CRC 110 A3.project
Meson-meson scattering in 2D QCD

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In collaboration with
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Literatures

1. In large N limit, tetraquark state is not ruled out.
   S.Weinberg PRL(2013)

2. Large-N expansion and the ‘t Hooft model.
   ‘t Hooft.Gerard Nucl.Phys. B(1974)

3. The form factor for a meson decay to an quark and an antiquark, and also the exact expression for the decay amplitude of a meson to two mesons in the framework of 1/N-expansion.
   Callan,Coote,Gross PRD(1976)

4. A meson-level effective Hamiltonian, including the cubic and quartic interaction terms.
   J.C.F Barbon, K Demeterfi Nucl.Phys. B(1995)

5. Numerical calculations for the mesons decay with both massive and massless quark occasion.
   E.Abdalla,R.Mohayee arxiv:hep-th/9610059(1997)

The 't Hooft model (1+1D QCD, large-N limit)

The Lagrangian

$$\mathcal{L} = -\frac{1}{4} tr\left(G_{\mu\nu} G^{\mu\nu}\right) + \sum_a \bar{q}^a (i\gamma \cdot D - m^a) q^a$$

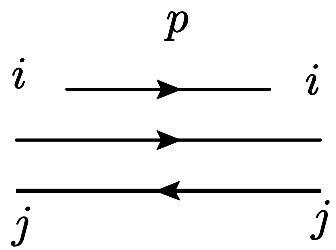
With a SU(N) gauge symmetry, where $g^2 N = const, N \to \infty$ (Large-N limit).

Light cone gauge

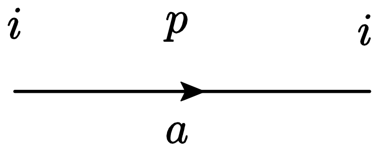
$$A^+ = 0$$

$$\mathcal{L} = \frac{1}{2} tr\left((\partial_- A^-)^2\right) + \sum_a \bar{q}^a (i\gamma^\mu \partial_\mu - g\gamma^+ A_+ - m^a) q^a$$

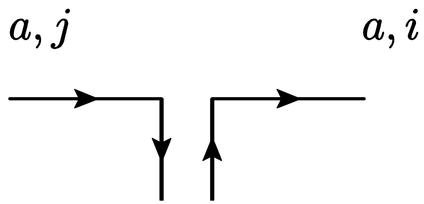
The Feynman Rules



$\frac{i}{p_-^2}$



$\frac{i}{\not{p} - m_a + i\epsilon}$



$-ig\gamma^+$

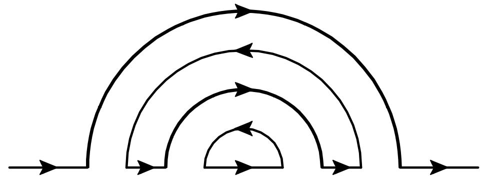
An infrared cutoff in p_- integration

$$\int_{-\infty}^{\infty} \frac{p_+}{2\pi} \left(\int_{-\infty}^{-\lambda} + \int_{\lambda}^{\infty} \frac{dp_-}{2\pi} \right)$$

Phys.Rev.D 13.1649 by C G.Callan, Jr.Nigel Coote and D J.Gross

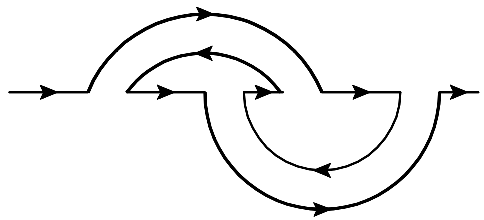
The Dyson-Shwinger equation for dressed quark propagator

