Only the *P*-wave components survive \iff *P*-wave projector from F_π only with $J^P = 1^-$ contribution,.
- Different interpolated currents lead to different partial wave contributions.

Hadronic dispersion relation

The imaginary part

$$\frac{1}{\pi} \operatorname{Im} F_{(\varepsilon)}(s, q^2) = \frac{\sqrt{s} [\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}} F_\pi^*(s) F_\perp^{(\ell=1)}(s, q^2) + \dots \quad (11)$$

- We are studying on pion form factors at low $s \lesssim 1.0 - 1.5 \text{ GeV}^2$, higher threshold states ($4\pi, K\bar{K}$, etc) contributions are expected to be suppressed.
- Borel-transforming Eq.(6) (replacing k^2 by M^2), Employing quark-hadron duality (semi-local) approximation:

$$\int_{s_0^{2\pi}}^{\infty} ds e^{-s/M^2} \operatorname{Im} F_{(\varepsilon)}(s, q^2) = \int_{s_0^{2\pi}}^{\infty} ds e^{-s/M^2} \operatorname{Im} F_{(\varepsilon)}^{\text{OPE}}(s, q^2) \quad (12)$$

B meson LCSR for $F_\perp(s, q^2)$

$$\begin{aligned}
 & f_B m_B \left[\int_0^{\sigma_0^{2\pi}} d\sigma e^{-s(\sigma, q^2)/M^2} \frac{\phi_+^B(\sigma m_B)}{\bar{\sigma}} + m_B \Delta V^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right] \\
 &= \int_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \frac{\sqrt{s} [\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}} F_\pi^\star(s) F_\perp^{(\ell=1)}(s, q^2) , \tag{13}
 \end{aligned}$$

- $\sigma_0^{2\pi} m_B^2 - \sigma_0^{2\pi} q^2 / \sigma_0^{\bar{2}\pi} \equiv s_0^{2\pi}$; ΔV^{BV} : three-particle DA contribution
- Similar for other invariant amplitudes $F_{(\parallel, 0)}(s, q^2)$ deduced by the Axial-Vector weak current.
- But for timelike-helicity FF $F_{(t)}(s, q^2)$, we should consider a new correlation function with PS heavy-light current $im_b \bar{u}(0) \gamma_5 b(0) \dots$

B meson LCSR^ss for $F_{\perp}(s, q^2)$

Comments on Eq.(13)

- The **s dependence** in $F_{\perp}^{(\ell=1)}(s, q^2)$ is convoluted with $F_{\pi}^*(s)$,
- **Reality condition:**

$$\text{Im}[\text{l.h.s of Eq.(13)}] = 0. \quad (14)$$

- ◆ Constraints $B \rightarrow \pi\pi$ FFs in a certain **ansatz and/or (resonance) model**;
- ◆ Resonance model for $F_{\perp}^{(\ell=1)}$ should **recover the $B \rightarrow \rho$ sum rules**
- ◆ B meson LCDAs and timelike pion vector FF are the main inputs;
- ◆ Check the validity and consistency of this new sum rules with $B \rightarrow \pi\pi$ calculated via an alternative method;

Resonance ansatz for $F_{\perp}(s, q^2)$

$$F_{\perp}^{(\ell=1)}(s, q^2) = \frac{\sqrt{s}\sqrt{\lambda}}{\sqrt{3}} \sum_{R=\rho, \rho', \rho''} \frac{g_{R\pi\pi} V^{B \rightarrow R}(q^2) e^{i\phi_R(s)}}{(m_B + m_R)[m_R^2 - s - i\sqrt{s}\Gamma_R(s)]}, \quad (15)$$

- ◆ $\langle \pi^+(k_1)\pi^0(k_2)|R\rangle = g_{R\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha;$
- ◆ $\langle R^+(k)|\bar{u}\gamma_\nu b|\bar{B}^0(p)\rangle = \epsilon_{\nu\alpha\beta\gamma} \epsilon^{*\alpha} q^\beta k^\gamma \frac{2V^{B \rightarrow R}(q^2)}{m_B + m_R};$
- ◆ $\phi_R(s)$ is the phase:
- tacitly assumed to be same for all $F_{\perp,\parallel,t,0}^{(\ell=1)}(s, q^2)$;
- simplest way (holds for each R): s -dependent but q^2 -independent:

$$\phi_R(s, q^2) \rightarrow \phi_R(s) = -\text{Arg} \left[\frac{F_\pi^*(s)}{m_R^2 - s - i\sqrt{s}\Gamma_R(s)} \right]. \quad (16)$$

Resonance ansatz for $F_\perp(s, q^2)$, go to narrow ρ appr.

