

NLO correction to $\rho\gamma^* \rightarrow \pi$ form factors in k_T factorization

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outline

Motivation

Factorization of $\rho\gamma^* \rightarrow \pi$

Numerical Results

Summary

Motivation

pseudoscalar meson

π

NLO

Perturbative QCD factorization of $\pi\gamma^*\rightarrow\gamma(\pi)$ and $B\rightarrow\gamma(\pi)l\nu$

Hsiang-nan Li

Phys. Rev. D64(2001) 014019

Next-to-leading-order corrections to exclusive processes in k_T factorization

$\pi\gamma^*\rightarrow\gamma$

S.Nandi, Hsiang-nan Li

Phys. Rev. D76(2003) 034008

Next-to-leading-order correction to pion form factor in k_T factorization

$\pi\gamma^*\rightarrow\pi$

Hsiang-nan Li, Yue-Long Shen, Yu-Ming Wang, Hao Zou

Phys. Rev. D83(2011) 054029

Next-to-leading-order corrections to $B\rightarrow\pi$ form factors in k_T factorization

$B\rightarrow\pi$

Hsiang-nan Li, Yue-Long Shen, Yu-Ming Wang

Phys. Rev. D85(2012) 074004

NLO twist-3 contributions to $B\rightarrow\pi$ form factors in k_T factorization

Shan Cheng, Ying-Ying Fan, Xin Yu, Cai-Dian Lü, Zhen-Jun Xiao

Phys. Rev. D89(2014) 094004

Motivation

vector meson

ρ

$\rho\gamma^*\rightarrow\pi(\rho)$ transition form factors in the perturbative QCD factorization approach
Ya-lan Zhang , Shan Cheng, Jun Hua, Zhen-Jun Xiao

Phys.Rev.D92(2015)094031

Revisiting the factorization theorem for $\rho\gamma^*\rightarrow\pi(\rho)$ at twist 3
Shan Cheng, Ya-lan Zhang ,Jun Hua, Hsiang-nan Li , Zhen-Jun Xiao

Phys.Rev.D95(2017)076005

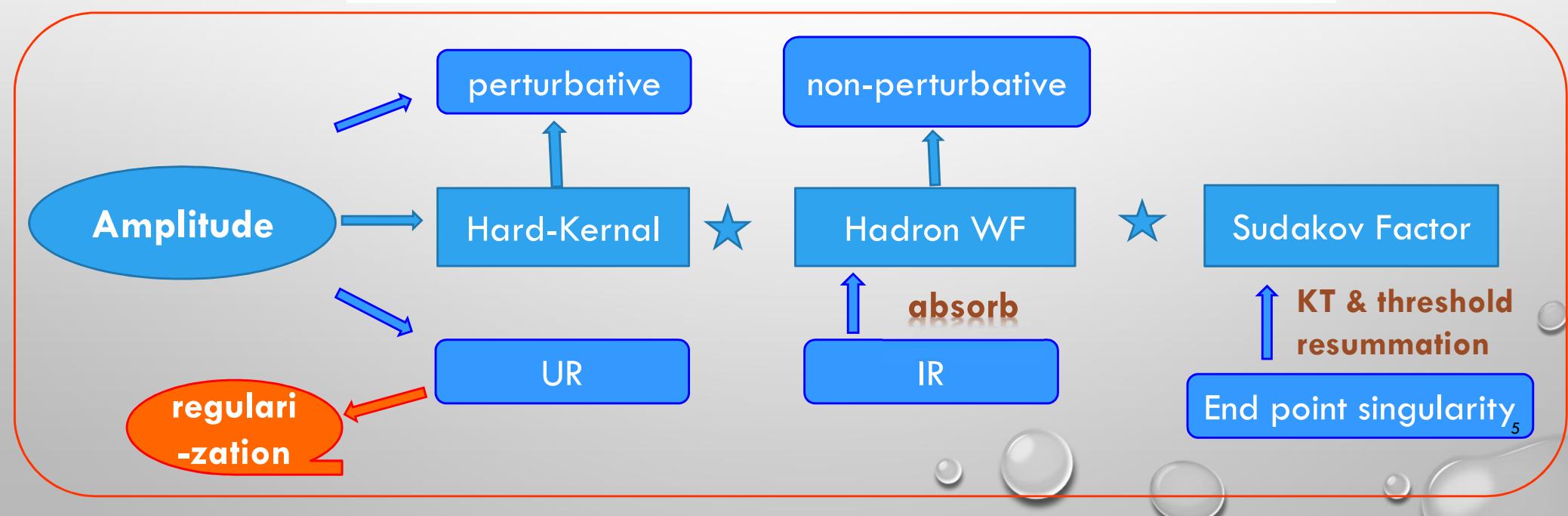
We've proved the factorization theorem of $\rho\gamma^*\rightarrow\pi(\rho)$

