Meson-meson scattering in 2D QCD

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HFCPV-2017

Wu Han

Tetraquark states

- Some tetraquark candidate:

  X(3872),2003, Z(4430),Y(4660),2007

  Zc(3900),2013

- In large N limit, the tetraquark states are rule out or not?

  S.Coleman, Aspects of Symmetry

  S. Weinberg,PRL(2013)

1/N Expansion

- There is a hidden candidate for a prossible expansion parameter in QCD.

- In 1974, 't Hooft(Nucl.Phys.B72,461) suggested to generalize QCD from SU(3) gauge group to SU(N) gauge group, which may be qualitatively and quantitatively close to the large N limit.

-E.Witten,Nucl.Phys.B160(1979)

The 't Hooft model (1+1D QCD, large-N limit)

The Lagrangian

$$\mathcal{L} = -\frac{1}{4} tr\left(G_{\mu\nu}G^{\mu\nu}\right) + \sum_a \bar{q}^a (i\gamma \cdot D - m^a) q^a$$

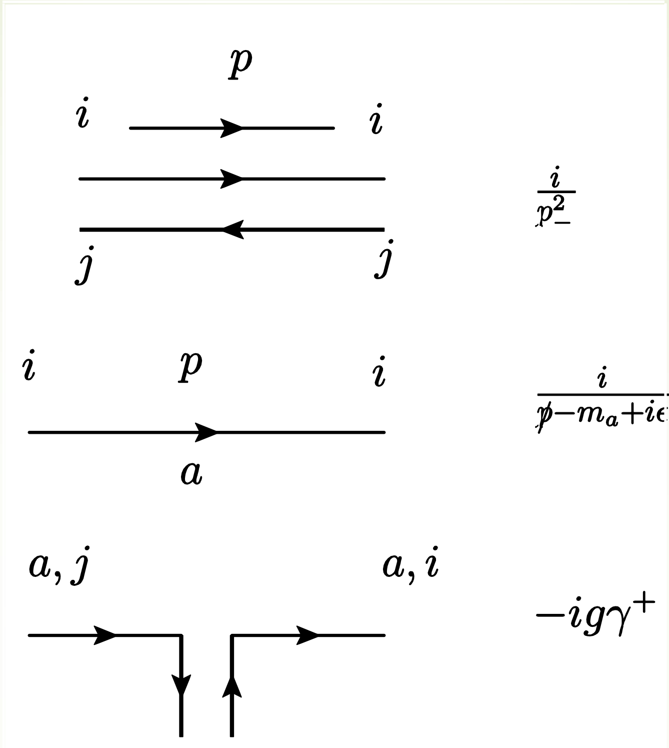
With a SU(N) gauge symmetry, where $g^2 N = const, N \rightarrow \infty$ (Large-N limit).

Light cone gauge

$$A^+ = 0$$

$$\mathcal{L} = \frac{1}{2} tr\left((\partial_- A^-)^2\right) + \sum_a \bar{q}^a (i\gamma^\mu \partial_\mu - g\gamma^+ A_+ - m^a) q^a$$

The Feynman Rules



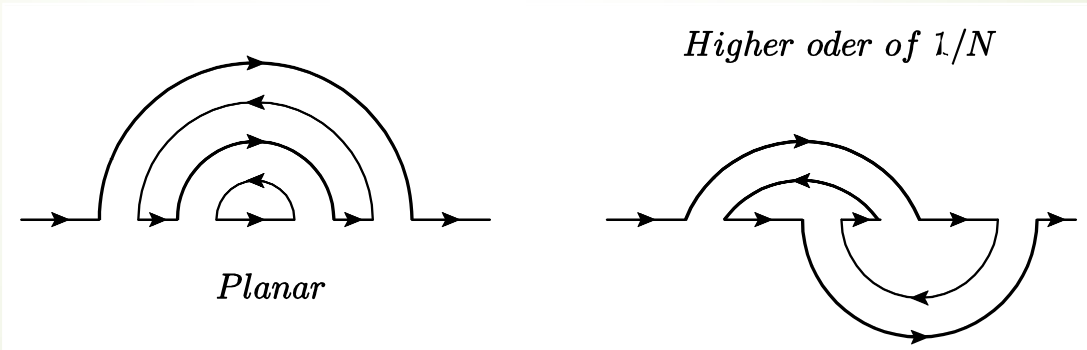
An infrared cutoff in p_- integration

$$\int_{-\infty}^{\infty}\frac{p_+}{2\pi}(\int_{-\infty}^{-\lambda}+\int_{\lambda}^{\infty}\frac{dp_-}{2\pi})$$

