

A.1: Flavor symmetries and final-state interactions in hadronic decays

F.-K. Guo, J. Haidenbauer, B. Kubis, S. Paul

thanks to B. Grube

HISKP (Theorie) & BCTP
Universität Bonn, Germany

CRC110 general meeting
Beijing, August 29th, 2017

Overview — Staff

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Claudio Valletta [TUM] (11/2017–)

Overview — selected results

- **Pion–pion final-state interactions**

- ▷ form factors and Omnès problem: $\bar{B}^0 \rightarrow J/\psi \pi\pi$
- ▷ flavour symmetries: $\pi\eta$ (and $K\bar{K}$)
- ▷ left-hand cuts: Z_b states and $\Upsilon(nS) \rightarrow \Upsilon(mS)\pi\pi$

- **Three-body systems**

- ▷ Khuri–Treiman equations: $\eta' \rightarrow \eta\pi\pi$
- ▷ diffractive 3π production at COMPASS

- **$\bar{N}N$ interactions at N³LO**

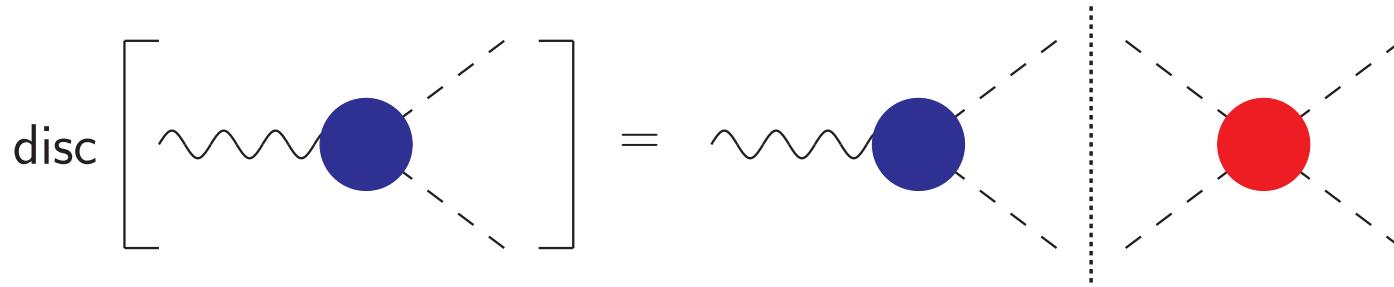
- see also:

Sergi Gonzàlez-Solís (ITP), talk on [Wednesday](#)

Dmitri Ryabchikov (TUM), talk on [Thursday](#)

Final-state interactions: Omnès formalism

- two-particles FSI: form factor; from unitarity:

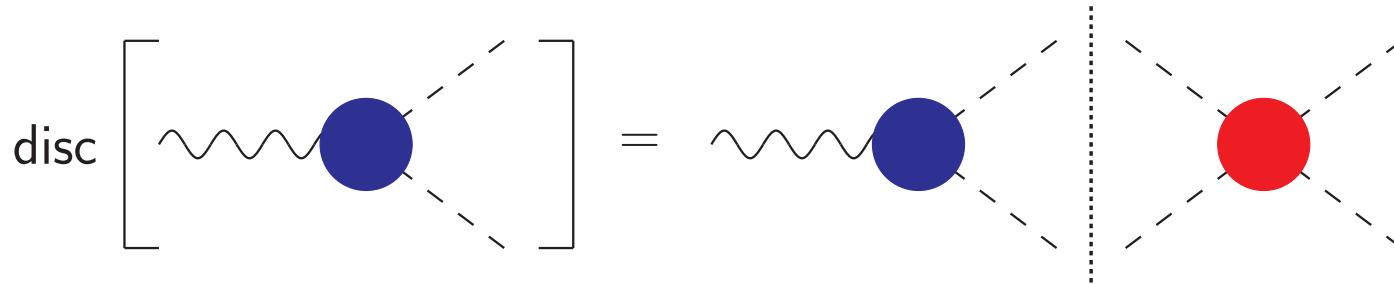


$$\frac{1}{2i} \text{disc } F_I(s) = \text{Im } F_I(s) = F_I(s) \times \theta(s - 4M_\pi^2) \times \sin \delta_I(s) e^{-i\delta_I(s)}$$

→ final-state theorem: phase of $F_I(s)$ is just $\delta_I(s)$ Watson 1954

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- solution to this homogeneous integral equation known:

$$F_I(s) = P_I(s)\Omega_I(s) \ , \quad \Omega_I(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^\infty ds' \frac{\delta_I(s')}{s'(s'-s)} \right\}$$

$P_I(s)$ polynomial, $\Omega_I(s)$ Omnès function Omnès 1958

$P_I(s)$ non-universal, needs to be fixed by symmetries, data, . . .

- today: high-accuracy $\pi\pi$ phase shifts available

Ananthanarayan et al. 2001, García-Martín et al. 2011, Caprini et al. 2012

Scalar form factors: coupled channels

- two scalar isoscalar pion form factors:

$$\langle \pi^+ \pi^- | \frac{1}{2}(\bar{u}u + \bar{d}d) | 0 \rangle = \mathcal{B}^n \Gamma_{\pi}^n(s) \quad \langle \pi^+ \pi^- | \bar{s}s | 0 \rangle = \mathcal{B}^s \Gamma_{\pi}^s(s)$$

- strong inelastic coupling to $\bar{K}K$ near $f_0(980)$
→ requires coupled-channel treatment $\pi\pi \leftrightarrow \bar{K}K$

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- three input functions:

- ▷ $\pi\pi$ S-wave phase shift $\delta(s)$ Caprini, Colangelo, Leutwyler 2012
- ▷ modulus $|g(s)|$ and phase $\psi(s)$ of $\pi\pi \rightarrow \bar{K}K$ amplitude Büttiker et al. 2004; Cohen et al. 1980, Etkin et al. 1982

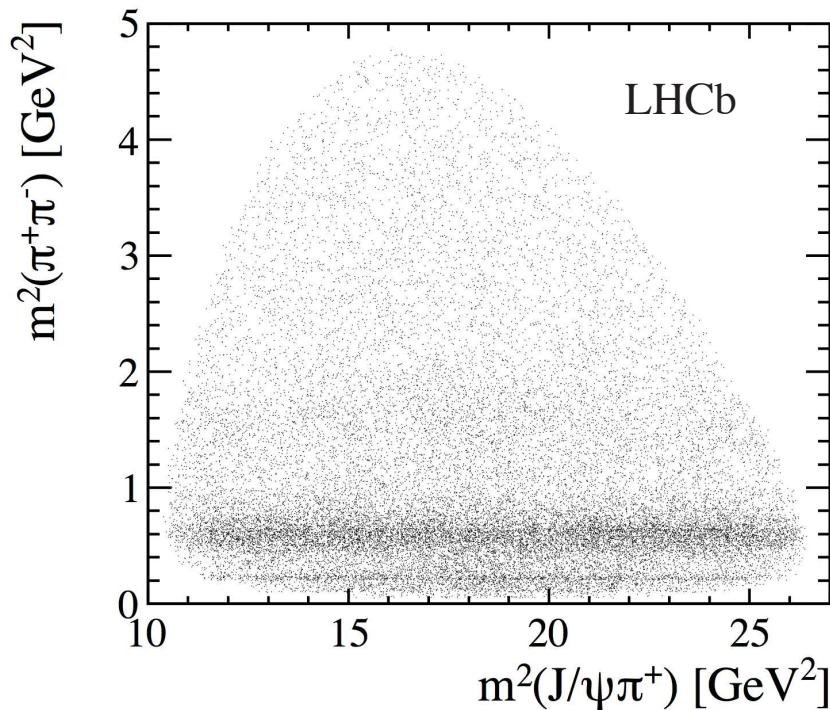
- solution in terms of Omnès matrix

$$\begin{pmatrix} \Gamma_\pi(s) \\ \frac{2}{\sqrt{3}}\Gamma_K(s) \end{pmatrix} = \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix} \begin{pmatrix} \Gamma_\pi(0) \\ \frac{2}{\sqrt{3}}\Gamma_K(0) \end{pmatrix}$$

