Higgs EFT at future lepton colliders

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DESY & IHEP

Higgs physics toward CDR meeting
July 19, 2017

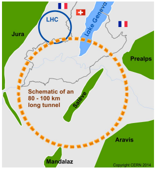
[arXiv:1704.02333] G. Durieux, C. Grojean, JG, K. Wang
and current work with N. Craig, S. Di Vita, G. Durieux, C. Grojean, Z. Liu, G. Panico, M. Riembau, T. Vantalon

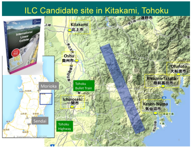
Introduction

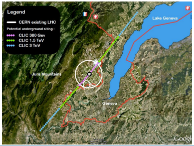
- Higgs and nothing else? What next?
- An e^+e^- collider is an obvious direction to go.
- Higgs factory ($e^+e^- \to hZ$ at 240-250 GeV, $e^+e^- \to \nu\bar{\nu}h$ at higher energies), and many more other measurements.
- The scale of new physics Λ is large $\Rightarrow$ EFT is a good description at low energy.
- A global analysis of the Higgs coupling constraints, in the EFT framework.
- Robust constraints on the triple Higgs coupling at both circular and linear colliders. (2nd part of this talk)

Future e^+e^- colliders

- Circular colliders
  - The Circular Electron-Positron Collider (CEPC) in China.
  - The Future Circular Collider (FCC-ee) at CERN.
  - 91 GeV(Z-pole), 160 GeV(WW), 240 GeV(hZ) and 350 GeV($t\bar{t}$).
  - Large luminosity.
  - A natural step towards a 100 TeV hadron collider.
- Linear colliders
  - The International Linear Collider (ILC) in Japan.
  - The Compact Linear Collider (CLIC) at CERN.
  - ILC: 250 GeV, 350 GeV, 500 GeV and possibly 1 TeV.
  - CLIC: 350(380) GeV, 1.4(1.5) TeV and 3 TeV.
  - Can go to higher $\sqrt{s}$, and also implement longitudinal beam polarizations.

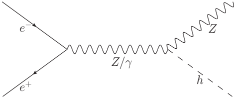




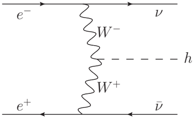


Higgs measurements

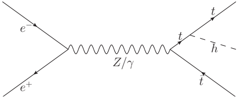
- $e^+e^- \to hZ$, cross section maximized at around 250 GeV.



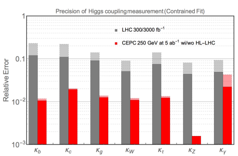
- $e^+e^- \to \nu\bar{\nu}h$, cross section increases with energy.

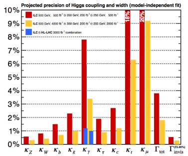


- At higher energies (not available at circular colliders)
  - $e^+e^- \to t\bar{t}h$ (top Yukawa).
  - $e^+e^- \to Zhh$ and $e^+e^- \to \nu\bar{\nu}hh$ (triple Higgs coupling).



κ framework vs. EFT





From the CEPC preCDR and "Physics Case for the ILC" ([arXiv:1506.05992])

- Conventionally, the constraints on Higgs couplings are obtained from global fits in the so-called "κ" framework.

$$g_h^{\rm SM} \to g_h^{\rm SM}(1+\kappa)\,.$$

- Anomalous couplings such as $hZ^{\mu\nu}Z_{\mu\nu}$ or $hZ_\mu\partial_\nu Z^{\mu\nu}$ are assumed to be zero.
- EFT framework
  - Assuming $v \ll \Lambda$, leading contribution from BSM physics are well-parameterized by D6 operators.
  - Gauge invariance is built in the parameterization.
- Lots of parameters! (Is it practical to perform a global fit?)

The "12-parameter" framework in EFT

- Assume the new physics
  - is CP-even,
  - does not generate dipole interaction of fermions,
  - only modifies the diagonal entries of the Yukawa matrix,
  - has no corrections to Z-pole observables and W mass (more justified if the machine will run at Z-pole).
- Additional measurements
  - Triple gauge couplings from $e^+e^- \to WW$. (The LEP constraints will be improved at future colliders.)
  - Angular observables in $e^+e^- \to hZ$.
  - $h \to Z\gamma$ is also important.
- Only 12 combinations of operators are relevant for the measurements considered (with the inclusion of the Yukawa couplings of t, c, b, τ, μ).
- All 12 EFT parameters can be constrained reasonably well in the global fit!

EFT basis

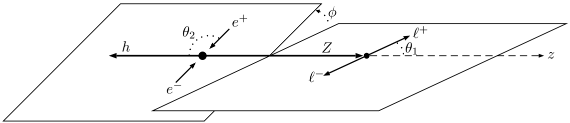
- We work in the Higgs basis (LHCHXSWG-INT-2015-001, A. Falkowski) with the following 12 parameters,

$$\delta c_Z\,,\quad c_{ZZ}\,,\quad c_{Z\Box}\,,\quad c_{\gamma\gamma}\,,\quad c_{Z\gamma}\,,\quad c_{gg}\,,\quad \delta y_t\,,\quad \delta y_c\,,\quad \delta y_b\,,\quad \delta y_\tau\,,\quad \delta y_\mu\,,\quad \lambda_Z\,.$$

- The Higgs basis is defined in the broken electroweak phase.
  - $\delta c_Z \leftrightarrow hZ^\mu Z_\mu, \quad c_{ZZ} \leftrightarrow hZ^{\mu\nu}Z_{\mu\nu}, \quad c_{Z\Box} \leftrightarrow hZ_\mu \partial_\nu Z^{\mu\nu}.$
- Couplings of h to W are written in terms of couplings of h to Z and γ.
- 3 aTGC parameters ($\delta g_{1,Z}$, $\delta\kappa_\gamma$, λ_Z), 2 written in terms of Higgs parameters.
- It can be easily mapped to the following basis with D6 operators.

