Improved Formalism of the SLH Model and Updated Vertices

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S.-P. He, **Y.-N. Mao**, C. Zhang, and S.-H. Zhu, arXiv: 1709.08929.

Introduction

- The Little Higgs framework was proposed to solve the "little hierarchy problem"; N. Arkani-Hamed, A. G. Cohen, E. Katz, and A. E. Nelson, JHEP 0207 (2002) 034.

- It contains a lot of models, including the "Simplest Little Higgs" model; D. E. Kaplan and M. Schmaltz, JHEP 0310 (2003) 039.

- In this model, the $Zh\eta$ vertex (η is a pseudoscalar) have been discussed since the model was born, and the η phenomenology is correlated with $\eta f\bar{f}$ vertices; W. Kilian, D. Rainwater, and J. Reuter, Phys. Rev. D71 (2005) 015008; etc.

- However, we accidentally meet some unintelligible problems during the phenomenology research of the SLH model, which make us turn to check the basic formalism of the SLH model, and at last we found out the key point and improve it;

A Brief Review of the SLH Model (Scalar Sector)

- Global Symmetry breaking $[\mathrm{SU}(3) \times \mathrm{U}(1)]^2 \to [\mathrm{SU}(2) \times \mathrm{U}(1)]^2$ at scale $f \gg v$;
- Gauge symmetry breaking $\mathrm{SU}(3) \times \mathrm{U}(1) \to \mathrm{SU}(2)_L \times \mathrm{U}(1) \to \mathrm{U}(1)_{\mathrm{em}}$;
- Ten Nambu-Goldstone boson are generated, in which eight are eaten by the massive gauge bosons, and two are left as physical scalars;
- One (h) is a 0^+ scalar, we treat it as the 125 GeV Higgs, the other (η) is a 0^- scalar;
- The two scalar triplets $\Phi_{1,2}$ transformed as $(\mathbf{1}, \mathbf{3})$ and $(\mathbf{3}, \mathbf{1})$ respectively.

- The nonlinear realization of the scalar triplets:

$$\Phi_1 = \mathrm{e}^{\mathrm{i}\Theta'}\mathrm{e}^{\mathrm{i}t_\beta\Theta}\begin{pmatrix} \mathbf{0}_{1\times 2} \\ f c_\beta \end{pmatrix}, \qquad \Phi_2 = \mathrm{e}^{\mathrm{i}\Theta'}\mathrm{e}^{-\mathrm{i}\Theta/t_\beta}\begin{pmatrix} \mathbf{0}_{1\times 2} \\ f s_\beta \end{pmatrix};$$

with the definitions of the matrix fields

$$\Theta \equiv \frac{1}{f}\left(\frac{\eta \mathbb{I}_{3\times 3}}{\sqrt{2}} + \begin{pmatrix} \mathbf{0}_{2\times 2} & \phi \\ \phi^\dagger & 0 \end{pmatrix}\right), \quad \text{and} \quad \Theta' \equiv \frac{1}{f}\left(\frac{G' \mathbb{I}_{3\times 3}}{\sqrt{2}} + \begin{pmatrix} \mathbf{0}_{2\times 2} & \varphi \\ \varphi^\dagger & 0 \end{pmatrix}\right).$$

- η is the pseudoscalar field; $\phi \equiv \left((v_h + h - \mathrm{i}G)/\sqrt{2}, G^-\right)^T$ is the usual Higgs doublet; G' and $\varphi \equiv (y^0, x^-)^T$ are all eaten by the five heavy gauge bosons.

- The covariant derivative term $(D_\mu\Phi_1)^\dagger(D^\mu\Phi_1)+(D_\mu\Phi_2)^\dagger(D^\mu\Phi_2)$;

- $D_\mu \equiv \partial_\mu - \mathrm{i}\mathbb{G}_\mu$, where the gauge field matrix

$$\mathbb{G} = \frac{A^3}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix} + \frac{A^8}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} & W^+ & Y^0 \\ W^- & & X^- \\ \bar{Y}^0 & X^+ & \end{pmatrix} + \frac{t_W B}{3\sqrt{1-t_W^2/3}}\mathbb{I}$$

- θ_W is the electro-weak mixing angle, $W^\pm$ and $X^\pm$ are charged, $Y(\bar{Y}) = (Y_1 \pm \mathrm{i}Y^2)/\sqrt{2}$

- A^3, A^8, B are linear combinations of γ, Z, Z' at LO of (v/f);

- Z, Z', Y^2 have further mixing beyond leading order of (v/f).

