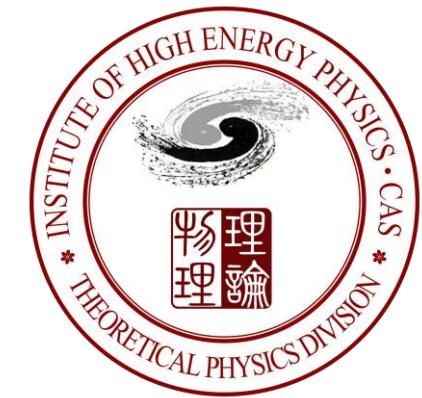


# $\rho$ meson unpolarized GPDs with a light-front constituent quark model

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**IHEP, CAS**

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# Outline

[arXiv:1707.03972](https://arxiv.org/abs/1707.03972) [Accepted by PRD]

- Light-front form
- GPDs & Hadron structure
- Summary

# Dirac's forms of relativistic dynamics

[ Diehl, Rev.Mod.Phys.21.392 ]

[ Burkardt, 1996 & Carbonell, 1998 & Brodsky 1998 ]

## instant form

Time variable:

$$t = x^0$$

Quantization surface:

$$t = 0$$

Hamiltonian:

$$H = P^0$$

Dispersion relation

$$p^0 = \sqrt{\vec{p}_\perp^2 + m^2}$$

$$g^{++} = g^{--} = 0 \quad g^{+-} = \frac{1}{2}$$

$$g^{i+} = g^{i-} = 0 \quad g^{ij} = -\delta^{ij} \quad \text{for } i=1,2,$$

$$p^2 = m^2 = p_+ p_- - \vec{p}_\perp^2 \quad d^4 k = \frac{1}{2} dk_\perp dk_+ dk_-$$

$$x = (x^+, x^-, x_\perp) \equiv (x^0 + x^3, x^0 - x^3, x^1, x^2)$$

## front form

$$x^+ = x^0 + x^3$$

$$x^+ = 0$$

$$P^- \equiv P^0 + P^3$$

$$p^- = (\vec{p}_\perp^2 + m^2)/p^+$$



$$H = P^- = \frac{m^2 + \vec{P}_\perp^2}{P^+} + V_{\text{int}}$$

Mass operator square  $M^2$

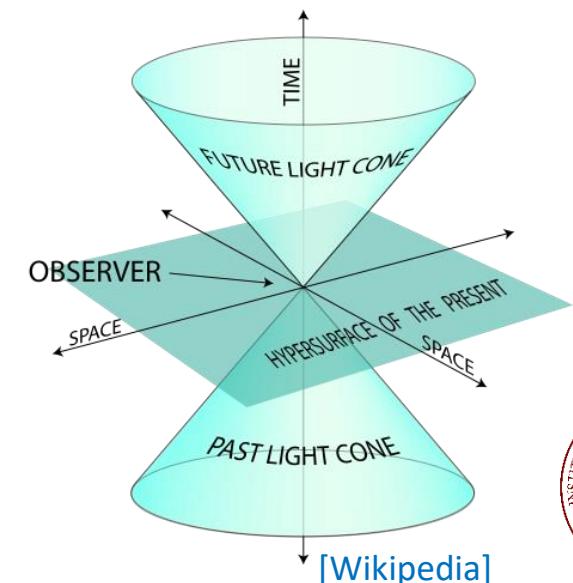
## point form

$$\tau = \sqrt{t^2 - \vec{x}^2 - a^2}$$

$$\tau = 0$$

$$P^\mu$$

$$p^\mu = mv^\mu$$

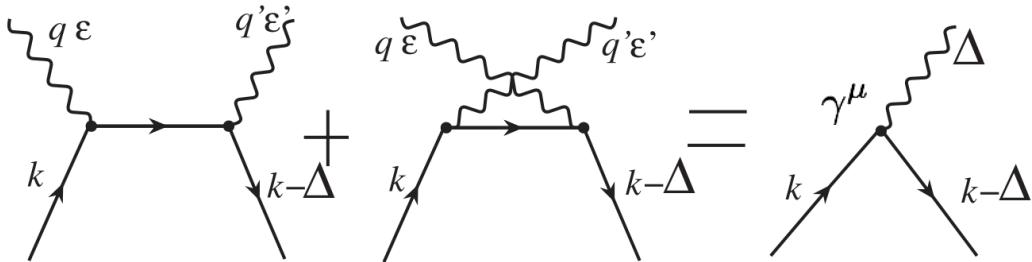


# GPDs & Hadron structure

- Definition of GPDs (General Parton Distributions)
- Connections between hadron structure functions
- Some basic properties
  - Sum rules
  - Symmetry properties
  - Forward limit
  - QCD Evolution

# Definition of GPDs (General Parton Distributions)

[Chueng-yong Ji, Phys.Rev.D.73.114013]  
 [M. Diehl, Eur. Phys. J. A (2016) 52: 149]



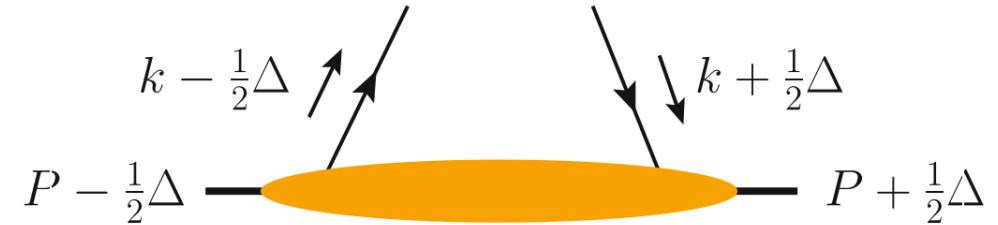
A GPD factorization formula:

$$\mathcal{A}(\xi, \Delta^2, Q^2) = \sum_i \int_{-1}^1 dx C_i(x, \xi; \log(Q/\mu)) H_i(x, \xi, \Delta^2; \mu)$$

*DVCS, TCS, meson production*

In the Deep Inelastic Limit, neglect any 4-products not involving  $q$ :

$$q^2, q \cdot a \gg a \cdot b, m^2, \varepsilon \cdot q, \varepsilon' \cdot q \quad \Gamma^\mu \rightarrow \gamma^\mu$$



Parton correlation function:

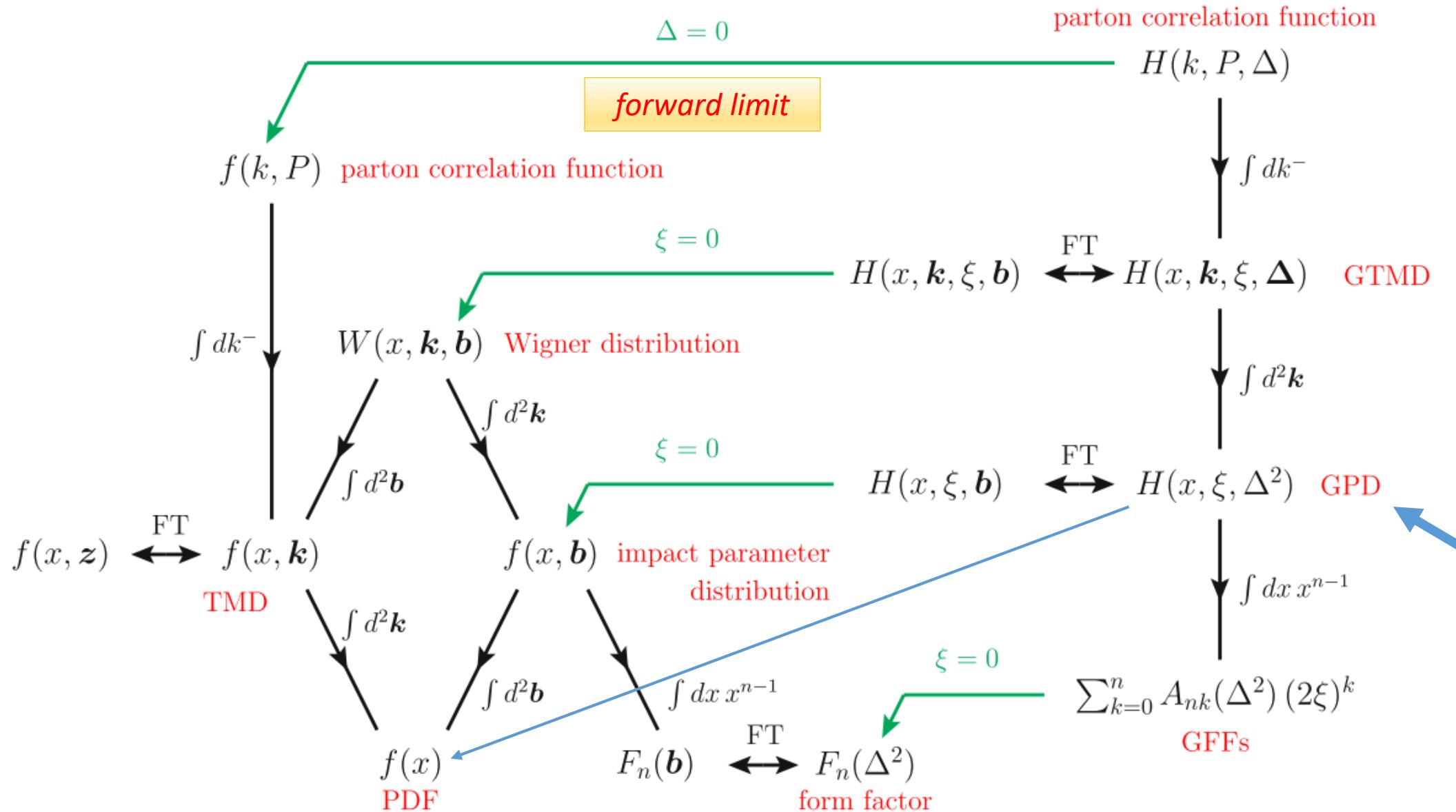
*flavor by flavor*

$$H(k, P, \Delta) = (2\pi)^{-4} \int d^4 z e^{izk} \times \langle p(P + \frac{1}{2}\Delta) | \bar{q}(-\frac{1}{2}z) \Gamma q(\frac{1}{2}z) | p(P - \frac{1}{2}\Delta) \rangle$$