Phys.Rev.D 13.1649 by C G.Callan, Jr.Nigel Coote and D J.Gross

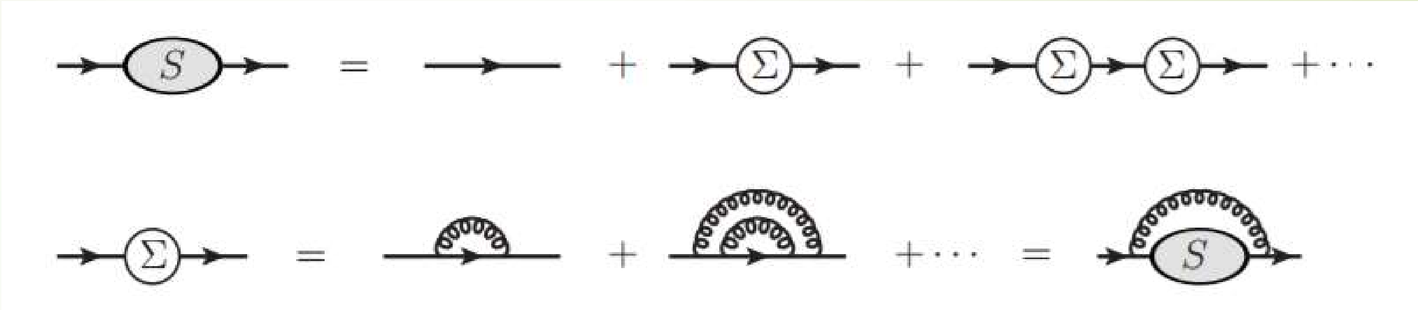
The planar diagram

Closed color loop provide a factor of N



The Dyson-Shwinger Equation

The leading order diagram for the DS equation



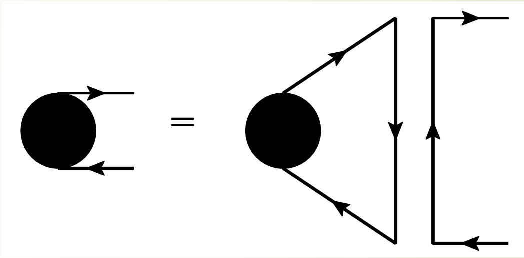
The coefficient of the γ^+ of the dressed quark propagator

$$\tilde{s}_a(p) = \frac{1}{2p^- - \dfrac{m_a^2 - \dfrac{g^2 N}{\pi}}{p^+} - \dfrac{g^2 N}{\pi}\dfrac{Sgn(p^+)}{\lambda} + i\epsilon\, Sgn(p^+)}$$

Callan et al. PRD (1976)

The Bethe-Salpeter Equation

The leading order diagram of Bethe-Salpeter Equation



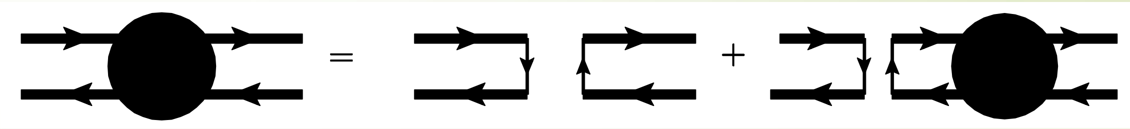
The 't Hooft Equation

$$\mu_k^2\varphi_k(x) = (\frac{\gamma_1}{x} + \frac{\gamma_2}{1-x})\varphi_k(x) + \int_0^1 dy \frac{\varphi_k(x) - \varphi_k(y)}{(x-y)^2}.$$

$$\gamma_a = \frac{\pi m_a^2}{g^2 N}$$

't Hooft.Gerard, Nucl.Phys. B75,461-470 (1974)

The leading order diagram of quark-antiquark scattering amplitude



Quantum theory of field. Volume 1,10.2 polology
S.Weinberg

$$G \to \frac{-2i\sqrt{\mathbf{q}^2 + m^2}}{q^2 + m^2 - i\epsilon}(2\pi)^7\delta^4(q_1 + \cdots + q_n)$$
$$\times \sum_\sigma M_{0|\mathbf{q},\sigma}(q_2 \cdots q_r) M_{\mathbf{q},\sigma|0}(q_{r+2} \cdots q_n)$$

