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Meson Form Factors in Chiral Perturbation Theory

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Outline

- Motivation

- Chiral Perturbation theory and its unitarization

  ChPT Lagrangian.

  Scattering amplitude at LO/NLO.

  Isospin decomposition of scattering amplitude.

  Unitarized scattering amplitudes.

- Meson Form Factors

  Scalar, Vector and Tensor Currents in Meson ChPT.

  Unitarized form factors.

  Dispersion relation improvement.

Motivation

- In heavy quark physics, we need to evaluate the branching fractions of FCNC transitions, which is sensitive to the potential NP.

- The $b \to s l^+ l^-$ process can be used to test SM. For example:

$$B \to K^*(\to K\pi)\mu^+\mu^-$$

- The hadron transformation matrix element is parameterized as:

$$\begin{aligned}\langle \phi_1\phi_2|\bar{q}\Gamma q|Hadron\rangle &\sim F^{Hadron\to\phi_1\phi_2}(m^2_{\phi_1\phi_2}, q^2)\\ &\sim F_{\phi_1\phi_2}(m^2_{\phi_1\phi_2})F^{Hadron\to Resonance}(q^2)\end{aligned}$$

Is non-perturbative at low energy scale for QCD but can be calculated by EFT

Can be calculated perturbatively

ChPT Lagrangian

- According to the spontaneous breaking of chiral symmetry, $SU(3)_L \times SU(3)_R \to SU(3)_V$, we can construct the most general, chirally invariant, effective Lagrangian at the lowest order:

$$\mathcal{L}_{ChPT}^{(2)} = \frac{F_0^2}{4} Tr(D_\mu U D^\mu U^\dagger) + \frac{F_0^2}{4} Tr(\chi U^\dagger + U \chi^\dagger)$$

- U is parameterized by the 8 Goldstone fields ϕ_a, which live in the quotient space $SU(3)_L \times SU(3)_R / SU(3)_V$:

$$U = exp(i\frac{\phi}{F_0}) \quad \phi = \sum_{a=1}^{8} \phi_a \lambda_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

Power Counting

- ChPT is essentially a non-renormalizable theory due to the existence of infinitely many terms with arbitrary-high dimensions that satisfying chiral symmetry.

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \ldots$$

- But for high dimension terms, their matrix elements are suppressed by a factor:

$$\left(\frac{p}{\Lambda}\right)^n$$

- *n = number of ∂ and double of quark mass term.* $M_\phi^2 \propto m_q$
- p is a typical small energy scale associated to the NG bosons. Λ is called the chiral symmetry breaking scale which is around 1 GeV.
- We need only finite terms to get an answer accurate to a given order.

Meson-Meson Scattering

- Expand the ChPT Lagrangian to $4-\phi$ order:

$$\mathcal{L}^{(2)}_{ChPT} = \frac{F_0^2}{4} Tr(D_\mu U D^\mu U^\dagger) + \frac{F_0^2}{4} Tr(\chi U^\dagger + U\chi^\dagger)$$

$$U = exp(i\frac{\phi}{F_0}) = 1 + i\frac{\phi}{F_0} + \frac{1}{2!}(i\frac{\phi}{F_0})^2 + \ldots$$

- Tree level scattering amplitudes:

$$\pi^+\pi^- \to \pi^0\pi^0 \quad : \quad \frac{-4m_\pi^2 + (m_\pi^2)_0 + 3s}{3F_0^2}$$

$$K^+\pi^+ \to K^+\pi^+ \quad : \quad \frac{(m_K^2)_0 + (m_\pi^2)_0 - 2s + t + u}{6F_0^2}$$

$$K^+K^- \to K^+K^- \quad : \quad \frac{4m_K^2 + 2(m_K^2)_0 - 3u}{3F_0^2}$$

$$K_0\bar{K}_0 \to K^+K^- \quad : \quad \frac{4m_K^2 + 2(m_K^2)_0 - 3u}{6F_0^2}$$

$$\bar{K}_0\eta \to \bar{K}_0\eta \quad : \quad \frac{6(m_K^2)_0 - 2(m_\pi^2)_0 - 3(s - 2t + u)}{12F_0^2}$$

$$\bar{K}_0\eta \to \bar{K}_0\pi_0 \quad : \quad \frac{2(m_K^2)_0 - 2(m_\pi^2)_0 + 3(s - 2t + u)}{12\sqrt{3}F_0^2}$$

$$\pi_0\eta \to \pi_0\eta \quad : \quad \frac{(m_\pi^2)_0}{3F_0^2}$$

$$\eta\eta \to \eta\eta \quad : \quad \frac{16(m_K^2)_0 - 7(m_\pi^2)_0}{9F_0^2}$$

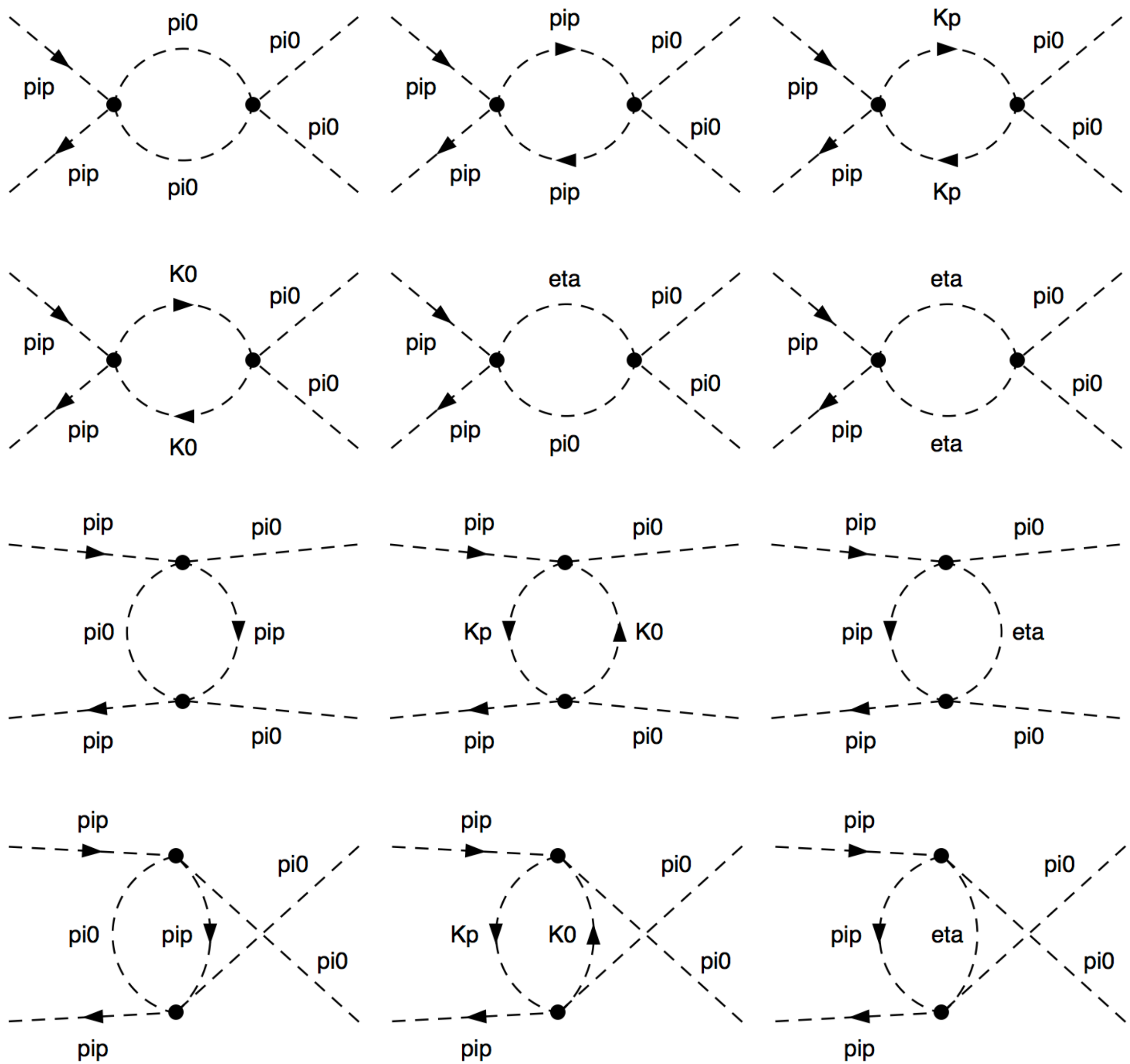
Meson-Meson Scattering

- One loop correction:

For example:

$\pi^+\pi^- \rightarrow \pi^0\pi^0$

With the help of FeynRules, FeynArt and FormCalc



Renormalization Program

- The divergence of loop correction has the form:

$$\frac{\Gamma_i}{32\pi^2}(L+1+c-\ln\frac{m_\phi^2}{\mu^2})$$

$$L=\frac{2}{4-d}-\gamma_E+ln4\pi$$

- In order to subtract the divergence, we need to introduce higher order Lagrangians:

$$\begin{aligned}\mathcal{L}_4 \quad = \quad & L_1\mathrm{Tr}[(D_\mu U)(D_\mu U)^\dagger]^2+L_2\mathrm{Tr}[(D_\mu U)(D_\nu U)^\dagger]\mathrm{Tr}[(D^\mu U)(D^\nu U)^\dagger] \\ & +L_3\mathrm{Tr}[(D_\mu U)(D^\mu U)^\dagger(D_\nu U)(D^\nu U)^\dagger]+L_4\mathrm{Tr}[(D_\mu U)(D^\mu U)^\dagger]\mathrm{Tr}[\chi U^\dagger+U\chi^\dagger] \\ & +L_5\mathrm{Tr}[(D_\mu U)(D^\mu U)^\dagger(\chi U^\dagger+U\chi^\dagger)]+L_6\mathrm{Tr}[\chi U^\dagger+U\chi^\dagger]^2 \\ & +L_7\mathrm{Tr}[\chi U^\dagger-U\chi^\dagger]^2+L_8\mathrm{Tr}[U\chi^\dagger U\chi^\dagger+\chi U^\dagger\chi U^\dagger] \\ & -iL_9\mathrm{Tr}[r_{\mu\nu}(D^\mu U)(D^\nu U)^\dagger+l_{\mu\nu}(D^\mu U)^\dagger(D^\nu U)]\end{aligned}$$

- L_1 ... L_9 are low energy coefficients.

