## BI.Partonic structure of nucleon and nuclei

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QMain themes (we are working now):

- Double parton distributions (DPDs) of the pion in effective chiral quark model (ATK, HDS,MVP)
- Soft pion theorems and ChPT for DPDs of the pion (ATK, HDS, MVP)
- Parton quasi Distributions (PqD) of the nucleon in the Chiral Quark-Soliton model (HDS, MVP)
- (new topic) Possibility of charmed baryon exotics (MVP+collaborators from Krakow, St.Petersburg, Incheon, Irkutsk)


## QTasks to be done

- DPDs of the nucleon in Chiral Quark-Soliton model.
-Application of the results to LHC hard scattering
-ChPT for nucleon DPDs
-ChPT for nucleon PqD


## Double parton distrbtn of the pion in effective quark model

Effective quark-goldstone boson action

$$
\begin{gathered}
S_{e f f}=\int d^{4} x \bar{\psi}\left(i \not \partial-M U^{\gamma_{5}}\right) \psi \\
U=\exp \left(i \pi^{a}(x) \tau^{a}\right), \quad U^{\gamma_{5}}=\exp \left(i \pi^{a}(x) \tau^{a} \gamma_{5}\right)=\frac{1+\gamma_{5}}{2} U+\frac{1-\gamma_{5}}{2} U^{\dagger}
\end{gathered}
$$

Double parton distributions

$$
\begin{aligned}
& F_{\mathrm{us}, a_{1} a_{2}}\left(x_{1}, x_{2}, \boldsymbol{z}_{1}, \boldsymbol{z}_{2}, \boldsymbol{y}\right)= 2 p^{+}\left(x_{1} p^{+}\right)^{-n_{1}}\left(x_{2} p^{+}\right)^{-n_{2}} \int \frac{d z_{1}^{-}}{2 \pi} \frac{d z_{2}^{-}}{2 \pi} d y^{-} e^{i\left(x_{1} z_{1}^{-}+x_{2} z_{2}^{-}\right) p^{+}} \\
& \times\langle p| \mathcal{O}_{a_{1}}\left(y, z_{1}\right) \mathcal{O}_{a_{2}}\left(0, z_{2}\right)|p\rangle \\
& \mathcal{O}_{a}(y, z)=\left.\bar{q}\left(y-\frac{1}{2} z\right) W^{\dagger}\left(y-\frac{1}{2} z, v_{L}\right) \Gamma_{a} W\left(y+\frac{1}{2} z, v_{L}\right) q\left(y+\frac{1}{2} z\right)\right|_{z^{+}=y^{+}=0} \\
& W(\xi, v)=\operatorname{Pexp}\left[i g t^{a} \int_{-\infty}^{0} d s v A^{a}(\xi+s v)\right]
\end{aligned}
$$

Double parton distrbtn of the pion in effective quark model (continued)

Leading diagram is zero
hence the quark interaction due to goldstone bosons exchange is important to describe correlations in DPDs. It is also subleading in large Nc limit and does NOT reduce to the product of quark distributions!

Presently all interaction diagrams are computed, it seems there is very interesting quark correlations in DPDs!

## Quark quasi-distributions in Chiral Quark-Soliton

 model.Parton quasi-distributions were suggested by X.-D. Ji [PRL (2013)] to access parton distribution on the lattice.
Actually, the PqDs were introduced in 1997 by
Diakonov, Petrov,Weiss+MVP [PRD 1997] to compute parton distributions in quark-soliton model

$$
\begin{aligned}
& D_{i}(x, v)=\int \frac{d^{3} k}{(2 \pi)^{3}} \delta\left(x-\frac{k^{3}}{P_{N}}\right) \int d^{3} x_{1} d^{3} x_{2} e^{-i \vec{k} \cdot\left(\vec{x}_{1}-\vec{x}_{2}\right)}\left\langle N_{v}\right| \bar{\psi}\left(\vec{x}_{2}, t\right) \Gamma_{i} \psi\left(\vec{x}_{1}, t\right)\left|N_{v}\right\rangle \\
& \Gamma=\gamma_{0}\left(1 \pm \gamma_{5}\right)!2
\end{aligned}
$$