$\mathcal{O}_H = \frac{1}{2}(\partial_\mu \lvert H^2 \rvert)^2$	$\mathcal{O}_{GG} = g_s^2 \lvert H \rvert^2 G^A_{\mu\nu} G^{A,\mu\nu}$
$\mathcal{O}_{WW} = g^2 \lvert H \rvert^2 W^a_{\mu\nu} W^{a,\mu\nu}$	$\mathcal{O}_{y_u} = y_u \lvert H \rvert^2 \bar{Q}_L \tilde{H} u_R$
$\mathcal{O}_{BB} = g'^2 \lvert H \rvert^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{y_d} = y_d \lvert H \rvert^2 \bar{Q}_L H d_R$
$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$	$\mathcal{O}_{y_e} = y_e \lvert H \rvert^2 \bar{L}_L H e_R$
$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon_{abc} W^{a\,\nu}_{\mu} W^b_{\nu\rho} W^{c\,\rho\mu}$

angular observables in $e^+e^- \to hZ$

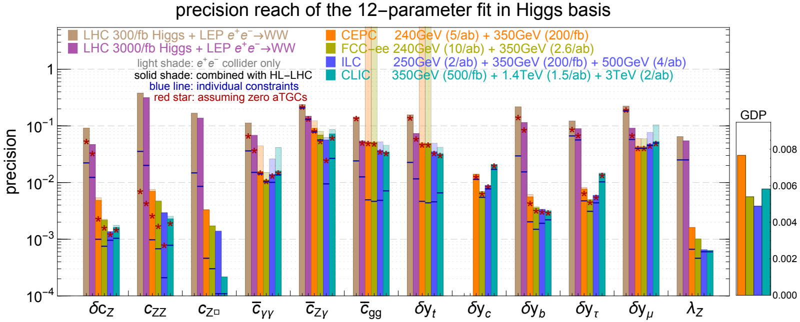


- Angular distributions in $e^+e^- \to hZ$ can provide information in addition to the rate measurement alone.
- Previous studies
  - [arXiv:1406.1361] M. Beneke, D. Boito, Y.-M. Wang
  - [arXiv:1512.06877] N. Craig, JG, Z. Liu, K. Wang
- 6 independent asymmetry observables from 3 angles

$$\mathcal{A}_{\theta_1}\ ,\ \ \mathcal{A}_{\phi}^{(1)}\ ,\ \ \mathcal{A}_{\phi}^{(2)}\ ,\ \ \mathcal{A}_{\phi}^{(3)}\ ,\ \ \mathcal{A}_{\phi}^{(4)}\ ,\ \ \mathcal{A}_{c\theta_1,c\theta_2}\ .$$

- Focusing on leptonic decays of Z (good resolution, small background, statistical uncertainty dominates).

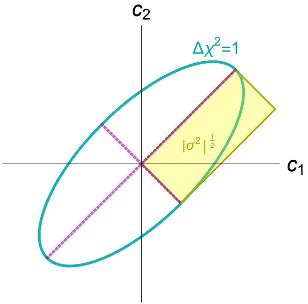
Results of the "12-parameter" fit



- Assuming the following run plans (no official plan for CEPC 350 GeV run yet)
  - CEPC 240 GeV(5/ab) + 350 GeV(200/fb)
  - FCC-ee 240 GeV(10/ab) + 350 GeV(2.6/ab) [1]
  - ILC 250 GeV(2/ab) + 350 GeV(200/fb) + 500 GeV(4/ab)
  - CLIC 350 GeV(500/fb) + 1.4 TeV(1.5/ab) + 3 TeV(2/ab)

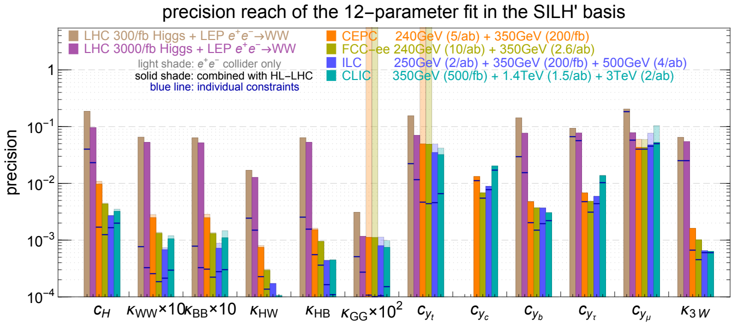
[1] The luminosities of FCC-ee have been recently updated to 240 GeV(5/ab) + 350 GeV(1.5/ab), see talks at the FCC week 2017.

GDP



- Global Determinant Parameter ($\text{GDP} \equiv \sqrt[2n]{\det \sigma^2}$).
- Ratios of GDPs are basis-independent.
- Anti-capitalism definition: smaller GDP $\rightarrow$ better precision!

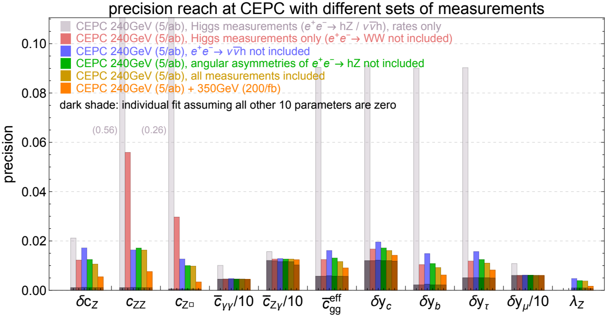
If you don't like the Higgs basis...