Renormalization Program

- L_i has the form:

$$L_i = L_i^r(\mu) - \frac{\Gamma_i}{32\pi^2}(L+1)$$

- They must satisfy the condition:

$$\langle \mathcal{L}^2_{tree} \rangle + \langle \mathcal{L}^2_{1loop} \rangle + \langle \mathcal{L}^4_{tree} \rangle = finite\ value\ and\ independent\ of\ scale\ \mu$$

- Which means:

$$L_i^r(\mu) + \frac{\Gamma_i}{32\pi^2} ln \frac{\mu^2}{m_\phi^2} = C$$

Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}
$\frac{3}{32}$	$\frac{3}{16}$	0	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{11}{144}$	0	$\frac{5}{48}$	$\frac{1}{4}$	$-\frac{1}{4}$

- The L_i we use:

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	
0.56	1.21	-2.79	-0.36	1.4	0.07	-0.44	0.78	7.1	$\times 10^{-3}$

A. Gomez Nicola and J. R. Pelaez. Meson meson scattering within one loop chiral perturbation theory and its unitarization. *Phys. Rev.*, D65:054009, 2002.

One Particle Eigenstates of Isospin

- Define the SU(2) isospin operator: $\hat{J}_i \equiv \hat{Q}^\dagger \frac{\tau_i}{2} \hat{Q} \quad \hat{Q} = \begin{pmatrix} \hat{u} & \hat{d} \end{pmatrix}^T$

$$\hat{u} = \begin{pmatrix} a_u \\ b_u^\dagger \end{pmatrix} \quad \hat{d} = \begin{pmatrix} a_d \\ b_d^\dagger \end{pmatrix} \quad \hat{J}_+ = \hat{J}_1 + i\hat{J}_2 \qquad \hat{J}_- = \hat{J}_1 - i\hat{J}_2$$

- We found that:

$$\hat{J}_-|\pi^+\rangle = -\sqrt{2}|\pi^0\rangle \qquad \hat{J}_-|\pi^0\rangle = \sqrt{2}|\pi^-\rangle$$

- So we can conclude:

$$\begin{aligned} |\pi, 1, +1\rangle &= -|\pi_+\rangle \\ |\pi, 1, 0\rangle &= |\pi_0\rangle \\ |\pi, 1, -1\rangle &= |\pi_-\rangle \end{aligned}$$

- Similarily:

$$\begin{aligned} \left|K, \frac{1}{2}, +\frac{1}{2}\right\rangle &= |K_+\rangle & \left|\bar{K}, \frac{1}{2}, +\frac{1}{2}\right\rangle &= |\bar{K}_0\rangle \\ \left|K, \frac{1}{2}, -\frac{1}{2}\right\rangle &= |K_0\rangle & \left|\bar{K}, \frac{1}{2}, -\frac{1}{2}\right\rangle &= -|K_-\rangle \end{aligned} \qquad |\eta, 0, 0\rangle = |\eta\rangle$$

Isospin Decomposition Of The Scattering Amplitude

- Two particle eigenstates of isospin:

$$
\begin{aligned}
\left|K^+\pi^+\right\rangle &= -\left|K,\frac{1}{2},\frac{1}{2}\right\rangle|\pi,1,1\rangle \\
&= -\left|K\pi,\frac{3}{2},\frac{3}{2}\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
\left|K^+\pi^-\right\rangle &= \left|K,\frac{1}{2},\frac{1}{2}\right\rangle|\pi,1,-1\rangle \\
&= \sqrt{\frac{1}{3}}\left|K\pi,\frac{3}{2},-\frac{1}{2}\right\rangle+\sqrt{\frac{2}{3}}\left|K\pi,\frac{1}{2},-\frac{1}{2}\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
\left|K^0\pi^0\right\rangle &= \left|K,\frac{1}{2},-\frac{1}{2}\right\rangle|\pi,1,0\rangle \\
&= \sqrt{\frac{2}{3}}\left|K\pi,\frac{3}{2},-\frac{1}{2}\right\rangle-\sqrt{\frac{1}{3}}\left|K\pi,\frac{1}{2},-\frac{1}{2}\right\rangle
\end{aligned}
$$

$$\left|K^0\pi^-\right\rangle = \left|K\pi,\tfrac{3}{2},-\tfrac{3}{2}\right\rangle$$

- Isospin decomposition of The scattering amplitude ($K\pi \to K\pi$ channel)

$$T(K^+\pi^+ \to K^+\pi^+)(s,t,u) = T^{\frac{3}{2}}_{K\pi\to K\pi}(s,t,u)$$

$$T^{\frac{1}{2}}_{K\pi\to K\pi}(s,t,u) = \frac{3}{2}T^{\frac{3}{2}}_{K\pi\to K\pi}(u,t,s) - \frac{1}{2}T^{\frac{3}{2}}_{K\pi\to K\pi}(s,t,u)$$

Unitarization of scattering ampltudes

- Unitarization of S matrix:

$$\hat{S}^\dagger \hat{S} \;=\; (1 - i\hat{T}^\dagger)(1 + 1\hat{T}) = 1 + i(\hat{T} - \hat{T}^\dagger) + \hat{T}^\dagger \hat{T} = 1$$

- Optical theorem:

$$2\mathrm{Im}\,\langle i|\,\hat{T}\,|i\rangle = \sum_X |\,\langle X|\,\hat{T}\,|i\rangle\,|^2$$

- For single channel scattering, s < inelastic threshold, only $ab \to ab$ scattering exists. Optical theorem in terms of partial-wave amplitudes:

$$\mathrm{Im}T_J(s) = \frac{|\vec{p}_{\mathrm{cm}}|}{8\pi\sqrt{s}}|T_J(s)|^2 \qquad T_J(s) = \frac{1}{2}\int_{-1}^{1} P_J(\cos\theta)T(s,\theta)d\cos\theta.$$

- For Multiple-Channel Scattering, s > inelastic threshold, We have generalized optical theorem for partial-wave amplitudes:

$$\mathrm{Im}[T_J(s)] = T_J(s)\Sigma(s)T_J^*(s) \qquad (\Sigma(s))_{ij} \equiv \delta_{ij}\frac{|\vec{p}_i|}{8\pi\sqrt{s}}$$

Unitarization of scattering ampltudes

$$\mathrm{Im}[T_J(s)] = T_J(s)\Sigma(s)T_J^*(s)$$

- This unitarity constraint can never be exactly satisfied in a finite-order perturbation theory. Expand T_J as a perturbative series:

$$T_J = T_J^{(2)} + T_J^{(4)} + ...$$

- The unitarity constraint can only be satisfied order by order:

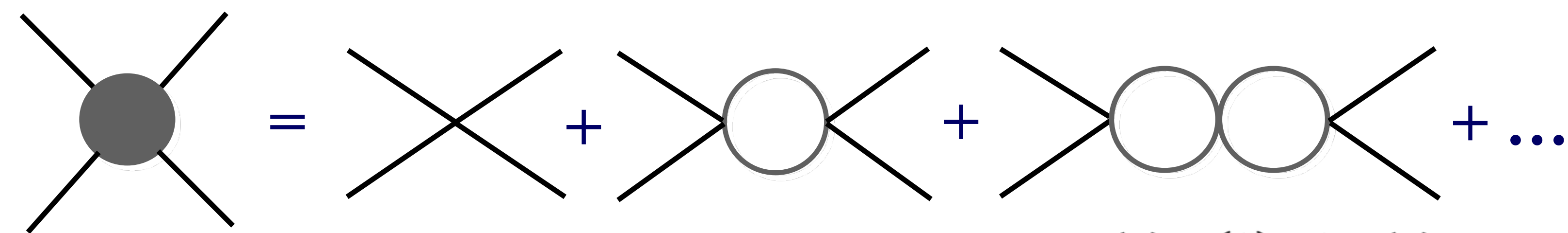
$$\begin{aligned} \mathrm{Im}T_J^{(2)} &= 0 \\ \mathrm{Im}T_J^{(4)} &= T_J^{(2)}\Sigma T_J^{(2)} \\ &\vdots \end{aligned}$$

Unitarization of scattering ampltudes

- We need a non-perturbative form of T_J that satisfy the exact unitarity relation:

$$\mathrm{Im}[T_J(s)] = T_J(s)\Sigma(s)T_J^*(s)$$

- Feyman diagram interpretation:



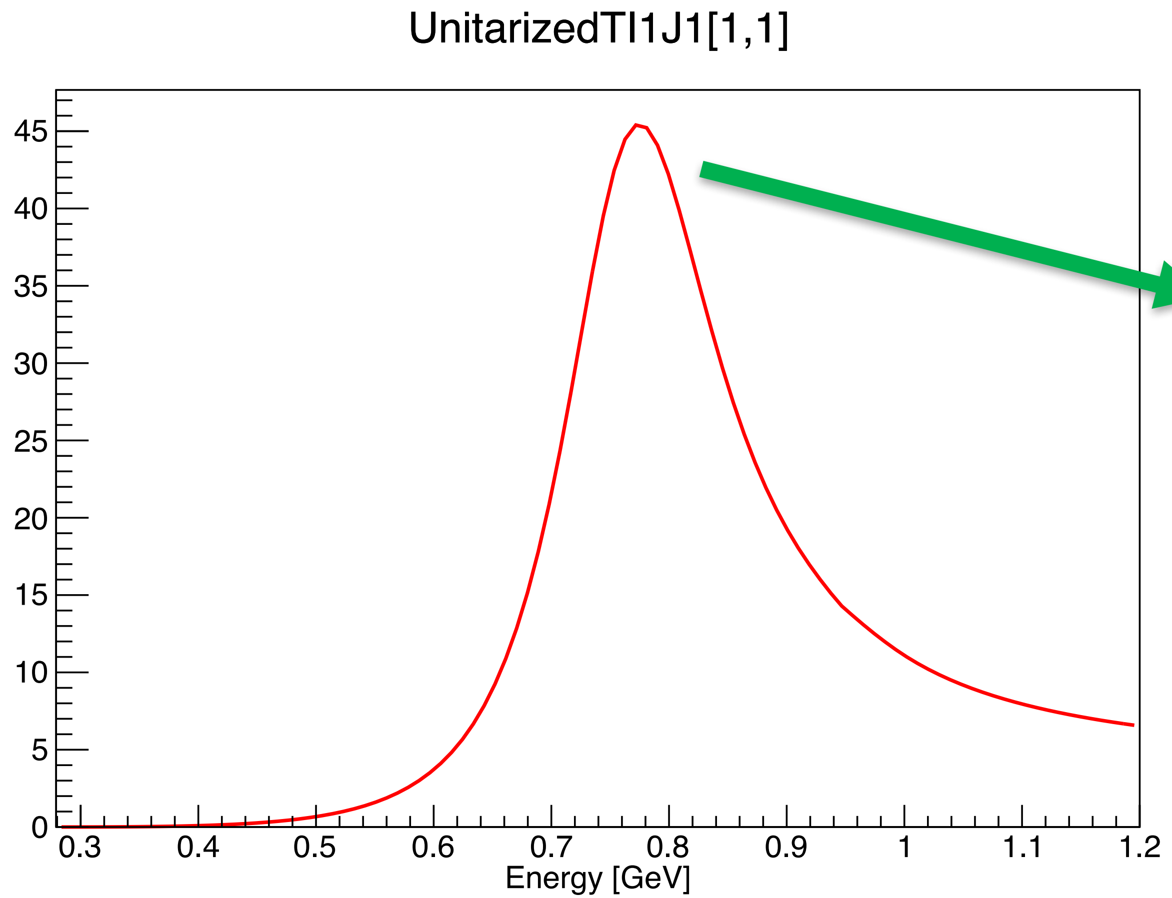
$$T_{unitarized} \quad = \quad T^{(2)} \quad + \quad T^{(4)} \quad + \quad T^{(4)}T^{(2)-1}T^{(4)} \quad + \ldots$$

