



Investigation of B_c decays into charmonium

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Outlines

- Introduction
- B_c decays into P-wave charmonium

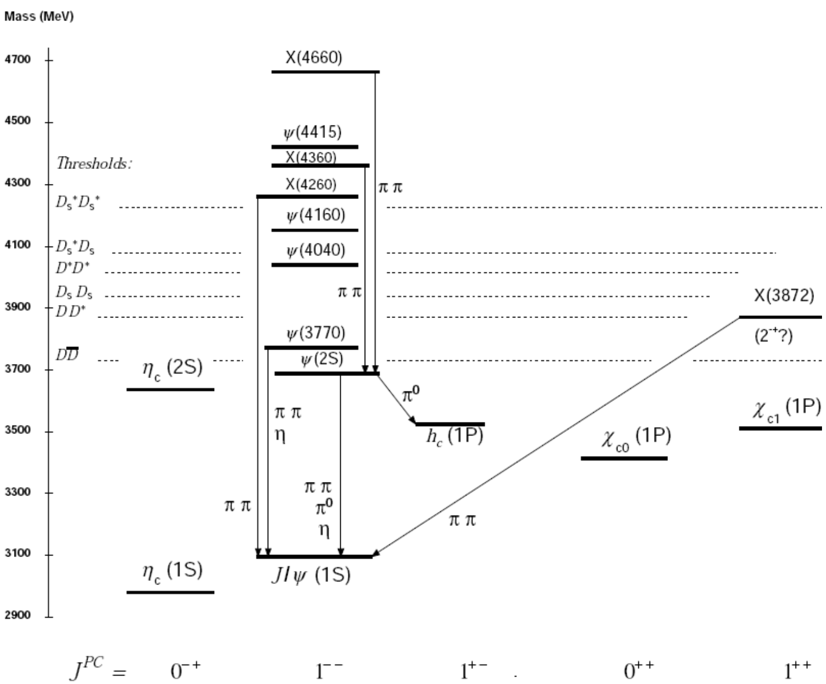
  -Relativistic corrections to the form factors
- B_c exclusive two-body and semileptonic decays into S-wave charmonium

  -The ratio $\mathcal{R} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi \pi^+)}{\mathcal{B}(B_c^+ \to J/\psi \mu^+ \nu_\mu)}$,

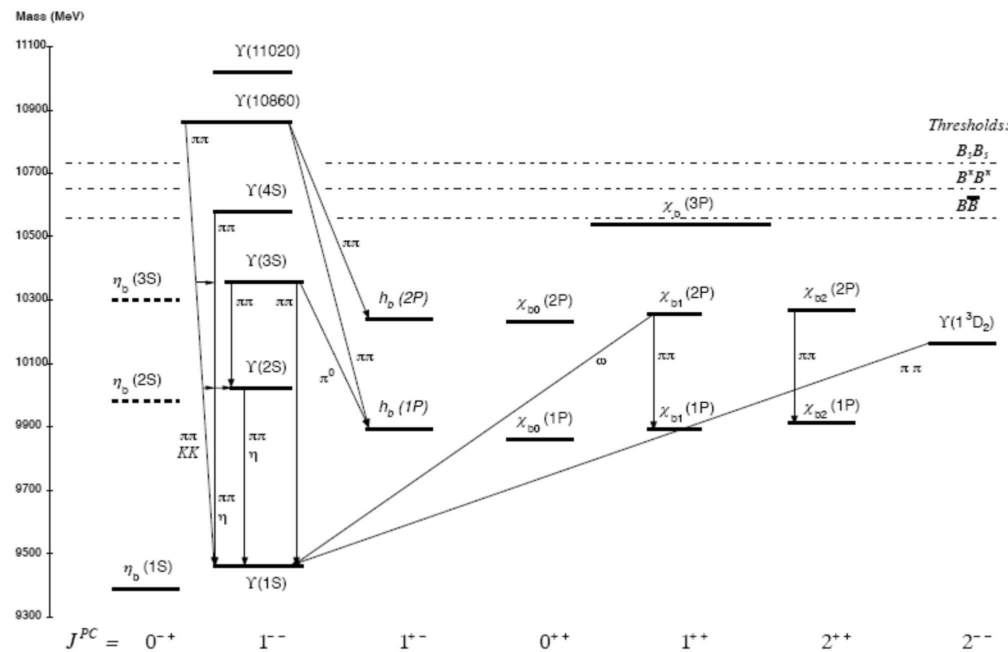
  -The anomaly $R_{J/psi}$ in B_c decays
- Summary

Introduction

- Heavy quarkonium family



Charmonium family



Bottomonium family

PDG 2016

Introduction

- B_c meson family

BOTTOM, CHARMED MESONS $(B = C = \pm 1)$

$B_c^+ = c\overline{b}$, $B_c^- = \overline{c}b$, similarly for B_c^*'s

B_c^+

$I(J^P) = 0(0^-)$
I, J, P need confirmation.

Quantum numbers shown are quark-model predictions.

