

Status of project B.4: Boxed hadrons

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Res. 1: Quark mass dependence of the QCD spectrum

J. Ruiz de Elvira, U.-G. Meißner, AR and G. Schierholz, arXiv:1706:09015

- The ratios of matrix elements in the quark model follow certain pattern

Example: the strangeness content of the nucleon should be small

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} \ll 1$$

- In QCD, the quark-model relations emerge in the large- N_c limit (sea quark effects suppressed)
- A substantial deviation of the ratios of matrix elements from the quark model values signals the exotic nature of the states under consideration.

Feynman-Hellmann theorem

Matrix elements from the quark mass dependence:

$$\frac{dm_N^2}{dm_q} = \langle N | \bar{q}q | N \rangle, \quad q = u, d, s$$

Baryon octet, $m_u = m_d = \hat{m}$, $m_s \neq \hat{m}$, $y_B = \frac{dm_B^2}{dm_s}$, $x_B = \frac{dm_B^2}{d\hat{m}}$

$$\gamma_B = \frac{2y_\Sigma - y_N - y_\Xi}{2(y_N + y_\Xi) - y_\Sigma} \quad [0]$$

$$\beta_B = \frac{y_N - y_\Xi}{2(y_N + y_\Xi) - y_\Sigma} \quad \left[-\frac{2}{3} \right]$$

$$\gamma'_B = \frac{2x_\Sigma - x_N - x_\Xi}{2(x_N + x_\Xi) - x_\Sigma} \quad [0]$$

$$\beta'_B = \frac{x_N - x_\Xi}{2(x_N + x_\Xi) - x_\Sigma} \quad \left[\frac{1}{3} \right]$$

Pseudoscalar, vector octets

... two independent ratios:

$$\gamma_P = \frac{2(y_\pi - y_K)}{4y_K - y_\pi} \quad \left[-\frac{1}{2} \right]$$

$$\gamma'_P = \frac{2(x_\pi - x_K)}{4x_K - x_\pi} \quad [1]$$

- For the tetraquark octet, the corresponding coefficients are

$$\gamma_P = 1, \quad \gamma'_P = -\frac{1}{5}$$

- Other $SU(3)$ multiplets (e.g. the decuplet) can be studied in a similar fashion

Use lattice data to study the exotic content of the simulated hadronic states!

A test with the use of the data from QCDSF coll.

- The test is performed at the $SU(3)$ symmetric point $m_1 = 0$

$$\bar{m} = (2\hat{m} + m_s)/3, \quad m_1 = \hat{m} - m_s$$

- (Approximately) linear dependence of the masses on m_1

$$M_\pi^2 = M_P^2(\bar{m}) + 2\alpha_P m_1/3, \quad M_K^2 = M_P^2(\bar{m}) - \alpha_P m_1/3$$
$$\lambda_P = \frac{1}{1 + \alpha_Z} \frac{dM_P^2}{d\bar{m}}, \quad 1 + \alpha_Z = Z_m^S/Z_m^{NS}$$

- Quark-model values near the symmetric point

$$\underbrace{\frac{2\alpha_P}{\lambda_P} = \frac{2\alpha_V}{\lambda_V} = \mathbf{1}}_{\text{pseudoscalar, vector}}, \quad \underbrace{\frac{3A_1}{\lambda_B} = \mathbf{1}, \frac{3A_2}{\lambda_B} = \mathbf{0}}_{\text{baryon octet}}, \quad \underbrace{\frac{3A}{\lambda_D} = \mathbf{1}}_{\text{decuplet}}$$

