

Lattice study of multi-channel charmed meson scattering

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Outline

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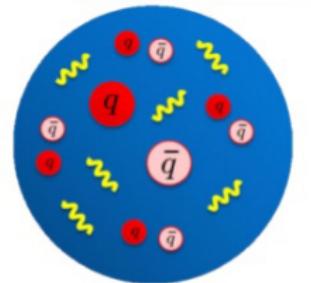
1 Motivation

2 Multi-channel scattering study on lattice

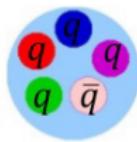
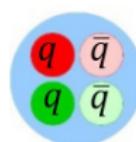
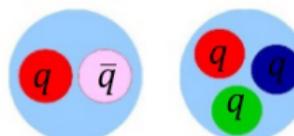
3 Summary and outlook

Motivation

- Quantum Chromodynamics (QCD) is the fundamental quantum field theory of strong interactions.



- Conventional hadrons:
mesons, baryons
- Exotic hadrons:
tetraquarks, pentaquarks



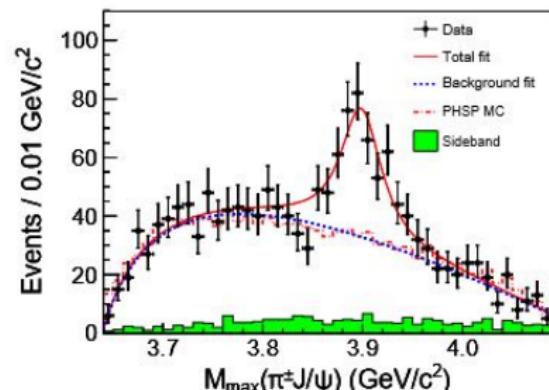
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Charged charmonium-like hadrons Z_c^\pm with unconventional quark content $\bar{c}c\bar{d}u$

$Z_c^\pm(3900)$

A charged resonance-like structure $Z_c^\pm(3900)$ was observed at **BES III**^[1], and was confirmed shortly by **Belle**^[2] and **CLEO**^[3] collaborations in 2013.

- $e^+e^- \rightarrow \pi^+\pi^- J/\psi$
- mass:
 $3899.0 \pm 3.6 \pm 4.9 \text{ Mev}/c^2$
- width:
 $46 \pm 10 \pm 20 \text{ Mev}$
- $I^G(J^{PC}) = 1^+(1^{+-})^{[4]}$
The same state as $[Z_c^\pm(3885)]$



Is $Z_c^\pm(3900)$ a molecular bound state formed by the D and \bar{D}^* mesons ?

[1] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 110, 252001 (2013).

[2] Z.Q. Liu et al. (The Belle Collaboration), Phys. Rev. Lett. 110, 252002 (2013).

[3] T. Xiao, S. Dobbs, A. Tomaradze, Kamal K. Seth, Phys. Lett. B(727, 366-370 (2013)).

[4] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 119, 072001 (2017).

Lattice study

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- $Z_c^\pm(3900)$ is close to the $(D\bar{D}^*)^\pm$ threshold.
- Interaction between the charmed mesons is non-perturbative in nature which requires a genuine non-perturbative framework such as [lattice QCD](#).
- Low energy $D\bar{D}^*$ study near threshold on the lattice in the single-channel $I^G(J^{PC}) = 1^+(1^{+-})$.
CLQCD, Y. Chen et al., Phys.Rev.D89,094506 (2014).
- Spectrum study using many interpolating operators and find no additional eigenstate related to $Z_c^\pm(3900)$.
Sasa Prelovsek, C.B. Lang, Luka Leskovec, Daniel Mohler, Phys, Rev, D.91.014504(2014).

Extract the scattering parameters

- Only threshold scattering parameters, i.e. a_0 and r_0 are relevant.
- Lüscher formula gives a direct relation of k^2 and the elastic scattering phase shift, where k^2 is defined by $\Delta E = \sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2} - m_1 - m_2$.
- For the s-wave:

$$qcot\delta_0(k) = \frac{1}{\pi^{3/2}} \mathcal{Z}_{00}(1; q^2), \quad q^2 = k^2(L/2\pi)^2$$

- When s-wave and p-wave mix:

$$[qcot\delta_0(k) - m_{00}(q^2)][q^3 cot\delta_1 k - m_{11}(q^2)] = m_{01}^2 q^2$$

where m_{00} , m_{11} and m_{01} are known functions of q^2 .

- Effective range expansion:

$$k^{2l+1} cot\delta_l(k) = a_l^{-1} + \frac{1}{2} r_l k^2 + \dots$$

Lattice setup

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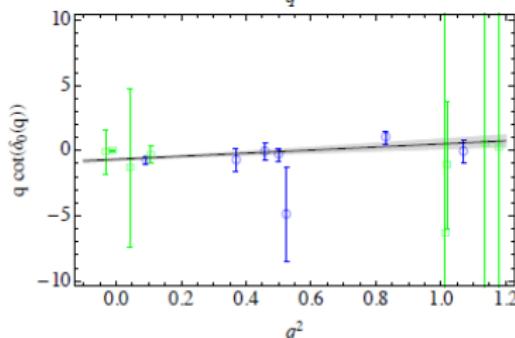
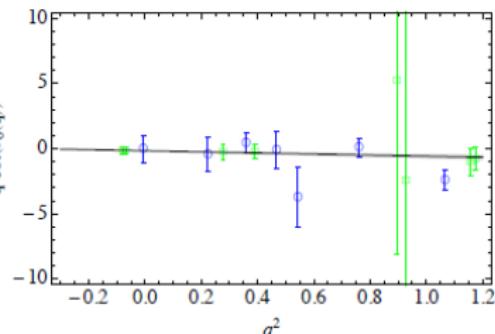
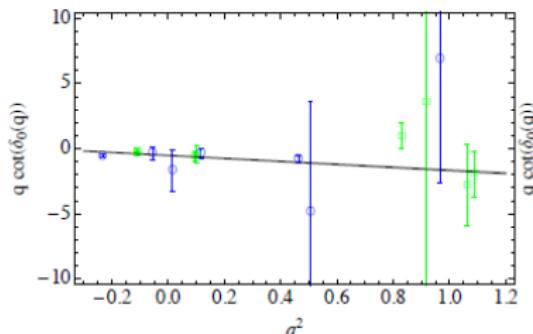
$N_f = 2$ twisted mass gauge field configurations generated by ETMC collaboration.