B_c^+ MASS

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
6275.1 ± 1.0 OUR AVERAGE			
6274.0 ± 1.8 ± 0.4	[1] AAIJ 14AQ	LHCB	pp at 7, 8 TeV

B_c^+ DECAY MODES × B($\overline{b} \to B_c$)

B_c^- modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)	Confidence level
	The following quantities are not pure branching ratios; rather the fraction $\Gamma_i/\Gamma \times$ B($\overline{b} \to B_c$).		
Γ_1	$J/\psi(1S)\ell^+\nu_\ell$ anything	$(5.2\ ^{+2.4}_{-2.1}) \times 10^{-5}$	
Γ_2	$J/\psi(1S)\mu^+\nu_\mu$		
Γ_3	$J/\psi(1S)\pi^+$	seen	
Γ_4	$J/\psi(1S)K^+$	seen	
Γ_5	$J/\psi(1S)\pi^+\pi^+\pi^-$	seen	
Γ_6	$J/\psi(1S)a_1(1260)$	$< 1.2 \times 10^{-3}$	90%
Γ_7	$J/\psi(1S)K^+K^-\pi^+$	seen	
Γ_8	$J/\psi(1S)\pi^+\pi^+\pi^+\pi^-\pi^-$	seen	
Γ_9	$\psi(2S)\pi^+$	seen	
Γ_{10}	$J/\psi(1S)D_s^+$	seen	
Γ_{11}	$J/\psi(1S)D_s^{*+}$	seen	
Γ_{12}	$J/\psi(1S)p\overline{p}\pi^+$	seen	
Γ_{13}	$D^*(2010)^+\overline{D}^0$	$< 6.2 \times 10^{-3}$	90%
Γ_{14}	D^+K^{*0}	$< 0.20 \times 10^{-6}$	90%
Γ_{15}	$D^+\overline{K}^{*0}$	$< 0.16 \times 10^{-6}$	90%
Γ_{16}	$D_s^+K^{*0}$	$< 0.28 \times 10^{-6}$	90%
Γ_{17}	$D_s^+\overline{K}^{*0}$	$< 0.4 \times 10^{-6}$	90%
Γ_{18}	$D_s^+\phi$	$< 0.32 \times 10^{-6}$	90%
Γ_{19}	K^+K^0	$< 4.6 \times 10^{-7}$	90%
Γ_{20}	$B_s^0\pi^+$ / B($\overline{b} \to B_s$)	$(2.37\ ^{+0.37}_{-0.35}) \times 10^{-3}$	

$B_c(2S)^\pm$

$I(J^P) = 0(0^-)$

OMITTED FROM SUMMARY TABLE
Quantum numbers neither measured nor confirmed.

$B_c(2S)^\pm$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
6842±4±5	57	[1] AAD 14AQ	ATLS	pp at 7, 8 TeV

[1] Observed in the decay mode $B_c(2S)^+ \to B_c^+\pi^+\pi^-$ ($B_c^+ \to J/\psi\pi^+$) with 5.2 standard deviations significance.

Introduction

- Heavy quarkonium and B_c meson properties

  （see the talk by Prof. Chao-His Chang）

1) It is composed of two heavy flavors

$$c, \qquad b, \qquad \bar{c}, \qquad \bar{b}$$

2) The quark relative velocity is small

$$v \approx \alpha_s(m_i v)$$

$$v^2 \approx 0.3 \text{ for the } J/\psi \qquad\qquad v^2 \approx 0.1 \quad \text{for the } \Upsilon$$

3) Multi-scale system

Quark mass:	M	
Momentum:	Mv	$M \gg Mv \gg Mv^2 \sim \Lambda_{QCD}$
Energy:	Mv^2	

Introduction

Theoretical studies for B_c into charmonium

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Introduction

- Nonrelativistic QCD (NRQCD)

$$\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \mathcal{L}_{\rm heavy} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\rm light} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \sum_{\text{light flavor}} \bar{q}\, i \not{D}\, q.$$

$$\mathcal{L}_{\rm heavy} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2m_Q}\right)\psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2m_Q}\right)\chi$$

$$\begin{aligned}
&\delta\mathcal{L}_{\rm bilinear} \\
&= \frac{c_1}{8m_Q^3}[\psi^\dagger(\mathbf{D}^2)^2\psi - \chi^\dagger(\mathbf{D}^2)^2\chi] \\
&+ \frac{c_2}{8m_Q^2}[\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - \mathbf{E}\cdot g\mathbf{D})\psi + \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - \mathbf{E}\cdot g\mathbf{D})\chi] \\
&+ \frac{c_3}{8m_Q^2}[\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi + \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi] \\
&+ \frac{c_4}{2m_Q}[\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi - \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi],
\end{aligned}$$

Operator	Estimate	Description
α_s	v	effective quark-gluon coupling constant
ψ	$(Mv)^{3/2}$	heavy-quark (annihilation) field
χ	$(Mv)^{3/2}$	heavy-antiquark (creation) field
D_t	Mv^2	gauge-covariant time derivative
$\mathbf{D}$	Mv	gauge-covariant spatial derivative
$g\mathbf{E}$	M^2v^3	chromoelectric field
$g\mathbf{B}$	M^2v^4	chromomagnetic field
$g\phi$ (in Coulomb gauge)	Mv^2	scalar potential
$g\mathbf{A}$ (in Coulomb gauge)	Mv^3	vector potential

$$\Gamma_H = \sum_n \frac{C_n(\mu)}{m_Q^{d_n-4}} \langle H|\mathcal{O}_n(\mu)|H\rangle\,,$$

$$\sigma_H = \sum_n \frac{C'_n(\mu)}{m_Q^{d_n-4}} \langle 0|\mathcal{O}_n^H(\mu)|0\rangle\,,$$