- Tetraquarks: $\frac{2\alpha_T}{\lambda_T} = -\frac{1}{2}$

Numerical results

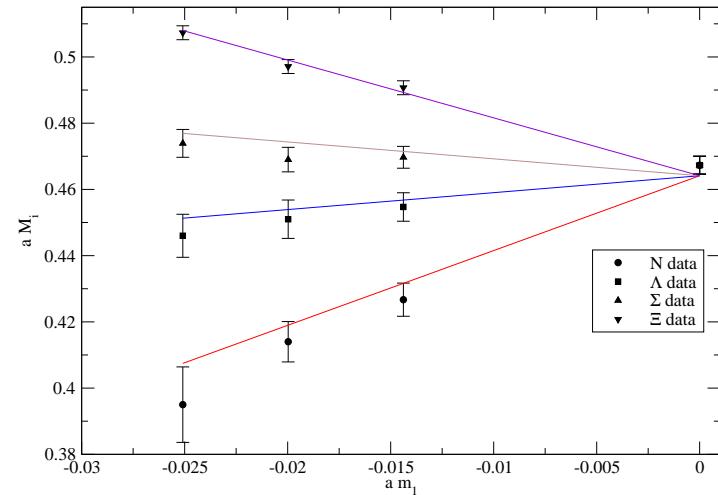
PS: $2\alpha_P/\lambda_P = 1$ [1]

V: $2\alpha_V/\lambda_V = 0.778 \pm 0.014$ [1]

N: $3A_1/\lambda_B = 0.635 \pm 0.080$ [1]

$3A_2/\lambda_B = 0.144 \pm 0.023$ [0]

Δ : $3A/\lambda_D = 0.590 \pm 0.201$ [1]



- The quark model values are reproduced up to $1/N_c \simeq 30\%$ accuracy
- Interesting to apply for the analysis of the **0⁺⁺ mesons!**

Unstable particles

- All candidates for exotica are resonances
- We generalize Feynman-Hellmann theorem for the resonances:

$$\frac{dM_a^2}{dm_q} = \langle p, a | \bar{q}q | p, a \rangle, \quad p^2 = M_a^2$$

- The position of the pole in the complex plane is given by M_a^2
- The notation $|p, a\rangle$ **does not correspond** to the eigenvector of the QCD Hamiltonian
- The resonance matrix element $\langle p, a | \bar{q}q | p, a \rangle$ is **defined** through the residue of the three-point function at the pole

Res. 2: Three particles in a finite volume

Motivation

- Roper resonance
- Three-particle decays: e.g., $\omega \rightarrow 3\pi$
- Nuclear physics on the lattice

Three-particle sector: continuum

- bound states
- elastic scattering, rearrangement reactions
- breakup...

Three-particle sector: finite volume

- energy levels below and above three-particle threshold

How does one extract information about the three-body dynamics
from the finite-volume spectrum?

The history

K. Polejaeva and AR, EPJA 48 (2012) 67

Finite volume energy levels determined solely by the S -matrix

M. Hansen and S. Sharpe, PRD 90 (2014) 116003; PRD 92 (2015) 114509

Quantization condition

R. Briceno and Z. Davoudi, PRD 87 (2013) 094507

Dimer formalism, quantization condition

P. Guo, PRD 95 (2017) 054508

Quantization condition in the 1+1-dimensional case

S. Kreuzer and H.-W. Hammer, PLB 694 (2011) 424; EPJA 43 (2010) 229; PLB 673

(2009) 260; S. Kreuzer and H. W. Grießhammer, EPJA 48 (2012) 93

Dimer formalism, numerical solution

- Complicated, not well suited for the analysis of the lattice data
- What is the convenient set of observables to be extracted from data?

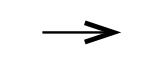
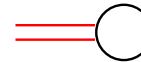
H.-W. Hammer, J-Y. Pang and AR, arXiv:1706.07700; arXiv:1707.02176

NREFT: dimer picture in the two-particle sector

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 - \frac{\nabla^2}{2m} \right) \psi + \mathcal{L}_2 + \mathcal{L}_3$$

$$\mathcal{L}_2 = \sigma T^\dagger T + \left(T^\dagger [f_0 \psi \psi + f_1 \psi \nabla^2 \psi + \dots] + \text{h.c.} \right)$$

