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Analysis of Exclusive Processes $e^+e^- \to VP$ and $e^+e^- \to TP$ in k_T Factorization

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OUTLINE:

- Motivation
- A brief introduction of k_T factorization
- Exclusive processes $e^+e^- \to VP$ & TP in k_T factorization
- Numerical results and discussion
- Summary

Motivation

- On the experimental side, some channels of the $e^+e^- \to VP$ and TP processes have been measured by CLEO-c collaboration at $\sqrt{s} = 3.67GeV$ and Belle and BARBAR collaboration at $\sqrt{s} = 10.58GeV$. This work can give a reliable prediction in other similar processes.

- On the theoretical side, these processes can provide an opportunity to investigate the time-like form factors :

➢ In the two-body hadronic B meson decays in PQCD approach, the sizable strong phases are produced from penguin annihilation amplitudes, which involve time-like form factors.

➢ The PQCD formalism for three-body B decay need to introduce the two-meson wave functions, whose parametrization involves time-like form factors associated with various currents.

Graphical k_T Factorization

Amplitude → Hard Scattering Kernel $\otimes$ Hadron Wave Function

Hard Scattering Kernel → Calculated Perturbative

Hadron Wave Function → Universal & Non-perturbative Input

Amplitude:

- UV Divergence → Regularization
- IR Divergence → Soft Divergences Cancelled each other; Collinear Divergences absorbed into Hadron WFs
- End-Point Singularity → Introduce the transverse momentum dependence

Double Log terms: Resummation

Basic ideas of the k_T factorization

- The amplitude can be expressed as the convolution of the non-perturbative hadron wave functions and the perturbative hard scattering kernel by both longitudinal and transverse momentum.

- Considering the **transverse momentum** of valence quarks;

Universal Hadron Wave Function, non-perturbative

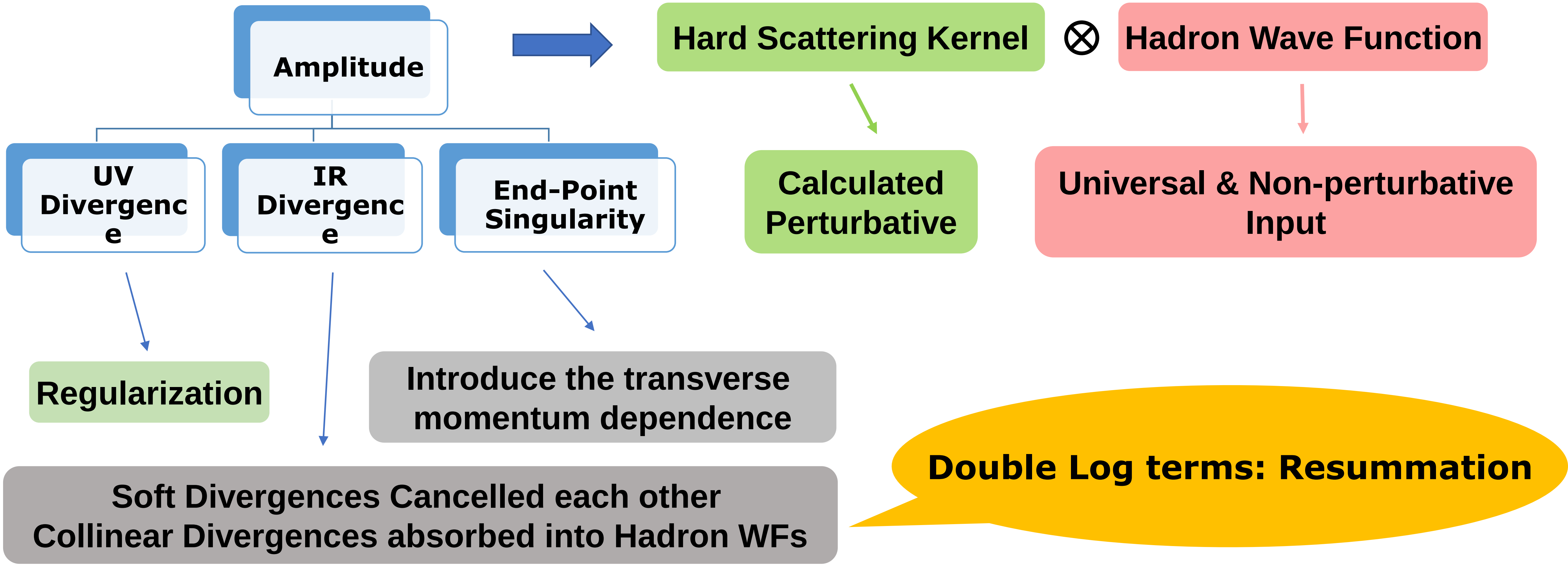
Hard Scattering Kernel, can be calculated perturbative