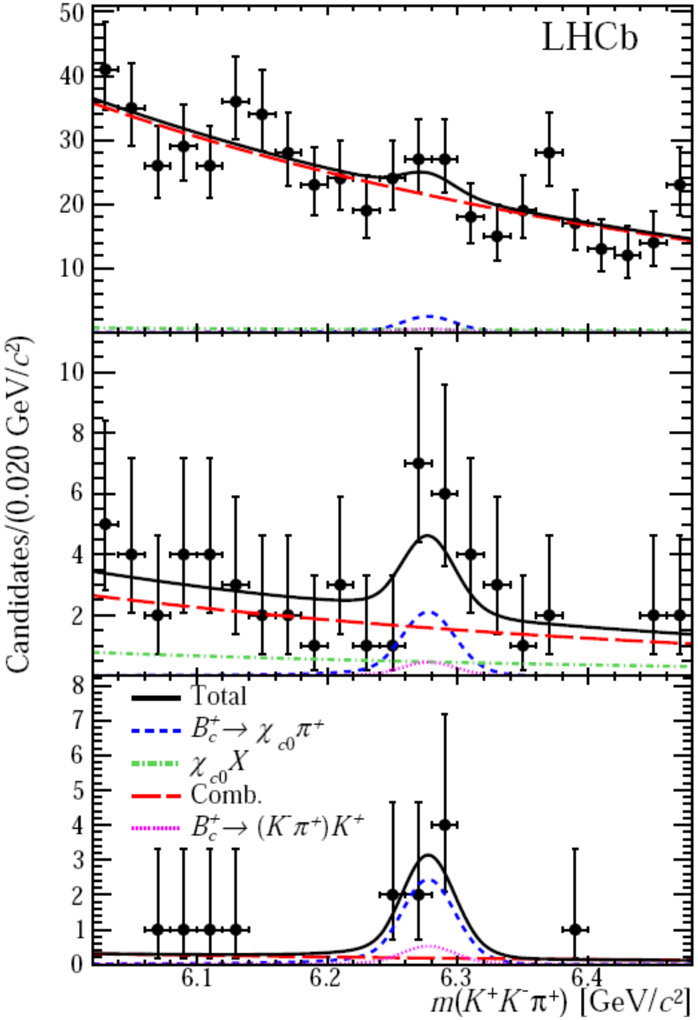
B_c decays into P-wave charmonium

- The first evidence for B_c decays into P-wave charmonium

$$\frac{\sigma(B_c^+)}{\sigma(B^+)} \times \mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) =$$

$$(9.8^{+3.4}_{-3.0}(\text{stat}) \pm 0.8(\text{syst})) \times 10^{-6}.$$

2.4 σ and 4.0 σ



LHCb, PRD94, 091102(2016).

B_c decays into P-wave charmonium

- Form factors of B_c decays into P-wave charmonium

$$\langle h_c(p,\varepsilon^*)|\bar{c}\gamma^\mu b|B_c(P)\rangle = -i[2m_{h_c}A_0^{h_c}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu - A_2^{h_c}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{h_c}}(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{h_c}^2}{q^2}q^\mu) + (m_{B_c}+m_{h_c})A_1^{h_c}(q^2)(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu)]\,,$$

$$\langle h_c(p,\varepsilon^*)|\bar{c}\gamma^\mu\gamma^5 b|B_c(P)\rangle = \frac{2V^{h_c}(q^2)}{m_{B_c}+m_{h_c}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho P_\sigma\,,$$

$$\langle \chi_{c0}(p,\varepsilon^*)|\bar{c}\gamma^\mu\gamma^5 b|B_c(P)\rangle = [f_0^{\chi_{c0}}(q^2)\frac{m_{B_c}^2-m_{\chi_{c0}}^2}{q^2}q^\mu + f_+^{\chi_{c0}}(q^2)(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{\chi_{c0}}^2}{q^2}q^\mu)](-i)\,.$$

$$\langle \chi_{c2}(p,\varepsilon^*)|\bar{c}\gamma^\mu b|B_c(P)\rangle = \frac{2V^{\chi_{c2}}(q^2)}{m_{B_c}(m_{B_c}+m_{\chi_{c2}})}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\nu\alpha}p_\rho P_\sigma P_\alpha\,.$$

$$\langle \chi_{c1}(p,\varepsilon^*)|\bar{c}\gamma^\mu b|B_c(P)\rangle = -i[2m_{\chi_{c1}}A_0^{\chi_{c1}}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu - A_2^{\chi_{c1}}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{\chi_{c1}}}(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{\chi_{c1}}^2}{q^2}q^\mu) + (m_{B_c}+m_{\chi_{c1}})A_1^{\chi_{c1}}(q^2)(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu)]\,,$$

$$\langle \chi_{c1}(p,\varepsilon^*)|\bar{c}\gamma^\mu\gamma^5 b|B_c(P)\rangle = \frac{2V^{\chi_{c1}}(q^2)}{m_{B_c}+m_{\chi_{c1}}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho P_\sigma.$$

$$\langle \chi_{c2}(p,\varepsilon^*)|\bar{c}\gamma^\mu\gamma^5 b|B_c(P)\rangle = [2m_{\chi_{c2}}A_0^{\chi_{c2}}(q^2)\frac{\varepsilon^{*\alpha}\cdot q}{q^2}q^\mu - A_2^{\chi_{c2}}(q^2)\frac{\varepsilon^{*\alpha}\cdot q}{m_{B_c}+m_{\chi_{c2}}}(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{\chi_{c2}}^2}{q^2}q^\mu) + (m_{B_c}+m_{\chi_{c2}})A_1^{\chi_{c2}}(q^2)(\varepsilon^{*\mu\alpha}-\frac{\varepsilon^{*\alpha}\cdot q}{q^2}q^\mu)]\frac{-iP_\alpha}{m_{B_c}}\,,$$