In the large Nc limit it is reduced to
$D_{i}(x, v)=-i N_{c} \int \frac{d^{3} k}{(2 \pi)^{3}} \delta\left(x-\frac{k^{3}}{P_{N}}\right) \int d^{3} x_{1} d^{3} x_{2} e^{-i \vec{k} \cdot\left(\vec{x}_{1}-\vec{x}_{2}\right)}\left(\Gamma_{i}\right)_{a b} G_{F}\left(\vec{x}_{1}, t_{1}, \vec{x}_{2}, t_{2}\right)_{b a}$
Where
$-_{-i G_{F}}=S[\vec{v}]\left[\Theta\left(t_{2}-t_{1}\right) \sum_{o c c} \Phi_{n}\left(\vec{x}_{1}\right) \Phi_{n}^{\dagger}\left(\vec{x}_{2}\right) \gamma_{0} \exp \left(-i E_{n}\left(t_{1}-t_{2}\right)\right)\right] S^{-1}[\vec{v}]$.
is Feynman boosted Green function in external soliton (meson mean) field.

# Quark quasi-distributions in Chiral Quark-Soliton model (continued) 

First results for the unpolarized (anti-) quark quasi-distribution


Plans: to compute polarized and transversity quasi-distributions and apply the results for the extrapolation of the lattice measurements

## ChPT for (double) parton distributions

## Chiral expansion for 3D quark distributions

$$
q\left(x, b_{\perp}\right)=\int \frac{d^{2} \Delta_{\perp}}{(2 \pi)^{2}} e^{i \vec{b}_{\perp} \vec{\Delta}_{\perp}} \int \frac{d \lambda}{2 \pi} e^{-i \lambda x(P \cdot n)}\left\langle\pi^{+}\left(p^{\prime}\right)\right| \bar{u}\left(\frac{\lambda}{2} n\right)(\gamma \cdot n) u\left(-\frac{\lambda}{2} n\right)\left|\pi^{+}(p)\right\rangle
$$

For large transverse distance one can apply ChPT because only dofs related to Goldstone bosons are relevant. The light-cone quark operator can be matched to the operator made of only Goldstone bosons

$$
O_{f g}(\lambda)=\frac{i F_{\pi}^{2}}{4} \int_{-1}^{1} d \beta \int_{|\beta|-1}^{1-|\beta|} \mathcal{L}(\alpha, \beta) U\left(\frac{\alpha+\beta}{2} \lambda n\right) n \cdot \stackrel{\leftrightarrow}{\partial} U^{\dagger}\left(\frac{\alpha-\beta}{2} \lambda n\right)
$$

Here $\mathcal{L}(\alpha, \beta)$ is the function which encodes the non-perturbative QCD dynamics
Tree level calculations of the pion matrix element with Weinberg Lagrangian gives

$$
q\left(x, b_{\perp}\right)=\delta\left(\vec{b}_{\perp}\right) \stackrel{o}{q}(x)+O\left(\frac{1}{b_{\perp}^{4} \Lambda_{\chi}^{2}}\right) \quad \text {, where } \quad \stackrel{o}{q}(x)=\int_{|x|-1}^{1-|x|} d \alpha \mathcal{L}(\alpha, x)
$$

is usual quark distribution in the pion

## ChPT for (double) parton distributions (continued)

## Chiral expansion for 3D quark distributions

$$
q\left(x, b_{\perp}\right)=\int \frac{d^{2} \Delta_{\perp}}{(2 \pi)^{2}} e^{i \vec{b}_{\perp} \vec{\Delta}_{\perp}} \int \frac{d \lambda}{2 \pi} e^{-i \lambda x(P \cdot n)}\left\langle\pi^{+}\left(p^{\prime}\right)\right| \bar{u}\left(\frac{\lambda}{2} n\right)(\gamma \cdot n) u\left(-\frac{\lambda}{2} n\right)\left|\pi^{+}(p)\right\rangle
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$$