- Results in the SILH'(-like) basis ($\mathcal{O}_{W,B} \to \mathcal{O}_{WW,WB}$)

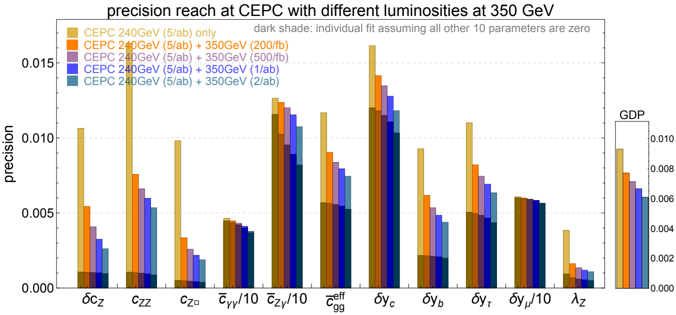
$$\mathcal{L}_{\rm D6} = \frac{c_H}{v^2}\mathcal{O}_H + \frac{\kappa_{WW}}{m_W^2}\mathcal{O}_{WW} + \frac{\kappa_{BB}}{m_W^2}\mathcal{O}_{BB} + \frac{\kappa_{HW}}{m_W^2}\mathcal{O}_{HW} + \frac{\kappa_{HB}}{m_W^2}\mathcal{O}_{HB} + \frac{\kappa_{GG}}{m_W^2}\mathcal{O}_{GG} + \frac{\kappa_{3W}}{m_W^2}\mathcal{O}_{3W} + \sum_{f=t,c,b,\tau,\mu}\frac{c_{y_f}}{v^2}\mathcal{O}_{y_f} \, .$$

The importance of combining all measurements



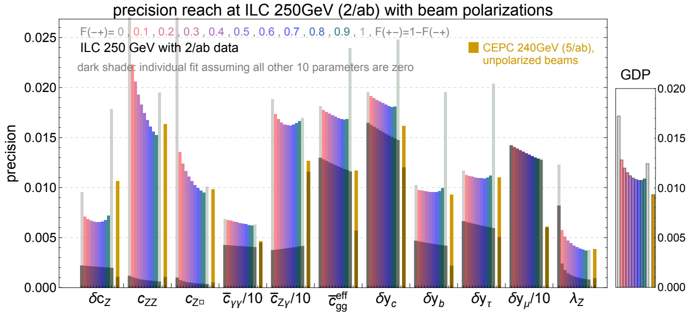
- The results are much worse if we only include the rates of Higgs measurements alone!
- There is some overlap in the information from different measurements.
- Measurements at different energies can be very helpful.

Impact of the 350 GeV run



- Advantages of the 350 GeV run
  - Much better measurement of the WW fusion process ($e^+e^- \to \nu\bar{\nu}h$).
  - Probing $e^+e^- \to hZ$ at a different energy.
  - Improving constraints on aTGCs ($e^+e^- \to WW$).
- Very helpful in resolving the degeneracies among parameters!

CEPC 240 GeV vs. ILC 250 GeV



- Beam polarization helps discriminate different parameters.
  - Two polarization configurations are considered, $P(e^-, e^+) = (-0.8, +0.3)$ and $(+0.8, -0.3)$.
  - $F(-+)$ in the range of 0.6-0.8 gives an optimal overall results.
- Large luminosity still wins (but it is important to include all possible measurements).

The Higgs self-coupling at e^+e^- colliders

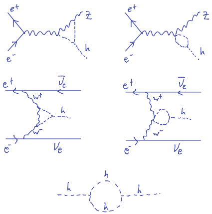
(current work with N. Craig, S. Di Vita, G. Durieux, C. Grojean, Z. Liu, G. Panico, M. Riembau, T. Vantalon)

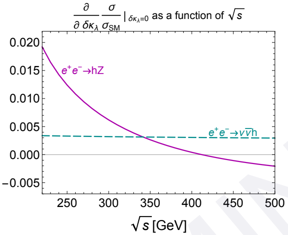
- Triple Higgs coupling

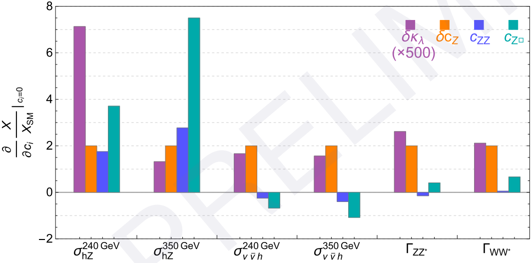
$$\kappa_\lambda \equiv \frac{\lambda_{hhh}}{\lambda_{hhh}^{\rm SM}}, \quad \delta\kappa_\lambda \equiv \kappa_\lambda - 1 \; = c_6 - \frac{3}{2} c_H, \quad \text{with } \mathcal{L} \supset -\frac{c_6 \lambda}{v^2}(H^\dagger H)^3$$

- HL-LHC: $\sim \mathcal{O}(1)$ determination. ($\kappa_\lambda \in [-0.8, \, 7.7]$ at 95% CL from Atlas projection for the $b\bar{b}\gamma\gamma$ channel, ATL-PHYS-PUB-2017-001)
- Linear colliders: direct measurements with $e^+e^- \to Zhh$, $e^+e^- \to \nu\bar{\nu}hh$.
  - ILC: 26.6% at 500 GeV (4 ab^{-1}) [C. F. Dürig, PhD thesis, Hamburg U. (2016)]
  - CLIC: 40%-54% at 1.4 TeV (1.5 ab^{-1}) and 22%-29% at 3 TeV (2 ab^{-1}) (Higgs Physics at CLIC [arXiv:1608.07538]).
  - Are these bounds robust under a global analysis in an EFT framework?
- Circular colliders: probe indirectly via the loop contribution in $e^+e^- \to hZ$ ([arXiv:1312.3322] M. McCullough).
  - TLEP (FCC-ee) 240 GeV: $|\delta\kappa_\lambda| \lesssim 28\%$ assuming all other Higgs couplings are SM-like.
  - What if other Higgs couplings are not SM-like?
- A global analysis in the 12+1 parameter framework!