$$q = P - p$$

B_c decays into P-wave charmonium

- LO results for the form factors

$$V^{h_c}(q^2)|_{LO} = \frac{8\sqrt{2}\pi\psi(0)_{B_c}\psi'(0)_{h_c}(z+1)^{3/2}(3z+1)C_A C_F \alpha_s}{z^{5/2}m_b^4 N_c(y-z+1)^2(y+z-1)^2},$$

$$A_0^{h_c}(q^2)|_{LO} = -\frac{16\sqrt{2}\pi\psi(0)_{B_c}\psi'(0)_{h_c}\left(z^2-1\right)C_A C_F \alpha_s\left(-y^2(3z+2)+5z^3+8z^2+9z+2\right)}{z^2\sqrt{z(z+1)}m_b^4 N_c\left((z-1)^2-y^2\right)^3},$$

$$A_1^{h_c}(q^2)|_{LO} = \frac{8\sqrt{2}\pi\psi(0)_{B_c}\psi'(0)_{h_c}(z+1)^{3/2}C_A C_F \alpha_s\left(-y^2+5z^2+2z+1\right)}{z^{5/2}m_b^4\left(y^2-(z-1)^2\right)^2(3zN_c+N_c)},$$

$$A_2^{h_c}(q^2)|_{LO} = -\frac{8\sqrt{2}\pi\psi(0)_{B_c}\psi'(0)_{h_c}\sqrt{z+1}(3z+1)C_A C_F \alpha_s\left(y^2(1-3z)+15z^3+17z^2+17z-1\right)}{z^{5/2}m_b^4 N_c\left((z-1)^2-y^2\right)^3},$$

$$f_+^{\chi_{c0}}(q^2)|_{LO} = -\frac{8\sqrt{\frac{2}{3}}\pi\psi(0)_{B_c}\psi'(0)_{\chi_{c0}}\sqrt{z(z+1)}C_A C_F \alpha_s\left(-y^4+2y^2\left(-2z^2+z+5\right)+9z^4+6z^3-6z-9\right)}{z^3 m_b^4 N_c\left((z-1)^2-y^2\right)^3},$$

$$f_0^{\chi_{c0}}(q^2)|_{LO} = -\frac{8\sqrt{6}\pi\psi(0)_{B_c}\psi'(0)_{\chi_{c0}}(z(z+1))^{3/2}C_A C_F \alpha_s\left(-y^2(5z+3)+9z^3+9z^2+11z+3\right)}{z^4\left(3z^2-2z-1\right)m_b^4 N_c\left(y^2-(z-1)^2\right)^2},$$

…

$$\psi'(0)_{h_c}\varepsilon^{*i} = \frac{1}{\sqrt{2N_c}}\langle h_c(\varepsilon^*)|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{D}^i)\chi|0\rangle.$$

$$\psi'(0)_{\chi_{c0}} = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{2N_c}}\langle\chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi|0\rangle.$$

$$z = m_c/m_b, \quad y = \sqrt{\frac{q^2}{m_b^2}}$$

B_c decays into P-wave charmonium

- Relativistic corrections to the form factors

Supposed:
$$\langle 0|\chi_b^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\psi_c|B_c\rangle \simeq |\mathbf{k}|^2\langle 0|\chi_b^\dagger\psi_c|B_c\rangle,$$

$$|\mathbf{k}|^2 = m_{red}^2|\mathbf{v}|^2 = m_b^2 m_c^2|\mathbf{v}|^2/(m_b+m_c)^2$$

$$\mathcal{A}(k) = \mathcal{A}(0) + \frac{\partial\mathcal{A}(k)}{\partial k^\mu}|_{k=0}\, k^\mu + \frac{1}{2!}\frac{\partial^2\mathcal{A}(k)}{\partial k^\mu\partial k^\nu}|_{k=0}\, k^\mu k^\nu$$

$$V^{h_c}(q^2)|_{RC1} = V^{h_c}(q^2)|_{LO}\frac{|\mathbf{k}|^2}{m_b^2}\frac{y^2\left(-3z^2+38z-3\right)+3z^4-60z^3+82z^2-28z+3}{24z^2(y-z+1)(y+z-1)},$$

$$\begin{aligned} f_+^{\chi_{c0}}(q^2)|_{RC1} = f_+^{\chi_{c0}}(q^2)|_{LO}\frac{|\mathbf{k}|^2}{m_b^2}[&\frac{-27z^8+918z^7+1338z^6+574z^5-220z^4-926z^3-1066z^2-566z-25}{24z^2\left((z-1)^2-y^2\right)\left(-y^4+2y^2\left(-2z^2+z+5\right)+9z^4+6z^3-6z-9\right)} \\ &+\frac{y^6\left(-\left(3z^2+2z+11\right)\right)-3y^4\left(3z^4+38z^3+156z^2+154z+1\right)}{24z^2\left((z-1)^2-y^2\right)\left(-y^4+2y^2\left(-2z^2+z+5\right)+9z^4+6z^3-6z-9\right)} \\ &+\frac{y^2\left(39z^6-490z^5-231z^4+708z^3+1209z^2+1030z+39\right)}{24z^2\left((z-1)^2-y^2\right)\left(-y^4+2y^2\left(-2z^2+z+5\right)+9z^4+6z^3-6z-9\right)}], \end{aligned}$$

$$\begin{aligned} f_0^{\chi_{c0}}(q^2)|_{RC1} = f_0^{\chi_{c0}}(q^2)|_{LO}\frac{|\mathbf{k}|^2}{m_b^2}[&\frac{-81z^7+2727z^6+4851z^5+5787z^4+4765z^3+2789z^2+641z+25}{72z^2\left((z-1)^2-y^2\right)\left(-y^2(5z+3)+9z^3+9z^2+11z+3\right)} \\ &-\frac{y^2\left(y^2\left(45z^3-267z^2-265z-25\right)-126z^5+2250z^4+3604z^3+2660z^2+906z+50\right)}{72z^2\left((z-1)^2-y^2\right)\left(-y^2(5z+3)+9z^3+9z^2+11z+3\right)}], \end{aligned}$$

. . .

