

Coupled-Channel Effects for the bottomonium and X(4260)

in CRC110 B2,
Based on [PRD94, 034021], [PRD95, 034018] & [arxiv:1705.00449]

study of the $\psi' \rightarrow J/\psi + \eta$ at the end of this talk [PRD95, 114031]

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Outline

1 Introduction

- Quark Model & QCD
- 3P_0 model & coupled-channel effects

2 Bottomonia

- Mass shift
- $P_{b\bar{b}}$ & fine splittings
- $S - D$ mixing of Υ
- Excited B meson loops

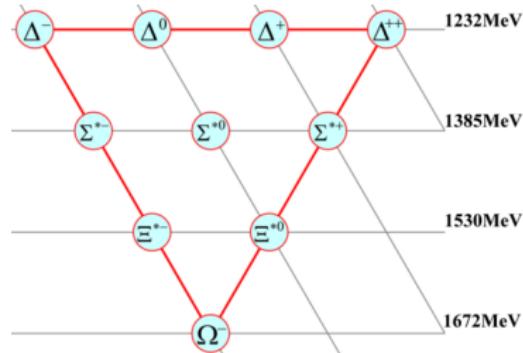
3 $X(4260)$

- Properties of the $X(4260)$
- Coupled-channel effects of the $X(4260)$

4 Summary

Quark Model & QCD

- Numerous hadrons ask for a classification method $K, \eta, \pi, \rho, \Sigma, \Lambda, \Xi, \Sigma^* \dots$
- Quark model (e.g. predicts m_Ω)
 - ▶ 6 flavors (u,d,s,c,b,t)
 - ▶ spatial \otimes spin \otimes flavor \otimes color (r,g,b)
- Quantum chromo-dynamics (QCD)
$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} + \sum_{j=1}^{N_f} \bar{\psi}^j (i\gamma^\mu D_\mu - m) \psi^j$$
 - ▶ Gluons transmit strong interactions
 - ▶ Asymptotic freedom
 - ▶ $\alpha_s < 1$, perturbation theory works



How to calculate when $\alpha_s > 1$?

Lattice QCD, Chiral perturbation theory, QCD inspired effective models

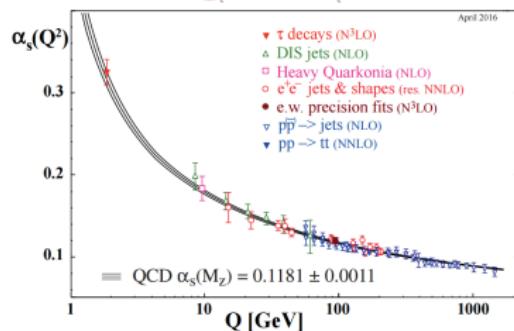
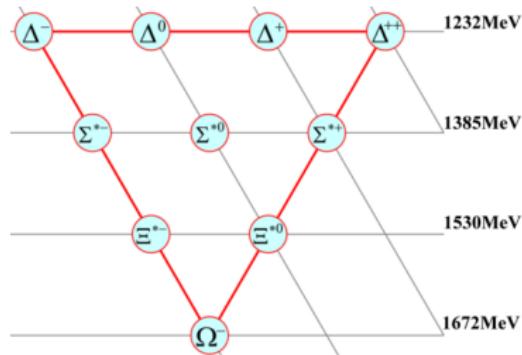
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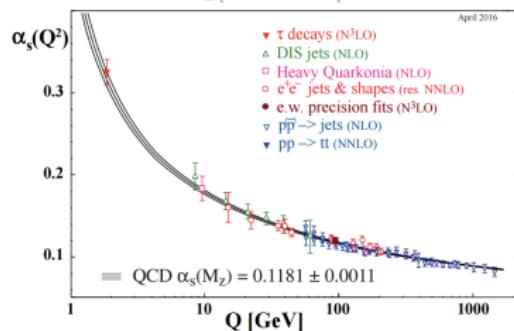
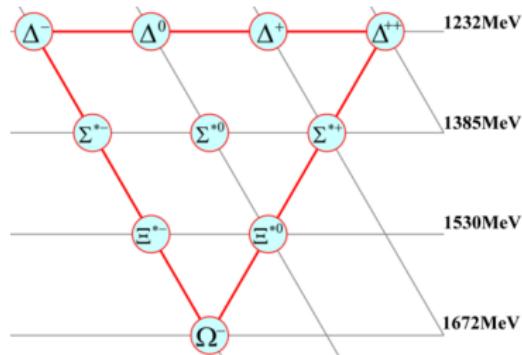
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Coupled-channel effects (CCEs)

static quenched quark model
dynamic $q\bar{q}$ creation

$\left. \begin{array}{c} \text{static quenched quark model} \\ \text{dynamic } q\bar{q} \text{ creation} \end{array} \right\} \Rightarrow \text{unquenched quark model}$

Cornell potential model

$$V(r) = -\frac{4}{3} \frac{\alpha}{r} + \lambda r + c + \text{spin-dependent}$$

3P_0 model

$q\bar{q}$ are created from vacuum

$$H_I = 2m_q \gamma \int d^3x \bar{\psi}_q \psi_q$$

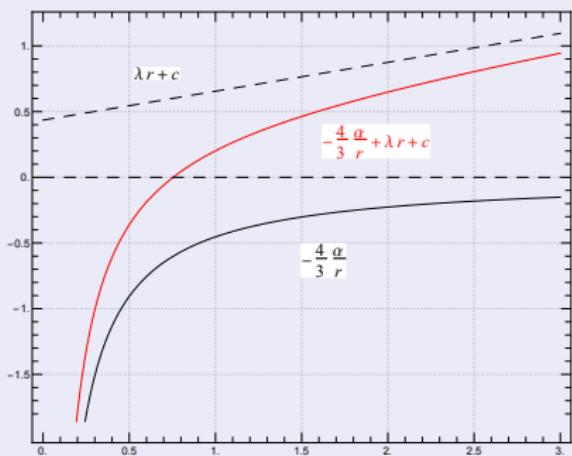
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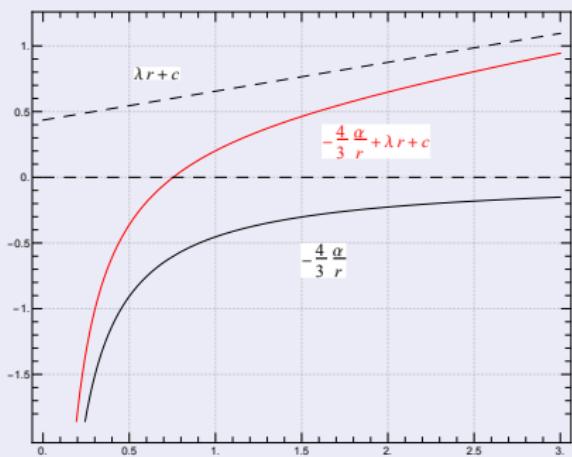
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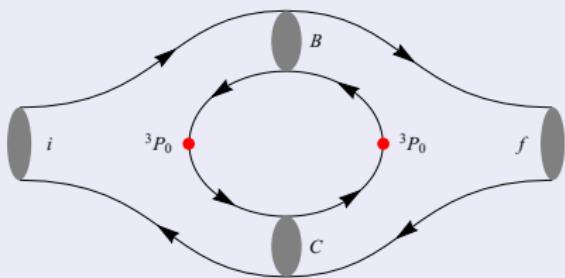


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$$\left. \begin{array}{l} J^{PC} = 0^{++} \\ P = (-)^{L+1} \\ C = (-)^{L+S} \end{array} \right\} \Rightarrow L = S = 1$$

$$2S+1 L_J = ^3P_0$$

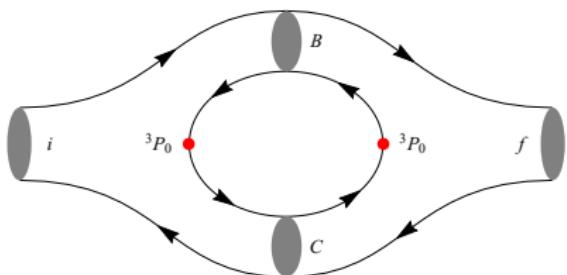
Impact of the 3P_0

CCEs contain **all** the effects of changes of H & $|A\rangle$

Hamiltonian: $H = H_0 + H_{BC} + H_I$

Mass shift ΔM : $\int d^3 p \frac{|\langle BC; p | H_I | \psi_0 \rangle|^2}{M - E_{BC} - i\epsilon}$

Decay Γ : $2\pi P_B \frac{E_B E_C}{m_A} |\langle BC; P_B | H_I | \psi_0 \rangle|^2$



Wave function: $|A\rangle = c_0 |\psi_0\rangle + \sum_{BC} \int d^3 p c_{BC}(p) |BC; p\rangle$

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Meson component probability: $1 - P_{Q\bar{Q}}$ or $P_{Q\bar{Q}} \times \int d^3 p \frac{|\langle BC; p | H_I | \psi_0 \rangle|^2}{(M - E_{BC})^2}$

i, f mixing e.g. ${}^3S_1 \rightarrow$ meson molecule $\rightarrow {}^3D_1$

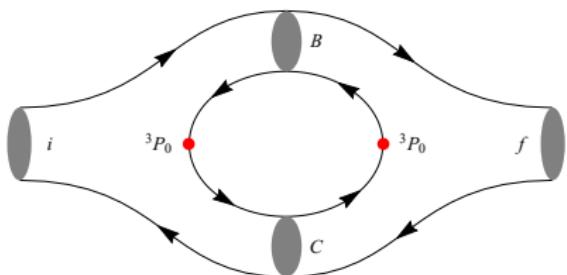
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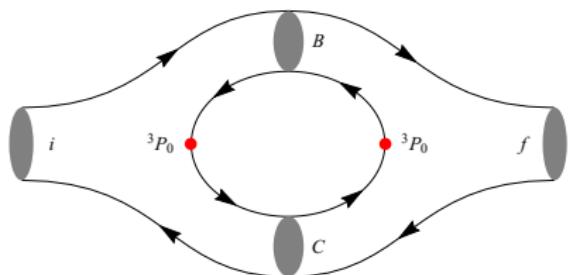
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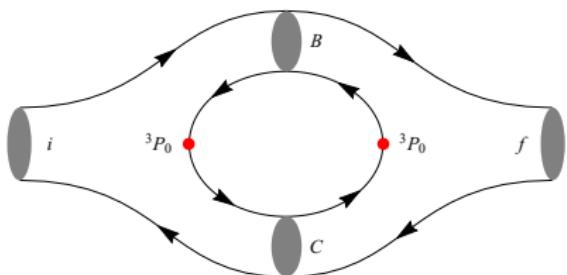
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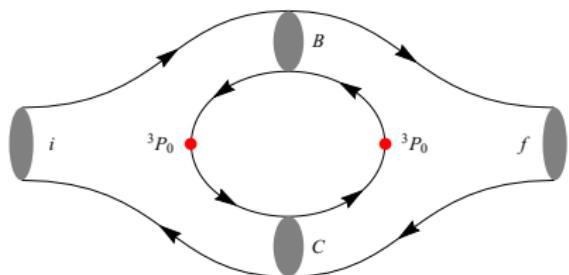
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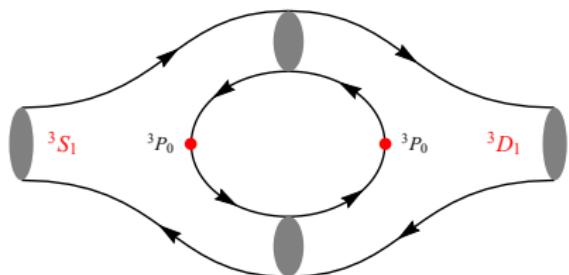
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- ΔM is negative if $M < m(B) + m(\bar{B})$
 - ▶ $M \rightarrow m(B) + m(\bar{B}) \Rightarrow M \searrow$
 - ▶ Explains $V_{scr} = \lambda r^{1-\frac{c-p^2}{pr}}$
[B-Q, Li PRD80, 014012]
 - ▶ V_{scr} works in the low red region
- ΔM is positive if $M \gg m(B) + m(\bar{B})$
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 - ▶ V_{scr} works in the high red region
- ΔM is chaotic in between
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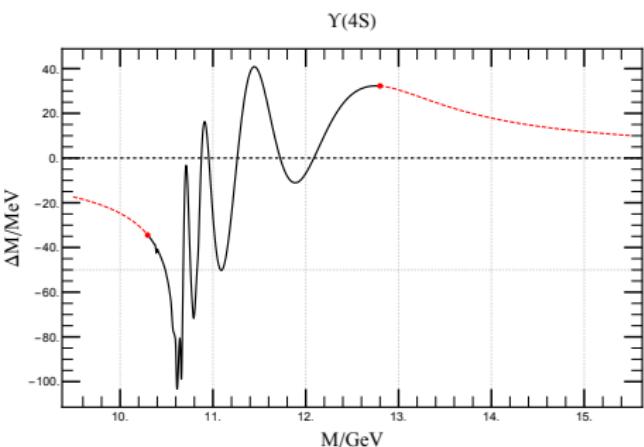


