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Nucleon-nucleon interaction in covariant chiral effective field theory

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Kai-Wen Li, Bing-Wei Long, Peter Ring

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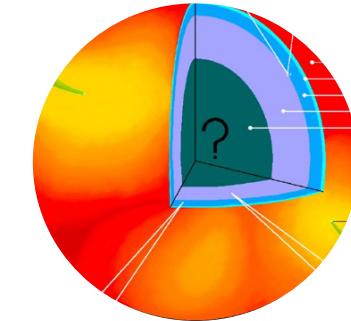
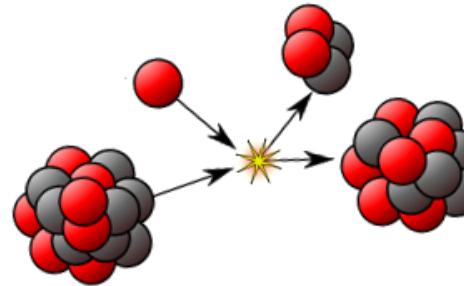
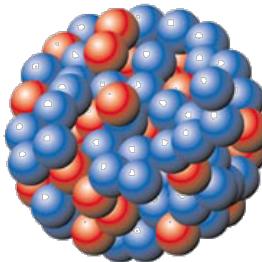
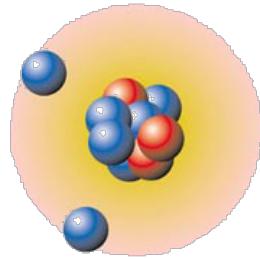
- Introduction
- Theoretical framework
- Results and discussion
- Summary and perspectives

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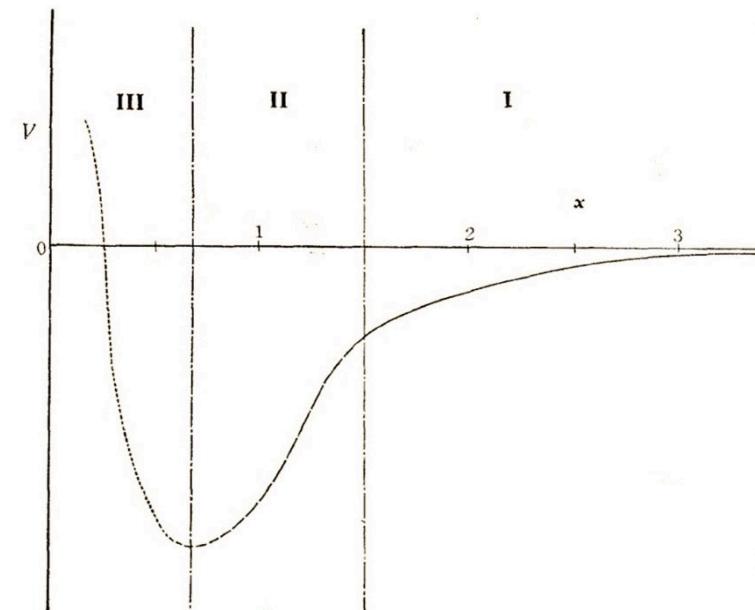
Basic for all nuclear physics

□ Precise understanding of the nuclear force



□ Complexity of the nuclear force (vs. electromagnetic force)

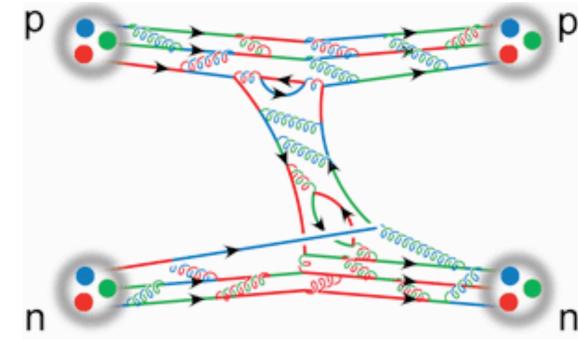
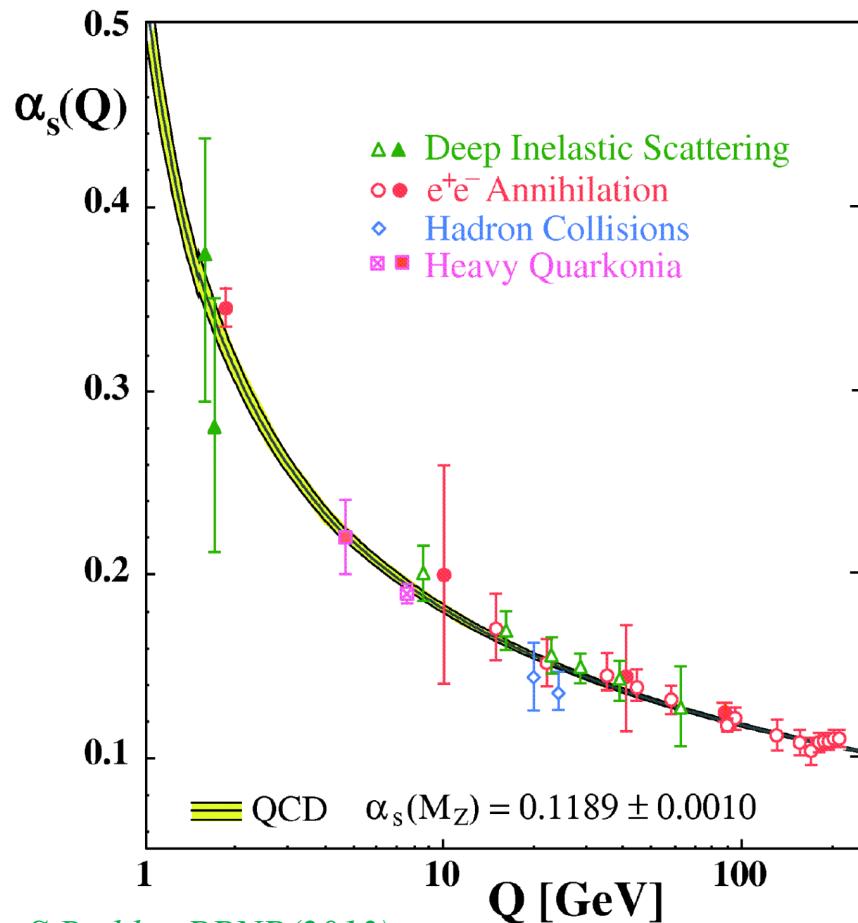
- Finite range
- Intermediate-range **attraction**
- Short-range **repulsion**-“hard core”
- Spin-dependent **non-central** force
 - Tensor interaction
 - Spin-orbit interaction
- Charge independent (approximate)



M. Taketani, Suppl.PTP3(1956)1

Nuclear force (NF) from QCD

- Residual quark-gluon strong interaction
- Understood from QCD



At low-energy region

- Running coupling constant $\alpha_s \geq 1$
- Nonperturbative QCD -- unsolvable

→ Phenomenological models
Lattice QCD simulation
Chiral effective field theory

NF from Chiral EFT

- Chiral effective field theory *S. Weinberg, Phys.A 1979*
 - Effective field theory (EFT) of **low-energy QCD**
 - **Model independent** to study the nuclear force *S. Weinberg, PLB 1990*

□ Main advantages of chiral nuclear force

- **Self-consistently include** many-body forces

$$V = V_{2N} + V_{3N} + \cdots + V_{iN} + \cdots$$

- **Systematically improve** NF order by order

$$V_{iN} = V_{iN}^{\text{LO}} + V_{iN}^{\text{NLO}} + V_{iN}^{\text{NNLO}} + \cdots$$

- **Systematically estimate** theoretical uncertainties

$$|V_{iN}^{\text{LO}}| > |V_{iN}^{\text{NLO}}| > |V_{iN}^{\text{NNLO}}| > \cdots$$