- Divide the $Zh\eta$ vertex into two parts: antisymmetric type $(h\partial_\mu\eta - \eta\partial_\mu h)$ and symmetric type $(h\partial_\mu\eta + \eta\partial_\mu h)$, we are interested in the antisymmetric type coupling since the symmetric type amplitude is proportional to the mass of the fermion linking to Z and in the SLH model it can be canceled by $h\eta f\bar{f}$ contact vertex;

- If we calculate naively through the covariant derivative term, we can get the usually antisymmetric interaction as $(\sqrt{2}m_Z \cot 2\beta/f)(h\partial_\mu\eta - \eta\partial_\mu h)Z^\mu \propto v/f$;

- However, this treatment is careless on the two-point transitions in the lagrangian, which may lead to additional contributions to a physical process, thus we cannot naively calculate a physical process with this vertex;

- The formalism must be improved if we want to remove all the two-point transitions.

The points are:

- Find a basis to remove all cross-terms in the scalar kinetic part of the lagrangian, the two point function must have the form $\mathrm{i}\delta^2\Gamma/\delta S_a\delta S_b = -\mathrm{i}(p_a^\mu p_{b,\mu}\delta_{ab} - m_{ab})$;

- The gauge fixing term must cancel all the two-point transitions like $V_\mu \partial^\mu S$;

- No additional two-point transitions generated from these operations, for example, no additional cross-terms arising from the gauge boson kinetic parts;

- All sectors can be diagonalized together;

Then we can perform calculations following the usually procedures.

Improved Formalism of the SLH model

- The kinetic part of CP-odd scalar sector is non-canonically normalized: $(\mathbb{K}_{ij}/2)\partial_\mu G_i \partial^\mu G_j$ with $\mathbb{K} \neq \mathbb{I}$ and $G_{i/j}$ denotes one of (η, G, G', y^2);

- Consider the linear space spanned by the four G_i, we need a new basis S_i, in which the fields are canonically normalized: $\mathcal{L} \supset (\delta_{ij}/2)\partial_\mu S_i \partial^\mu S_j$;

- Define the inner product $\langle S_i|S_j\rangle = \delta_{ij}$, it is easy to show $\langle G_i|G_j\rangle = (\mathbb{K}^{-1})_{ij}$;

- This relation is important in the following calculations.

- The VEVs in $\Phi_{1,2}$ will lead to the two-point vector-scalar transitions, parameterized as $V_p^\mu \mathbb{F}_{pi} \partial_\mu G_i$, where $\mathbb{F}$ is a 4×3 matrix, with mass dimension 1;

- It must be canceled by the gauge fixing terms thus $\mathcal{L}_{\text{G.F.}} \supset (\partial_\mu V_p^\mu) \mathbb{F}_{pi} G_i$;

- Define another basis $\bar{G}_p = \mathbb{F}_{pi} G_i$, we have $\langle \eta | \bar{G}_p \rangle = 0$ and $\langle \bar{G}_p | \bar{G}_q \rangle = (\mathbb{M}_V^2)_{pq}$, where $\mathbb{M}_V^2$ is the mass matrix in the basis (Z, Z', Y^2);

- We can use a matrix $\mathbb{R}$ to diagonalize it $(\mathbb{R} \mathbb{M}_V^2 \mathbb{R}^T)_{pq} = m_p^2 \delta_{pq}$;

- It is natural to define $\tilde{G}_p = \mathbb{R}_{pq} \bar{G}_q / m_p = (\mathbb{R}\mathbb{F})_{pi} G_i / m_p$ thus $\langle \tilde{G}_p | \tilde{G}_q \rangle = \delta_{pq}$;

- Now we have a canonically normalized basis $(\eta/\sqrt{(\mathbb{K}^{-1})_{11}}, \tilde{G}_p)$;
- The two-point transition becomes $m_p \tilde{V}_p^\mu \partial_\mu \tilde{G}_p$ with $\tilde{V}_p^\mu = \mathbb{R}_{pq} V_q$;
- Choose the gauge fixing term $\mathcal{L}_{\text{G.F.}} = -\sum_p (1/2\xi_p)(\partial_\mu \tilde{V}_p^\mu - \xi_p m_p \tilde{G}_p)^2$;
- $\tilde{G}_p$ is the corresponding Goldstone eaten by $\tilde{V}_p$ with its mass $\sqrt{\xi_p} m_p$;
- Realization of $\Phi_{1,2}$ keeps no mass mixing between η and $\tilde{G}_p$.