Factorization of $\rho\gamma^* \rightarrow \pi$

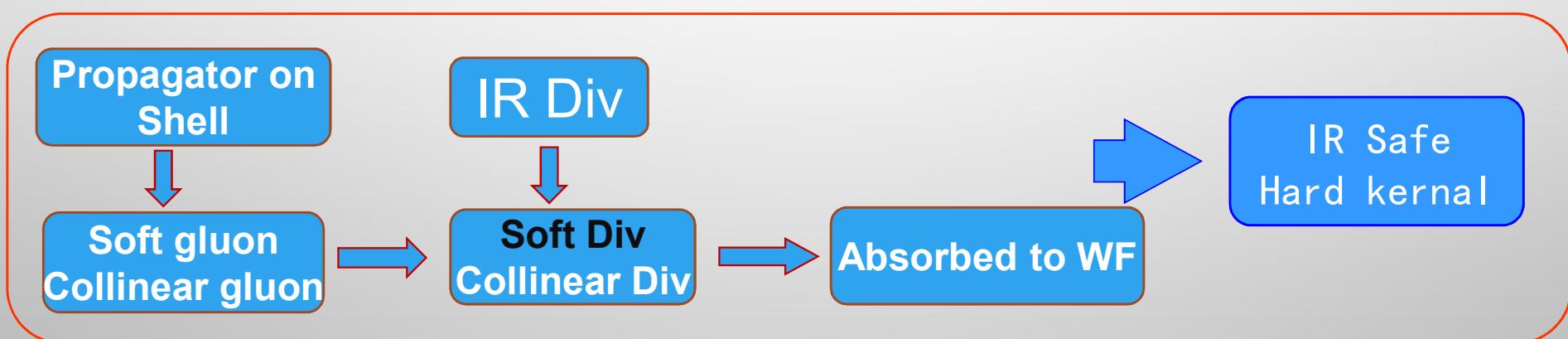
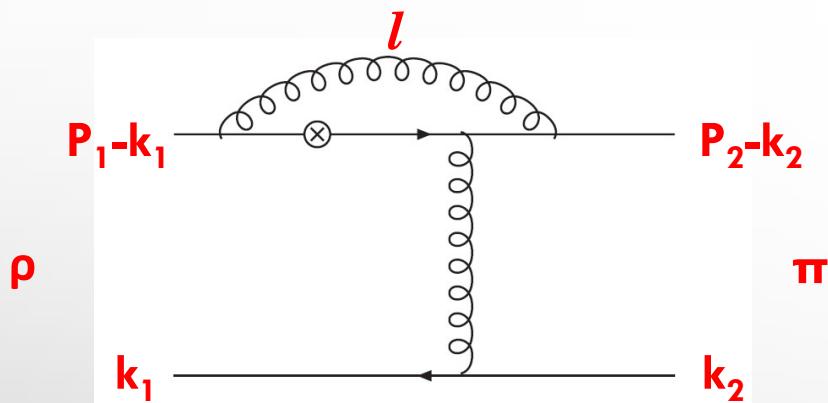
- **K_T FACTORIZATION**

$$\mathcal{A}(B \rightarrow M_2 M_3) \sim \int dx_1 dx_2 dx_3 db_1 db_2 db_3$$

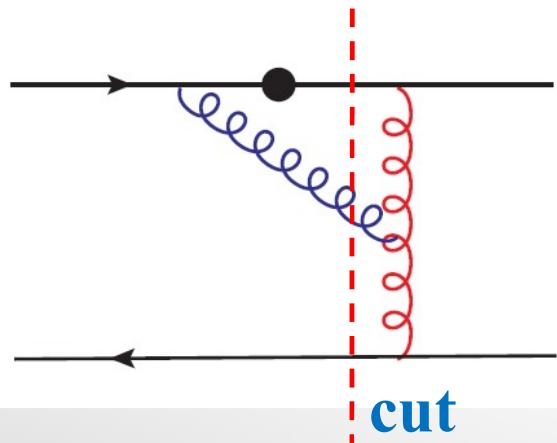
$$Tr[C(t)\Phi_B(x_1, b_1)\Phi_{M_2}(x_2, b_2)\Phi_{M_3}(x_3, b_3)H(x_i, b_i, t)S_t(x_i)e^{-s(t)}]$$



Factorization of $\rho\gamma^* \rightarrow \pi$



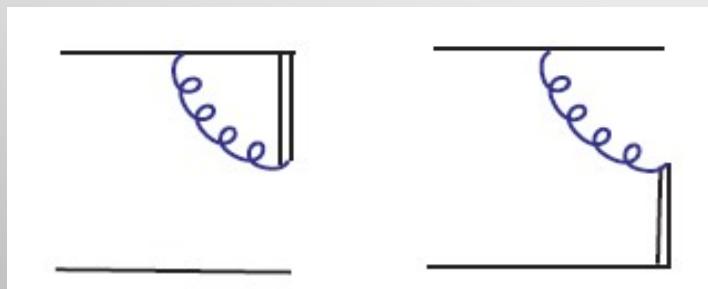
Factorization of $\rho\gamma^* \rightarrow \pi$



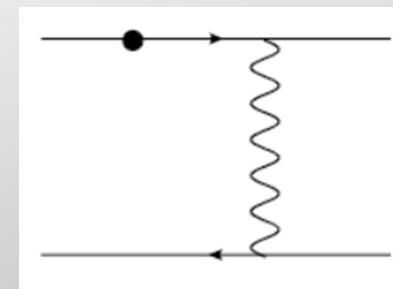
Fierz identity

$$I_{ij} I_{lk} = \frac{1}{4} I_{ik} I_{lj} + \frac{1}{4} (\gamma_5)_{ik} (\gamma_5)_{lj} + \frac{1}{4} (\gamma^\alpha)_{ik} (\gamma_\alpha)_{lj} \\ + \frac{1}{4} (\gamma_5 \gamma^\alpha)_{ik} (\gamma_\alpha \gamma_5)_{lj} + \frac{1}{8} (\gamma_5 \delta^{\alpha\beta})_{ik} (\delta_{\alpha\beta} \gamma_5)_{lj}$$

Effective WF



Hard kernel

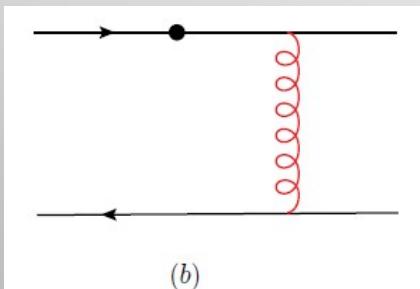


Factorization of $\rho\gamma^*\rightarrow\pi$

$$H^{(1)}(x_1, k_{1T}, x_2, k_{2T}, Q^2) = \underline{G^{(1)}(x_1, k_{1T}, x_2, k_{2T}, Q^2)} - \int dx'_1 d^2 k'_{1T} \underline{\Phi^{(1)}(x_1, k_{1T}; x'_1, k'_{1T}) H^{(0)}(x'_1, k'_{1T}, x_2, k_{2T}, Q^2)}$$

$$- \int dx'_2 d^2 k'_{2T} H^{(0)}(x_1, k_{1T}, x'_2, k'_{2T}, Q^2) \Phi^{(1)}(x_2, k_{2T}; x'_2, k'_{2T}),$$