Current status of chiral NF

□ Nonrelativistic (NR) chiral NF

- NN interaction
 - up to NLO *U. van Kolck et al., PRL, PRC1992-94; N. Kaiser, NPA1997*
 - up to NNLO *U. van Kolck et al., PRC1994; E. Epelbaum, et al., NPA2000*
 - up to **N³LO** *R. Machleidt et al., PRC2003; E. Epelbaum et al., NPA2005*
 - up to **N⁴LO** *E. Epelbaum et al., PRL2015, D.R. Entem, et al., PRC2015*
 - up to **N⁵LO** (dominant terms) *D.R. Entem, et al., PRC2015*
 - 3N interaction
 - up to NNLO *U. van Kolck, PRC1994*
 - up to N³LO *S. Ishikwas, et al, PRC2007; V. Bernard et al, PRC2007*
 - up to **N⁴LO** *H. Krebs, et al., PRC2012-13*
 - 4N interaction
 - up to N³LO *E. Epelbaum, PLB 2006, EPJA 2007*
- P. F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339*
E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

Chiral NN potential is of high precision

	Phenomenological forces			NR Chiral nuclear force				
	Reid93	AV18	CD-Bonn	LO	NLO	NNLO	N³LO	N⁴LO
No. of para.	50	40	38	2+2	9+2	9+2	24+2	24+3
χ^2/datum <i>np data</i> 0-290 MeV	1.03	1.04	1.02	94	36.7	5.28	1.23/ 1.27	1.14/ 1.10

P.Reinert's talk, Bochum-Juelich (2017)

D.Entem, et al., PRC96(2017)024004

Chiral force has been extensively applied in the study of nuclear structure and reactions within the non-relativistic few-/many-body theories.

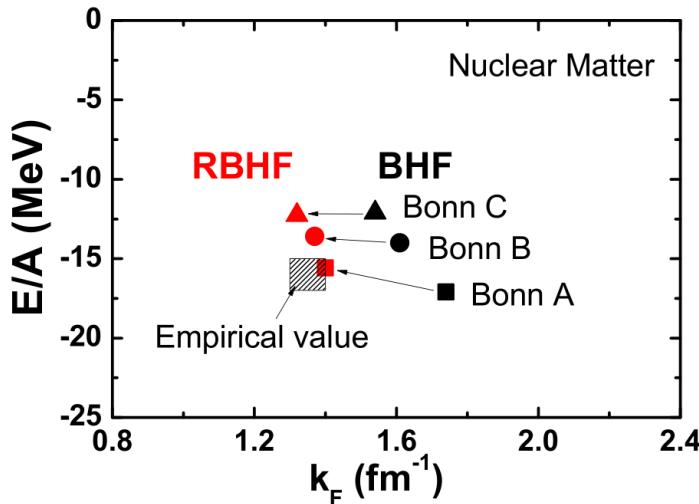
E. Epelbaum, et al., PRL 106(2011) 192501, PRL109(2012)252501, PRL112(2014)102501; S. Elhatisari, et al., Nature 528 (2015) 111, arXiv:1702.05177; G. Hagen, et al., PRL109(2012)032502; H. Hergert, et al., PRL110(2013)24501; G.R. Jansen, et al., PRL113(2014)102501; S.K.Bogner, et al., PRL113(2014)142501; J.E. Lynn, et al., PRL113(2014)192501; V. Lapoux, et al., PRL117(2016)052501.....

Motivation for the relativistic chiral force

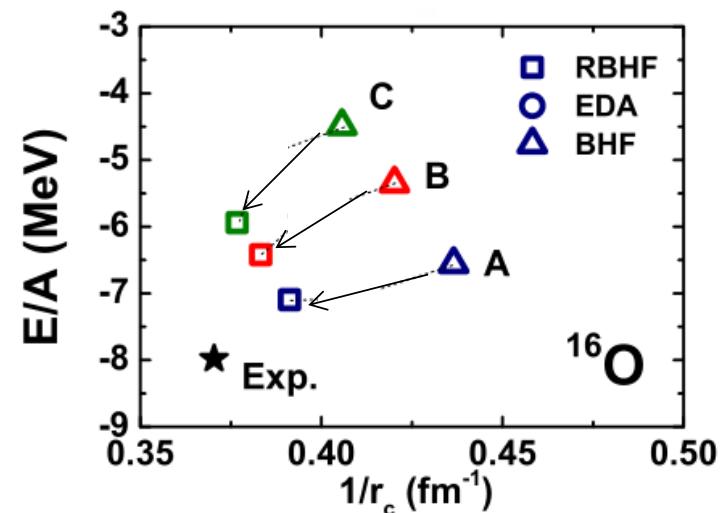
- The success of **covariant density functional theory (CDFT)** in the nuclear structure studies.

*P. Ring, PPNP (1996), D. Vretenar et al., Phys.Rept.(2005),
J. Meng, PPNP(2006), Phys.Rept.(2015), IRNP(2016)*

- Relativistic Brueckner-Hartree-Fock theory in nuclear matter and finite nuclei (**input: relativistic Bonn**)



R. Brockmann & R. Machleidt, PRC(1990)



S.H. Shen, et al., CPL(2016), PRC(2017)

Relativistic nuclear force based on ChEFT is needed

Motivation for the relativistic chiral force

- The success of **covariant density functional theory (CDFT)** in the nuclear structure studies.

*P. Ring, PPNP (1996), D. Vretenar et al., Phys.Rept.(2005),
J. Meng, PPNP(2006), Phys.Rept.(2015), IRNP(2016)*

- Relativistic Brueckner-Hartree-Fock theory in nuclear matter and finite nuclei (**input: relativistic Bonn**)

We extend **covariant ChEFT** to the nucleon-nucleon sector and construct a **relativistic nuclear force** up to next-to-leading order.

R. Brockmann & R. Machleidt, PRC(1990)

S.H. Shen, et al., CPL(2016), PRC(2017)

Relativistic nuclear force based on ChEFT is needed

OUTLINE

□ Introduction

□ Theoretical framework

- NN potential concepts
- Relativistic chiral force up to NLO

□ Results and discussion

□ Summary and perspectives

NN potential concept

- Often-thought as nonrelativistic quantity

- Appear in the **Schrödinger** equation

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(t, \mathbf{r}) + V(\mathbf{r}) \Psi(t, \mathbf{r}) = i\hbar \frac{\partial}{\partial t} \Psi(t, \mathbf{r}).$$

- (or) Appear in the **Lippmann-Schwinger** equation

$$T(\mathbf{p}', \mathbf{p}) = V(\mathbf{p}', \mathbf{p}) + \int \frac{d\mathbf{k}}{(2\pi)^3} V(\mathbf{p}', \mathbf{k}) \frac{m_N}{\mathbf{p}^2 - \mathbf{k}^2 + i\epsilon} T(\mathbf{k}, \mathbf{p}).$$

- Generalize the definition of potential

- An interaction quantity appearing in a **three-dimensional scattering equation** can be referred as a **NN potential**.

⇒ Relativistic potential

*M.H. Partovi, E.L. Lomon, PRD2 (1970) 1999
K. Erkelenz, Phys.Rept. 13C(1974) 191*

Bethe-Salpeter equation

□ For the relativistic nucleon-nucleon scattering

$$\overline{p} \quad \textcolor{red}{T} \quad \overline{p'} = \overline{p} \quad \textcolor{green}{A} \quad \overline{p'} + \overline{p} \quad \textcolor{red}{T} \quad \textcolor{blue}{G}_k \quad \textcolor{green}{A} \quad \overline{p'}$$
$$W = \sqrt{s}/2$$

Bethe-Salpeter equation with an operator form:

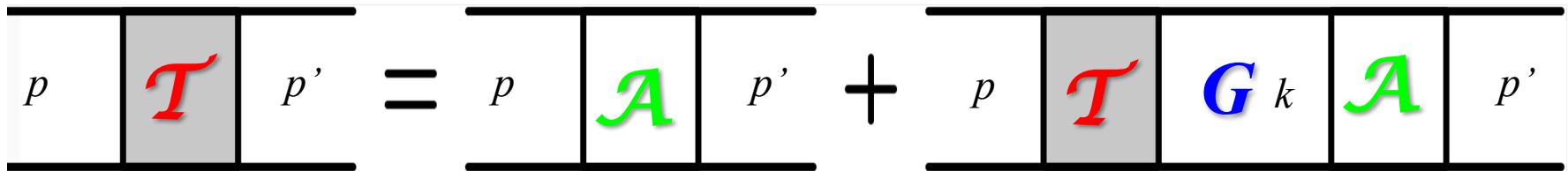
$$\mathcal{T}(p', p | W) = \mathcal{A}(p', p | W) + \int \frac{d^4 k}{(2\pi)^4} \mathcal{A}(p', p | W) G(k | W) T(k, p | W),$$