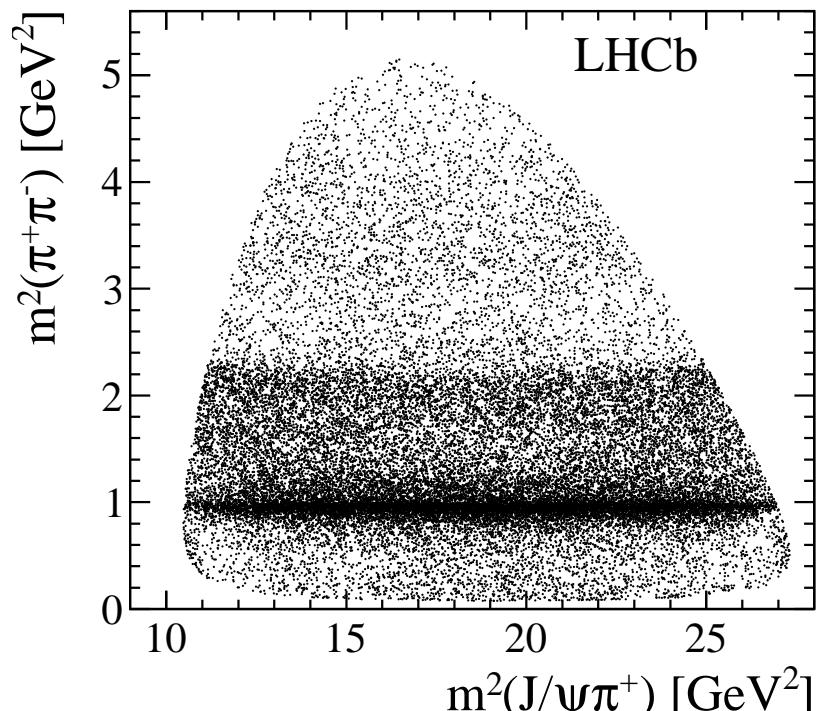
Scalar form factors from decays: $\bar{B}_{d/s}^0 \rightarrow J/\psi \pi\pi$

- no scalar source in SM \rightarrow test scalar form factors in decays
- experimental evidence: only $\pi\pi$ dynamics important

$$\bar{B}_d^0 \rightarrow J/\psi \pi^+ \pi^-$$



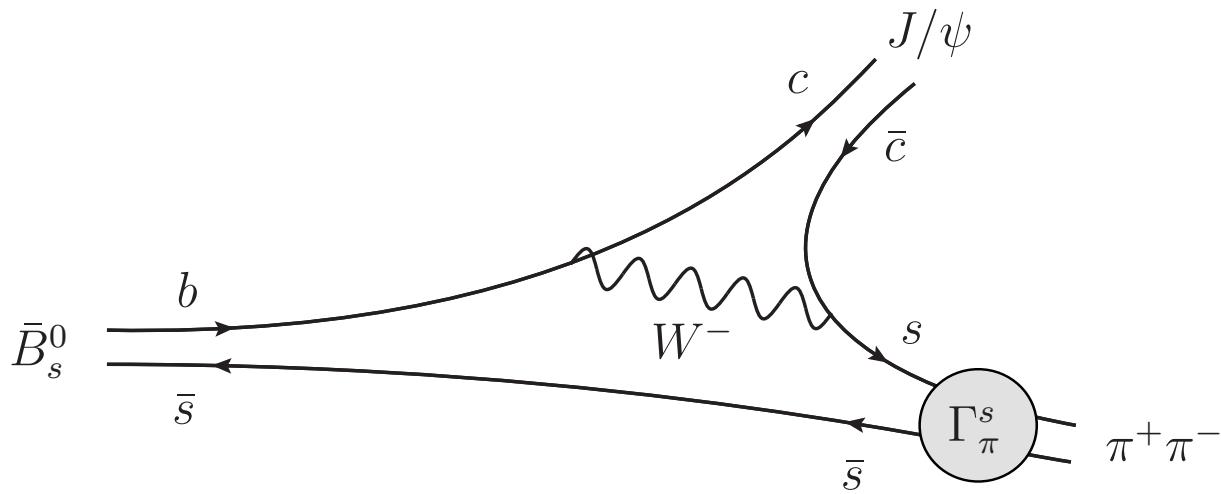
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LHCb 2014

Scalar form factors from decays: $\bar{B}_{d/s}^0 \rightarrow J/\psi \pi\pi$

- no scalar source in SM \rightarrow test scalar form factors in decays
- experimental evidence: only $\pi\pi$ dynamics important
- $\bar{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-$: clean $\bar{s}s$ source \rightarrow S-wave dominated



- $\bar{B}_d^0 \rightarrow J/\psi \pi^+ \pi^-$: analogous $\bar{d}d$ source
 \rightarrow both isoscalar S-wave and isovector P-wave contribute

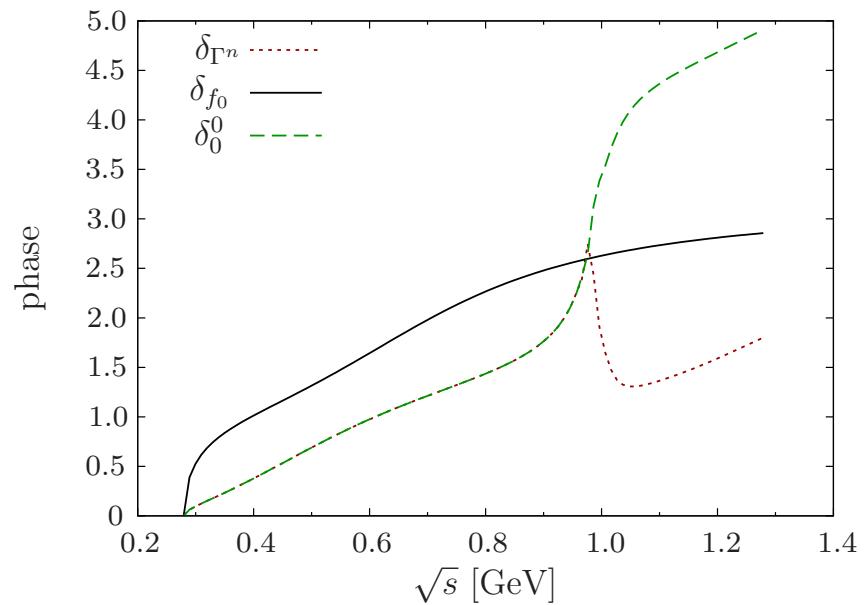
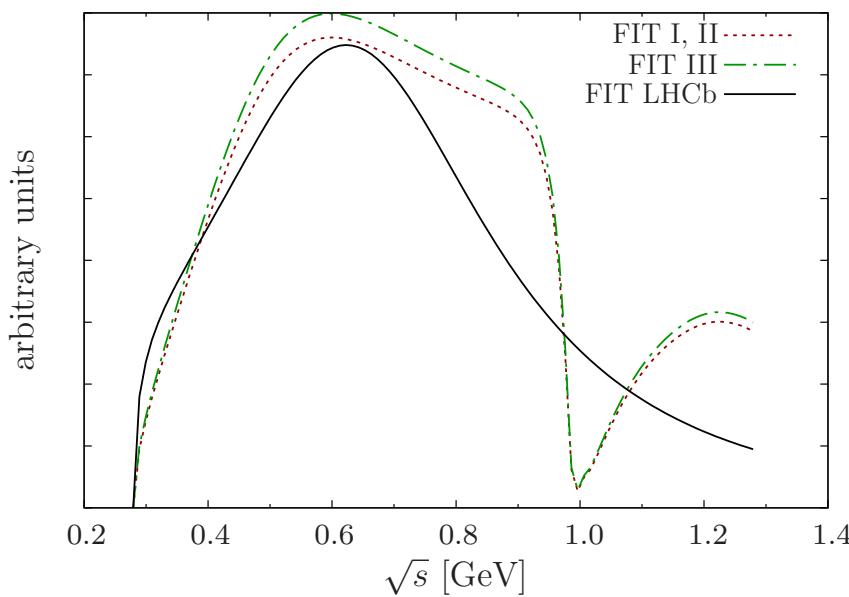
$\bar{B}_d^0 \rightarrow J/\psi \pi^+ \pi^-$: fit results (S-wave)