Closed color loop provide a factor of N

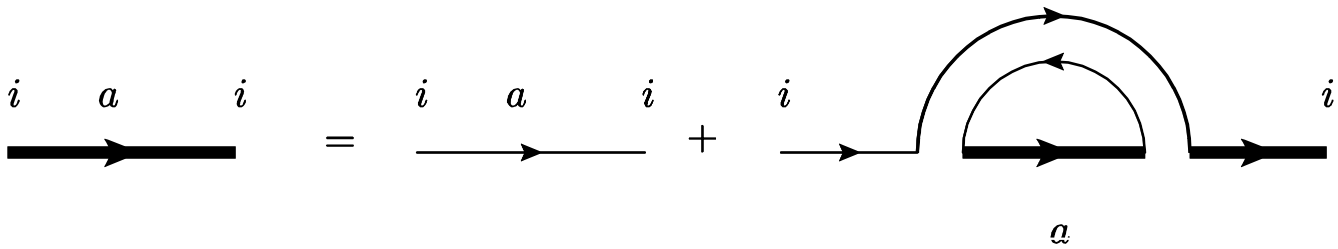


Planar

Higher oder of 1/N



The leading order diagram for the DS equation

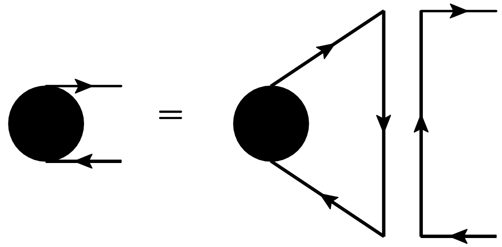


The coefficient of the γ^+ of the dressed quark propagator

$$\widetilde{s_a}(p) = \frac{1}{2p^- - \dfrac{m_a^2 - \dfrac{g^2N}{\pi}}{p^+} - \dfrac{g^2N}{\pi}\dfrac{Sgn(p^+)}{\lambda} + i\epsilon\, Sgn(p^+)}$$

Callan et al. PRD (1976)

The leading order diagram of Bethe-Salpeter Equation



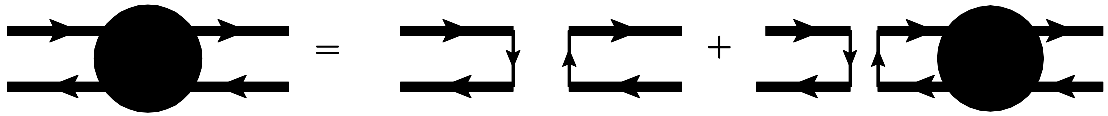
The ’t Hooft Equation

$$\mu_k^2 \varphi_k(x) = (\frac{\gamma_1}{x} + \frac{\gamma_2}{1-x})\varphi_k(x) + \int_0^1 dy \frac{\varphi_k(x) - \varphi_k(y)}{(x-y)^2}.$$

$$\gamma_a = \frac{\pi m_a^2}{g^2 N}$$

‘t Hooft.Gerard, Nucl.Phys. B75,461-470 (1974)

The leading order diagram of quark-antiquark scattering amplitude



Quantum theory of field. Volume 1,10.2 polology
S.Weinberg

$$G \to \frac{-2i\sqrt{\mathbf{q}^2+m^2}}{q^2+m^2-i\epsilon}(2\pi)^7\delta^4(q_1+\cdots+q_n)$$
$$\times\sum_\sigma M_{0;\mathbf{q},\sigma}(q_2\cdots q_r)M_{\mathbf{q},\sigma|0}(q_{r+2}\cdots q_n)$$

The quark-antiquark

$$T(x',x;r)=\frac{ig^2}{r_-^2(x'-x)^2}-\sum_k\frac{i}{(r^2-r_k^2)}\left\{\phi_k^*(x')\frac{2g}{\lambda}\left(\frac{g^2N}{\pi}\right)^{1/2}\left[\theta(x'(1-x'))+\frac{\lambda}{2|r_-|}\left(\frac{\gamma_a-1}{x'}+\frac{\gamma_b-1}{1-x'}-\mu_k^2\right)\right]\right\}$$
$$\times\left\{\phi_k(x)\frac{2g}{\lambda}\left(\frac{g^2N}{\pi}\right)^{1/2}\left[\theta(x(1-x))+\frac{\lambda}{2|r_-|}\left(\frac{\gamma_a-1}{x}+\frac{\gamma_b-1}{1-x}-\mu_k^2\right)\right]\right\}$$