$$\mathcal{A} = \langle M_1 M_2 | \mathcal{H}_{\text{eff}} | 0 \rangle \sim \int d^4k_1 d^4k_2 \ \mathrm{Tr}\left[\Phi_{M_1}(k_1)\Phi_{M_2}(k_2) H(k_1, k_2, Q, \mu)\right]$$

$$\Rightarrow \int_0^1 dx_1 dx_2 \int d^2\mathbf{k_{T1}} d^2\mathbf{k_{T2}} \ \mathrm{Tr}\left[\ \Phi_{M_1}(x_1, \mathbf{k_{T1}}, P_1, \mu)\ \Phi_{M_2}(x_2, \mathbf{k_{T2}}, P_2, \mu)\ H(x_1, x_2, \mathbf{k_{T1}}, \mathbf{k_{T2}}, Q, \mu)\right]$$

$$\Rightarrow \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b_1}}{(2\pi)^2} \frac{d^2\mathbf{b_2}}{(2\pi)^2} \ \mathrm{Tr}\left[\ \mathcal{P}_{M_1}(x_1, \mathbf{b_1}, P_1, \mu)\ \mathcal{P}_{M_2}(x_2, \mathbf{b_2}, P_2, \mu)\ H(x_1, x_2, \mathbf{b_1}, \mathbf{b_2}, Q, \mu)\right]$$

Graphical k_T Factorization

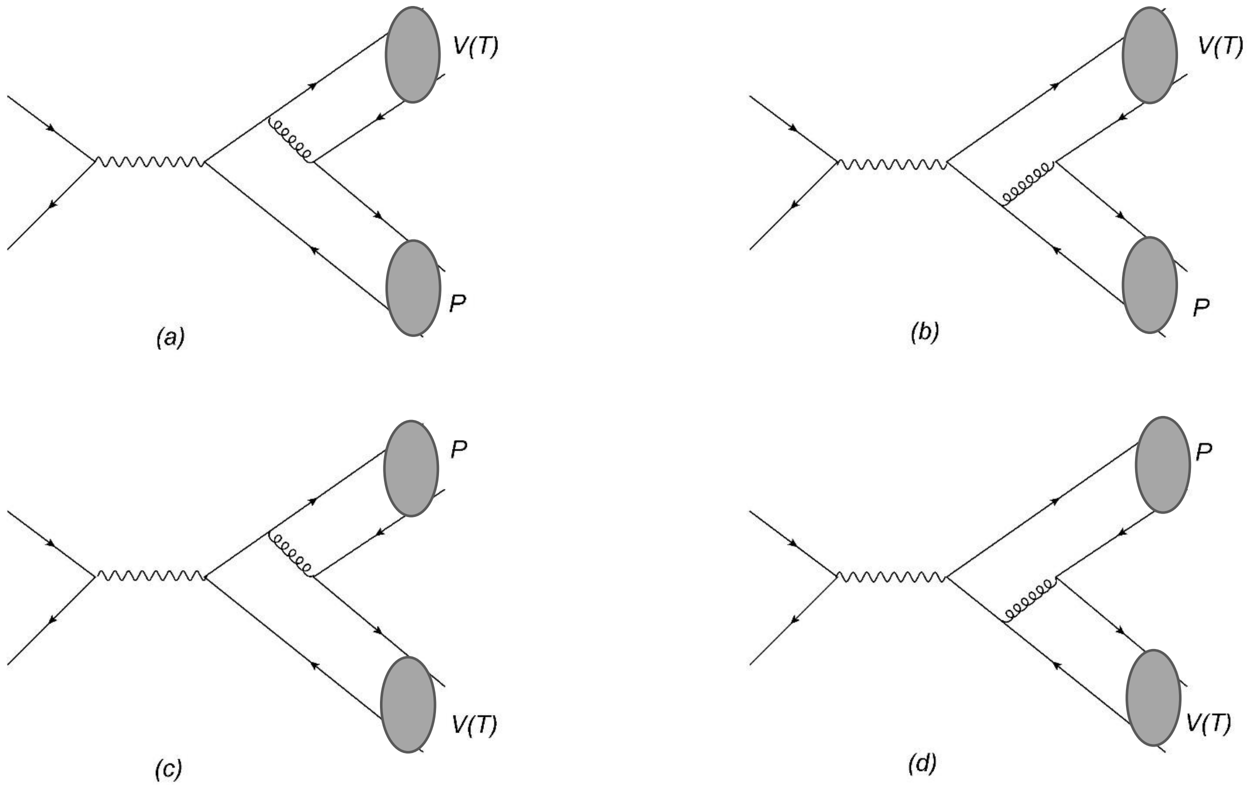


Basic ideas of the k_T factorization

- The double logarithm, arising from the overlap of the soft and collinear divergence, should be resumed into the Sudakov factor, and single logarithms from ultraviolet divergences, can be summed using the renormalization group equation (RGE) method.
- In the threshold region with $x \to 0$, the double logarithm produced by QCD loop correction to the electromagnetic vertex can be resumed into another universal Sudakov factor S_t(x).