The Dirac matrix  $\Gamma$  selects the twist and the parton spin degrees of freedom.

*Gauge  $A^+ = 0$*

# Connections between hadron structure functions



# Definition of GPDs (J=1)

[Berger, Phys.Rev.Lett.87.142302]

- Unpolarized

$$V_{\lambda' \lambda} = \frac{1}{2} \int \frac{d\omega}{2\pi} e^{ix(Pz)} \langle p', \lambda' | \bar{q}(-\frac{1}{2}z) \not{q} q(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z=\omega n}$$

$$= \sum_i \epsilon'^{\nu} V_{\nu \mu}^{(i)} \epsilon^{\mu} H_i^q(x, \xi, t)$$

$$V_{\nu \mu} = \{g_{\mu\nu}, P_{\mu}n_{\nu}, n_{\mu}P_{\nu}, P_{\mu}P_{\nu}, n_{\mu}n_{\nu}\}$$

- Polarized

$$A_{\lambda' \lambda} = \frac{1}{2} \int \frac{d\omega}{2\pi} e^{ix(Pz)} \langle p', \lambda' | \bar{q}(-\frac{1}{2}z) \not{q} \gamma_5 q(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z=\omega n}$$

$$= \sum_i \epsilon'^{\nu} A_{\nu \mu}^{(i)} \epsilon^{\mu} \tilde{H}_i^q(x, \xi, t)$$

- Symmetry properties:

$$P = \frac{p'+p}{2}, \quad t = \Delta^2 = (p'-p)^2,$$

$$n^2 = 0, \text{ (lightlike four-vector)}$$

$$\xi = (n \cdot \Delta) / (n \cdot P), \text{ skewness parameter},$$

$$\epsilon = \epsilon(p, \lambda), \epsilon' = \epsilon'(p', \lambda'), \text{ polarizations},$$

$$H_i(x, \xi, t) = H_i(x, -\xi, t) \quad (I = 1, 2, 3, 5)$$

$$H_4(x, \xi, t) = -H_4(x, -\xi, t)$$

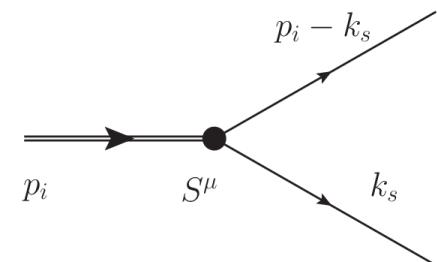
$$\textcolor{red}{T} \quad \tilde{H}_i(x, \xi, t) = \tilde{H}_i(x, -\xi, t) \quad (I = 1, 2, 4)$$

$$\tilde{H}_3(x, \xi, t) = -\tilde{H}_3(x, -\xi, t)$$

$$\textcolor{red}{G} \quad H_{\rho^+}^d(x, \xi, t) = -H_{\rho^+}^u(x, -\xi, t)$$

# Definition of GPDs: Isospin combinations

- **Lagrangian:**  $\mathcal{L}_{\rho \rightarrow q\bar{q}} = -i(M/f_\rho)\bar{q}S^\mu\tau q \cdot \rho_\mu = -i(M/f_\rho) [\bar{u}S^\mu u\rho_\mu^0 + \sqrt{2}\bar{u}S^\mu d\rho_\mu^+ + \sqrt{2}\bar{d}S^\mu u\rho_\mu^- + \bar{d}S^\mu d\rho_\mu^0]$
- **Quark field doublets:**  $q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}, \quad \tau_3 q(x) = \begin{pmatrix} u(x) \\ -d(x) \end{pmatrix}$
- **5 unpolarized GPDs: Isospin combinations**



$$\begin{aligned} & \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix(Pz)} \langle \rho^b(p', \lambda') | \bar{q}(-\frac{1}{2}z) \not{\epsilon} \tau_3 q(\frac{1}{2}z) | \rho^a(p, \lambda) \rangle \Big|_{z=\lambda n} = i\epsilon_{3ab} \left\{ - (\epsilon'^* \cdot \epsilon) H_{1,\rho^b}^{I=1} \right. \\ & + \frac{(\epsilon \cdot n)(\epsilon'^* \cdot P) + (\epsilon'^* \cdot n)(\epsilon \cdot P)}{P \cdot n} H_{2,\rho^b}^{I=1} - \frac{2(\epsilon \cdot P)(\epsilon'^* \cdot P)}{m^2} H_{3,\rho^b}^{I=1} \\ & \left. + \frac{(\epsilon \cdot n)(\epsilon'^* \cdot P) - (\epsilon'^* \cdot n)(\epsilon \cdot P)}{P \cdot n} H_{4,\rho^b}^{I=1} + \left[ m^2 \frac{(\epsilon \cdot n)(\epsilon'^* \cdot n)}{(P \cdot n)^2} + \frac{1}{3} (\epsilon'^* \cdot \epsilon) \right] H_{5,\rho^b}^{I=1} \right\} \end{aligned}$$

Isospin combinations:  $H_{i,\rho^\pm}^{I=1}(x, \xi, t) = \frac{1}{2}[H_{i,\rho^\pm}^u(x, \xi, t) - H_{i,\rho^\pm}^d(x, \xi, t)]$

G parity:  $H_{\rho^+}^d(x, \xi, t) = -H_{\rho^+}^u(x, -\xi, t)$

# Sum rules

- Form factor decomposition of Local current

$$\begin{aligned} I_{\lambda'\lambda}^\mu &= \langle p', \lambda' | \bar{q}(0) \gamma^\mu q(0) | p, \lambda \rangle \\ &= \epsilon'^*\epsilon^\alpha \left[ - \left( G_1^q(t) g_{\beta\alpha} + G_3^q(t) \frac{P_\beta P_\alpha}{2M^2} \right) P^\mu + G_2^q(t) \left( g_\alpha^\mu P_\beta + g_\beta^\mu P_\alpha \right) \right] \end{aligned}$$

FFs in flavor

- Sum rules

$$\int_{-1}^1 dx H_i^q(x, \xi, t) = G_i^q(t) \quad (i = 1, 2, 3) ,$$

$$\int_{-1}^1 dx H_i^q(x, \xi, t) = 0 \quad (i = 4, 5) .$$

- Conventional Form factors

$$G_C(t) = G_1(t) + \frac{2}{3}\eta G_Q(t) ,$$

$$G_M(t) = G_2(t) ,$$

$$G_Q(t) = G_1(t) - G_2(t) + (1 + \eta)G_3(t) ,$$

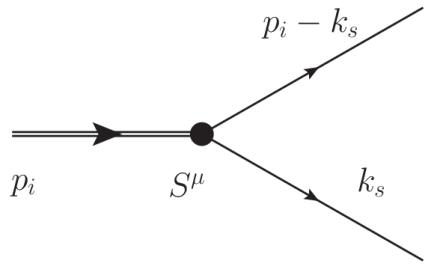
[Berger, Phys.Rev.Lett.87.142302]

[Broniowski, Phys.Rev.D.77.034023, *ibid.* 78.094011]

[Frederico, Phys.Rev.D.80.054021]

Bypassing the ambiguity  
of angular conditions

# Definition of GPDs: Phenomenal vertex



$$x' = \frac{-k_s^+}{p_i^+}$$
$$\kappa_\perp = k_{s\perp} - \frac{k_s^+}{p_i^+} p_{i\perp}$$

*Phenomenal vertex:*  $S^\mu = \Gamma^\mu \Lambda(k_s, p)$

*Bethe-Salpeter amplitude(BSA):*  $\Lambda(k_s, p) = \frac{c}{[k_s^2 - m_R^2 + i\epsilon][(p - k_s)^2 - m_R^2 + i\epsilon]}$