图: Loops limited to ground state B mesons

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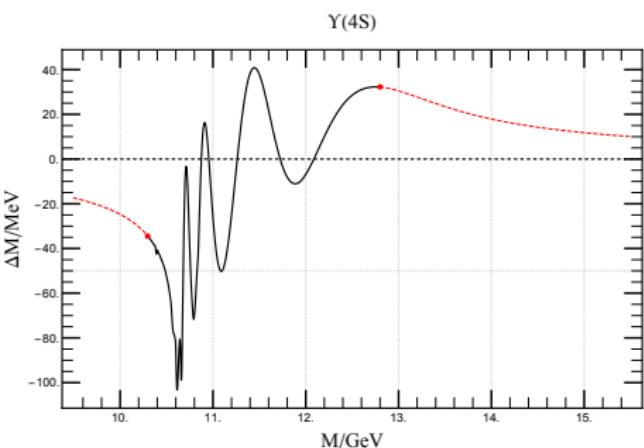


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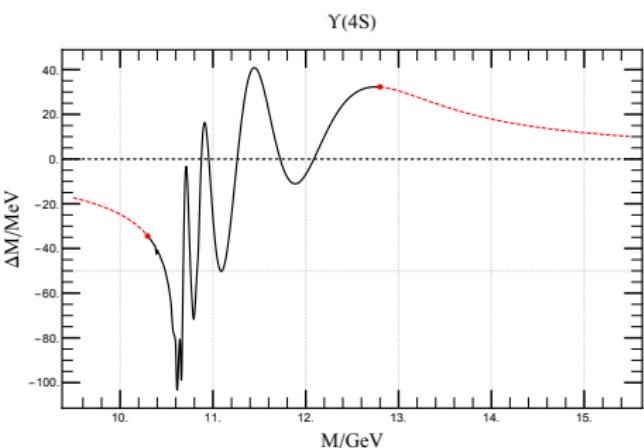


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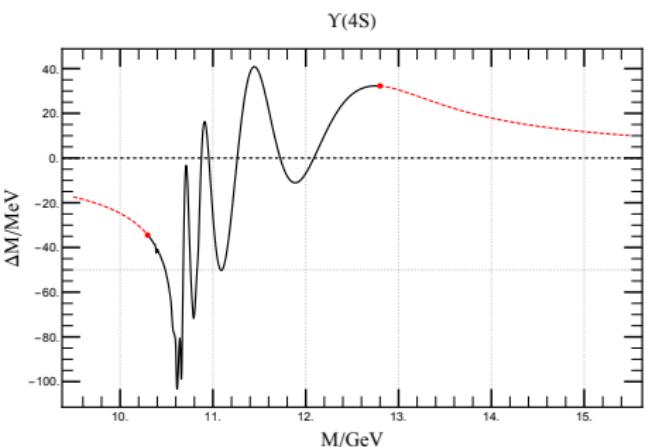
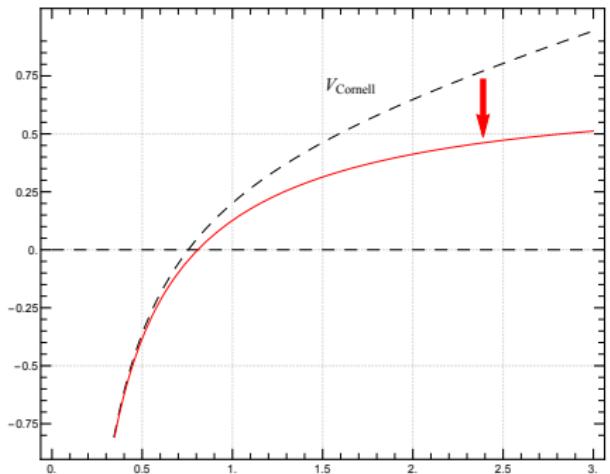


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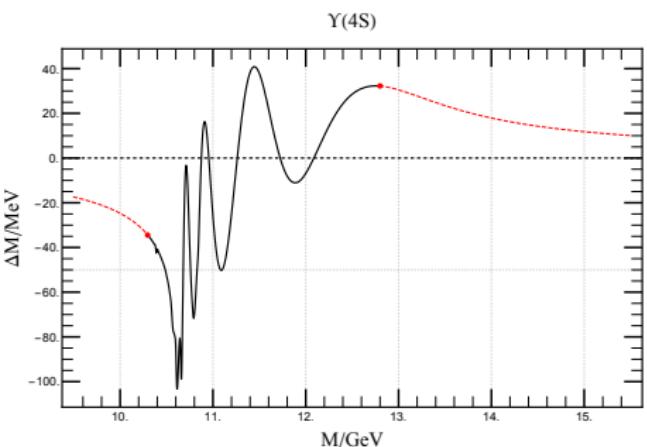
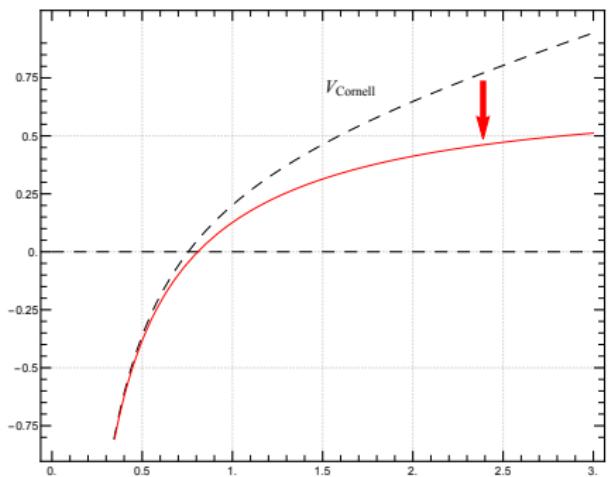


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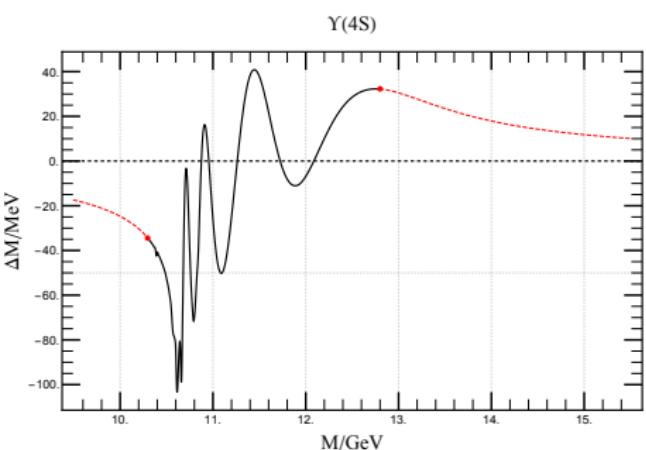


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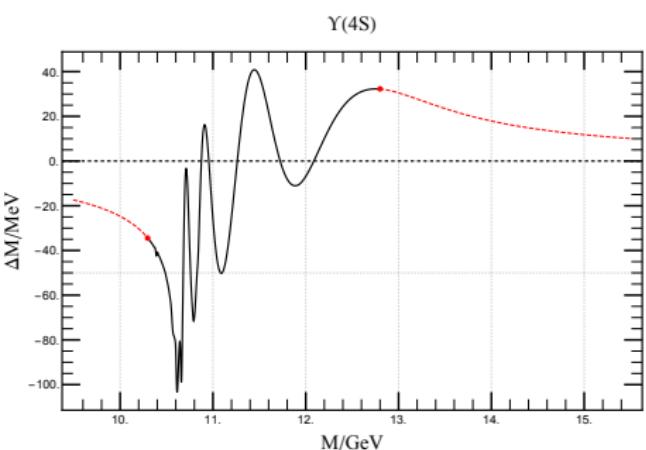


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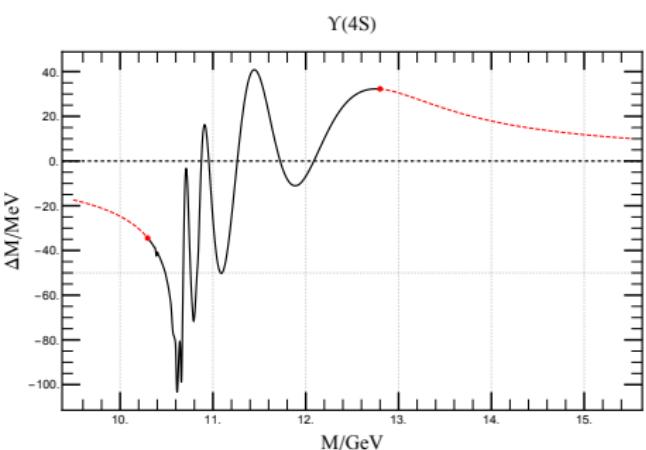


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- ΔM is positive if $M \gg m(B) + m(\bar{B})$
 - ▶ may not really happen (\because more channels are opened)
- ΔM is chaotic in between
 - ▶ due to $\int \psi$ & different channels
 - ▶ V_{scr} even worse in this region

Mass shift ΔM & SHO approximation

$$\Delta M \equiv M - M_0 = \sum_{BC} \int d^3 p \frac{|\langle BC; p | H_I | \psi_0 \rangle|^2}{M - E_{BC} - i\epsilon}$$

SHO only works in the low energy region

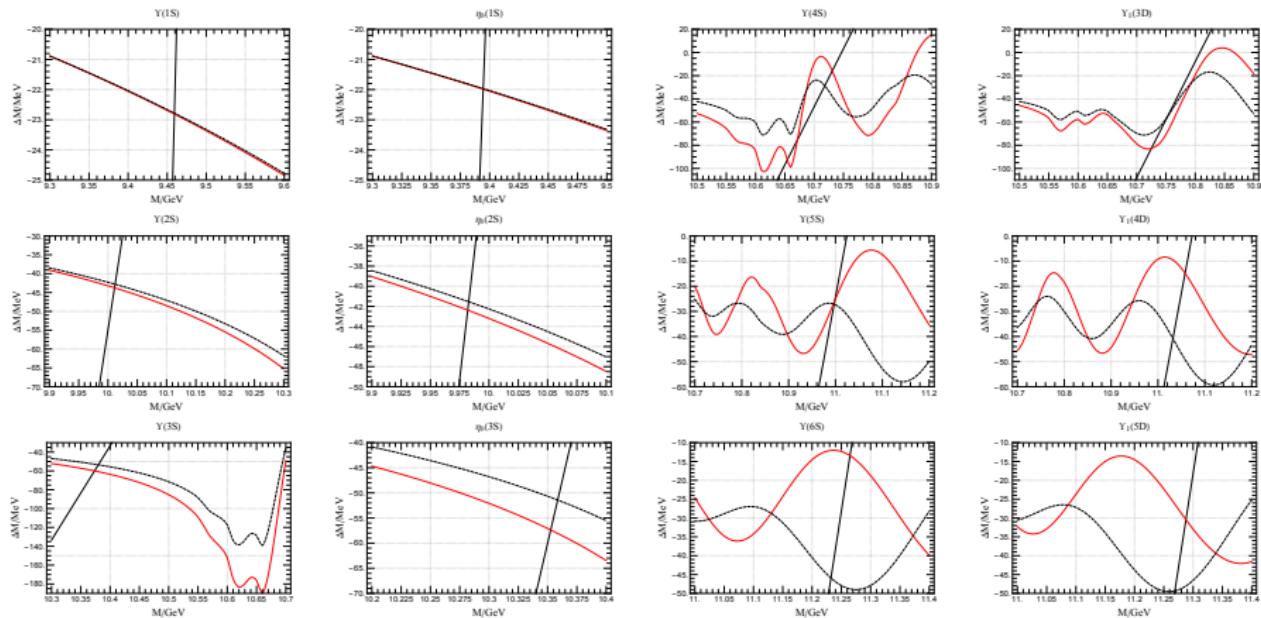


图: ΔM in GEM V.S. SHO

$P_{b\bar{b}}$ & fine splittings

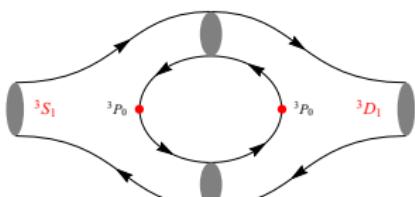
- More informative when compare $P_{b\bar{b}}$ with fine splittings δm in (n, L) multiplets (see also [T.J. Burns, PRD84, 034021])
 - $\delta m = \langle \psi | V_{L,S} | \psi \rangle$ (in leading order)
 - renormalize $\langle \psi | \psi \rangle = 1 \rightarrow P_{b\bar{b}}$
 - if $P_{b\bar{b}}$ s are equal for all states in same (n, L) multiplets
 - $\delta m \rightarrow P_{b\bar{b}} \delta m$
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States	BB		$BB^* + h.c.$		$B^* B^*$		$B_s B_s$		$B_s B_s^* + h.c.$		$B_s^* B_s^*$		$P_{b\bar{b}}$	
	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO
$h_b(3P)$	0	0	19.75	18.19	9.04	7.7	0	0	0.67	0.54	0.54	0.45	70.0	73.12
$\chi_{b0}(3P)$	34.08	38.84	0	0	8.07	6.21	0.31	0.22	0	0	0.62	0.48	56.92	54.26
$\chi_{b1}(3P)$	0	0	21.9	20.1	7.54	6.44	0	0	0.64	0.51	0.54	0.46	69.38	72.5