The quark-antiquark

$$T(x', x; r) = \frac{ig^2}{r_-^2(x'-x)^2} - \sum_k \frac{i}{(r^2 - r_k^2)} \left\{\phi_k^*(x') \frac{2g}{\lambda}\left(\frac{g^2N}{\pi}\right)^{1/2}\left[\theta(x'(1-x')) + \frac{\lambda}{2|r_-|}\left(\frac{\gamma_a - 1}{x'} + \frac{\gamma_b - 1}{1-x'} - \mu_k^2\right)\right]\right\}$$
$$\times \left\{\phi_k(x) \frac{2g}{\lambda}\left(\frac{g^2N}{\pi}\right)^{1/2}\left[\theta(x(1-x)) + \frac{\lambda}{2|r_-|}\left(\frac{\gamma_a - 1}{x} + \frac{\gamma_b - 1}{1-x} - \mu_k^2\right)\right]\right\}$$

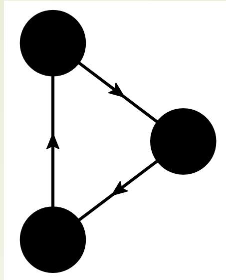
Callan et al. PRD13,1649(1976

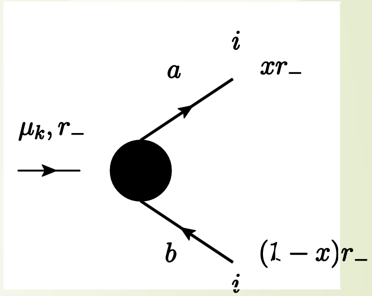
The form factor

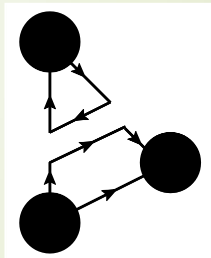
$$\Phi_k^{1,2}(x) = \varphi_k(x)\frac{g^2}{|r_-|}\sqrt{\frac{N_c}{\pi}}\left[\theta(x(1-x))\frac{2|r_-|}{\lambda} + \frac{\gamma_1 - 1}{x} + \frac{\gamma_2 - 1}{1-x} - \mu_k^2\right].$$

Decay amplitude

The leading order diagram of the 1 meson → 2 mesons Decay





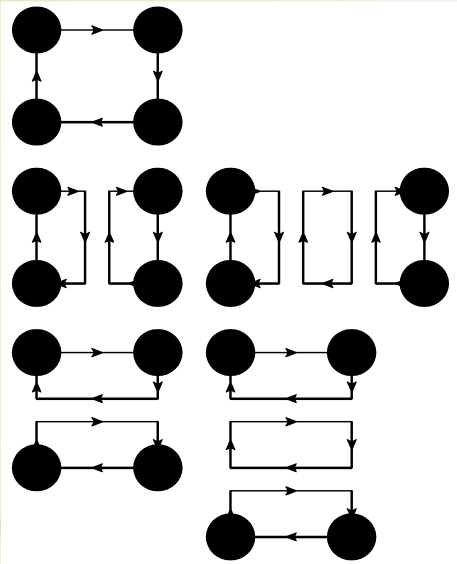


A meson with momentum r_- decay to a pair of quark propagators are described by the form factor $\Phi_k^{a,b}(x)$

This kind of diagram is counted by the form factor

Analytically solved in Callan et al. PRD13,1649(1976) and J.C.F Barbon et al.,Nucl.Phys. B434,109 (1995)
Some numerical results is shown in J.C.F Barbon et al.,Nucl.Phys. B434,109 (1995) and E.Abdalla,R.Mohayee,arxiv:hep-th/9610059(1997)