$$\mathcal{P}_i(x_j, \mathbf{b_j}, P_j, \mu) = \exp\left[-s(x_j, b_j, Q) - s(1-x_j, b_j, Q) - 2\int_{1/b_j}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu}))\right] \bar{\mathcal{P}}_i(x_j, \mathbf{b}_j, \mu)$$

Exclusive processes $e^+e^- \to VP$ and TP in k_T factorization



Dominant contributions

Leptonic part $\otimes$ Hadronic part

$\Downarrow$

Time-like Form Factor

Calculated in perturbative QCD approach based on k_T factorization

Time-like Form Factors:

$$\langle V(P_1,\epsilon_T)P(P_2)|j_\mu^{\rm em}|0\rangle = F_{\rm VP}(s)\epsilon_{\mu\nu\alpha\beta}\epsilon_T^\nu P_1^\alpha P_2^\beta.$$

$$\langle T(P_1,\lambda)P(P_2)|j_\mu^{\rm em}|0\rangle = F_{\rm TP}(s)\epsilon_{\mu\nu\alpha\beta}\boxed{\xi^\nu(\lambda)}P_1^\alpha P_2^\beta.$$

For a tensor meson, the polarization tensor $\epsilon_{\mu\nu}(\lambda)$ satisfying $\epsilon_{\mu\nu}(\lambda)P_1^\mu = 0$, so it's convenient to introduce a new polarization vector $\xi(\lambda)$:

$$\xi_\mu(\lambda) = \frac{\epsilon_{\mu\nu}(\lambda)q^\nu}{P_1\cdot q}m_T$$

The polarization tensor $\epsilon_{\mu\nu}(\lambda)$ can be constructed via the **polarization vectors of a massive vector** state by using of the Clebsch-Gordan coefficients:

$$\epsilon_{\mu\nu}(\pm 2) = \epsilon_\mu(\pm)\epsilon_\nu(\pm),$$

$$\epsilon_{\mu\nu}(\pm 1) = \sqrt{\frac{1}{2}}[\epsilon_\mu(\pm)\epsilon_\nu(0) + \epsilon_\mu(0)\epsilon_\nu(\pm)],$$

$$\epsilon_{\mu\nu}(\pm 0) = \sqrt{\frac{1}{6}}[\epsilon_\mu(+)\epsilon_\nu(-) + \epsilon_\mu(-)\epsilon_\nu(+)] + \sqrt{\frac{2}{3}}\epsilon_\mu(0)\epsilon_\nu(0).$$

Then the cross sections of processes $e^+e^- \rightarrow VP$ and TP can be expressed as

$$\sigma(e^+e^- \to VP) = \frac{\pi\alpha_{\rm em}^2}{6}|F_{\rm VP}|^2\Phi^{3/2}(s)$$

$$\eta = 1 - m_T^2/Q^2$$

$$\sigma(e^+e^- \to TP) = \frac{\pi\alpha_{\rm em}^2}{3}\left(\frac{s\eta}{2m_T^2 + s\eta}\right)^2|F_{\rm TP}|^2\Phi^{3/2}(s)$$

with

$$\Phi(s) = \left[1 - \frac{(m_{V(T)} + m_P)^2}{s}\right]\left[1 - \frac{(m_{V(T)} - m_P)^2}{s}\right]$$

The time-like form factor can be expressed as the convolution of the hadron wave functions and the hard scattering kernel by both longitudinal and transverse momentum.

$$F(Q^2) = \int_0^1 dx_1 dx_2 \int d^2\mathbf{k}_{\mathbf{T1}} d^2\mathbf{k}_{\mathbf{T2}} \Phi_{M_1}(x_1, \mathbf{k}_{\mathbf{T1}}, P_1, \mu) H(x_1, x_2, \mathbf{k}_{\mathbf{T1}}, \mathbf{k}_{\mathbf{T2}}, Q, \mu) \Phi_{M_2}(x_2, \mathbf{k}_{\mathbf{T2}}, P_2, \mu)$$

$$= \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b}_1}{(2\pi)^2} \frac{d^2\mathbf{b}_2}{(2\pi)^2} \boxed{\mathcal{P}_{M_1}(x_1, \mathbf{b}_1, P_1, \mu)} H(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, Q, \mu) \boxed{\mathcal{P}_{M_2}(x_2, \mathbf{b}_2, P_2, \mu)}$$