B_c decays into P-wave charmonium

- Heavy bottom quark limit

Supposed:

$$m_b \to \infty, \quad z = m_c/m_b \to 0$$

$$V^{h_c}(0)|_{LO}^{m_b\to\infty} = \frac{8\sqrt{2}\pi\psi(0)_{B_c}\psi'(0)_{h_c}C_AC_F\alpha_s}{z^{5/2}m_b^4N_c},$$

$$V^{\chi_{c1}}(0)|_{LO}^{m_b\to\infty} = \frac{40\pi\psi(0)_{B_c}\psi'(0)_{\chi_{c1}}C_AC_F\alpha_s}{z^{5/2}m_b^4N_c},$$

$$V^{\chi_{c2}}(0)|_{LO}^{m_b\to\infty} = \frac{96\sqrt{2}\pi\psi(0)_{B_c}\psi'(0)_{\chi_{c2}}C_AC_F\alpha_s}{z^{3/2}m_b^4N_c}$$

$$V^{h_c}(0)|_{RC1}^{m_b\to\infty} = -\frac{1}{8}\frac{|\mathbf{k}|^2}{z^2m_b^2}V^{h_c}(0)|_{LO}^{m_b\to\infty},$$

$$V^{\chi_{c1}}(0)|_{RC1}^{m_b\to\infty} = -\frac{13}{40}\frac{|\mathbf{k}|^2}{z^2m_b^2}V^{\chi_{c1}}(0)|_{LO}^{m_b\to\infty},$$

$$V^{\chi_{c2}}(0)|_{RC1}^{m_b\to\infty} = -\frac{17}{72}\frac{|\mathbf{k}|^2}{z^2m_b^2}V^{\chi_{c2}}(0)|_{LO}^{m_b\to\infty},$$

$$f_+^{\chi_{c0}}(0)|_{RC1}^{m_b\to\infty} = \frac{25}{216}\frac{|\mathbf{k}|^2}{z^2m_b^2}f_+^{\chi_{c0}}(0)|_{LO}^{m_b\to\infty},$$

$$A_2^H(0)|_{LO}^{m_b\to\infty} = A_1^H(0)|_{LO}^{m_b\to\infty} = V^H(0)|_{LO}^{m_b\to\infty},$$

$$A_2^H(0)|_{RC1}^{m_b\to\infty} = A_1^H(0)|_{RC1}^{m_b\to\infty} = V^H(0)|_{RC1}^{m_b\to\infty},$$

$$f_+^{\chi_{c0}}(0)|_{LO}^{m_b\to\infty} = \frac{24\sqrt{6}\pi\psi(0)_{B_c}\psi'(0)_{\chi_{c0}}C_AC_F\alpha_s}{z^{5/2}m_b^4N_c},$$

$$f_0^{\chi_{c0}}(0) = f_+^{\chi_{c0}}(0)\,,$$

…

B_c decays into P-wave charmonium

- Phenomenological discussions

Form factors	NRQCD LO	NRQCD LO+RC	LFQM [20]	QCD SR [13]
$V^{h_c}(0)$	$0.20^{+0.02}_{-0.02}$	$0.20^{+0.02}_{-0.02}$	0.12	0.48
$A_0^{h_c}(0)$	$1.32^{+0.16}_{-0.12}$	$1.68^{+0.20}_{-0.18}$	0.64	0.03
$A_1^{h_c}(0)$	$0.12^{+0.01}_{-0.01}$	$0.13^{+0.01}_{-0.01}$	0.14	0.08
$A_2^{h_c}(0)$	$-2.02^{+0.25}_{-0.29}$	$-2.60^{+0.34}_{-0.41}$	-1.14	0.21
$f_0^{\chi_{c0}}(0) = f_+^{\chi_{c0}}(0)$	$1.03^{+0.11}_{-0.09}$	$1.33^{+0.16}_{-0.13}$	0.47	0.67
$V^{\chi_{c1}}(0)$	$2.73^{+0.35}_{-0.30}$	$3.37^{+0.48}_{-0.40}$	0.64	0.47
$A_0^{\chi_{c1}}(0)$	$0.10^{+0.01}_{-0.00}$	$0.15^{+0.01}_{-0.02}$	0.13	0.03
$A_1^{\chi_{c1}}(0)$	$0.55^{+0.05}_{-0.04}$	$0.67^{+0.07}_{-0.06}$	0.24	0.08
$A_2^{\chi_{c1}}(0)$	$1.35^{+0.15}_{-0.14}$	$1.60^{+0.20}_{-0.17}$	0.53	0.21
$V^{\chi_{c2}}(0)$	$4.38^{+0.68}_{-0.58}$	$4.94^{+0.81}_{-0.68}$	0.68	
$A_0^{\chi_{c2}}(0)$	$1.38^{+0.19}_{-0.16}$	$0.98^{+0.11}_{-0.10}$	0.86	
$A_1^{\chi_{c2}}(0)$	$1.50^{+0.21}_{-0.18}$	$1.85^{+0.28}_{-0.23}$	0.81	
$A_2^{\chi_{c2}}(0)$	$1.71^{+0.25}_{-0.20}$	$1.95^{+0.29}_{-0.25}$	0.68	