The leading order 2 mesons → 2 mesons Scattering in the 1/N-expression



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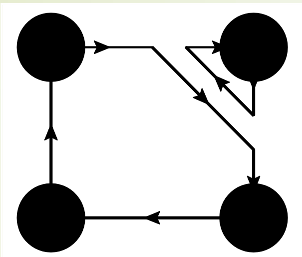
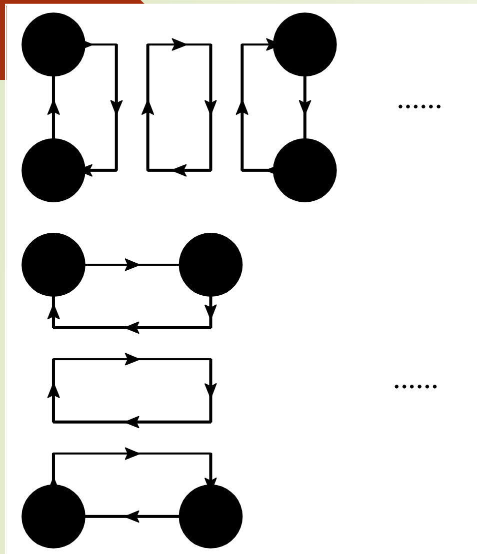
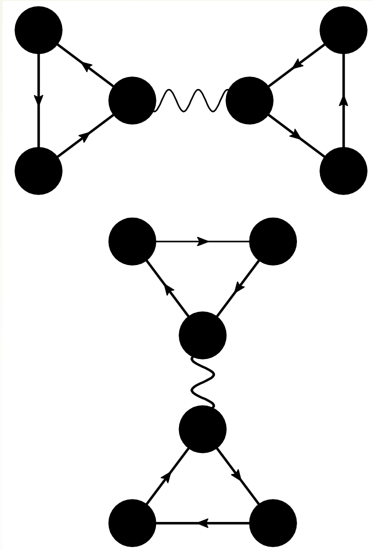
The leading order diargram of the scattering