In the hadron wave function, the double logarithms arising from the overlap of soft and collinear divergences, can be resumed into the **Sudakov factor**

$$\mathcal{P}_{M_i}(x_i, \mathbf{b}_i, P_i, \mu) = \exp[s(x_i, b_i, Q) + s(1-x_i, b_i, Q) + 2\int_{1/b_i}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu}))] \mathcal{P}_{M_i}(x_i, \mathbf{b}_i, 1/b_i)$$

$$s(\xi, b, Q) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln\left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}} - 1\right) - \frac{A^{(1)}}{2\beta_1}\left(\hat{q} - \hat{b}\right) - \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q}\left[\frac{\ln(2\hat{b}) + 1}{\hat{b}} - \frac{\ln(2\hat{q}) + 1}{\hat{q}}\right]$$
$$- \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln\left(\frac{e^{2\gamma - 1}}{2}\right)\right] \ln\left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right]$$

The time-like form factor can be expressed as the convolution of the hadron wave functions and the hard scattering kernel by both longitudinal and transverse momentum.

$$F(Q^2) = \int_0^1 dx_1 dx_2 \int d^2\mathbf{k_{T1}} d^2\mathbf{k_{T2}} \Phi_{M_1}(x_1, \mathbf{k_{T1}}, P_1, \mu) H(x_1, x_2, \mathbf{k_{T1}}, \mathbf{k_{T2}}, Q, \mu) \Phi_{M_2}(x_2, \mathbf{k_{T2}}, P_2, \mu)$$

$$= \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b_1}}{(2\pi)^2} \frac{d^2\mathbf{b_2}}{(2\pi)^2} \mathcal{P}_{M_1}(x_1, \mathbf{b_1}, P_1, \mu) \boxed{H(x_1, x_2, \mathbf{b_1}, \mathbf{b_2}, Q, \mu)} \mathcal{P}_{M_2}(x_2, \mathbf{b_2}, P_2, \mu)$$

The single logarithms from ultraviolet divergences, can be resumed using the **renormalization group equation** method:

$$H(x_1, x_2, \mathbf{b_1}, \mathbf{b_2}, Q, \mu) = \exp\left[-4 \int_\mu^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{u})) \right] \times H(x_1, x_2, \mathbf{b_1}, \mathbf{b_2}, Q, t)$$

t is the largest mass scale involved in the hard scattering:

$$t = \max(\sqrt{x_2}Q, 1/b_1, 1/b\_2).$$

The time-like form factor can be expressed as the convolution of the hadron wave functions and the hard scattering kernel by both longitudinal and transverse momentum.

$$F(Q^2) = \int_0^1 dx_1 dx_2 \int d^2\mathbf{k}_{\mathbf{T1}} d^2\mathbf{k}_{\mathbf{T2}} \Phi_{M_1}(x_1, \mathbf{k}_{\mathbf{T1}}, P_1, \mu) H(x_1, x_2, \mathbf{k}_{\mathbf{T1}}, \mathbf{k}_{\mathbf{T2}}, Q, \mu) \Phi_{M_2}(x_2, \mathbf{k}_{\mathbf{T2}}, P_2, \mu)$$

$$= \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b}_1}{(2\pi)^2} \frac{d^2\mathbf{b}_2}{(2\pi)^2} \mathcal{P}_{M_1}(x_1, \mathbf{b}_1, P_1, \mu) H(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, Q, \mu) \mathcal{P}_{M_2}(x_2, \mathbf{b}_2, P_2, \mu)$$

In **the threshold region** with $x \to 0$, the double logarithm produced by QCD loop correction to the electromagnetic vertex can be resumed into another **universal Sudakov factor S_t(x)**.

$$S_t(x, Q) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)} [x(1 - x)]^c$$

$$c(Q^2) = 0.04Q^2 - 0.51Q + 1.87$$

H. n. Li and S. Mishima,
Phys. Rev. D 80, 074024 (2009)

Combing all the above ingredients, we obtain the factorization formula for the LO diagrams:

$$F_a = 16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_a) h(\bar{x}_1, x_2, b_1, b_2) S_t(x_2) \Big\{ r_1 \big[\phi_1^{p(a)}(x_1, b_1) - \phi_1^v(x_1, b_1)\big] \phi_2^A(x_2, b_2) \Big\}$$

$$F_b = 16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_b) h(x_2, \bar{x}_1, b_2, b_1) S_t(\bar{x}_1)$$
$$\times \Big\{ r_1 \bar{x}_1 \big[\phi_1^{p(a)}(x_1, b_1) + \phi_1^v(x_1, b_1)\big] \phi_2^A(x_2, b_2) - 2 r_2 \phi_1^T(x_1, b_1) \phi_2^P(x_2, b_2) \Big\}$$

$$F_c = -16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 E(t_c) h(\bar{x}_2, x_1, b_2, b_1) S_t(x_1)$$
$$\times \Big\{ r_1 x_1 \big[\phi_1^{p(a)}(x_1, b_1) - \phi_1^v(x_1, b_1)\big] \phi_2^A(x_2, b_2) + 2 r_2 \phi_1^T(x_1, b_1) \phi_2^P(x_2, b_2) \Big\}$$

$$F_d = -16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \boxed{E(t_d) h(x_1, \bar{x}_2, b_1, b_2)} S_t(\bar{x}_2) \Big\{ r_1 \big[\phi_1^{p(a)}(x_1, b_1) + \phi_1^v(x_1, b_1)\big] \phi_2^A(x_2, b_2) \Big\}$$

with **h** and **E** are defined by:

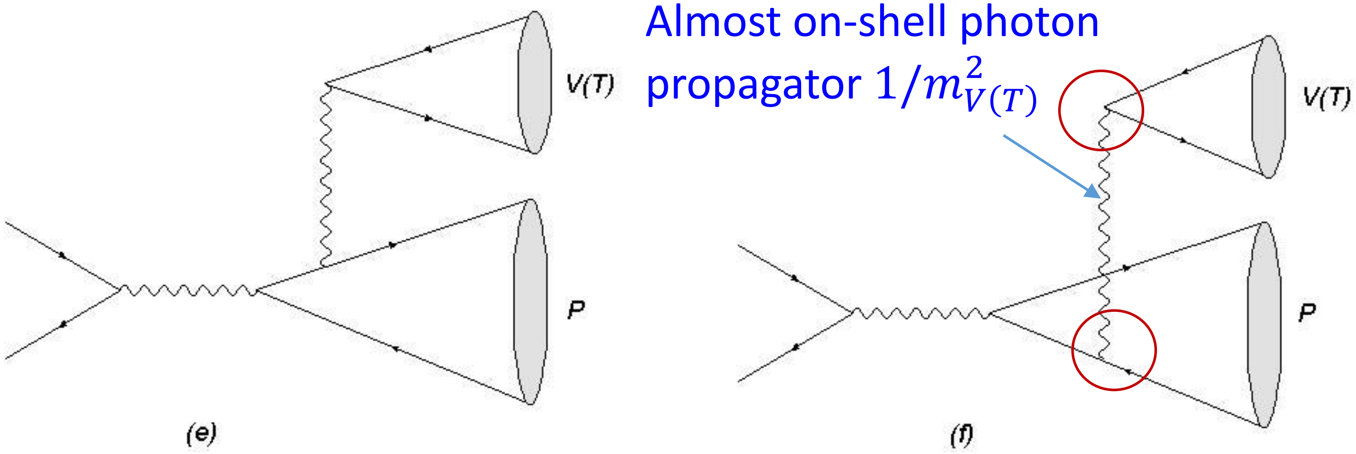
$$E(t_a) \equiv \alpha_s(t_a) \exp\big[-S_1(t_a) - S_2(t_a)\big]$$

$$h(x_1, x_2, b_1, b_2) \equiv \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\beta b_2) \Big[\theta(b_2 - b_1) J_0(b_1\alpha) H_0^{(1)}(b_2\alpha) + \theta(b_1 - b_2) J_0(b_2\alpha) H_0^{(1)}(b_1\alpha)\Big] \; S_t(x_2)$$

And the **factorization scales** are:

$$t_a = \max(\sqrt{x_2}Q, 1/b_1, 1/b_2) \qquad t_b = \max(\sqrt{\bar{x}_1}Q, 1/b_1, 1/b_2)$$
$$t_c = \max(\sqrt{x_1}Q, 1/b_1, 1/b_2) \qquad t_d = \max(\sqrt{\bar{x}_2}Q, 1/b_1, 1/b_2)$$

Exclusive processes $e^+e^- \to VP$ and TP in k_T factorization



Almost on-shell photon propagator $1/m_{V(T)}^2$

Gluon propagator in the first four diagrams

$$\frac{\alpha_{em}^2}{m_{V(T)}^2} \sim 1/s$$

Enhanced diagrams for the neutral vector (tensor) mesons production

$$F_e = F_f = \frac{12\pi\alpha_{em}^2 f_P f_{V(T)}}{m_{V(T)} s}(1 + a_2^P)$$

Then the form factors for the explicit channels of $e^+e^- \to VP$ processes:

$$
\begin{aligned}
F_{\rho^+\pi^-} &= F_{\rho^-\pi^+} = \frac{1}{3}\big[F_a(\rho\pi)+F_b(\rho\pi)\big],\\
F_{\rho^0\pi^0} &= \frac{1}{3}\big[F_a(\rho\pi)+F_b(\rho\pi)\big]+\frac{1}{6}\big[F_e(\rho\pi)+F_f(\rho\pi)\big],\\
F_{K^{*+}K^-} &= \frac{2}{3}\big[F_a(K^*K)+F_b(K^*K)\big]-\frac{1}{3}\big[F_c(K^*K)+F_d(K^*K)\big],\\
F_{K^{*-}K^+} &= -\frac{1}{3}\big[F_a(K^*K)+F_b(K^*K)\big]+\frac{2}{3}\big[F_c(K^*K)+F_d(K^*K)\big],\\
F_{K^{*0}\bar{K}^0} &= F_{\bar{K}^{*0}K^0} = -\frac{1}{3}\big[F_a(K^*K)+F_b(K^*K)\big]-\frac{1}{3}\big[F_c(K^*K)+F_d(K^*K)\big],\\
F_{\omega\pi^0} &= \big[F_a(\omega\pi)+F_b(\omega\pi)\big]+\frac{1}{18}\big[F_e(\omega\pi)+F_f(\omega\pi)\big],\\
F_{\phi\pi^0} &= \frac{\sqrt{2}}{18}\big[F_e(\phi\pi)+F_f(\phi\pi)\big].
\end{aligned}
$$