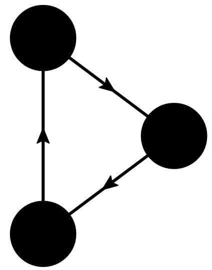
Callan et al. PRD13,1649(1976)

The form factor

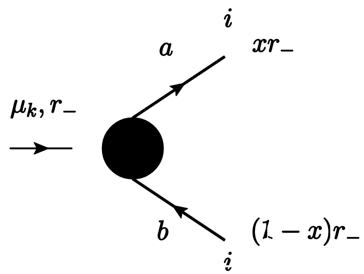
$$\Phi_k^{1,2}(x)=\varphi_k(x)\frac{g^2}{|r_-|}\sqrt{\frac{N_c}{\pi}}\left[\theta(x(1-x))\frac{2|r_-|}{\lambda}+\frac{\gamma_1-1}{x}+\frac{\gamma_2-1}{1-x}-\mu_k^2\right].$$

Decay amplitude

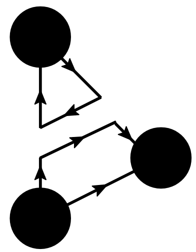
The leading order diagram of the 1 meson → 2 mesons Decay



A meson with momentum r_{-} decay to a pair of quark propagators are described by the form factor $\Phi_k^{a,b}(x)$

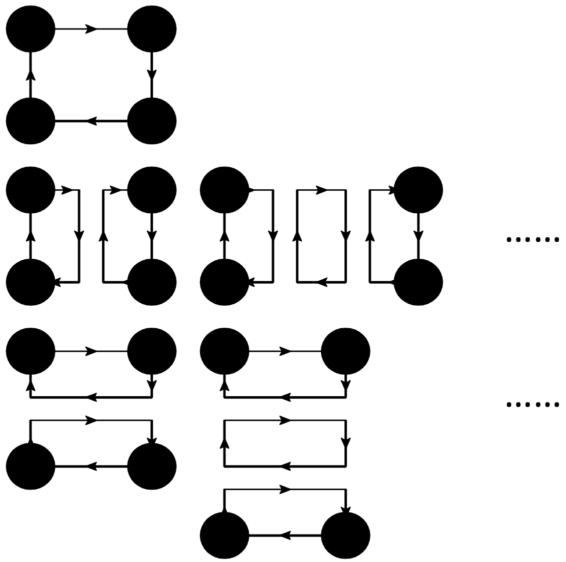


This kind of diagram is counted by the form factor

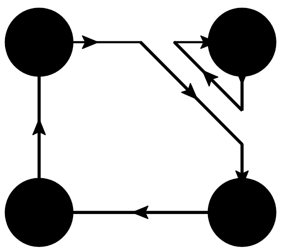


Analytically solved in Callan et al. PRD13,1649(1976) and J.C.F Barbon et al.,Nucl.Phys. B434,109 (1995)
Some numerical results is shown in J.C.F Barbon et al.,Nucl.Phys. B434,109 (1995) and E.Abdalla,R.Mohayee,arxiv:hep-th/9610059(1997)