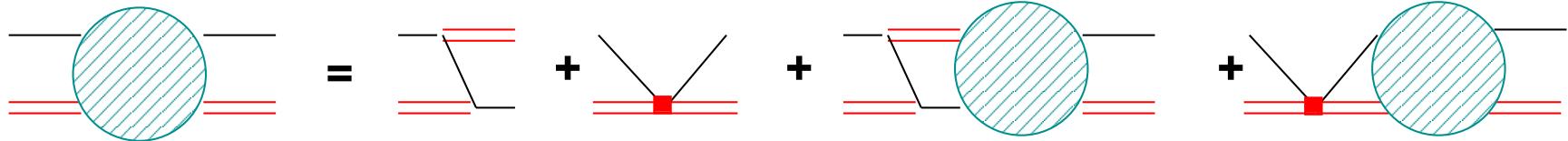
$$\begin{aligned} \mathcal{L}_3 &= h_0 T^\dagger T \psi^\dagger \psi + h_2 T^\dagger T (\psi^\dagger \nabla^2 \psi + \text{h.c.}) \\ &+ h_4 T^\dagger T (\psi^\dagger \nabla^4 \psi + \text{h.c.}) + h'_4 T^\dagger T \nabla^2 \psi^\dagger \nabla^2 \psi + \dots \end{aligned}$$

dimer:  +  + ... \rightarrow  +  + ...

f_0, f_1, \dots matched to $p \cot \delta(p) = -\frac{1}{a} + \frac{r}{2} p^2 + \dots$

- Higher partial waves can be included: dimers with arbitrary spin $T_{i_1 \dots i_{2k}}$
- Can be generalized to the non-rest frames

The scattering equation



$$\mathcal{M}(\mathbf{p}, \mathbf{q}; E) = Z(\mathbf{p}, \mathbf{q}; E) + \int_{\mathbf{k}}^{\Lambda} Z(\mathbf{p}, \mathbf{q}; E) \tau(\mathbf{k}; E) \mathcal{M}(\mathbf{k}, \mathbf{q}; E)$$

$$Z(\mathbf{p}, \mathbf{q}; E) = \frac{1}{\mathbf{p}^2 + \mathbf{q}^2 + \mathbf{p}\mathbf{q} - mE} + H_0 + H_2(\mathbf{p}^2 + \mathbf{q}^2) + \dots$$

H_0, H_2, \dots are related to the couplings h_0, h_2, \dots

$$\tau^{-1}(\mathbf{k}; E) = k^* \cot \delta(k^*) + \underbrace{\sqrt{\frac{3}{4} \mathbf{k}^2 - mE}}_{=k^*}$$

Finite volume

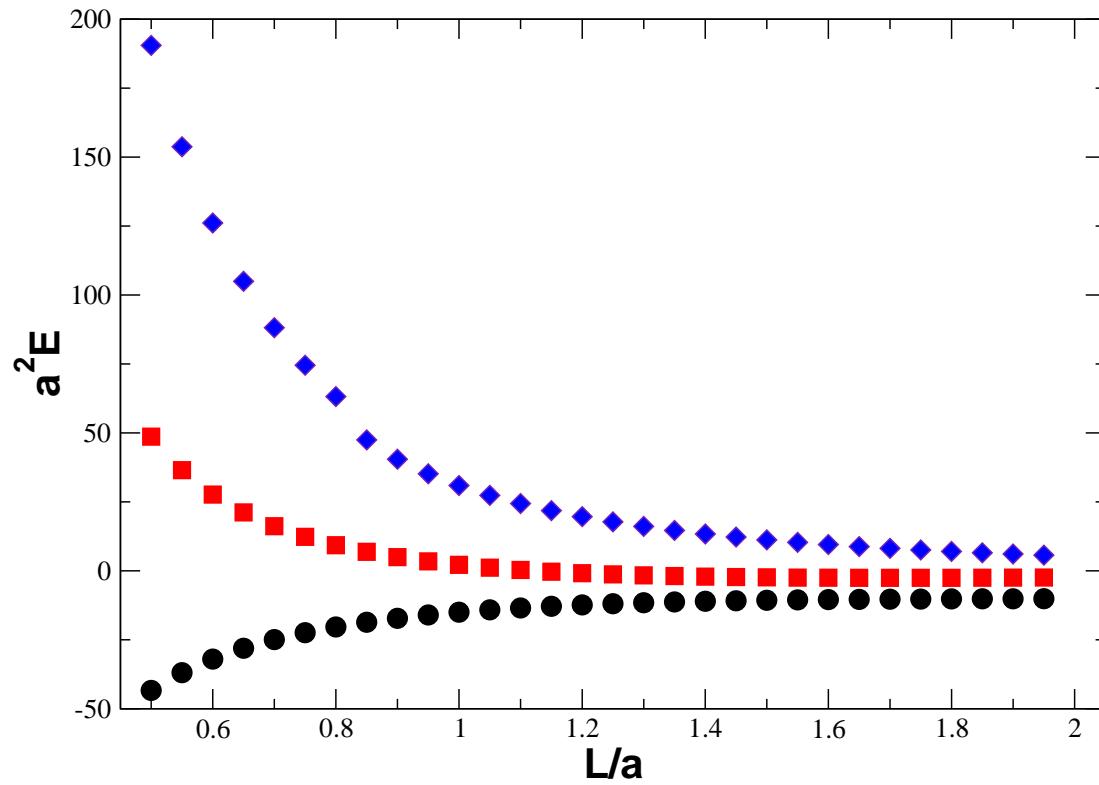
$$\mathbf{k} = \frac{2\pi}{L} \mathbf{n}, \quad \mathbf{n} \in \mathbb{Z}^3, \quad \int_{\mathbf{k}}^{\Lambda} \rightarrow \frac{1}{L^3} \sum_{\mathbf{k}}^{\Lambda}$$