Adopting $\rho \rightarrow \pi\pi$ width:

$$\begin{aligned}\Gamma_\rho(s) &= \frac{g_{\rho\pi\pi}^2 [\beta_\pi(s)]^3 \sqrt{s}}{48\pi} \theta(s - 4m_\pi^2) = \Gamma_\rho^{\text{tot}} \left[\frac{\beta_\pi(s)}{\beta_\pi(m_\rho^2)} \right]^3 \frac{\sqrt{s}}{m_\rho} \theta(s - 4m_\pi^2), \\ F_\pi^\star(s) &= \frac{f_\rho g_{\rho\pi\pi} m_\rho}{\sqrt{2}(m_\rho^2 - s + i\sqrt{s}\Gamma_\rho(s))},\end{aligned}\quad (18)$$

R.H.S of Eq.(13) \Rightarrow

$$\begin{aligned}&\frac{2f_\rho m_\rho V^{B \rightarrow \rho}(q^2)}{(m_B + m_\rho)} \int_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \left(\frac{1}{\pi} \frac{\Gamma_\rho(s)\sqrt{s}}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)} \right) \\ &\xrightarrow{\Gamma_\rho^{\text{tot}} \rightarrow 0} \frac{2f_\rho m_\rho V^{B \rightarrow \rho}(q^2)}{(m_B + m_\rho)} e^{-m_\rho^2/M^2}\end{aligned}\quad (19)$$

which is exactly the formula for $B \rightarrow \rho$ FFs.

Determining of LCSR parameters: M^2 , $s_0^{2\pi}$

Borel parameter $M^2 = 1.25 \pm 0.25 \text{ GeV}^2$:

[P. Colangelo and A. Khodjamirian, arXiv:hep-ph/0010175]

- Convergence(30%) of OPE is manifested by the relatively small three-parton DA contribution (power counting);
- Higher spectral density contribution does not exceed 40% of the total integral, (quark-hadron duality).

Cut threshold $s_0^{2\pi} = 1.5 \pm 0.1 \text{ GeV}^2$

- A separate investigation with using vector $F_\pi(s)$ data from Belle
[The Belle Collaboration, Phys.Rev.D78(2008)072006; Phys.Rev.D93(2016)032003,]
- Employing 2pSRs (SVZ) for the isospin-1 light-quark vector currents,
[M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl.Phys.B147(1979)385]
 $\rho \rightarrow \pi\pi$, substitute $F_\pi(s)$ in the hadronic part.

Numerically for $F_\perp(s, q^2)$ with B DAs

Our sum rules in Eq.(13), in a compact version

$$\begin{aligned} I_V^{\text{OPE}}(q^2, M^2, s_0^{2\pi}) &= \sum_R X_V^R V^R(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \left[F_\pi^\star(s) D^R(s) e^{i\phi_R(s)} \right] \\ &\equiv \sum_R X_V^R V^R(q^2) \mathcal{I}^R(M^2, s_0^{2\pi}), \end{aligned} \quad (20)$$

- $\mathcal{I}_V^{\text{OPE}}(q^2, M^2, s_0^{2\pi})$ is the L.H.S,
- $X_V^R = 1/(m_B + m_R)$,
 $V^R(q^2)$ are parameterized in the standard z -series formula.

[A. Khodjamirian, T. Mannel, A.A. Pivovarov and Y.M. Wang, JHEP 09 (2010) 089]

- Notice the large hierarchy of magnitude $|\mathcal{I}^R(1.0, 1.5)|$

$$\text{Abs}\{\mathcal{I}^\rho, \mathcal{I}^{\rho'}, \mathcal{I}^{\rho''}\} \cdot 10^2 = \{2.6, 0.43, 0.25\}, \quad (21)$$

will generate large uncertainties for FF V fitting.

- Another ansatz to fix the relative weight of different R to $V^R(q^2)$.

Numerically with B DAs

Model-0: single- ρ with width

	$V^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$A_0^{B\rho}(0)$
narrow- ρ	0.34	0.26	0.21	0.30
$F_\pi^{(\rho)}$	0.36 ± 0.17	0.27 ± 0.13	0.22 ± 0.15	0.30 ± 0.06
F_π	0.41 ± 0.11	0.31 ± 0.08	0.25 ± 0.10	0.34 ± 0.04
ρ -DAs	0.33 ± 0.03	0.26 ± 0.03	0.23 ± 0.04	0.36 ± 0.04

- The finite width of ρ does not impact the $B \rightarrow \rho$ form factors significant, but the higher resonances in F_π does (15% – 20%).

Numerically with B DAs

Model-I: assess ρ' contribution

- Assume the $B \rightarrow \rho$ FFs are well determined from the LCSR with ρ -meson DAs, $g_{\rho\pi\pi} = 5.94 \leftarrow \text{Eq.(17)}$

R	$g_{R\pi\pi} V^{BR}(0)$	$g_{R\pi\pi} A_1^{BR}(0)$	$g_{R\pi\pi} A_2^{BR}(0)$	$g_{R\pi\pi} A_0^{BR}(0)$,
ρ	2.0 ± 0.2	1.6 ± 0.2	1.4 ± 0.2	2.1 ± 0.2
ρ'	3.0 ± 2.5	1.5 ± 1.4	1.0 ± 2.2	-0.3 ± 0.4

- Large uncertainties generate because of the suppressed sensitivity in the ρ' region, a quite appreciable ρ' contribution.