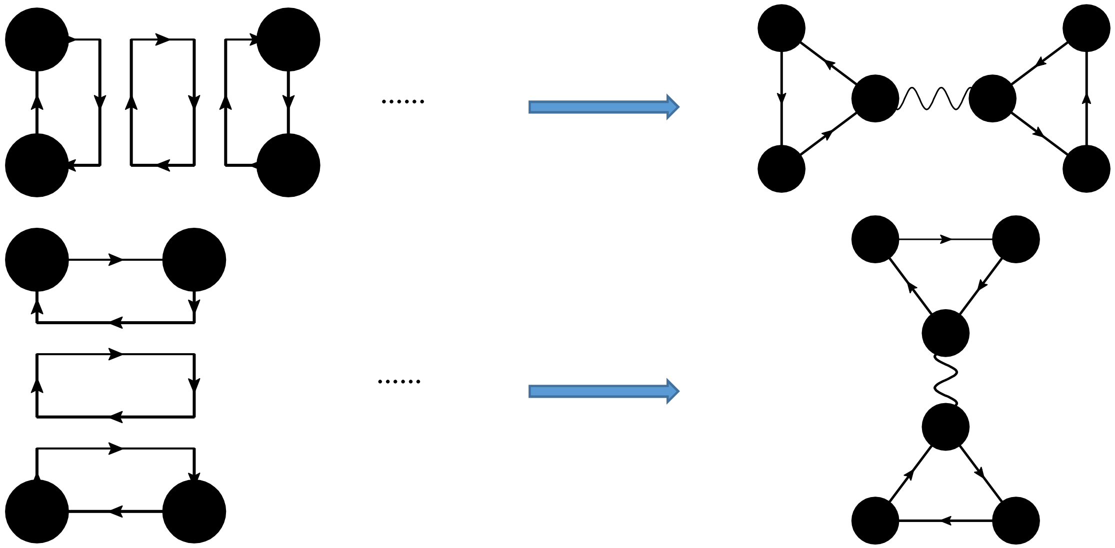
The leading order 2 mesons → 2 mesons Scattering in the 1/N-expression