Triple Higgs coupling in single Higgs processes

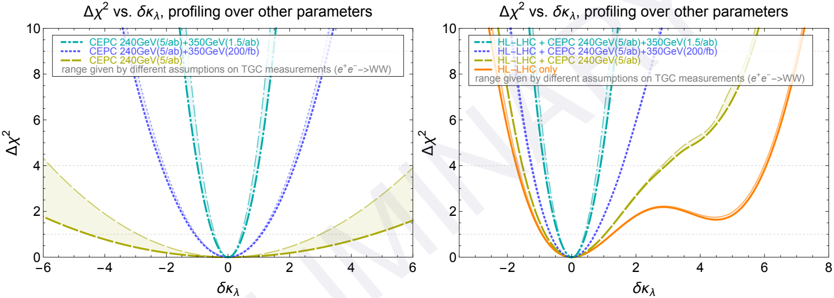






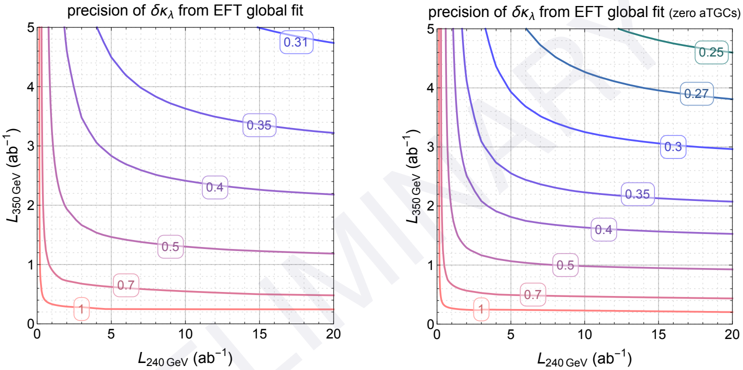
- One loop corrections to all Higgs couplings (production and decay).
- 240 GeV: hZ near threshold (more sensitive to $\delta\kappa_\lambda$)
- at 350 GeV:
  - WW fusion
  - hZ at a different energy
- $h\to WW^*/ZZ^*$ also have some discriminating power (but turned out to be not enough).

χ^2 vs. $\delta\kappa_\lambda$ from global fits at CEPC



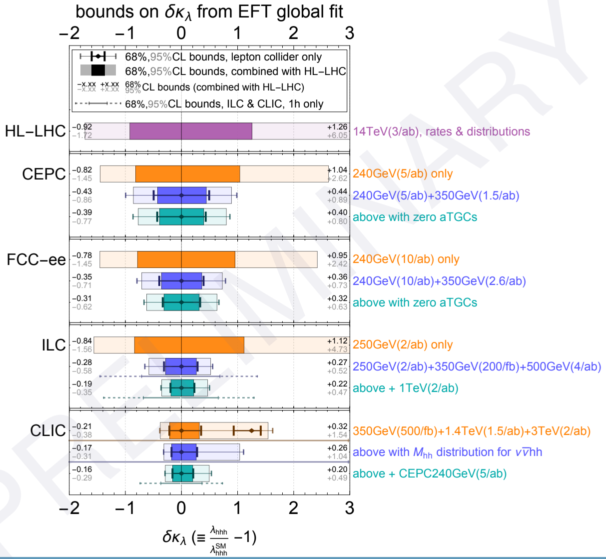
- 240 GeV + 350 GeV much better than 240 GeV alone!
- 240 GeV alone can improve on the top of HL-LHC bounds.
- HL-LHC: Both single and double Higgs measurements, inclusive and differential.
  [arXiv:1704.01953] Di Vita, Grojean, Panico, Riembau, Vantalon
  [arXiv:1502.00539] Azatov, Contino, Panico, Son

The precision reach of $\delta\kappa_\lambda$ at circular colliders



- The precision reach of $\delta\kappa_\lambda$ in the luminosity plane (luminosities at 240 GeV and 350 GeV).
- $e^+e^- \to WW$ measured very well $\Rightarrow$ setting aTGCs to zero is a good approximation.