μ	β	$a[\text{fm}]$	$V/(a^4)$	$m_\pi [\text{Mev}]$	$m_\pi L$	N_{conf}
0.003	4.05	0.067	$32^3 \times 64$	300	3.3	200
0.006	4.05	0.067	$32^3 \times 64$	420	4.6	200
0.008	4.05	0.067	$32^3 \times 64$	485	5.3	200

Results of $Z_c^\pm(3900)$ and $Z_c^\pm(4025)$ for single channel both utilize the same ensembles.

$Z_c^\pm(3900)$

[CLQCD, Y. Chen et al., Phys.Rev.D89,094506 (2014).]

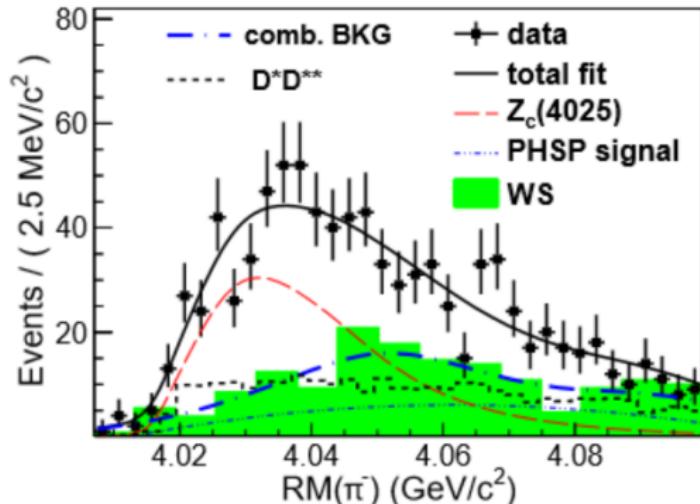


	$a_0(\text{fm})$
$\mu = 0.003$	-0.67(1)
$\mu = 0.006$	-2.1(1)
$\mu = 0.008$	-0.51(7)

Conclusion: No evidence for a $D\bar{D}^*$ bound state.

$Z_c^\pm(4025)$

- $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$
- mass:
 $4026.3 \pm 2.6 \pm 3.7 \text{ Mev}/c^2$
- width:
 $24.8 \pm 5.6 \pm 7.7 \text{ Mev}$
- $I^G(J^{PC}) = 1^+(1^{+-})$



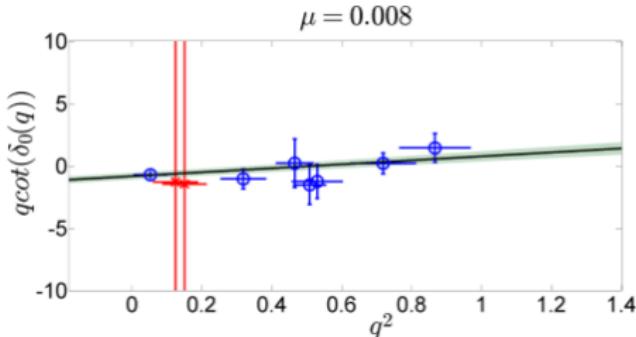
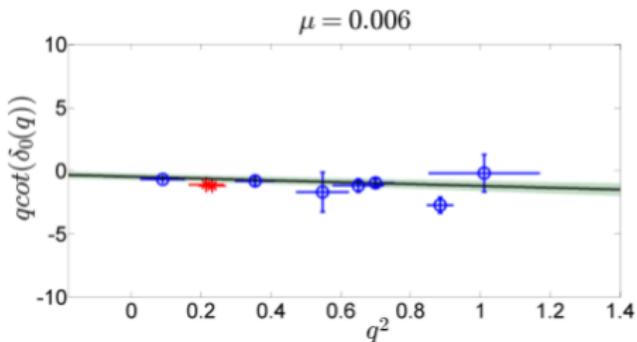
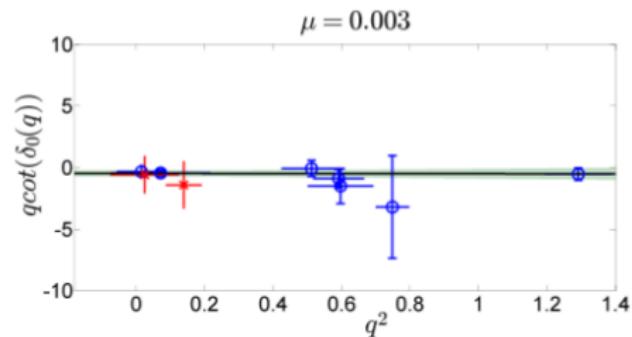
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M.Ablikim et al.(BESIII Collaboration), Phys.Rev.Lett 112, 132001(2014).

S.Prelovsek et al. PRD91, 014504(2015).

$Z_c^\pm(4025)$

[CLQCD, Y. Chen et al., Phys.Rev.D92,054507 (2015).]



μ	B_0
0.003	-0.47(35)
0.006	-0.46(23)
0.008	-0.83(17)

$$B_l = [L/(2\pi)]^{2l+1} a_l^{-1}$$

No evidence for a $D^* \bar{D}^*$ bound state.

- **Questions:**

Will a bound state appear at physical pion mass?

Will the results change if the coupled channel effects are taken into account?

How significant are the effects of finite volume and finite lattice spacing?

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2 Multi-channel scattering study on lattice