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## ChPT for (double) parton distributions (continued)

3D quark distributions at large $b$

$$
\begin{aligned}
& q\left(x, b_{\perp}\right)=\delta(x) \frac{2}{3 \pi \Lambda_{\chi}^{2} b_{\perp}^{4}}+O\left(\frac{\ln \left(b_{\perp}^{2} \Lambda_{\chi}^{2}\right)}{\Lambda_{\chi}^{4} b_{\perp}^{6}}\right) \quad \begin{array}{l}
\text { Leading order of large b expansion } \\
\text { one loop }
\end{array} \\
& q^{\mathrm{NL}}\left(x, b_{\perp}\right) \sim \frac{\ln \left(b_{\perp}^{2} \Lambda_{\chi}^{2}\right)}{\Lambda_{\chi}^{4} b_{\perp}^{6}}\left(1+O\left(\frac{1}{\ln \left(b_{\perp}^{2} \Lambda_{\chi}^{2}\right)}\right)\right)\left[\delta^{\prime}(x)+q^{\text {smooth }}(x)\right] \begin{array}{ll} 
& \begin{array}{l}
\text { Next correction. Note } \\
\text { the delta function contribution }
\end{array} \\
\text { for } \mathrm{x} \sim \mathrm{I} / \mathrm{b} \wedge 2 \text { it is of the same } \\
\text { order as LO !! }
\end{array}
\end{aligned}
$$

Three loop correction gives $I / b^{\wedge} 8$, but with more singular second derivative of delta function. Coefficients in front of singular contribution are leading logs coefficients!

We computed LL coeffs (see below) and obtained that pion radius grows exponentialy with rapidity, in contrast to Gribov diffusion in which the radius ${ }^{\wedge} 2$ grows linearely with rapidity.

## ChPT for (double) parton distributions (continued)

The singular in $\times$ contributions provide leading large $b$ contribution

$$
\begin{aligned}
& q\left(x, b_{\perp}\right)=\int \frac{d^{2} \Delta_{\perp}}{(2 \pi)^{2}} e^{i \vec{b}_{\perp} \vec{\Delta}_{\perp}} \sum_{n=1}^{\infty} \delta^{(n-1)}(x) \frac{C_{n}}{n!}\langle\stackrel{o}{\beta}(n-1) \\
& \\
& \times \int_{-1}^{1} d \eta\left[\frac{m_{\pi}^{2}+\frac{\Delta_{\perp}^{2}}{4}\left(1-\eta^{2}\right)}{\Lambda_{\chi}^{2}} \ln \left(\frac{\Lambda_{\chi}^{2}}{m_{\pi}^{2}+\frac{\Delta_{\perp}^{2}}{4}\left(1-\eta^{2}\right)}\right)\right]^{n}
\end{aligned}
$$



Coefficients Cn are related to LLs in pi-pi scattering amplitude in massless limit.

$$
\begin{array}{rlrl}
q\left(x, b_{\perp}\right) & =\frac{1}{\pi b_{\perp}^{2} \ln \left(b_{\perp}^{2} \Lambda_{\chi}^{2}\right)} \int_{0}^{1} \frac{d \beta}{\beta} Q(\beta) & \Lambda_{\chi} & =4 \pi F_{\pi} \approx 6 \mathrm{fm}^{-1} \\
& \times\left[\frac{x}{\beta} \frac{b_{\perp}^{2} \Lambda_{\chi}^{2}}{A \ln \left(b_{\perp}^{2} \Lambda_{\chi}^{2}\right)} K_{2}\left(2 \sqrt{\frac{x}{\beta} \frac{b_{\perp}^{2} \Lambda_{\chi}^{2}}{A \ln \left(b_{\perp}^{2} \Lambda_{\chi}^{2}\right)}}\right)\right] & A & =\frac{3}{2} \\
& \times\left(1+O\left(\frac{1}{\ln \left(\Lambda_{\chi}^{2} b_{\perp}^{2}\right)}\right)\right)
\end{array}
$$