Quark diagram



Effective diagram

TL23

$$H_{b,23}^{(0)} = \frac{eg_s^2 C_F}{2} \frac{[\not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha [\gamma_5 m_\pi^0 \phi_\pi^p] \gamma_\alpha (\not{p}_2 - \not{k}_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2}$$

Factorization of $\rho\gamma^*\rightarrow\pi$

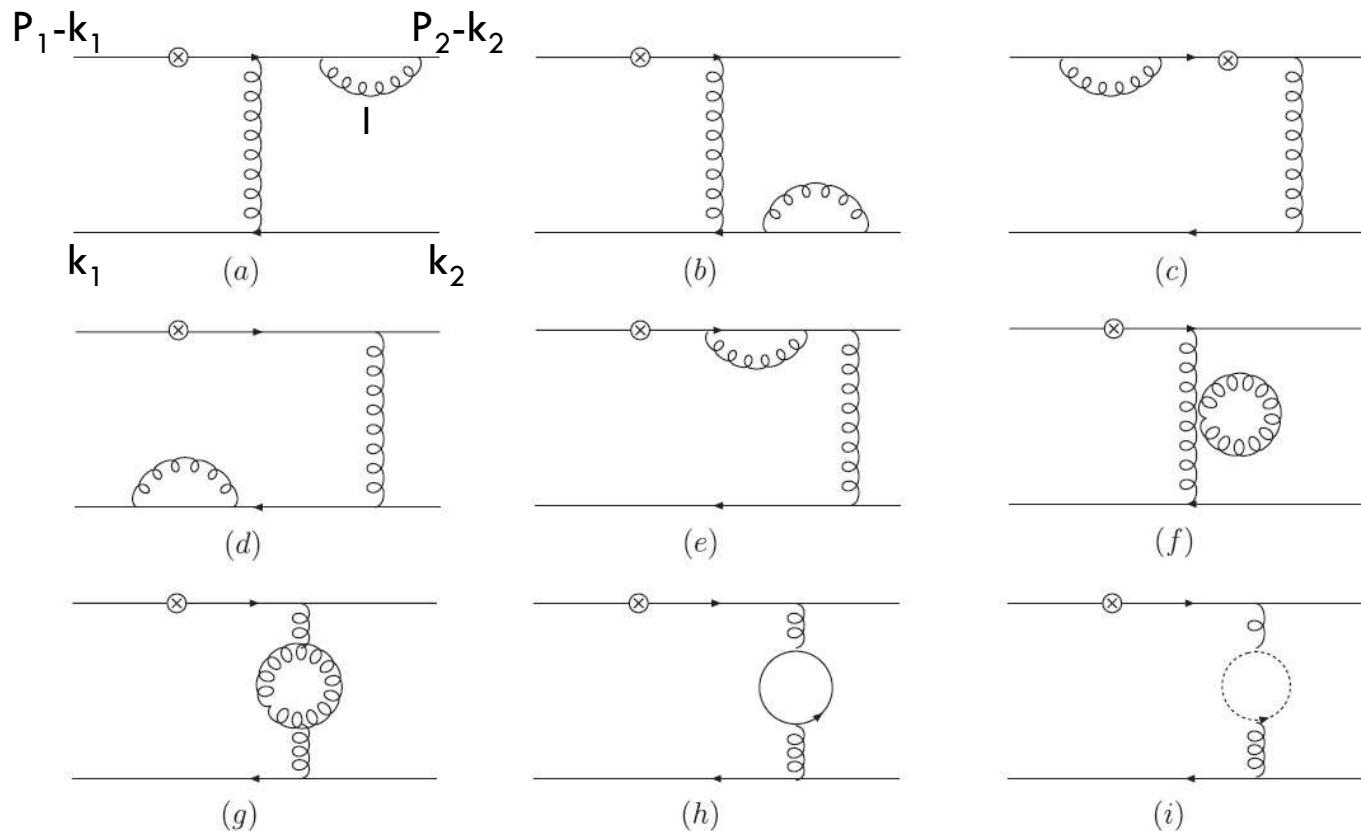


FIG. 2. Self-energy diagrams.

Factorization of $\rho\gamma^* \rightarrow \pi$

$$G_{2a,2b}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left(\frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{\delta_2 Q^2 e^{\gamma_E}} + 2 \right) \otimes H^{(0)}$$

$$G_{2c,2d}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left(\frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{\delta_1 Q^2 e^{\gamma_E}} + 2 \right) \otimes H^{(0)}$$

$$G_{2e}^{(1)} = -\frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{x_1 Q^2 e^{\gamma_E}} + 2 \right) \otimes H^{(0)}$$

$$G_{2f+2g+2h+2i}^{(1)} = -\frac{\alpha_s}{4\pi} \left(\frac{5}{3} N_c - \frac{2}{3} N_f \right) \left(\frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{\delta_{12} Q^2 e^{\gamma_E}} \right) \otimes H^{(0)}$$

