Status report on project B7: Chiral Symmetry in Nuclear Physics

N. Kaiser (TU Munich)

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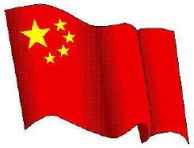
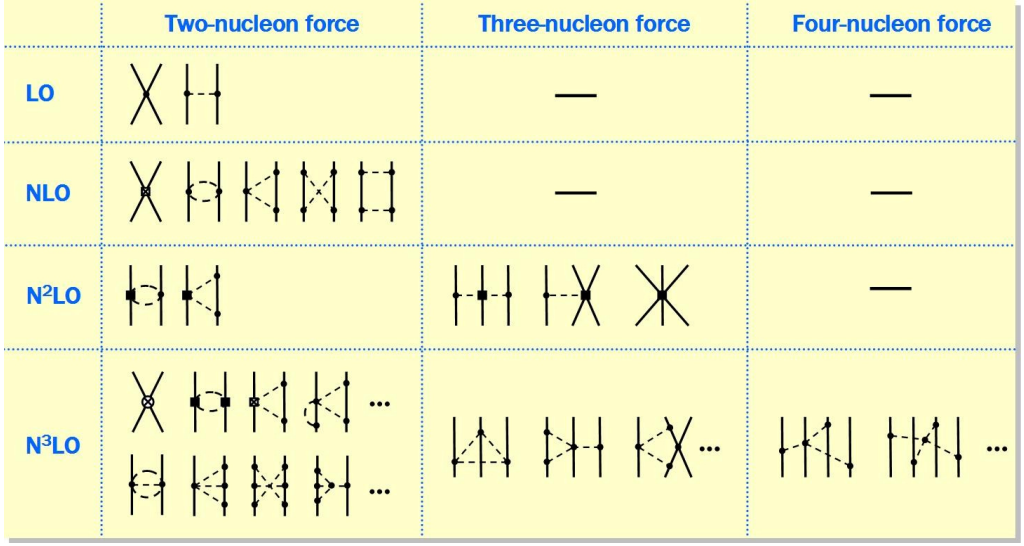


- PIs: E. Epelbaum (Bochum), N. Kaiser (Munich), J. Meng (Peking)
- PhD student: Corbinian Wellenhofer (until July 2017)
  Thermodynamics of nuclear matter from chiral interactions
- PhD student: Susanne Strohmeier (since Sept. 2016)
  Elastic NN-scattering with coupled $N\Delta$-channels at NLO (2π-exch.)
- PhD student: Dominik Gerstung (since May 2017)
  Baryonic forces and hyperons in nuclear matter from SU(3) chiral EFT
- Postdoc: Dr. Qibo Chen (start Sept. 2017) [Jan.$-$March 2017 at TUM]
  EFT for triaxially deformed nuclei: rotations + valence N + vibrations

Thermodynamic equation of state of nuclear matter

- Publications: C. Wellenhofer, J.W. Holt, N. Kaiser,
  PRC 89, 064009 ('14); PRC 92, 015801 ('15); PRC 93, 055802 ('16)
- EoS relevant for many astrophysical phenomena + heavy-ion collisions
  (neutron star structure and evolution, core-collapse supernova, n-star mergers)
- Low-momentum chiral interactions → MBPT applicable to nuclear matter

Hierarchy of nuclear forces in chiral effective field theory



- N^3LO two-body potential + N^2LO three-nucleon force, regularized by cutoff Λ, low-energy constants $c_D(\Lambda)$, $c_E(\Lambda)$ fitted to few-N observables
- 3N-force approximated by density dependent in-medium NN-potential

Thermodynamic equation of state of nuclear matter

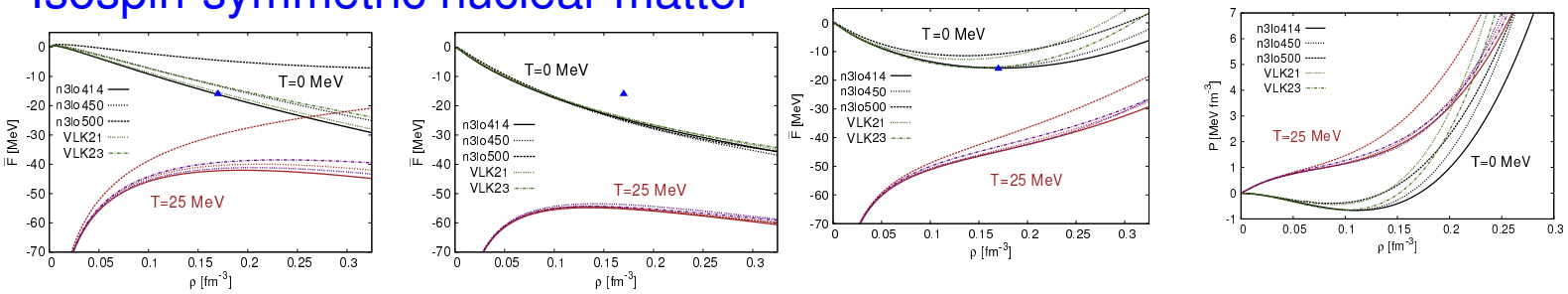
- Perturbation series for free energy density (Kohn-Luttinger-Ward formalism)