- \mathcal{T} : Invariant scattering amplitude
- \mathcal{A} : **Interaction kernel** (**sum all the irreducible diagrams**)
- \mathbf{G} : Two-nucleon's Green function

$$G(k | W) = i \frac{1}{[\gamma^\mu (W + k)_\mu - m_N + i\epsilon]^{(1)} [\gamma^\mu (W - k)_\mu - m_N + i\epsilon]^{(2)}},$$

Bethe-Salpeter equation

□ For the relativistic nucleon-nucleon scattering



$$W = \sqrt{s}/2$$

Bethe-Salpeter equation with an operator form:

$$\mathcal{T}(p', p|W) = \mathcal{A}(p', p|W) + \int \frac{d^4 k}{(2\pi)^4} \mathcal{A}(p', p|W) G(k|W) T(k, p|W),$$

- \mathcal{T} : Invariant scattering amplitude
- \mathcal{A} : Interaction kernel (sum all the irreducible diagrams)
- G : Two-nucleon's Green function

It is hard to solve the BS equation, one always perform the 3-dimensional reduction.

Reduction of BS equation

- Introduce a three dimensional Green function \mathbf{g}
 - Maintain the same **elastic unitarity** of \mathbf{G} at physical region
 - We choose the Kadyshevsky propagator

$$g = 2\pi \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\mathbf{k})\Lambda_+^{(2)}(-\mathbf{k})}{\sqrt{s} - 2E_k + i\epsilon} \delta[k_0 - (E_k - \frac{\sqrt{s}}{2})].$$

- To replace \mathbf{G} with \mathbf{g} , one can introduce the effective interaction kernel \mathcal{V}

$$\mathcal{T} = \mathcal{A} + \mathcal{A}G\mathcal{T}. \quad \left\{ \begin{array}{l} \mathcal{T} = \mathcal{V} + \mathcal{V} g \mathcal{T}. \\ \mathcal{V} = \mathcal{A} + \mathcal{A} (G - g) \mathcal{V}. \end{array} \right.$$

Reduction of BS equation

- BS equation reduces to the Kadyshevsky equation:

$$\begin{aligned}
 \mathcal{T} &= \mathcal{V} + \mathcal{V} g \mathcal{T} \\
 &= \mathcal{V} + \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{dk_0}{2\pi} \mathcal{V} \times 2\pi \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\mathbf{k}) \Lambda_+^{(2)}(-\mathbf{k})}{\sqrt{s} - 2E_k + i\epsilon} \delta[k_0 - (E_k - \frac{\sqrt{s}}{2})] \times \mathcal{T} \\
 &= \mathcal{V} + \int \frac{d\mathbf{k}}{(2\pi)^3} \mathcal{V} \frac{m_N^2}{E_k^2} \frac{\Lambda_+^{(1)}(\mathbf{k}) \Lambda_+^{(2)}(-\mathbf{k})}{\sqrt{s} - 2E_k + i\epsilon} \mathcal{T}, \quad \text{with } k_0 = E_k - \frac{\sqrt{s}}{2}.
 \end{aligned}$$

- Sandwiched by Dirac spinors:

$$T(\mathbf{p}', \mathbf{p}) = V(\mathbf{p}', \mathbf{p}) + \int \frac{d^3k}{(2\pi)^3} V(\mathbf{p}', \mathbf{k}) \frac{m_N^2}{2E_k^2} \frac{1}{E_p - E_k + i\epsilon} T(\mathbf{k}, \mathbf{p}),$$

V. Kadyshevsky, NPB (1968).

- Relativistic potential definition:

$$\begin{aligned}
 V(\mathbf{p}', \mathbf{p}) &= \bar{u}(\mathbf{p}', s_1) \bar{u}(-\mathbf{p}', s_2) \times \\
 &\quad \mathcal{V}(p'_0 = E_{p'} - \sqrt{s}/2, \mathbf{p}'; p_0 = E_p - \sqrt{s}/2, \mathbf{p}|W) \times u(\mathbf{p}, s_1) u(\mathbf{p}', s_2).
 \end{aligned}$$

Calculate potential in ChEFT

- To obtain the potential

$$V(\mathbf{p}', \mathbf{p}) = \bar{u}_1 \bar{u}_2 \mathcal{V}(p, p') u_1 u_2.$$

- Solve the iterated equation perturbatively

$$\mathcal{V} = \mathcal{A} + \mathcal{A}(G - g)\mathcal{V}.$$

$$\mathcal{V}^{(2)} = \mathcal{A}^{(2)},$$

$$\mathcal{V}^{(4)} = \mathcal{A}^{(4)} + \boxed{\mathcal{A}^{(2)}(G - g)\mathcal{A}^{(2)}},$$

....

Large cancellation, neglected

K. Erkelenz, ZPA 1973, Phys.Rept. 1974

R. Machleit, Phys.Rept. 1987

- Interaction kernel, \mathcal{A} , can be calculated by using
covariant chiral perturbation theory order by order.

Interaction kernel in covariant ChEFT

□ Perturbative expansion

$$\mathcal{A} = \sum_i C[g_i(\mu)] \left(\frac{Q}{\Lambda_\chi} \right)^{n_\chi}$$

- Expansion parameters

$$(Q/\Lambda_\chi)^{n_\chi} \quad \text{light --- } Q \sim p, m_\pi, \quad \text{heavy --- } \Lambda_\chi \sim 1 \text{ GeV}$$

- Chiral dimension n_χ (naïve dimensional analysis)

$$n_\chi = 4L - 2N_\pi - N_n + \sum_k k V_k$$

- We have the **power counting** to collect the effective Lagrangians and corresponding diagrams.

Interaction kernel up to NLO

□ Covariant chiral Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

• LO contact Lagrangian

$$\begin{aligned}\mathcal{L}_{NN}^{(0)} = & -\frac{1}{2} [\mathbf{C}_S(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + \mathbf{C}_A(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) + \mathbf{C}_V(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + \\ & \mathbf{C}_{AV}(\bar{\Psi}\gamma_5\gamma_\mu\Psi)(\bar{\Psi}\gamma_5\gamma^\mu\Psi) + \mathbf{C}_T(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi).]\end{aligned}$$

H. Polinder, J. Haidenbauer, U.-G. Meißner; NPA779, 244 (2006)

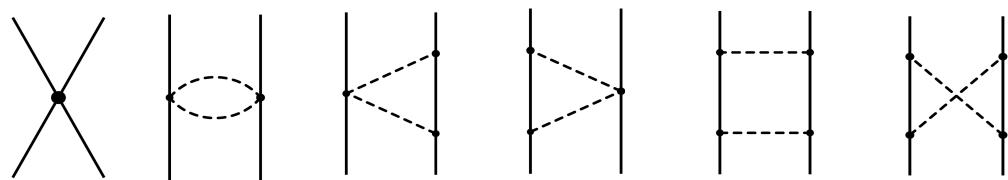
• NLO contact Lagrangian --- **to be constructed**

□ Feynman diagrams

$$(\mathbf{Q}/\Lambda_\chi)^0$$



$$(\mathbf{Q}/\Lambda_\chi)^2$$



Relativistic chiral NF up to NLO

$$V_{\text{LO}} = \bar{u}_1 \bar{u}_2 \left(\begin{array}{c|c} \diagup & \diagdown \\ \diagdown & \diagup \end{array} \right) u_1 u_2$$

$$V_{\text{NLO}} = \bar{u}_1 \bar{u}_2 \left[\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \\ \text{Diagram 6} \end{array} \right] u_1 u_2$$