$$\delta_1 = \frac{k_{1T}^2}{Q^2} \quad \delta_2 = \frac{k_{2T}^2}{Q^2}$$

$$\delta_{12} = \frac{x_1 x_2 Q^2 + |k_{1T} - k_{2T}|^2}{Q^2}$$

$$x_{1,2} \gg \delta_{12} \gg \delta_{1,2}$$

Factorization of $\rho\gamma^*\rightarrow\pi$

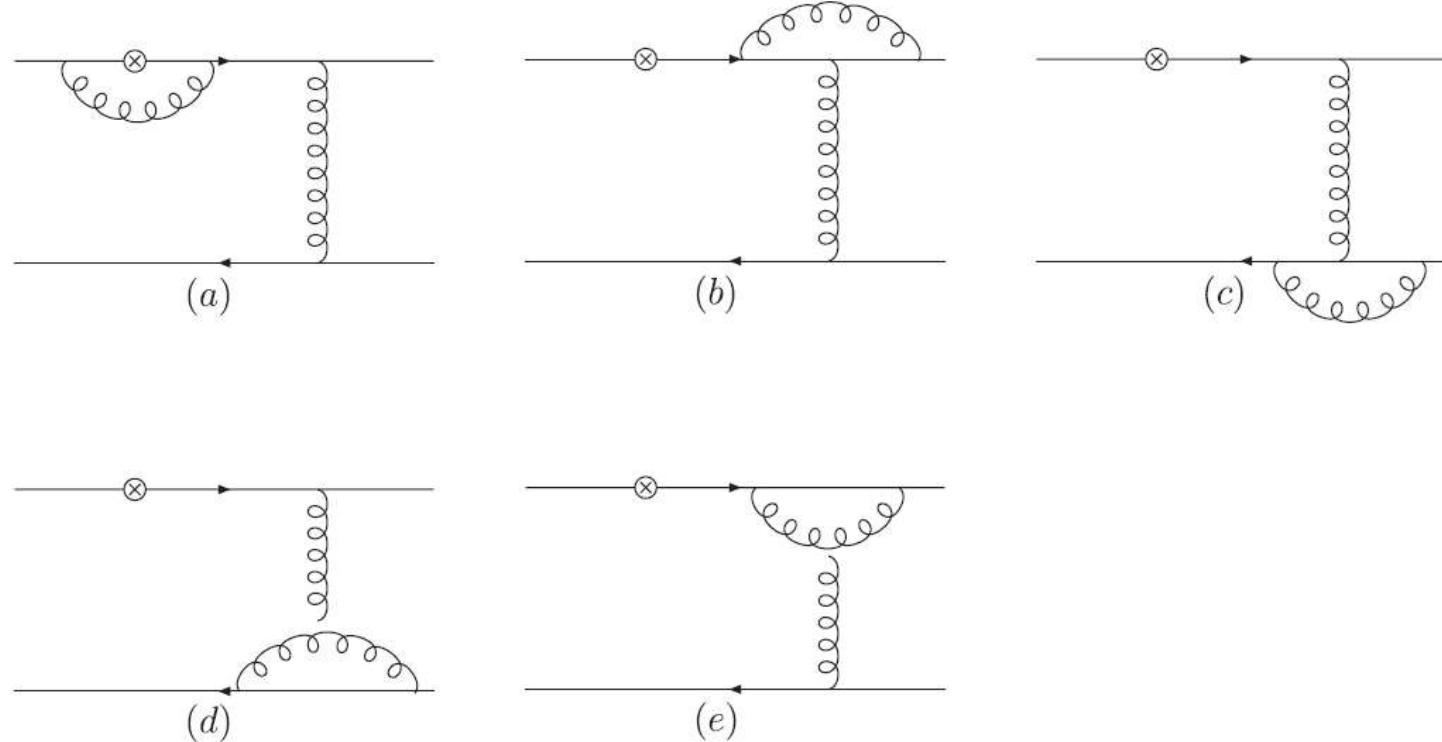


FIG. 3. Vertex-correction diagrams.

Factorization of $\rho\gamma^* \rightarrow \pi$

$$G_{3a}^{(1)} = \frac{\alpha_s C_F}{4\pi} \left\{ \frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{Q^2 e^{\gamma_E}} - 2 \ln \delta_1 \ln x_1 - 2 \ln \delta_1 - 2 \ln x_1 - \frac{\pi^2}{3} - \frac{3}{2} \right\} \otimes H^{(0)}$$

$$G_{3b}^{(1)} = -\frac{\alpha_s}{8\pi N_C} \left\{ \frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{x_1 Q^2 e^{\gamma_E}} + 2 \right\} \otimes H^{(0)}$$

$$G_{3c}^{(1)} = -\frac{\alpha_s}{8\pi N_C} \left\{ \frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{\delta_{12} Q^2 e^{\gamma_E}} - \ln \frac{\delta_{12}}{\delta_1} \ln \frac{\delta_{12}}{\delta_2} + \ln \frac{\delta_1 \delta_2}{\delta_{12}^2} - \frac{\pi^2}{3} + \frac{3}{2} \right\} \otimes H^{(0)}$$

$$G_{3d}^{(1)} = \frac{\alpha_s N_C}{8\pi} \left\{ \frac{3}{\varepsilon} + 3 \ln \frac{4\pi\mu^2}{\delta_{12} Q^2 e^{\gamma_E}} + 2 \ln \left(\frac{\delta_{12}^2}{\delta_1 \delta_2} \right) + 5 \right\} \otimes H^{(0)}$$

$$G_{3e}^{(1)} = \frac{\alpha_s N_C}{8\pi} \left\{ \frac{3}{\varepsilon} + 3 \ln \frac{4\pi\mu^2}{x_1 Q^2 e^{\gamma_E}} + \ln \frac{x_1}{\delta_2} \left(1 - \ln \frac{x_1}{\delta_{12}} \right) + 2 \ln \frac{x_1}{\delta_{12}} - \frac{2\pi^2}{3} + 5 \right\} \otimes H^{(0)}$$