3 Summary and outlook

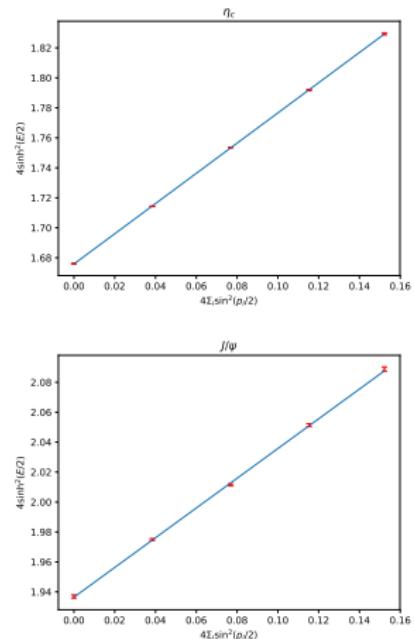
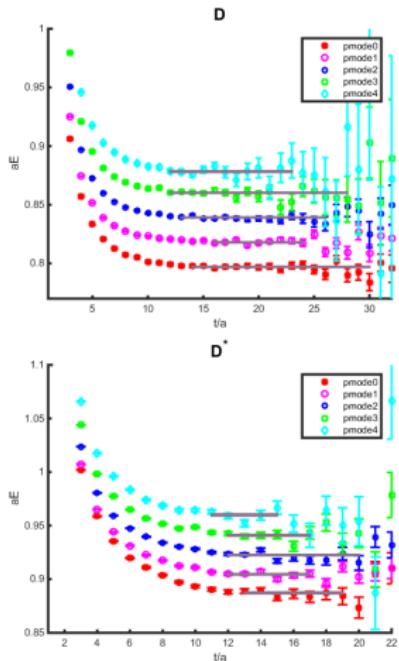
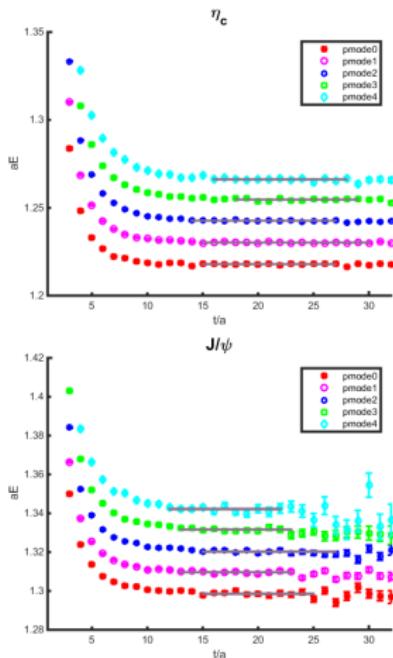
Multi-channel scattering study on lattice

- $N_f = 2 + 1 + 1(u, d; c, s)$ dynamical quark flavors configurations with various lattice spacing, volume, and pion mass, generated by ETMC Collaboration^[a,b].
 - ▶ Lattice spacing: 0.06 fm \sim 0.09 fm (0.0863fm)
 - ▶ Volume: 2 fm \sim 3 fm (2.72fm)
 - ▶ Pion mass: Physical \sim 400 MeV (320Mev)
- Stochastic LapH smearing \longrightarrow all-to-all propagators.
- Enlarged operator basis.
- Coupled channels with $I = 1, J^{PC} = 1^{+-}$: $J/\Psi\pi, D\bar{D}^*, D^*\bar{D}^*, \eta_c\rho$

[a] ETM, R. Baron et al., JHEP 06, 111 (2010), arXiv:1004.5284.

[b] ETM, R. Baron et al., Comput.Phys.Commun. 182, 299 (2011), arXiv:1005.2042.

Single-particle energies and dispersion relation



Two-particle operators and correlators

- Osterwalder-Seiler action:

$$S_{OS}^{\pi/2}[q_f, \bar{q}_f, U] = a^4 \sum_x \bar{q}_f(x) [\gamma \cdot \tilde{\nabla} - i\gamma_5 W_{cr}(r_f) + m_f] q_f(x)$$

- Interpolating operators

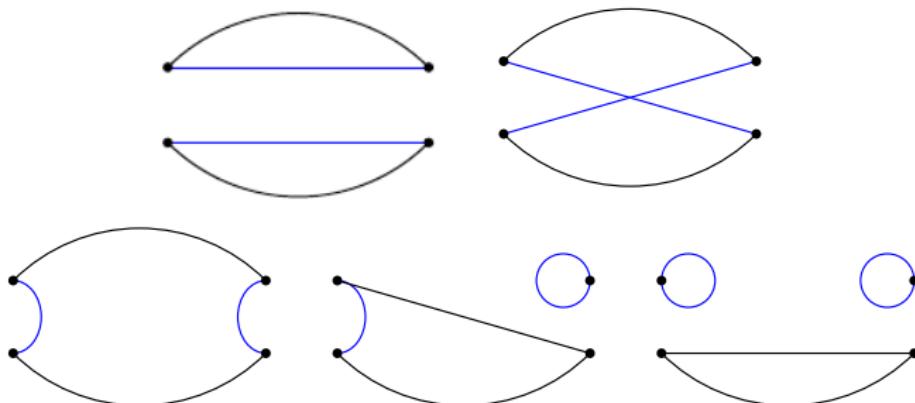
$$\mathcal{O}^{DD^*} = \bar{d}\gamma_i c(\vec{k}) \bar{c}\gamma_5 u(-\vec{k}) + \bar{c}\gamma_i u(\vec{k}) \bar{d}\gamma_5 c(-\vec{k})$$

$$\mathcal{O}^{D^* D^*} = \epsilon_{ijk} \bar{d}\gamma_j c(\vec{k}) \bar{c}\gamma_k u(-\vec{k})$$

$$\mathcal{O}^{J/\Psi\pi} = \bar{c}\gamma_i c(\vec{k}) \bar{d}\gamma_5 u(-\vec{k})$$

$$\mathcal{O}^{\eta_c\rho} = \bar{c}\gamma_5 c(\vec{k}) \bar{d}\gamma_i u(-\vec{k}) \quad (k^2 = 0, 1, 2, 3)$$

- Wick contractions



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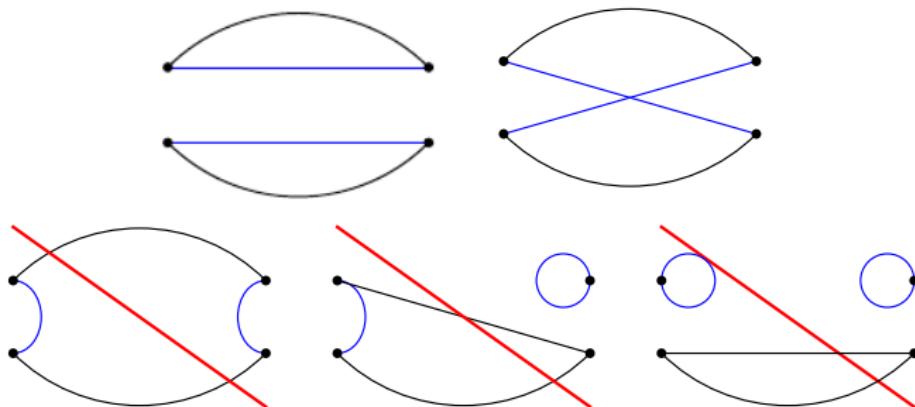
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- Wick contractions



Two-particle operators and correlators

- Construct the matrix of correlators

$$C_{ij}(t) = \langle \Omega | \mathcal{O}_i(t + t_{src}) \mathcal{O}_j^\dagger(t_{src}) | \Omega \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}.$$

where overlap factors: $Z_j^{(n)}(t) = e^{E_n t/2} \frac{|C_{jk}(t)v_k^n t|}{|C(t)^{\frac{1}{2}} v^{(n)} t|}$