The leading order diargram of the scattering

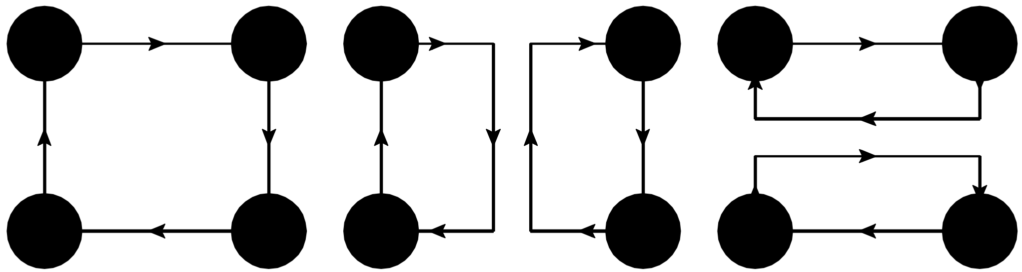


Counted by the form factor

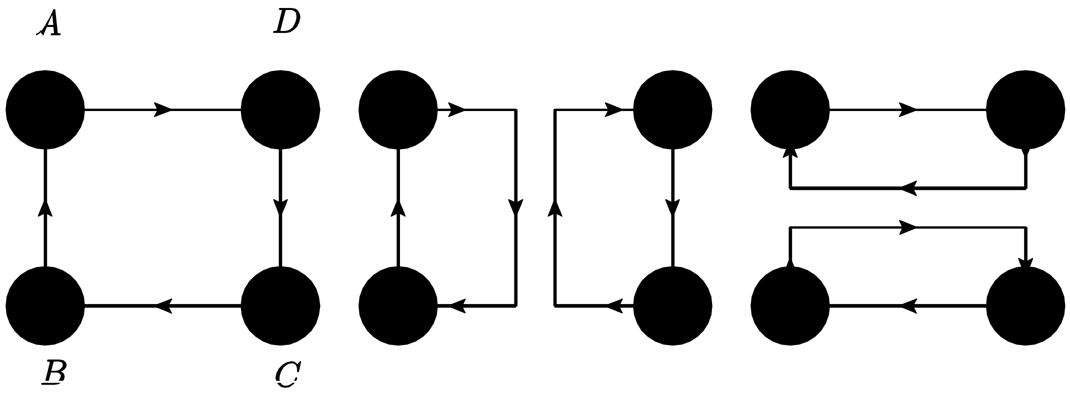


We didn’t calculate the one meson exchange diagram

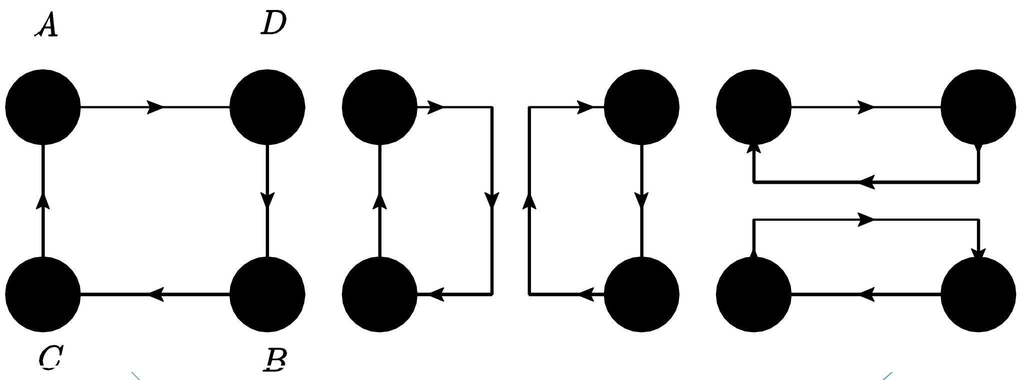
We only calculated the contact diagram



For a particular scattering: A+B→C+D



No $\frac{1}{\lambda}$ in each diagram



The $\frac{1}{\lambda}$ of the 1^{st} diagram is canceled by the other 2 diagrams

The result are $\frac{1}{\lambda}$ independent

Some analytic results

1.Two-flavor:

$$A(q^a\overline{q}^b)+B(q^b\overline{q}^a)\rightarrow C(q^a\overline{q}^b)+D(q^b\overline{q}^a)$$

$$\omega_1=\frac{r_{B-}}{r_{C-}}$$

$$\omega_2=\frac{r_{D-}}{r_{C-}}$$

$$i\mathcal{M}=(1+\mathcal{P})(1+\mathcal{C})i\mathcal{M}_0.$$

$$i\mathcal{M}_0 \;=\; \theta(\omega_2-\omega_1)i4g^2\omega_1\int_0^1 dx\int_0^1 dy\frac{1}{(y\omega_1-\omega_2-x)^2}\varphi_A(\frac{\omega_2-\omega_1+x}{\omega_2-\omega_1+1})\varphi_B(y)\varphi_C(x)\varphi_D(\frac{y\omega_1}{\omega_2}),$$

2.Single-flavor:

$$A(q^a\overline{q}^a)+B(q^a\overline{q}^a)\rightarrow C(q^a\overline{q}^a)+D(q^a\overline{q}^a)$$

$$i\mathcal{M}=(1+\mathcal{R})(1+\mathcal{P})(1+\mathcal{C})i\mathcal{M}_0+(1+\mathcal{R})i\mathcal{M}_1,$$