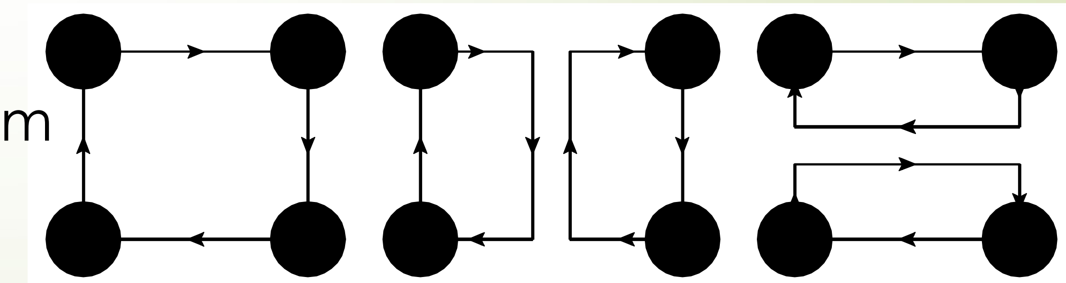
Counted by the form factor



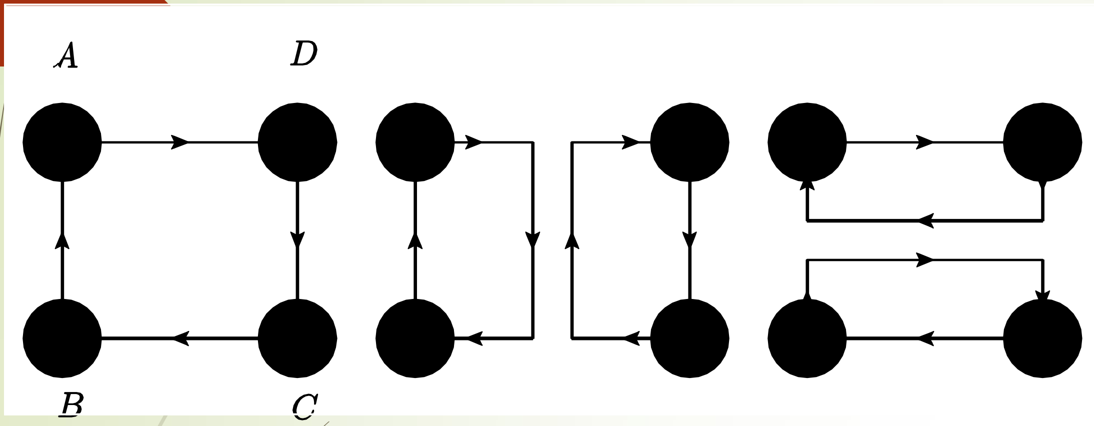


We didn’t calculate the one meson exchange diagram

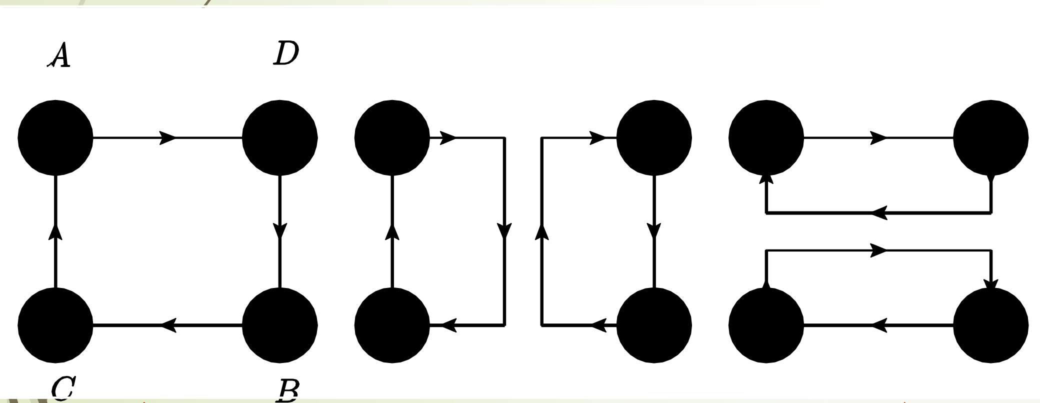
We only calculated the contact diagram



For a particular scattering: A+B→C+D



No $\frac{1}{\lambda}$ in each diagram



The result are $\frac{1}{\lambda}$ independent

The $\frac{1}{\lambda}$ of the 1st diagram is canceled by the other 2 diagrams

Some analytic results

1.Two-flavor:

$$A(q^a\overline{q}^b)+B(q^b\overline{q}^a) \to C(q^a\overline{q}^b)+D(q^b\overline{q}^a)$$

$$\omega_1 = \frac{r_{B-}}{r_{C-}}$$

$$\omega_2 = \frac{r_{D-}}{r_{C-}}$$

$$i\mathcal{M} = (1+\mathcal{P})(1+\mathcal{C})i\mathcal{M}_0.$$

$$i\mathcal{M}_0 = \theta(\omega_2-\omega_1)i4g^2\omega_1\int_0^1 dx\int_0^1 dy\frac{1}{(y\omega_1-\omega_2-x)^2}\varphi_A(\frac{\omega_2-\omega_1+x}{\omega_2-\omega_1+1})\varphi_B(y)\varphi_C(x)\varphi_D(\frac{y\omega_1}{\omega_2}),$$

2.Single-flavor:

$$A(q^a\overline{q}^a)+B(q^a\overline{q}^a) \to C(q^a\overline{q}^a)+D(q^a\overline{q}^a)$$

$$i\mathcal{M} = (1+\mathcal{R})(1+\mathcal{P})(1+\mathcal{C})i\mathcal{M}_0 + (1+\mathcal{R})i\mathcal{M}_1,$$

$$\begin{aligned}
i\mathcal{M}_1 = & \Big\{-\theta(1-\omega_1)i4g^2\int_0^1 dxP\int_0^1 dy\frac{\omega_1\omega_2}{[(y-1)\omega_1+(1-x)\omega_2]^2}\varphi_A(\frac{x\omega_2}{1+\omega_2-\omega_1})\varphi_B(y)\varphi_C(y\omega_1)\varphi_D(x) \\
& +(B\leftrightarrow C, A\leftrightarrow D, \omega_1\to 1/\omega_1, \omega_2\to\frac{1+\omega_2-\omega_1}{\omega_1})\Big\}+ \\
& \Big\{-\theta(\omega_2-\omega_1)i4g^2\int_0^1 dxP\int_0^1 dy\frac{\omega_1}{(y\omega_1-x)^2}\varphi_A(\frac{x+\omega_2-\omega_1}{1+\omega_2-\omega_1})\varphi_B(y)\varphi_C(x)\varphi_D(\frac{(y-1)\omega_1+\omega_2}{\omega_2}) \\
& +(A\leftrightarrow C, B\leftrightarrow D, \omega_1\to\frac{\omega_2}{1+\omega_2-\omega_1}, \omega_2\to\frac{\omega_1}{1+\omega_2-\omega_1})\Big\}+ \\
& \Big\{-\theta(\omega_2-\omega_1)\theta(\omega_1-1)i\frac{4\pi}{N_c}\int_0^1 dx\left[2r_{C+}r_{C-}+2r_{D+}r_{C-}+\frac{M_a^2}{x-\omega_1}+\frac{M_a^2}{x-1}-\frac{M_a^2}{x-\omega_1+\omega_2}-\frac{M_a^2}{x}\right] \\
& \times\varphi_A(\frac{x-\omega_1+\omega_2}{1+\omega_2-\omega_1})\varphi_B(x/\omega_1)\varphi_C(x)\varphi_D(\frac{x-\omega_1+\omega_2}{\omega_2}) + (A\leftrightarrow D, B\leftrightarrow C, \omega_1\to 1/\omega_1, \omega_2\to\frac{1+\omega_2-\omega_1}{\omega_1}) \\
& +(A\leftrightarrow B, C\leftrightarrow D, \omega_1\to\frac{1+\omega_2-\omega_1}{\omega_2}, \omega_2\to 1/\omega_2) + (A\leftrightarrow C, B\leftrightarrow D, \omega_1\to\frac{\omega_2}{1+\omega_2-\omega_1}, \omega_2\to\frac{\omega_1}{1+\omega_2-\omega_1})\Big\},
\end{aligned}$$