*Meson vertex:*  $\Gamma^\mu = \gamma^\mu - \frac{(k_q + k_{\bar{q}})^\mu}{M_0 + 2m}$

$$M_0^2 = \frac{\kappa_\perp^2 + m^2}{1 - x'} + \frac{\kappa_\perp^2 + m^2}{x'}$$

Dispersion relation

*Kinematic invariant mass:*

[Choi, Phys.Rev.D.70.053015]

# Residuals

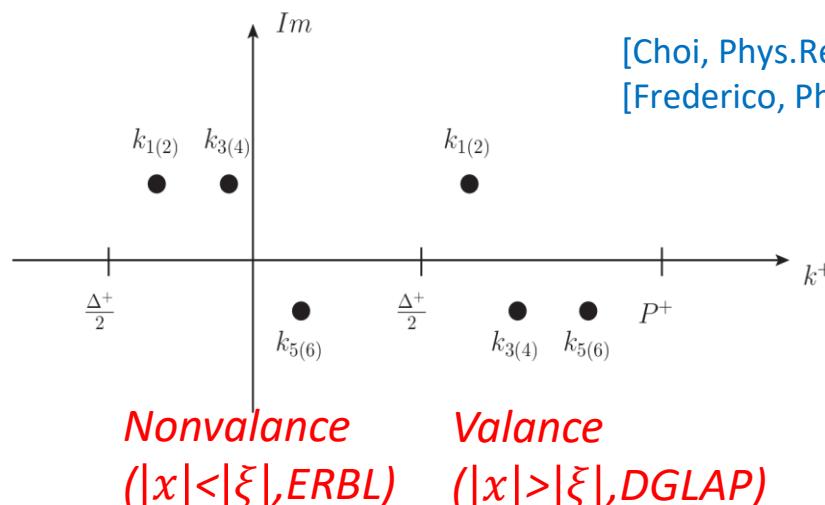
[Choi, Phys.Rev.D.70.053015]  
 [Frederico, Phys.Rev.D.80.054021]

- Six pole (Valence)

$$k_{1(2)}^- = P^- + (k - P)_{on(R)}^- - i \frac{\epsilon}{k^+ - P^+} ,$$

$$k_{3(4)}^- = \frac{\Delta^-}{2} + (k - \frac{\Delta}{2})_{on(R)}^- - i \frac{\epsilon}{k^+ - \frac{\Delta^+}{2}} ,$$

$$k_{5(6)}^- = -\frac{\Delta^-}{2} + (k + \frac{\Delta}{2})_{on(R)}^- - i \frac{\epsilon}{k^+ + \frac{\Delta^+}{2}} .$$



- Nonvalence kinematic invariant mass

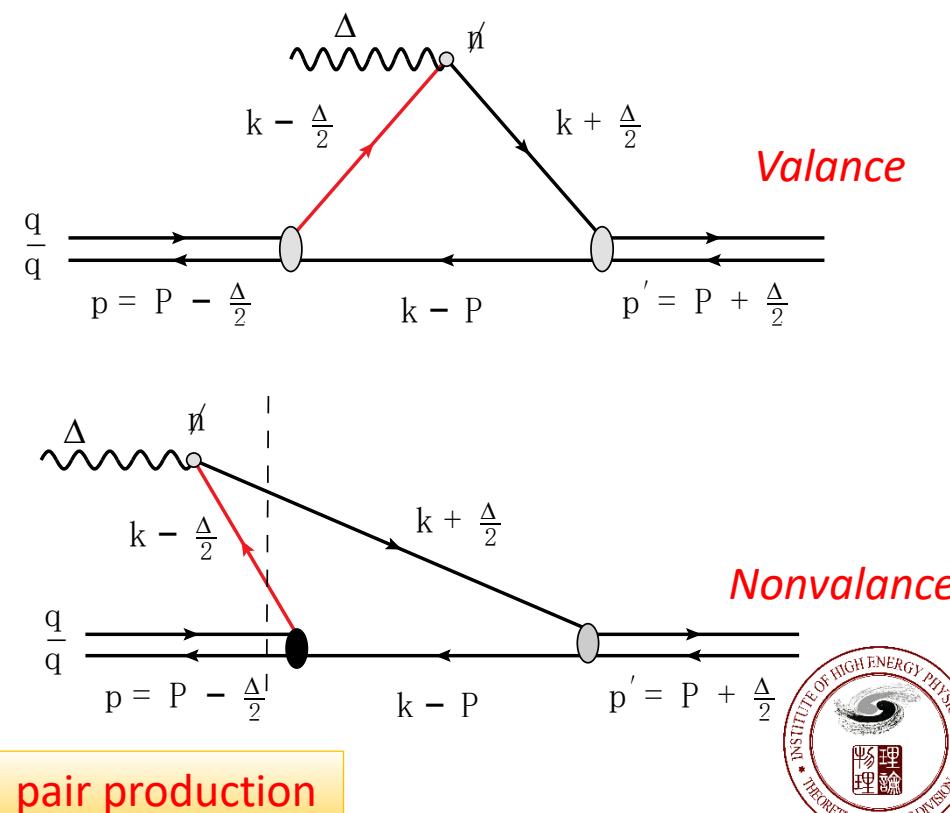
$$M_{0i(v)}^2 = \frac{\kappa_\perp^2 + m^2}{1 - x'} + \frac{\kappa_\perp^2 + m^2}{x'}$$

$$\rightarrow \frac{\kappa_\perp^2 + m^2}{x' - 1} + \frac{\kappa_\perp^2 + m^2}{x'} = M_{0i(nv)}^2$$

$x \rightarrow 0, 1$     intrinsic momentum go infinite!

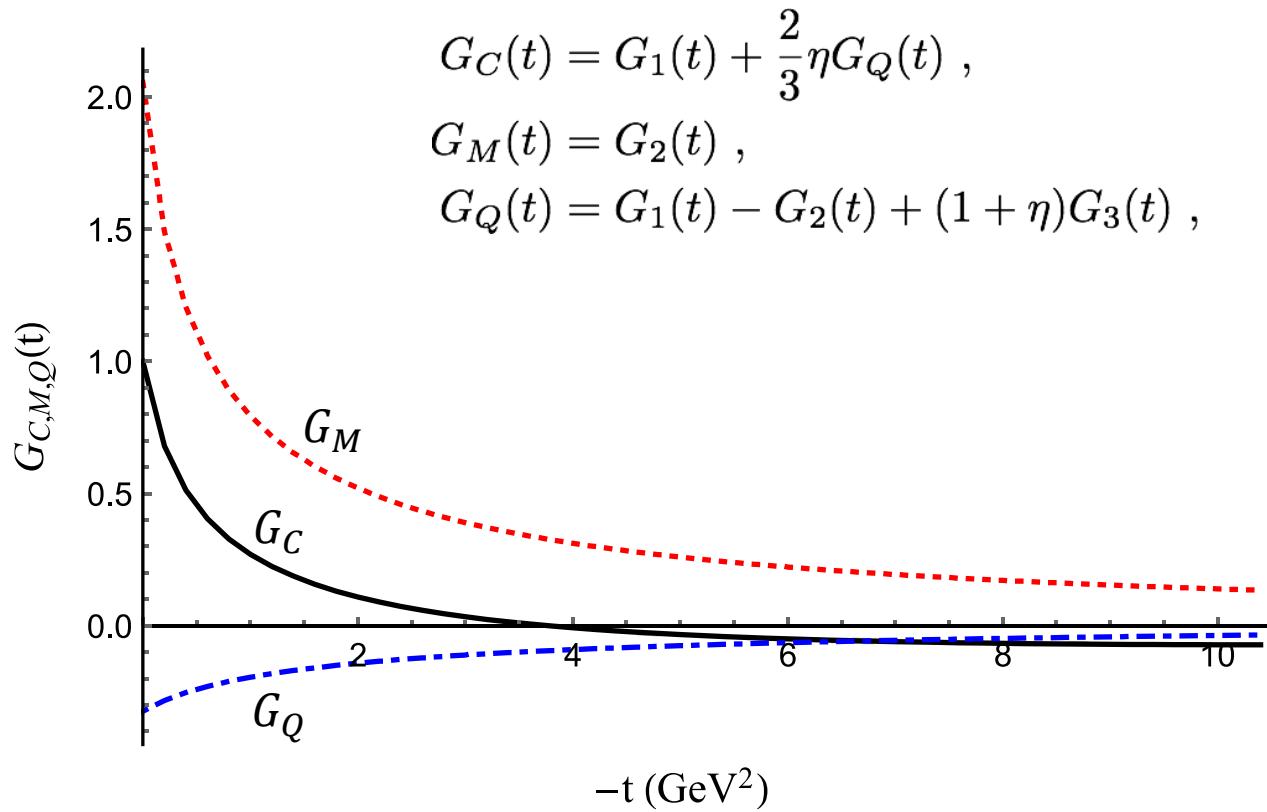
$$x = \frac{n \cdot k}{n \cdot P} = \frac{k^+}{P^+}$$

$$x' = \frac{1 - x}{1 - |\xi|}$$



# Our results: FFs $G_{C,M,Q}$

- Form factors



[ Melo, Phys.Rev.C.55.2043 ]