$S - D$ mixing

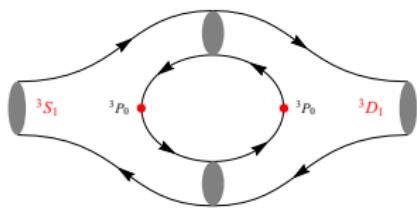


$$\begin{pmatrix} M_S^0 + \Delta M_S & \Delta M_{SD} \\ \Delta M_{DS} & M_D^0 + \Delta M_D \end{pmatrix} \begin{pmatrix} c_S \\ c_D \end{pmatrix} = M \begin{pmatrix} c_S \\ c_D \end{pmatrix}$$

$$\Delta M_f = \int d^3 p \frac{|\langle \psi_f | H_I | BC \rangle|^2}{M - E_{BC} - i\epsilon} \quad (f = S, D)$$

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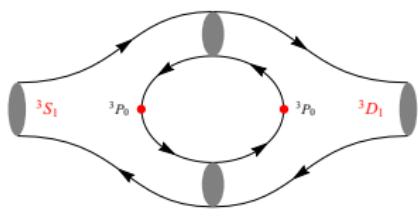
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2nd order perturbation [T. Barnes and E.S. Swanson (2007)]:

$$c_D = \frac{1}{M_D^0 - M_S^0} \int d^3 p \frac{\langle \psi_D | H_I | BC \rangle \langle BC | H_I | \psi_S \rangle}{M_S^0 - E_{BC}}$$

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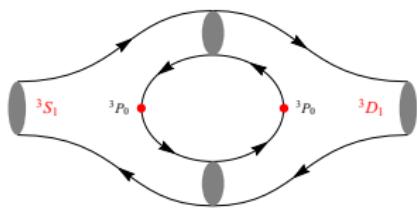
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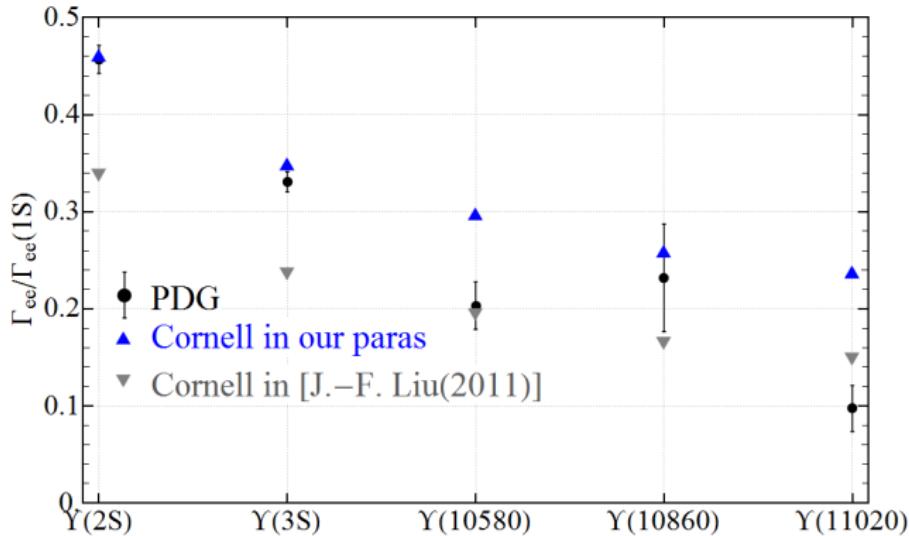
	<i>S</i> wave Theory (keV)	<i>D</i> wave Theory (keV)	Exp (keV)
$\Gamma_{ee}(\Upsilon(10580))$	0.372	1.435×10^{-3}	0.272 ± 0.029
$\Gamma_{ee}(\Upsilon(10860))$	0.314	1.697×10^{-3}	0.31 ± 0.07
$\Gamma_{ee}(\Upsilon(11020))$	0.270	1.878×10^{-3}	0.130 ± 0.03

表: Theory value from [A. M. Badalian(2010)], Exp from [PDG(2016)]

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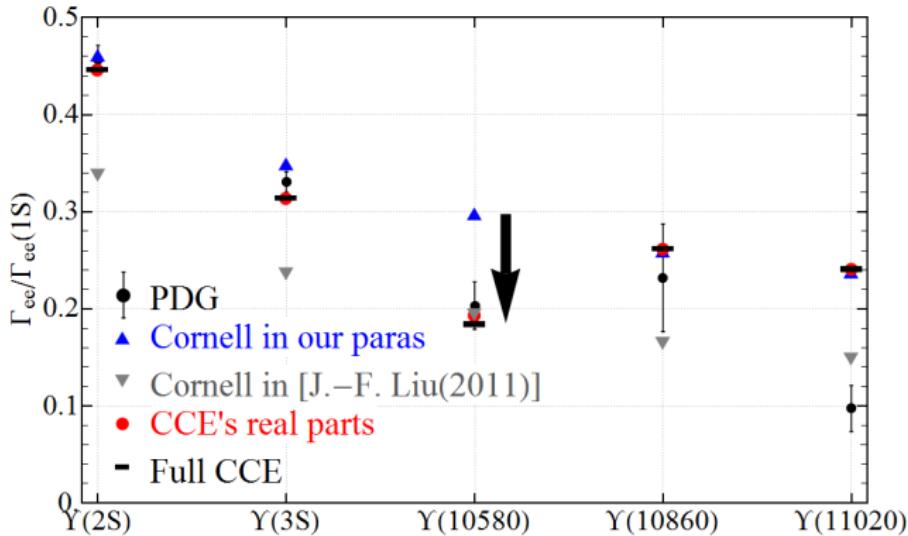
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Υ 's radiative decay

E_1 decay width:

$$\Gamma(n^{2S+1}L_J \rightarrow n'^{2S'+1}L'_{J'} + \gamma) = \frac{4}{3} C_{fi} \delta_{SS'} e_b^2 \alpha |\langle f|r|i\rangle|^2 E_\gamma^3$$

$$\langle f|r|i\rangle = \int_0^\infty R_f(r) R_i(r) r^3 dr,$$

$$C_{fi} = \max(L, L') (2J' + 1) \left\{ \begin{array}{ccc} L' & J' & S \\ J & L & 1 \end{array} \right\}^2$$

Define $r_\gamma \equiv \frac{\Gamma(\Upsilon \rightarrow \chi_{b2}(1P) + \gamma)}{\Gamma(\Upsilon \rightarrow \chi_{b0}(1P) + \gamma)}$, eliminate $\langle f|r|i\rangle$

$$r_\gamma(S) = 5 \left(\frac{E_{\gamma 2}}{E_{\gamma 0}} \right)^3, \quad r_\gamma(D) = \frac{1}{20} \left(\frac{E_{\gamma 2}}{E_{\gamma 0}} \right)^3$$

	<i>S</i> wave Theo	<i>D</i> wave Theory	Exp [PDG 2016]
$r_\gamma(2S)$	1.57	0.0157	1.91 ± 0.29
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- $\Upsilon(2S), \Upsilon(3S)$ are pure *S* wave ($\theta \approx 0$)
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Push the calculation to the excited B meson loops

- In heavy sector, *systematic* study of all the meson loops are still missing
- Channels' number explodes ($6 \rightarrow 42 \nearrow$) & $\int \psi$ becomes harder
- Focus on the ΔM & do not fit γ of 3P_0 's Hamiltonian

First, up to $B(1P)$:

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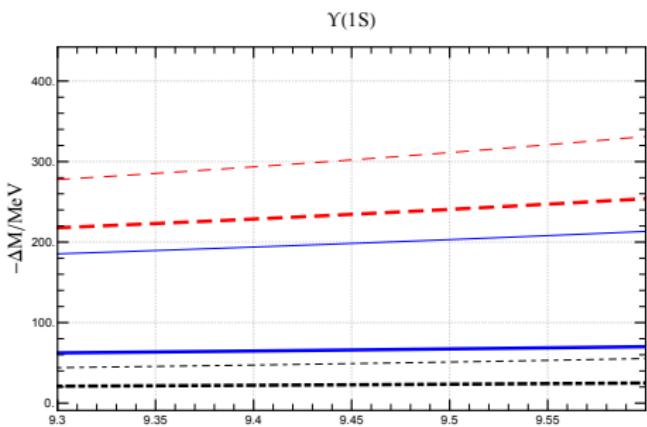
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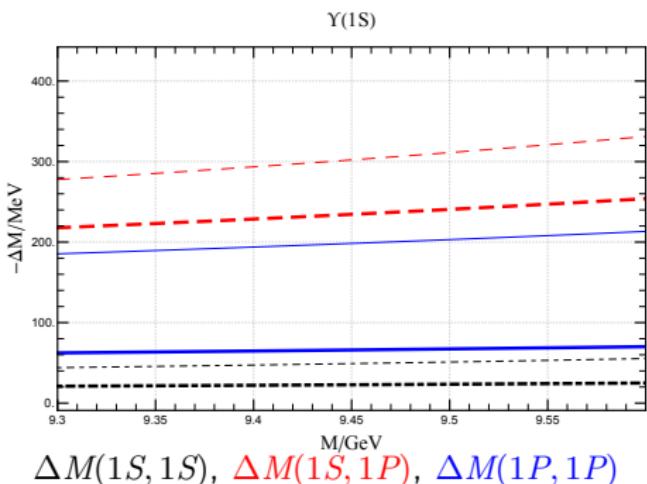
Our paras-thick curve

[J.-F. Liu(2011)]'s paras-thin curve

⇒ Refit the parameters?

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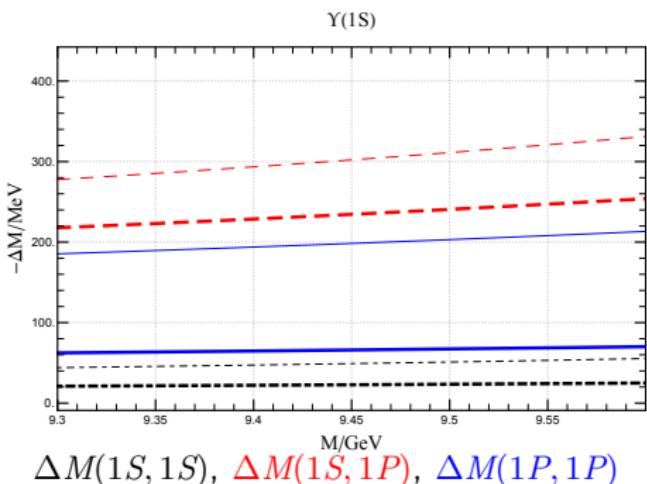
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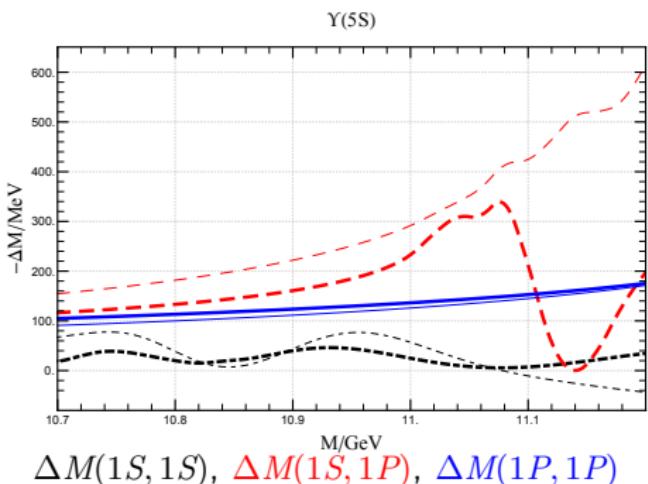
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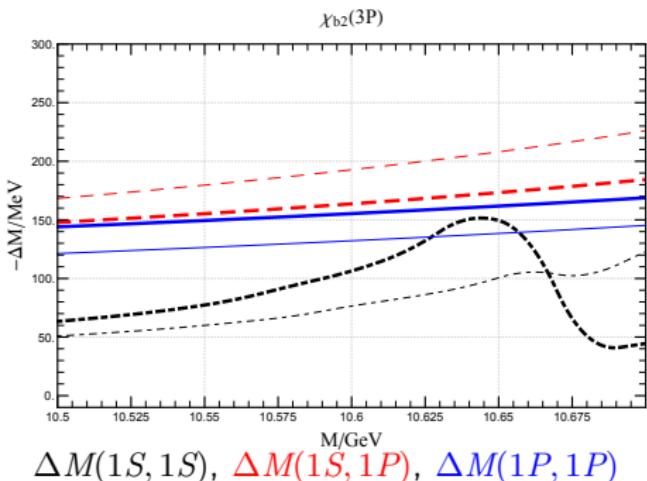
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- Not only for Υ but for all $b\bar{b}$,
 $\Delta M(1S, 1P)$ is generally the largest