- Where we only need S-channel scattering.

$$T_J^{unitarized} = T_J^{(2)}(T_J^{(2)} - T_J^{(4)})^{-1}T_J^{(2)}$$

An amazing result

- Unitarized scattering amplitude, I=1, J=1, $\pi\pi \to \pi\pi$ channel:



The peak around 780 MeV implies the ρ meson

Scalar Form Factor

- Scalar current:

$$S_{ij}(x) \equiv \bar{Q}_i(x) Q_j(x) \qquad Q = \begin{pmatrix} u & d & s \end{pmatrix}^T$$

- For QCD Lagrangian, we have:

$$\mathcal{L} = \bar{Q}_L i \not{D} Q_L + \bar{Q}_R i \not{D} Q_R - \bar{Q}_R X_q Q_L - \bar{Q}_L X_q^\dagger Q_R - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$S_{ij} = -\left(\frac{\partial \mathcal{L}}{\partial (X_q)_{ij}} + \frac{\partial \mathcal{L}}{\partial (X_q^\dagger)_{ij}} \right) \Bigg|_{X_q = M} \qquad M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

- With similar procedure for ChPT, we have:

$$\mathcal{L}^{(2)}_{mass} = \frac{F_0^2 B_0}{2} Tr(\mathcal{M} U^\dagger + U \mathcal{M}^\dagger)$$

$$S^{(2)}_{ij} = -\left(\frac{\partial \mathcal{L}^{(2)}}{\partial \mathcal{M}_{ij}} + \frac{\partial \mathcal{L}^{(2)}}{\partial \mathcal{M}^\dagger_{ij}} \right) \Big|_{\mathcal{M} = M}$$

$$= -\frac{F_0^2 B_0}{2} (U^\dagger + U)_{ji}$$

Scalar Form Factor

- At lowest order:

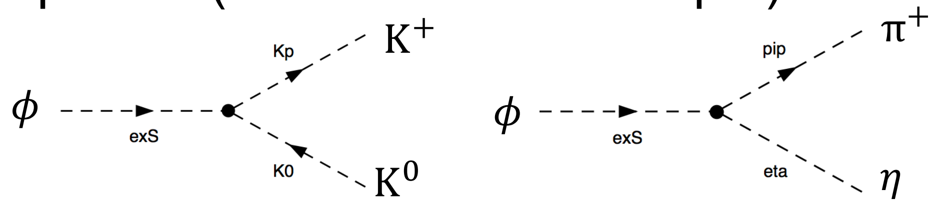
$$
\begin{aligned}
\bar{u}u &= B_0[-F_0^2 + K_+K_- + \frac{\pi_0\eta}{\sqrt{3}} + \frac{1}{6}\eta^2 + \frac{1}{2}\pi_0^2 + \pi_+\pi_-] + \ldots \\
\bar{d}d &= B_0[-F_0^2 + K_0\bar{K}_0 - \frac{\pi_0\eta}{\sqrt{3}} + \frac{1}{6}\eta^2 + \frac{1}{2}\pi_0^2 + \pi_+\pi_-] + \ldots \\
\bar{s}s &= B_0[-F_0^2 + K_0\bar{K}_0 + K_+K_- + \frac{2}{3}\eta^2] + \ldots \\
\bar{u}d &= B_0[K_0K_- + \sqrt{\frac{2}{3}}\eta\pi_-] + \ldots \\
\bar{u}s &= B_0[\bar{K}_0\pi_- - \frac{K_-\eta}{\sqrt{6}} + \frac{K_-\pi_0}{\sqrt{2}}] + \ldots \\
\bar{d}s &= B_0[-\frac{\bar{K}_0\eta}{\sqrt{6}} - \frac{\bar{K}_0\pi_0}{\sqrt{2}} + K_-\pi_+] + \ldots
\end{aligned}
$$

Scalar Form Factor

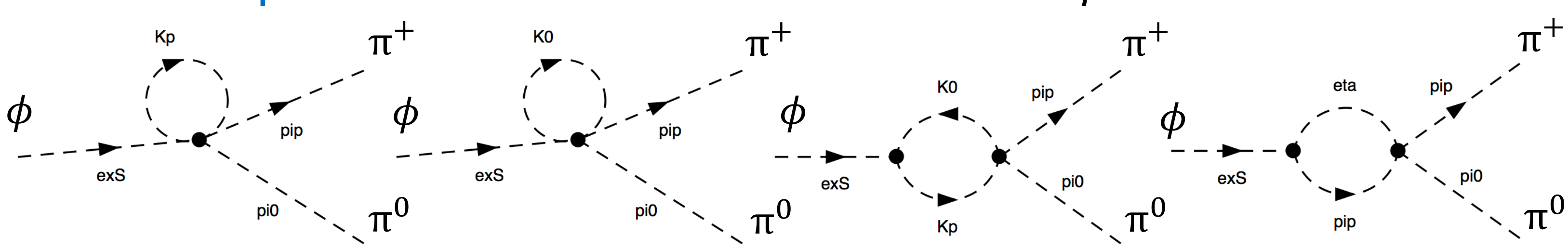
- Define the scalar form factor as: $F_i^S(s) = \langle a_i(p_{a_i}) b_i(p_{b_i}) | \bar{q} q | 0 \rangle$
- Introduce an auxiliary scalar field ϕ :

$$F_i^S(s) = \langle a_i(p_{a_i}) b_i(p_{b_i}) | T | \phi \rangle = \langle a_i(p_{a_i}) b_i(p_{b_i}) | \bar{q} q \phi | \phi \rangle$$

- Tree level amplitude ($\bar{u}d$ current for example)



- For 1-loop correction we should also include $\phi \to \pi^+ \pi^0$:



- To subtract divergence, we need to introduce currents from higher order Lagrangian: $S_{ij}^{(4)} = -(\frac{\partial \mathcal{L}^{(4)}}{\partial \mathcal{M}_{ij}} + \frac{\partial \mathcal{L}^{(4)}}{\partial \mathcal{M}_{ij}^\dagger}) |_{\mathcal{M}=M}$

$$\langle \mathcal{S}^2_{tree} \rangle + \langle \mathcal{S}^2_{1loop} \rangle + \langle S^2_{field\ renorm} \rangle + \langle \mathcal{S}^4_{tree} \rangle = finite\ value\ and\ independent\ of\ scale\ \mu$$

Unitarization of Scalar Form Factor

- Unitary constraint: $2\mathrm{Im}\langle a_i b_i|\hat{T}|\phi\rangle = \sum_X \langle X|\hat{T}|a_i b_i\rangle^* \langle X|\hat{T}|\phi\rangle$

$$\mathrm{Im}F^{\bar{q}'q}(s) = T_0^*(s)\Sigma(s)F^{\bar{q}'q}(s)$$

- Only J=0 contributes.
- By power counting we can expand F into series:

$$F = F^{(0)} + F^{(2)} + \ldots$$

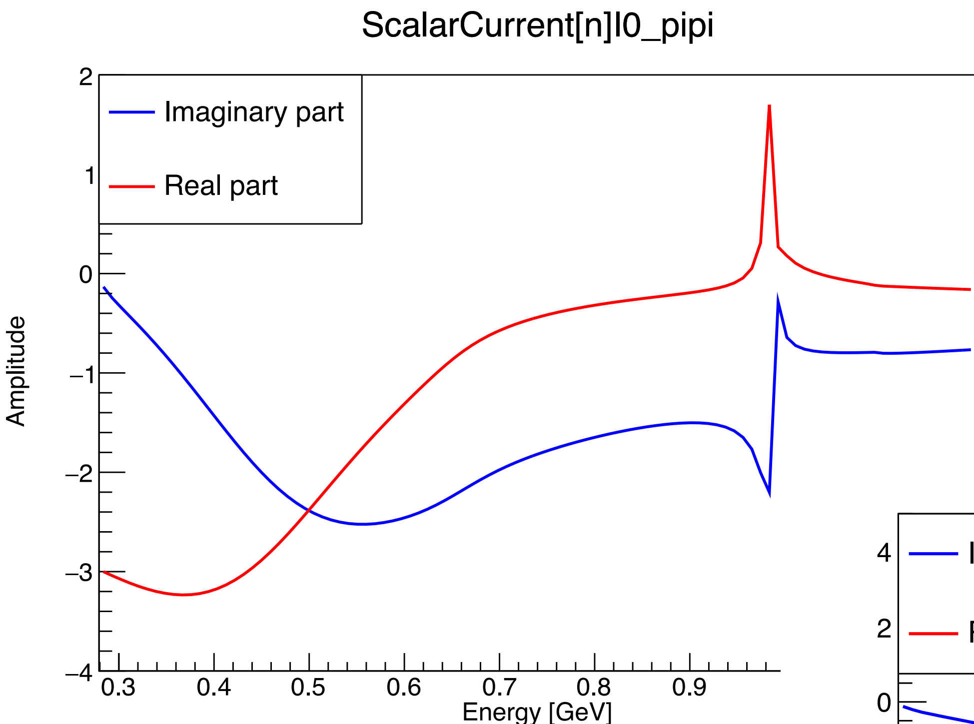
- But such series only satisfy perturbative unitary relation:

$$\begin{aligned} \mathrm{Im}F^{(0)} &= 0 \\ \mathrm{Im}F^{(2)} &= T_0^{(0)}\Sigma F^{(0)} \\ &\vdots \end{aligned}$$