A summary of the (future) bounds on $\delta\kappa_\lambda$

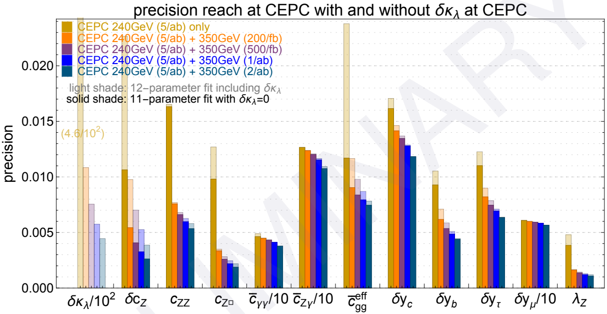


Conclusion (and how to write the CDR)

- It makes sense to go beyond the "κ" frame and study Higgs physics in the EFT framework.
- We can obtain strong and robust constraints on the coefficients of the relevant dimension-6 operators!
- Choice of basis (my personal view)
  - Higgs basis: closet thing to "κ" frame, yet essentially a D6 EFT.
  - A convenient choice of D6 operators works equally fine, and the translation is straightforward.
- The 350 GeV run is good!
  - Very helpful in discriminating hVV type couplings with different Lorentz structures.
  - Crucial for obtaining robust constraints on the triple Higgs coupling.
  - How much luminosity do we want?
- Unanswered questions...
  - What's the impact of a future Z-pole run?
  - How well can aTGCs be constrained from $e^+e^- \to WW$? (Experimental studies desired.)
  - Include Higgs invisible/exotic decay?

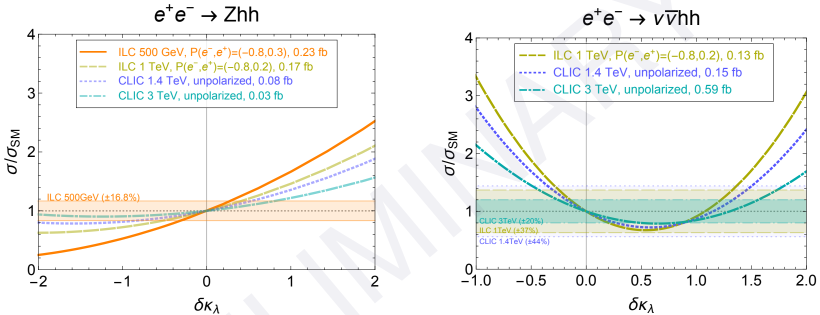
backup slides

Impact of $\delta\kappa_\lambda$ on the other parameters



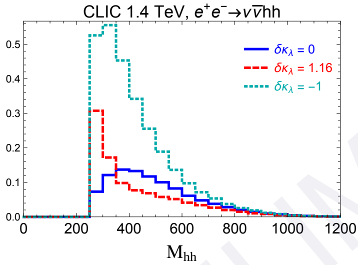
- Adding one more parameter could worsen the bounds on others.
- The effect is under control if the degeneracies are well-resolved.
- The HL-LHC bounds on $\delta\kappa_\lambda$ can also help.

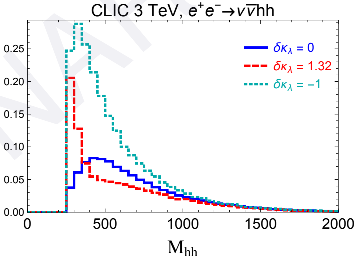
Double-Higgs measurements



- Diagram with λ_{hhh} interferes with diagrams without λ_{hhh}.
- $e^+e^- \to \nu\bar{\nu}hh$ has destructive interference! Important to keep both the linear and the square term.

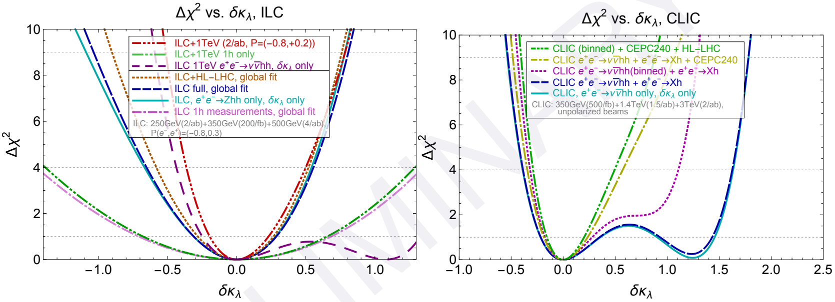
M_{hh} of $\nu\bar{\nu}hh$





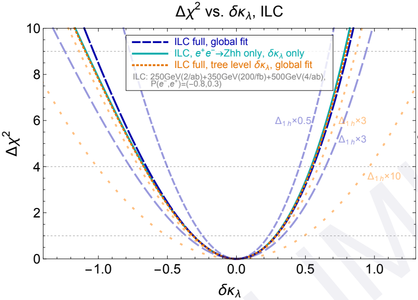
- The M_{hh} distribution can help lift the "2nd minimum."

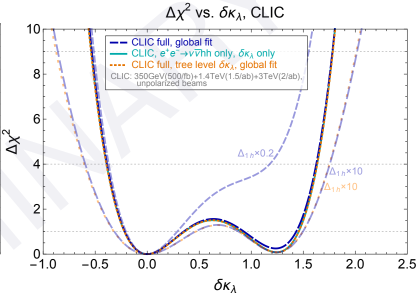
χ^2 vs. $\delta\kappa_\lambda$ for different scenarios



- The double-Higgs measurements still provide the best reach on $\delta\kappa_\lambda$.
- Other parameters are well constrained by single Higgs measurements.
- CLIC: The 2nd minimum can be lift by the M_{hh} distribution or combining with CEPC 240 GeV.