Scattering equation and Phase shifts

- Perform the partial wave projection, one can obtain the Kadyshevsky equation in $|LSJ\rangle$ basis

$$T_{L',L}^{SJ}(\mathbf{p}', \mathbf{p}) = V_{L',L}^{SJ}(\mathbf{p}', \mathbf{p}) + \sum_{L''} \int_0^{+\infty} \frac{k^2 dk}{(2\pi)^3} V_{L',L}^{SJ}(\mathbf{p}', \mathbf{k}) \frac{M_N^2}{2(\mathbf{k}^2 + M_N^2)} \frac{1}{\sqrt{\mathbf{p}^2 + M_N^2} - \sqrt{\mathbf{k}^2 + M_N^2} + i\epsilon} T_{L'',L}^{SJ}(\mathbf{k}, \mathbf{p}).$$

V. Kadyshevsky, NPB (1968).

- Cutoff renormalization for scattering equation
 - Potential regularized by an **exponential regulator function**

$$V(\mathbf{p}', \mathbf{p}) \rightarrow V(\mathbf{p}', \mathbf{p}) \exp[-(|\mathbf{p}'|/\Lambda)^{2n} - (|\mathbf{p}|/\Lambda)^{2n}]. \quad n = 2$$

- On-shell S matrix and phase shift δ

E.Epelbaum et al., NPA(2000)

$$S_{L'L}^{SJ} = \delta_{L'L} - \frac{i}{8\pi^2} \frac{M_N^2 |\mathbf{p}|}{E_p} T_{L'L}^{SJ}. \quad S = \exp(2i\delta)$$

For couple channel: Stapp parameterization

Results and discussion for LO potential

XLR, K.-W. Li, L.-S. Geng, B. Long, P. Ring, J. Meng,
arXiv: 1611.08475

Relativistic chiral potential at LO

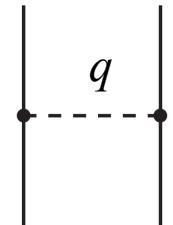
□ Contact potential (momentum space):

$$V_{\text{CTP}} = C_S(\bar{u}_2 u_2)(\bar{u}_1 u_1) + C_A(\bar{u}_2 \gamma_5 u_2)(\bar{u}_1 \gamma_5 u_1) \\ + C_V(\bar{u}_2 \gamma_\mu u_2)(\bar{u}_1 \gamma^\mu u_1) + C_{AV}(\bar{u}_2 \gamma_\mu \gamma_5 u_2)(\bar{u}_1 \gamma^\mu \gamma_5 u_1) \\ + C_T(\bar{u}_2 \sigma_{\mu\nu} u_2)(\bar{u}_1 \sigma_{\mu\nu} u_1).$$



□ One-pion-exchange potential (momentum space):

$$V_{\text{OPEP}} = -\frac{g_A^2}{4f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\bar{u}_1 \gamma^\mu \gamma_5 q_\mu u_1)(\bar{u}_2 \gamma^\nu \gamma_5 q_\nu u_2)}{(E_{p'} - E_p)^2 - \mathbf{q}^2 - m_\pi^2}.$$



Retardation effect included

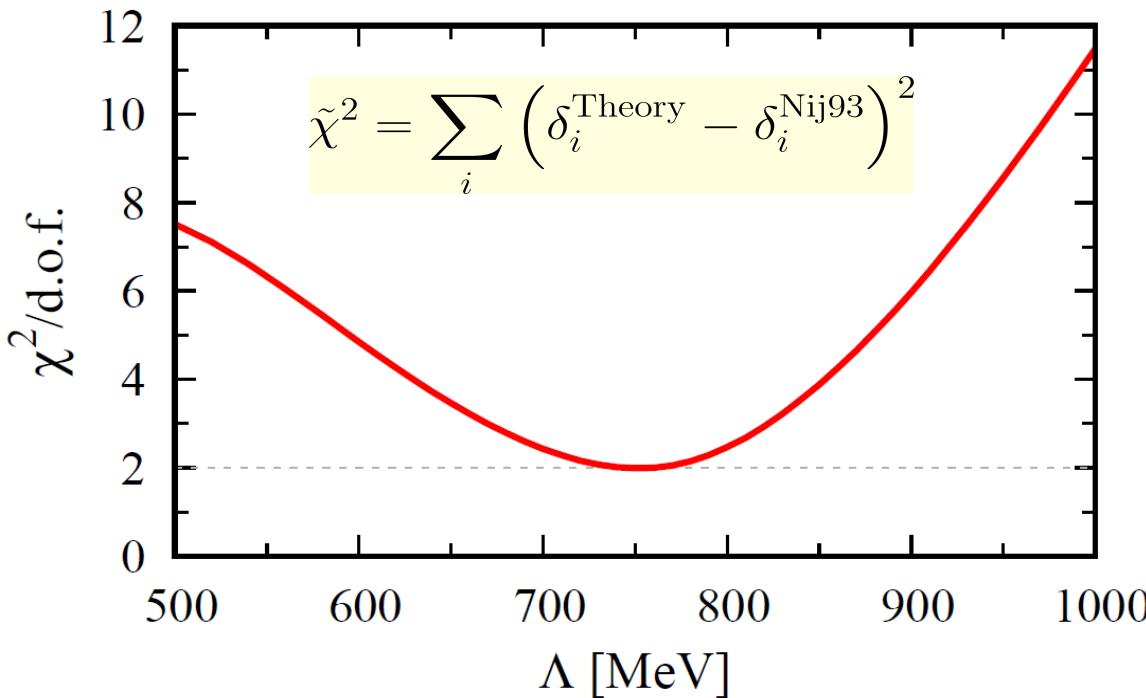
- In the static limit ($m_N \rightarrow \infty$), the NR results can be recovered

$$V^{\text{NonRel.}} = \frac{(C_S + C_V)}{C_S^{\text{HB}}} - \frac{(C_{AV} - 2C_T)}{C_T^{\text{HB}}} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{4f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{\mathbf{q} + m_\pi^2 + i\epsilon} + \mathcal{O}\left(\frac{1}{M_N}\right).$$