Up to $B(1P)$



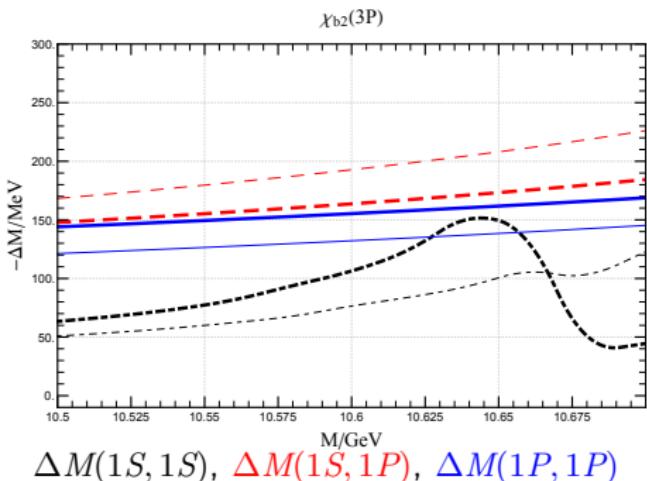
Our paras-thick curve

[J.-F. Liu(2011)]'s paras-thin curve

⇒ Refit the parameters?

- $\Delta M(1S, 1P)$ is generally the largest
- for $\Upsilon(1S)$:
 $\Delta M(1S, 1P) + \Delta M(1P, 1P) \approx 14 \Delta M(1S, 1S)$
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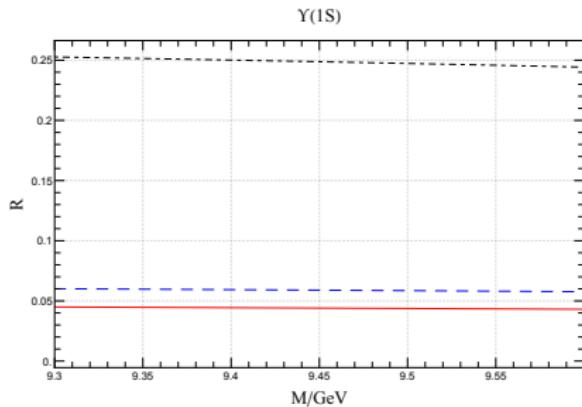
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Push to $B(2S)$



no form factor, $r = 0.408$ fm,
 $r = 0.335$ fm [J. Ferretti(2013)]

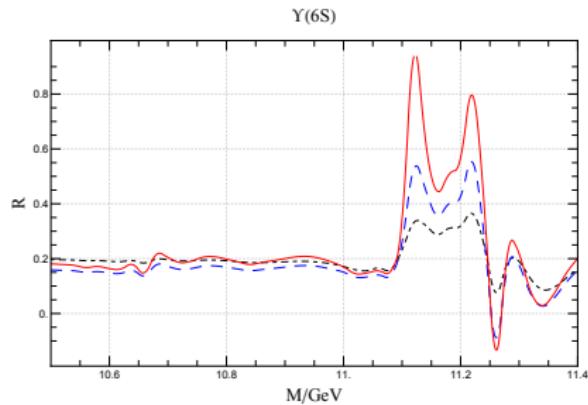
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$$\int d^3 p \frac{|\langle BC; p | H_I e^{-2r^2 p^2/3} | \psi_0 \rangle|^2}{M - E_{BC} - i\epsilon}$$

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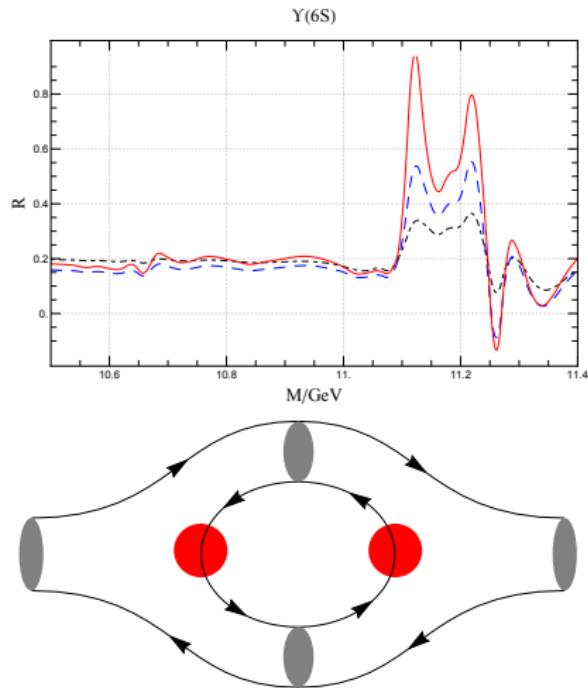
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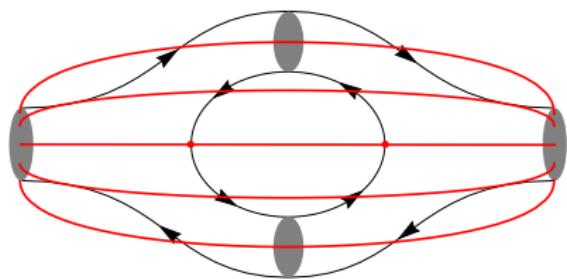
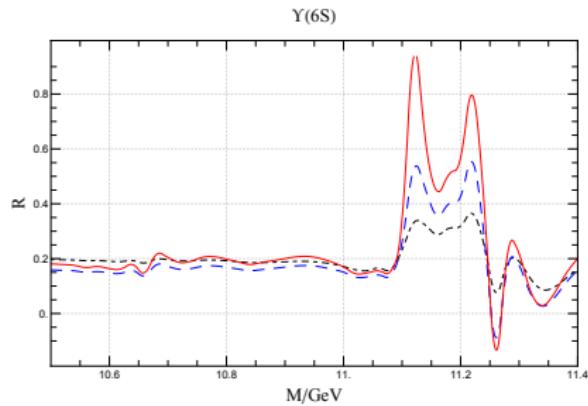
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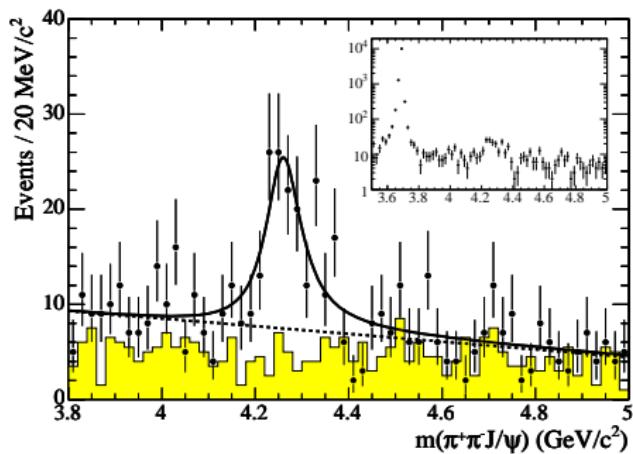
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In 2005, BaBar found a peak around 4.26GeV in $J/\psi\pi\pi$ invariant spectrum in $e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi\pi\pi$. Later confirmed by CLEO and Belle

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- (i) How to accommodate its mass in potential model?
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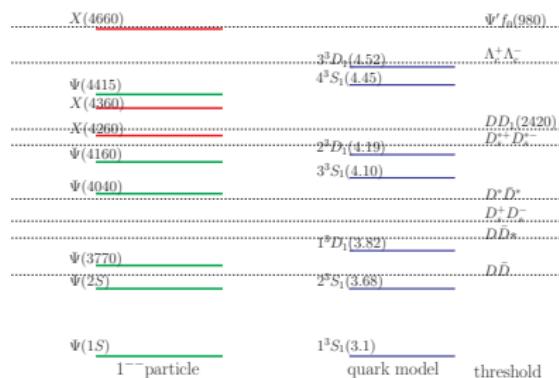


图: Results from [S. Godfrey and N. Isgur(1985)], Graph from [L.Y. Dai et al (2015)]

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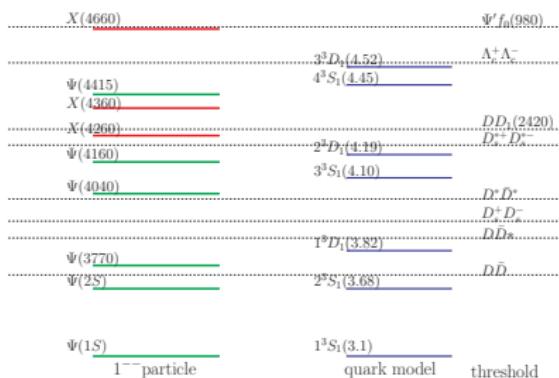


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- HQSS predicts [Close and Swanson, PRD 72, 094004]

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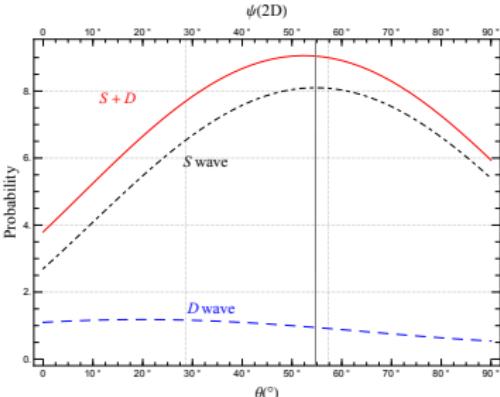
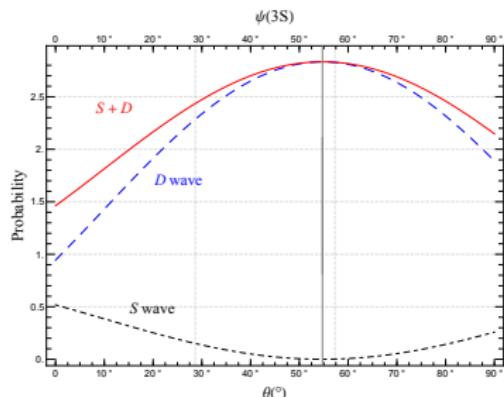
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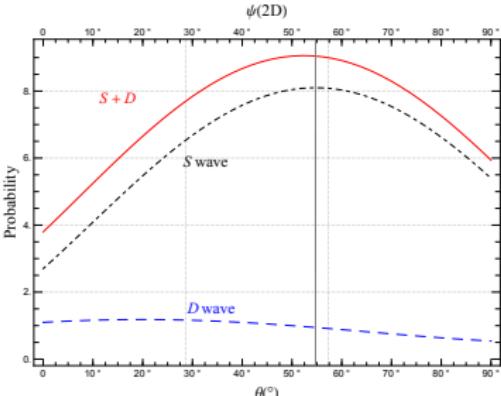
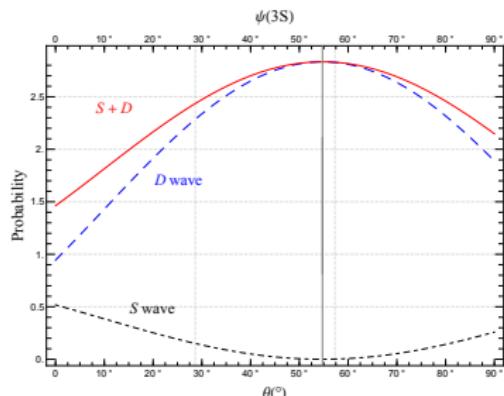
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$$|X(4260)\rangle = c_1|{}^3S_1\rangle + c_2|D_1\bar{D}\rangle + C_3|\text{other } D \text{ mesons}\rangle$$

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	Coupled Channels	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$
Our paras	$D_1 - D$	2.83	1.48	9.05	3.32
	$D'_1 - D$	0.75	0.62	0.68	0.36
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Coupled Channels	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$
$D - D_0^*$	0	0	0	0
$D^* - D_0^*$	0.215	0.238	0.028	0.07
$D - D'_1$	0.265	0.42	0.076	0.109
$D^* - D'_1$	0.325	0.317	0.075	0.118
$D - D_1$	1	1	1	1
$D^* - D_1$	0.616	0.564	0.149	0.18
$D - D_2^*$	0.629	0.59	0.065	0.093
$D^* - D_2^*$	0.992	0.914	0.225	0.339

表: Relative couplings in our parameters

Results Analysis

$$|X(4260)\rangle = c_1|{}^3S_1\rangle + c_2|D_1\bar{D}\rangle + C_3|\text{other } D \text{ mesons}\rangle$$

$$|X(4260)\rangle = c'_1|{}^3D_1\rangle + C_2|D_1\bar{D}\rangle + c_3|\text{other } D \text{ mesons}\rangle$$

- $D_1 D$ couples stronger to 3D_1 charmonium $\psi(nD)$. $C_2 > c_2$ (somewhat model independent)
- How large are the other D meson's contributions?
 $m(D)$ s have different values, HQSS is violated, more realistic.
- For 3S_1 , $D_1 D$ is not always dominant, $C_3 > c_2$ (model dependency shows up)
- For 3D_1 , $C_2 > c_3$
- The $D_1 D$ molecule scenario favors ${}^3D_1 c\bar{c}$ kernel, agree with experimentally measured R ratio.