$$
\begin{aligned}
F_{\rho^0\eta_q} &= \big[F_a(\rho\eta_q)+F_b(\rho\eta_q)\big]+\frac{5}{18}\big[F_e(\rho\eta_q)+F_f(\rho\eta_q)\big],\\
F_{\rho^0\eta_s} &= -\frac{\sqrt{2}}{6}\big[F_e(\rho\eta_s)+F_f(\rho\eta_s)\big],\\
F_{\omega\eta_q} &= \frac{1}{3}\big[F_a(\omega\eta_q)+F_b(\omega\eta_q)\big]+\frac{5}{54}\big[F_e(\omega\eta_q)+F_f(\omega\eta_q)\big],\\
F_{\omega\eta_s} &= -\frac{\sqrt{2}}{18}\big[F_e(\omega\eta_s)+F_f(\omega\eta_s)\big],\\
F_{\phi\eta_q} &= -\frac{5\sqrt{2}}{54}\big[F_e(\phi\eta_q)+F_f(\phi\eta_q)\big],\\
F_{\phi\eta_s} &= -\frac{2}{3}\big[F_a(\phi\eta_s)+F_b(\phi\eta_s)\big]-\frac{1}{27}\big[F_e(\phi\eta_s)+F_f(\phi\eta_s)\big].
\end{aligned}
$$

$$
\begin{aligned}
F_{V(T)\eta} &= \cos\theta F_{V(T)\eta_q}-\sin\theta F_{V(T)\eta_s},\\
F_{V(T)\eta'} &= \sin\theta F_{V(T)\eta_q}+\cos\theta F_{V(T)\eta_s},
\end{aligned}
$$

And the form factors for the explicit channels of $e^+e^- \to TP$ processes:

$$
\begin{aligned}
F_{a_2^+\pi^-} &= -F_{a_2^-\pi^+} = \left[F_a(a_2\pi) + F_b(a_2\pi)\right], \\
F_{a_2^0\pi^0} &= \frac{1}{6}\left[F_e(a_2\pi) + F_f(a_2\pi)\right], \\
F_{K_2^{*+}K^-} &= \frac{2}{3}\left[F_a(K_2^*K) + F_b(K_2^*K)\right] - \frac{1}{3}\left[F_c(K_2^*K) + F_d(K_2^*K)\right], \\
F_{K_2^{*-}K^+} &= -\frac{1}{3}\left[F_a(K_2^*K) + F_b(K_2^*K)\right] + \frac{2}{3}\left[F_c(K_2^*K) + F_d(K_2^*K)\right], \\
F_{K_2^{*0}\bar{K}^0} &= F_{\bar{K}_2^{*0}K^0} = -\frac{1}{3}\left[F_a(K_2^*K) + F_b(K_2^*K)\right] - \frac{1}{3}\left[F_c(K_2^*K) + F_d(K_2^*K)\right], \\
F_{a_2^0\eta_q} &= \frac{5}{18}\left[F_e(a_2\eta_q) + F_f(a_2\eta_q)\right], \\
F_{a_2^0\eta_s} &= -\frac{\sqrt{2}}{6}\left[F_e(a_2\eta_s) + F_f(a_2\eta_s)\right].
\end{aligned}
$$

Models of the transverse momentum dependent wave functions

At present, the intrinsic transverse momentum dependence of WF is still unknown from the first principle of QCD. As an illustration, we use a simple model in which the dependence of the WF on the longitudinal and transverse momentum can be factorized into two parts:

$$\psi(x, \mathbf{k}_T) = \phi(x) \times \Sigma(\mathbf{k}_T)$$

The transverse WF can be chosen as

1. $\Sigma() = 1;$
2. $\Sigma(b) = \exp\left(-\frac{b^2}{4\beta^2}\right)$, with $\beta^2 = 4\mathrm{GeV}^2;$
3. $\Sigma(x, b) = \exp\left[-\frac{x(1-x)b^2}{4a^2}\right]$, with $a = 1\mathrm{GeV}.$

Phys. Lett. B **315**,463 (1993)
Phys. Lett. B **319**,545(E) (1993)
Phys. Lett. B **449**,299 (1999)