Numerically with B DAs

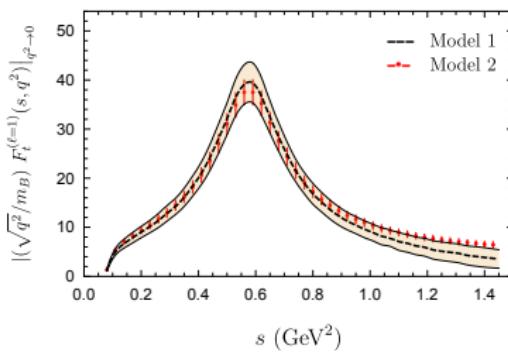
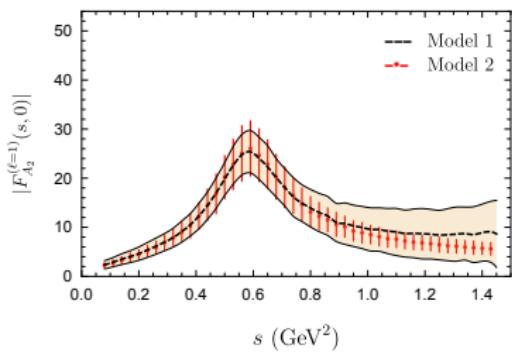
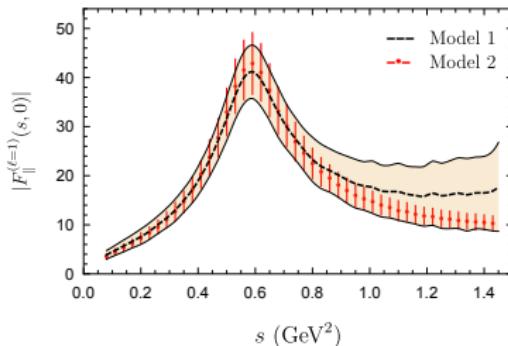
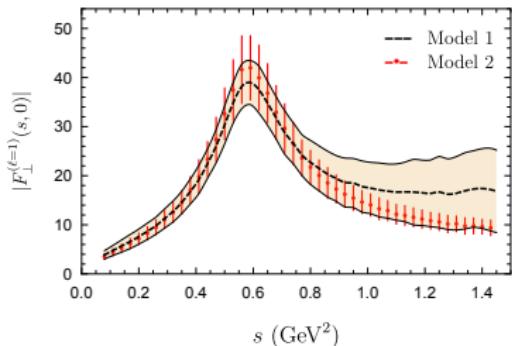
Model-II: $\rho + \rho' + \rho''$

- Assume the relative size of contributions from R is the same as in F_π

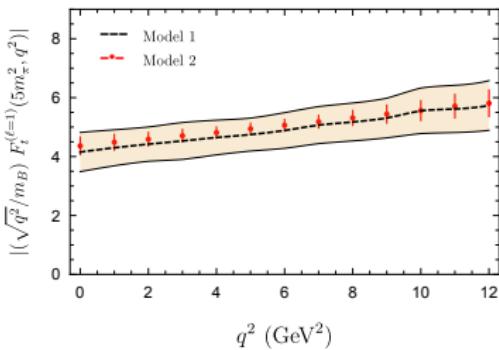
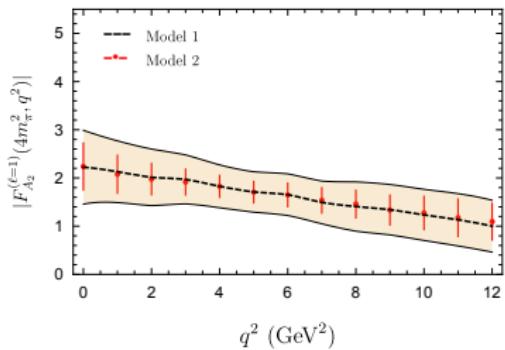
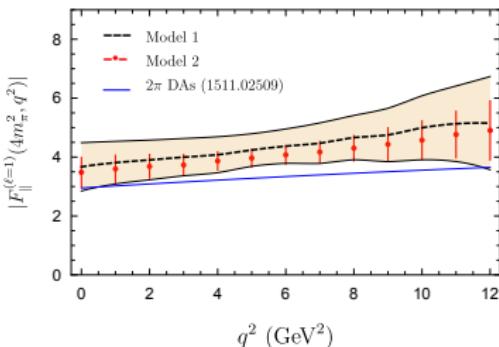
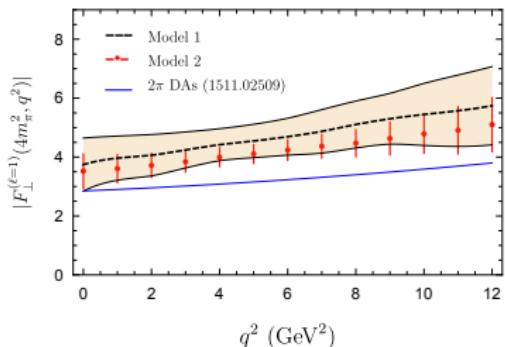
R	$g_{R\pi\pi} V^{BR}(0)$	$g_{R\pi\pi} A_1^{BR}(0)$	$g_{R\pi\pi} A_2^{BR}(0)$	$g_{R\pi\pi} A_0^{BR}(0)$
ρ	2.4 ± 0.4	1.8 ± 0.3	1.5 ± 0.3	1.9 ± 0.1
ρ'	0.35 ± 0.06	0.27 ± 0.04	0.22 ± 0.05	0.29 ± 0.02
ρ''	0.09 ± 0.01	0.07 ± 0.01	0.05 ± 0.01	0.07 ± 0.01