Coupled Channels	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$
$D - D_0^*$	0	0	0	0
$D^* - D_0^*$	0.809	1.073	0.073	0.167
$D - D'_1$	1.367	1.635	0.133	0.336
$D^* - D'_1$	1.117	1.651	0.148	0.378
$D - D_1$	1	1	1	1
$D^* - D_1$	0.816	0.922	0.185	0.269
$D - D_2^*$	0.7	0.716	0.069	0.1
$D^* - D_2^*$	1.384	1.632	0.267	0.537

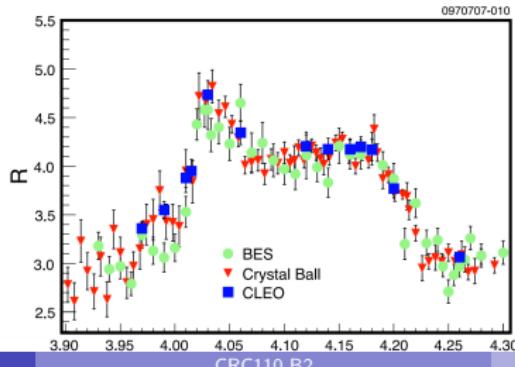
表: Relative couplings in [T. Barnes, E. S. Swanson (2007)]'s parameters

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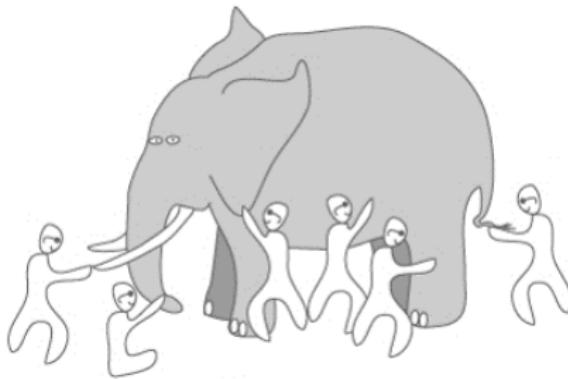


Summary

- CCEs are thoroughly & precisely evaluated in 3P_0 model
 - ▶ threshold effect breaks fine splitting in (n, L) multiplets
 - ▶ big $S - D$ mixing not favored
 - ▶ B meson components suppress $\Gamma_{ee}(\Upsilon)$
 - ▶ $\Upsilon \rightarrow \chi_{bj} + \gamma$ helps to distinguish $P_{b\bar{b}}$ & θ_{S-D}
 - ▶ Defect of 3P_0 model: convergence issue when summing all loops
 - ▶ $D_1 D$ scenario is reasonable for $X(4260)$ if charmonium core is 3D_1

Summary

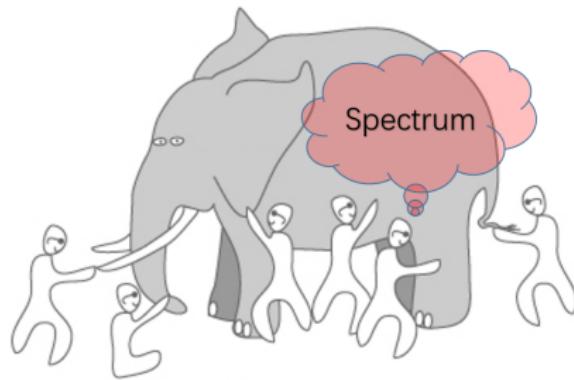
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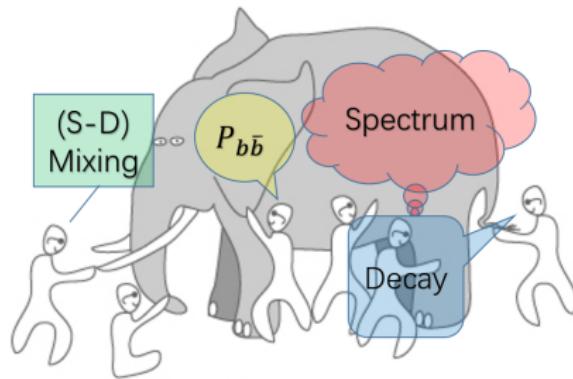
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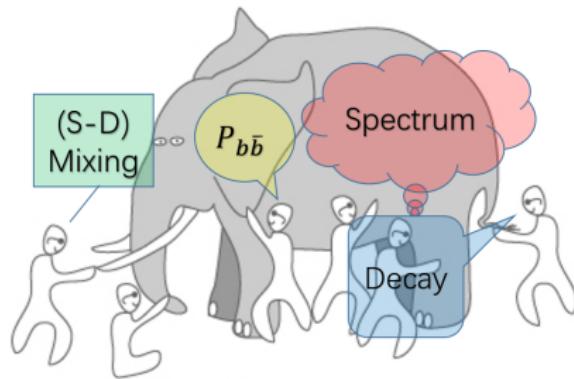
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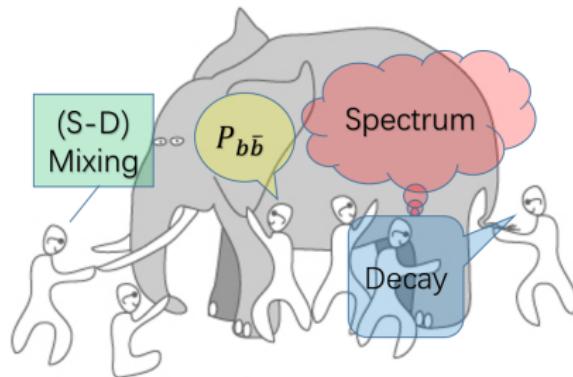
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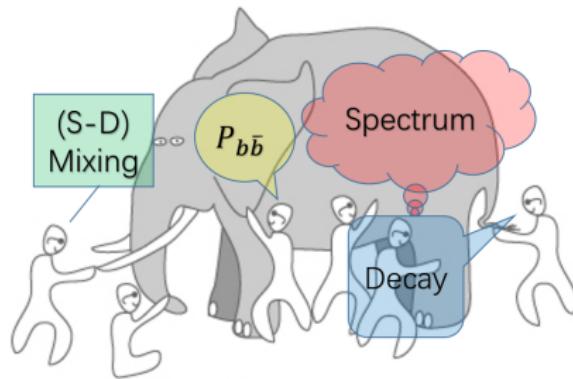
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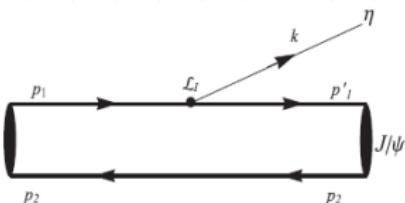


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Effective Model for HQSS Violating Decays, $\psi \rightarrow J/\psi\eta$ & $\psi \rightarrow h_c(1P)\eta$

M. Naeem Anwar, Yu Lu and Bing-Song Zou, Phys. Rev. D 95, 114031 (2017)

- Motivated by Nambu-Jona-Lasinio (NJL) Model, we constructed an effective model to create light meson(s) in heavy quarkonium transitions
- With small $S - D$ mixing among $J^{PC} = 1^{--}$
- Successfully described the corresponding available data Ψ
- Made several predictions for
 $\psi(3D, 4S, 4D, 5S, 5D, 6S) \rightarrow J/\psi\eta$ and $h_c(1P)\eta$
- Studied spectroscopic quantum numbers for Y(4360), Y(4390) and Y(4660)
- Based on the current exp. data, Y(4360) as potential candidate for $\psi(3D)$
- New data from BESIII is available for $Y(4360) \rightarrow h_c(1P)\eta$
- $\psi \rightarrow J/\psi\pi^+\pi^-$ and $\psi \rightarrow h_c\pi^+\pi^-$ study ongoing, stay tune!



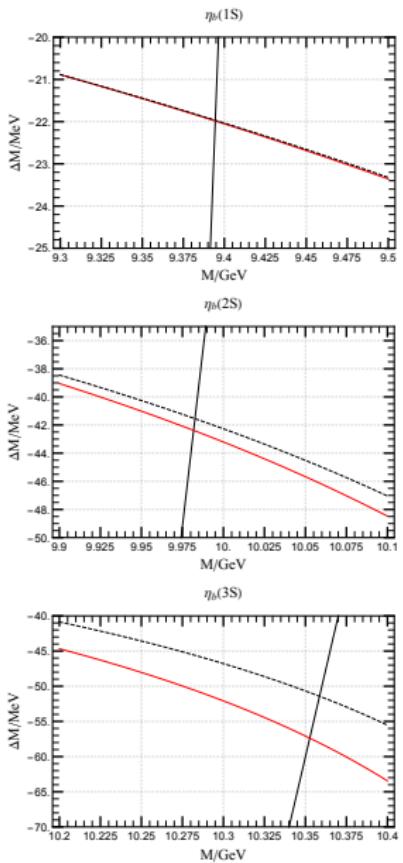
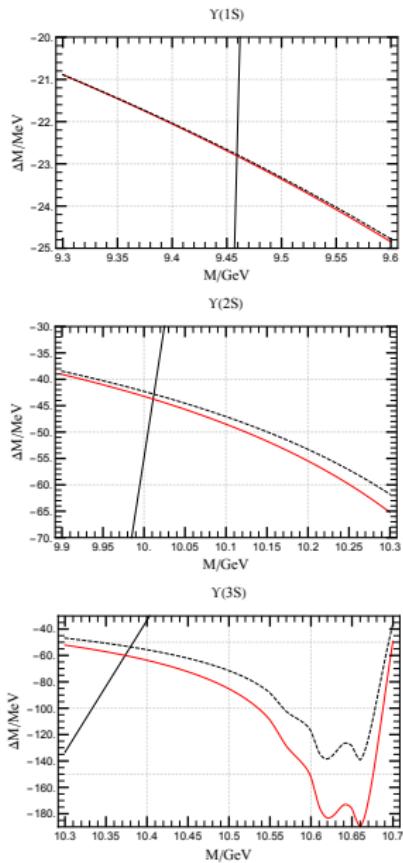
5 附录

6 Bottomonia

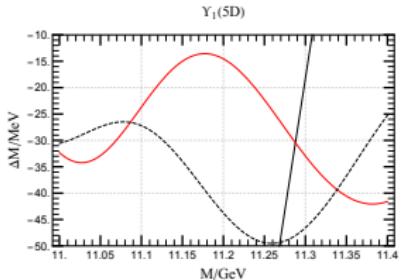
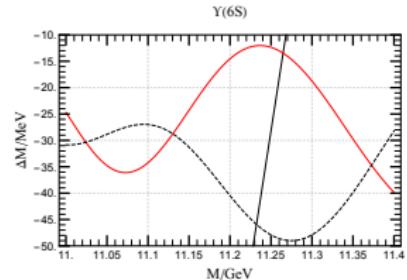
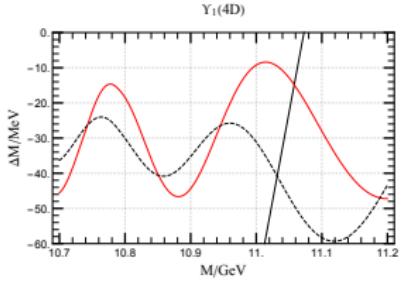
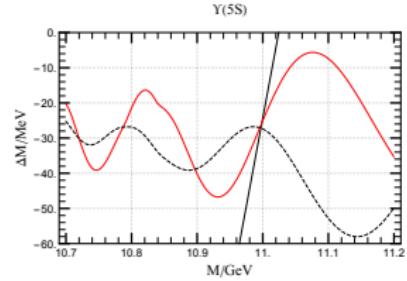
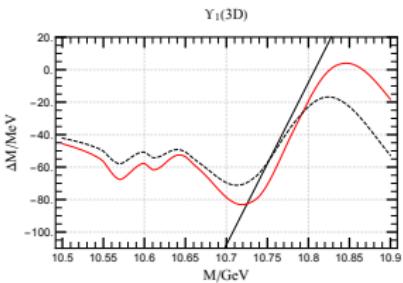
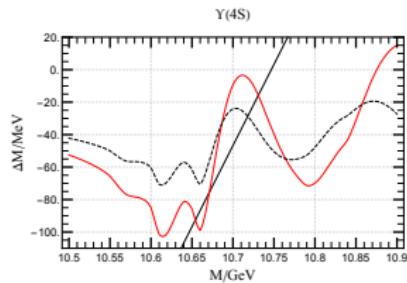
- mass shift plot
- GEM
- tables