[ Gudino, Int. J. Mod. Phys. Conf. Ser. 35, 1460463, (2014) ]

- low-energy observables

$$G_C(0) = 1 ,$$

$$G_M(0) = 2M\mu ,$$

$$G_Q(0) = M^2 Q_\rho ,$$

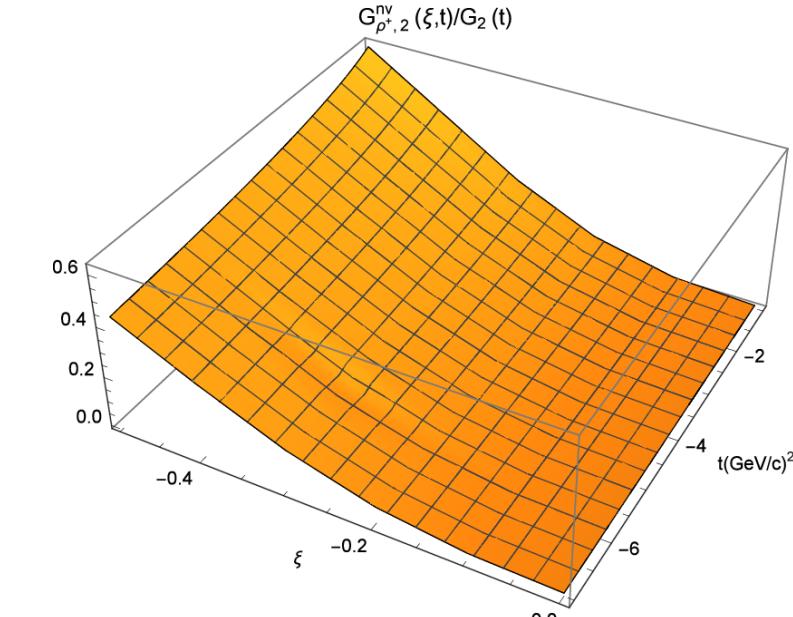
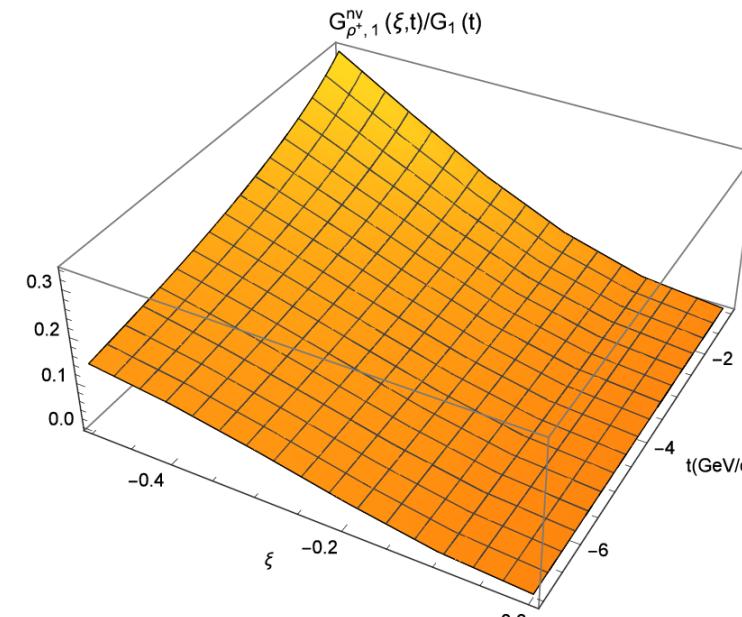
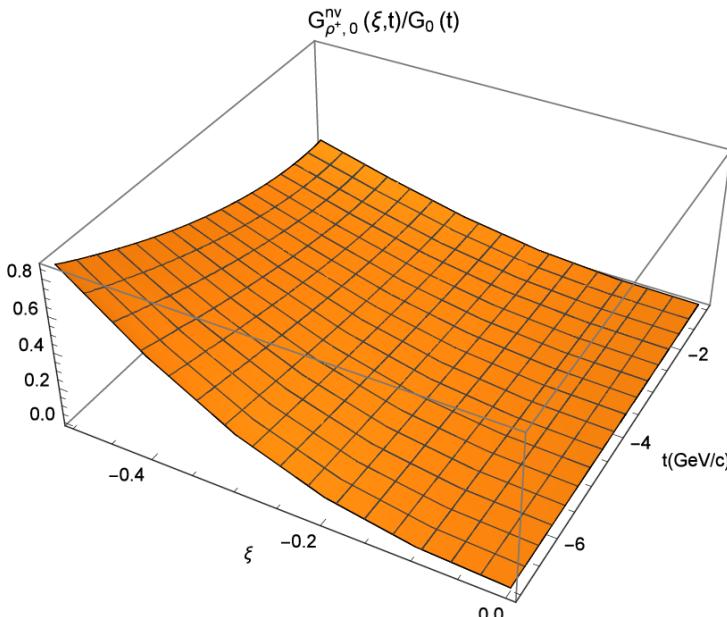
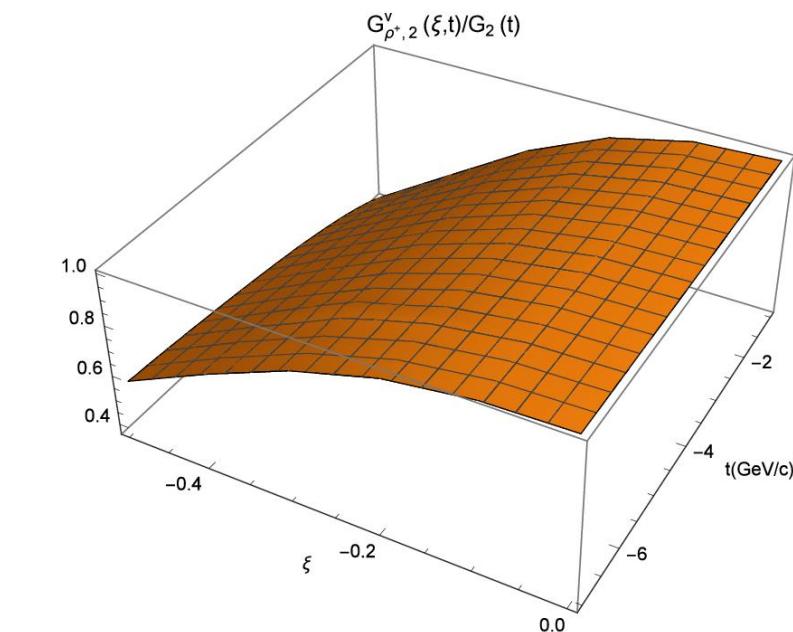
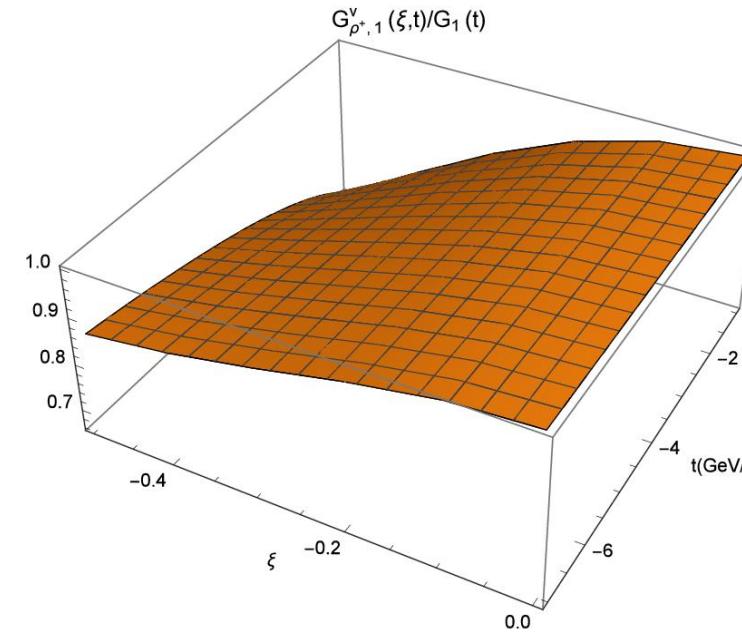
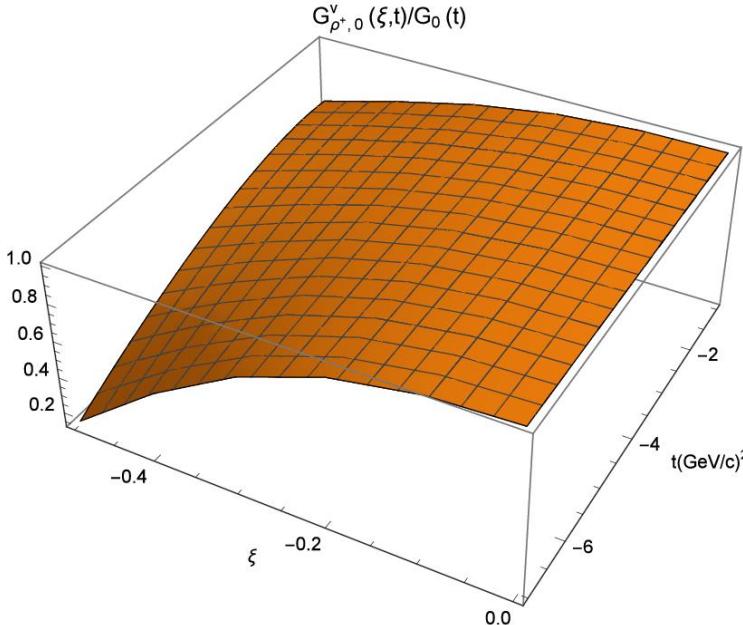
$$\langle r^2 \rangle = \lim_{t \rightarrow 0} \frac{6[G_C(t) - 1]}{t} .$$