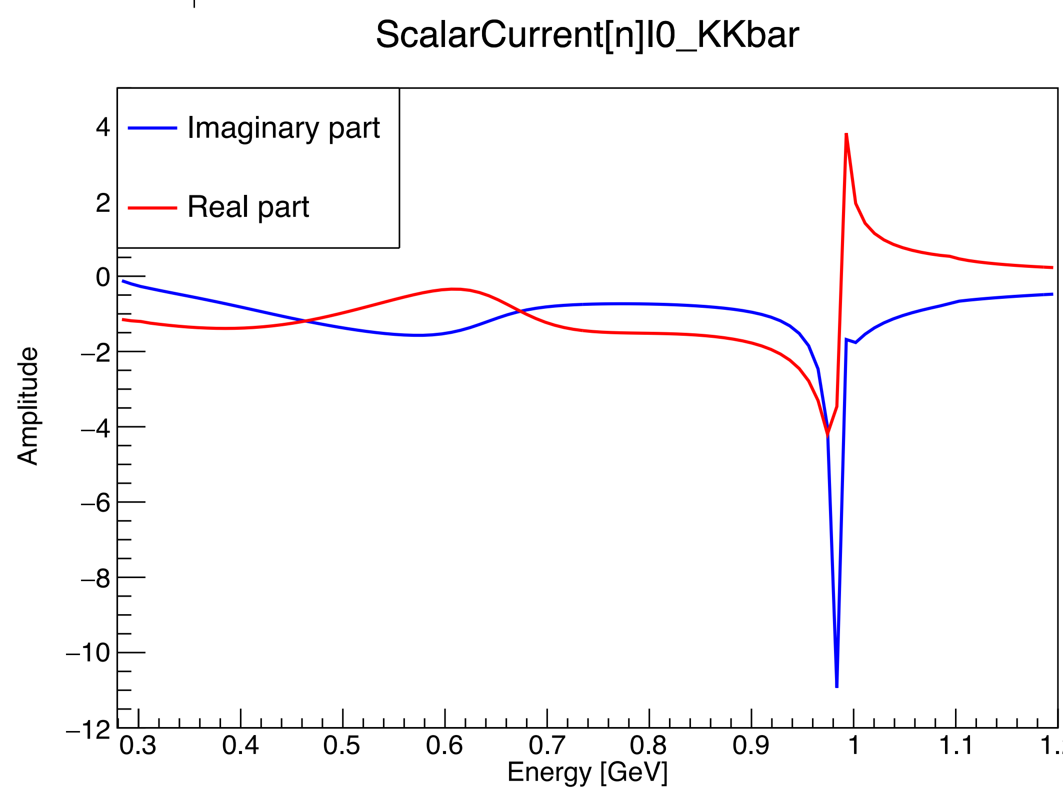
- To satisfy non-perturbative unitary relation, F must be:

$$F = F^{(0)} + T_0^{(2)}(T_0^{(2)} - T_0^{(4)})^{-1}F^{(2)}$$

Scalar Form Factors above thredshold



$$n = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$



Dispersion relation improvement

- The unitarization approach may generate some spurious substructures such as peaks below the thredshold that do not correspond to any physical resonance. This is the so-called Adler zero:

$$F = F^{(0)} + T_0^{(2)} \underline{\left(T_0^{(2)} - T_0^{(4)}\right)}^{-1} F^{(2)}$$

The denominator can be very small at the minimum of $\det\left[T^{(2)}(s_0) - T^{(4)}(s_0)\right]$

- To extend our unitarized form factors to the region below the threshold, we use the dispersion relation:

Evaluated above thredshold

$$\mathrm{Re}F(s) = \frac{1}{\pi}\int_{s_{\mathrm{th}}}^{\infty} dz \frac{\mathrm{Im}F(z)}{z-s}$$

Evaluated at the whole region

Dispersion relation improvement

- To reproduce the asymptotic 1/s-behavior, we need some modification on the high energy behavior of $\mathrm{Im}F(s)$:

$$\mathrm{Im}\tilde{F}^{(1)}(s) \equiv (1-\sigma(s))\mathrm{Im}F^{(1)}(s) + \sigma(s)\frac{\alpha^{(1)}}{s}$$

$$\sigma(s) = \frac{1}{2}\left(\tanh\left\{\frac{4(s-s_0)}{\delta s}\right\}+1\right) \qquad \sigma(-\infty) = 0 \text{ and } \sigma(+\infty) = 1$$

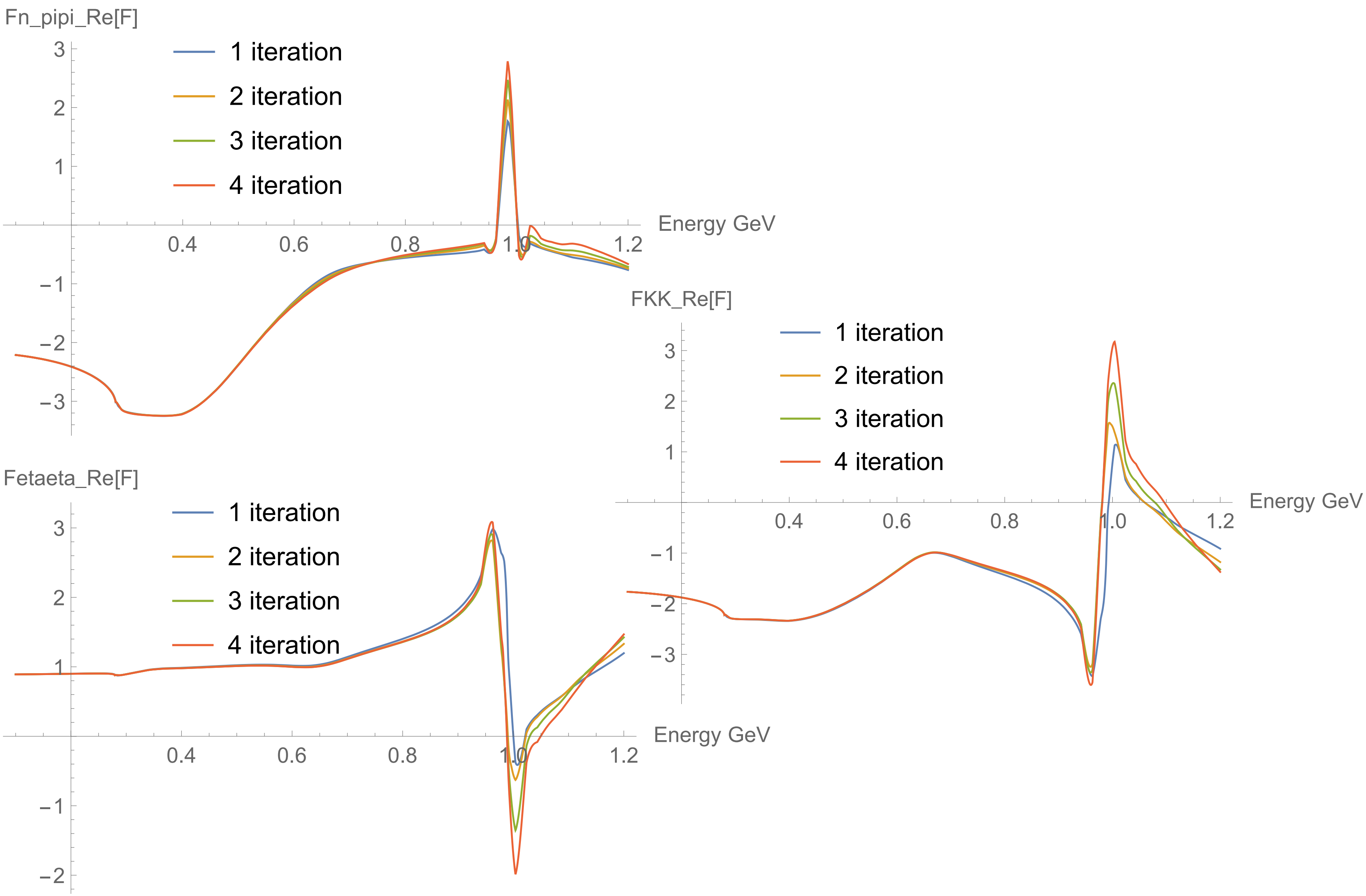
- Do iteration:

$$ImF^{(n+1)}(s) = Re[T^*(s)\Sigma F^{(n)}(s)]$$

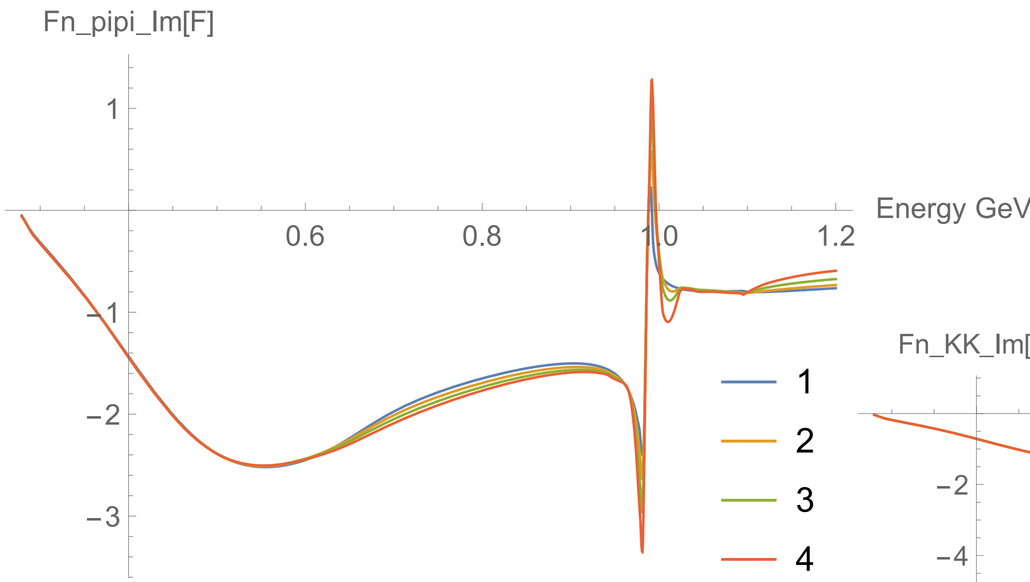
$$ReF^{(n+1)}(s) = \frac{1}{\pi}\int_{s_{th}}^{\infty} dz \frac{Im\tilde{F}^{(n+1)}(z)}{z-s}$$