7 $X(4260)$

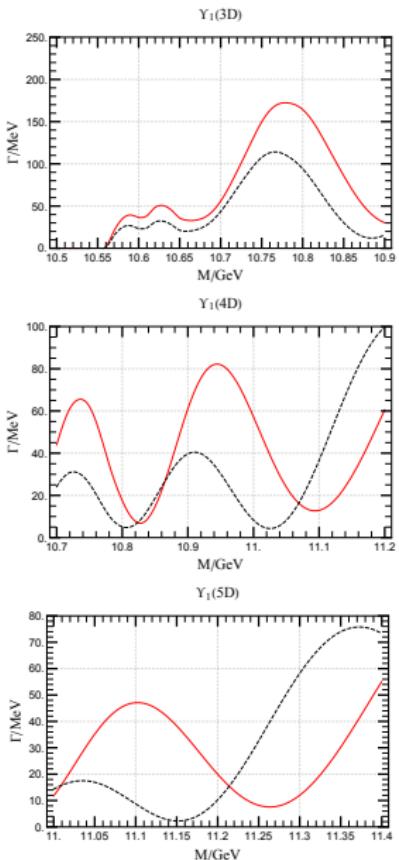
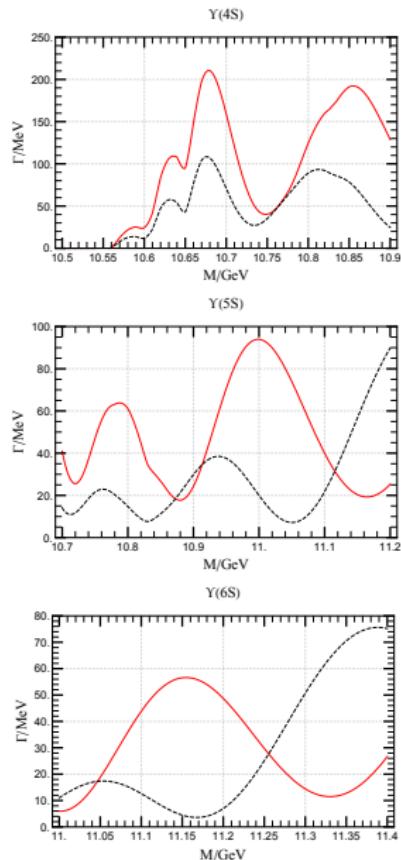
some mass shift



some mass shift



some mass shift



Gaussian expansion method (GEM)

- $\int d^3 p \frac{|\langle BC; p | H_I | \psi_0 \rangle|^2}{M - E_{BC}}$ plays the central role
- Usually, simple harmonic oscillator (SHO) wave function approximates ϕ

$$\phi(r) = \sum_{i=1}^n c_i (-1)^n (-i)^L \beta_i^{L+\frac{3}{2}} \sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}} L_n^{L+\frac{1}{2}}(\beta_i^2 r^2) e^{-\beta_i^2 r^2/2} r^L$$

- GEM is both simple and accurate (We use GEM in all of the calculations)

$$\phi(r) = \sum_{i=1}^n c_i \beta_i^{(L+3/2)} e^{-\beta_i^2 r^2/2} r^L$$

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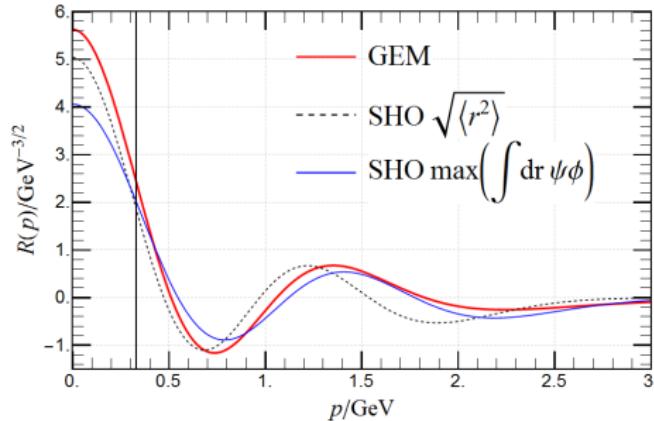
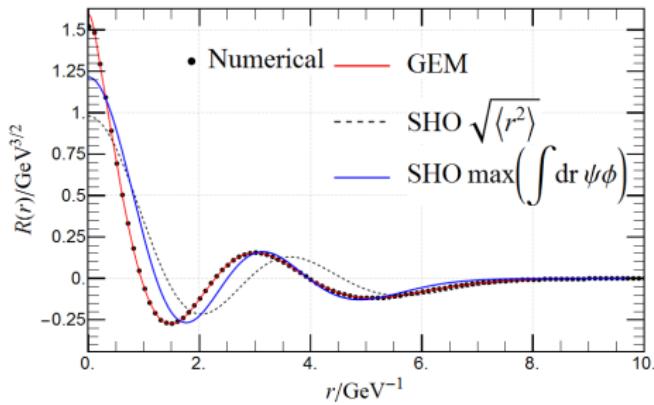
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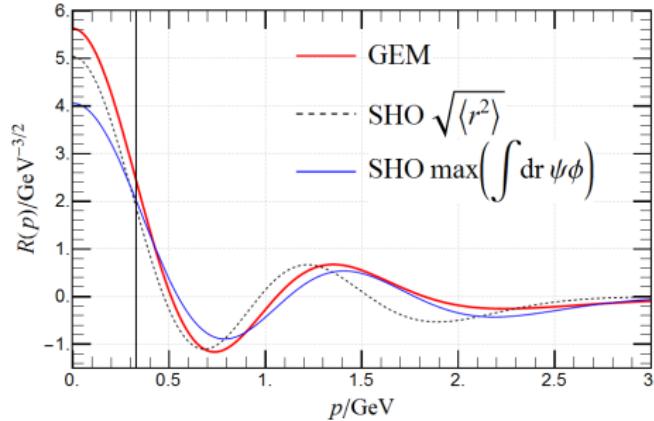
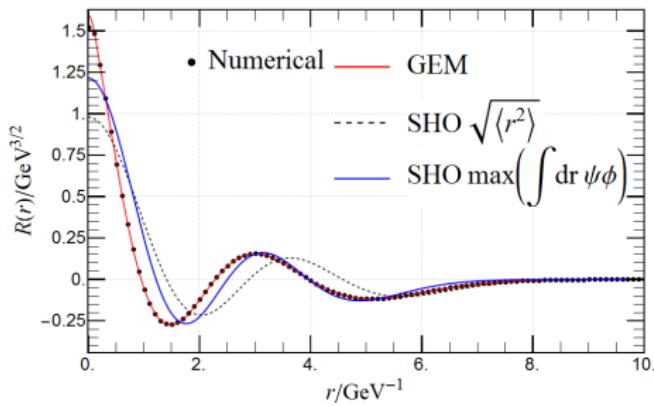
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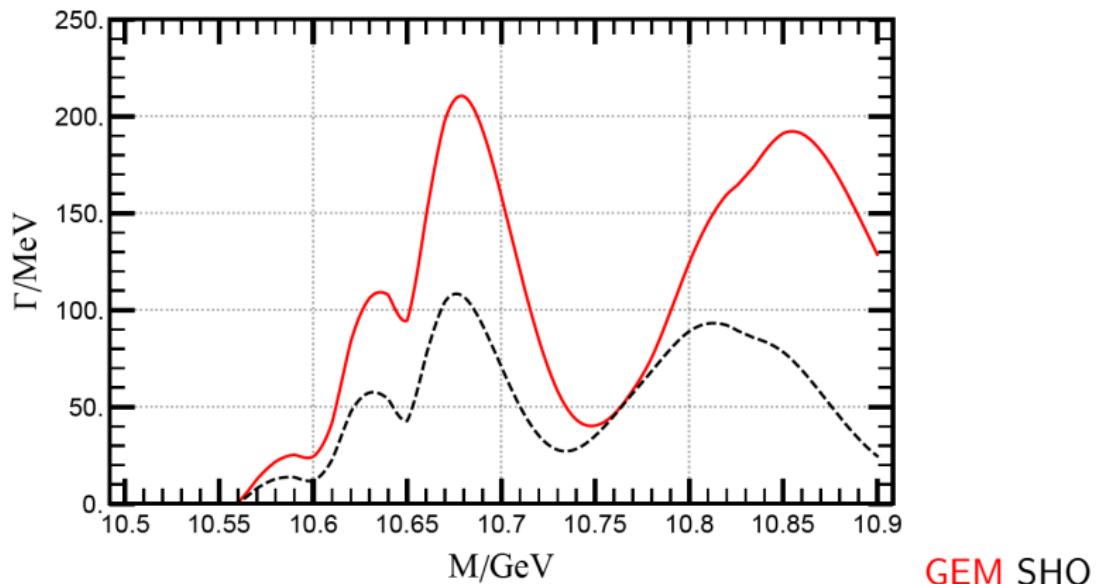
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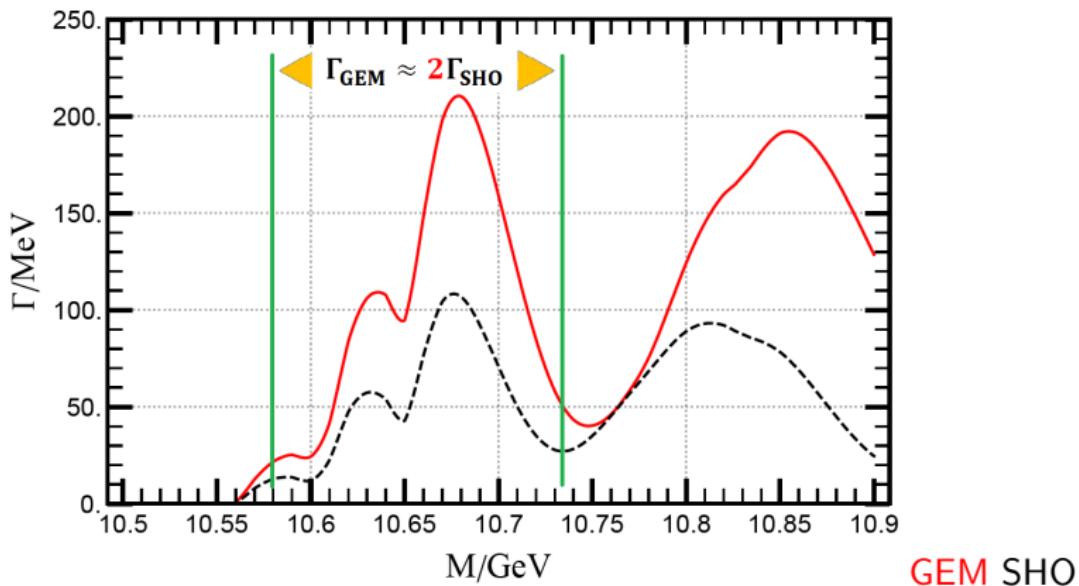
GEM VS SHO

SHO is bad for highly excited states
 $\Upsilon(4S)$



GEM SHO

SHO is bad for highly excited states
 $\Upsilon(4S)$



Parameters

$\alpha = 0.34$	$\lambda = 0.22 \text{ GeV}^2$	$c = 0.435 \text{ GeV}$
$m_b = 4.5 \text{ GeV}$	$m_u = m_d = 0.33 \text{ GeV}$	$m_s = 0.5 \text{ GeV}$
$\sigma = 3.838 \text{ GeV}$	$\gamma = 0.205$	

表: 3P_0 parameters. Color & flavor factors suppressed. Convention is the same as Ref.[T. Barnes(2007)]