$$F(\rho, T) = F_0(\mu_0, T) + \Omega_1(\mu_0, T) + \left\{ \Omega_2(\mu_0, T) - \frac{1}{2} \frac{(\partial\Omega_1/\partial\mu_0)^2}{\partial^2\Omega_0/\partial\mu_0^2} \right\} + \dots$$

$$\rho = \partial\Omega/\partial\mu = \partial\Omega_0/\partial\mu_0, \qquad \mu = \mu_0 + \mu_1 + \dots, \qquad \Omega_2 = \Omega_2^{\text{normal}} + \Omega_2^{\text{anomalous}}$$

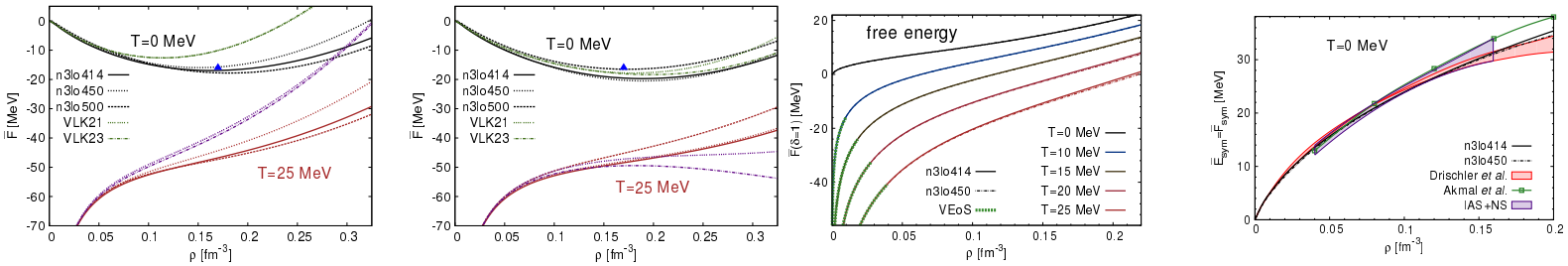
μ_0 one-body effective chemical potential, $\Omega_2^{\text{anomalous}} \sim T^{-1} \int f_1 f_2 (1 - f_2) f_3$

Isospin-symmetric nuclear matter



(a) NN first order, no 3N (b) NN second order, no 3N

Pure neutron matter



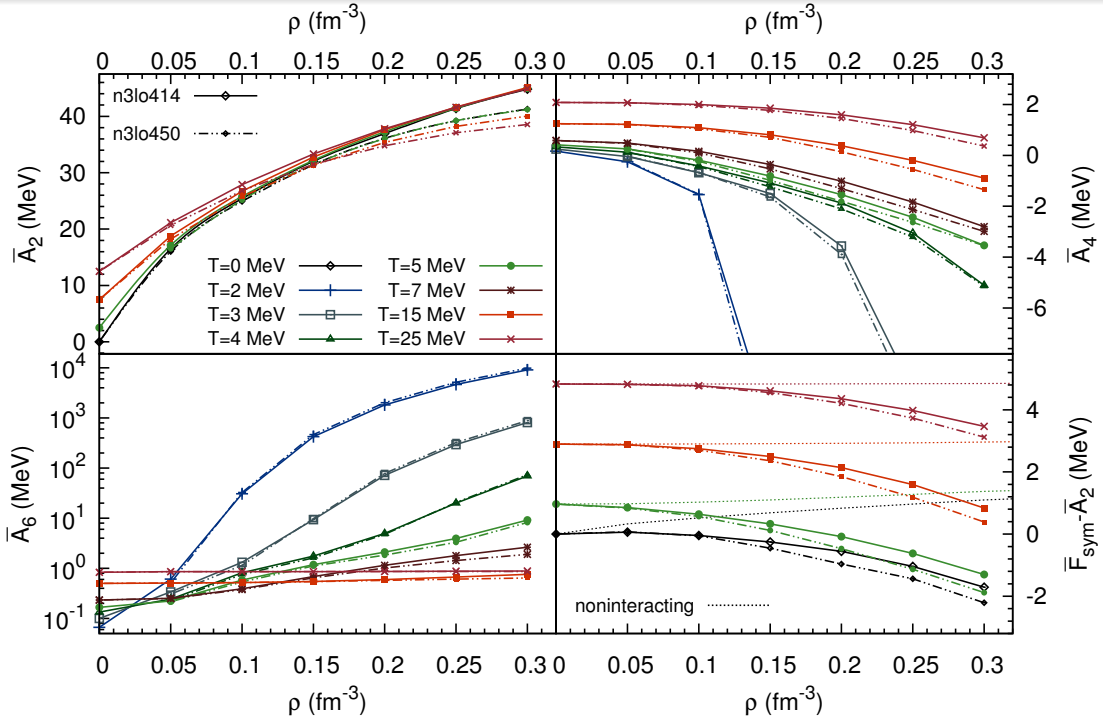
(c) NN second order, 3N first order (d) NN second order, 3N second order