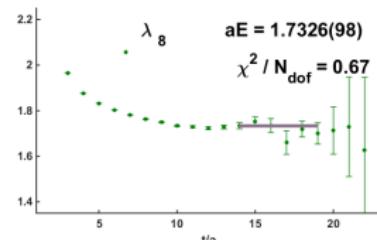
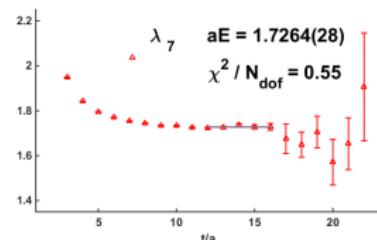
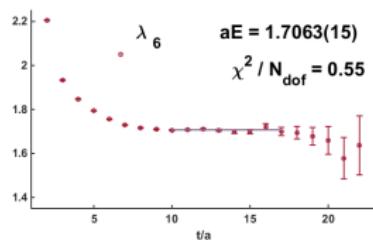
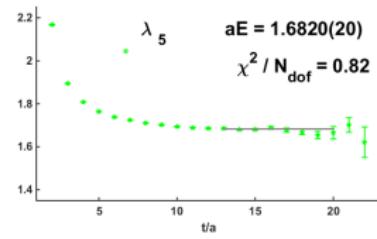
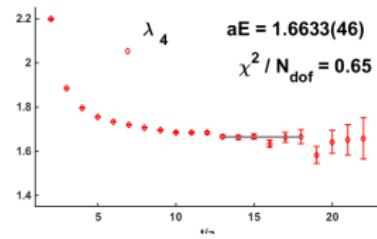
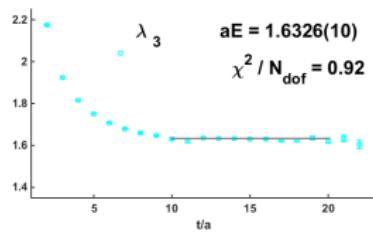
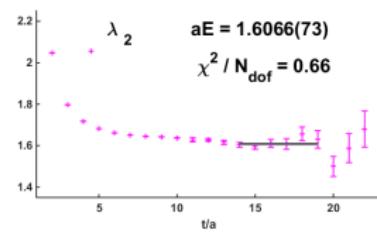
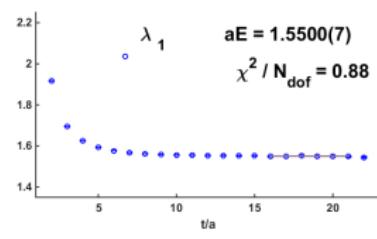
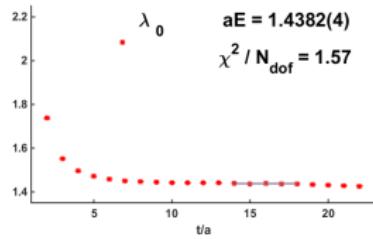
- Solve the generalized eigenvalue problem (GEVP)

$$C_{ij}(t)v_j^n = \lambda_n(t)C_{ij}(t_0)v_j^n$$

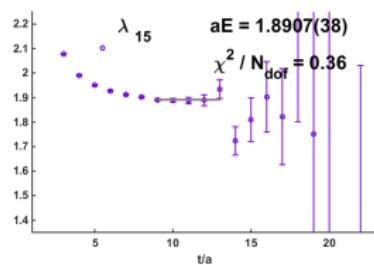
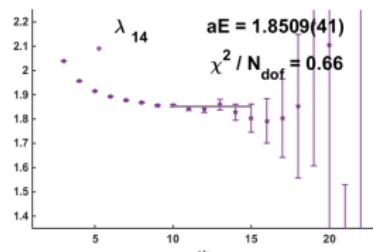
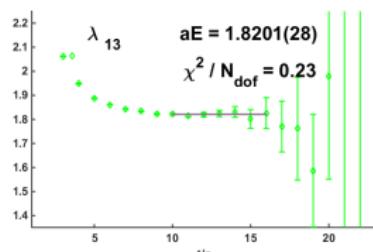
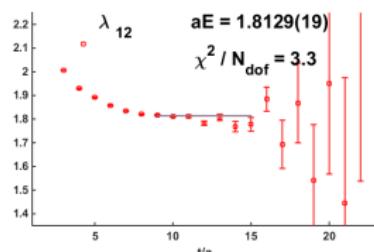
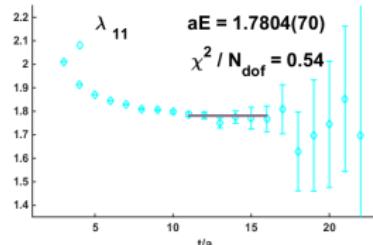
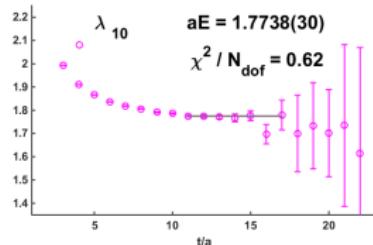
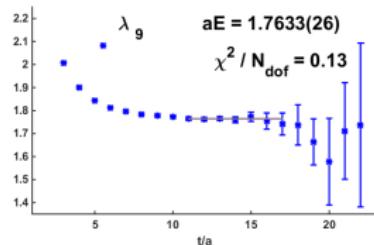
- Eigenvalues:

$$\lambda_n(t) \rightarrow e^{-E_n t} (1 + O(e^{-\Delta E t}))$$

Extract the energies



Extract the energies



Problems

- In multi-channel case,

Multi-channel Lüscher formula instead of single-channel Lüscher formula.

The S-matrix elements require more parameters, all are functions of energy.

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Multi-channel Lüscher formula instead of single-channel Lüscher formula.

The S-matrix elements require more parameters, all are functions of energy.

- Contractions take too much time.

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1 Motivation

2 Multi-channel scattering study on lattice

3 Summary and outlook

- **Summary:**

- ▶ Make a first step of studying coupled $J/\Psi\pi$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\eta_c\rho$ scattering on lattice;
- ▶ Extract the energies of single particles and the interacting two-meson states in the center-of-mass frame.

- **Outlook:**

- ▶ Extrapolate the scattering parameters from the data using multi-channel Lüscher formalism and the effective range expansion;
- ▶ Consider several more moving frames and more ensembles;
- ▶ Estimate finite volume effects;
- ▶ Estimate discretization effects by using different values of the lattice spacing;
- ▶ Investigate a range of pion masses.