S. Weinberg, PLB1990

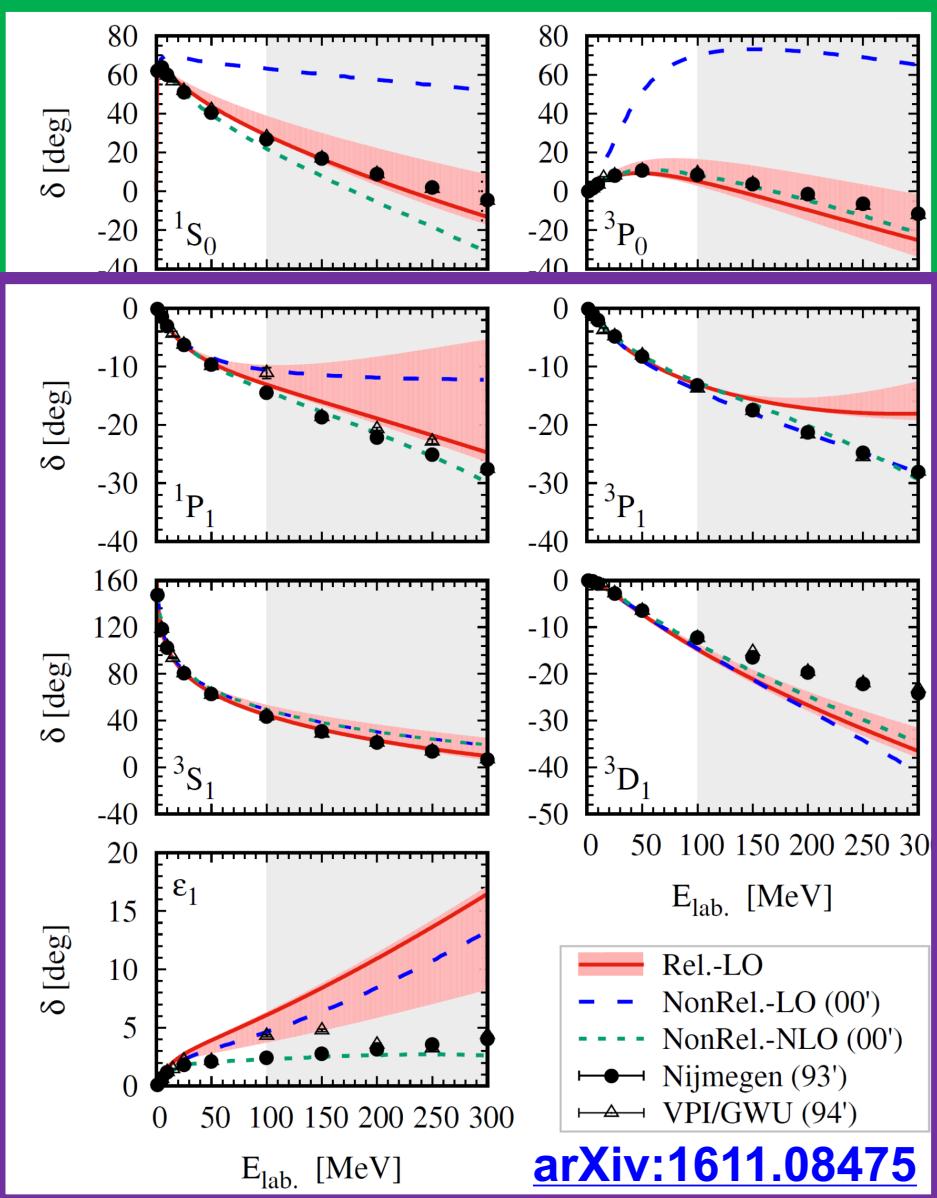
Numerical details

- 5 LECs $C_{S,A,V,AV,T}$ are determined by fitting
 - **NPWA:** p - n scattering phase shifts of Nijmegen 93
V. Stoks et al., PRC48(1993)792
 - 7 partial waves: $J=0, 1$ $^1S_0, ^3P_0, ^1P_1, ^3P_1, ^3D_1, ^3S_1, \epsilon_1$
 - 42 data points: 6 data points for each partial wave
($E_{\text{lab}} = 1, 5, 10, 25, 50, 100$ MeV)



LECs	Values [10^4 GeV $^{-2}$]
C_S	-0.125
C_A	0.040
C_V	0.248
C_{AV}	0.221
C_T	0.059

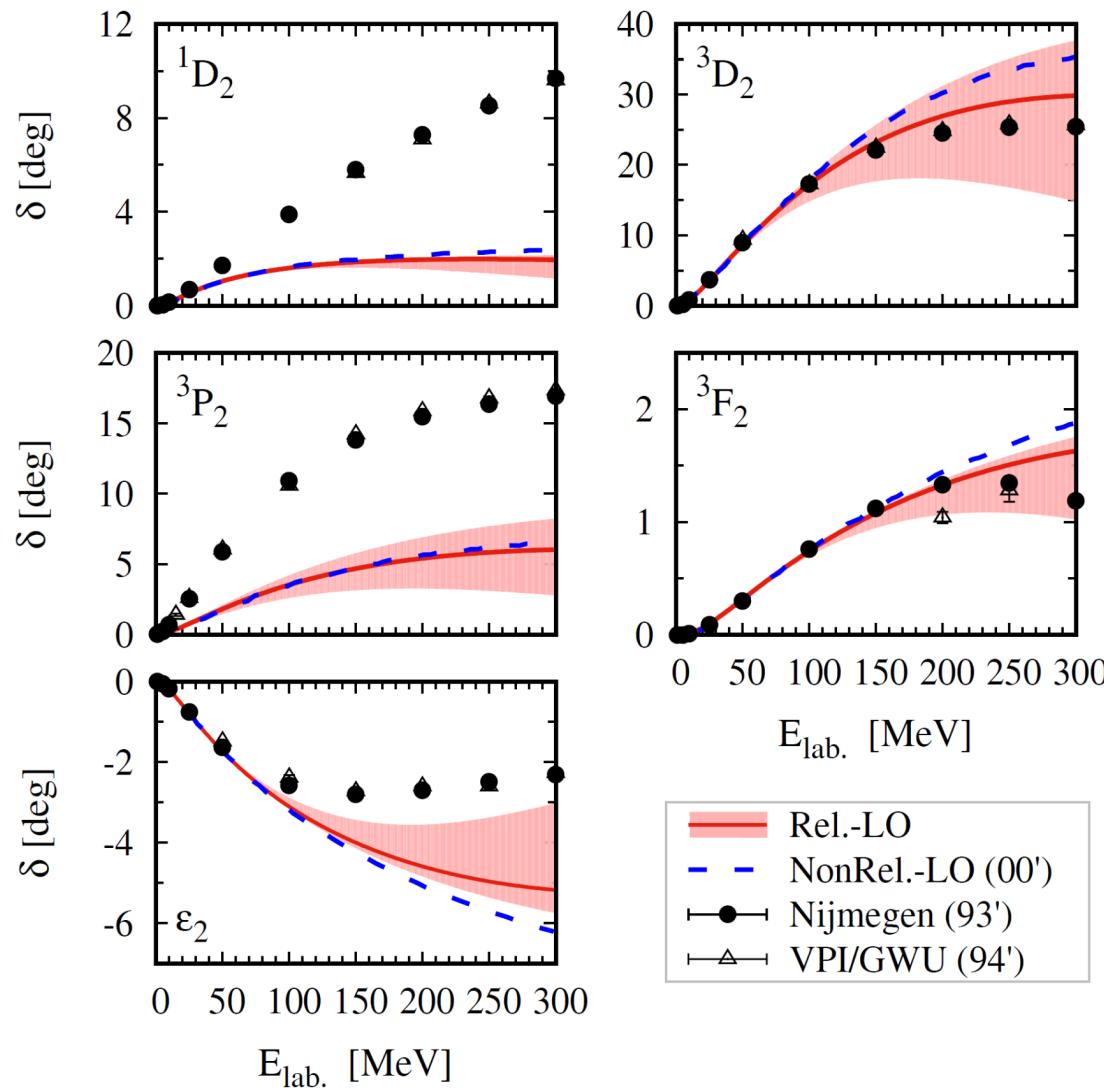
Description of J=0, I partial waves



- Red variation bands:
cutoff 500~1000 MeV
- Improve description of $^1S_0, ^3P_0$ phase shifts
- Quantitatively similar to the nonrelativistic case for $J=1$ partial waves

Higher partial waves

Only OPEP contributes



- The relativistic results are almost **the same** as the non-relativistic case.
- **Relativistic correction of OPEP is small !**

1S0 wave phenomena

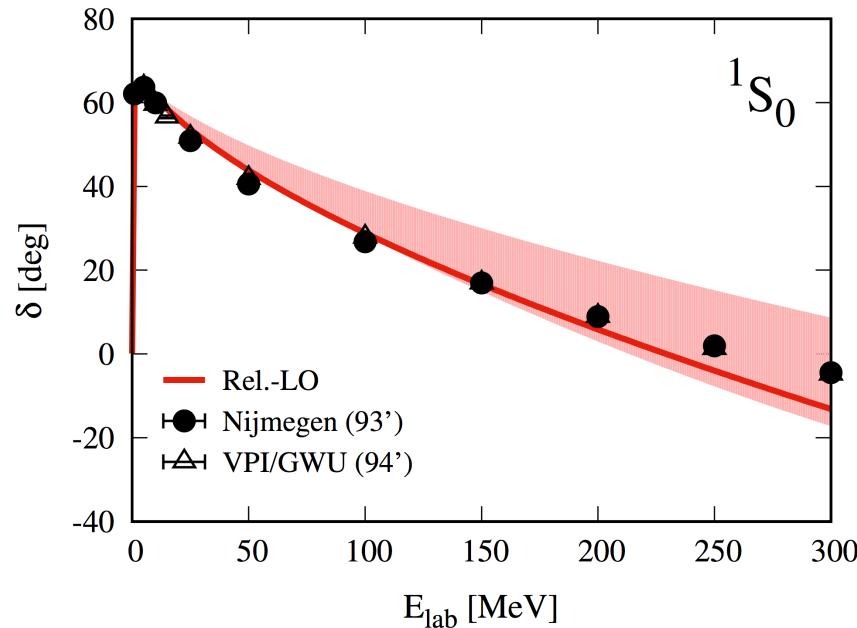
- Interesting phenomena of 1S0 wave
 - Large variance of phase shift from 60 to -10
(zero point: $k_0=340.5$ MeV)
 - Virtual bound state at very low-energy region
(pole position: $-i10$ MeV)
 - Significantly large scattering length ($a=-23.7$ fm)