	This work	Melo1997	Exp. [Gudino2014]
$\langle r^2 \rangle$ (fm $^2$ )	0.52	0.37	*
$\mu$	2.06	2.19	2.1(5)
$Q_2$ (fm $^2$ )	0.021	0.050	*

m (constituent mass)	mR (regulator mass)
0.403GeV	1.61GeV

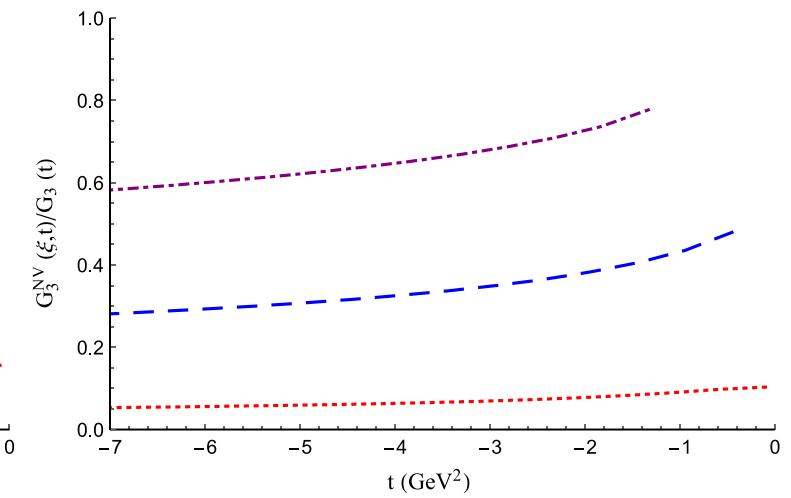
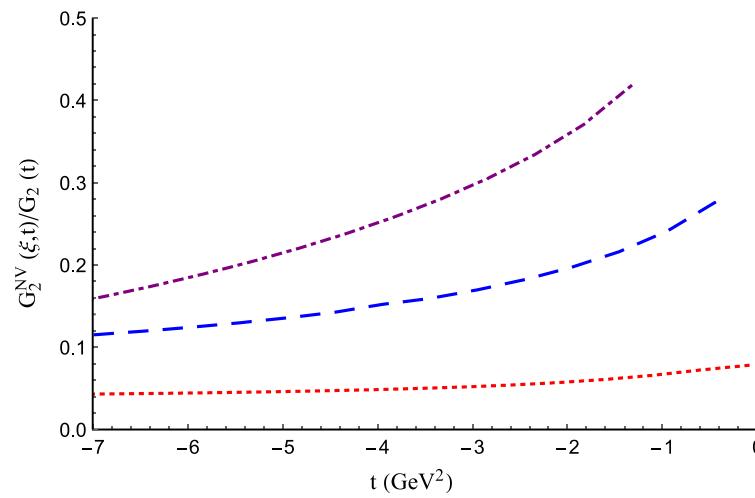
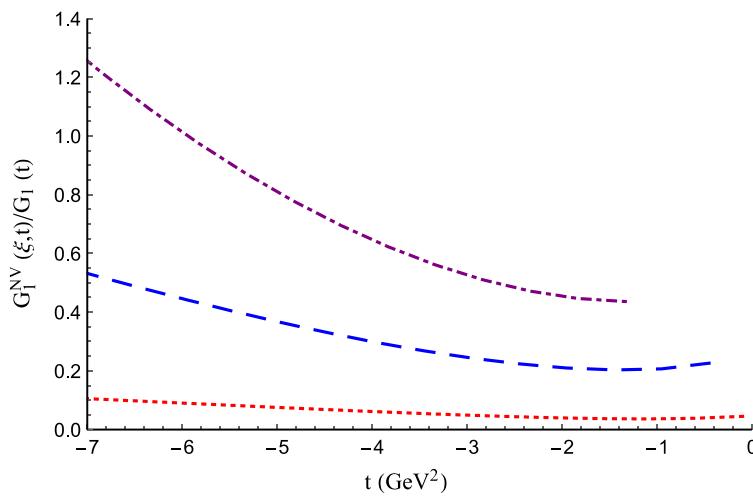
# FFs $G_{1,2,3}$ : Valence/Nonvalence contributions

$$|\xi| < \frac{1}{\sqrt{1 - 4M^2/t}}$$



# FFs $G_{1,2,3}$ : Nonvalence contributions

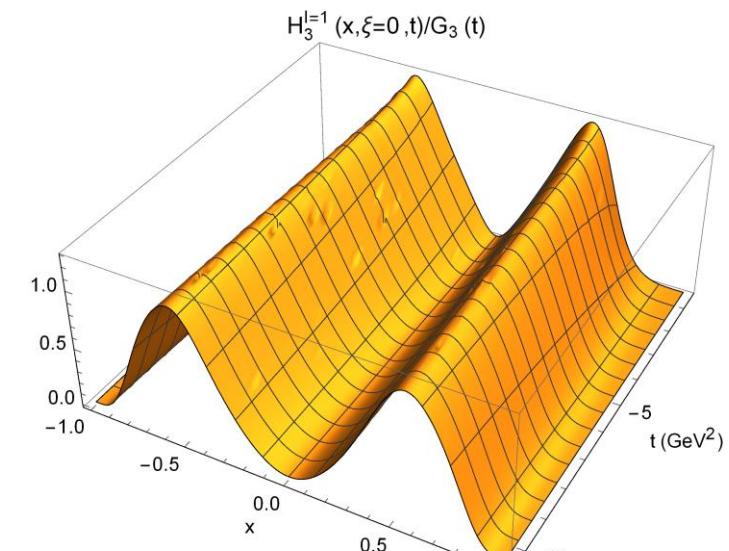
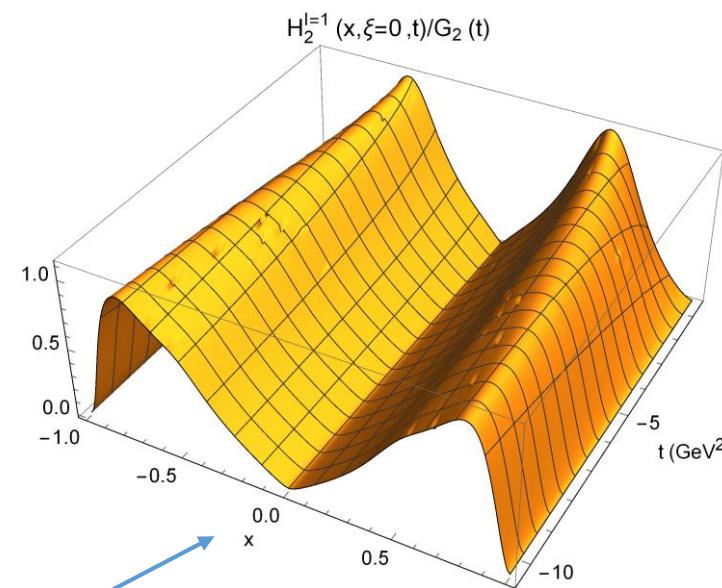
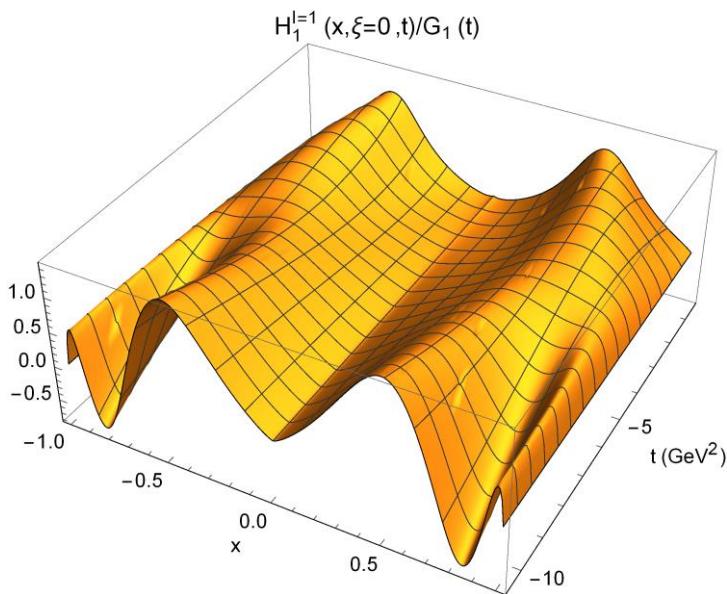
$$|\xi| < \frac{1}{\sqrt{1 - 4M^2/t}}$$