Thanks!

Extract the energies

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- Compute correlation function on lattice

$$\langle O'(x)O(y) \rangle = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}U O'(x)O(y) e^{-S(U,\bar{\psi},\psi)}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}U e^{-S(U,\bar{\psi},\psi)}} \quad (1)$$

Extract the energies

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- Integrate the fermion degree out

$$\langle O'(x)O(y) \rangle = \frac{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}U O'(x)O(y) e^{-S_g[U_\mu]} e^{-\bar{\psi} \mathbf{M} \psi}}{\int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}U e^{-S_g[U_\mu]} e^{-\bar{\psi} \mathbf{M} \psi}} \quad (2)$$

$$= \frac{1}{Z} \int \mathcal{D}U (F[U_\mu]) e^{-S_g[U_\mu]} \det \mathbf{M}[U_\mu] \quad (3)$$

Extract the energies

- Compute correlation function on lattice

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$$= \frac{1}{Z} \int \mathcal{D}U (F[U_\mu]) e^{-S_g[U_\mu]} \det \mathbf{M}[U_\mu] \quad (3)$$

- Fermion action

$$S_f(U, \psi, \bar{\psi}) = \sum_{x,y} \bar{\psi}_y M_{yx}[U] \psi_x \quad (4)$$

Lattice numerical computation

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$$\langle O'(x)O(y) \rangle = \frac{1}{Z} \int \mathcal{D}U_\mu (F[U_\mu]) e^{-S_g[U_\mu]} \det \mathbf{M}[U_\mu]$$

- Generate configurations

$$p(U_\mu) \propto e^{-S_g[U_\mu]} \det \mathbf{M}[U_\mu] \quad (5)$$

Lattice numerical computation

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- Approximate expectation using statistical average

$$\langle F(U_\mu) \rangle = \frac{1}{N} \sum_{i=1}^N (F(U_i)) \quad (6)$$

Lattice numerical computation

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-

Construct correlation function: $C(t) = \langle 0 | O_i(t) O_j^+(t) | 0 \rangle = \sum_n C_n e^{-E_n t}$

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↑
Construct operators: $O[U, \psi, \bar{\psi}]$

Lattice numerical computation

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Table: XYZ particles

particle	$m(MeV)$	$\Gamma(MeV)$	J^{PC}	Year	founder
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	2003	OK
$Y(4260)$	4263_{-9}^{+8}	95 ± 14	1^{--}	2005	OK
$Z(4430)^+$	4443_{-18}^{+24}	107_{-71}^{+113}	?	2007	Belle
$Z_c(3900)^{+[1]}$	$3899.0 \pm 3.6 \pm 4.9$	$46 \pm 10 \pm 20$	1^+	2013	OK
$Z_c(4025)^{+[2]}$	4026.3 ± 2.6	24.8 ± 5.6	1^+	2013	BESSIII
$Z(4430)^{-[3]}$	4485_{-22-11}^{+22+28}	200_{-46-35}^{+41+26}	1^+	2013	Belle
$Z(4430)^{-[3]}$	$4475 \pm 7_{-25}^{+15}$	$172 \pm 13_{-34}^{+37}$	1^+	2014	OK,LHCb

[1] CLQCD, Y. Chen (Beijing, Inst. High Energy Phy.) et al., Phys.Rev.D89,094506 (2014).

[2] CLQCD, Y. Chen (Beijing, Inst. High Energy Phy.) et al., Phys.Rev.D92,054507 (2015).

[3] CLQCD, T. Chen (Peking U) et al., Phys.Rev.D93,114501 (2016).

$Z_c^\pm(3900)$

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- $Z_c^\pm(3900)$: $I^G(J^{PC}) = 1^+(1^{+-})$.

Table: scattering parameters

	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
$a_0[fm])$	-0.66(1)	-2.13(13)	-0.51(7)
$r_0[fm])$	-0.78(3)	-0.27(7)	0.82(27)

- Conclusion: NO evidence for a $D\bar{D}^*$ bound state.

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Configurations and Stochastic LapHs

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 - Lattice spacing: 0.06 fm \sim 0.09 fm
 - Volume: 2 fm \sim 3 fm
 - Pion mass: Physical \sim 400 MeV
- Stochastic LapH smearing \longrightarrow all-to-all propagators.
Indices: $(color * L_v^3 * L_t * Dirac, \dots) \longrightarrow (N_e * L_t * Dirac, \dots)$.
- Coupled channels.

[a] ETM, R. Baron et al., JHEP 06, 111 (2010), arXiv:1004.5284.

[b] ETM, R. Baron et al., Comput.Phys.Commun. 182, 299 (2011), arXiv:1005.2042.

Configurations and Stochastic LapHs

- $N_f = 2 + 1 + 1(u, d; c, s)$ dynamical quark flavors configurations with various lattice spacing, volume, and pion mass, generated by European Twisted Mass Collaboration(ETMC)^[a,b].
 - Lattice spacing: 0.06 fm \sim 0.09 fm
 - Volume: 2 fm \sim 3 fm
 - Pion mass: Physical \sim 400 MeV
- Stochastic LapH smearing \longrightarrow all-to-all propagators.
Indices: $(color * L_v^3 * L_t * Dirac, \dots) \longrightarrow (N_e * L_t * Dirac, \dots)$.
- Enlarged operator basis.
- Coupled channels.

[a] ETM, R. Baron et al., JHEP 06, 111 (2010), arXiv:1004.5284.

[b] ETM, R. Baron et al., Comput.Phys.Commun. 182, 299 (2011), arXiv:1005.2042.

One-particle operators

.....