These typical energy scales are smaller than
chiral symmetry breaking scale (~ 1 GeV)

- ⇒ The 1S0 phenomena **should be roughly reproduced simultaneously** at the **lowest order** of chiral nuclear force

$1S0$ in relativistic chiral force (LO)

- A good description of $1S0$ phase shift:



- Predicted results: (reproduced simultaneously)

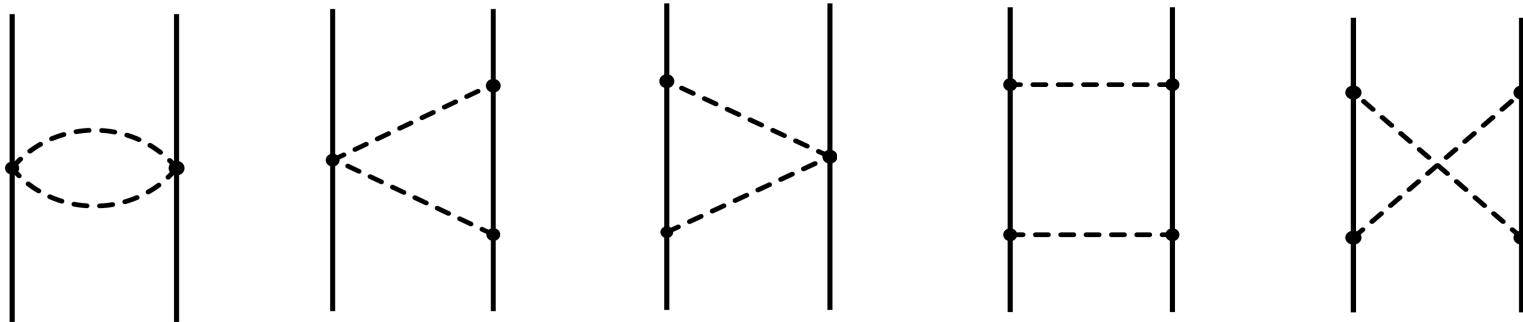
	Nijmegen PWA	Global-Fit
Λ [MeV]	—	750^{500}_{1000}
scattering length a [fm]	-23.7	$-20.3^{-19.8}_{-16.2}$
effective range r [fm]	2.70	$2.45^{2.41}_{2.24}$
virtual pole position $i\gamma$ [MeV]	$-i10$	$-i9.2^{-i9.4}_{-i11.4}$

Work in progress: Construction NLO potential

In collaboration with:
L.-S. Geng, J. Meng, E. Epelbaum

NLO corrections for chiral force

□ Two pion exchange:



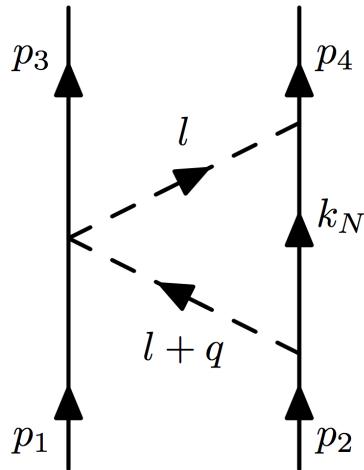
- Except football diagram, the expresses are **very complicated** with 3-/4-point functions
- Introduce the **power counting breaking** terms
- Keep the four **external legs off-shell** (cannot use Dirac eq.)

□ Contact potential:

- Construct the effective Lagrangian with two derivatives

Take left-triangle diagram for example

$$(\frac{\sqrt{s}}{2} + p'_0, \vec{p}') \quad (\frac{\sqrt{s}}{2} - p'_0, -\vec{p}')$$



$$(\frac{\sqrt{s}}{2} + p_0, \vec{p}) \quad (\frac{\sqrt{s}}{2} - p_0, -\vec{p})$$

□ In the momentum space

$$V = \frac{ig_A^2}{8f_\pi^4} \vec{\tau}_1 \cdot \vec{\tau}_2 \int \frac{d^4l}{(2\pi)^4} \frac{(\bar{u}_3 \gamma^\mu (2l + q)_\mu u_1) (\bar{u}_4 \gamma^\nu \gamma_5 l_\nu (\not{p}_4 - \not{l} + M_N) \gamma^\rho \gamma_5 (l + q)_\rho u_2)}{(l^2 - m_\pi^2 + i\epsilon)[(l + q)^2 - m_\pi^2 + i\epsilon][(p_4 - l)^2 - M_N^2 + i\epsilon]}$$

- Perform the one loop integration in **FeynCalc (D-dimension)**

□ Transform to the Helicity basis

- Apply four identities related to Dirac spinor to simplify the tensor structures

$$\begin{aligned} \not{p}_1 u_1(\vec{p}, \lambda) &= [m_N + \gamma^0(p_{1,0} - E_p)] u_1(\vec{p}, \lambda), \\ \not{p}_2 u_2(-\vec{p}, \lambda) &= [m_N + \gamma^0(p_{2,0} - E_p)] u_2(-\vec{p}, \lambda), \\ \bar{u}_3(\vec{p}', \lambda) \not{p}_3 &= \bar{u}_3(\vec{p}', \lambda) [m_N + \gamma^0(p_{3,0} - E_{p'})], \\ \bar{u}_4(-\vec{p}', \lambda) \not{p}_4 &= \bar{u}_4(-\vec{p}', \lambda) [m_N + \gamma^0(p_{4,0} - E_{p'})]. \end{aligned}$$

**“(off-shell)
Dirac equations”**

➡ Left triangle contributions

M.J. Zuilhof et al., PRC26, 1277 (1982)

$$\begin{aligned} V = \frac{ig_A^2}{8F_\pi^4} \vec{\tau}_1 \cdot \vec{\tau}_2 &[(\bar{u}_3 u_1)(\bar{u}_4 u_2) \times F_{LT}^1(A_0, B_0, C_0...) + (\bar{u}_3 u_1)(\bar{u}_4 \gamma^0 u_2) \times F_{LT}^2(A_0, B_0, C_0...)] \\ &+ (\bar{u}_3 \gamma^0 u_1)(\bar{u}_4 u_2) \times F_{LT}^3(A_0, B_0, C_0...) + (\bar{u}_3 \gamma^0 u_1)(\bar{u}_4 \gamma^0 u_2) \times F_{LT}^4(A_0, B_0, C_0...) \\ &+ (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2) \times F_{LT}^5(A_0, B_0, C_0...) + (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma_\mu \gamma^0 u_2) \times F_{LT}^6(A_0, B_0, C_0...) \\ &+ (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^0 \gamma_\mu \gamma^0 u_2) \times F_{LT}^7(A_0, B_0, C_0...) + (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma_\mu u_2) \times F_{LT}^8(A_0, B_0, C_0...)] \end{aligned}$$