- number of fit parameters: 3–4 (fixed D-wave added)
- compare LHCb: 14 Breit–Wigner parameters
→ comparable fit quality ($\sqrt{s} \leq 1.02$ GeV)

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extracted S-wave:

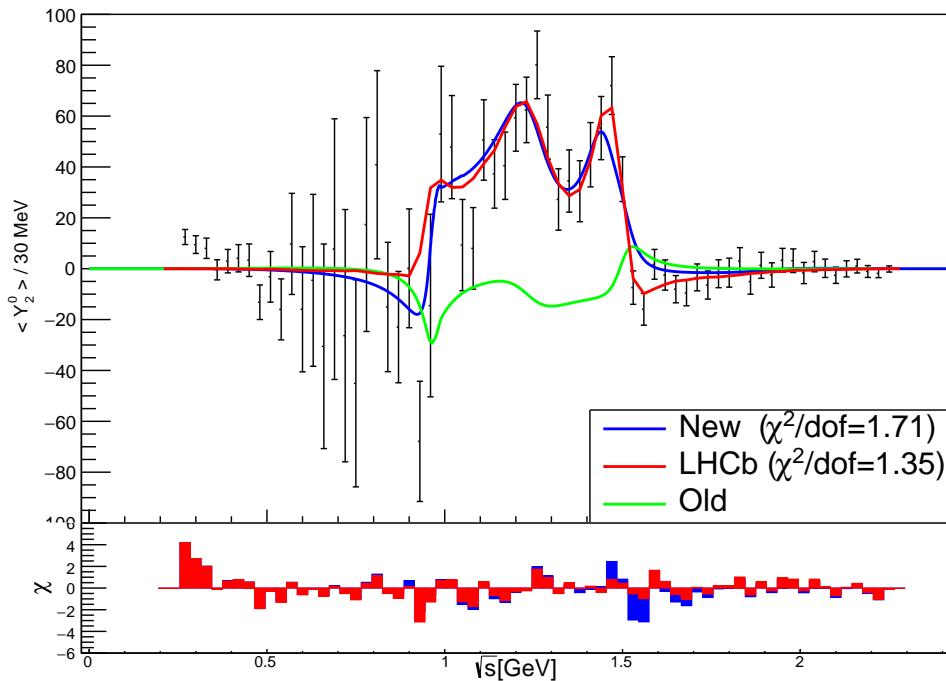
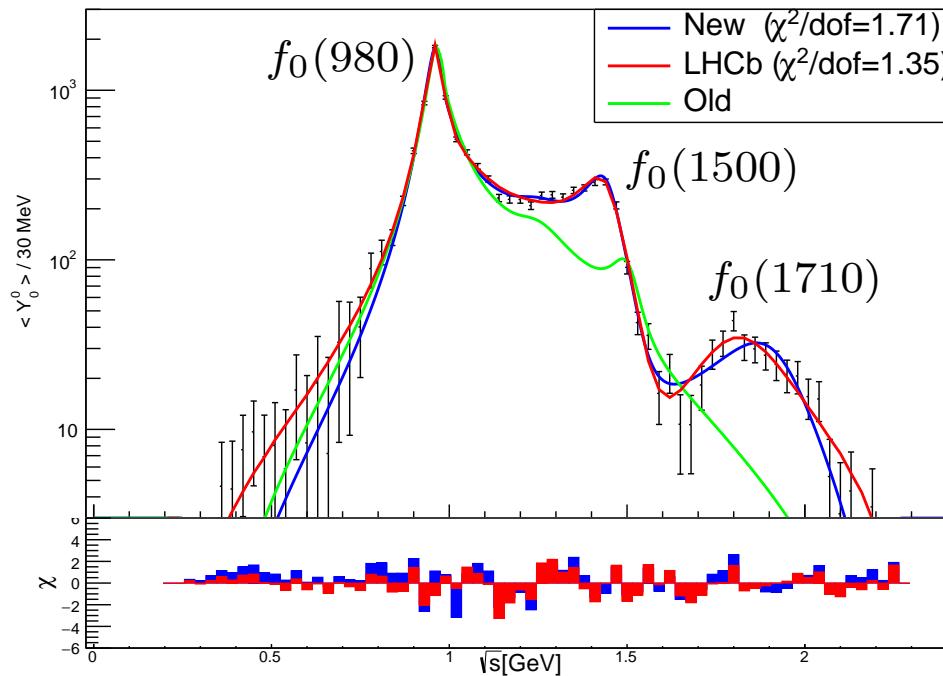


- drastic differences in modulus and phase Daub, Hanhart, BK 2015
→ avoid Breit–Wigner parametrisations of the $f_0(500)!$

$\bar{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-$: extension to higher energies

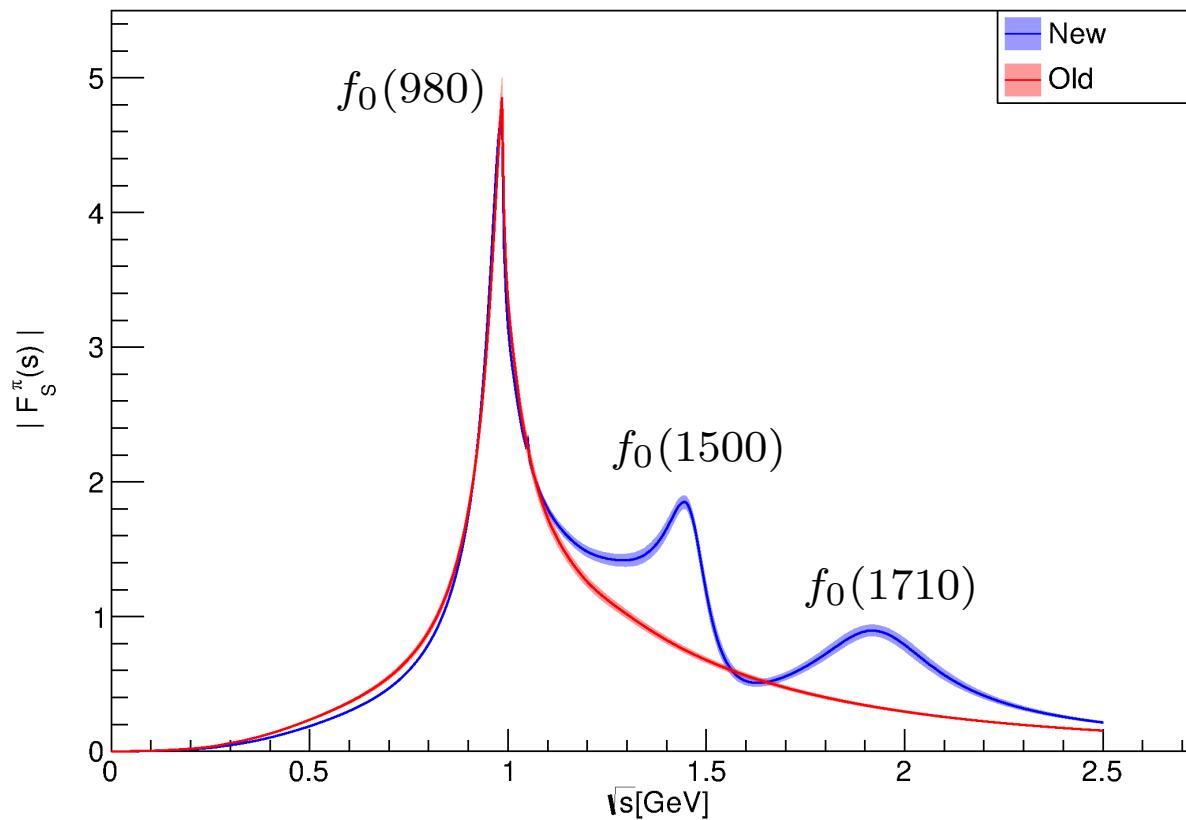
- further inelastic channels ($4\pi \simeq \rho\rho$) coupled through resonances analyticity and unitarity respected
- fit to angular moments:

Hanhart 2012



$\bar{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-$: extension to higher energies

- further inelastic channels ($4\pi \simeq \rho\rho$) coupled through resonances analyticity and unitarity respected Hanhart 2012
- extracted **strange scalar form factor**:

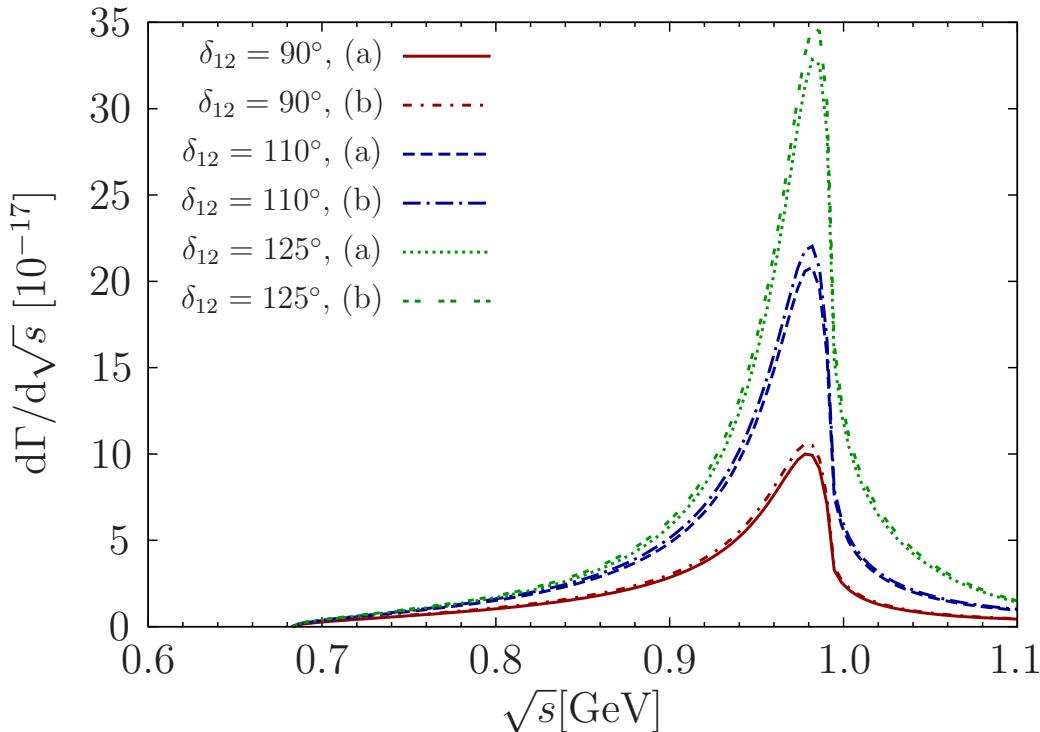


- extraction of **pole positions** still to be done

Ropertz, Hanhart, BK

$\bar{B}_d^0 \rightarrow J/\psi \pi^0 \eta$ prediction

- $\bar{d}d = -\frac{1}{2}(\bar{u}u - \bar{d}d) + \frac{1}{2}(\bar{u}u + \bar{d}d) \rightarrow$ known relative strength of isoscalar and isovector components
- scalar-isovector form factors \rightarrow coupled-channel $\pi\eta / \bar{K}K$
 - ▷ incorporates chiral constraints + resonance information [$a_0(980)$, $a_0(1450)$] + unitarity Albaladejo, Moussallam 2015
 - ▷ measurement of $\bar{B}_d^0 \rightarrow J/\psi \pi^0 \eta \rightarrow$ learn about $\pi\eta$ amplitude

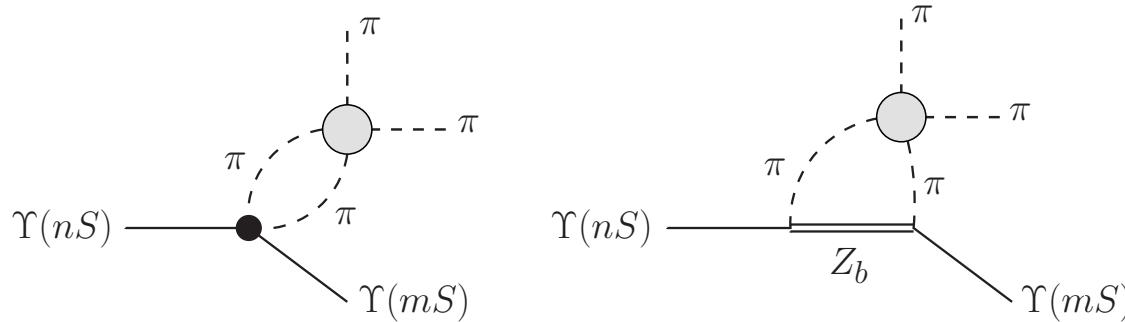


Albaladejo, Daub, Hanhart,
BK, Moussallam 2017

$\Upsilon(nS) \rightarrow \Upsilon(mS)\pi\pi$

- inclusion of Z_b exchanges as left-hand cuts:

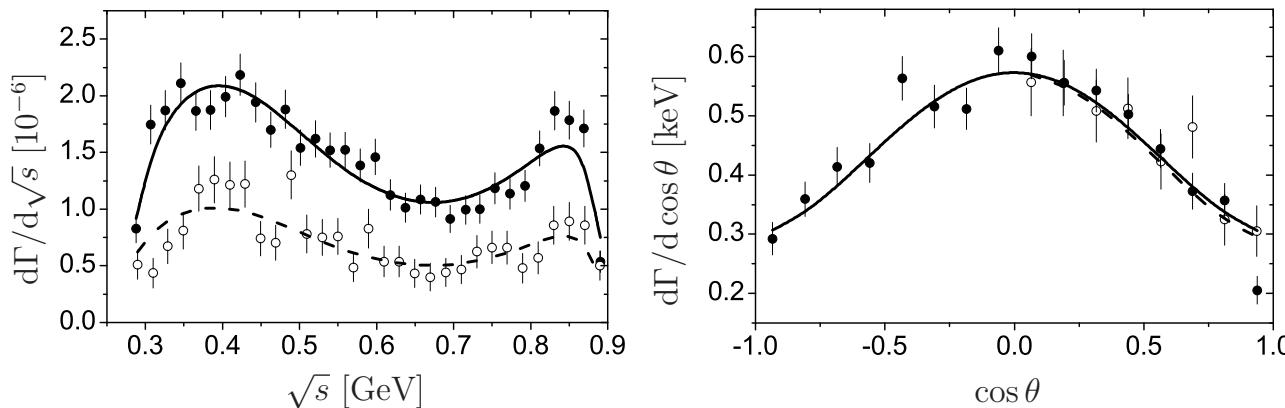
↔ project B.3!



- formally: $\hat{M}(s)$ partial-wave projection of Z_b exchanges

$$M(s) + \hat{M}(s), \quad M(s) = \Omega(s) \left\{ P^{n-1}(s) + \frac{s^n}{\pi} \int_{4M_\pi^2}^\infty \frac{dx}{x^2} \frac{\hat{M}(x) \sin \delta(x)}{|\Omega(x)|(x-s)} \right\}$$

→ e.g. two peaks in $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ reproduced:

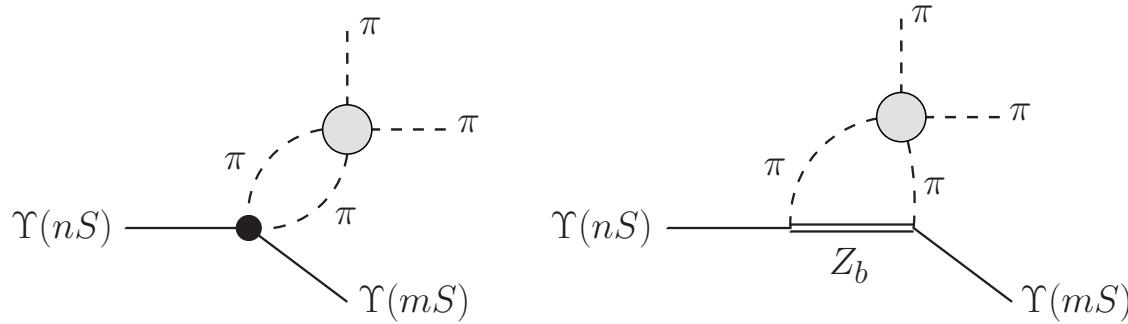


Chen, Daub, Guo, BK, Meißner, Zou 2015

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- generalisation for $\Upsilon(4S)$ decays:

- ▷ add open-flavour ($B^{(*)}\bar{B}^{(*)}$) loops to left-hand cuts
 - ▷ generalise to $\pi\pi/K\bar{K}$ coupled channels for S-waves
- prediction for $\Upsilon(4S) \rightarrow \Upsilon(1S)K\bar{K}$

Chen, Cleven, Daub, Guo, Hanhart, BK, Meißner, Zou 2016

$\eta' \rightarrow \eta\pi\pi$ Dalitz plot

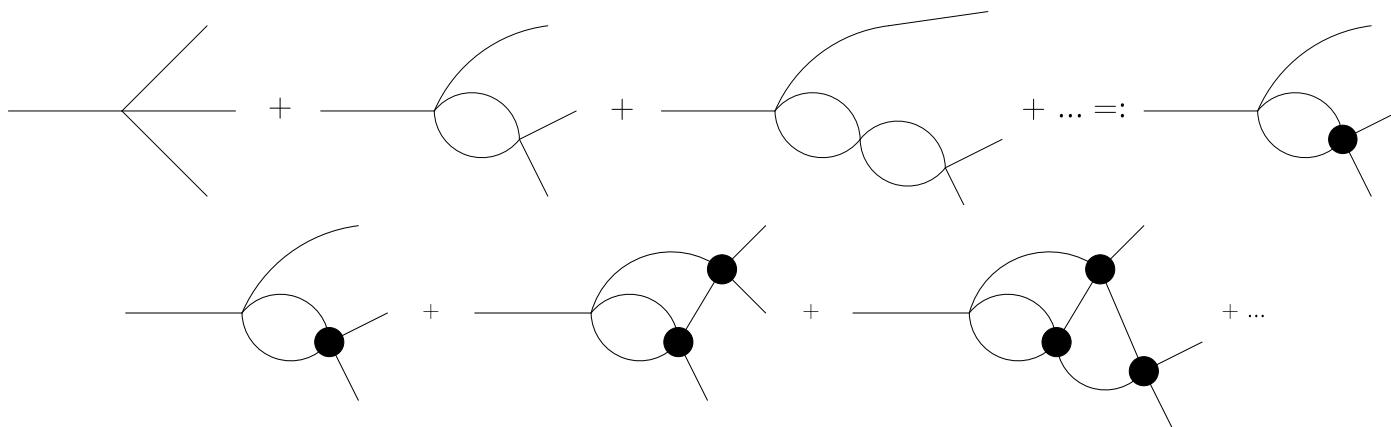
- solve Khuri–Treiman equations Isken, BK, Schneider, Stoffer 2017
input: S-wave phase shifts $\delta_0 \equiv \delta_{\pi\pi}$ and $\delta_1 \equiv \delta_{\pi\eta}$

$$\mathcal{A}(s, t, u) = \mathcal{A}_0(s) + \mathcal{A}_1(t) + \mathcal{A}_1(u),$$

$$\mathcal{A}_0(s) = \Omega_0(s) \left\{ \alpha + \beta s + \frac{s^2}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{dx}{x^2} \frac{\hat{\mathcal{A}}_0(x) \sin \delta_0(x)}{|\Omega_0(x)|(x-s)} \right\}$$

$$\mathcal{A}_1(t) = \Omega_1(t) \left\{ \gamma t + \frac{t^2}{\pi} \int_{t_{\text{thr}}}^{\infty} \frac{dx}{x^2} \frac{\hat{\mathcal{A}}_1(x) \sin \delta_1(x)}{|\Omega_1(x)|(x-t)} \right\}$$

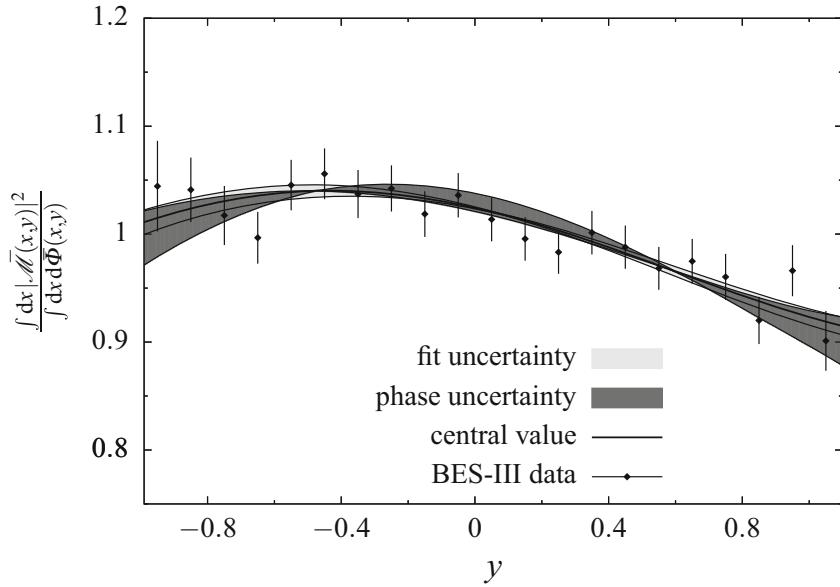
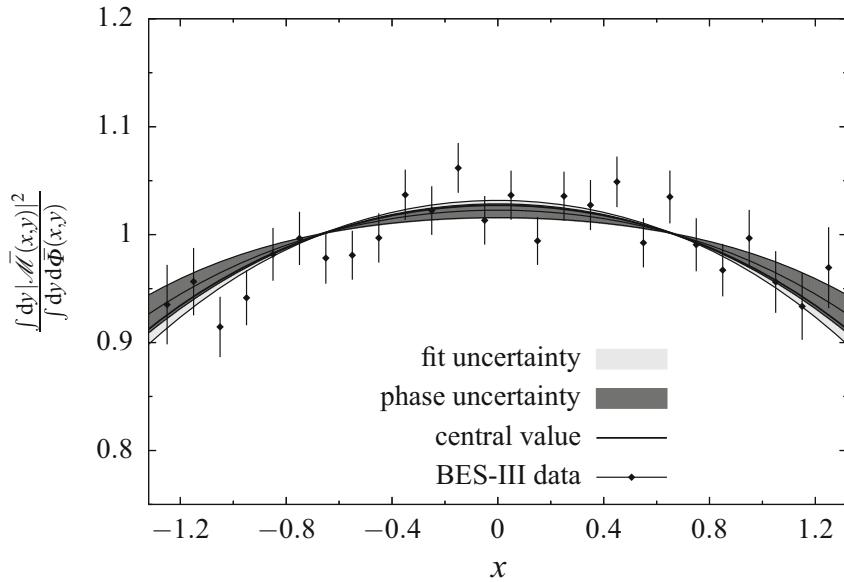
- $\hat{\mathcal{A}}_{0/1}$: partial-wave projections of crossed-channel amplitudes:



→ crossed-channel rescattering fully taken into account

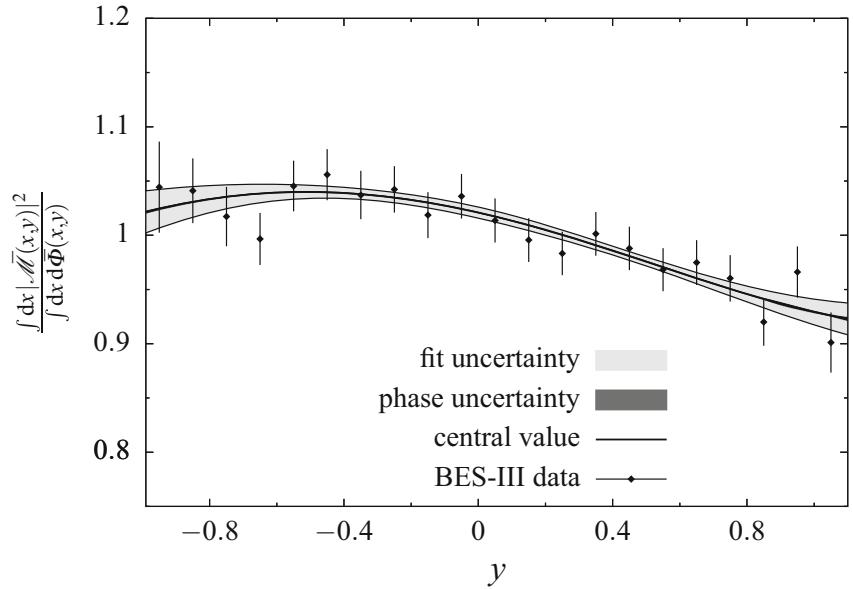
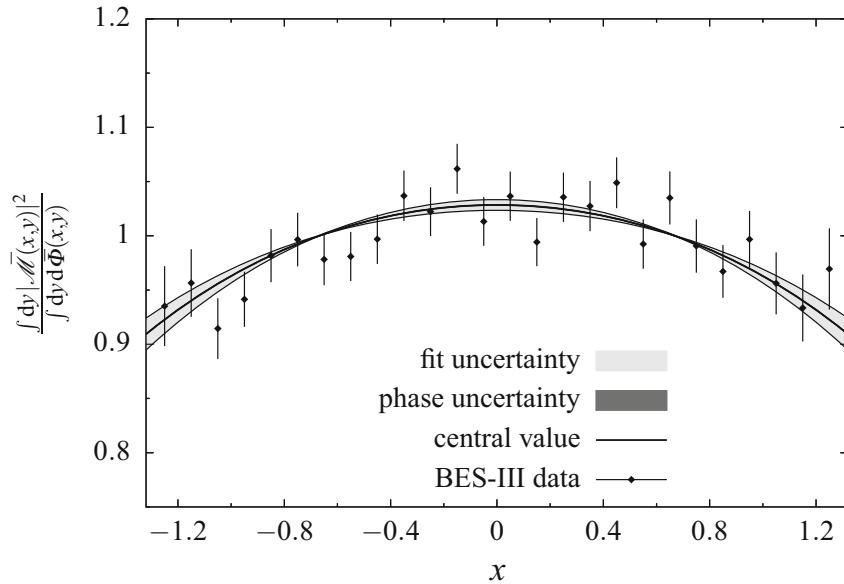
$\eta' \rightarrow \eta\pi\pi$ Dalitz plot

- 3 or 4 subtraction constants: Isken, BK, Schneider, Stoffer 2017
more predictive vs. less dependence on phase shift uncertainty
- one-dimensional projections vs. data: BESIII 2010



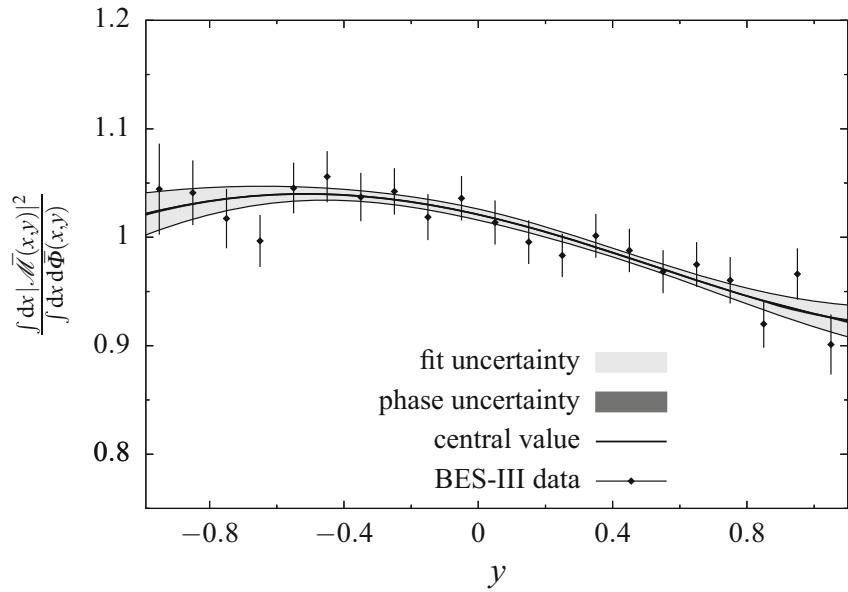
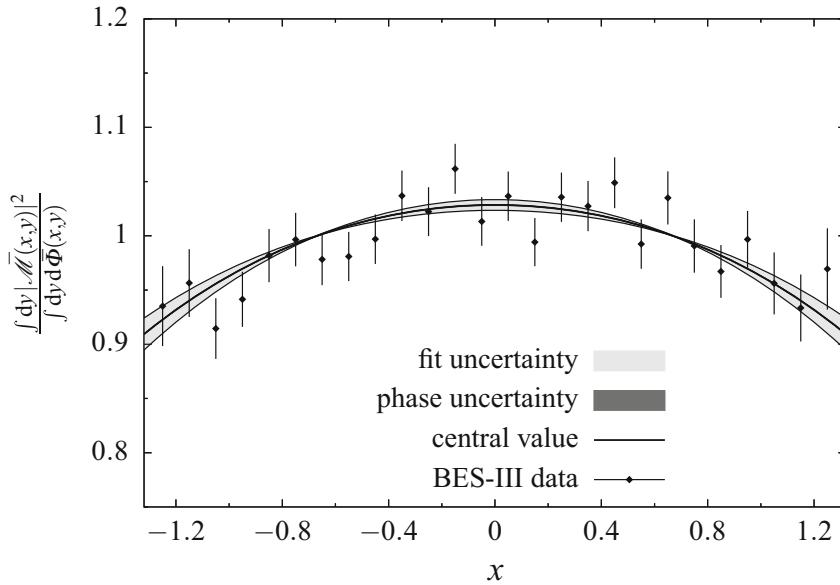
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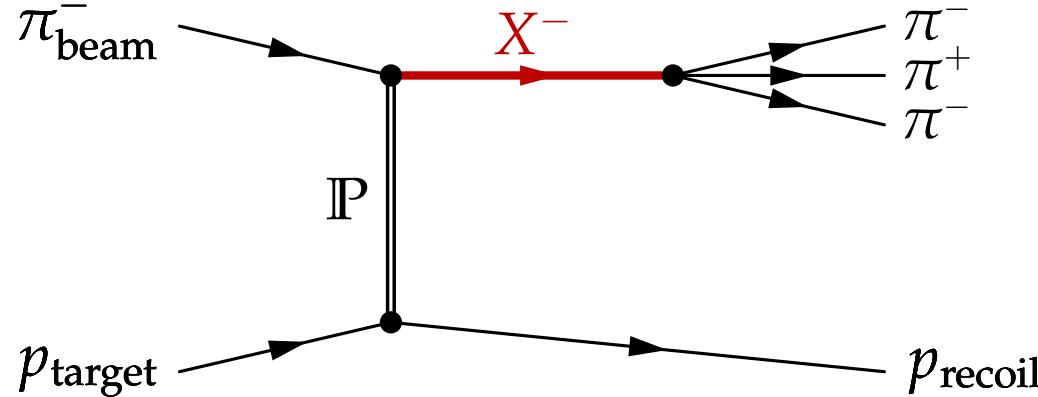
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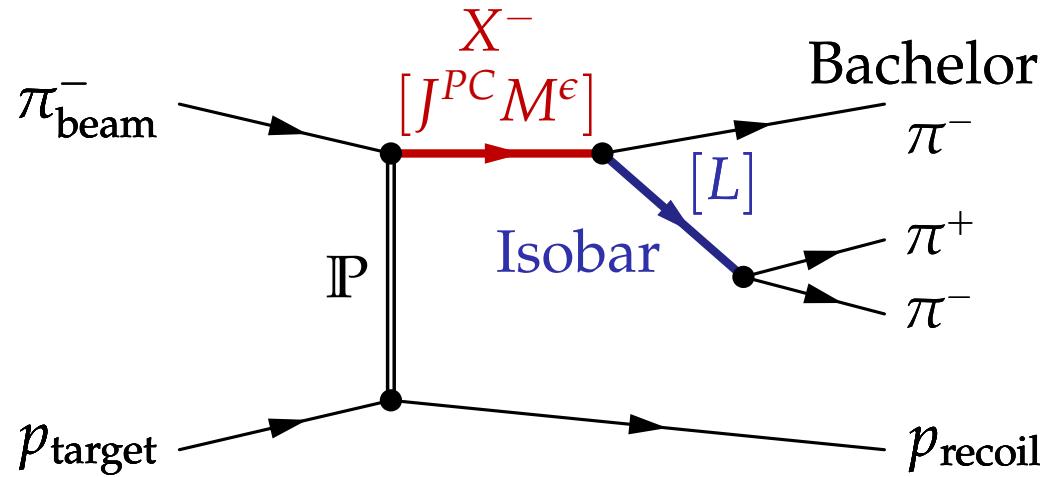
- Dalitz plot parameters well reproduced, higher ones predicted
- analysis tool for future high-precision Dalitz plots A2@MAMI, BESIII
- ingredient for forthcoming $\eta' \rightarrow 3\pi$ analysis BESIII