Branching ratios (10^{-3})	NRQCD LO+RC	LFQM	QCD SR
$B_c^\pm \to h_c + \pi^\pm$	$6.17^{+1.56}_{-1.25}$	0.90	0.002
$B_c^\pm \to \chi_{c0} + \pi^\pm$	$4.20^{+1.46}_{-1.02}$	0.53	1.07
$B_c^\pm \to \chi_{c1} + \pi^\pm$	$0.05^{+0.00}_{-0.02}$	0.04	0.002
$B_c^\pm \to \chi_{c2} + \pi^\pm$	$0.74^{+0.17}_{-0.14}$	0.57	
$B_c^\pm \to h_c + K^\pm$	$0.47^{+0.12}_{-0.09}$	0.07	0.0002
$B_c^\pm \to \chi_{c0} + K^\pm$	$0.32^{+0.11}_{-0.08}$	0.04	0.08
$B_c^\pm \to \chi_{c1} + K^\pm$	$0.004^{+0.000}_{-0.001}$	0.003	0.0002
$B_c^\pm \to \chi_{c2} + K^\pm$	$0.056^{+0.013}_{-0.011}$	0.043	

$$\frac{\sigma(B_c^+)}{\sigma(B^+)} \times \mathcal{B}(B_c^+ \to \chi_{c0}\pi^+) =$$

$$(9.8^{+3.4}_{-3.0}(\text{stat}) \pm 0.8(\text{syst})) \times 10^{-6}.$$

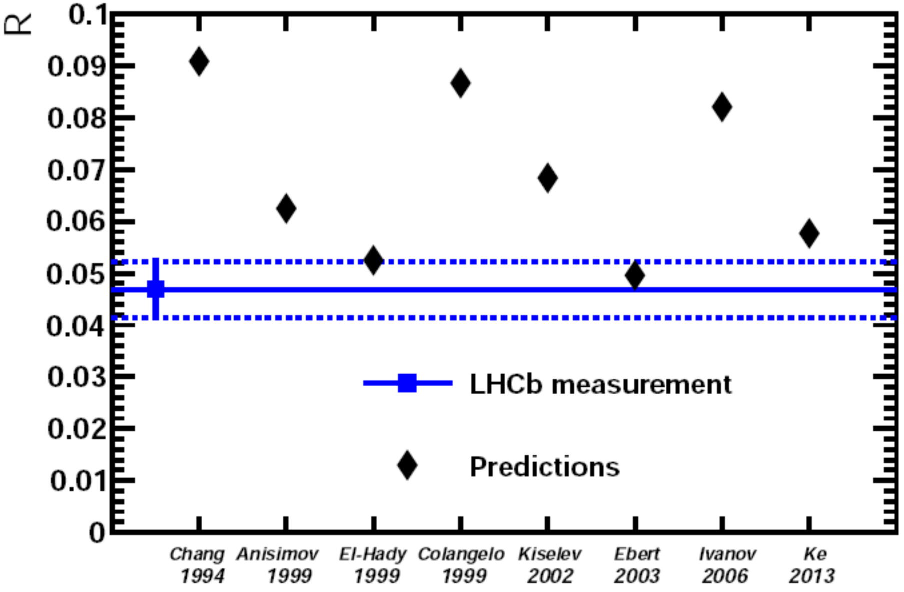
$$\frac{\sigma(B_c^+)}{\sigma(B^+)} \simeq 2.3 \times 10^{-3}$$

B_c exclusive two-body and semileptonic decays into S-wave charmonium

- The measurements for B_c decays into J/psi

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi \pi^+)}{\mathcal{B}(B_c^+ \to J/\psi \mu^+ \nu_\mu)},$$

$$\mathcal{R} = 0.0469 \pm 0.0028 \pm 0.0046.$$

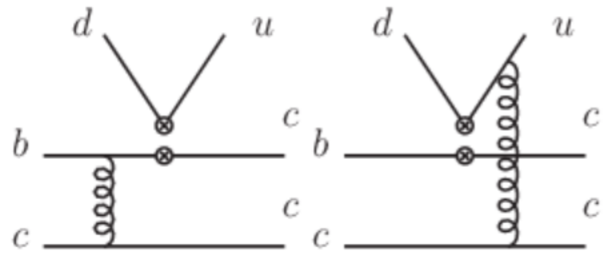


LHCb，PRD90,032009(2014)

B_c exclusive two-body and semileptonic decays into S-wave charmonium

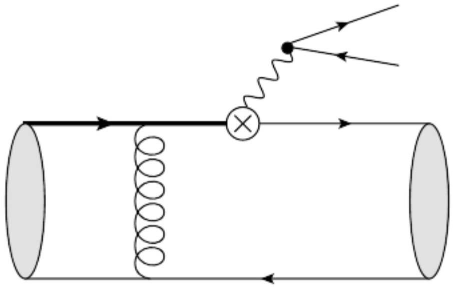
The properties for B_c decays into J/psi

- The exclusive two-body decays include the factorizable and nonfactorizable diagrams



$$\begin{aligned}\mathcal{A}(B_c^- \to J/\psi(\eta_c)\pi^-) &= \langle J/\psi(\eta_c)\pi^- |\mathcal{H}_{\rm eff}| B_c^-\rangle \\ &= \frac{G_F}{\sqrt{2}} V_{ud}^* V_{cb} \left(C_0(\mu)\langle Q_0(\mu)\rangle + C_8(\mu)\langle Q_8(\mu)\rangle\right) .\end{aligned}$$

- The semileptonic decays depend on the form factors.