- Uncertainties decrease, ρ, ρ', ρ'' contributions are in the strong hierarchy .

Numerically with B DAs (s -dependence)



Numerically with B DAs (q^2 -dependence)



Numerically with B DAs (Intermediate conclusion)

- ▶ At large recoil region $q^2 = 0$, evolution on dipion mass k^2 shape peaks around ρ mass.
- ▶ At low dipion mass $k^2 = 4m_\pi^2/0.1\text{GeV}^2$, evolutions of FFs on q^2 are gentle with the acceptable uncertainties.
- ▶ Radially excited resonant states in our model contribute $\sim 20\%$.
- ▶ We can also fit out $B \rightarrow \rho', \rho''$ FFs if we know exactly how large of the couplings $g_{\rho'\pi\pi}, g_{\rho''\pi\pi}$.
- ▶ An optimal choice of the relative contributions from different resonances (numerics technology) should be found out.

With dipion DAs

With dipion DAs

Correlation Function: [C. Hambrock and A. Khodjamirian, Nucl.Phys.B905(2016)373]

$$\begin{aligned} F_\mu(k_1, k_2, q) &= i \int d^4x e^{iq \cdot x} \langle \pi^+(k_1) \pi^0(k_2) | T\{ j_\mu^{V-A}(x), j_5(0) \} | 0 \rangle \\ &\downarrow \quad \text{Lorentz decomposition} \\ &\equiv \varepsilon_{\mu\nu\rho\sigma} q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}, \end{aligned} \quad (22)$$

- $j_\mu^{V-A}(x) \equiv \bar{u}(x) \gamma_\mu (1 - \gamma_5) b(x), \quad j_5(0) \equiv i m_b \bar{b}(0) \gamma_5 d(0);$
- Kinematics: $k = k_1 + k_2, \quad \bar{k} = k_1 - k_2, \quad p = k + q$
Four independent invariant variables: $p^2, q^2, k^2, q \cdot k;$
- $p^2, q^2 \ll m_b^2$, to guarantee the validity of OPE near the LC ($x^2 \sim 0$),
- $k^2 \lesssim 1 \text{ GeV}^2 \ll m_b^2$, to avoid generic $\mathcal{O}(k^2 x^2)$ terms in LC expansion.

OPE Calculation with dipion DAs

Dipion DAs, [M.V. Polyakov, Nucl.Phys.B555(1999)231]

$$\begin{aligned} & \langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x, 0]d(0)|0\rangle \\ = & -\sqrt{2}k_\mu \int_0^1 du e^{iu(k\cdot x)} \Phi_{\parallel}^{I=1}(u, \zeta, k^2), \end{aligned} \quad (23)$$

$$\begin{aligned} & \langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x, 0]d(0)|0\rangle \\ = & 2\sqrt{2} \frac{k_{1\mu}k_{2\nu} - k_{1\nu}k_{2\mu}}{2\zeta - 1} \int_0^1 du e^{iu(k\cdot x)} \Phi_{\perp}^{I=1}(u, \zeta, k^2), \end{aligned} \quad (24)$$

- Chiral-even and -odd LC expansion respectively with gauge factor $[x, 0]$.
- u quark carries longitudinal momentum fraction,
 $2q \cdot \vec{k}$ determines the LC momentum distribution carried by two pions.
- Normalization condition:
 $\int_0^1 \Phi_{\parallel}^{I=1}(u, \zeta, k^2) = (2\zeta - 1)F_{\pi}^{em}(k^2)$, $\int_0^1 \Phi_{\perp}^{I=1}(u, \zeta, k^2) = (2\zeta - 1)F_{\pi}^t(k^2)$.
 $F_{\pi}^{em}(0) = 1$ and $F_{\pi}^t(0) = 1/f_{2\pi}^{\perp}$.
- Higher twist proportional to $1/\gamma_\mu\gamma_5$ are neglected here,
 γ_5 vanishes because of P -parity conservation.