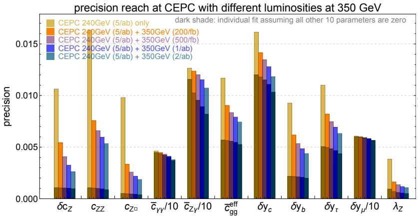
Impact of the single Higgs measurements

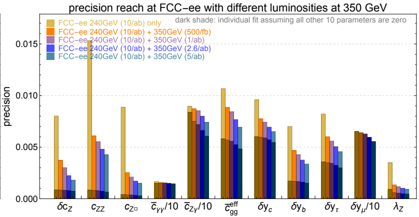


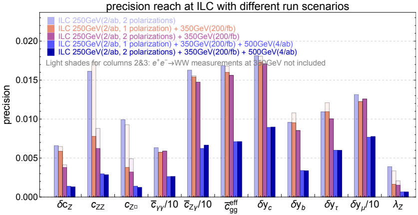


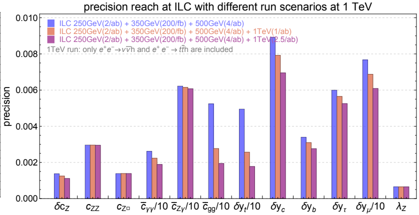
- What if the single Higgs measurements are much better or much worse?
- Much better: can further improve the bounds on $\delta\kappa_\lambda$ from double-Higgs measurements.
- Much worse: can significantly worsen the bounds on $\delta\kappa_\lambda$ from double-Higgs measurements.

Impact of the Higher energy runs

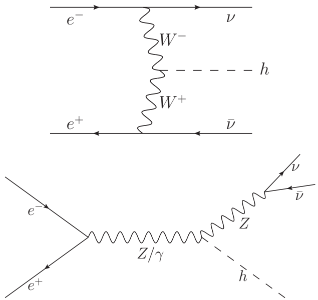


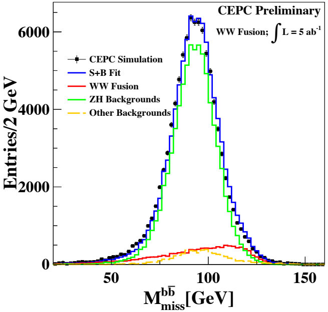






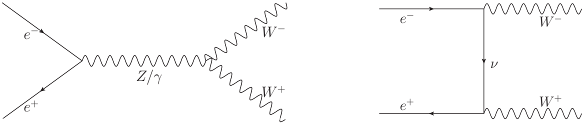
$e^+e^- \to \nu\bar{\nu}h$





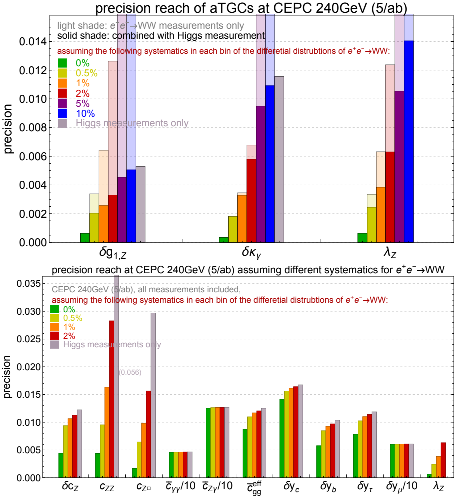
- It is hard to separate the WW fusion process from $e^+e^- \to hZ, Z \to \nu\bar{\nu}$ at 240 GeV.
- It is not consistent to focus on one process and treat the other one as SM-like!
- For CEPC/FCC-ee 240 GeV, we analyze the combined $e^+e^- \to \nu\bar{\nu}h$ process, assuming new physics can contribute to both processes.

$e^+e^- \to WW$



- $e^+e^- \to WW$ offers a great way to probe the anomalous triple gauge couplings (aTGCs, parameterized by $\delta g_{1,Z}$, $\delta\kappa_\gamma$, λ_Z).
- $\delta g_{1,Z}$ and $\delta\kappa_\gamma$ are related to Higgs observables.
- CEPC with $5\,\mathrm{ab}^{-1}$ data at 240 GeV can produce $\sim 9 \times 10^7$ $e^+e^- \to WW$ events.
- With such large statistics, the aTGCs can be very well constrained ([1507.02238] Bian, Shu, Zhang), but with two potential issues:
  - Systematic uncertainties can be important!
  - If $e^+e^- \to WW$ is measured more precisely than the Z-pole measurements, is it still ok to assume the fermion gauge couplings are SM-like?

The interplay between Higgs and TGC



- $\delta g_{1,Z}$, $\delta\kappa_\gamma \leftrightarrow$ c_{ZZ} , $c_{Z\Box}$, $c_{\gamma\gamma}$, $c_{Z\gamma}$
- We try different assumptions on the systematic uncertainties (in each bin with the differential distribution divided into 20 bins).
- Detailed study of $e^+e^- \to WW$ required to estimate the systematic uncertainties!

TGC at ILC 500 GeV

ILC				
	uncertainty	correlation matrix		
		$\delta g_{1,Z}$	$\delta\kappa_\gamma$	λ_Z
$\delta g_{1,Z}$	6.1×10^{-4}	1	0.634	0.477
$\delta\kappa_\gamma$	6.4×10^{-4}		1	0.354
λ_Z	7.2×10^{-4}			1

- Linear colliders (large $\sqrt{s}$, beam polarizations) could potentially constrain the aTGCs very well.
- Estimated precisions of aTGCs from the $e^+e^- \to WW$ measurements at ILC assuming $500\,\text{fb}^{-1}$ data at 500 GeV and a beam polarization of $P(e^-, e^+) = (\pm 0.8, \pm 0.3)$. [I. Marchesini, PhD thesis, Hamburg U. (2011)]