- n3lo414, n3lo450 good perturbative behavior, meet empirical satur. point
- 1st order liquid-gas phase transition, critical temperature $T_c \simeq 17\,\text{MeV}$
- Low-density n-matter: agreement with virial expansion [PLB 638, 153 ('06)]

Thermodynamic equation of state of nuclear matter

- Expansion in isospin-asymmetry parameter $\delta = (\rho_n - \rho_p)/\rho$

$$F_{as}(\rho_p, \rho_n, T) = F(\rho, T) + A_2(\rho, T)\,\delta^2 + \left[A_4(\rho, T)\,\delta^4 + A_6(\rho, T)\,\delta^6\right] + \ldots$$

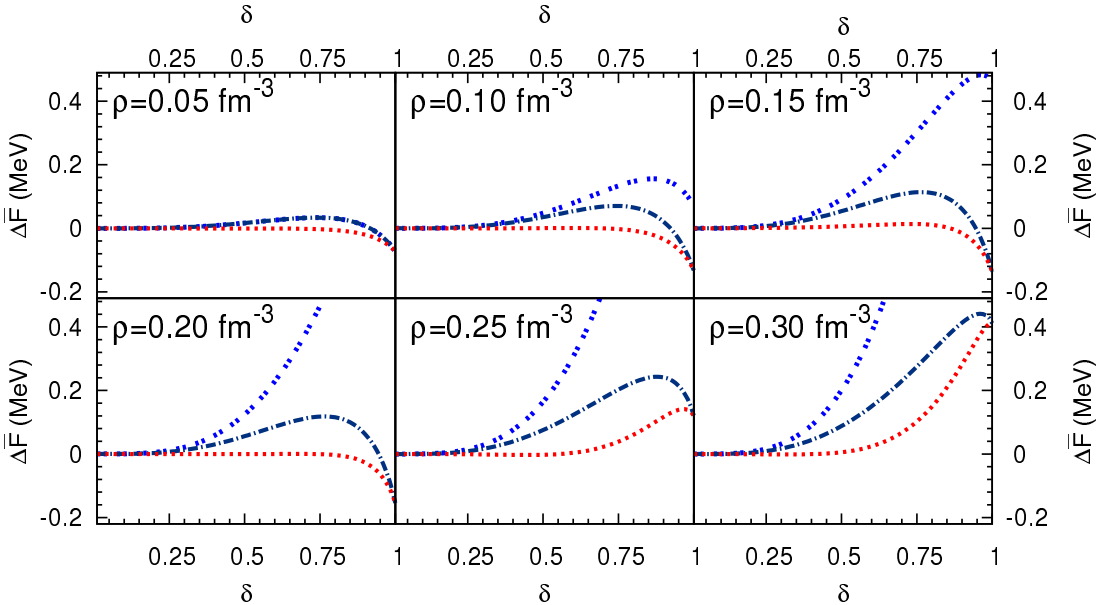


- Quartic/sextic expansion coefficients show divergent behavior at small T
- S-wave contact interaction at 2nd order reveals nonanalytical log-term