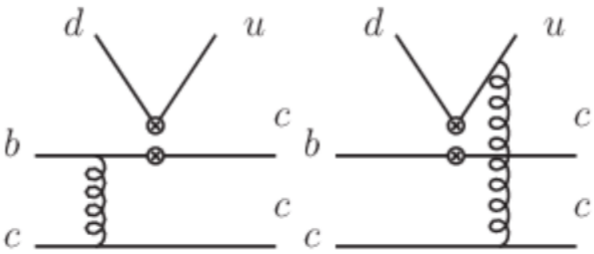
$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 \lambda(q^2)^{1/2} |V_{cb}|^2 q^2}{192\pi^3 m_{B_c}^3} |H_0(q^2)|^2,$$

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 \lambda(q^2)^{1/2} |V_{cb}|^2 q^2}{192\pi^3 m_{B_c}^3} (|H_+(q^2)|^2 + |H_-(q^2)|^2),$$

B_c exclusive two-body and semileptonic decays into S-wave charmonium

- The properties for B_c decays into J/psi

  - The exclusive two-body decays include the factorizable and nonfactorizable diagrams



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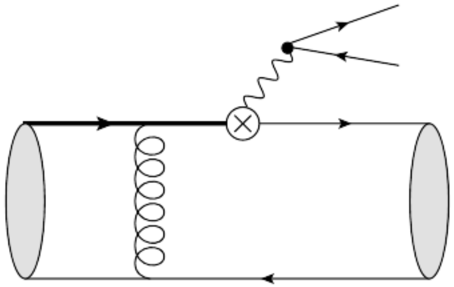
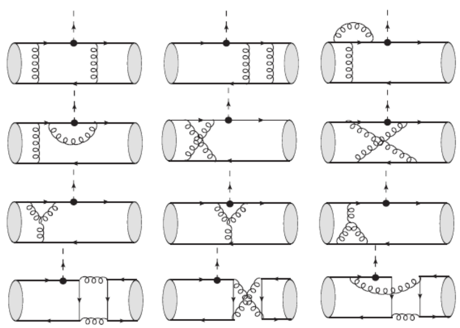
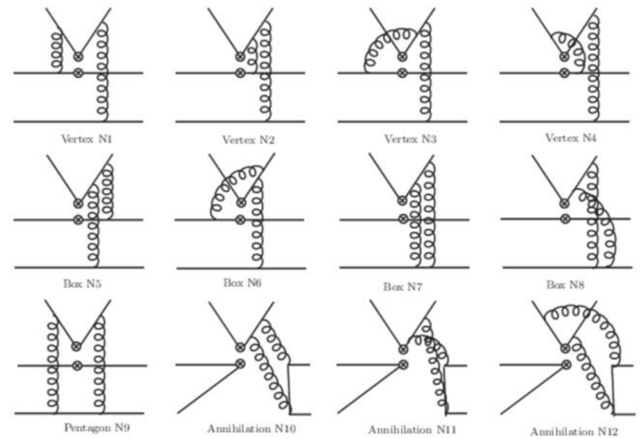


B_c exclusive two-body and semileptonic decays into S-wave charmonium

- NLO corrections indicated form factors diagrams dominate the contributions in two-body decays.

  - NLO QCD corrections





...

  - The relativistic corrections

$$V^{RC}(q^2) = |\mathbf{k}|^2 V^{LO}(q^2)\frac{-y^2\left(24z^2+27z+5\right)-12z^4+87z^3+171z^2+69z+5}{6m_b^2z^2(z+1)(3z+1)\left((z-1)^2-y^2\right)},$$

$$A_0^{RC}(q^2) = |\mathbf{k}|^2 A_0^{LO}(q^2)\frac{-3y^4-2y^2\left(14z^3+5z^2-3\right)-4z^5+85z^4+348z^3+214z^2-3}{24m_b^2z^3(z+1)^2\left((z-1)^2-y^2\right)},$$

$$A_1^{RC}(q^2) = |\mathbf{k}|^2 A_1^{LO}(q^2)\left(\frac{-45z^6+721z^5+1554z^4+1954z^3+807z^2+125z+4}{12m_b^2z^2(3z+1)\left((z-1)^2-y^2\right)\left(-y^2(2z+1)+4z^3+5z^2+6z+1\right)}\right.$$
$$\left.-\frac{y^2\left(y^2\left(3z^2-7z-4\right)+48z^4+580z^3+512z^2+132z+8\right)}{12m_b^2z^2(3z+1)\left((z-1)^2-y^2\right)\left(-y^2(2z+1)+4z^3+5z^2+6z+1\right)}\right),$$

...

B_c exclusive two-body and semileptonic decays into S-wave charmonium

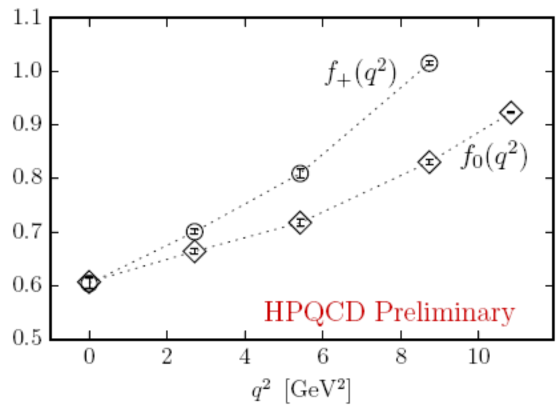
- The semileptonic decays depend on the form factors curves.