Table: single meson operators

meson	$I(J^P)$	physical basic	twisted basic
D^+	$\frac{1}{2}(0^-)$	$i\bar{d}\gamma_5 c$	$i\bar{d}\gamma_5 c$
\bar{D}^0	$\frac{1}{2}(0^-)$	$i\bar{c}'\gamma_5 u$	$i\bar{c}'\gamma_5 u$
D^{*+}	$\frac{1}{2}(1^-)$	$i\bar{d}\gamma_i c$	$-i\bar{d}\gamma_i\gamma_5 c$
\bar{D}^{*0}	$\frac{1}{2}(1^-)$	$i\bar{c}'\gamma_i u$	$-i\bar{c}'\gamma_i\gamma_5 u$

Two-particle operators

- The operators are constructed in T_1 irreducible representation of the O_h group.
- $D\bar{D}^* \implies D^{*+}\bar{D}^0 + \bar{D}^{*0}D^+$

$$\mathcal{O}_\alpha^1(t) = \sum_{R \in G} [D^{*+}(R\vec{p}_\alpha, t+1)\bar{D}^0(-R\vec{p}_\alpha, t) + \bar{D}^{*0}(R\vec{p}_\alpha, t+1)D^+(-R\vec{p}_\alpha, t)] \quad (7)$$

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- $D^*\bar{D}^* \implies \varepsilon_{ijk}D_j^{*+}\bar{D}_k^{*0}$

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- $\vec{n} = (0, 0, 0), (0, 0, 1), (0, 1, 1)$

Table: Multi-hadron operators

$D(k)D^*(-k)$	$k^2 = 0, 1, 2, 3$
$D^*(k)D^*(-k)$	$k^2 = 0, 1, 2, 3$

Two point correlation function

- Two point correlation function
- $T \rightarrow \infty, C(\vec{p}, t) \propto C \cosh(E(\vec{p})(t - T/2))$

$$E(\vec{p}) = \ln(r(t) + \sqrt{r^2(t) - 1}) \quad (9)$$

$$r(t) = \frac{C(\vec{p}, t-1) + C(\vec{p}, t+1)}{C(\vec{p}, t-1)} \quad (10)$$

- $T \rightarrow \infty, t < T/2, C(\vec{p}, t) \propto C' e^{-E(\vec{p})t},$

$$E_0(t) = \ln \frac{C(t)}{C(t+1)} \quad (11)$$

Four point correlation function

- Construct the matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^+(0) | 0 \rangle = \sum_n A_n e^{-E_n t} \quad (12)$$

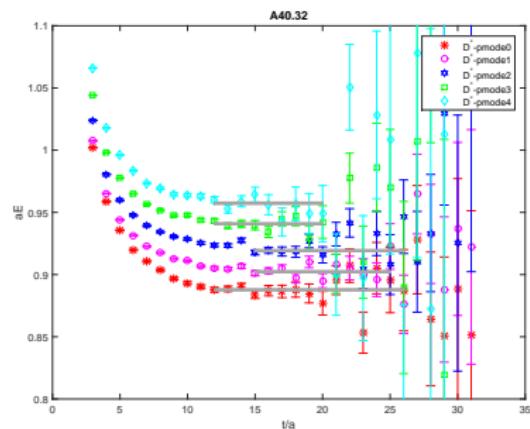
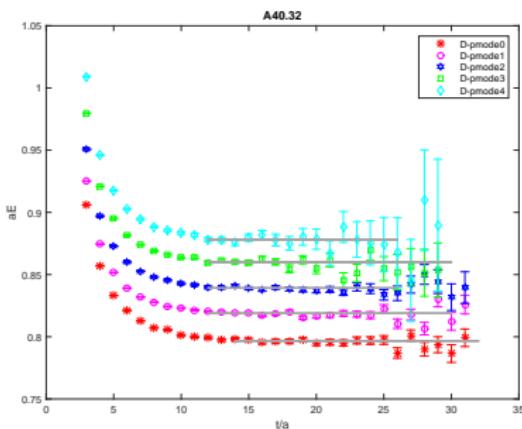
- Solve the generalized eigenvalue problem(GEVP)

$$C_{ij}(t) v_j^n = \lambda_n(t) C_{ij}(t_0) v_j^n \quad (13)$$

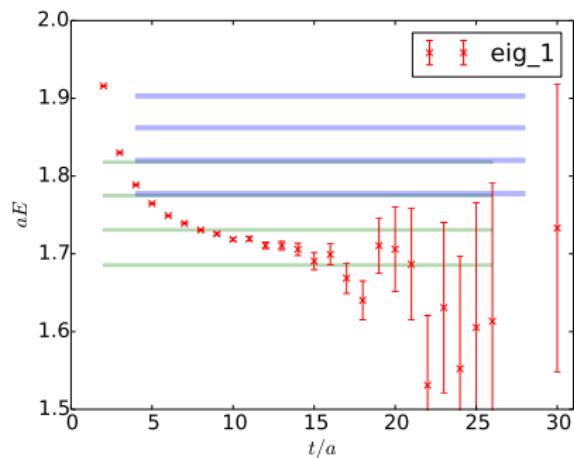
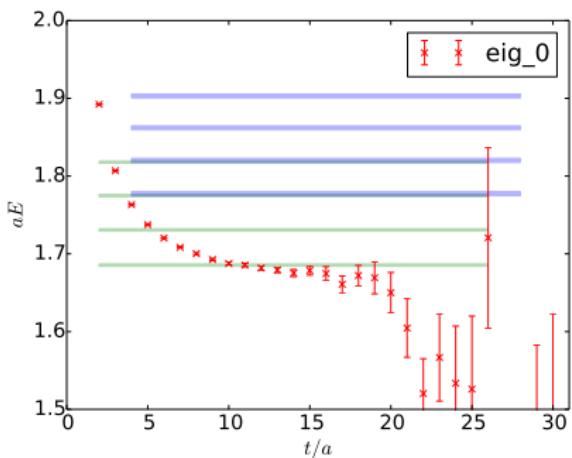
- Eigenvales: $\lambda_n(t) \rightarrow e^{-E_n t} (1 + O(e^{-\Delta E t}))$

Extract the energies

As an example, the energies of two point correlation function of D and D^* for ensemble A40.32/charm225 are shown below. The statistical errors are calculated by using jackknife method.