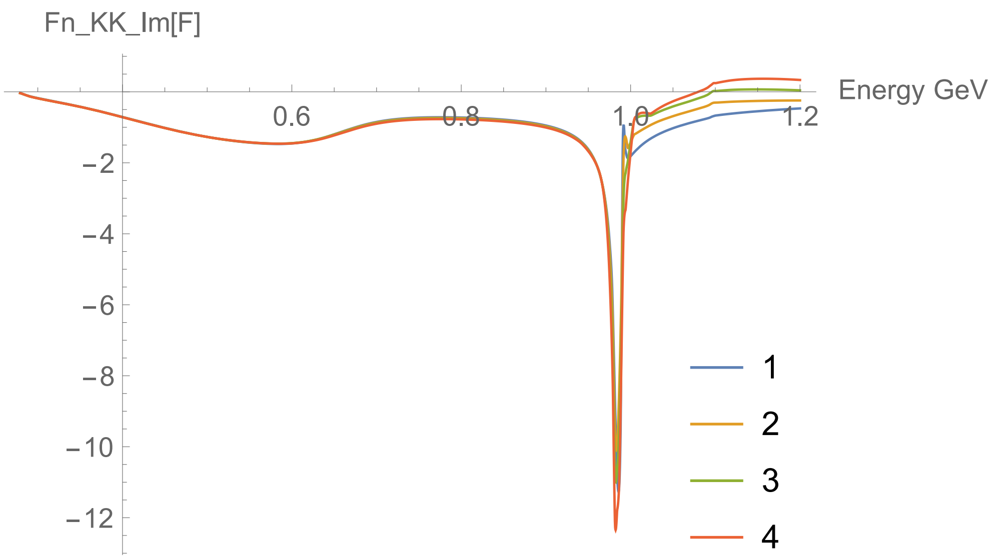
$$F^{(n+1)}(s) = ReF^{(n+1)}(s) + iImF^{(n+1)}(s)$$

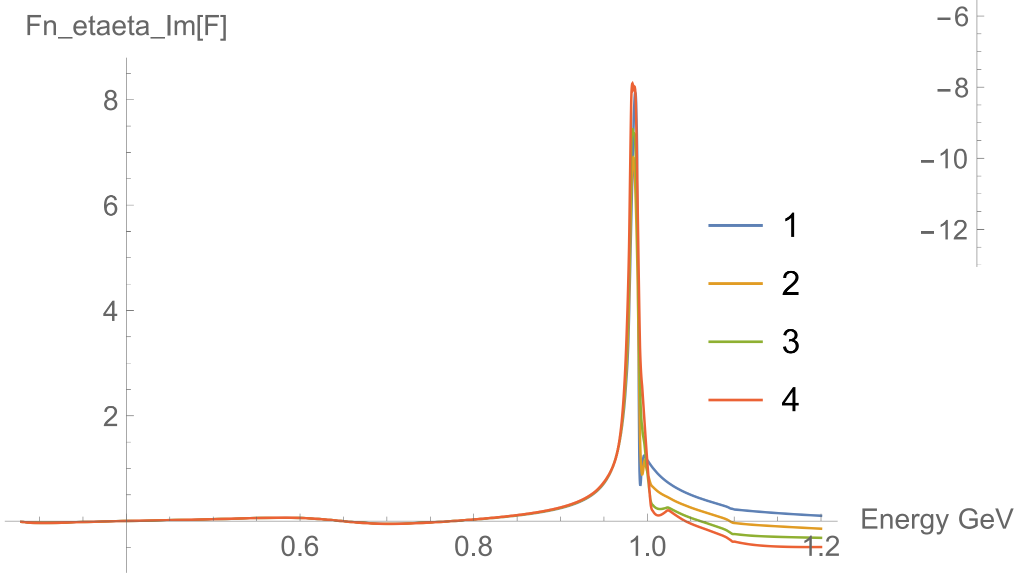
Scalar form factors of current n (real part)



Scalar form factors of current n (image part)







Vector Current

- Vector current corresponds to SU(3):

$$V_\mu^a = \bar{Q} T^a \gamma_\mu Q$$

- QCD Lagrangian:

$$\mathcal{L}_{ext} = \bar{q}_R \gamma^\mu r_\mu^a T^a q_R + \bar{q}_L \gamma^\mu T^a l_\mu^a q_L$$

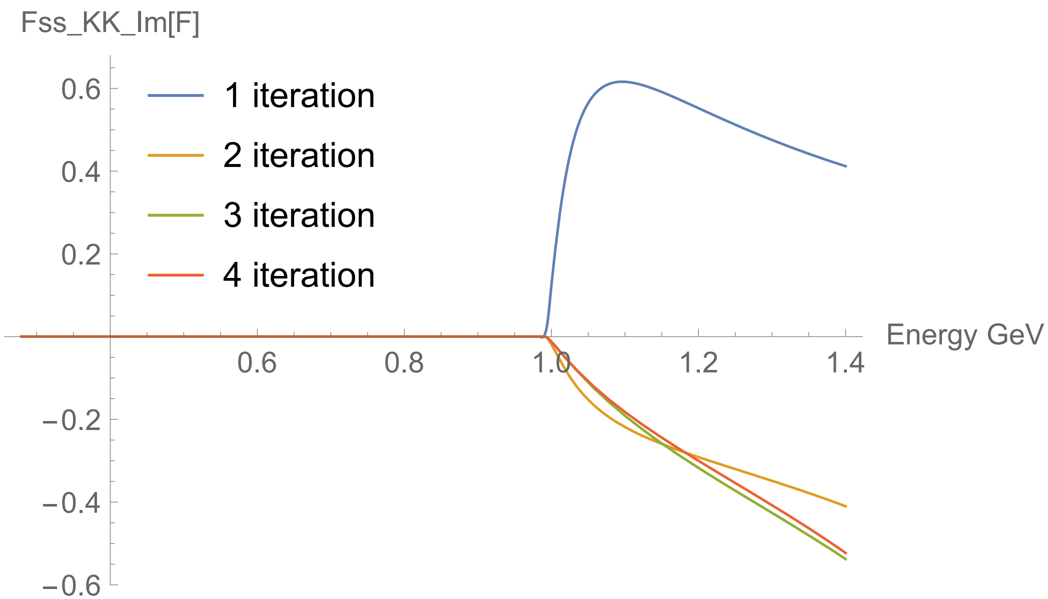
$$V_\mu^a = \left(\frac{\partial}{\partial r_a^\mu} + \frac{\partial}{\partial l_a^\mu} \right) \mathcal{L}$$

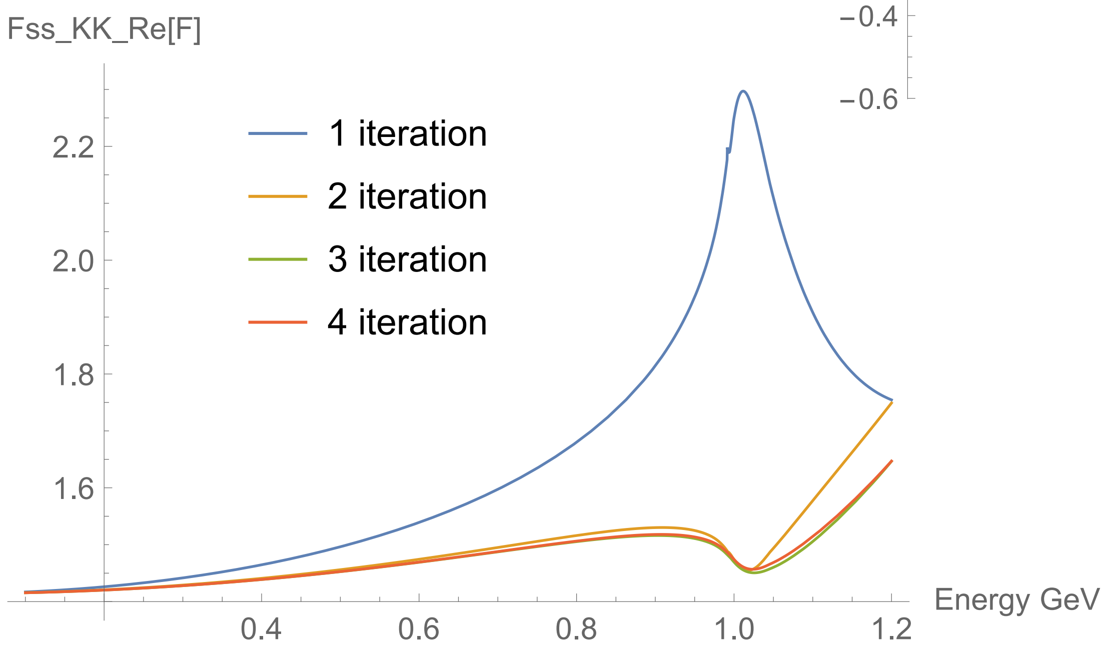
- Vector current in lowest order ChPT:

$$V_{a\mu}^{(2)} = -\frac{iF_0^2}{4} \mathrm{Tr}[\lambda^a [U, \partial_\mu U^\dagger]]$$

$$\begin{aligned} \bar{u}\gamma^\mu d &= V_1^\mu + iV_2^\mu \\ &= -iK_0 \overleftrightarrow{\partial^\mu} K_- + i\sqrt{2}\pi_0 \overleftrightarrow{\partial^\mu} \pi_- + \ldots \\ \bar{u}\gamma^\mu s &= V_4^\mu + iV_5^\mu \\ &= i\pi_- \overleftrightarrow{\partial^\mu} \bar{K}_0 - i\sqrt{\frac{3}{2}} K_- \overleftrightarrow{\partial^\mu} \eta - \frac{i}{\sqrt{2}} K_- \overleftrightarrow{\partial^\mu} \pi_0 + \ldots \end{aligned}$$

Vector form factors of current ss





Tensor Current

- Since thers's no tensor source term in both QCD and ChPT Lagrangian, we need to add it by hand:

$$u \equiv \sqrt{U} = \exp\{\frac{i\lambda^a \phi^a(x)}{2F_0}\} \qquad u_\mu \equiv i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger)$$

$$t_{\pm}^{\mu\nu} \equiv u^\dagger t^{\mu\nu} u^\dagger \pm u t^{\dagger\mu\nu} u$$