C: $(A\leftrightarrow C, B\leftrightarrow D, \omega_1\to\frac{\omega_2}{1+\omega_2-\omega_1}, \omega_2\to\frac{1}{1+\omega_2-\omega_1})$

P: $(A\leftrightarrow B, C\leftrightarrow D, \omega_1\to\frac{1+\omega_2-\omega_1}{\omega_2}, \omega_2\to\frac{1}{\omega_2})$

R: $(C\leftrightarrow D, \omega_1\to\frac{\omega_1}{\omega_2}, \omega_2\to\frac{1}{\omega_2})$

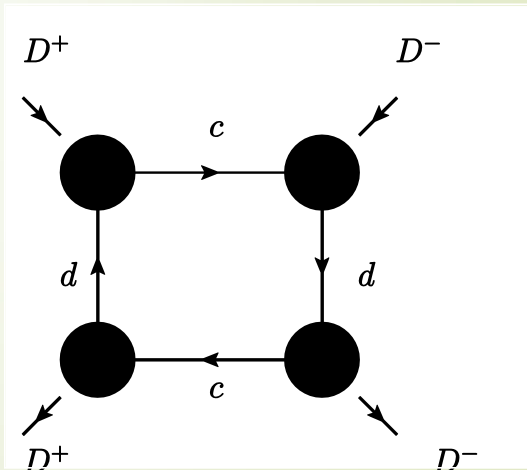
Example for scattering process with nonvanishing amplit

A 2-flavor example

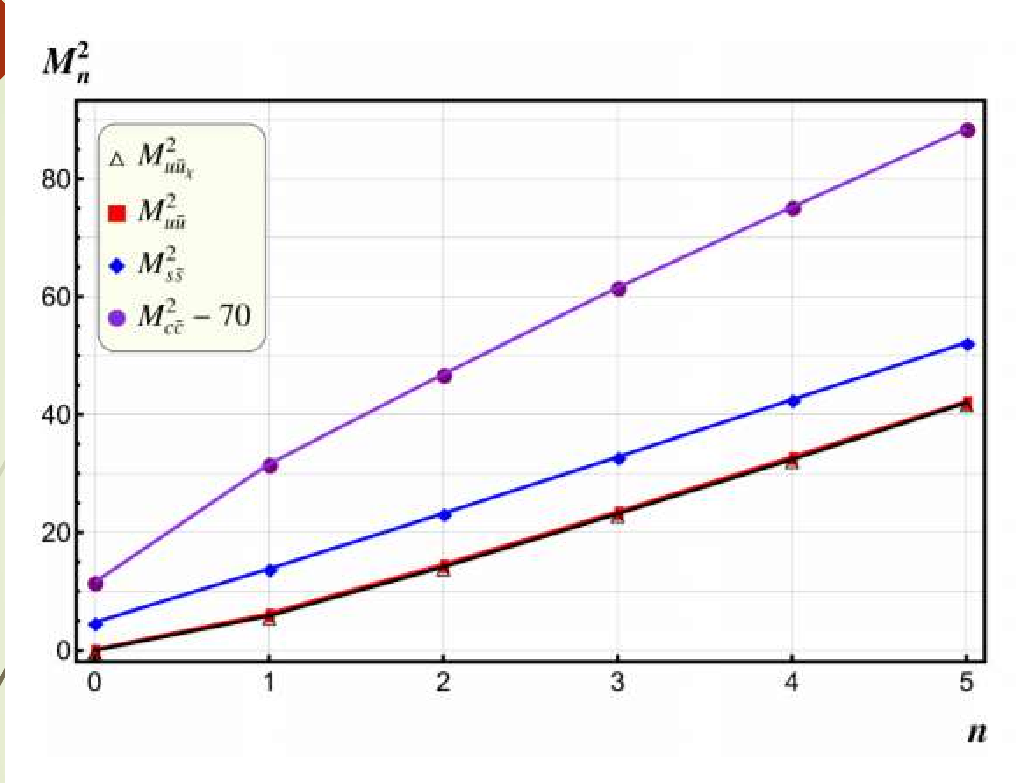
$$K^+(u\bar{s}) + K^-(s\bar{u}) \to K^+(u\bar{s}) + K^-(s\bar{u})$$