Phys. Rev. D **74**, 014027 (2006)

Numerical Results

Channel	$\sqrt{s} = 3.67$ GeV σ_{S1}(pb)	σ_{S2}(pb)	σ_{S3}(pb)	$\sigma_{\exp}$(pb)	$\sqrt{s} = 10.58$ GeV σ_{S1}(fb)	σ_{S2}(fb)	σ_{S3}(fb)	$\sigma_{\exp}$(fb)
$\rho^+\pi^-$	6.80 ± 1.18	3.38 ± 0.53	3.95 ± 0.63	$4.8^{+1.5+0.5}_{-1.2-0.5}$	0.66 ± 0.10	0.53 ± 0.08	0.60 ± 0.09	
$\rho^0\pi^0$	3.38 ± 0.60	1.69 ± 0.27	1.99 ± 0.32	$3.1^{+1.0+0.4}_{-1.2-0.4}$	0.25 ± 0.05	0.20 ± 0.04	0.23 ± 0.04	
$K^*(892)^-K^+$	10.13 ± 0.91	5.27 ± 0.50	5.39 ± 0.35	$1.0^{+1.1+0.5}_{-0.7-0.5}$	1.15 ± 0.10	0.94 ± 0.08	1.02 ± 0.08	$0.18^{+0.14}_{-0.12} \pm 0.02$
$K^*(892)^0\bar{K}^0$	61.94 ± 13.76	31.34 ± 6.15	31.85 ± 6.25	$23.5^{+4.6+3.1}_{-3.9-3.1}$	6.65 ± 1.20	5.39 ± 0.93	5.88 ± 1.02	$7.48 \pm 0.67 \pm 0.51$
$\omega\pi^0$	24.94 ± 4.59	12.41 ± 2.08	15.18 ± 2.59	$15.2^{+2.8+1.5}_{-2.4-1.5}$	2.38 ± 0.40	1.90 ± 0.31	2.16 ± 0.35	
$\phi\pi^0$	1.2×10^{-4}	1.2×10^{-4}	1.2×10^{-4}	< 2.2	2.2×10^{-3}	2.2×10^{-3}	2.2×10^{-3}	
$\rho^0\eta$	14.37 ± 2.10	7.21 ± 0.96	8.10 ± 1.06	$10.0^{+2.2+1.0}_{-1.9-1.0}$	1.10 ± 0.13	0.89 ± 0.11	1.03 ± 0.12	
$\rho^0\eta'$	8.22 ± 1.19	4.10 ± 0.54	4.57 ± 0.59	$2.1^{+4.7+0.2}_{-1.6-0.2}$	1.03 ± 0.11	0.83 ± 0.09	0.93 ± 0.10	
$\omega\eta$	1.31 ± 0.20	0.65 ± 0.09	0.77 ± 0.11	$2.3^{+1.8+0.5}_{-1.0-0.5}$	0.10 ± 0.01	0.081 ± 0.011	0.094 ± 0.012	
$\omega\eta'$	0.75 ± 0.11	0.37 ± 0.05	0.43 ± 0.06	< 17.1	0.094 ± 0.011	0.076 ± 0.009	0.086 ± 0.010	
$\phi\eta$	17.82 ± 3.34	9.21 ± 1.51	8.23 ± 1.32	$2.1^{+1.9+0.2}_{-1.2-0.2}$	2.11 ± 0.30	1.75 ± 0.23	1.84 ± 0.25	$2.9 \pm 0.5 \pm 0.1$
$\phi\eta'$	21.97 ± 4.13	11.36 ± 1.87	10.20 ± 1.65	< 12.6	2.81 ± 0.42	2.31 ± 0.33	2.47 ± 0.35	

Results of $e^+e^- \to VP$ cross sections at $\sqrt{s} = 3.67\ GeV$ and $\sqrt{s} = 10.58\ GeV$ denoted by different transverse momentum distributions functions $S1$, $S2$ and $S3$, respectively.

Numerical Results

Channel	$\sqrt{s}=3.67$ GeV				$\sqrt{s}=10.58$ GeV			
	σ_{S1}(pb)	σ_{S2}(pb)	σ_{S3}(pb)	$\sigma_{\rm exp}$(pb)	σ_{S1}(fb)	σ_{S2}(fb)	σ_{S3}(fb)	$\sigma_{\rm exp}$(fb)
$a_2^+\pi^-$	43.88 ± 13.98	20.34 ± 6.59	28.96 ± 8.62		6.66 ± 1.73	4.96 ± 1.30	6.06 ± 1.58	
$a_2^0\pi^0$	0	0	0		0	0	0	
$K_2^*(1430)^-K^+$	60.57 ± 15.89	27.81 ± 7.45	33.81 ± 8.98		11.48 ± 2.45	8.48 ± 1.79	9.98 ± 2.15	$8.36\pm0.95\pm0.62$
$K_2^*(1430)^0\bar{K}^0$	3.2×10^{-2}	1.1×10^{-2}	1.3×10^{-2}		8.8×10^{-3}	6.0×10^{-3}	7.3×10^{-3}	$1.65^{+0.86}_{-0.78}\pm0.27$
$a_2^0\eta$	0	0	0		0	0	0	
$a_2^0\eta'$	0	0	0		0	0	0	
$f_2\pi^0$	0	0	0		0	0	0	
$f_2'\pi^0$	0	0	0		0	0	0	