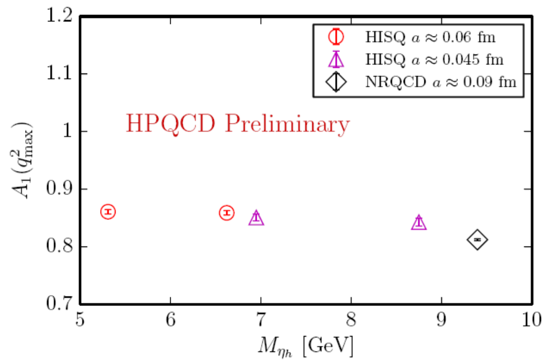
  - Pole models
  $$f'(q^2) = \frac{f(0)}{1 - q^2/m_{\rm pole}^2 - \beta q^4/m_{\rm pole}^4}.$$

    based on pole models, NRQCD prediction gives $\mathcal{R} \simeq 0.48$

    which is consistent with the data

  - Lattice calculation





A. Lytle, 16th ICBP, France

B_c exclusive two-body and semileptonic decays into S-wave charmonium

- The data challenging the lepton universality in B/B_c decays. （see the talk by Guy Wormser)

■ R_D

$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)} \tau \bar{\nu})}{\Gamma(B \to D^{(*)} e/\mu \bar{\nu})}.$$

2.3 sigma / 3.4 sigma

	R_D	R_{D^*}
Experimental average	$0.407 \pm 0.039 \pm 0.024$	$0.304 \pm 0.013 \pm 0.007$
SM prediction	0.300 ± 0.010	0.252 ± 0.005

A. K. Alok et al., arXiv:1710.04127

■ $R_{J/psi}$

$$R_{J/\psi} = \frac{\Gamma(B_c \to J/\psi \tau \bar{\nu})}{\Gamma(B_c \to J/\psi \mu \bar{\nu})} = 0.71 \pm 0.17 \pm 0.18,$$

over 2 sigma

LHCb-PAPER-2017-035.

The standard model predictions from NRQCD, PQCD, RQM, LFQM, QCD SR lie in the range (0.2-0.3).

B_c exclusive two-body and semileptonic decays into S-wave charmonium

The model independent investigation of $R_{J/psi}$

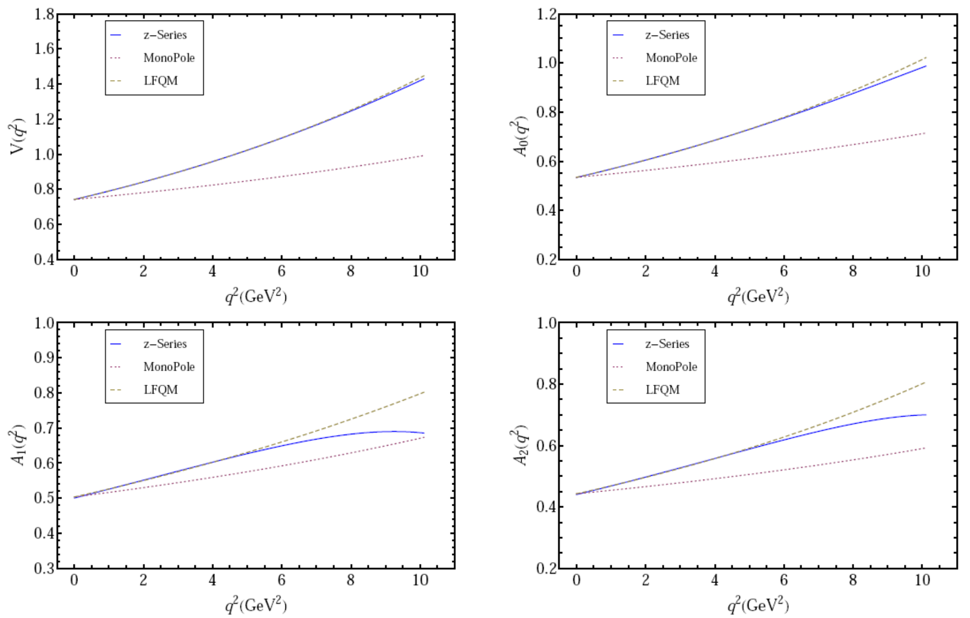
■ Model independent determination of form factors (Unitary constraints)

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{k=0}^{\infty} a_k \; z(t;t_0)^k$$

$$\frac{1+z(t;t_0)}{1-z(t;t_0)} = \sqrt{\frac{t_+ - t}{t_+ - t_0}}$$

$$|\phi_I^X(t)|^2 = \frac{1}{48\pi\,\chi_I^X(n)} \frac{(t-t_+)^2}{(t_+ - t_0)^{1/2}} \frac{(t-t_-)^{3/2}}{t^{n+2}} \frac{t-t_0}{t}.$$

$$t_+ = (m_{B_c} + m_{J/\psi})^2, \quad t_- = (m_{B_c} - m_{J/\psi})^2$$



It still can not explain the data.

Distinguishing the SM and BSM contributions precisely is important!

Summary

- Some decay channels of B_c into P-wave charmonium have large branching ratios, which can be tested in LHCb experiment.
- The uncertainties of form factors are reduced. The branching ratios except the $R_{J/psi}$ can be understand.
- More precise calculation and model-independent determination are needed to understand the anomalies.

Thank You
for your attention !