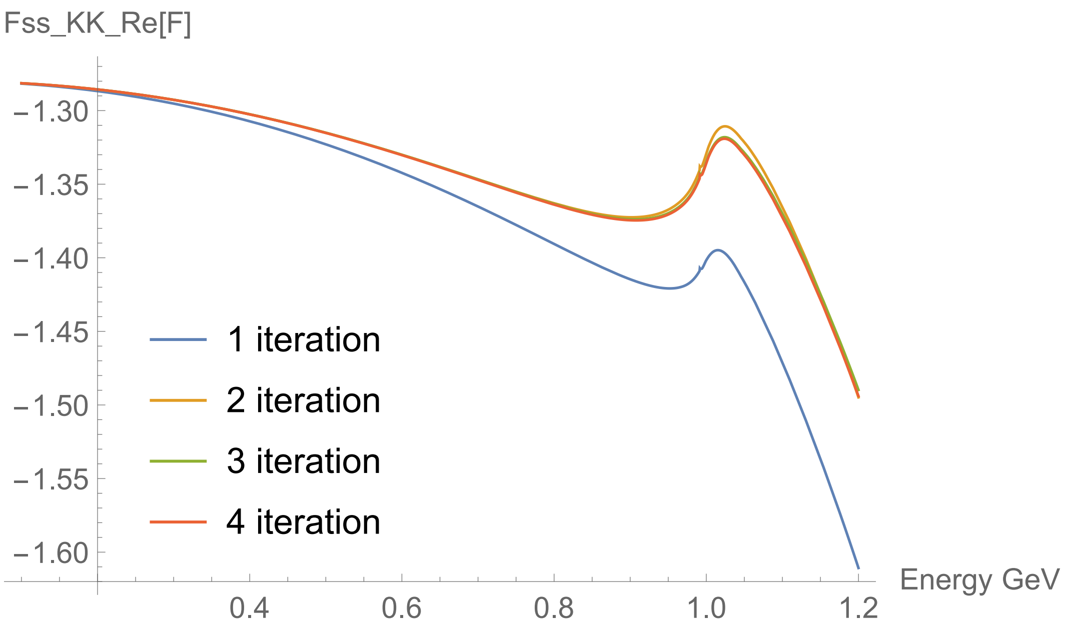
$$\boxed{\mathcal{L}^{(4)} \supset -i\Lambda_2 \mathrm{Tr}[t_+^{\mu\nu} u_\mu u_\nu]}$$

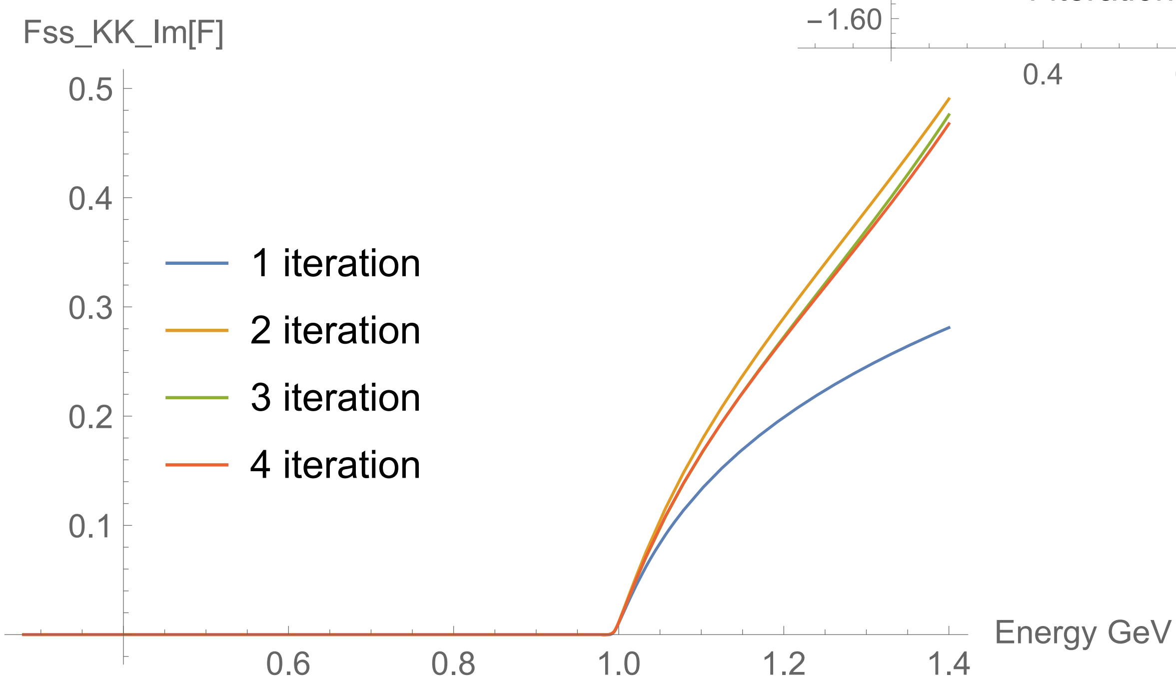
$$T_{ij}^{\mu\nu}(x) \equiv \bar{Q}_i(x)\sigma^{\mu\nu}Q_j(x) = \frac{1}{2}\left(P_L^{\mu\nu\lambda\rho}\frac{\partial\mathcal{L}}{\partial(t_{\lambda\rho})_{ij}} + P_R^{\mu\nu\lambda\rho}\frac{\partial\mathcal{L}}{\partial(t_{\lambda\rho}^\dagger)_{ij}}\right)$$

$$\bar{u}\sigma^{\mu\nu}u = \frac{i\Lambda_2}{F_0^2}(-\partial_\mu K^+\partial_\nu K^- + \partial_\mu K^-\partial_\nu K^+ - \partial_\mu\pi^+\partial_\nu\pi^- + \partial_\mu\pi^-\partial_\nu\pi^+) + \ldots$$

$$\bar{d}\sigma^{\mu\nu}d = \frac{i\Lambda_2}{F_0^2}(\partial_\mu\bar{K}^0\partial_\nu K^0 - \partial_\mu K^0\partial_\nu\bar{K}^0 + \partial_\mu\pi^+\partial_\nu\pi^- - \partial_\mu\pi^-\partial_\nu\pi^+) + \ldots$$

Tensor form factors of current ss





Conclusions

- ## Meson-Meson Scattering

  T with 1-loop correction.

  T_J with isospin decomposition.

  Unitarized T_J.

- ## Meson Form Factors

  Scalar, Vector and Tensor Currents in Meson ChPT.

  Unitarized form factors.

  Dispersion relation improvement.

Thank you for your attention !

In The Language of Group

- Assume the 8 Goldstone bosons ϕ_i form an 8-dimension space M.
- Define an action of G on $\vec{\phi}$, which has properties:

$$\varphi(e, \vec{\Phi}) = \vec{\Phi}, \quad \forall \vec{\Phi} \in M, \quad e \ identity \ of \ G$$

$$\boxed{\varphi(g_1, \varphi(g_2, \vec{\Phi})) = \varphi(g_1 g_2, \vec{\Phi}),} \ \forall \vec{\Phi} \in M, \quad g_1, g_2 \ \in G$$

homomorphism

- After symmetry breaking, the symmetry group becomes H, under which the ground state $\phi_i = 0$ (origin of M) should be invariant.

$$\varphi(h, 0) = 0, \quad \forall h \ \in H$$

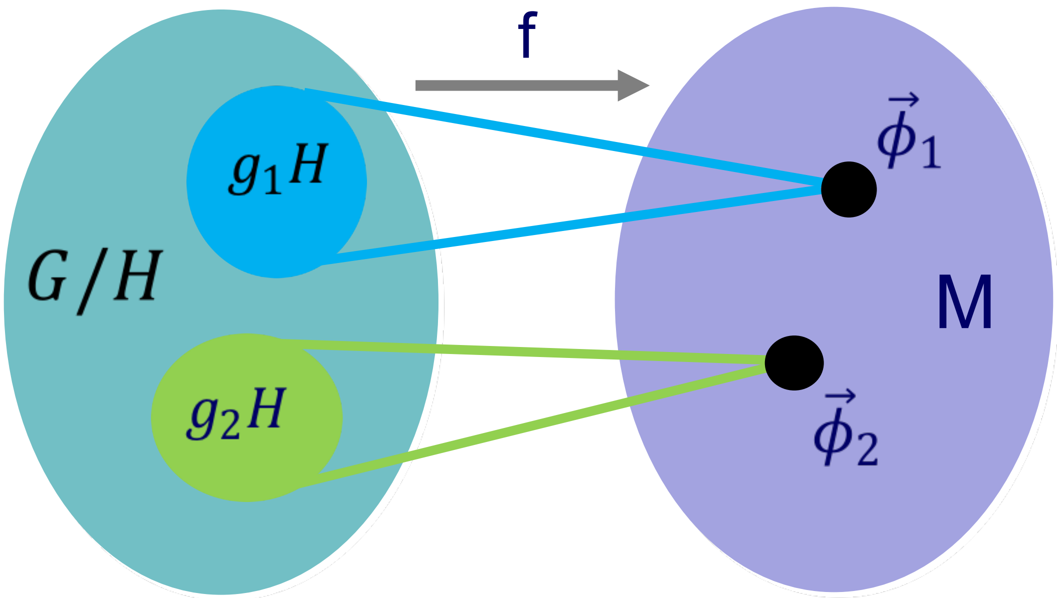
In The Language of Group

- Consider the quotient G/H:

$$G/H = \{gH | g \in G\}, \quad gH = \{gh | h \in H\}$$

- The map f: $G/H \rightarrow M$, $\vec{\Phi} = \varphi(gh, 0) \ \forall gh \ \in gH$, f has two properties:

1. Elements of the same coset gH map the origin to the same point in M:
2. The mapping is injective with respect to the elements of G/H



In The Language of Group

- Consider the quotient G/H:

$$G/H = \{gH|g \in G\}, \quad gH = \{gh|h \in H\}$$

- The map $G/H \to M$: $\vec{\Phi} = \varphi(gh, 0)$ has two properties:

1. Elements of the same coset gH map the origin to the same point in M:

$$\varphi(gh, 0) = \varphi(g, \varphi(h, 0)) = \varphi(g, 0) \quad \forall gh \ \in gH, \quad \forall g \ \in G$$

2. The mapping is injective with respect to the elements of G/H

If $g_1, g_2 \in G$ and $g_2 \notin g_1 H$, asume $\varphi(g_1, 0) = \varphi(g_2, 0)$. Then:

$$0 = \varphi(e, 0) = \varphi(g_1^{-1} g_1, 0) = \varphi(g_1^{-1}, \varphi(g_1, 0)) = \varphi(g_1^{-1}, \varphi(g_2, 0)) = \varphi(g_1^{-1} g_2, 0)$$

Which implies $g_1^{-1} g_2 \in H$ or $g_2 \in g_1 H$ in contradiction to the assumption.