That eq. provides model independent asymptotic of 3D quark distribution for

$$
b_{\perp} \rightarrow \infty \text { and } x \sim \frac{\Lambda_{\chi}^{2}}{b_{\perp}^{2}}
$$

## ChPT for (double) parton distributions (continued)

## Pion radius

$$
b_{\perp}^{2}(x)=\int d^{2} b_{\perp} b_{\perp}^{2} q\left(x, b_{\perp}\right)
$$

Normalized quark density: $\quad \rho\left(x, b_{\perp}\right)=\frac{q\left(x, b_{\perp}\right)}{q(x)}$ corresponds to conditional probabilty $\rho\left(x, b_{\perp}\right) \rightarrow \delta\left(\vec{b}_{\perp}\right) \quad$ at $x \rightarrow 1$ so we can interpret as probability distribution for a stochastic motion of a "particle"
The role of the time is played by the rapidity: $\eta=\ln \left(\frac{1}{x}\right)$
Mean distance for a "particle" is defined as: $\quad d_{\perp}^{2}(x)=\int d^{2} b_{\perp} b_{\perp}^{2} \rho\left(x, b_{\perp}\right)=\frac{b_{\perp}^{2}(x)}{q(x)}$
Gribov diffusion: $\quad d_{\perp}^{2}(x)=D \ln \left(\frac{1}{x}\right)=D \eta$ it is realized in a Regge-like model for GPDs

$$
q\left(x, b_{\perp}\right)=\int \frac{d^{2} \Delta_{\perp}}{(2 \pi)^{2}} e^{-i \vec{b}_{\perp} \vec{\Delta}_{\perp}} x^{\alpha^{\prime} \Delta_{\perp}^{2}} q(x)
$$

## ChPT for (double) parton distributions (continued)

## Chiral inflation of the pion radius

If one uses our resummation, the result for the radius (massless limit):

$$
b_{\perp}^{2}(x)=\frac{2}{3\left(4 \pi F_{\pi}\right)^{2}} \frac{1}{x}
$$

That implies that the mean distance of corresponding stochastic process rises exponentially with "time" (rapidity)

$$
d_{\perp}^{2}(x) \sim e^{\left(1-\alpha_{0}\right) \eta}
$$

For massive case this grows stops at $\quad x \sim \frac{m_{\pi}^{2}}{\left(4 \pi F_{\pi}\right)^{2}} \sim 10^{-2}$
Note that the total pion radius $b_{\perp}^{2}=\int_{0}^{1} d x b_{\perp}^{2}(x) \begin{aligned} & \text { is logarithmically divergent, that } \\ & \text { corresponds to well known result of ChPT }\end{aligned}$

$$
b_{\perp}^{2}=\frac{2}{3\left(4 \pi F_{\pi}\right)^{2}} \ln \left(\frac{\Lambda_{\chi}^{2}}{m_{\pi}^{2}}\right)
$$

## ChPT for (double) parton distributions (continued)

The same program can be carried out for DPDs
I) Derive the effective chiral operator for

$$
\bar{q}\left(z_{1}\right) \Gamma_{1} q\left(z_{2}\right) \bar{q}\left(z_{3}\right) \Gamma_{2} q(0)
$$

note that the corresponding effective operator is NOT a product of effective operators for usual parton distributions!
2) Develop the all order resummation of chiral logs for that operator. Interesting task: logs in higher (than 4-point) Green functions are required.

a

b


C

## Summary

- Despite of slow hiring the project started and new interesting results and ideas are obtained
- It is clear how to proceed further
- The closer collaboration with Chinese group is still missing. This meeting perfectly serves to exchange ideas and to establish collaboration.