$$\mathcal{M}_L(\mathbf{p}, \mathbf{q}; E) = Z(\mathbf{p}, \mathbf{q}; E) + \frac{8\pi}{L^3} \sum_{\mathbf{k}}^{\Lambda} Z(\mathbf{p}, \mathbf{q}; E) \tau_L(\mathbf{k}; E) \mathcal{M}_L(\mathbf{k}, \mathbf{q}; E)$$

$$\tau_L^{-1}(\mathbf{k}; E) = k^* \cot \delta(k^*) - \frac{4\pi}{L^3} \sum_{\mathbf{l}} \frac{1}{\mathbf{k}^2 + \mathbf{l}^2 + \mathbf{k}\mathbf{l} - mE}$$

- Poles of $\mathcal{M}_L \rightarrow$ finite-volume energy spectrum
- $k^* \cot \delta(k^*)$ fitted in the two-particle sector; H_0, H_2, \dots should be fitted to the three-particle energies
- S -matrix in the infinite volume \rightarrow equation with H_0, H_2, \dots
- The finite volume spectrum is determined alone by the S -matrix!

The finite-volume spectrum



- The spectrum both below and above the three-particle threshold is given

Milestones

2016/2 Analysis of the lattice data on the quark mass dependence ✓

Framework for calculation of compton scattering amplitude on the lattice ✓

2017 Extraction of the phase shift and inelasticities on the lattice (optical potential) ✓
Three-body quantization condition ✓

Publications

- H.-W. Hammer, J.-Y. Pang and A. Rusetsky, arXiv:1707.02176
- J. Ruiz de Elvira, U.-G. Meißner, A. Rusetsky and G. Schierholz, arxiv:1706.09015
- H.-W. Hammer, J.-Y. Pang and A. Rusetsky, arXiv:1706.07700
- A. Agadjanov, U.-G. Meißner and A. Rusetsky, Phys. Rev. D95 (2017) 031502
- Z.-H. Guo, L. Liu, U.-G. Meißner, J. Oller and A. Rusetsky, Phys. Rev. D95 (2017) 054004

Summary, outlook

- Using three-body quantization condition for the analysis of lattice data → A.2
- Three-particle Lellouch-Lüscher formula
- Inclusion of relativistic effects, higher partial waves, spin, partial wave mixing, etc
- Three-nucleon interactions: inclusion of the long-range forces
- Three-particle bound state: unequal masses, higher partial waves (see Yu Meng's talk!)

In progress in coll. with colleagues from Darmstadt and GWU

→ B.4 is on a good track, more results following soon!