LO and twist-2 appro.

- Dipion DAs are presented in **double expansion** of Legendre and Gegenbauer polynomials: $C_l^{1/2}(2\zeta - 1)$ & $C_n^{3/2}(2u - 1)$;
Partial wave & eigenfunction of evolution equation:

$$\Phi_{\perp/\parallel}(u, \zeta, k^2) = \frac{6u(1-u)}{f_{2\pi}^{\perp}/1} \sum_{n=0,2,\dots}^{\infty} \sum_{l=1,3,\dots}^{n+1} B_{nl}^{\perp/\parallel}(k^2) C_n^{3/2}(2u-1) C_l^{1/2}(2\zeta-1),$$

$$C_l^{1/2}(2\zeta - 1) = \beta_\pi P_l^{(0)}\left(\frac{2\zeta - 1}{\beta_\pi}\right), \quad (25)$$

- $B_{nl}^{\perp/\parallel}(k^2)$: renormalizable coefficients, $B_{01}^{\perp/\parallel}(0) = 1$, $B_{01}^{\parallel}(k^2) = F_\pi^{em}(k^2)$.
- $n \geq 2$ at low k^2 determine the non-asymptotic part of DAs, logarithmically decreasing at large scale.
- With truncating at a given n_{max} , l is restricted to $n_{max} + 1$.
- unitarity relation, $B_{nl}^{\perp}(k^2)$ are complex functions at $k^2 > 4m_\pi^2$.

OPE Calculation with dipion DAs

At twist-2 accuracy

- Rewriting in a generic dispersion form in $p^2 = p^2 - s + s \rightarrow s$.
- $s = s(u) = (m_b^2 - \bar{u}q^2 + u\bar{u}k^2)/u$.
- $F^{(A,k)}(s, q^2, k^2, \zeta)$:
 $q \cdot \bar{k}$ generate a cut at the real axis to avoid imaginary part,
 $(\sqrt{q^2} - \sqrt{k^2})^2 < p^2 < (\sqrt{q^2} + \sqrt{k^2})^2$,
non physical intermediate state, a typical kinematic singularity.
- After Borel trans., this cut contribution to the dispersion integral is enhanced with respecting to the b -quark spectral.
- We need a new correlator to touch this amplitude:
pseudoscalar $b \rightarrow u$ transition current ($j_\mu^{V-A} \rightarrow j_5$).
- Multiplying q_μ on Eq.(9), solid at leading power precision.

Hadronic dispersion relation with dipion DAs

$$F_\mu(q, k_1, k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2 - p^2} + \dots, \quad (26)$$

$$\Pi_5(p^2, q^2, k^2, \zeta) = \frac{\sqrt{q^2} F_t(q^2, k^2, \zeta) f_B m_B^2}{m_B^2 - p^2} + \dots, \quad (27)$$

results at leading twist:

$$\frac{F_\perp(q^2, k^2, \zeta)}{\sqrt{k^2} \sqrt{\lambda_B}} = -\frac{m_b}{\sqrt{2} f_B m_B^2 (2\zeta - 1)} \int_{u_0}^1 \frac{du}{u} e^{\frac{-s+m_B^2}{M^2}} \Phi_\perp(u, \zeta, k^2), \quad (28)$$

$$\frac{F_\parallel(q^2, k^2, \zeta)}{\sqrt{k^2}} = -\frac{m_b}{\sqrt{2} f_B m_B^2 (2\zeta - 1)} \int_{u_0}^1 \frac{du}{u^2} e^{\frac{-s+m_B^2}{M^2}} (m_b^2 - q^2 + u^2 k^2) \Phi_\perp(u, \zeta, k^2), \quad (29)$$

$$\sqrt{q^2} F_t(q^2, k^2, \zeta) = -\frac{m_b^2}{\sqrt{2} f_B m_B^2} \int_{u_0}^1 \frac{du}{u^2} e^{\frac{-s+m_B^2}{M^2}} (m_b^2 - q^2 + k^2 u^2) \Phi_\parallel^{l=1}(u, \zeta, k^2), \quad (30)$$

$$\sqrt{q^2} F_0(q^2, k^2, \zeta) = \frac{1}{m_B^2 - q^2 - k^2} [\sqrt{\lambda_B} \sqrt{q^2} F_t(q^2, k^2, \zeta) + 2\sqrt{k^2} q^2 (2\zeta - 1) F_\parallel(q^2, k^2, \zeta)]. \quad (31)$$

- $\langle \bar{B}^0(p)|\bar{b}im_b\gamma_5 d|0\rangle = f_B m_B^2$, $2\zeta - 1 = \beta_\pi \cos[\theta_\pi]$.