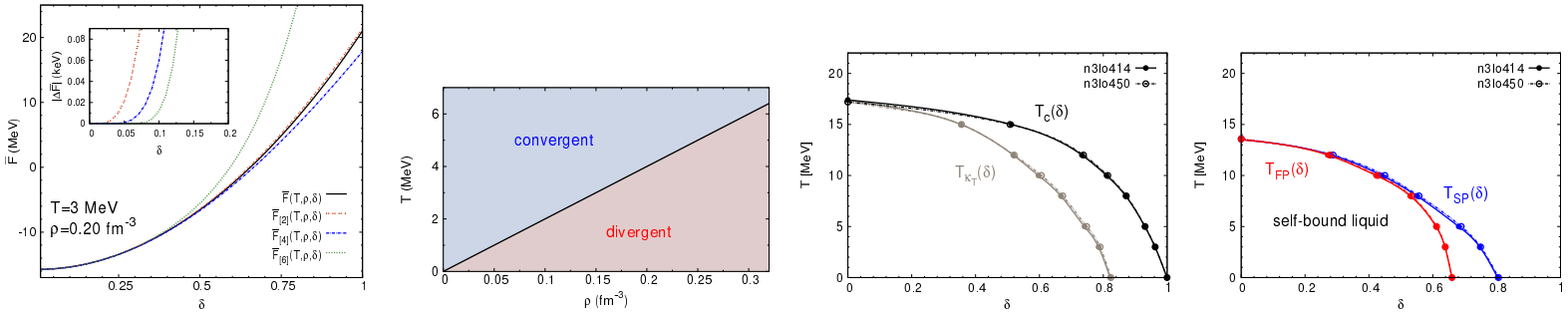
$$\bar{E}_{\rm as}(\rho_p, \rho_n) = \frac{k_f^4}{5\pi^2 M}\left\{\frac{3}{7}(a_s^2 + a_t^2)(11 - 2\ln 2) + \frac{4\delta^2}{3}\left[a_s^2(3 - \ln 2) - a_t^2(2 + \ln 2)\right]\right.$$
$$\left. + \frac{\delta^4}{81}\left[a_s^2\left(10\ln\frac{|\delta|}{3} + 2\ln 2 - \frac{41}{6}\right) + a_t^2\left(30\ln\frac{|\delta|}{3} + 2\ln 2 + \frac{3}{2}\right)\right]\right\} + \ldots$$

Thermodynamic equation of state of nuclear matter

- Origin of $\delta^4 \ln(|\delta|/3)$: energy denominator vanishes at phase-space boundary, feature goes away at finite T or after inclusion of pairing



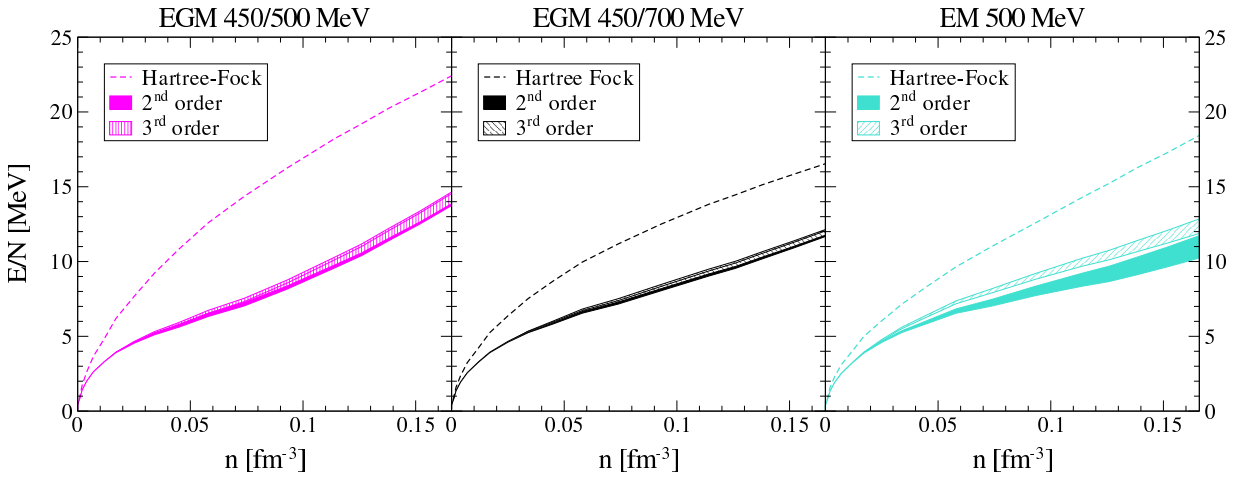
- Sequence of approximations: quadratic, quartic without and with $\ln(|\delta|)$



- Critical temperature $T_c(\delta)$ (binodal) and mech. instability $T_\kappa(\delta)$ (spinodal)
- Isospin destillation: coexisting liquid and gas phases have different ρ_p/ρ_n

Third-order many-body contributions from p^2-contact interaction

- Publication: N. Kaiser, Eur. Phys. J. A 53, 104 ('17)
- By using chiral low-momentum interactions, nuclear matter can be calculated reliably in many-body perturbation theory, also at finite T
- Neutron matter up to third-order ladder diagrams [PRC88, 025802 ('15)]