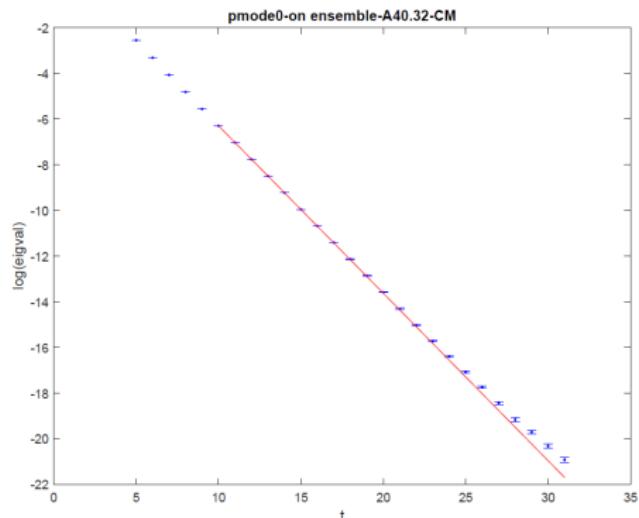


Four point correlation function



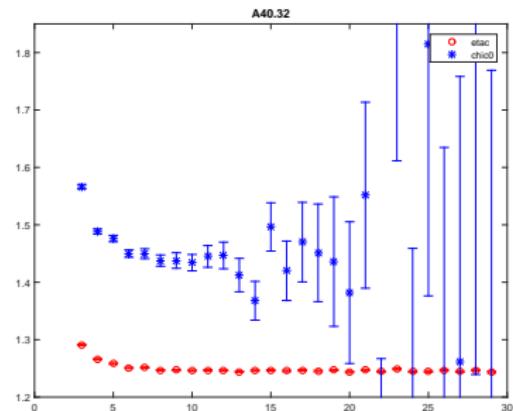
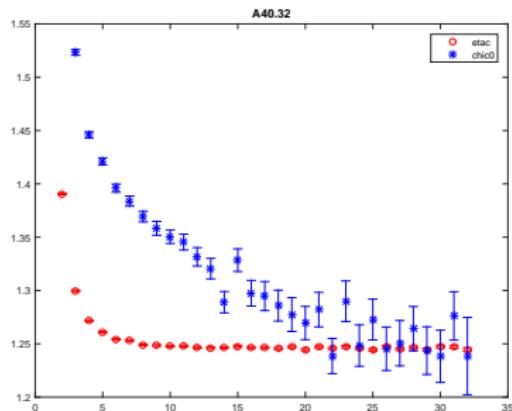
Four point correlation function

Figure: Fit of eigenvalues of four point correlation function matrix



Why coupled channels

CLQCD, T. Chen (Peking U) et al., 10.1140/epjc/s10052-016-4212-8 (2016).



Following work

- Next step,

Extrapolate the scattering parameters from the data using *Lüscher formula* and the effective range expansion.

Following work

- Next step,

Extrapolate the scattering parameters from the data using *Lüscher formula* and the effective range expansion.

- At last,

Estimate finite volume effects;

Estimate discretization effects by using different values of the lattice spacing;

Investigate a range of pion masses;

One-particle operators

.....

Table: single meson operator

meson	$I(J^P)$	physical basic	twisted basic
J/Ψ	1	$i\bar{c}'\gamma_i c$	$-i\bar{c}'\gamma_i\gamma_5 c$

Two-particle operators

- Two-particle operators in physical basic

$$D^{*+}\bar{D}^0 + \bar{D}^{*0}D^+ = (i\bar{d}\gamma_i c)(i\bar{c}'\gamma_5 u) + (i\bar{c}'\gamma_i u)(i\bar{d}\gamma_5 c)$$
$$\varepsilon_{ijk} D_j^{*+} \bar{D}_k^{*0} = \varepsilon_{ijk} (i\bar{d}\gamma_j c)(i\bar{c}'\gamma_k u)$$

- Two-particle operators in twisted basic

$$D^{*+}\bar{D}^0 + \bar{D}^{*0}D^+ = (-i\bar{d}\gamma_i\gamma_5 c)(i\bar{c}'\gamma_5 u) + (-i\bar{c}'\gamma_i\gamma_5 u)(i\bar{d}\gamma_5 c)$$
$$\varepsilon_{ijk} D_j^{*+} \bar{D}_k^{*0} = \varepsilon_{ijk} (-\bar{d}\gamma_j\gamma_5 c)(-\bar{c}'\gamma_k\gamma_5 u)$$

$$\mathcal{O}_1^{T_1} = (-\bar{d}\gamma_2\gamma_5 c)(-\bar{c}'\gamma_3\gamma_5 u) - (-\bar{d}\gamma_3\gamma_5 c)(-\bar{c}'\gamma_2\gamma_5 u)$$
$$\mathcal{O}_2^{T_1} = (-\bar{d}\gamma_3\gamma_5 c)(-\bar{c}'\gamma_1\gamma_5 u) - (-\bar{d}\gamma_1\gamma_5 c)(-\bar{c}'\gamma_3\gamma_5 u)$$
$$\mathcal{O}_3^{T_1} = (-\bar{d}\gamma_1\gamma_5 c)(-\bar{c}'\gamma_2\gamma_5 u) - (-\bar{d}\gamma_2\gamma_5 c)(-\bar{c}'\gamma_1\gamma_5 u)$$

Stochastic LapHs

- All color contractions and spatial sums are carried out in evaluating the hadron functions. Each hadron function for a given choice of noises takes up very little space on disk since each is an array over time and dilution indices only. In this way, a large number of correlation matrices can be evaluated very efficiently.

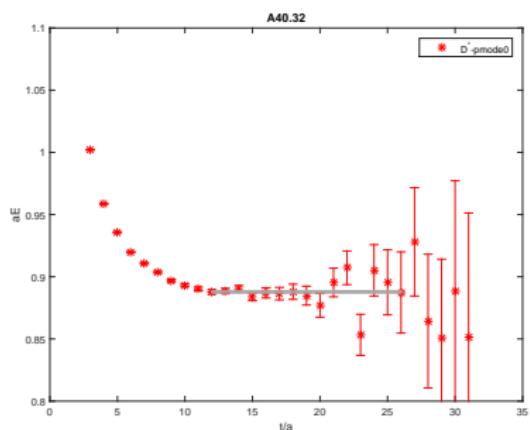
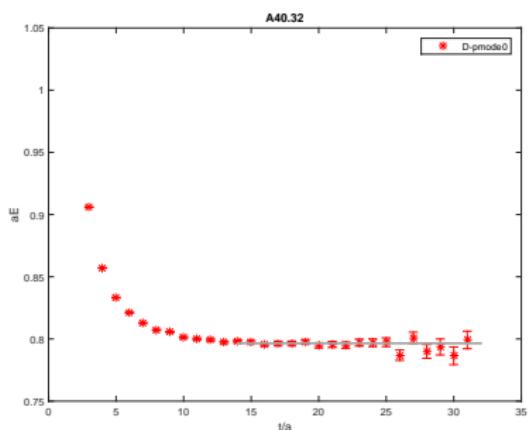
Extract the energies