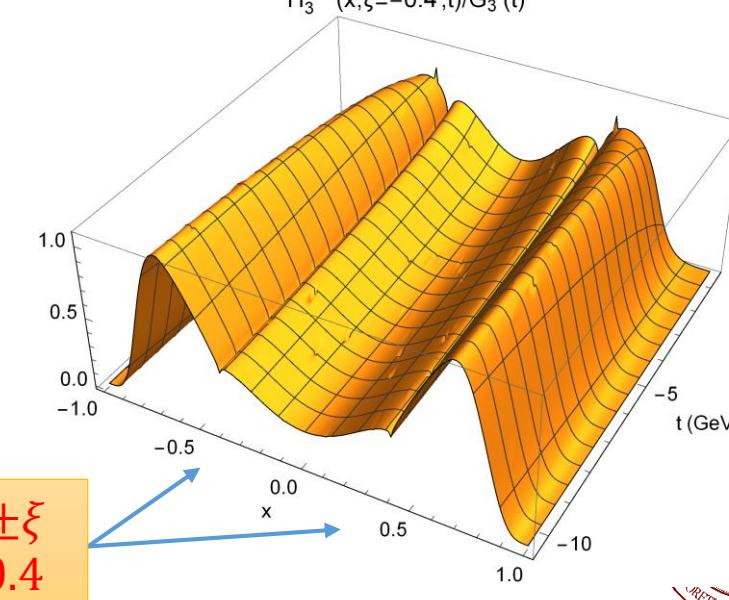
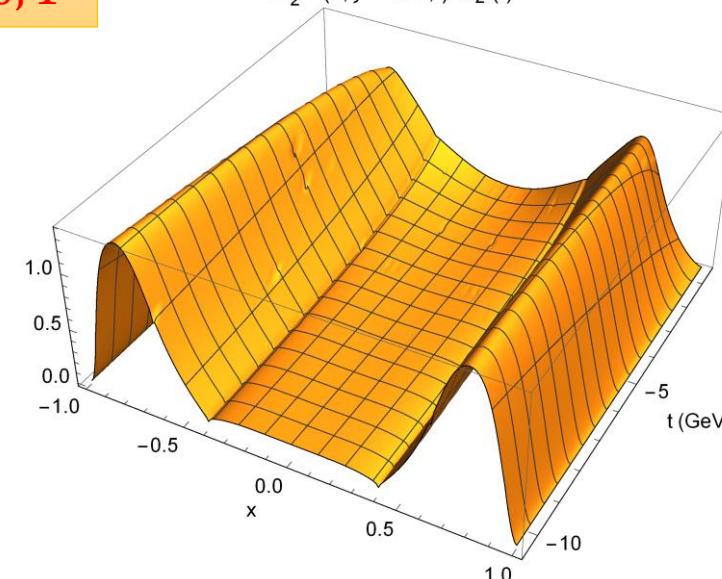
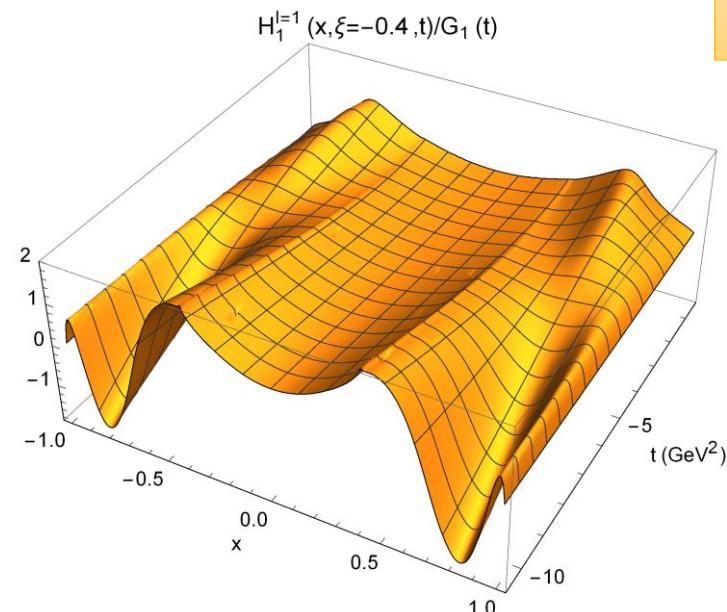
The nonvalence contributions to FFs  $G_{1,2,3}$  at  $\xi = -0.2$  (dotted red line),  
 $-0.4$  (dashed blue line),  $-0.6$  (dot-dashed purple line), respectively.