Asymmetry observables

$$\mathcal{A}_{\theta_1} = \frac{1}{\sigma}\int_{-1}^{1} d\cos\theta_1 \,\mathrm{sgn}(\cos(2\theta_1))\,\frac{d\sigma}{d\cos\theta_1}\,,$$

$$\mathcal{A}_{\phi}^{(1)} = \frac{1}{\sigma}\int_{0}^{2\pi} d\phi\,\mathrm{sgn}(\sin\phi)\,\frac{d\sigma}{d\phi}\,,$$

$$\mathcal{A}_{\phi}^{(2)} = \frac{1}{\sigma}\int_{0}^{2\pi} d\phi\,\mathrm{sgn}(\sin(2\phi))\,\frac{d\sigma}{d\phi}\,,$$

$$\mathcal{A}_{\phi}^{(3)} = \frac{1}{\sigma}\int_{0}^{2\pi} d\phi\,\mathrm{sgn}(\cos\phi)\,\frac{d\sigma}{d\phi}\,,$$

$$\mathcal{A}_{\phi}^{(4)} = \frac{1}{\sigma}\int_{0}^{2\pi} d\phi\,\mathrm{sgn}(\cos(2\phi))\,\frac{d\sigma}{d\phi}\,, \tag{1}$$

$$\mathcal{A}_{c\theta_1,c\theta_2} = \frac{1}{\sigma}\int_{-1}^{1} d\cos\theta_1\,\mathrm{sgn}(\cos\theta_1)\int_{-1}^{1} d\cos\theta_2\,\mathrm{sgn}(\cos\theta_2)\,\frac{d^2\sigma}{d\cos\theta_1\,d\cos\theta_2}\,, \tag{2}$$

The “12-parameter” framework in the Higgs basis

- The relevant terms in the EFT Lagrangian are

$$\mathcal{L} \supset \mathcal{L}_{hVV} + \mathcal{L}_{hff} + \mathcal{L}_{\rm tgc} \,, \tag{3}$$

- the Higgs couplings with a pair of gauge bosons

$$\begin{aligned}\mathcal{L}_{hVV} = \frac{h}{v}\Big[& (1+\delta c_W)\frac{g^2 v^2}{2} W^+_\mu W^-_\mu + (1+\delta c_Z)\frac{(g^2+g'^2)v^2}{4} Z_\mu Z_\mu \\ & + c_{WW}\frac{g^2}{2} W^+_{\mu\nu}W^-_{\mu\nu} + c_{W\Box}\, g^2 (W^-_\mu \partial_\nu W^+_{\mu\nu} + \text{h.c.}) \\ & + c_{gg}\frac{g_s^2}{4} G^a_{\mu\nu}G^2_{\mu\nu} + c_{\gamma\gamma}\frac{e^2}{4} A_{\mu\nu}A_{\mu\nu} + c_{Z\gamma}\frac{e\sqrt{g^2+g'^2}}{2} Z_{\mu\nu}A_{\mu\nu} \\ & + c_{ZZ}\frac{g^2+g'^2}{4} Z_{\mu\nu}Z_{\mu\nu} + c_{Z\Box}\, g^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box}\, g g' Z_\mu \partial_\nu A_{\mu\nu} \Big] \,. \end{aligned} \tag{4}$$

The “12-parameter” framework in the Higgs basis

- Not all the couplings are independent, for instance one could write the following couplings as

$$
\begin{aligned}
\delta c_W &= \delta c_Z + 4\delta m\,, \\
c_{WW} &= c_{ZZ} + 2 s^2_{\theta_W} c_{Z\gamma} + s^4_{\theta_W} c_{\gamma\gamma}\,, \\
c_{W\Box} &= \frac{1}{g^2 - g'^2}\left[g^2 c_{Z\Box} + g'^2 c_{ZZ} - e^2 s^2_{\theta_W} c_{\gamma\gamma} - (g^2 - g'^2) s^2_{\theta_W} c_{Z\gamma}\right]\,, \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2}\left[2g^2 c_{Z\Box} + (g^2 + g'^2) c_{ZZ} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{Z\gamma}\right]\,, \qquad (5)
\end{aligned}
$$

- we only consider the diagonal elements in the Yukawa matrices relevant for the measurements considered,

$$
\mathcal{L}_{hff} = -\frac{h}{v} \sum_{f=t,c,b,\tau,\mu} m_f (1 + \delta y_f) \bar{f}_R f_L + \text{h.c.}\,. \qquad (6)
$$

TGC

$$
\begin{aligned}
\mathcal{L}_{\rm tgc} = \; & igs_{\theta_W} A^\mu (W^{-\nu} W^+_{\mu\nu} - W^{+\nu} W^-_{\mu\nu}) \\
& + ig(1+\delta g_1^Z) c_{\theta_W} Z^\mu (W^{-\nu} W^+_{\mu\nu} - W^{+\nu} W^-_{\mu\nu}) \\
& + ig \left[(1+\delta\kappa_Z) c_{\theta_W} Z^{\mu\nu} + (1+\delta\kappa_\gamma) s_{\theta_W} A^{\mu\nu} \right] W^-_\mu W^+_\nu \\
& + \frac{ig}{m_W^2} (\lambda_Z c_{\theta_W} Z^{\mu\nu} + \lambda_\gamma s_{\theta_W} A^{\mu\nu}) W^{-\rho}_v W^+_{\rho\mu} \,, \qquad (7)
\end{aligned}
$$