In terms of QCD

- For QCD:

$$G = SU(3) \times SU(3) = \{(L, R) | L \in SU(3), R \in SU(3)\}$$

$$H = \{(V, V) | V \in SU(3)\}$$

- A coset can be expressed as:

$$for\ \forall g = (L, R) \ \in G$$

$$gH = (L, R)H = (L, R)(L^\dagger, L^\dagger)H = (1, RL^\dagger)H$$

- Thus a coset can be characterized by a SU(3) matrix: $RL^\dagger$
- Under transformation $\bar{g}$:
- $$\bar{g}gH = (\bar{L}, \bar{R}RL^\dagger)H = (1, \bar{R}RL^\dagger\bar{L}^\dagger)(\bar{L}, \bar{L})H = (1, \bar{R}(RL^\dagger)\bar{L}^\dagger)H$$

$$if\ U = RL^\dagger \quad then\ U \to \bar{R}U\bar{L}^\dagger$$

Locally Invariant QCD Lagrangian

- Vector currents of QCD:

$$\mathcal{L}_{QCD} = \mathcal{L}^0_{QCD} + \mathcal{L}_{ext}$$

$$\mathcal{L}_{ext} = \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q$$

$$v^\mu = v^\mu_a \frac{\lambda_a}{2} \quad a^\mu = a^\mu_b \frac{\lambda_b}{2}$$

- Define:

$$r_\mu = v_\mu + a_\mu, \quad l_\mu = v_\mu - a_\mu$$

$$\mathcal{L}_{ext} = \bar{q}_R\gamma^\mu r_\mu q_R + \bar{q}_L\gamma^\mu l_\mu q_L$$

- Under $SU(3)\times SU(3)$ transformation:

$$r_\mu \to R r_\mu R^\dagger + iR\partial_\mu R^\dagger$$

$$l_\mu \to L l_\mu L^\dagger + iL\partial_\mu L^\dagger$$

One Particle Eigenstates of Isospin

Meaon wave functions:

$$\begin{aligned}
|\pi_+\rangle &= |u\bar{d}\rangle \\
|\pi_-\rangle &= |d\bar{u}\rangle \\
|\pi_0\rangle &= \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \\
|K_+\rangle &= |u\bar{s}\rangle \\
|K_-\rangle &= |s\bar{u}\rangle \\
|K_0\rangle &= |d\bar{s}\rangle \\
|\bar{K}_0\rangle &= |s\bar{d}\rangle \\
|\eta\rangle &= \frac{1}{\sqrt{6}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle\right)
\end{aligned}$$

- Define the SU(2) isospin operator:

$$\hat{J}_i \equiv \hat{Q}^\dagger \frac{\tau_i}{2}\hat{Q} \qquad \hat{Q} = \left(\begin{array}{cc} \hat{u} & \hat{d} \end{array}\right)^T$$

$$\hat{u} = \begin{pmatrix} a_u \\ b_u^\dagger \end{pmatrix} \qquad \hat{d} = \begin{pmatrix} a_d \\ b_d^\dagger \end{pmatrix}$$

$$\hat{J}_+ = \hat{J}_1 + i\hat{J}_2 = \hat{u}^\dagger\hat{d} = a_u^\dagger a_d + b_u b_d^\dagger$$

$$\hat{J}_- = \hat{J}_1 - i\hat{J}_2 = \hat{d}^\dagger\hat{u} = a_d^\dagger a_u + b_d b_u^\dagger$$

$$\begin{aligned}
\hat{J}_-|\pi^+\rangle &= (a_d^\dagger a_u + b_d b_u^\dagger)|u\bar{d}\rangle \\
&= |d\bar{d}\rangle + |\bar{u}u\rangle \\
&= |d\bar{d}\rangle - |\bar{u}u\rangle = -\sqrt{2}|\pi^0\rangle
\end{aligned}$$

$$\hat{J}_-|\pi^0\rangle = \sqrt{2}|d\bar{u}\rangle = \sqrt{2}|\pi^-\rangle$$

One Particle Eigenstates of Isospin

- Since for isospin eigenstates:

$$\begin{aligned}\hat{J}_+ \left|I, I_3\right\rangle &= \sqrt{(I-I_3)(I+I_3+1)}\left|I, I_3+1\right\rangle \\ \hat{J}_- \left|I, I_3\right\rangle &= \sqrt{(I+I_3)(I-I_3+1)}\left|I, I_3-1\right\rangle\end{aligned}$$

- We can conclude that:

$$\begin{aligned}\left|\pi, 1, +1\right\rangle &= -\left|\pi_+\right\rangle \\ \left|\pi, 1, 0\right\rangle &= \left|\pi_0\right\rangle \\ \left|\pi, 1, -1\right\rangle &= \left|\pi_-\right\rangle\end{aligned}$$

- Similarily:

$$\begin{aligned}\left|K, \frac{1}{2}, +\frac{1}{2}\right\rangle &= \left|K_+\right\rangle \\ \left|K, \frac{1}{2}, -\frac{1}{2}\right\rangle &= \left|K_0\right\rangle\end{aligned}
\qquad
\begin{aligned}\left|\bar{K}, \frac{1}{2}, +\frac{1}{2}\right\rangle &= \left|\bar{K}_0\right\rangle \\ \left|\bar{K}, \frac{1}{2}, -\frac{1}{2}\right\rangle &= -\left|K_-\right\rangle\end{aligned}
\qquad
\left|\eta, 0, 0\right\rangle = \left|\eta\right\rangle$$

Unitarization of scattering ampltudes

- We need a non-perturbative form of T_J that satisfy the exact unitarity relation: $\mathrm{Im}[T_J(s)] = T_J(s)\Sigma(s)T_J^*(s)$
- This relation means:

$$\mathrm{Im}[T_J^{-1}] = -\Sigma$$

$$T_J = \left(\mathrm{Re}[T_J^{-1}] - i\Sigma\right)^{-1}$$

- Expand T_J in series:

$$\begin{aligned}
T_J &= T_J^{(2)} + T_J^{(4)} + ... \\
\Longrightarrow T_J^{-1} &= \left(T_J^{(2)} + T_J^{(4)} + ..\right)^{-1} \\
&= \left(\left(1 + T_J^{(4)}T_J^{(2)-1} + ...\right)T_J^{(2)}\right)^{-1} \\
&= T_J^{(2)-1}\left(1 + T_J^{(4)}T_J^{(2)-1} + ...\right)^{-1} \\
&= T_J^{(2)-1}\left(1 - T_J^{(4)}T_J^{(2)-1} + ...\right)
\end{aligned}$$

Unitarization of scattering ampltudes

$$T_J = \left(\mathrm{Re}[T_J^{-1}] - i\Sigma\right)^{-1}$$

- Since $T_J^{(2)-1}$ is real,

$$\mathrm{Re}[T_J^{-1}] = T_J^{(2)-1}\left(1 - \mathrm{Re}[T_J^{(4)}]T_J^{(2)-1} + ...\right)$$

$$\begin{aligned}
T_J &= \left[T_J^{(2)-1}\left(1 - \mathrm{Re}[T_J^{(4)}]T_J^{(2)-1} + ...\right) - i\Sigma\right]^{-1} \\
&= \left[T_J^{(2)-1}\left(T_J^{(2)} - \mathrm{Re}[T_J^{(4)}] - iT_J^{(2)}\Sigma T_J^{(2)} + ...\right) T_J^{(2)-1}\right]^{-1} \\
&= T_J^{(2)}\left(T_J^{(2)} - \mathrm{Re}[T_J^{(24}] - iT_J^{(2)}\Sigma T_J^{(2)} + ...\right)^{-1} T_J^{(2)} \\
&= T_J^{(2)}\left(T_J^{(2)} - \mathrm{Re}[T_J^{(4)}] - i\mathrm{Im}[T_J^{(4)}] + ...\right)^{-1} T_J^{(2)} \\
&= T_J^{(2)}\left(T_J^{(2)} - T_J^{(4)} + ...\right)^{-1} T_J^{(2)} \\
&\approx T_J^{(2)}\left(T_J^{(2)} - T_J^{(4)}\right)^{-1} T_J^{(2)}
\end{aligned}$$

Unitarization of Scalar Form Factor

- Unitary constraint: $$2\mathrm{Im}\,\langle a_i b_i|\,\hat{T}\,|\phi\rangle = \sum_X \langle X|\,\hat{T}\,|a_i b_i\rangle^* \langle X|\,\hat{T}\,|\phi\rangle$$

- In terms of partial wave amplitude:

$$2\mathrm{Im}F_i^{\bar{q}'q}(s) = \sum_j \frac{|\vec{p}_j|}{4\pi\sqrt{s}}(T_0^*(s))_{ji}F_j^{\bar{q}'q}(s)$$

$$\mathrm{Im}F^{\bar{q}'q}(s) = T_0^*(s)\Sigma(s)F^{\bar{q}'q}(s)$$

- Only J=0 contributes.
- By power counting we can expang F into series:

$$F = F^{(0)} + F^{(2)} + \ldots$$

- But such F only satisfy perturbative unitary relation:

$$\begin{aligned} \mathrm{Im}F^{(0)} &= 0 \\ \mathrm{Im}F^{(2)} &= T_0^{(0)}\Sigma F^{(0)} \\ &\vdots \end{aligned}$$

Unitarization of Scalar Form Factor

- To satisfy non-perturbative unitary relation, we find that F must has the following form: (A is a real vector)

$$F = T_0 A$$

$$Since \quad T_0^* \Sigma T_0 A = (Im T_0) A = Im F$$

- Unitarized T_0 is: $T_0 = T_0^{(2)} (T_0^{(2)} - T_0^{(4)})^{-1} T_0^{(2)}$

  Then:
$$\begin{aligned} F &= T_0^{(2)} (T_0^{(2)} - T_0^{(4)})^{-1} T_0^{(2)} A \\ &= T_0^{(2)} (T_0^{(2)} - T_0^{(4)})^{-1} (A^{(0)} + A^{(2)} + ...) \\ &= T_0^{(2)} ((1 - T_0^{(4)} T_0^{(2)-1}) T_0^{(2)})^{-1} (A^{(0)} + A^{(2)} + ...) \\ &= A^{(0)} + (A^{(2)} + T_0^{(4)} T_0^{(2)-1} A^{(0)}) + ... \\ &= F^{(0)} + F^{(2)} + ... \end{aligned}$$