- Comparing with the naive treatment: η mass eigenstate is proportional to η, however, all the three original Goldstones degrees of freedom (G, G', y^2) contain η component;

- When we consider the interactions including η, we must consider the same interaction with G, G', y^2 together from the original lagrangian;

- Divide $\mathbb{F}$ into two parts: $\mathbb{F} = (\tilde{f}, \tilde{\mathbb{F}})$, where $\tilde{f}_p = \mathbb{F}_{p1}$ is a 1×3 vector, whose components are the coefficients of $V_\mu^p \partial^\mu \eta$ transition;

- Thus in G_j, the coefficient of η component is $(\tilde{\mathbb{F}}^{-1}\tilde{f})_j$;

- Physical coefficient $\tilde{c}_\eta = \sqrt{(\mathbb{K}^{-1})_{11}}\left(c_\eta - (\tilde{\mathbb{F}}^{-1}\tilde{f})_j c_j\right)$.

Results of $Zh\eta$ Vertex

- Follow to procedure above, we can calculate $Zh\eta$ vertex directly;
- It is easy to check, at $\mathcal{O}(v/f)$, $\tilde{c}_{Zh\eta} = 0$, which is different from the previous result;
- Calculate the matrix elements to $\mathcal{O}(v^3/f^3)$, we have the result

$$\tilde{c}_{Zh\eta} = \frac{m_Z}{2\sqrt{2}c_W^2 t_{2\beta} v} \left(\frac{v}{f}\right)^3$$

- In that result, we must consider the mixing among (Z, Z', Y^2).

Results on $\eta f\bar{f}$ Vertex

- Yukawa lagrangian with anomaly free embedding: F. del Águila, J. I. Illana, and M. D. Jenkins, JHEP 1103 (2011) 080; O. C. W. Kong, Report No. NCU-HEP-k009.

$$
\begin{aligned}
\mathcal{L}_y \;=\; & \mathrm{i}\lambda_N^j \bar{N}_{R,j}\Phi_2^\dagger L_j - \frac{\mathrm{i}\lambda_\ell^{jk}}{\Lambda}\bar{\ell}_{R,j}\,\mathrm{det}\left(\Phi_1,\Phi_2,L_k\right) \\
& +\mathrm{i}\left(\lambda_t^a\bar{u}_{R,3}^a\Phi_1^\dagger + \lambda_t^b\bar{u}_{R,3}^b\Phi_2^\dagger\right)Q_3 - \mathrm{i}\frac{\lambda_{b,j}}{\Lambda}\bar{d}_{R,j}\,\mathrm{det}\left(\Phi_1,\Phi_2,Q_3\right) \\
& +\mathrm{i}\left(\lambda_{d,n}^a\bar{d}_{R,n}^a\Phi_1^T + \lambda_{d,n}^b\bar{d}_{R,n}^b\Phi_2^T\right)Q_n - \mathrm{i}\frac{\lambda_u^{jk}}{\Lambda}\bar{u}_{R,j}\,\mathrm{det}\left(\Phi_1^*,\Phi_2^*,Q_k\right)
\end{aligned}
$$

- Fermion doublets are enlarged to triplets, $L=(\nu_L,\ell_L,\mathrm{i}N_L)^T$, $Q_1=(d_L,-u_L,\mathrm{i}D_L)$, $Q_2=(s_L,-c_L,\mathrm{i}S_L)$, $Q_3=(t_L,b_L,\mathrm{i}T_L)$.

- For the SM fermions, we parameterize the Yukawa couplings to η as

$$\mathcal{L} \supset -\sum_f c_{\eta,f} \frac{i m_f}{v} \eta \bar{f} \gamma^5 f;$$

- For $f = \nu, \ell, u, c, b$, we have $c_{\eta,f} = 0$ to all order of (v/f);

- For $f = d, s, t$, we have $c_{\eta,f} \neq 0$ due to the left-handed mixing between SM and corresponding additional quarks:

$$c_{\eta,f} = \pm \left(\frac{v}{\sqrt{2} f} \right) \frac{c_{2\beta} + c_{2\theta}}{s_{2\beta}}$$

- In the equation above, choose "+" for t, "−" for d and s, θ is the right-handed mixing angle between SM and the corresponding additional quarks.

Conclusions and Discussions

- We improved the formalism of the SLH model, performed the canonically normalization and canceled all two-point transitions carefully;
- With the improved formalism, we recalculated the $Zh\eta$ and $\eta f\bar{f}$ vertices, which are quite different from the previous results in a lot of papers;
- This procedure can be applied to other nonlinear realized models (in linear realized models, we don't face this problem, because the kinetic part can be canonically normalized automatically thus we can easily find a correct basis);
- The η phenomenologies change a lot comparing with the previous results, as Y.-N. Mao, arXiv: 1703.10123; K. Cheung, S.-P. He, Y.-N. Mao, and C. Zhang, in preparation.