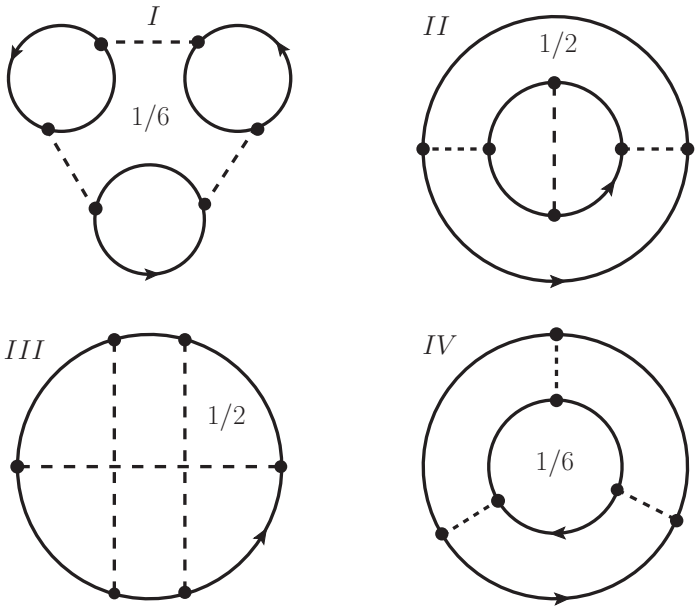
- Often neglected due to its complexity: 3rd order particle-hole diagram
- Analytical calculation for general p^2-contact interaction (7+2 parameters)

$$V_{\rm ct} = t_0(1+x_0P_\sigma) + \frac{t_1}{2}(1+x_1P_\sigma)(\vec{q}_{\rm out}^{\,2}+\vec{q}_{\rm in}^{\,2}) + t_2(1+x_2P_\sigma)\,\vec{q}_{\rm out}\cdot\vec{q}_{\rm in}$$
$$+iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}_{\rm out}\times\vec{q}_{\rm in}) + V_{\rm ten}(t_4,t_5)$$

Spin and isospin exchange operators: $P_\sigma=(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)/2$, $P_\tau=(1+\vec{\tau}_1\cdot\vec{\tau}_2)/2$, $\vec{q}_{\rm in}=(\vec{p}_1-\vec{p}_2)/2$, $\vec{q}_{\rm out}=(\vec{p}_1{}'-\vec{p}_2{}')/2$ momentum differences in initial/final state, completed to general p^2-contact interaction by adding two tensor terms

Third-order many-body contributions from p^2-contact interaction

- Direct and exchange-type 3-ring diagrams: $I+II+III+IV = (dir-exc)^3/6$



- Three loops factorize into cube of Euclidean polarization function

$$\Pi(\omega,\vec{q}\,) = \int \frac{d^3l}{(2\pi)^3}\,\frac{1}{i\omega+\vec{l}\cdot\vec{q}/M}\Big\{\theta(k_f-|\vec{l}-\vec{q}/2|)-\theta(k_f-|\vec{l}+\vec{q}/2|)\Big\} = \frac{Mk_f}{4\pi^2 s}\,Q_0(s,\kappa)$$

$$Q_0(s,\kappa) = s - s\kappa\arctan\frac{1+s}{\kappa} - s\kappa\arctan\frac{1-s}{\kappa} + \frac{1}{4}(1-s^2+\kappa^2)\ln\frac{(1+s)^2+\kappa^2}{(1-s)^2+\kappa^2}$$

setting $|\vec{q}\,| = 2sk_f$ and $\omega = 2s\kappa k_f^2/M$

- Momentum-dependence of $V_{\rm ct}$ introduces further polarization functions

$$\Pi[\vec{l}\,] = -\frac{Mk_f^2}{4\pi^2 s}\,i\kappa\,Q_0(s,\kappa)\,\hat{q}\,,\quad \Pi[l_il_j] = \frac{Mk_f^3}{4\pi^2 s}\left\{\frac{\delta_{ij}}{3}\,Q_1(s,\kappa)+\left(\hat{q}_i\hat{q}_j-\frac{\delta_{ij}}{3}\right)Q_2(s,\kappa)\right\}$$

Third-order many-body contributions from p^2-contact interaction

- Resulting 3-ring energy per particle for isospin-symmetric nuclear matter

$$\bar{E}(k_f)^{3\mathrm{ph}} = \frac{M^2 k_f^5}{32\pi^7}\Big\{ t_0^3(1-6x_0^2)\mathcal{N}_1 + k_f^2\, t_0^2 t_1(1-2x_0^2-4x_0x_1)\mathcal{N}_2$$
$$+k_f^2\, t_0^2 t_2\big[5+4x_2+2x_0^2(1+2x_2)\big]\mathcal{N}_3 + k_f^4\, t_0 t_1^2(4x_0x_1+2x_1^2-1)\mathcal{N}_4$$
$$+k_f^4\, t_0 t_1 t_2\Big[\frac{5}{2}+x_0x_1(1+2x_2)+2x_2\Big]\mathcal{N}_5 + k_f^4\, t_0 t_2^2\Big[\frac{5}{2}+4x_2+x_2^2\Big]\mathcal{N}_6$$
$$+k_f^6\, t_1^2 t_2\Big[\frac{5}{2}+x_1^2+2x_2(1+x_1^2)\Big]\mathcal{N}_7 + k_f^6\, t_1 t_2^2\Big[\frac{5}{2}+4x_2+x_2^2\Big]\mathcal{N}_8$$
$$+k_f^6\, t_1^3(1-6x_1^2)\mathcal{N}_9 + k_f^6\, t_2^3\Big[\frac{5}{4}+3x_2+\frac{39}{14}x_2^2+x_2^3\Big]\mathcal{N}_{10}$$
$$+k_f^4\, W_0^2\Big[t_0(1+x_0)\mathcal{N}_{11} + k_f^2\, t_1(1+x_1)\mathcal{N}_{12} + k_f^2\, t_2(1+x_2)\mathcal{N}_{13}\Big]\Big\}$$