$$\begin{aligned}
i\mathcal{M}_1 \;=\; & \Big\{-\theta(1-\omega_1)i4g^2\int_0^1 dxP\int_0^1 dy\frac{\omega_1\omega_2}{[(y-1)\omega_1+(1-x)\omega_2]^2}\varphi_A(\frac{x\omega_2}{1+\omega_2-\omega_1})\varphi_B(y)\varphi_C(y\omega_1)\varphi_D(x)\\
&+(B\leftrightarrow C,A\leftrightarrow D,\omega_1\rightarrow 1/\omega_1,\omega_2\rightarrow\frac{1+\omega_2-\omega_1}{\omega_1})\Big\}+\\
&\Big\{-\theta(\omega_2-\omega_1)i4g^2\int_0^1 dxP\int_0^1 dy\frac{\omega_1}{(y\omega_1-x)^2}\varphi_A(\frac{x+\omega_2-\omega_1}{1+\omega_2-\omega_1})\varphi_B(y)\varphi_C(x)\varphi_D(\frac{(y-1)\omega_1+\omega_2}{\omega_2})\\
&+(A\leftrightarrow C,B\leftrightarrow D,\omega_1\rightarrow\frac{\omega_2}{1+\omega_2-\omega_1},\omega_2\rightarrow\frac{\omega_1}{1+\omega_2-\omega_1})\Big\}+\\
&\Big\{-\theta(\omega_2-\omega_1)\theta(\omega_1-1)i\frac{4\pi}{N_c}\int_0^1 dx\left[2r_{C+}r_{C-}+2r_{D+}r_{C-}+\frac{M_a^2}{x-\omega_1}+\frac{M_a^2}{x-1}-\frac{M_a^2}{x-\omega_1+\omega_2}-\frac{M_a^2}{x}\right]\\
&\times\varphi_A(\frac{x-\omega_1+\omega_2}{1+\omega_2-\omega_1})\varphi_B(x/\omega_1)\varphi_C(x)\varphi_D(\frac{x-\omega_1+\omega_2}{\omega_2})+(A\leftrightarrow D,B\leftrightarrow C,\omega_1\rightarrow 1/\omega_1,\omega_2\rightarrow\frac{1+\omega_2-\omega_1}{\omega_1})\\
&+(A\leftrightarrow B,C\leftrightarrow D,\omega_1\rightarrow\frac{1+\omega_2-\omega_1}{\omega_2},\omega_2\rightarrow 1/\omega_2)+(A\leftrightarrow C,B\leftrightarrow D,\omega_1\rightarrow\frac{\omega_2}{1+\omega_2-\omega_1},\omega_2\rightarrow\frac{\omega_1}{1+\omega_2-\omega_1})\Big\},
\end{aligned}$$

C: $(A\leftrightarrow C,B\leftrightarrow D,\omega_1\rightarrow\frac{\omega_2}{1+\omega_2-\omega_1},\omega_2\rightarrow\frac{1}{1+\omega_2-\omega_1})$

P: $(A\leftrightarrow B,C\leftrightarrow D,\omega_1\rightarrow\frac{1+\omega_2-\omega_1}{\omega_2},\omega_2\rightarrow\frac{1}{\omega_2})$

R: $(C\leftrightarrow D,\omega_1\rightarrow\frac{\omega_1}{\omega_2},\omega_2\rightarrow\frac{1}{\omega_2})$

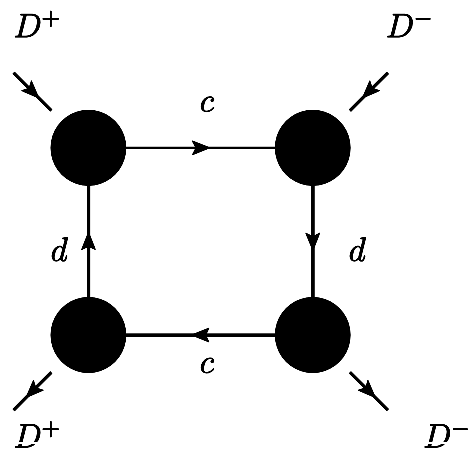
Example for scattering process with nonvanishing amplitude

A 2-flavor example

$$D^+(c\bar{d}) + D^-(d\bar{c}) \to D^+(c\bar{d}) + D^-(d\bar{c})$$

1.The line between two incoming(outgoing) mesons should connect an quark and an antiquark of a same flavor.