Factorization of $\rho\gamma^*\rightarrow\pi$

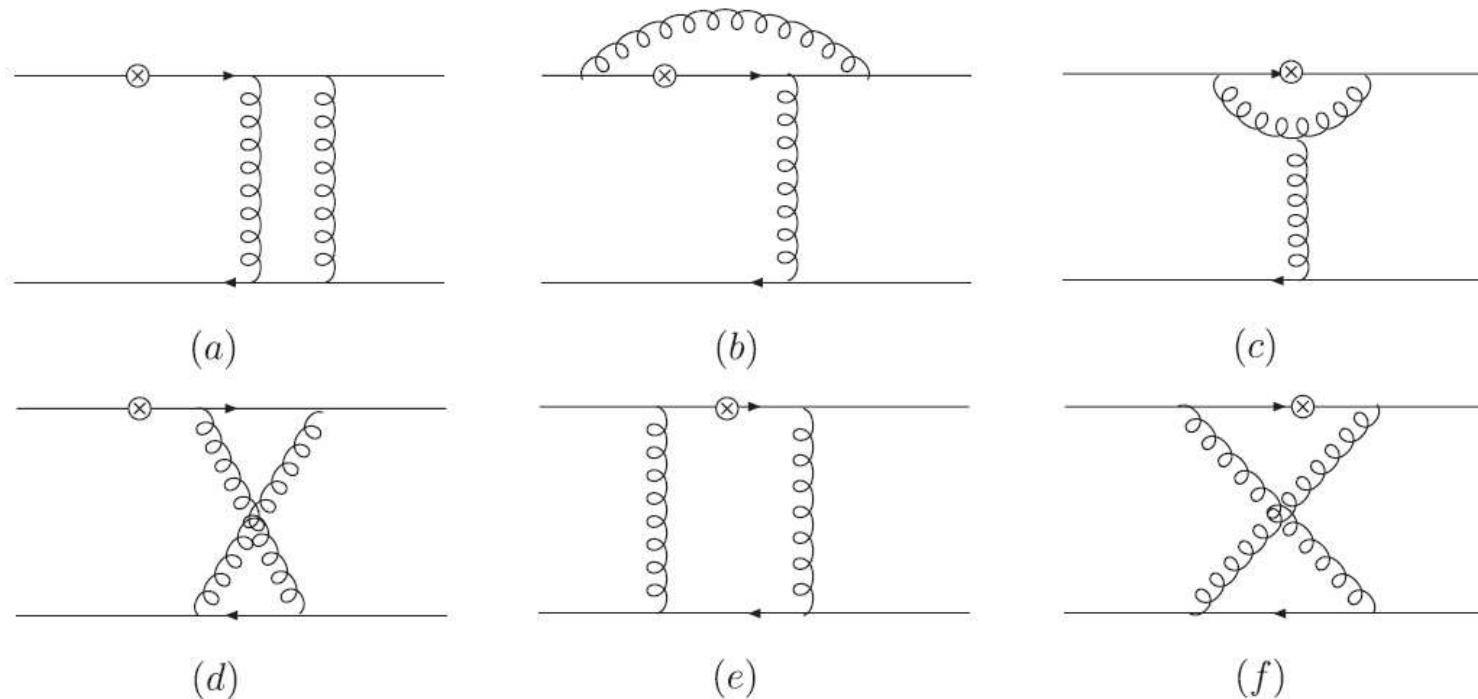


FIG. 4. Box and pentagon diagrams.

Factorization of $\rho\gamma^* \rightarrow \pi$

$$G_{4b}^{(1)} = \frac{\alpha_s}{8\pi N_C} \left\{ \ln \delta_1 \ln \delta_2 - \ln \delta_1 \ln x_1 - \ln x_1 (1 - \ln x_1) + \ln \delta_2 + \frac{\pi^2}{6} \right\} \otimes H^{(0)}$$

$$G_{4c}^{(1)} = \frac{\alpha_s N_C}{8\pi} \{-3 \ln x_2 + 2 \ln \delta_1 - 2\} \otimes H^{(0)}$$

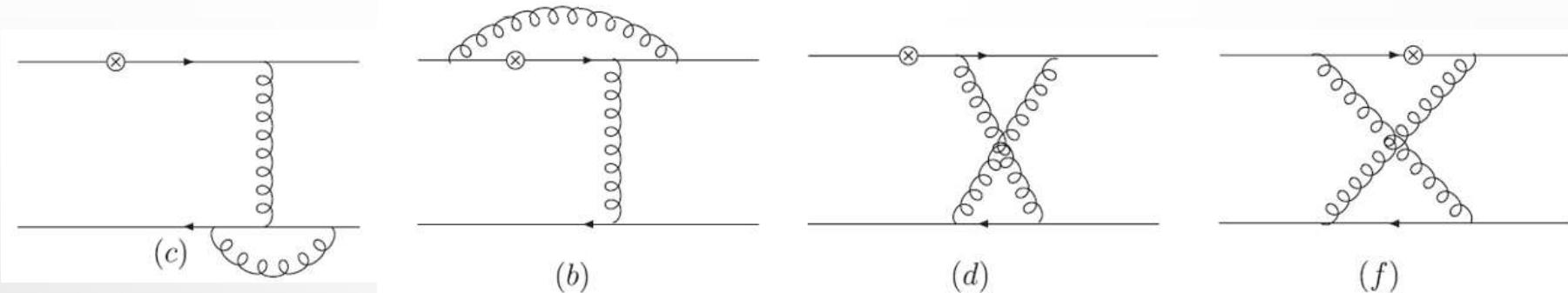
$$\dots \frac{\alpha_s N_C}{8\pi} [x_1 \leftrightarrow x_2, \delta_1 \leftrightarrow \delta_2] \otimes \overline{H}^{(0)}$$

$$G_{4d}^{(1)} = -\frac{\alpha_s}{8\pi N_C} \left\{ \ln \frac{\delta_{12}}{\delta_1} \ln \frac{x_1}{\delta_2} + \frac{\pi^2}{6} \right\} \otimes H^{(0)}$$

$$G_{4f}^{(1)} = -\frac{\alpha_s}{8\pi N_C} \left\{ \ln \frac{\delta_{12}}{x_1 \delta_1} \ln \frac{\delta_{12}}{\delta_2} - \frac{\pi^2}{4} - \frac{1}{2} \right\} \otimes H^{(0)}$$

$$\dots - \frac{\alpha_s}{8\pi N_C} [x_1 \leftrightarrow x_2, \delta_1 \leftrightarrow \delta_2] \otimes \overline{H}^{(0)}$$

Factorization of $\rho\gamma^* \rightarrow \pi$



$$G_{3c}^{(1)} = -\frac{\alpha_s}{8\pi N_c} \left\{ \frac{1}{\varepsilon} + \ln \frac{4\pi\mu^2}{\delta_{12} Q^2 e^{\gamma_E}} - \ln \frac{\delta_{12}}{\delta_1} \ln \frac{\delta_{12}}{\delta_2} + \ln \frac{\delta_1 \delta_2}{\delta_{12}^2} - \frac{\pi^2}{3} + \frac{3}{2} \right\} \otimes H^{(0)}$$

$$G_{4b}^{(1)} = \frac{\alpha_s}{8\pi N_c} \left\{ \ln \delta_1 \ln \delta_2 - \ln \delta_1 \ln x_1 - \ln x_1 (1 - \ln x_1) + \ln \delta_2 + \frac{\pi^2}{6} \right\} \otimes H^{(0)}$$

$$G_{4d}^{(1)} = -\frac{\alpha_s}{8\pi N_c} \left\{ \ln \frac{\delta_{12}}{\delta_1} \ln \frac{x_1}{\delta_2} + \frac{\pi^2}{6} \right\} \otimes H^{(0)}$$

$$G_{4f}^{(1)} = -\frac{\alpha_s}{8\pi N_c} \left\{ \ln \frac{\delta_{12}}{x_1 \delta_1} \ln \frac{\delta_{12}}{\delta_2} - \frac{\pi^2}{4} - \frac{1}{2} \right\} \otimes H^{(0)}$$