# Our results: unpolarized GPDs $H_{i=1,2,3}(x, \xi_0, t)$

$$|\xi| < \frac{1}{\sqrt{1 - 4M^2 / t}}$$



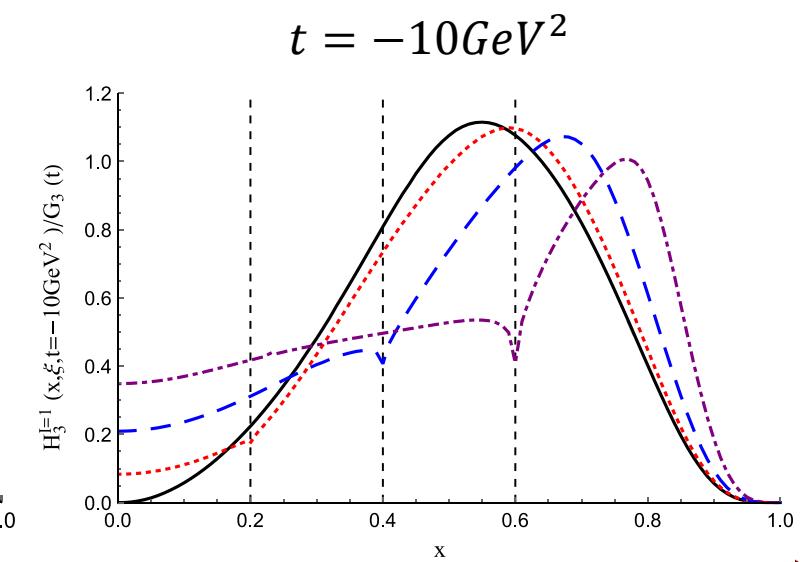
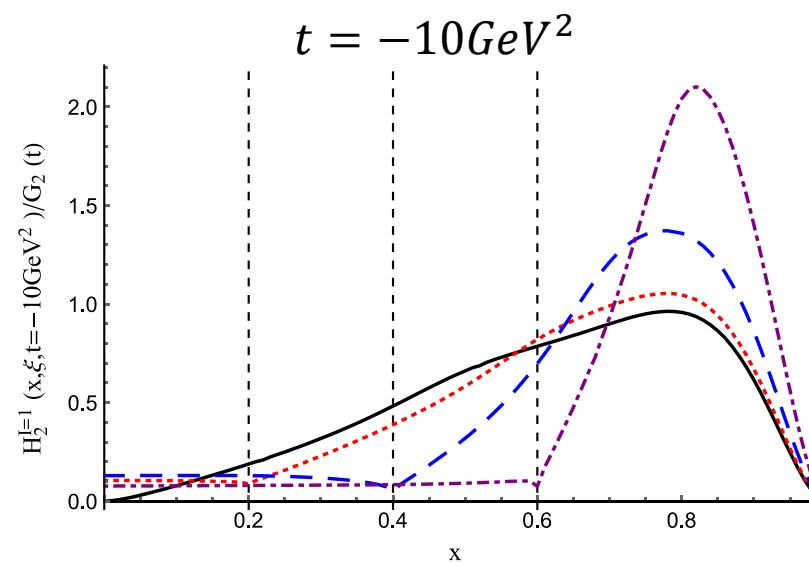
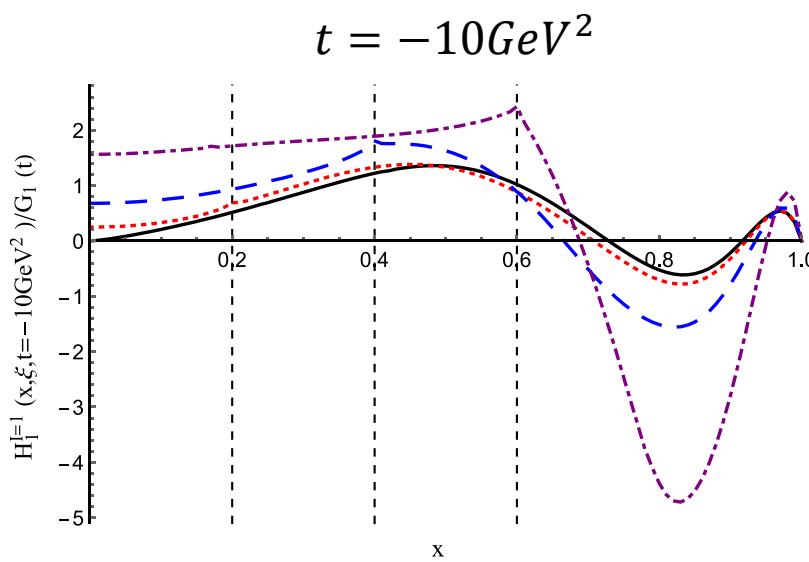
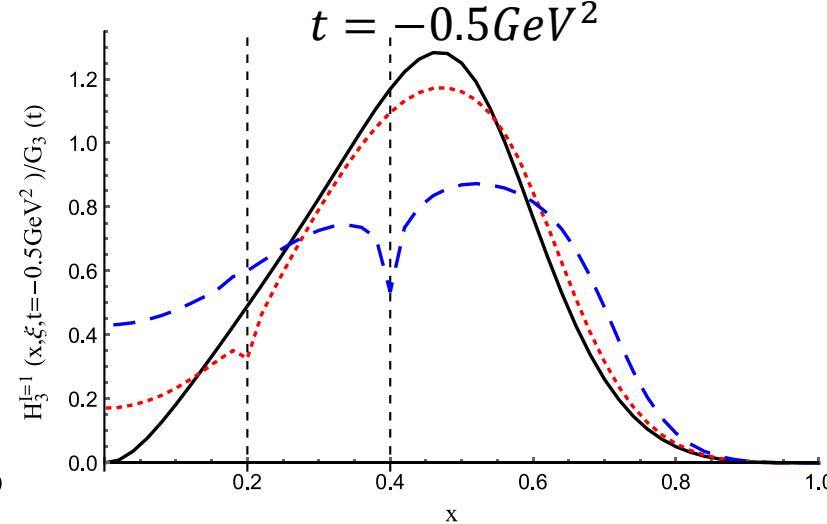
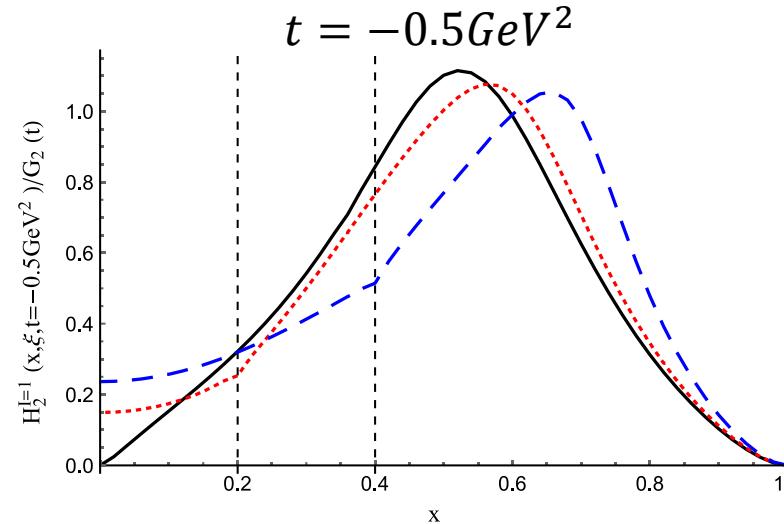
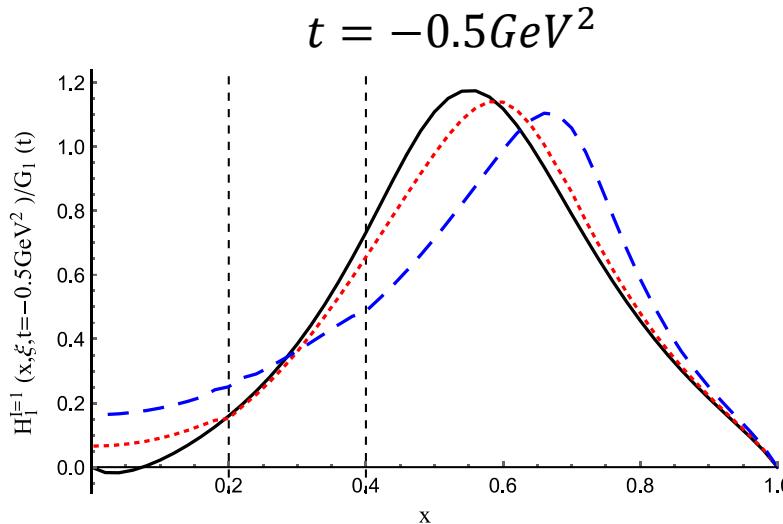
$x = 0, 1$



$x = \pm \xi$   
 $= \mp 0.4$

# Our results: unpolarized GPDs $H_{i=1,2,3}(x, \xi_0, t_0)$

$$|\xi| < \frac{1}{\sqrt{1-4M^2/t}}$$



$\xi = 0$  (solid black line),  $-0.2$  (dotted red line),  $-0.4$  (dashed blue line),  $-0.6$  (dot-dashed purple line)

# Forward limit

[Berger, Phys.Rev.Lett.87.142302]

- GPDs in forward limit

$$H_1(x, 0, 0) = \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3},$$

$$H_5(x, 0, 0) = q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2},$$

$$\tilde{H}_1(x, 0, 0) = q_\uparrow^1(x) - q_\uparrow^{-1}(x)$$

for  $x > 0$ .

- DIS structure functions

$$F_1(x) = \frac{1}{2} \sum_q e_q^2 \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3} + \{q \rightarrow \bar{q}\},$$

$$b_1(x) = \frac{1}{2} \sum_q e_q^2 \left[ q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2} \right] + \{q \rightarrow \bar{q}\}$$

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 [q_\uparrow^1(x) - q_\uparrow^{-1}(x)] + \{q \rightarrow \bar{q}\}.$$

Quark densities:

$$q^\lambda(x) = q_\uparrow^\lambda(x) + q_\downarrow^\lambda(x)$$

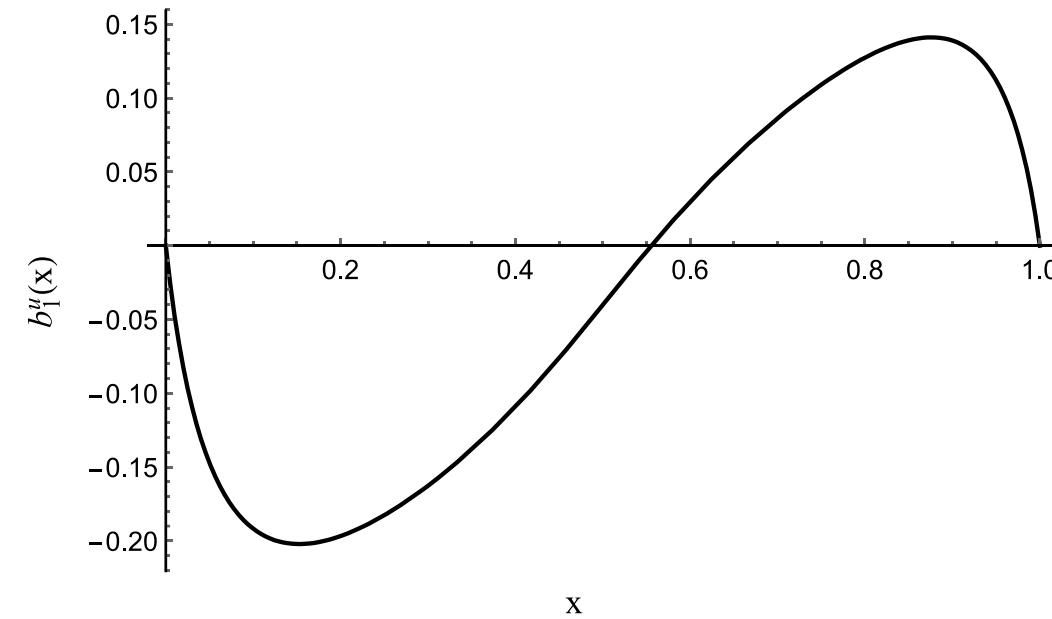
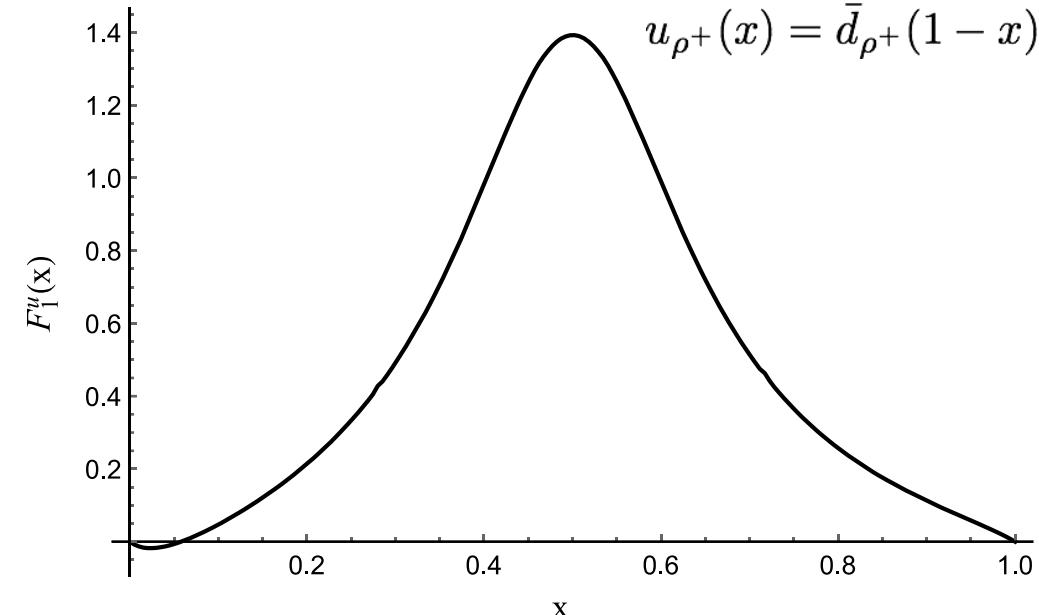
$$q_\uparrow^\lambda = q_\downarrow^{-\lambda}$$