Every Channel Shift

初态	$B\bar{B}$		$B\bar{B}^* + h.c.$		$B^*\bar{B}^*$		$B_s\bar{B}_s$		$B_s\bar{B}_s^* + h.c.$		$B_s^*\bar{B}_s^*$	
	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO
$\eta_b(1^1S_0)$	0	0	7.8	7.8	7.6	7.6	0	0	3.3	3.3	3.3	3.3
$\eta_b(2^1S_0)$	0	0	16.5	16.1	15.7	15.4	0	0	5.2	5.1	5.0	4.9
$\eta_b(3^1S_0)$	0	0	24.5	21.8	22.3	20.0	0	0	5.4	4.9	5.1	4.7
$\Upsilon(1^3S_1)$	1.4	1.4	5.4	5.4	9.2	9.2	0.6	0.6	2.3	2.3	3.9	3.9
$\Upsilon(2^3S_1)$	3.0	2.9	11.4	11.1	18.9	18.5	0.9	0.9	3.5	3.5	5.9	5.9
$\Upsilon(3^3S_1)$	4.8	4.2	17.2	15.2	27.1	24.3	1.0	0.9	3.7	3.4	6.1	5.6
$\Upsilon(4^3S_1)$	-0.7	3.7	-2.4	16.0	85.4	-0.6	1.0	1.0	3.6	3.3	5.7	5.2
$\Upsilon(5^3S_1)$	-0.5	2.8	2.8	6.8	17.8	10.1	0.8	0.7	1.7	2.7	3.1	4.0
$\Upsilon(6^3S_1)$	1.5	3.5	2.4	14.2	1.5	21.2	0.6	0.6	2.8	2.3	4.7	4.1
$\Upsilon_1(1^3D_1)$	4.0	4.3	3.7	4.0	27.8	29.2	1.0	1.1	1.0	1.1	8.7	9.3
$\Upsilon_1(2^3D_1)$	9.0	8.7	7.4	7.3	35.2	35.2	1.4	1.4	1.2	1.3	8.1	8.2
$\Upsilon_1(3^3D_1)$	7.5	2.3	6.1	7.4	57.8	35.4	2.3	2.0	1.5	1.4	7.4	7.2
$\Upsilon_1(4^3D_1)$	0.1	6.2	-1.6	3.6	6.8	22.5	1.2	1.0	1.3	1.1	7.0	6.4
$\Upsilon_1(5^3D_1)$	3.3	5.6	1.8	6.5	19.1	30.0	0.5	0.9	0.8	0.8	5.0	5.5
$h_b(1^1P_1)$	0	0	13.5	14.0	13.0	13.4	0	0	4.8	5.0	4.6	4.8
$h_b(2^1P_1)$	0	0	21.9	21.6	20.3	20.2	0	0	5.6	5.6	5.3	5.3
$h_b(3^1P_1)$	0	0	38.0	33.5	29.5	26.3	0	0	5.4	5.0	5.0	4.6
$\chi_{b0}(1^3P_0)$	4.1	4.3	0	0	21.4	22.2	1.3	1.4	0	0	7.8	8.1
$\chi_{b0}(2^3P_0)$	9.3	9.0	0	0	31.1	31.0	2.1	2.1	0	0	8.4	8.5
$\chi_{b0}(3^3P_0)$	25.5	22.4	0	0	40.7	36.9	2.3	2.0	0	0	7.6	7.2
$\chi_{b1}(1^3P_1)$	0	0	10.8	11.2	15.5	16.0	0	0	3.7	3.9	5.6	5.9
$\chi_{b1}(2^3P_1)$	0	0	19.7	19.4	22.1	22.0	0	0	4.8	4.8	6.0	6.0
$\chi_{b1}(3^3P_1)$	0	0	37.4	32.6	29.7	26.9	0	0	4.8	4.4	5.4	5.2
$\chi_{b2}(1^3P_2)$	3.4	3.5	9.8	10.1	13.6	14.2	1.2	1.3	3.5	3.7	4.7	5.0
$\chi_{b2}(2^3P_2)$	5.3	5.2	14.6	14.4	23.2	23.0	1.3	1.3	3.8	3.8	5.8	5.9
$\chi_{b2}(3^3P_2)$	12.3	11.2	23.3	20.7	36.2	32.0	1.3	1.2	3.6	3.3	5.6	5.2
$\Upsilon_2(1^3D_2)$	0	0	16.0	17.0	19.8	20.9	0	0	4.6	5.0	6.2	6.6

表: $-\Delta M$

	M_0	$-\Delta M$				M_{theory}				M_{exp}
		GEM	SHO	Liu's	Ferretti's	GEM	SHO	Liu's	Ferretti's	
$\eta_b(1^1S_0)$	9416.5	22.0	22.0	55.5	64	9394.5	9394.5	9391.8	9391	9398.
$\eta_b(2^1S_0)$	10024.2	42.4	41.5	66.2	101	9981.8	9982.6	10004.9	9980	9999.
$\eta_b(3^1S_0)$	10410.0	57.4	51.4	66.4	129	10352.7	10358.6	10337.9	10338	-
$\Upsilon(1^3S_1)$	9482.0	22.8	22.8	58.2	69	9459.2	9459.2	9460.3	9489	9460.3
$\Upsilon(2^3S_1)$	10054.9	43.8	42.8	68.0	108	10011.2	10012.1	10026.2	10022	10023.3
$\Upsilon(3^3S_1)$	10433.4	60.0	53.5	68.2	146	10373.4	10379.9	10351.9	10358	10355.2
$\Upsilon(4^3S_1)$	10746.7	92.6	28.7	76.3	-	10654.2	10718.0	10602.7	-	10579.4
$\Upsilon(5^3S_1)$	11024.3	25.7	27.2	84.2	-	10998.6	10997.1	10819.9	-	10876.
$\Upsilon(6^3S_1)$	11278.2	13.5	45.9	85.5	-	11264.8	11232.3	11022.6	-	11019.
$\Upsilon_1(1^3D_1)$	10181.9	46.1	49.1	96.8	159	10135.7	10132.8	10138.1	10112	-
$\Upsilon_1(2^3D_1)$	10515.9	62.3	62.0	88.4	-	10453.6	10453.9	10420.4	-	-
$\Upsilon_1(3^3D_1)$	10807.9	82.6	55.7	93.4	-	10725.2	10752.2	10650.9	-	-
$\Upsilon_1(4^3D_1)$	11072.8	14.8	40.8	-	-	11057.9	11031.9	-	-	-
$\Upsilon_1(5^3D_1)$	11318.2	30.4	49.3	-	-	11287.8	11268.9	-	-	-
$h_b(1^1P_1)$	9921.7	35.8	37.3	85.7	115	9885.9	9884.4	9915.5	9885	9899.3
$h_b(2^1P_1)$	10315.4	53.1	52.7	78.8	146	10262.3	10262.7	10259.1	10247	10259.8
$h_b(3^1P_1)$	10637.9	77.9	69.4	79.8	114	10560.1	10568.5	10523.2	10591	-
$x_{b0}(1^3P_0)$	9886.1	34.6	36.0	81.8	108	9851.4	9850.0	9875.3	9879	9859.44
$x_{b0}(2^3P_0)$	10284.2	50.9	50.6	75.0	137	10233.4	10233.6	10227.9	10226	10232.5
$x_{b0}(3^3P_0)$	10608.7	76.1	68.6	75.7	186	10532.6	10540.2	10495.9	10495	-
$x_{b1}(1^3P_1)$	9915.4	35.5	37.0	84.8	114	9879.9	9878.4	9906.8	9879	9892.78
$x_{b1}(2^3P_1)$	10310.0	52.6	52.3	77.9	144	10257.4	10257.7	10252.4	10244	10255.5
$x_{b1}(3^3P_1)$	10632.9	77.4	69.0	78.8	121	10555.6	10563.9	10517.3	10580	10512.1
$x_{b2}(1^3P_2)$	9934.9	36.4	37.8	87.3	117	9898.5	9897.1	9929.6	9900	9912.21
$x_{b2}(2^3P_2)$	10327.6	54.1	53.7	80.4	149	10273.5	10273.9	10270.1	10257	10268.7
$x_{b2}(3^3P_2)$	10649.8	82.2	73.6	82.1	138	10567.6	10576.2	10532.4	10578	-
$\Upsilon_2(1^3D_2)$	10187.8	46.6	49.5	97.7	161	10141.2	10138.3	10144.6	10121	10163.7

Decay

initial states	$B\bar{B}$		$B\bar{B}^* + h.c.$		$B^*\bar{B}^*$		$B_s\bar{B}_s$		$B_s\bar{B}_s^* + h.c.$		$B_s^*\bar{B}_s^*$		Γ_{theory}	Γ_{exp}	
	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	GEM	SHO	
4S	21.14	12.53	0	0	0	0	0	0	0	0	0	0	21.14	12.53	20.5 ± 2.5
3D	34.06	24.15	0	0	0	0	0	0	0	0	0	0	34.06	24.15	
5S	5.12	3.45	4.82	11.06	1.9	4.1	0.86	0.15	0.63	0.41	4.53	0.53	17.87	19.71	55 ± 28
4D	10.81	7.22	3.99	5.44	18.14	18.08	1.21	0.31	0.32	0.2	2.82	0.88	37.3	32.13	
6S	2.91	1.32	3.35	6.36	0.07	6.53	0.33	0.0	1.01	0.12	0.15	0.17	7.81	14.5	79 ± 16
5D	6.54	3.04	2.85	3.32	9.16	10.14	0.38	0.01	0.38	0.07	1.13	0.24	20.44	16.81	

$P_{b\bar{b}}$ & ΔM

States	δM_0	$\bar{P}_{b\bar{b}}$	$\bar{P}_{b\bar{b}} \times \delta M_0$	δM	$\bar{P}_{b\bar{b}}$	$\bar{P}_{b\bar{b}} \times \delta M_0$	δM	δM_{exp}
		GEM		SHO				
$\Upsilon(1S), \eta_b(1S)$	65.5	98.7	64.7	64.7	98.7	64.7	64.7	62.3
$\Upsilon(2S), \eta_b(2S)$	30.7	95.5	29.3	29.4	95.9	29.4	29.5	24.3
$\Upsilon(3S), \eta_b(3S)$	23.4	89.0	20.8	20.7	91.1	21.3	21.3	-
$\chi_{b0}(1P), h_b(1P)$	-35.6	97.2	-34.6	-34.5	97.1	-34.6	-34.4	-39.9
$\chi_{b1}(1P), h_b(1P)$	-6.3	97.0	-6.1	-6.0	97.0	-6.1	-6.0	-6.5
$\chi_{b2}(1P), h_b(1P)$	13.2	96.9	12.8	12.6	96.8	12.8	12.7	12.9
$\chi_{b0}(2P), h_b(2P)$	-31.2	93.0	-29.0	-28.9	93.4	-29.2	-29.1	-27.3
$\chi_{b1}(2P), h_b(2P)$	-5.4	92.5	-5.0	-4.9	93.0	-5.0	-5.0	-4.3
$\chi_{b2}(2P), h_b(2P)$	12.2	92.1	11.2	11.2	92.7	11.3	11.2	8.8
$\chi_{b0}(3P), h_b(3P)$	-29.2	56.9	-16.6	-27.5	54.3	-15.8	-28.3	-
$\chi_{b1}(3P), h_b(3P)$	-5.0	69.4	-3.5	-4.5	72.5	-3.6	-4.6	-
$\chi_{b2}(3P), h_b(3P)$	11.9	-	-	7.5	-	-	7.7	-

S-D Mixing

		2S	1D	3S	2D	4S	3D	5S	4D	6S	5D
	M_0	10.055	10.182	10.433	10.516	10.747	10.808	11.024	11.073	11.278	11.318
GEM	M_{pure}	10.011	10.136	10.373	10.454	10.654	10.725	10.999	11.058	11.265	11.288
	M_c	10.011	10.136	10.373	10.454	10.651 +0.047i	10.731 +0.032i	10.999 +0.047i	11.058 +0.01i	11.265 +0.012i	11.288 +0.005i
	M_r	10.011	10.136	10.373	10.454	10.653	10.734	10.999	11.058	11.265	11.288
	θ°	0.01	0.018	0.109	0.274	9.179	44.085	1.371	1.785	0.742	0.304
SHO	M_{pure}	10.012	10.133	10.38	10.454	10.718	10.752	10.997	11.032	11.232	11.269
	M_c	10.012	10.133	10.38	10.454	10.716 +0.021i	10.754 +0.055i	10.997 +0.011i	11.032 +0.002i	11.232 +0.009i	11.269 +0.021i
	M_r	10.012	10.133	10.38	10.454	10.717	10.754	10.997	11.032	11.232	11.269
	θ°	0.01	0.017	0.098	0.235	15.6	4.832	1.564	0.283	1.148	1.213

$X(4260)$'s parameters

α	λ	c	σ
0.55	0.175 GeV ²	-0.419 GeV	1.45 GeV
m_c	m_s	m_u	m_d
1.7	0.5	0.33	0.33
$\psi(1S)$	$\psi(2S)$	$\psi(3S)$	$\psi(4S)$
3.112	3.755	4.194	4.562
$\psi(1D)$	$\psi(2D)$	$\psi(3D)$	$\psi(4D)$
3.878	4.270	4.613	4.926

表: The parameters (taken from [B.-Q. Li(2009)]) of Cornell potential model and the predicted spectrum. The units of mass is GeV.