2. The line Between one incoming and one outgoing mesons should connect two quark(antiquark) of a same flavor.

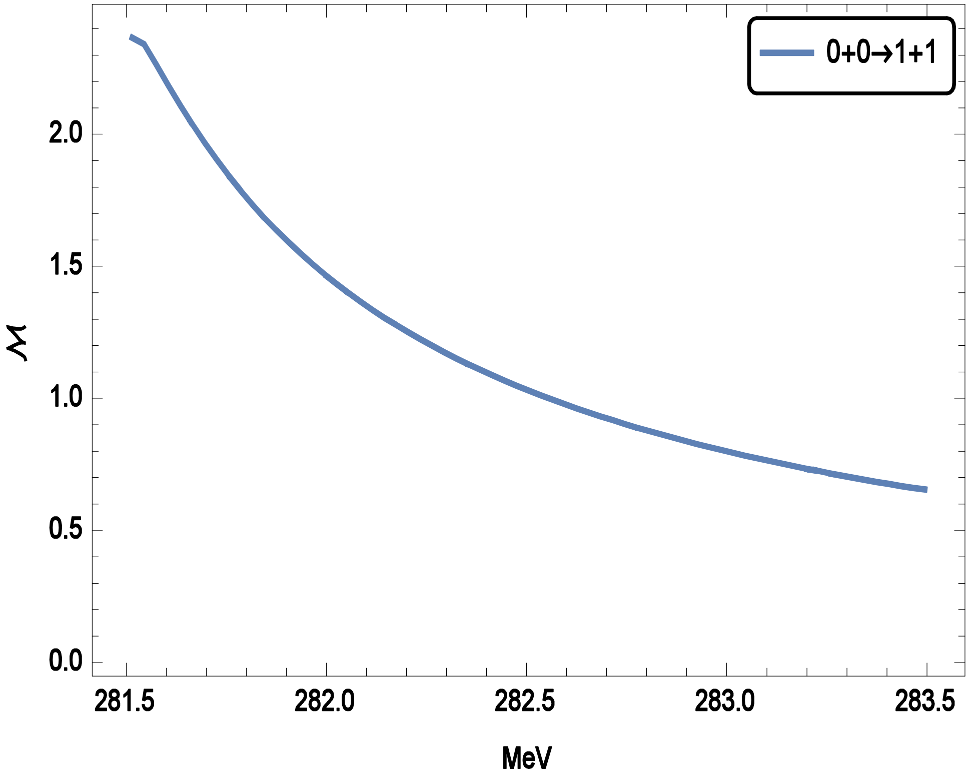


Numerical description

(only 1 free parameter in the center-of-mass frame)

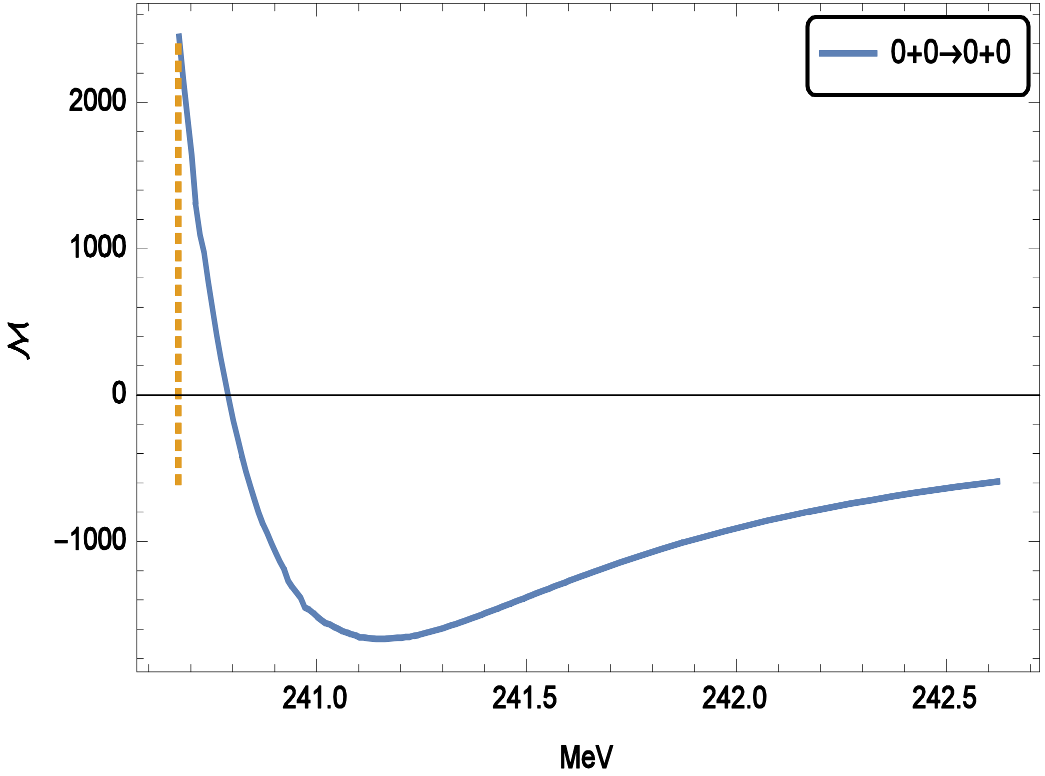
1.Two-flavor: $A(q^a\bar{q}^b)+B(q^b\bar{q}^a) \to C(q^a\bar{q}^b)+D(q^b\bar{q}^a)$