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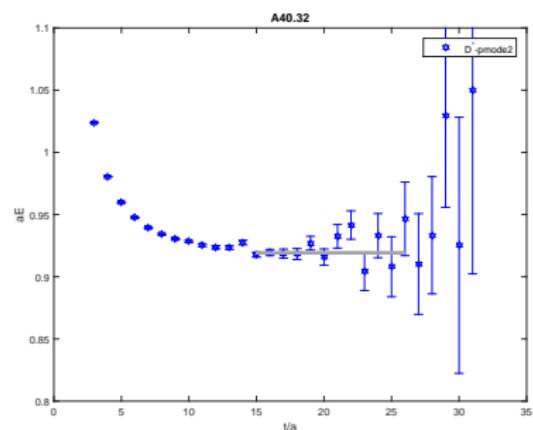
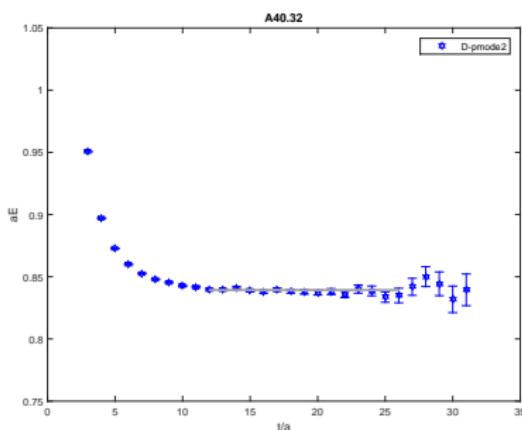
Table: Energies of single D^* meson

t_{min}	t_{max}	E	ΔE	χ^2/dof
12	26	0.8878	0.0010	0.9
15	25	0.9023	0.0012	0.9
15	26	0.9193	0.0013	1.0
12	20	0.9410	0.0011	0.9
12	20	0.9575	0.0017	0.9

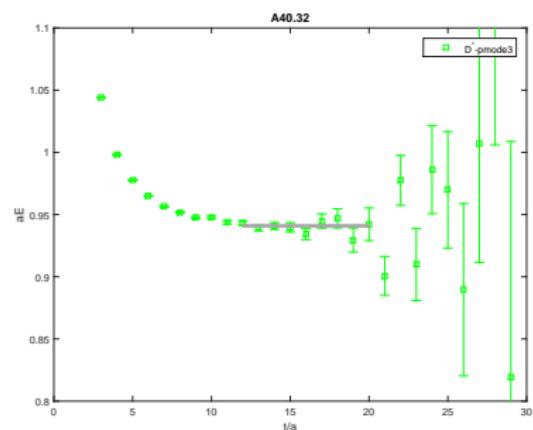
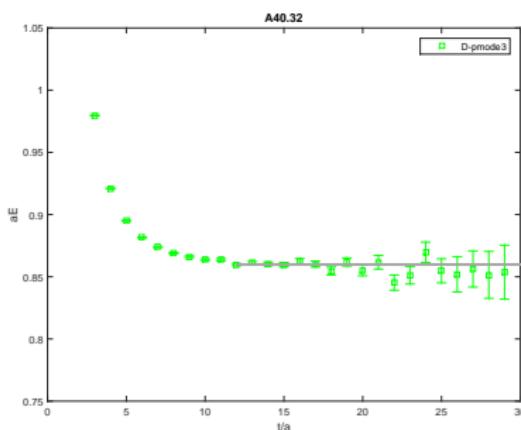
Extract the energies



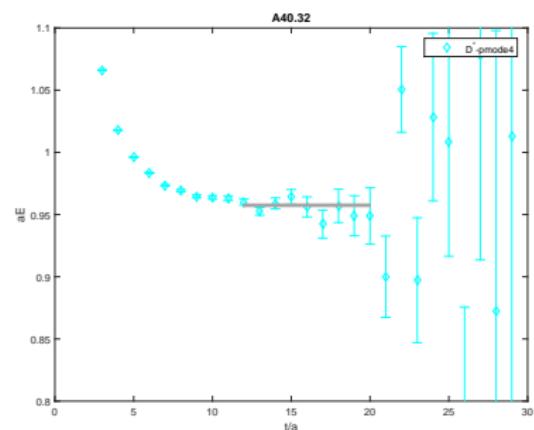
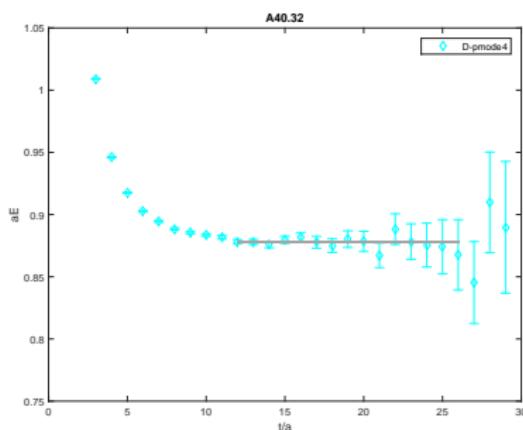
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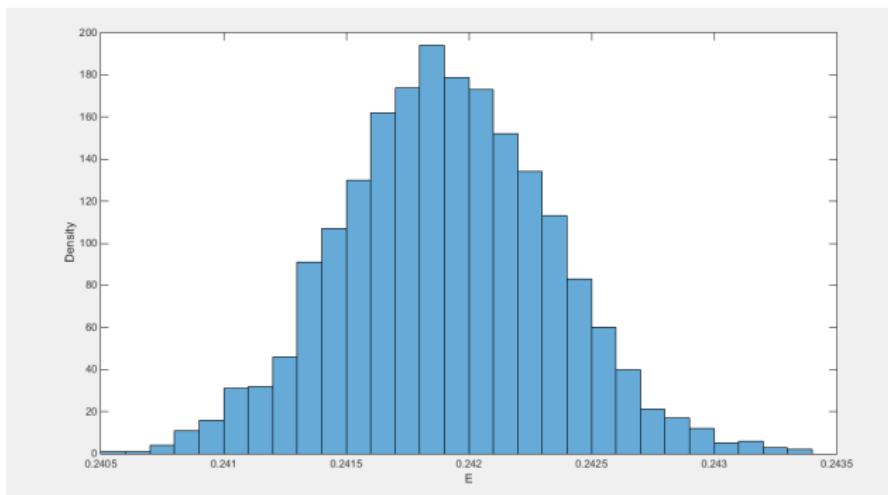


Extract the energies



Statistical errors

- Jackknife: $\delta E = \sqrt{\frac{N_c-1}{N_c} \sum_i (E_i^* - \bar{E}^*)^2}$
- Bootstrap: $\delta E = \sqrt{\frac{1}{N_b-1} \sum_i (E_i^* - \bar{E}^*)^2}$, where $N_b = 2000$ bootstrap samples.



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Thanks!