Factorization of $\rho\gamma^*\rightarrow\pi$

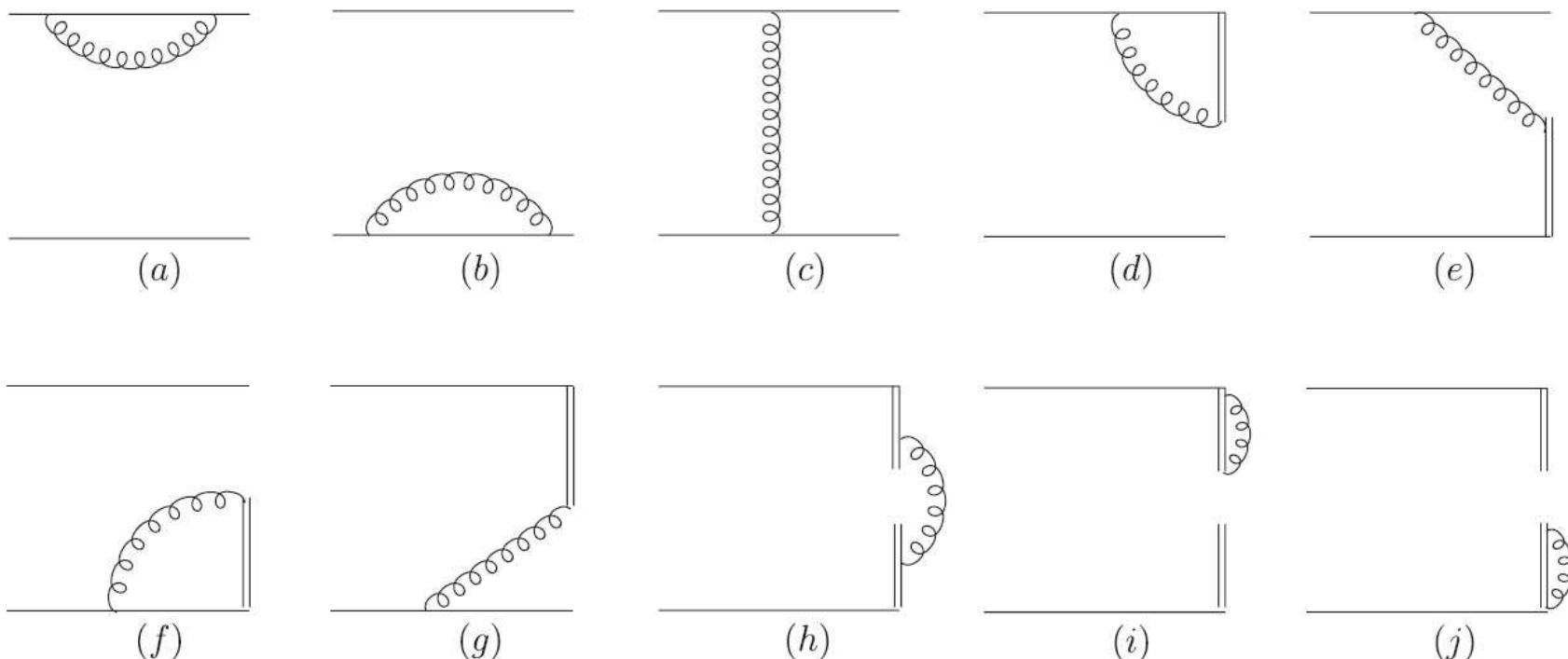


FIG. 5. Effective diagrams.

Factorization of $\rho\gamma^* \rightarrow \pi$

$$\phi_{\rho,a}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[\frac{1}{\varepsilon} + \ln \frac{4\pi u^2}{\delta_1 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)}$$

$$\phi_{\rho,b}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left[\frac{1}{\varepsilon} + \ln \frac{4\pi u^2}{\delta_1 Q^2 e^{\gamma_E}} + 2 \right] H^{(0)}$$

$$\phi_{\rho,c}^{(1)} \otimes H^{(0)} = 0$$

$$\phi_{\rho,d}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\varepsilon} + \ln \frac{4\pi u^2}{\xi_1^2 Q^2 e^{\gamma_E}} - \ln^2 \frac{k_{1T}^2}{\xi_1^2} - \ln \frac{k_{1T}^2}{\xi_1^2} - \frac{\pi^2}{3} + 2 \right] H^{(0)}$$

$$\phi_{\rho,e}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{k_{1T}^2}{x_1 \xi_1^2} + \frac{2\pi^2}{3} \right] H^{(0)}$$

$$\phi_{\rho,f}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\varepsilon} + \ln \frac{4\pi u^2}{\xi_1^2 Q^2 e^{\gamma_E}} - \ln^2 \frac{k_{1T}^2}{x_1^2 \xi_1^2} - \ln \frac{k_{1T}^2}{x_1^2 \xi_1^2} - \frac{\pi^2}{3} \right] H^{(0)}$$

$$\phi_{\rho,g}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{k_{1T}^2}{x_1^2 \xi_1^2} - \frac{\pi^2}{3} \right] H^{(0)}$$

$$(\phi_{\rho,h}^{(1)} + \phi_{\rho,i}^{(1)} + \phi_{\rho,j}^{(1)}) \otimes H^{(0)} = \frac{\alpha_s C_F}{2\pi} \left[\frac{1}{\varepsilon} + \ln \frac{4\pi u^2}{\delta_{12} Q^2 e^{\gamma_E}} \right] H^{(0)}$$