- $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ for $V = W^\pm, Z, A,$. Imposing Gauge invariance one obtains $\delta\kappa_Z = \delta g_{1,Z} - t^2_{\theta_W} \delta\kappa_\gamma$ and $\lambda_Z = \lambda_\gamma$.
- 3 aTGCs parameters $\delta g_{1,Z}$, $\delta\kappa_\gamma$ and λ_Z, 2 of them related to Higgs observables by

$$
\delta g_{1,Z} = \frac{1}{2(g^2 - g'^2)} \left[-g^2(g^2+g'^2) c_{Z\Box} - g'^2(g^2+g'^2) c_{ZZ} + e^2 g'^2 c_{\gamma\gamma} + g'^2(g^2-g'^2) c_{Z\gamma} \right]
$$

$$
\delta\kappa_\gamma = -\frac{g^2}{2} \left(c_{\gamma\gamma} \frac{e^2}{g^2+g'^2} + c_{Z\gamma} \frac{g^2-g'^2}{g^2+g'^2} - c_{ZZ} \right) . \qquad (8)
$$

CEPC/FCC-ee Higgs rate measurements

	CEPC				FCC-ee			
	[240 GeV, 5 ab^{-1}]		[350 GeV, 200 fb^{-1}]		[240 GeV, 10 ab^{-1}]		[350 GeV, 2.6 ab^{-1}]	
production	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$
σ	0.50%	-	2.4%	-	0.40%	-	0.67%	-
	$\sigma \times$ BR				$\sigma \times$ BR			
$h \to b\bar{b}$	0.21%*	0.39%$^\diamond$	2.0%	2.6%	0.20%	0.28%$^\diamond$	0.54%	0.71%
$h \to c\bar{c}$	2.5%	-	15%	26%	1.2%	-	4.1%	7.1%
$h \to gg$	1.2%	-	11%	17%	1.4%	-	3.1%	4.7%
$h \to \tau\tau$	1.0%	-	5.3%	37%	0.7%	-	1.5%	10%
$h \to WW^*$	1.0%	-	10%	9.8%	0.9%	-	2.8%	2.7%
$h \to ZZ^*$	4.3%	-	33%	33%	3.1%	-	9.2%	9.3%
$h \to \gamma\gamma$	9.0%	-	51%	77%	3.0%	-	14%	21%
$h \to \mu\mu$	12%	-	115%	275%	13%	-	32%	76%
$h \to Z\gamma$	25%	-	144%	-	18%	-	40%	-

Table: For $e^+e^- \to \nu\bar{\nu}h$, the precisions marked with a diamond $^\diamond$ are normalized to the cross section of the inclusive channel which includes both the WW fusion and $e^+e^- \to hZ, Z \to \nu\bar{\nu}$, while the unmarked ones include WW fusion only.

ILC Higgs rate measurements

ILC

	[250 GeV, 2 ab^{-1}]		[350 GeV, 200 fb^{-1}]		[500 GeV, 4 ab^{-1}]			[1 TeV, 1 ab^{-1}]		[1 TeV, 2.5 ab^{-1}]	
production	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	Zh	$\nu\bar{\nu}h$	$t\bar{t}h$	$\nu\bar{\nu}h$	$t\bar{t}h$	$\nu\bar{\nu}h$	$t\bar{t}h$
σ	0.71%	-	2.1%	-	1.1%	-	-	-	-	-	-
	$\sigma\times\mathrm{BR}$										
$h\to b\bar{b}$	0.42%	3.7%	1.7%	1.7%	0.64%	0.25%	9.9%	0.5%	6.0%	0.3%	3.8%
$h\to c\bar{c}$	2.9%	-	13%	17%	4.6%	2.2%	-	3.1%	-	2.0%	-
$h\to gg$	2.5%	-	9.4%	11%	3.9%	1.4%	-	2.3%	-	1.4%	-
$h\to\tau\tau$	1.1%	-	4.5%	24%	1.9%	3.2%	-	1.6%	-	1.0%	-
$h\to WW^*$	2.3%	-	8.7%	6.4%	3.3%	0.85%	-	3.1%	-	2.0%	-
$h\to ZZ^*$	6.7%	-	28%	22%	8.8%	2.9%	-	4.1%	-	2.6%	-
$h\to\gamma\gamma$	12%	-	44%	50%	12%	6.7%	-	8.5%	-	5.4%	-
$h\to\mu\mu$	25%	-	98%	180%	31%	25%	-	31%	-	20%	-
$h\to Z\gamma$	34%	-	145%	-	49%	-	-	-	-	-	-

CLIC Higgs rate measurements

CLIC

	[350 GeV, 500 fb^{-1}]		[1.4 TeV, 1.5 ab^{-1}]		[3 TeV, 2 ab^{-1}]
production	Zh	$\nu\bar{\nu}h$	$\nu\bar{\nu}h$	$t\bar{t}h$	$\nu\bar{\nu}h$
σ	1.6%	-	-	-	-
	$\sigma \times \mathrm{BR}$				
$h \to b\bar{b}$	0.84%	1.9%	0.4%	8.4%	0.3%
$h \to c\bar{c}$	10.3%	14.3%	6.1%	-	6.9%
$h \to gg$	4.5%	5.7%	5.0%	-	4.3%
$h \to \tau\tau$	6.2%	-	4.2%	-	4.4%
$h \to WW^*$	5.1%	-	1.0%	-	0.7%
$h \to ZZ^*$	-	-	5.6%	-	3.9%
$h \to \gamma\gamma$	-	-	15%	-	10%
$h \to \mu\mu$	-	-	38%	-	25%
$h \to Z\gamma$	-	-	42%	-	30%

Table: We also include the estimations for $\sigma(hZ) \times \mathrm{BR}(h \to b\bar{b})$ at high energies in [arXiv:1701.04804] (Ellis et al.), which are 3.3% (6.8%) at 1.4 TeV (3 TeV). For simplicity, the measurements of ZZ fusion ($e^+e^- \to e^+e^- h$) are not included in our analysis.