Dipion LCSR for $F_{\perp,\parallel}(s, q^2)$:

[C. Hambrock and A. Khodjamirian, Nucl.Phys.B905(2016)373]

[S. Cheng, A. Khodjamirian and J. Virto, Phys.Rev.D96(2017)051901(R)]

LO and twist-2 appro.

$$F_{\perp}^{(I)} = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^{\perp}} \frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B} e^{\frac{m_B^2}{M^2}} \sum_{n=0,2,\dots} \sum_{l'=1,3}^{n+1} I_{ll'} B_{nl'}^{\perp}(k^2) J_n^{\perp}(q^2, k^2, M^2, s_0^B), \quad (32)$$

$$F_{\parallel}^{(I)} = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^{\perp}} \frac{m_b^3}{m_B^2 f_B} e^{\frac{m_B^2}{M^2}} \sum_{n=0,2,\dots} \sum_{l'=1,3}^{n+1} I_{ll'} B_{nl'}^{\parallel}(k^2) J_n^{\parallel}(q^2, k^2, M^2, s_0^B), \quad (33)$$

$$\begin{aligned} \sqrt{q^2} F_t^{(I)}(k^2, q^2) &= -\frac{6m_b^2}{\sqrt{2}f_B m_B^2} \frac{\beta_\pi(k^2)}{\sqrt{2l+1}} \exp\left(\frac{m_B^2 - s}{M^2}\right) \\ &\times \sum_{n=l-1, l+1, \dots}^{\infty} B_{nl}^{\parallel}(k^2) \int_{u_0}^1 \frac{du}{u} \bar{u} (m_b^2 - q^2 + u^2 k^2) C_n^{3/2}(u - \bar{u}), \end{aligned} \quad (34)$$

$$I_{ll'} \equiv -\frac{\sqrt{2l+1}(l-1!)}{2(l+1)!} \int_{-1}^1 \frac{dz}{z} \sqrt{1-z^2} P_l^{(1)}(z) P_{l'}^{(0)}(z), \quad (35)$$

$$J_n^{\perp}(q^2, k^2, M^2, s_0^B) = \int_{u_0}^1 du e^{\frac{-s}{M^2}} 6(1-u) C_n^{3/2}(2u-1), \quad (36)$$

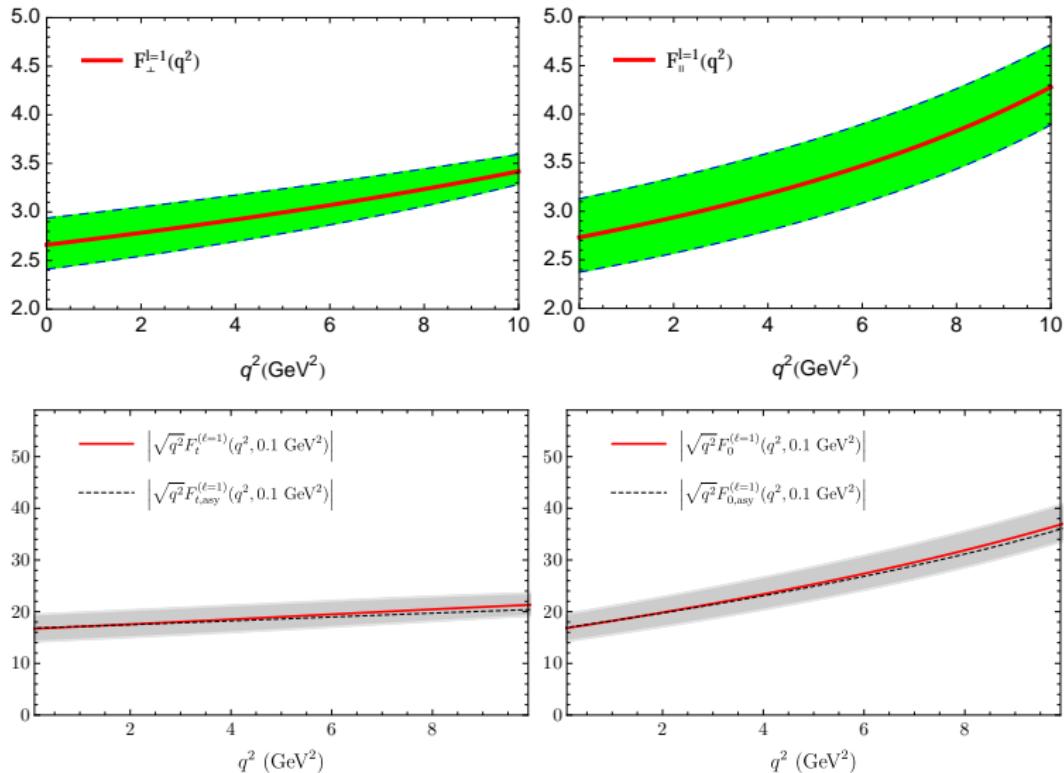
$$J_n^{\parallel}(q^2, k^2, M^2, s_0^B) = \int_{u_0}^1 du e^{\frac{-s}{M^2}} 6(1-u) C_n^{3/2}(2u-1) \left(1 - \frac{q^2 - u^2 k^2}{m_b^2}\right), \quad (37)$$