- Four-loop coefficients $\mathcal{N}_j$ computed accurately as double-integrals

$\mathcal{N}_1 = 4.1925784$, $\mathcal{N}_2 = -0.4633512$, $\mathcal{N}_3 = -2.259163$, $\mathcal{N}_4 = 2.902123$,
$\mathcal{N}_5 = 2.12658$, $\mathcal{N}_6 = 0.43897$, $\mathcal{N}_7 = 0.48756$, $\mathcal{N}_8 = -0.27614$, $\mathcal{N}_9 = -1.01924$,
$\mathcal{N}_{10} = 0.315484$, $\mathcal{N}_{11} = -2.24420$, $\mathcal{N}_{12} = -2.30577$, $\mathcal{N}_{13} = 2.53887$.

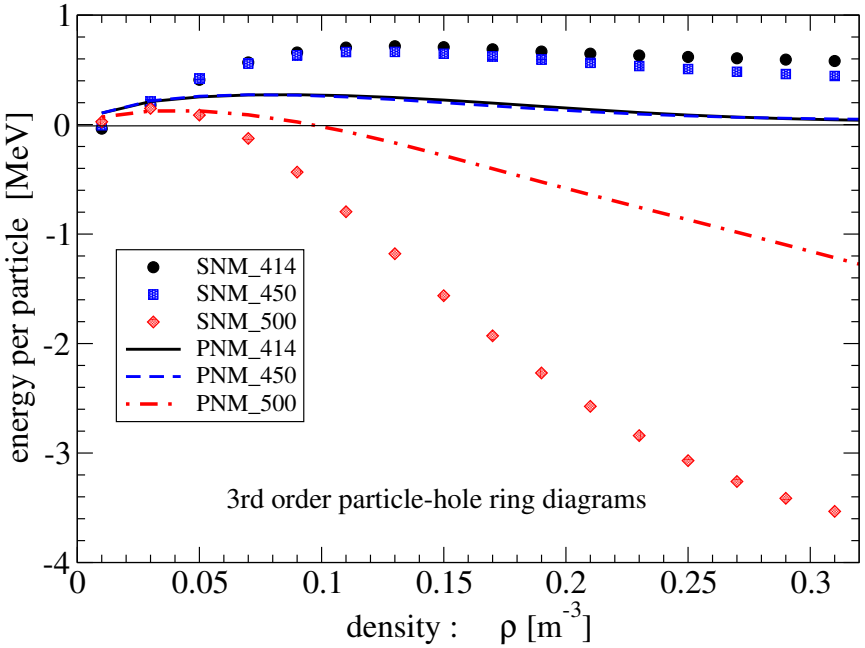
- Dimensional regularization of $\mathcal{N}_{j\neq 0}$ by subtracting power divergencies, verified by rederiving analytical results $\pi(r_j+\ln 2)$ for 2nd order contribut.
- 3rd order ladder diagrams from V_{ct} computed with complex in-medium loop $R+i\pi I$, proper real-valued integrand for energy density $R^2-\pi^2 I^2/3$

Third-order ring diagram with chiral NN-potentials

- Real thing: 3rd order particle-hole ring energy from chiral NN-interaction
- Partial-wave based numerical computation tested for model-interactions:

$$V_{\text{central}} = -\frac{g^2}{m^2+q^2}\,, \qquad V_{\text{tensor}} = -g^2\,\vec{\tau}_1\cdot\vec{\tau}_2\,\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{(m^2+q^2)^2}$$

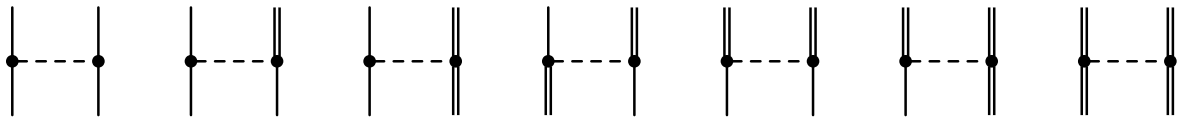
- For pure spin-orbit interaction: 3rd order ring diagrams vanish identically, spin-trace zero $\leftrightarrow$ delicate cancelations in multiple partial-wave sums