Factorization of $\rho\gamma^* \rightarrow \pi$

Quark diagram

$$G^{(1)} = \frac{\alpha_s C_F}{8\pi} \left\{ \frac{21}{2} \left(\frac{1}{\varepsilon} + \ln \frac{4\pi u^2}{Q^2 e^{\gamma_E}} \right) - 4 \ln x_1 \ln \delta_1 - 2 \ln x_2 \ln \delta_2 + \frac{9}{4} \ln x_1 \ln x_2 - \frac{1}{4} \ln^2 x_1 \right. \\ \left. - 4 \ln x_1 - \frac{27}{4} \ln x_2 - \frac{3}{2} \ln \delta_{12} + \frac{163}{8} - \frac{27}{16} \pi^2 \right\} H^{(0)}$$

Effective diagram

$$\phi_\rho^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left[\frac{3}{\varepsilon} + 3 \ln \frac{4\pi u^2}{\delta_1 Q^2 e^{\gamma_E}} + \underbrace{(2 \ln x_1 + 3) \ln \frac{\xi_1^2}{\delta_1 Q^2}}_{=} + 2 \ln \frac{\xi_1^2}{\delta_{12} Q^2} + \ln x_1 (\ln x_1 + 2) + 2 - \frac{\pi^2}{3} \right] H^{(0)}$$

$$H^{(0)} \otimes \phi_\pi^{(1)} = \frac{\alpha_s C_F}{8\pi} \left[\frac{2}{\varepsilon} + 2 \ln \frac{4\pi u^2}{\delta_2 Q^2 e^{\gamma_E}} + \underbrace{(2 \ln x_2 + 2) \ln \frac{\xi_2^2}{\delta_2 Q^2}}_{=} + 2 \ln \frac{\xi_2^2}{\delta_{12} Q^2} + \ln x_2 (\ln x_2 + 2) + 2 - \frac{\pi^2}{3} \right] H^{(0)}$$

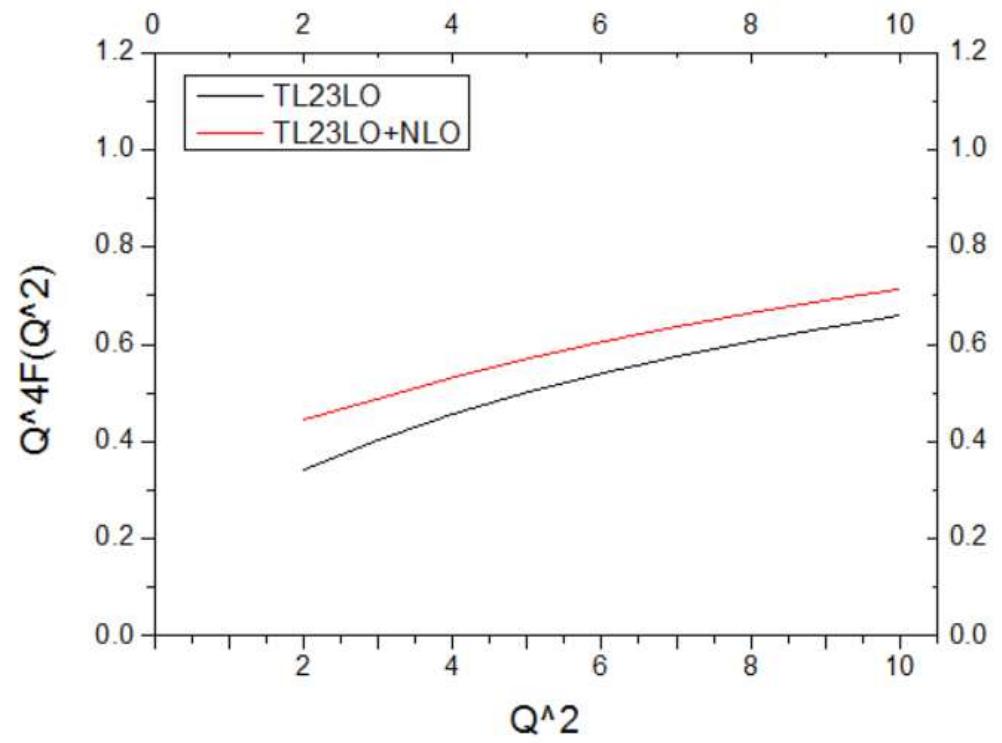
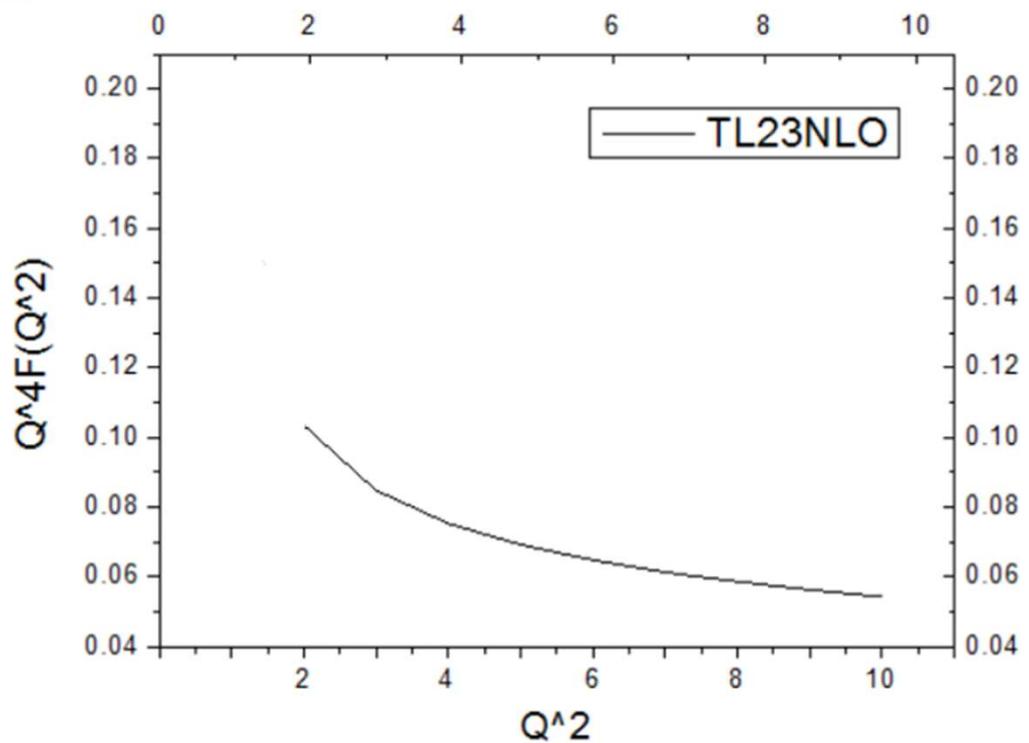
Factorization of $\rho\gamma^*\rightarrow\pi$