Forbidden due to the C-parity and U-spin symmetry

Broken by the SU(3) symmetry breaking effect

Results of $e^+e^- \to TP$ cross sections at $\sqrt{s}=3.67\ GeV$ and $\sqrt{s}=10.58\ GeV$ denoted by different transverse momentum distributions functions $S1$, $S2$ and $S3$, respectively.

R ratio

To investigate the $SU(3)$ symmetry breaking effect in the $e^+e^- \to K^*K$ processes

$$R_{VP} = \frac{\sigma(e^+e^- \to K^*(892)^0 \bar{K}^0)}{\sigma(e^+e^- \to K^*(892)^- K^+)}, \quad R_{TP} = \frac{\sigma(e^+e^- \to K_2^*(1430)^0 \bar{K}^0)}{\sigma(e^+e^- \to K_2^*(1430)^- K^+)}.$$

In the framework of k_T factorization:

$$R = \left| \frac{(F_a + F_b) + (F_c + F_d)}{2(F_a + F_b) - (F_c + F_d)} \right|^2 = \left| \frac{1 + \frac{F_c + F_d}{F_a + F_b}}{2 - \frac{F_c + F_d}{F_a + F_b}} \right|^2.$$

If SU(3) symmetry works well, $R_{VP} = 4$ and $R_{TP} = 0$.

In our results:

$$R_{VP}(\sqrt{s} = 3.67GeV) \simeq 5.99, \quad R_{VP}(\sqrt{s} = 10.58GeV) \simeq 5.76,$$

$$R_{TP} \lesssim 10^{-4}.$$

R ratio

To investigate the $SU(3)$ symmetry breaking effect in the $e^+e^- \to K^*K$ processes

$$R_{VP} = \frac{\sigma(e^+e^- \to K^*(892)^0\bar{K}^0)}{\sigma(e^+e^- \to K^*(892)^-K^+)}, \quad R_{TP} = \frac{\sigma(e^+e^- \to K_2^*(1430)^0\bar{K}^0)}{\sigma(e^+e^- \to K_2^*(1430)^-K^+)}.$$

Our results

$$R_{VP}(\sqrt{s} = 3.67GeV) \simeq 5.99,$$

$$R_{VP}(\sqrt{s} = 10.58GeV) \simeq 5.76,$$

$$R_{TP} \lesssim 10^{-4}.$$

Experimental results

$$R_{VP}^{Exp}(\sqrt{s} = 3.67GeV) = 23.5^{+17.1}_{-26.1} \pm 12.2.$$

CLEO-c results

		10.52GeV	10.58GeV	10.876GeV
R_{VP}^{Exp}	$>$	4.3,	20.0,	5.4,
R_{TP}^{Exp}	$<$	1.1,	0.4,	0.6.

Belle results

We neglected the contributions from $\psi(2S)$ and $\Upsilon(4S)$ resonance

The $1/s^n$ dependence of the cross section

Others' work:

Phys. Lett. B 425, 365 (1998)	$\propto 1/s^2$
Phys. Rev. D 75, 094020 (2007)	$\propto 1/s^3$
Phys. Rev. D 22, 2157 (1980) Phys. Rev. D 24, 2848 (1981)	$\propto 1/s^4$

Our results:

$$e^+e^- \to VP \quad n = 4.1$$

$$e^+e^- \to TP \quad n = 3.9$$

$\Rightarrow$ We favor the $1/s^4$ scaling.

The experimental results:

$$\mathrm{e^+e^-} \to \mathrm{K^*(892)^0\overline{K}^0} \qquad n = 3.83 \pm 0.07$$

$$\mathrm{e^+e^-} \to \omega\pi^0 \qquad n = 3.75 \pm 0.12$$

Summary

- Analysis of the exclusive processes $e^+e^- \to VP$ and TP in k_T factorization at $\sqrt{s}$ =3.67GeV and 10.58GeV.

- Perturbative QCD approach based on the k_T factorization.
  - Hard scattering kernel: high energy scale, calculated perturbatively;
  - Hadron wave function: universal $\begin{cases} longitudinal \\ transverse \end{cases}$: different models

- Our results are in good agreement with the experimental results:
  - Cross section;
  - R-ratio: SU(3) symmetry breaking effect;
  - s-dependence of the cross section.

THANK YOU!