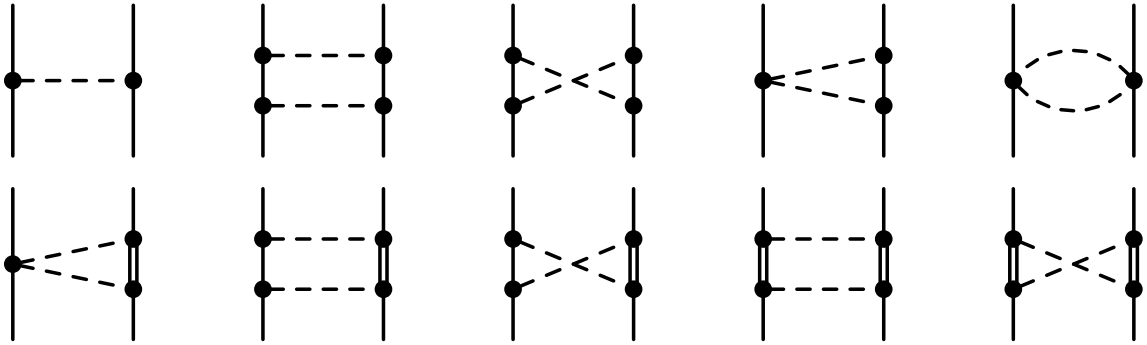
- For N^{3}LO chiral NN-potential with low resolution scale $\Lambda = (410-500)$ MeV, 3rd order particle-hole contribution bounded by a few MeV up to $\rho = 2\rho_0$
- Convergence of MBPT well under control [J. Holt, N. Kaiser, PRC 95, 034325 ('17)]

Elastic NN-scattering with coupled NΔ-channels

- Work in progress: PhD of Susanne Strohmeier (collabor. with Bochum)
- $\Delta(1232)$-isobar very important in πN-dynamics (2π-exch. NN-potential)
- Coupled $N\Delta$-channels efficient to deal with nuclear many-body forces



Corresponding 1π- and 2π-exchange diagrams



- Planar $N\Delta$- and $\Delta\Delta$-boxes include reducible parts: to be subtracted

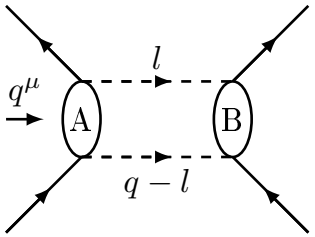
$$\int \frac{dl_0}{2\pi i} \frac{1}{(l_0-\Delta)(-l_0+i0)(l_0^2-\omega_1^2)(l_0^2-\omega_2^2)} = \frac{1}{\Delta\omega_1^2\omega_2^2} - \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2+\Delta(\omega_1+\omega_2)}{2\omega_1^2\omega_2^2(\omega_1+\omega_2)(\omega_1+\Delta)(\omega_2+\Delta)},$$

$$\int \frac{dl_0}{2\pi i} \frac{1}{(l_0-\Delta)(-l_0-\Delta)(l_0^2-\omega_1^2)(l_0^2-\omega_2^2)} = \frac{1}{2\Delta\omega_1^2\omega_2^2} - \frac{\omega_1^2+\omega_1\omega_2+\omega_2^2+\Delta(\omega_1+\omega_2)}{2\omega_1^2\omega_2^2(\omega_1+\omega_2)(\omega_1+\Delta)(\omega_2+\Delta)}$$

- Irreducible part of planar $N\Delta$-box and $\Delta\Delta$-box = minus crossed $N\Delta$-box

Elastic NN-scattering with coupled NΔ-channels

- Unitarity: Loop-potentials are determined by their imaginary parts:



$$\int d\Phi_2 AB\, l^\alpha l^\beta l^\gamma = X(\mu)(-g^{\alpha\beta}q^\gamma - g^{\alpha\gamma}q^\beta - g^{\beta\gamma}q^\alpha) + Y(\mu)\, q^\alpha q^\beta q^\gamma + \ldots$$

$$X(\mu) = \frac{|\vec{l}|}{16\pi\mu}\int_{-1}^{1} dx\, AB\, \frac{|\vec{l}|^2}{4}(1-x^2), \quad Y(\mu) = \frac{|\vec{l}|}{16\pi\mu}\int_{-1}^{1} dx\, AB\, \frac{2m_\pi^2 + |\vec{l}|^2(5-3x^2)}{4\mu^2}$$