Dipion LCSR for $F_{\perp,\parallel}(s, q^2)$:

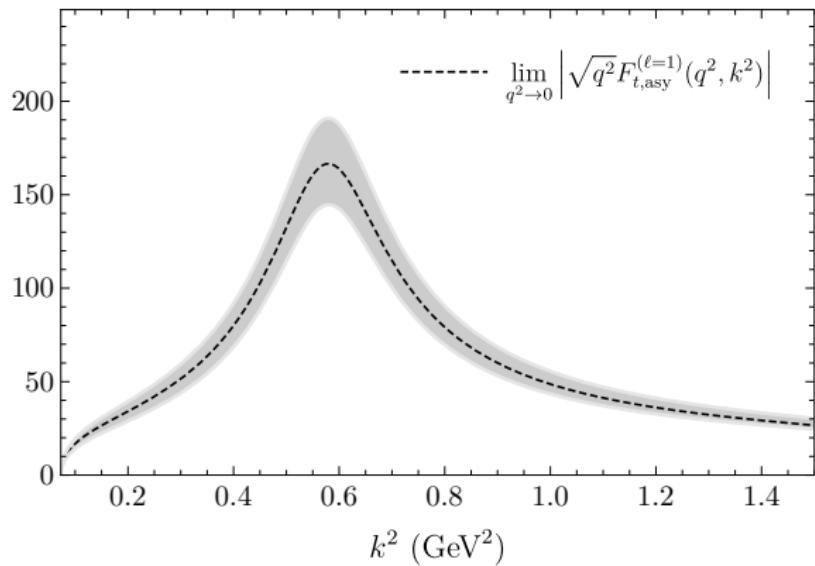
Input paras

- $I_{ll'} = 0$ when $l > l'$, $I_{11} = 1/\sqrt{3}$, $I_{13} = -1/\sqrt{3}$, $I_{15} = 4/(5\sqrt{3})$;
 $l = 1$, asymptotic DAs, partial P -wave term remains in the FFs.
- The complexity of $B_{nl}^\perp(k^2)$, only predictable at low $k^2 = 4m_\pi^2$ within the instanton model of QCD vacuum, no phase.
- Short-distance part of the correlator: renormalization scale $\mu = 3 \text{ GeV}$ without NLO gluon radiative correction.
- Two-point QCD sum rules' prediction for $f_B = 207_{-17}^{+9} \text{ MeV}$.
- $M^2 = 16.0 \pm 4.0 \text{ GeV}^2$ corresponding to $s_0^B = 37.5 \pm 2.5 \text{ GeV}^2$.
- ◆ How large of P -wave contribution to $B \rightarrow \pi\pi$ FFs (higher partial wave) ?
- ◆ How much ρ contained in P -wave $B \rightarrow \pi\pi$ FFs ? Resonance model.
- ◆ ρ meson: $a_2^\perp = 0.2 \pm 0.1$, $a_{n>2} = 0$, $f_\rho^\perp = 160 \pm 10 \text{ MeV}$.

Numerically with dipion DAs $F_{\perp,\parallel}(q^2, k^2 = 4m_\pi^2/0.1\text{GeV}^2)$



Numerically with dipion DAs $F_{t,0}(q^2 = 0, k^2)$



Numerically with dipion DAs (Intermediate conclusion)

- ▶ At LO and twist-2 approximation.
- ▶ $F_{\perp,\parallel}(q^2, k^2)$:
At $k^2 = 4m_\pi^2$, $F_{\perp,\parallel}(q^2)$ has a smooth evolution;
high partial waves give tiny contribution $\sim 2\%$.
- ▶ $\sqrt{q^2}F_{t,0}(q^2, k^2)$:
At $k^2 = 0.1\text{GeV}^2$, $\sqrt{q^2}F_{t,0}(q^2)$ is one order larger than $F_{\perp,\parallel}(q^2)$;
high partial waves $< 10\%$.
At $q^2 = 0\text{GeV}^2$, $\sqrt{q^2}F_t(k^2) = \sqrt{q^2}F_0(k^2)$;
 $\sim 10\%$ contribution from ρ' , ρ'' and NR background;
high partial wave $< 10\%$.