1.The line between two incoming(outgoing) mesons should connect an quark and an antiquark of a same flavor.

2. The line Between one incoming and one outgoing mesons should connect two quark(antiquark) of a same flavor.



Meson Spectra



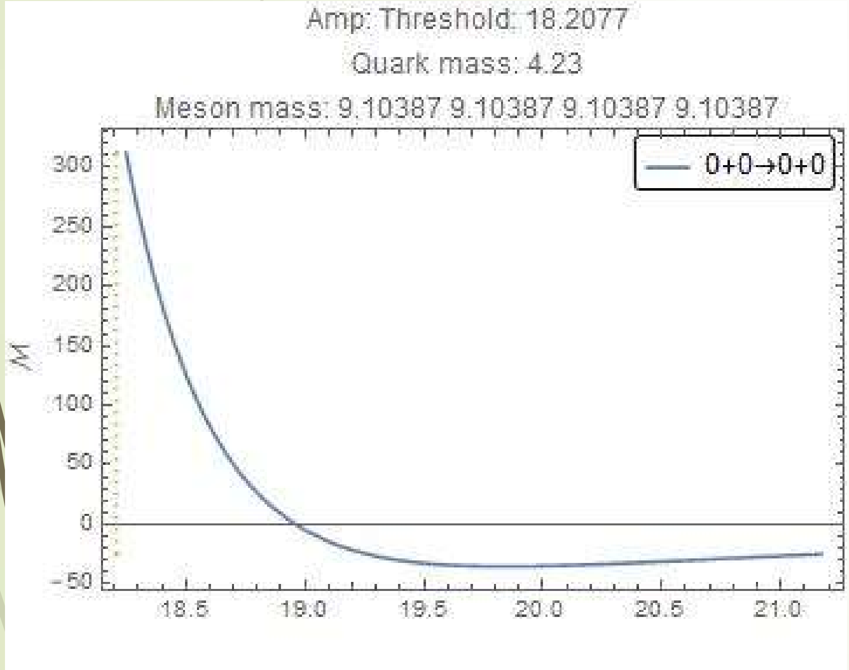
Yu Jia, et al.
arXiv:1708.0939v1[hep-ph](2017)

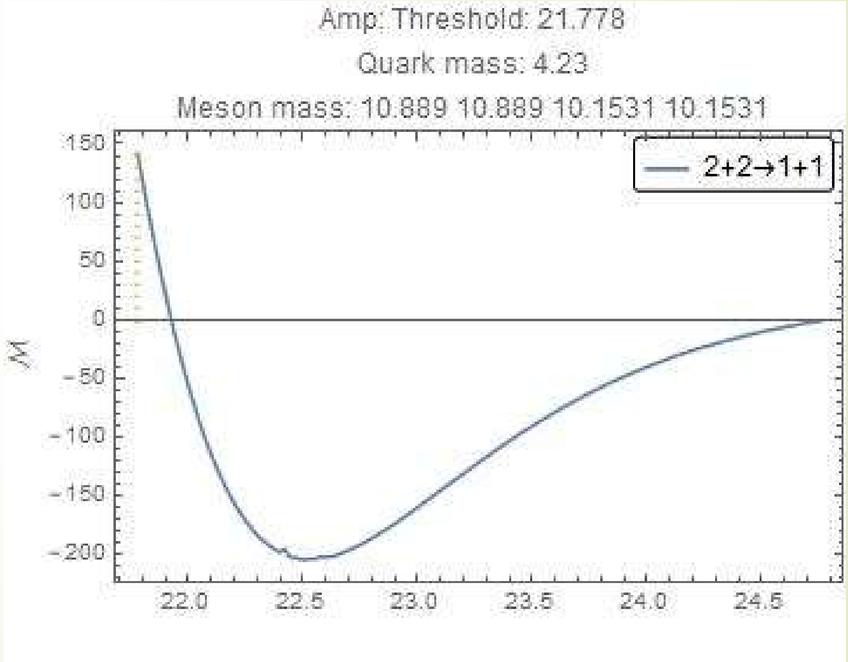
$$\bar{g} = \sqrt{\frac{g^2 N}{2\pi}} = 340Mev, m_u = 0.045\bar{g}, m_c = 4.23\bar{g}, m_s = 0.749\bar{g}$$