Diffractive production of 3π final states at COMPASS



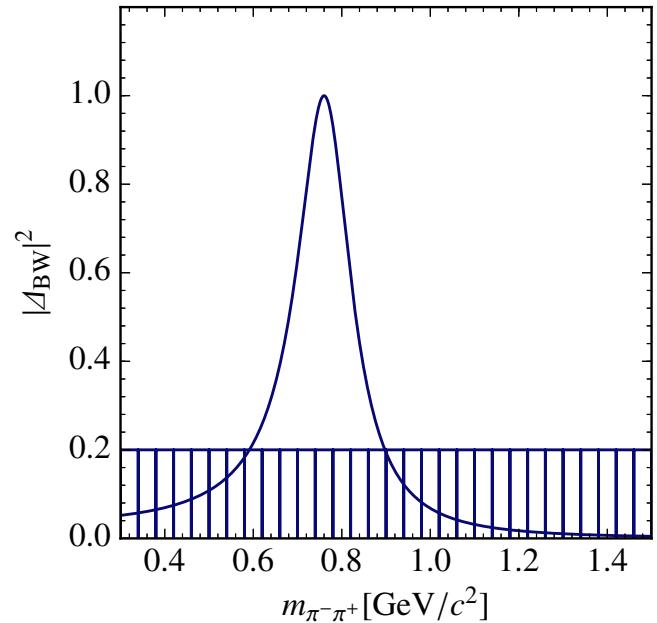
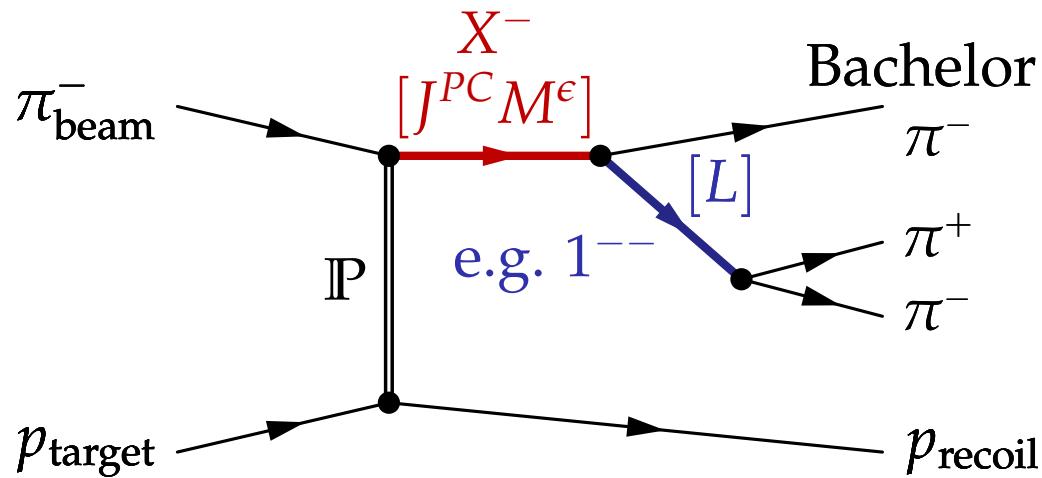
- 190 GeV pion beam on proton target, $\approx 50\text{M}$ events
- study of a_J and π_J states

Diffractive production of 3π final states at COMPASS



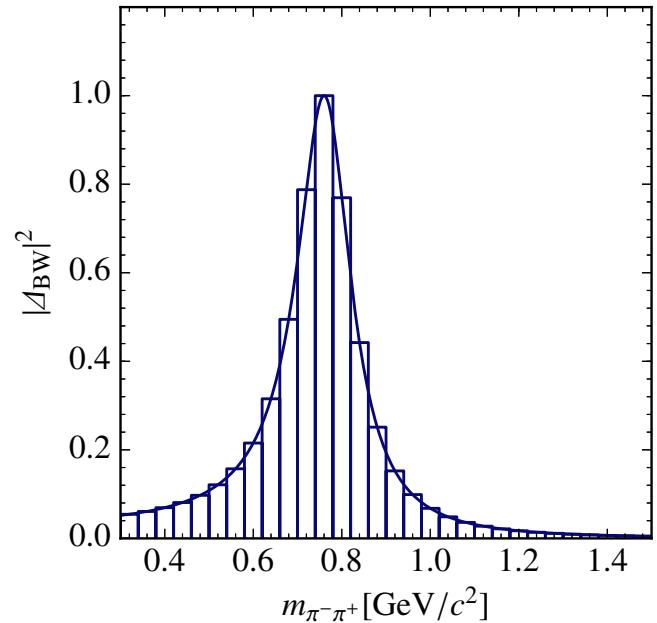
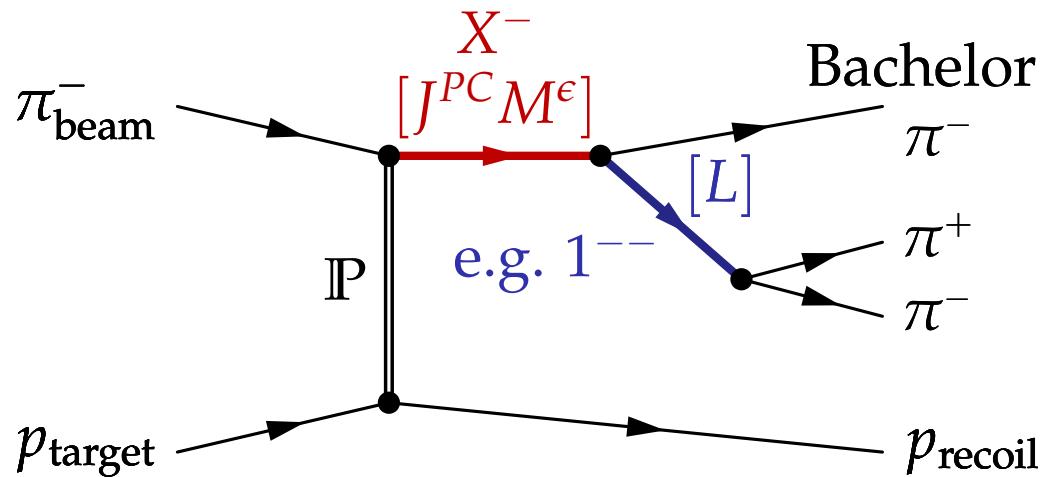
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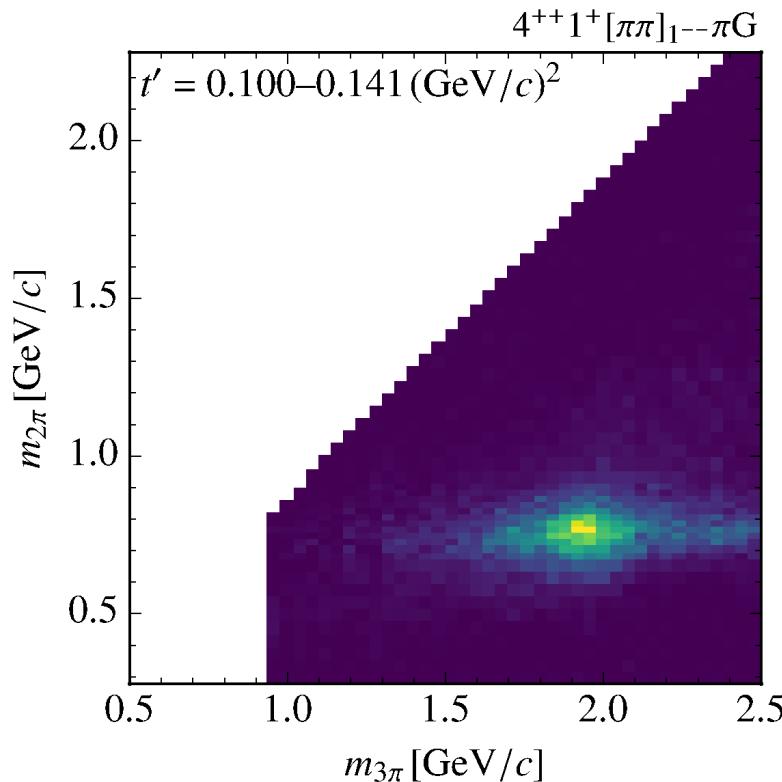