Contract form of TPE contributions

- Using the aforementioned “**(off-shell) Dirac eqs.**”, one can express TPE diagrams in terms of **twenty tensor structures**

$$\begin{aligned} O_1 &= 1^{(1)} 1^{(2)}, \quad O_2 = 1^{(1)} \gamma_0^{(2)}, \quad O_3 = \gamma_0^{(1)} 1^{(2)}, \quad O_4 = \gamma_0^{(1)} \gamma_0^{(2)}, \quad O_5 = \gamma_\mu^{(1)} \gamma^\mu{}^{(2)}, \\ O_6 &= \gamma_\mu^{(1)} (\gamma_0 \gamma^\mu)^{(2)}, \quad O_7 = \gamma_\mu^{(1)} (\gamma^\mu \gamma_0)^{(2)}, \quad O_8 = (\gamma_0 \gamma_\mu)^{(1)} \gamma^\mu{}^{(2)}, \quad O_9 = (\gamma_\mu \gamma_0)^{(1)} \gamma^\mu{}^{(2)}, \\ O_{10} &= (\gamma_0 \gamma_\mu)^{(1)} (\gamma_0 \gamma^\mu)^{(2)}, \quad O_{11} = (\gamma_0 \gamma_\mu)^{(1)} (\gamma^\mu \gamma_0)^{(2)}, \quad O_{12} = (\gamma_\mu \gamma_0)^{(1)} (\gamma_0 \gamma^\mu)^{(2)}, \quad O_{13} = (\gamma_\mu \gamma_0)^{(1)} (\gamma^\mu \gamma_0)^{(2)}, \\ O_{14} &= (\gamma_\mu)^{(1)} (\gamma_0 \gamma^\mu \gamma_0)^{(2)}, \quad O_{15} = (\gamma_0 \gamma_\mu \gamma_0)^{(1)} (\gamma^\mu)^{(2)}, \quad O_{16} = (\gamma_0 \gamma_\mu)^{(1)} (\gamma_0 \gamma^\mu \gamma_0)^{(2)}, \quad O_{17} = (\gamma_\mu \gamma_0)^{(1)} (\gamma_0 \gamma^\mu \gamma_0)^{(2)}, \\ O_{18} &= (\gamma_0 \gamma_\mu \gamma_0)^{(1)} (\gamma_0 \gamma^\mu)^{(2)}, \quad O_{19} = (\gamma_0 \gamma_\mu \gamma_0)^{(1)} (\gamma^\mu \gamma_0)^{(2)}, \quad O_{20} = (\gamma_0 \gamma_\mu \gamma_0)^{(1)} (\gamma_0 \gamma^\mu \gamma_0)^{(2)}. \end{aligned}$$

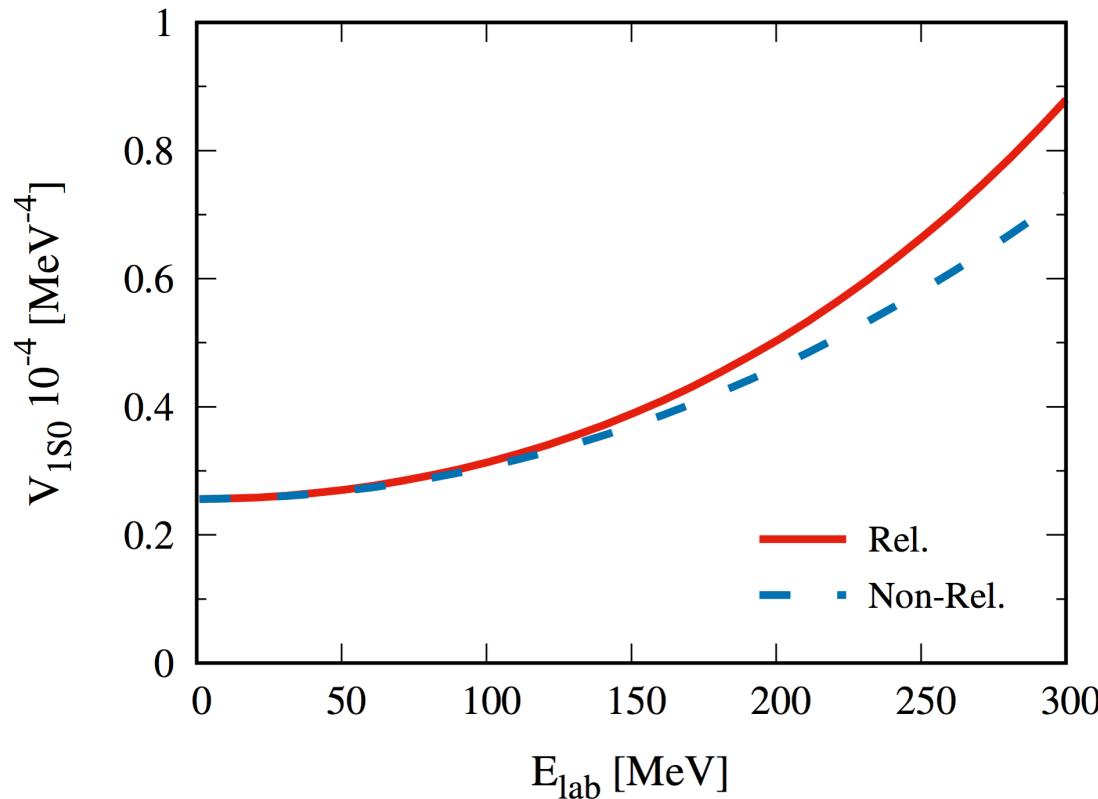
$$V_{\text{TPE}} = \sum_n \color{red} o_n \color{black} F_n(A_0, B_0, C_0, D_0, \dots)$$



- Superposition of **PaVe functions**
- Evaluated by **LoopTools / Package-X**
- Contain **power-counting breaking (PCB) terms**

Football diagram contribution

- Half off shell potential V_{1S0} (Rel. vs. NR)



- Relativistic correction is small
- Only pion propagators in football diagram

- It will be interesting to show the results of triangle and box diagrams, after removing the PCB terms
 - nucleon propagator appears

NLO contact Lagrangian

- Chiral dimension of building blocks:

- Clifford algebra and fields

$$1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} \sim \mathcal{O}(p^0) \quad \psi, \bar{\psi} \sim \mathcal{O}(p^0)$$

$$\mathcal{L}_{NN}^{(0)} = -\frac{1}{2} [\mathbf{C}_S(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + \mathbf{C}_A(\bar{\Psi}\gamma_5\Psi)(\bar{\Psi}\gamma_5\Psi) + \mathbf{C}_V(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^\mu\Psi) + \mathbf{C}_{AV}(\bar{\Psi}\gamma_5\gamma_\mu\Psi)(\bar{\Psi}\gamma_5\gamma^\mu\Psi) + \mathbf{C}_T(\bar{\Psi}\sigma_{\mu\nu}\Psi)(\bar{\Psi}\sigma^{\mu\nu}\Psi).]$$

H. Polinder, J. Haidenbauer, U.-G. Meißner, NPA779, 244 (2006)

- **Partial derivative** --- to increase the chiral order

- acting on the whole bilinear

$$\partial^\mu (\bar{\psi}\psi) \sim \bar{u}_1 i(p_3^\mu - p_1^\mu) u_1 \sim \mathcal{O}(p^1)$$

- acting on the inside of bilinear (*contracted pair*)

$$(\bar{\psi}\partial_\mu\psi)(\bar{\psi}\partial^\mu\psi) \sim -\mathbf{p}_1 \cdot \mathbf{p}_2 (\bar{\psi}\psi)(\bar{\psi}\psi) \sim \mathcal{O}(p^0)$$

We need subtract the mass terms: *D. Djukanovic, et al., FBS41(2007)141*

$$(\bar{\psi}\partial_\mu\psi)(\bar{\psi}\partial^\mu\psi) \sim [-\mathbf{p}_1 \cdot \mathbf{p}_2 + m_N^2] (\bar{\psi}\psi)(\bar{\psi}\psi) \sim \mathcal{O}(p^2)$$

Summary

- We performed an exploratory study to construct the **relativistic nuclear force** up to leading order in **covariant ChEFT**
 - Relativistic chiral force can **improve the description of 1S_0 and 3P_0** phase shifts at LO
 - For the phase shifts of partial waves with high angular momenta ($J>=1$), the relativistic results are **quantitatively similar to** the nonrelativistic counter parts.
- We are now working on the NLO studies
 - Calculate the two-pion exchange potentials (**almost finished**)
 - Construct the contact Lagrangians with two derivatives
 - Expect to achieve a better description of phase shifts  **Stay tuned**