M.Burdardt, Phys.Rev.D(2000)

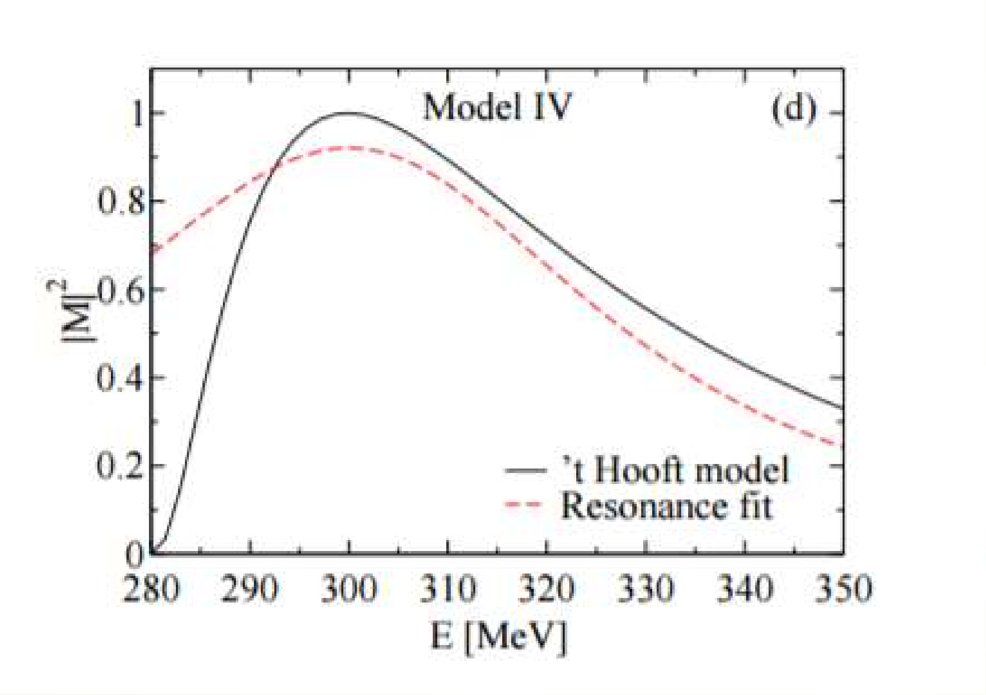
Numerical description
(only 1 free parameter in the center-of-mass frame)

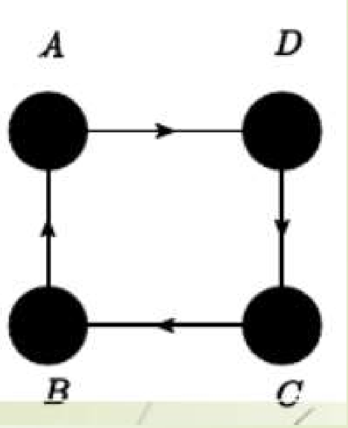
$$c\bar{c}+c\bar{c} \rightarrow c\bar{c}+c\bar{c}$$





Z Batiz et al. PRC(2004), incomplete





$$|\mathcal{M}|^2 = \tilde{g}^4 \frac{1}{(s - m_r^2)^2 + m_i^4}$$

$$|\mathcal{M}|^2 \approx \bar{g}^4 \frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4}$$

Summary

1. Calculate the leading order of the meson-meson scattering amplitude analyticly via the Feynman Diagram and the form fector, which is showed $\frac{1}{\lambda}$ independent. Classify the possible 2-2 processes of different flavors with nonvanishing amplitude.

2. The numerical result of the charmonium-charmonium scattering shows two different kind of spectrum:

   the spectrum of bound state is monotonously decreasing from the threshold;

   the spectrum of two 2^{nd} excited charmonium to two 1^{st} excited charmonium is a hypobolic curve.

Thank you