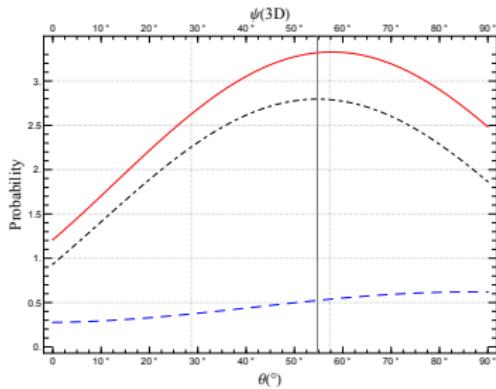
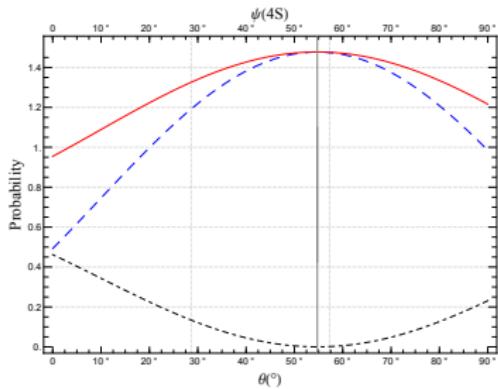
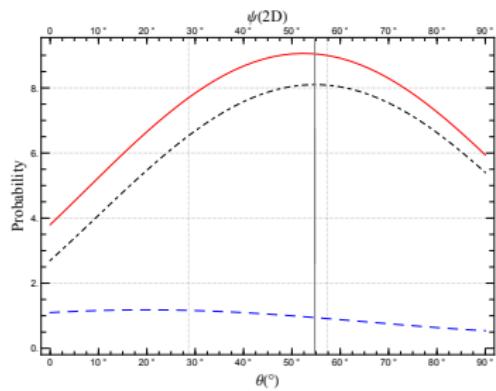
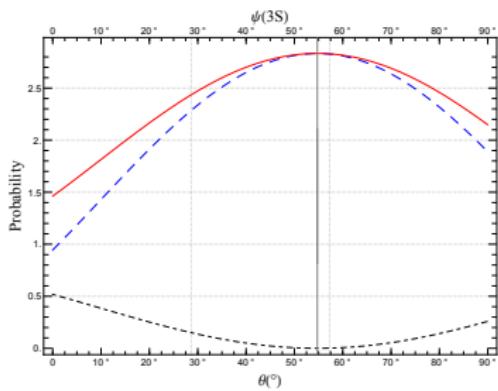
Summed coupling

	Coupled Channels	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$
Our paras	$D(1P) - D$	11.44	5.98	14.64	6.34
	$D_s(1P) - D_s$	1.36	0.57	1.14	0.5
	$D(1P) - D(1P)$	3.94	1.95	3.37	1.81
	$D_s(1P) - D_s(1P)$	0.87	0.46	0.76	0.43
	total coupling	17.61	8.97	19.91	9.09
SHO	$D(1P) - D$	7.59	2.86	9.7	3.03
	$D_s(1P) - D_s$	0.92	0.43	0.94	0.42
	$D(1P) - D(1P)$	3.02	1.79	3.27	1.88
	$D_s(1P) - D_s(1P)$	0.76	0.48	0.78	0.47
	total coupling	12.29	5.56	14.69	5.8

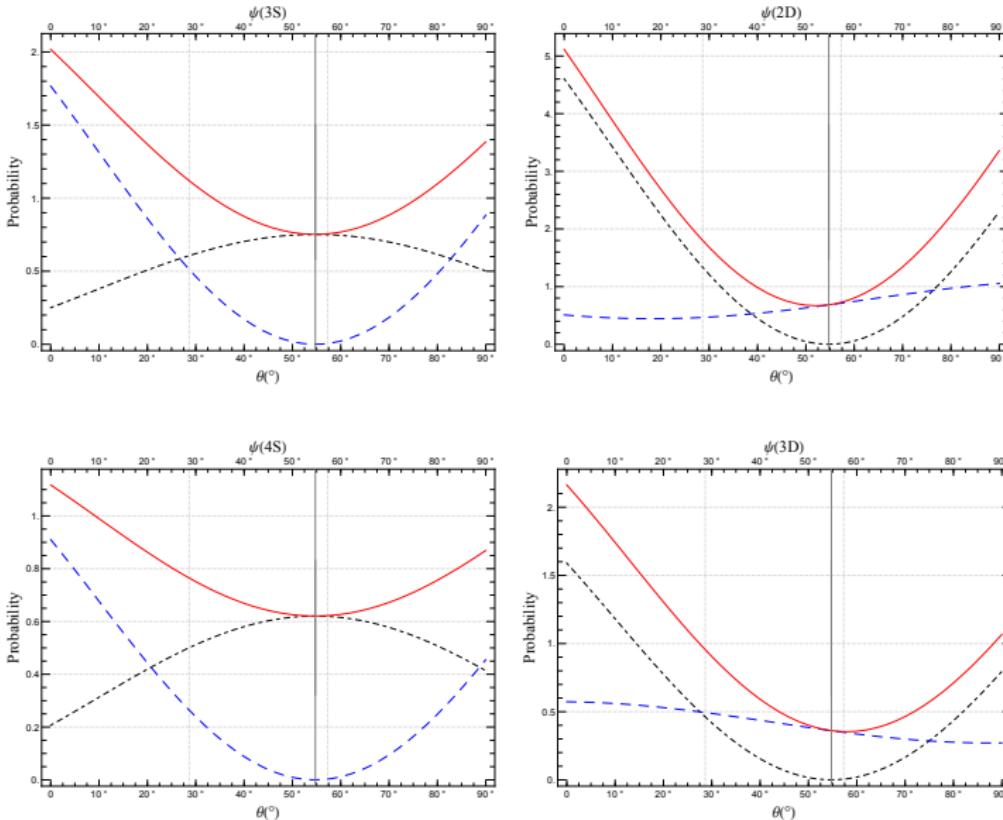
normalized ratio

Coupled Channels	Our Paras				T. Barnes, E. S. Swanson (2007)'s Paras			
	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$	$\psi(3S)$	$\psi(4S)$	$\psi(2D)$	$\psi(3D)$
$D - D_0^*$	0	0	0	0	0	0	0	0
$D^* - D_0^*$	0.215	0.238	0.028	0.07	0.809	1.073	0.073	0.167
$D - D_1(2430)$	0.265	0.42	0.076	0.109	1.367	1.635	0.133	0.336
$D^* - D_1(2430)$	0.325	0.317	0.075	0.118	1.117	1.651	0.148	0.378
$D - D_1(\mathbf{2420})$	1	1	1	1	1	1	1	1
$D^* - D_1(2420)$	0.616	0.564	0.149	0.18	0.816	0.922	0.185	0.269
$D - D_2^*$	0.629	0.59	0.065	0.093	0.7	0.716	0.069	0.1
$D^* - D_2^*$	0.992	0.914	0.225	0.339	1.384	1.632	0.267	0.537
$D_s - D_s^0$	0	0	0	0	0	0	0	0
$D_s^* - D_s^0$	0.043	0.041	0.004	0.01	0.11	0.168	0.01	0.024
$D_s - D_{s1}(2536)$	0.035	0.035	0.013	0.02	0.095	0.151	0.023	0.06
$D_s^* - D_{s1}(2536)$	0.054	0.056	0.014	0.022	0.16	0.27	0.024	0.066
$D_s^* - D_{s1}(2460)$	0.08	0.052	0.033	0.027	0.1	0.118	0.039	0.054
$D_s^* - D_{s1}(2460)$	0.089	0.066	0.02	0.021	0.132	0.178	0.028	0.05
$D_s - D_{s2}^*$	0.05	0.036	0.005	0.005	0.071	0.094	0.006	0.011
$D_s^* - D_{s2}^*$	0.129	0.102	0.037	0.047	0.209	0.306	0.052	0.119
$D_1(2430) - D_1(2430)$	0.155	0.122	0.059	0.07	0.273	0.513	0.102	0.226
$D_1(2430) - D_0^*$	0.072	0.045	0.006	0.005	0.097	0.175	0.011	0.024
$D_1(2430) - D_1^*(2420)$	0.128	0.112	0.023	0.035	0.225	0.434	0.034	0.105
$D_1(2430) - D_2^*$	0.206	0.266	0.044	0.083	0.566	1.077	0.094	0.277
$D_0^* - D_0^*$	0.027	0.018	0.016	0.017	0.034	0.062	0.027	0.064
$D_0^* - D_1(2420)$	0.052	0.042	0.01	0.012	0.08	0.159	0.017	0.043
$D_0^* - D_2^*$	0.172	0.12	0.045	0.055	0.273	0.522	0.046	0.19
$D_1(2420) - D_1(2420)$	0.16	0.154	0.049	0.069	0.331	0.614	0.097	0.211
$D_1(2420) - D_2^*$	0.264	0.21	0.067	0.081	0.48	0.922	0.1	0.265
$D_2^* - D_2^*$	0.157	0.23	0.053	0.119	0.5	0.918	0.104	0.324
$D_{s1}(2536) - D_{s1}(2536)$	0.027	0.026	0.011	0.014	0.062	0.124	0.019	0.045
$D_{s1}(2536) - D_{s0}^*$	0.013	0.009	0.001	0.001	0.02	0.037	0.001	0.003
$D_{s1}(2536) - D_{s1}^*(2460)$	0.028	0.027	0.005	0.009	0.059	0.122	0.009	0.03
$D_{s1}(2536) - D_{s2}^*$	0.051	0.063	0.011	0.021	0.147	0.306	0.023	0.075
$D_{s0}^* - D_{s0}^*$	0.007	0.005	0.004	0.005	0.01	0.018	0.007	0.017
$D_{s0}^* - D_{s1}(2460)$	0.014	0.013	0.003	0.004	0.027	0.056	0.005	0.013
$D_{s0}^* - D_{s2}^*$	0.034	0.029	0.01	0.015	0.07	0.14	0.016	0.051
$D_{s1}(2460) - D_{s1}(2460)$	0.038	0.038	0.01	0.014	0.087	0.168	0.018	0.041
$D_{s1}(2460) - D_{s2}^*$	0.053	0.05	0.014	0.02	0.121	0.246	0.024	0.067

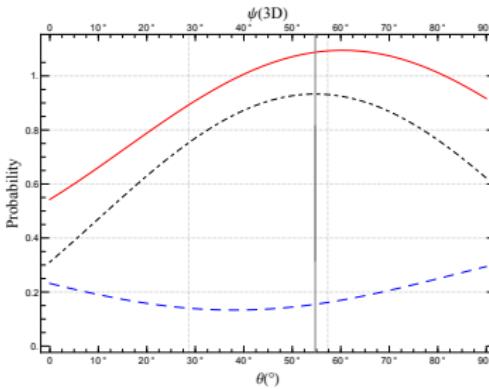
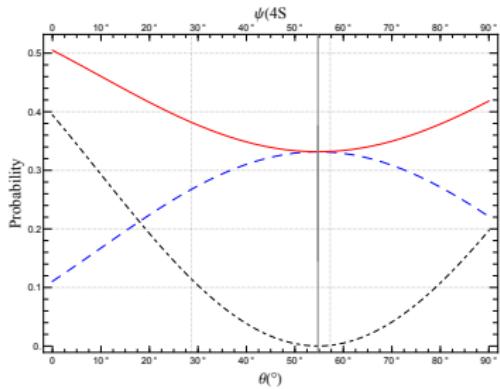
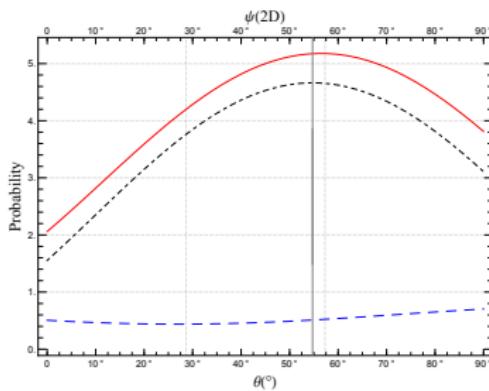
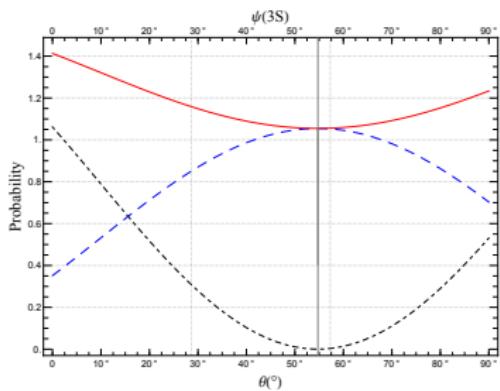
$D_1 D$ plot



$D_1(2430)D$ plot



$D_1 D$ plot



$D_1(2430)D$ plot