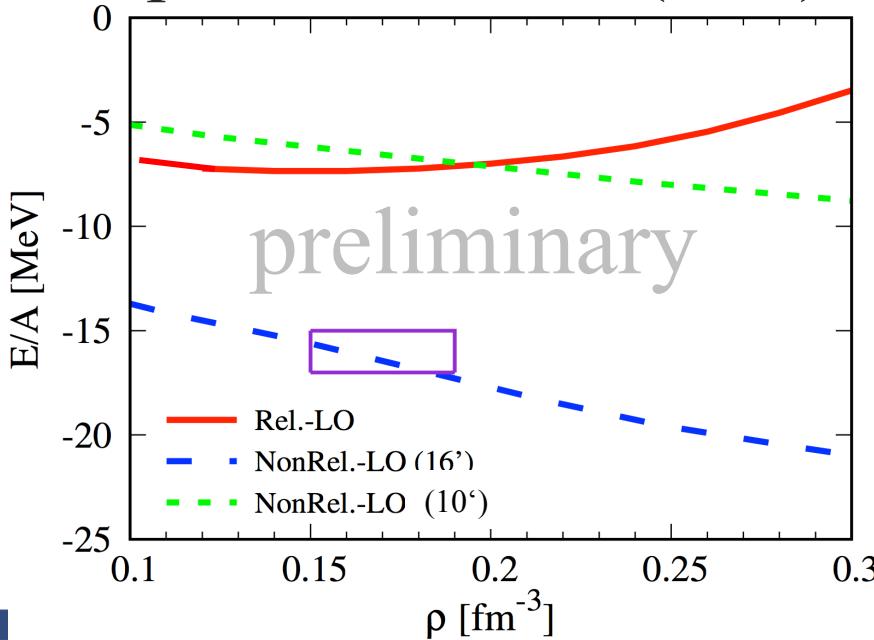
Outlook: application to nuclear matter

- ☐ Relativistic Brueckner-Hartree-Fock theory
 - Kadyshevsky equation in nuclear matter (angle average)

$$G(\mathbf{p}', \mathbf{p} | \mathbf{P}) = V(\mathbf{p}', \mathbf{p}) + \int \frac{d^3 k}{(2\pi)^3} V(\mathbf{p}', \mathbf{k}) \frac{{M^*}^2}{2{E^*}_{\mathbf{P}/2+\mathbf{k}}^2} \frac{\bar{Q}(\mathbf{k}, \mathbf{P})}{{E^*}_{\mathbf{P}/2+\mathbf{p}} - {E^*}_{\mathbf{P}/2+\mathbf{k}}} G(\mathbf{k}, \mathbf{p} | \mathbf{P})$$

- **G matrix**: effective interaction in nuclear matter
- $\mathbf{M}^* = \mathbf{M}_N \mathbf{U}_S$: effective mass; $Q(\mathbf{k}, \mathbf{P})$: Pauli operator

- ☐ Equation of state (EoS) for symmetric NM



- Saturated around $\rho = 0.15 \text{ fm}^{-3}$
- $E/A = -7.4 \text{ MeV}$

R. Machleidt et al., PRC81, 024001 (2010)

J.N. Hu et al., arXiv:1612.05433

**Thank you very much
for your attention!**

Back up slides

Motivation for the relativistic formulation

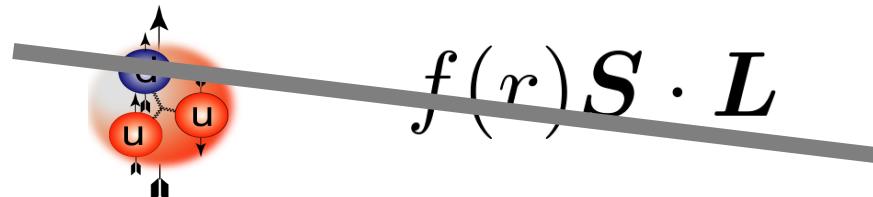
□ Relativistic effects in nuclear physics

- **Kinematical effect:** safely neglected or perturbatively treated

NR approximation: $\sqrt{p^2 + m_N^2} = m_N \sqrt{1 + 0.102}$

- **Dynamical effect:** nucleon spin, spin-orbit splitting, anti-nucleon ...

NR approximation:



□ Relativistic (dynamical) effects are important

- Nuclear system:

- Covariant density functional theory (CDFT)

P. Ring, PPNP (1996),

D.Vretenar et al., Phys.Rept.(2005), J. Meng, IRNP(2016)

- One-nucleon system:

- Covariant ChEFT with extended-on-mass-shell (EOMS) scheme

J. Gegelia, PRD(1999), T. Fuchs, PRD(2003)

Hint at a more efficient formulation

□ V_{1S0} : $1/m_N$ expansion

$$V_{1S0} = 4\pi \left[C_{1S0} + (C_{1S0} + \hat{C}_{1S0}) \left(\frac{\vec{p}^2 + \vec{p}'^2}{4M_N^2} + \dots \right) \right] \\ + \frac{\pi g_A^2}{2f_\pi^2} \int_{-1}^1 \frac{dz}{\vec{q}^2 + m_\pi^2} \left[\vec{q}^2 - \left(\frac{(\vec{p}^2 - \vec{p}'^2)^2}{4M_N^2} + \dots \right) \right].$$

- Relativistic corrections are suppressed
- One has to be careful with **the new contact term**, **the momentum dependent term**, which is desired to achieve a reasonable description of the phase shifts of 1S0 channel.

Only two LECs fit:

$$V_{\text{CTP}}^{\text{NonRel.}} = (C_S + C_V) - (C_{AV} - 2C_T)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \mathcal{O}\left(\frac{1}{M_N}\right).$$

- Take CS and CAV as free parameters
- Best fit result:
 - $\chi^2/\text{d.o.f.} = \textcolor{red}{84.5}$

	Relativistic Chiral NF	Non-relativistic Chiral NF	
Chiral order	LO	LO	NLO*
No. of LECs	5	2	9
$\chi^2/\text{d.o.f.}$	2.9	147.9	~2.5

Errors and correlation matrix

TABLE I: The best fit results of five LECs appearing in the contact terms (in unit of 10^4GeV^{-2}) with the momentum cutoff $\Lambda = 747 \text{ MeV}$.

LECs	C_S	C_A	C_V	C_{AV}	C_T
Best fit	0.13515 ± 0.00307	-0.055963 ± 0.018217	-0.26857 ± 0.01151	-0.24427 ± 0.01141	-0.062538 ± 0.001319

	C_S	C_A	C_V	C_{AV}	C_T
C_S	1.00	0.21	-0.93	-0.58	-0.39
C_A	0.23	1.00	-0.15	0.45	0.21
C_V	-0.93	-0.15	1.00	0.77	0.69
C_{AV}	-0.57	0.45	0.77	1.00	0.89
C_T	-0.39	0.21	0.69	0.89	1.00

T _{lab} [MeV]	1	50	100	150	200	250	300
P _{cm} [MeV]	21.67	153.22	216.68	265.38	306.43	342.60	375.30
V _{cm}	0.023 $\textcolor{blue}{c}$	0.16 $\textcolor{blue}{c}$	0.23 $\textcolor{blue}{c}$	0.28 $\textcolor{blue}{c}$	0.33 $\textcolor{blue}{c}$	0.36 $\textcolor{blue}{c}$	0.40 $\textcolor{blue}{c}$
E _{corr(2n)} [MeV]	0.25	12.5	25	37.5	50	62.5	75

$$p_{cm} = \sqrt{\frac{m_N T_{lab}}{2}} \quad V_{cm} = \frac{p_{cm}}{m_N} c$$

$$E_T^{\text{corr}} = \frac{p_{cm}^2}{2m_N}$$

Strategies to construct NLO Lagrangian

$$\mathcal{O}_{\Gamma_A \Gamma_B}^{(n)} \sim (\bar{\psi} i \overleftrightarrow{\partial}^{\mu_1} i \overleftrightarrow{\partial}^{\mu_2} \cdots i \overleftrightarrow{\partial}^{\mu_n} \Gamma_A^\alpha \psi) (\bar{\psi} i \overleftrightarrow{\partial}_{\mu_1} i \overleftrightarrow{\partial}_{\mu_2} \cdots i \overleftrightarrow{\partial}_{\mu_n} \Gamma_{B\alpha} \psi)$$

$$\mathcal{O}_{\Gamma_A \Gamma_B}^{(n)} \sim [(p_1 + p_3) \cdot (p_2 + p_4)]^n$$

- Keep $n=1$ terms *L. Girlanda, et al., PRC81(2010)034005*
 - perform non-rel. expansion