- Thus we have:

$$A^{(0)} = F^{(0)} \qquad A^{(2)} = F^{(2)} - T_0^{(4)} (T_0^{(2)})^{-1} F^{(0)}$$

Unitarization of Scalar Form Factor

- If we choose: $A = T_0^{(2)-1}(A^{(0)} + A^{(2)})$

$$F = T_0^{(2)}(T_0^{(2)} - T_0^{(4)})^{-1}[(1 - T_0^{(4)}(T_0^{(2)})^{-1})F^{(0)} + F^{(2)}]$$

$$\begin{aligned}
&(1 - T_0^{(4)}(T_0^{(2)})^{-1})F^{(0)} + F^{(2)} \\
= \; & F^{(0)} - (\mathrm{Re}T_0^{(4)} + i\mathrm{Im}T_0^{(4)})(T_0^{(2)})^{-1}F^{(0)} + \mathrm{Re}F^{(2)} + i\mathrm{Im}F^{(2)} \\
= \; & F^{(0)} - \mathrm{Re}T_0^{(4)}(T_0^{(2)})^{-1}F^{(0)} - iT_0^{(2)}\Sigma T_0^{(2)}(T_0^{(2)})^{-1}F^{(0)} + \mathrm{Re}F^{(2)} + i\mathrm{Im}F^{(2)} \\
= \; & F^{(0)} - \mathrm{Re}T_0^{(4)}(T_0^{(2)})^{-1}F^{(0)} - iT_0^{(2)}\Sigma F^{(0)} + \mathrm{Re}F^{(2)} + i\mathrm{Im}F^{(2)} \\
= \; & F^{(0)} - \mathrm{Re}T_0^{(4)}(T_0^{(2)})^{-1}F^{(0)} - i\mathrm{Im}F^{(2)} + \mathrm{Re}F^{(2)} + i\mathrm{Im}F^{(2)} \\
= \; & F^{(0)} - \mathrm{Re}T_0^{(4)}(T_0^{(2)})^{-1}F^{(0)} + \mathrm{Re}F^{(2)}
\end{aligned}$$

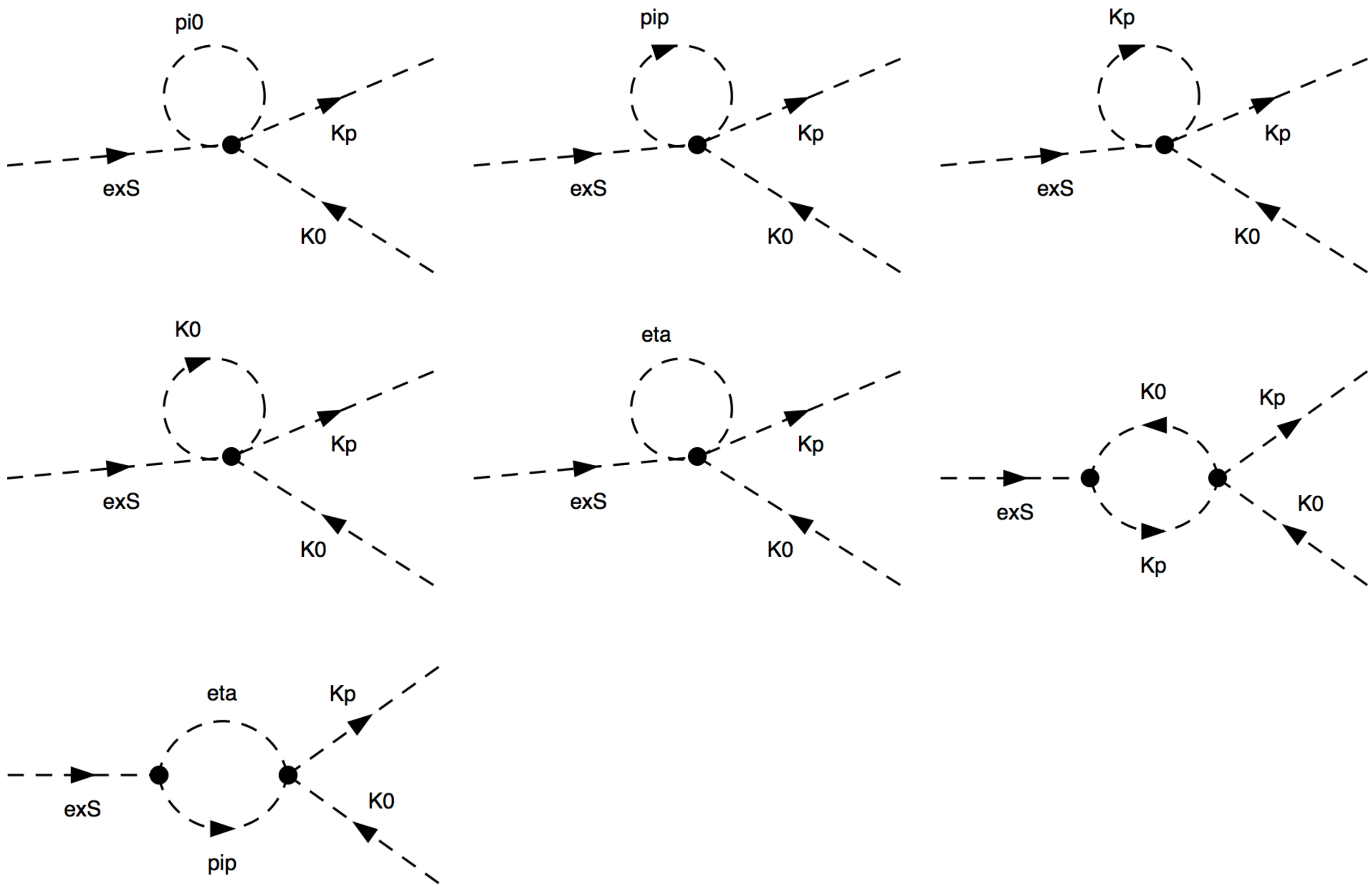
Is real !

- Then we have the simplified form:

$$\begin{aligned}
F &= T_0^{(2)}(T_0^{(2)} - T_0^{(4)})^{-1}[(1 - T_0^{(4)}(T_0^{(2)})^{-1})F^{(0)} + F^{(2)}] \\
&= F^{(0)} + T_0^{(2)}(T_0^{(2)} - T_0^{(4)})^{-1}F^{(2)}
\end{aligned}$$

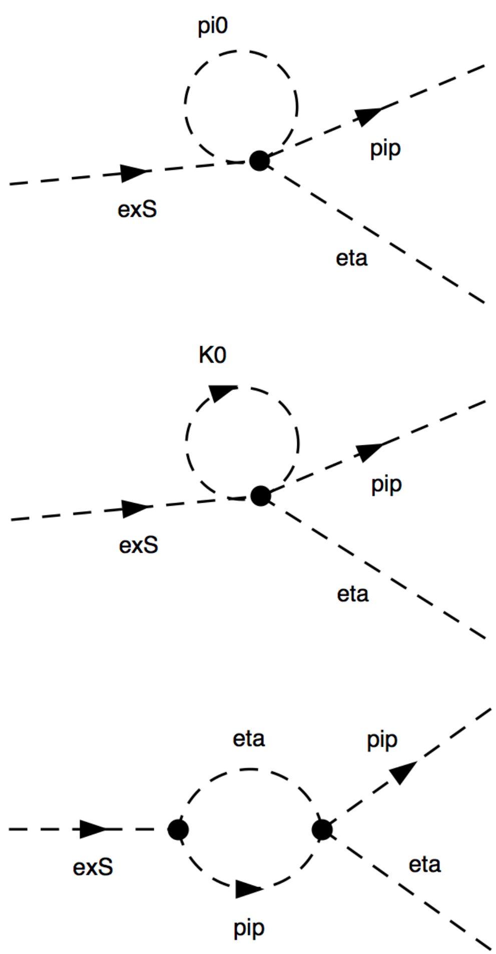
Scalar Form Factor

- 1-loop correction $\phi \to K^{+}K^{0}$:



Scalar Form Factor

- 1-loop correction $\phi \to \pi^+\eta$:



Scalar Form Factor

- To subtract divergence, we need to introduce currents from higher order Lagrangian:

$$S_{ij}^{(4)} = -(\frac{\partial \mathcal{L}^{(4)}}{\partial \mathcal{M}_{ij}} + \frac{\partial \mathcal{L}^{(4)}}{\partial \mathcal{M}_{ij}^{\dagger}}) \mid_{\mathcal{M}=M}$$

$$\begin{aligned} = \quad & -2B_0L_4\mathrm{Tr}[(\partial_\mu U)(\partial^\mu U^\dagger)](U^\dagger + U)_{ji} - 2B_0L_5(U^\dagger(\partial_\mu U)(\partial^\mu U^\dagger) + (\partial_\mu U)(\partial^\mu U^\dagger)U)_{ji} \\ & -8B_0^2L_6\mathrm{Tr}[MU^\dagger + UM^\dagger](U^\dagger + U)_{ji} - 8B_0^2L_7\mathrm{Tr}[MU^\dagger - UM^\dagger](U^\dagger - U)_{ji} \\ & -8B_0^2L_8(UM^\dagger U + U^\dagger MU^\dagger)_{ji} \end{aligned}$$

- To satisfy:

$$\langle \mathcal{S}_{tree}^2 \rangle + \langle \mathcal{S}_{1loop}^2 \rangle + \langle \mathcal{S}_{tree}^4 \rangle = finite\ value\ and\ independent\ of\ scale\ \mu$$

Unitarization of scattering ampltudes

- Partial-wave decomposition:

Extract out the angular dependence of the scattering amplitude T:

$$T(s,\theta) = \sum_{J=0}^{\infty} (2J+1) P_J(\cos\theta) T_J(s)$$

Using the orthogonality relation of Legendre polynomials:

$$T_J(s) = \frac{1}{2} \int_{-1}^{1} P_J(\cos\theta) T(s,\theta) d\cos\theta.$$

- Optical theorem:

$$\hat{S}^\dagger \hat{S} = (1 - i\hat{T}^\dagger)(1 + 1\hat{T}) = 1 + i(\hat{T} - \hat{T}^\dagger) + \hat{T}^\dagger \hat{T} = 1$$

$$2\mathrm{Im} \langle i| \hat{T} |i\rangle = \sum_X | \langle X| \hat{T} |i\rangle |^2$$