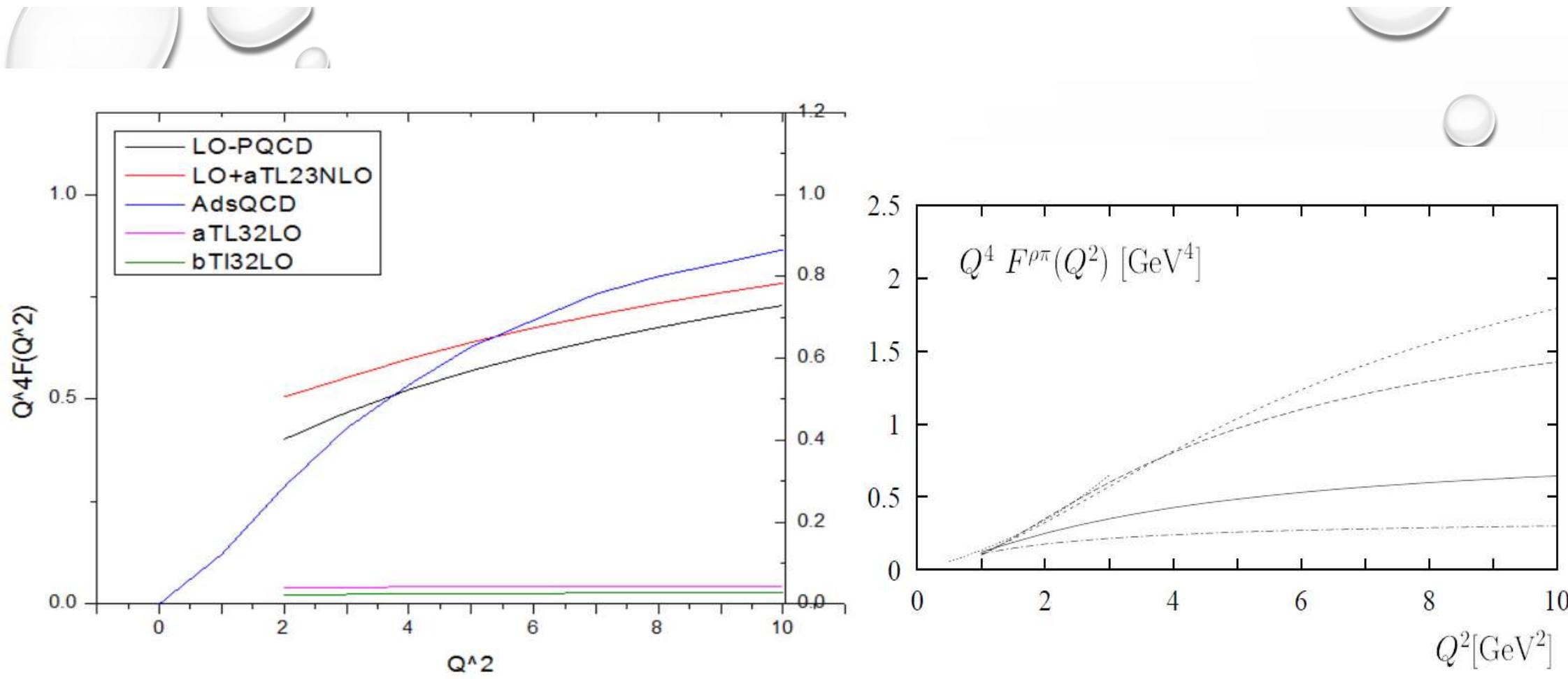
$$H^{(1)}(x_1, k_{1T}, x_2, k_{2T}, Q^2) = G^{(1)}(x_1, k_{1T}, x_2, k_{2T}, Q^2) - \int dx'_1 d^2 k'_{1T} \Phi^{(1)}(x_1, k_{1T}; x'_1, k'_{1T}) H^{(0)}(x'_1, k'_{1T}, x_2, k_{2T}, Q^2)$$
$$- \int dx'_2 d^2 k'_{2T} H^{(0)}(x_1, k_{1T}, x'_2, k'_{2T}, Q^2) \Phi^{(1)}(x_2, k_{2T}; x'_2, k'_{2T}),$$

IR Safe Hard-Kernal

$$H^{(1)} = \frac{\alpha_s(\mu_f) C_F}{8\pi} \left\{ \frac{21}{2} \ln \frac{u^2}{Q^2} - 8 \ln \frac{u_f^2}{Q^2} + \frac{9}{4} \ln x_1 \ln x_2 - \frac{7}{4} \ln^2 x_1 - \ln^2 x_2 - 8 \ln x_1 \right.$$
$$\left. - \frac{35}{4} \ln x_2 + \frac{9}{2} \ln \delta_{12} + \frac{115}{8} - \frac{49}{48} \pi^2 \right\} H^{(0)}$$

Numerical Results





gamma* rho0 ---> pi0 Transition Form Factor in Extended AdS/QCD Models

Fen Zuo (Beijing, Inst. Theor. Phys.) , Yu Jia, Tao Huang (Beijing, Inst. High Energy Phys. & Unlisted, CN)

Oct 2009 - 16 pages

Eur.Phys.J. C67 (2010) 253-261

Summary

We calculated NLO corrections to the $\rho\gamma^* \rightarrow \pi$ form factors at twist3 the in k_T factorization theorem .

The NLO corrections are about 10% of the form factors .

Thank You !