Back up: Some Calculation Details

Some calculations are extremely cumbersome, we show results to $\mathcal{O}(v^3/f^3)$

$$\mathbb{K} = \mathbb{I}_{4\times 4} + \xi \begin{pmatrix} \mathbf{0} & \mathbb{A} \\ \mathbb{A}^T & \mathbf{0} \end{pmatrix} + \xi^2 \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbb{B} \end{pmatrix} + \xi^3 \begin{pmatrix} \mathbf{0} & \mathbb{C} \\ \mathbb{C}^T & \mathbf{0} \end{pmatrix} + \mathcal{O}(\xi^4)$$

in the basis (η, G', G, y^2) and $\xi \equiv v/f$. $\mathbb{A}, \mathbb{B}, \mathbb{C}$ are 2×2 Matrixes

$$\mathbb{A} = \begin{pmatrix} \frac{\sqrt{2}}{t_{2\beta}} & -\sqrt{2} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} -\frac{5+3c_{4\beta}}{12 s_{2\beta}^2} & \frac{2}{3t_{2\beta}} \\ \frac{2}{3t_{2\beta}} & 0 \end{pmatrix}, \quad \mathbb{C} = \begin{pmatrix} -\frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3} & \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2} \\ \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2} & -\frac{2\sqrt{2}}{3t_{2\beta}} \end{pmatrix}$$

We also have $(\mathbb{K}^{-1})_{11} = 1 + (2/t_{2\beta}^2)\xi^2 + \mathcal{O}(\xi^4)$.

$$\frac{\tilde{f}}{gf} = \left(\frac{1}{\sqrt{2}c_W t_{2\beta}}\xi^2, \frac{\rho}{t_{2\beta}}\xi^2, -\xi + \frac{5+3c_{4\beta}}{6s_{2\beta}^2}\xi^3\right)^T + \mathcal{O}(\xi^4);$$

$$\frac{\tilde{\mathbb{F}}}{gf} = \begin{pmatrix} -\frac{\xi^2}{2\sqrt{2}c_W} & \frac{\xi}{2c_W} - \frac{(5+3c_{4\beta})\xi^3}{24c_W s_{2\beta}^2} & \frac{\xi^3}{3c_W t_{2\beta}} \\ \sqrt{\frac{2}{3-t_W^2}} - \frac{\kappa(1+c_{2W})\xi^2}{\sqrt{2}c_{2W}} & \kappa\xi - \frac{\kappa(5+3c_{4\beta})\xi^3}{12s_{2\beta}^2} & -\frac{2\kappa\xi^3}{3c_{2W}t_{2\beta}} \\ \frac{-2\xi^3}{3t_{2\beta}} & \frac{\sqrt{2}\xi^2}{3t_{2\beta}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \mathcal{O}(\xi^4);$$

The definitions $\rho \equiv \sqrt{\frac{1+2c_{2W}}{1+c_{2W}}}$ and $\kappa \equiv \frac{c_{2W}}{2c_W^2\sqrt{3-t_W^2}}$

To $\mathcal{O}(\xi^3)$, the off-diagonal elements of $\mathbb{R}$ can be written as

$$\mathbb{R}_{pq} = \frac{(\mathbb{M}_V^2)_{pq}}{(\mathbb{M}_V^2)_{pp} - (\mathbb{M}_V^2)_{qq}}$$

The matrix

$$\mathbb{R} = \begin{pmatrix} 1 & -\frac{\kappa\rho^2\xi^2}{2c_W} & -\frac{\sqrt{2}\xi^3}{3c_W t_{2\beta}} \\ \frac{\kappa\rho^2\xi^2}{2c_W} & 1 & -\frac{2\sqrt{2}(1+2c_{2W})\kappa\xi^3}{3c_{2W}t_{2\beta}} \\ \frac{\sqrt{2}\xi^3}{3c_W t_{2\beta}} & \frac{2\sqrt{2}(1+2c_{2W})\kappa\xi^3}{3c_{2W}t_{2\beta}} & 1 \end{pmatrix} + \mathcal{O}(\xi^4)$$

The End, Thank You!

广告

- 原计划的报告题目因内容过多临时决定更换，附上原计划报告的文章链接：
- Y.-N. Mao, “Spontaneous CP-violation in the Simplest Little Higgs Model and Its Future Collider Tests: the Scalar Sector”, arXiv: 1703.10123.
- 我对味物理不熟悉，诚征熟悉味物理的合作者进行关于SLH模型的味物理研究
- Email: maoyn@ihep.ac.cn