Amp: Threshold: 281.514 MeV
Quark mass: 60 80 Mev
Meson mass: 140.322 140.322 140.757 140.757 Mev

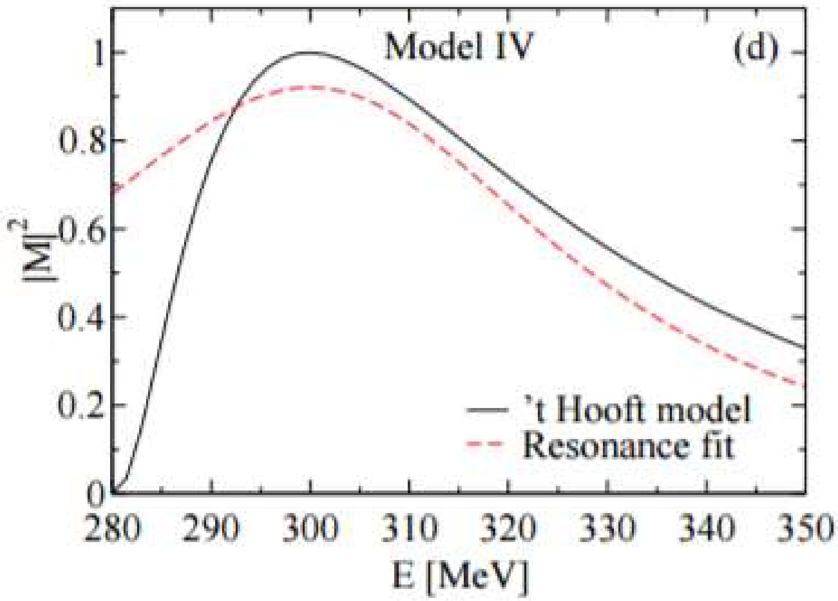


2.Single-flavor: $A(q^a\bar{q}^a)+B(q^a\bar{q}^a) \to C(q^a\bar{q}^a)+D(q^a\bar{q}^a)$

Amp: Threshold: 240.671 MeV
Quark mass: 60 Mev
Meson mass: 120.335 120.335 120.335 120.335 Mev



Z Batiz et al. PRC(2004), incomplete



g	2500
m_{01}	6.0
m_{02}	5.0

$$|\mathcal{M}|^2 = \tilde{g}^4 \frac{1}{(s - m_r^2)^2 + m_i^4}$$

$$|\mathcal{M}|^2 \approx \bar{g}^4 \frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4}$$

Summary and outlook

1. Calculate the leading order of the meson-meson scattering amplitude analyticly via the Feynman Diagram and the form fector, which is showed $\frac{1}{\lambda}$ independent.

2. Classify the possible 2-2 processes of different flavors with nonvanishing amplitude.

3. A numerical description of the result. So far we don't see whether there would be tetraquark states or not.

4. We are going to calculate the full amplitude of the meson-meson scattering.

Thank you