Conclusion

Comparison of LCSR with B DAs and dipion DAs

- ▶ They give similar plots for $B \rightarrow \pi^+ \pi^0$ FFs, at the same order .
- ▶ They both predict sizable contribution (10% order) from ρ', ρ'', \dots and/or NR background for P -wave $B \rightarrow \pi^+ \pi^0$ FFs.
- ▶ B meson LCSR does not suggest any higher partial wave contribution.
- ▶ For LCSR with dipion DAs, the higher partial wave contributions exist but tiny.
- ▶ B meson LCSR is more powerful: k^2, q^2, ϕ .
- ▶ We should go further for the dipion DAs: sub-leading twist, large dipion invariant mass, strong phase.

Prospect

Prospect

- ▶ Further improvements for this approach:
 - Improving the accuracy of λ_B .
 - Gathering more data on time-like pion FF: \mathcal{A} & ϕ .
 - Forwarding to the NLO correction on the OPE side.
- ▶ dipion DAs at subleading twist, evolution on invariant mass
- ▶ Considering the iso-scalar $\pi\pi$ system (f_0)
- ▶ Calculating $B \rightarrow$ Scalar FFs in B meson LCSR s .
- ▶ Extending the approach to $B_{(s)} \rightarrow K\pi$ FFs.

The End, Thanks.

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The pion timelike form factor in

$$F_\pi(s) = \frac{BW_\rho^{GS}(s) + |\beta|e^{i\phi_\beta} BW_{\rho'}^{GS}(s) + |\gamma|e^{i\phi_\gamma} BW_{\rho''}^{GS}(s)}{1 + |\beta|e^{i\phi_\beta} + |\gamma|e^{i\phi_\gamma}}, \quad (38)$$

$$BW_R^{GS}(s) = \frac{m_R^2 + m_R \Gamma_R d}{m_R^2 - s + f(s) - i\sqrt{s} \Gamma_R(s)}, \quad (39)$$

Parameters measured in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

Resonance	m_R (MeV)	Γ_R (MeV)	Weight factor
ρ	$774.6 \pm 0.2 \pm 0.5$	$148.1 \pm 0.4 \pm 1.7$	1.0
ρ'	$1446 \pm 7 \pm 28$	$434 \pm 16 \pm 60$	$ \beta = 0.15 \pm 0.05^{+0.15}_{-0.04}$ $\phi_\beta = 202 \pm 4^{+41}_{-8}$
ρ''	$1728 \pm 17 \pm 89$	$164 \pm 21^{+80}_{-26}$	$ \gamma = 0.037 \pm 0.006^{+0.065}_{-0.009}$ $\phi_\gamma = 24 \pm 9^{+118}_{-28}$

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z -parameterization for $B \rightarrow R$ decay FFs

$$\begin{aligned}\mathcal{F}^{B \rightarrow R}(q^2) &= \frac{\mathcal{F}^{B \rightarrow R}(0)}{1 - q^2/m_{\mathcal{F}}^2} \left\{ 1 + b_{\mathcal{F}}^R \left[z(q^2) - z(0) + \frac{1}{2}(z(q^2)^2 - z(0)^2) \right] + \dots \right\} \\ &\equiv \frac{\kappa_{\mathcal{F}}^R + \eta_{\mathcal{F}}^R \zeta_R(q^2)}{1 - q^2/m_{\mathcal{F}}^2}, \quad \mathcal{F} = V, A_0, A_1, A_2,\end{aligned}\tag{40}$$

(41)

$$\begin{aligned}z(q^2) &= \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \\ t_{\pm} &\equiv (m_B \pm m_R)^2, \quad t_0 = t_+(1 - \sqrt{1 - t_-/t_+}), \\ \kappa_{\mathcal{F}}^R &\equiv g_{R\pi\pi} \mathcal{F}^{B \rightarrow R}(0), \quad \eta_{\mathcal{F}}^R \equiv g_{R\pi\pi} \mathcal{F}^{B \rightarrow R}(0) b_V^R,\end{aligned}\tag{42}$$

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Orthogonality of Legendre Polynomials

$$\int_{-1}^1 dx P_l^n(x) P_k^n(x) = \frac{2}{2l+1} \frac{(l+n)!}{(l-n)!} \delta_{kl}, \quad (43)$$

$$\int_{-1}^1 dx \frac{P_l^m(x) P_l^n(x)}{1-x^2} = \frac{(l+m)!}{m(l-m)!} \delta_{mn}, \quad m = n \neq 0, \quad (44)$$

$$P_0^0(x) = 1, \quad P_0^1(x) = x, \quad P_1^1(x) = \sqrt{1-x^2}, \quad \dots \quad (45)$$

Poisson Kernel

$$\eta_y(x) = \frac{1}{\pi} \frac{y}{x^2 + y^2} = \int_{-\infty}^{\infty} d\zeta e^{2\pi i \zeta x - |y\zeta|} \quad (46)$$

$$\int_{-\infty}^{\infty} dx e^{-2\pi i (\zeta_1 - \zeta_2)x} = \delta(\zeta_1 - \zeta_2), \quad \int_{-\infty}^{\infty} dx e^{-2\pi i \zeta x} = 1, \quad (47)$$