For  $x < 0$ , antiquark distributions at  $-x$  with an overall minus sign in H1 and H5.

Callan-Gross relation

$$F_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_1^u(x, 0, 0)$$

$$b_1^{q\uparrow(\downarrow)}(x) = \frac{1}{2} H_5^u(x, 0, 0)$$



- Single-flavor  $F_1^q$ ,  $b_1^q$

[Broniowski, Phys.Rev.D.77.034023, *ibid.* 78.094011]

# QCD Evolution

- DGLAP evolution function at LO

$$\frac{\langle V_1(\mu) \rangle}{\langle V_1(\mu_0) \rangle} = \left( \frac{\alpha(\mu)}{\alpha(\mu_0)} \right)^{\gamma_n^{NS}/(2\beta_0)}$$

with

$$V_1 = \int_0^1 dx x H^{u+d}(x, 0, 0)$$

$$\alpha(\mu) = \left( \frac{4\pi}{\beta_0} \right) \frac{1}{\ln[\mu^2 / \Lambda_{QCD}^2]}, \quad \beta_0 = \frac{11}{3} C_A - \frac{11}{3} N_F,$$

$$\gamma_n^{NS} = -2C_F \left[ 3 + \frac{2}{(n+1)(n+2)} - 4 \sum_{k=1}^{n+1} \frac{1}{k} \right],$$

$$C_A = N_F = 3, \quad C_F = \frac{4}{3},$$

$$\gamma_1^{NS} / (2\beta_0) = 32/81,$$

$$\Lambda_{QCD} = 0.226 \text{ GeV}$$

[Broniowski. Phys.Rev.D 77.034023, *ibid.* 78.094011]  
 [Broniowski. *j.physletb*.2003.09.009]  
 [Best. Phys.Rev.D 56.2743]

Lattice[Best1997]:  
 $\langle x(\mu = 2.4) \rangle = 0.612$   
 Effective quark model:  
 $\langle x(\mu_0) \rangle = 1$   
 $\mu_0$ : Quark model point  
 (no gluon & sea quark)

Quark model point (our scale):

$$\mu_0 = 528^{+77}_{-62} \text{ MeV}$$

typical expansion parameter:

$$\frac{\alpha(\mu_0)}{2\pi} = 0.131^{+18}_{-23}$$

In our model, valence quark(  $q\bar{q}$  ) probability:

$$P_{val} \sim 70\%$$

By Fock decomposition, other constituent contributes.

- Lattice calculation[1]

$$V_1^u = 0.33(2) , \quad V_2^u = 0.17(5) , \quad V_3^u = 0.06(4) , \\ d_2 = 0.29_{-23}^{+22} , \quad d_3 = -0.001(15) , \quad d_4 = -0.01(6) .$$

- Our model predicts

$$V_1^u = 0.34(2) , \quad V_2^u = 0.15(1) , \quad V_3^u = 0.08(1) , \\ d_2 = 0.044(3) , \quad d_3 = 0.048(5) , \quad d_4 = 0.039(5) ,$$

# Summary

[ arXiv:1707.03972 ]

- Light-Front constituent model
- Connections between hadron structure functions
- $\rho$  meson FFs(V/Nv); unpolarized GPDs H;
- Single-flavor DIS structure functions  $F_1, b_1$
- QCD Evolution
  - Deuteron
  - DDs, DPDs,

*Thanks!*

# Backups: Extract GPDs $H_{i=1 \sim 5}$

$$\begin{aligned}
V^{u;\mu\nu}(x, \xi, t) &= \frac{M^2}{f_\rho^2} \frac{1}{2(2\pi)^3 \sqrt{\omega_{p'} \omega_p}} \int \frac{d^4 k}{(2\pi)^4} \delta [x P^+ - k^+] \\
&\times (-) Tr \left\{ \frac{i(k - P + m)}{(k - P)^2 - m^2 + i\epsilon} \gamma^\nu \frac{i(k + \frac{\Delta}{2} + m)}{(k + \frac{\Delta}{2})^2 - m^2 + i\epsilon} \not{n} \frac{i(k - \frac{\Delta}{2} + m)}{(k - \frac{\Delta}{2})^2 - m^2 + i\epsilon} \gamma^\mu \right\} \\
&\times \frac{c}{[(k - P)^2 - m_R^2 + i\epsilon][(k + \frac{\Delta}{2})^2 - m_R^2 + i\epsilon]} \times \frac{c}{[(k - P)^2 - m_R^2 + i\epsilon][(k - \frac{\Delta}{2})^2 - m_R^2 + i\epsilon]} \\
&= -g^{\mu\nu} H_1^u + \frac{n^\mu P^\nu + P^\mu n^\nu}{n \cdot P} H_2^u - \frac{2P^\mu P^\nu}{M^2} H_3^u + \frac{n^\mu P^\nu - P^\mu n^\nu}{n \cdot P} H_4^u + \left\{ \frac{M^2 n^\mu n^\nu}{(n \cdot P)^2} + \frac{1}{3} g^{\mu\nu} \right\} H_5^u ,
\end{aligned}$$

$$\begin{pmatrix} g_{\mu\nu} \\ n_\mu n_\nu \\ n_\mu P_\nu \\ n_\nu P_\mu \\ P_\mu P_\nu \end{pmatrix} \cdot V^{u;\mu\nu} = \begin{pmatrix} -4 & 2 & -\frac{2P^2}{M^2} & 0 & \frac{4}{3} \\ 0 & 0 & -\frac{2(n \cdot P)^2}{M^2} & 0 & 0 \\ -(n \cdot P) & n \cdot P & -\frac{2(n \cdot P)P^2}{M^2} & -(n \cdot P) & \frac{n \cdot P}{3} \\ -(n \cdot P) & n \cdot P & -\frac{2(n \cdot P)P^2}{M^2} & n \cdot P & \frac{n \cdot P}{3} \\ -P^2 & 2P^2 & -\frac{2P^4}{M^2} & 0 & M^2 + \frac{P^2}{3} \end{pmatrix} \cdot \begin{pmatrix} H_1^u \\ H_2^u \\ H_3^u \\ H_4^u \\ H_5^u \end{pmatrix}$$

# Backups: Extract GPDs $H_{i=1 \sim 5}$

$$\begin{pmatrix} H_1^u \\ H_2^u \\ H_3^u \\ H_4^u \\ H_5^u \end{pmatrix} = \begin{pmatrix} \frac{1}{6} \left( \frac{P^2}{M^2} - 3 \right) & \frac{P^2(P^2 - M^2)}{2M^2(n \cdot P)^2} & \frac{M^2 - P^2}{2M^2(n \cdot P)} & \frac{M^2 - P^2}{2M^2(n \cdot P)} & \frac{1}{3M^2} \\ -\frac{1}{2} & -\frac{3P^2}{2(n \cdot P)^2} & \frac{1}{n \cdot P} & \frac{1}{n \cdot P} & 0 \\ 0 & -\frac{M^2}{2(n \cdot P)^2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2(n \cdot P)} & -\frac{1}{2(n \cdot P)} & 0 \\ \frac{P^2}{2M^2} & \frac{3P^4}{2M^2(n \cdot P)^2} & -\frac{3P^2}{2M^2(n \cdot P)} & -\frac{3P^2}{2M^2(n \cdot P)} & \frac{1}{M^2} \end{pmatrix} \cdot \begin{pmatrix} g_{\mu\nu} \\ n_\mu n_\nu \\ n_\mu P_\nu \\ n_\nu P_\mu \\ P_\mu P_\nu \end{pmatrix} \cdot V^{u;\mu\nu}$$