- Perform partial-wave decomposition [J. Golak et al., EPJA 43, 241 ('10)]

$$H(L', S', L, S, J) = \frac{\sqrt{\pi(2L+1)}}{2J+1} \sum_{m_J=-J}^{J} \sum_{m=-L'}^{L'} \int_0^\pi d\theta\, \sin\theta\, Y_{L'm}(\theta, 0)\, C(L0, Sm_J, Jm_J)$$
$$\times C(L'm, S'(m_J - m), Jm_J)\langle S' m_J - m | V(\vec{p}\,', \vec{p}) | S m_J\rangle$$

$$\vec{p} = p(0,0,1), \qquad \vec{p}\,' = p'(\sin\theta, 0, \cos\theta)$$

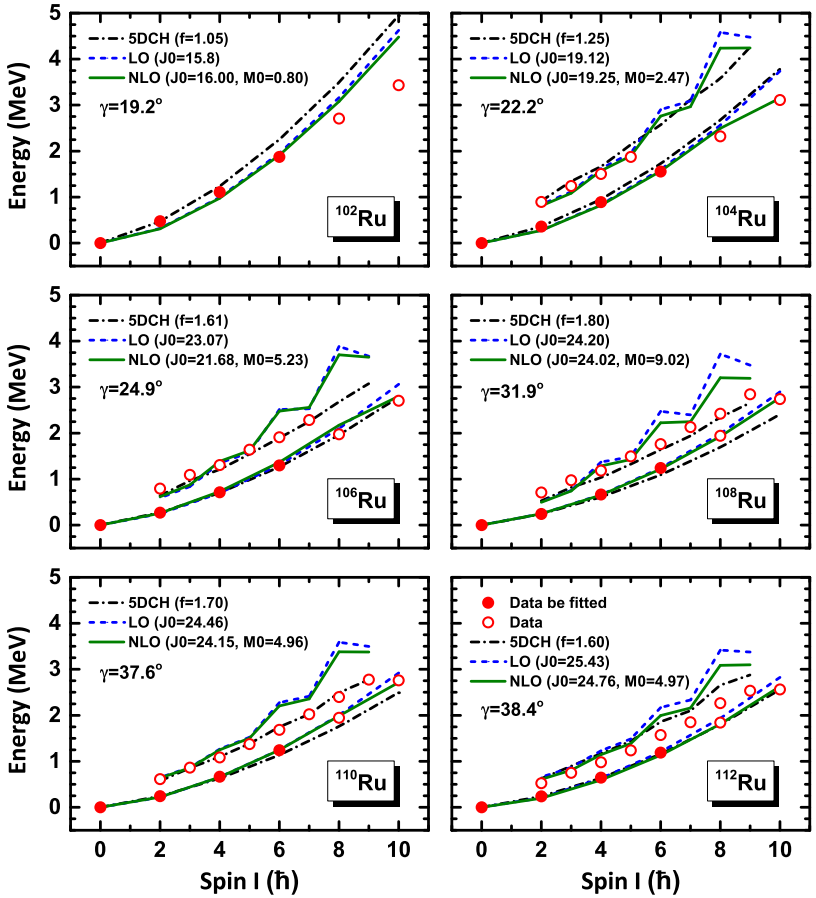
- Construct all 4-baryon contact-terms up to NLO (invoking evtl. large N_c)
- Solve regularized coupled-channel LS-equation (Kadyshevsky eqs.)
- Extensive coupling of $|LSJ\rangle$ states with $\Delta S = 0, 2$ and $\Delta L = 0, 2, 4, 6$
- Fits to empirical NN phase-shifts, determine strength of short-distance operators, quality of NLO vs. LO, estimate theoretical uncertainties ...

Effective field theory for triaxially deformed nuclei

- Sino-german collab. [Q. Chen, N. Kaiser, Ulf-G. Meißner, J. Meng, arXiv:1707.04353]
- Extend EFT approach (T. Papenbrock et al.) to triaxially deformed nuclei
- Hamiltonian for collective nuclear rotations at next-to-leading order

$$H = H_{\mathrm{LO}} + \Delta H_{\mathrm{NLO}} = \frac{I_1^2}{2J_1} + \frac{I_2^2}{2J_2} + \frac{I_3^2}{2J_3} - \left(\frac{M_1 I_1^4}{4J_1^4} + \frac{M_2 I_2^4}{4J_2^4} + \frac{M_3 I_3^4}{4J_3^4} \right)$$

I_k angular momenta, J_k principal moments of inertia, M_k non-rigidity parameters



- Moments of inertia of irrotational type: $J_k, M_k = J_0, M_0 \sin(\gamma - 2\pi k/3)$
- Ground-state and γ bands in even-even isotopes $^{102}_{44}\mathrm{Ru} - {}^{112}_{44}\mathrm{Ru}$
- NLO describe data better than LO
- Further prediction of $K = 4$ bands
- For $I > 6$: systematic inclusion of coupling to coll. vibrational motion
- Odd nuclei: particle-rotor coupling
- Chiral 2-bands in triaxial rot. nuclei
- More details: see Qibo Chen's talk