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 - ▷ replace fixed parametrizations for $\pi^-\pi^+$ isobars by sets of step-like functions → determined by PWA fit
 - ▷ challenge: mathematical ambiguities → resolved by additional constraints

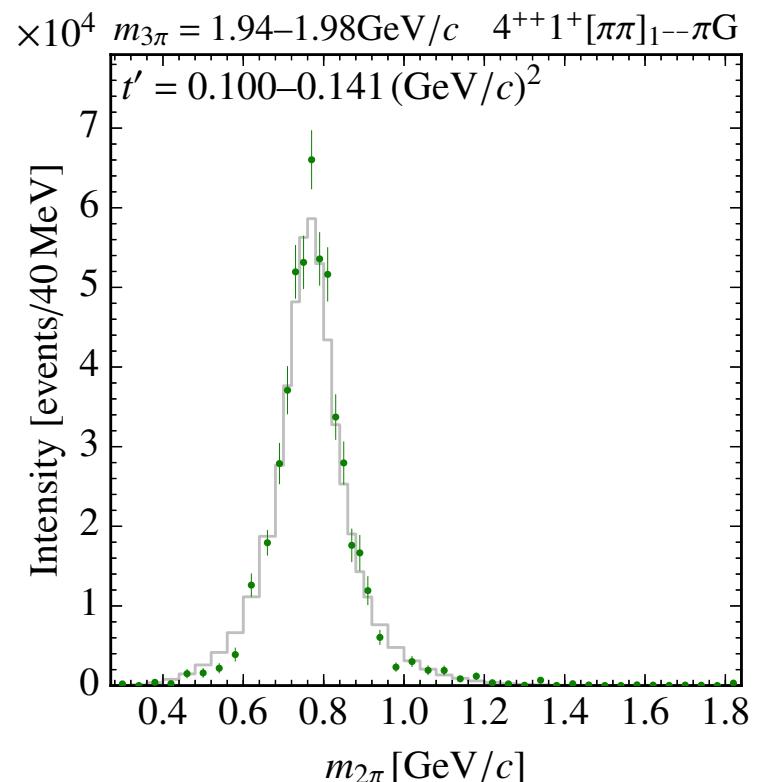
Diffractive production of 3π final states at COMPASS

work in progress (material not officially released by COMPASS)

intensity



intensity at $a_4(2040)$



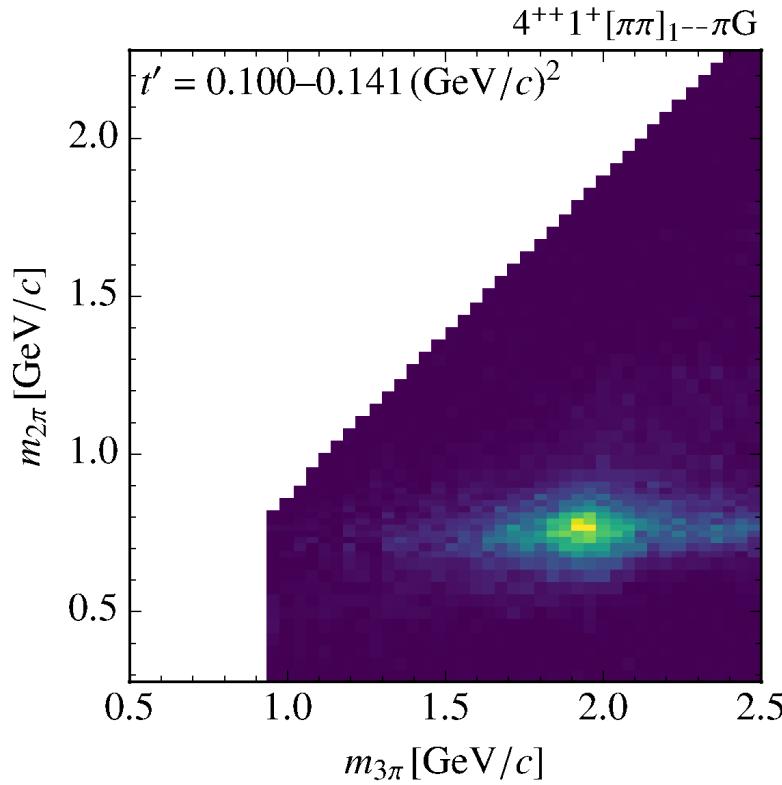
- example: 3π system with $J^{PC} = 4^{++}$
 $\longrightarrow \pi\pi$ subsystem with $J^{PC} = 1^{--}$ and π in G -wave
- peak = $a_4(2040) \rightarrow \rho(770) + \pi$

Grube, Krinner, Paul, Ryabchikov, Wallner

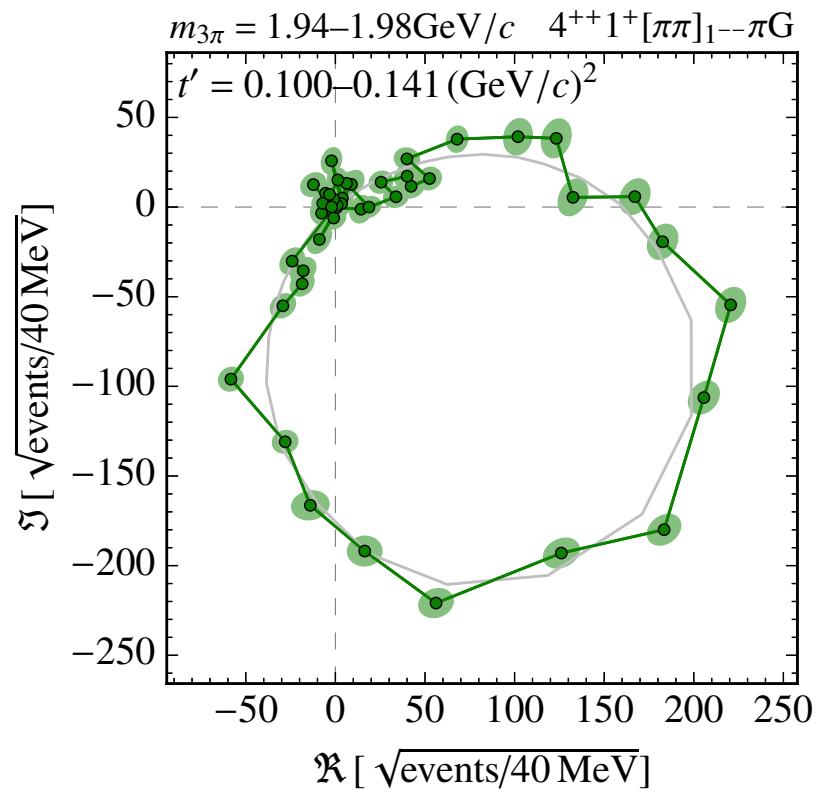
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Argand diagram at $a_4(2040)$

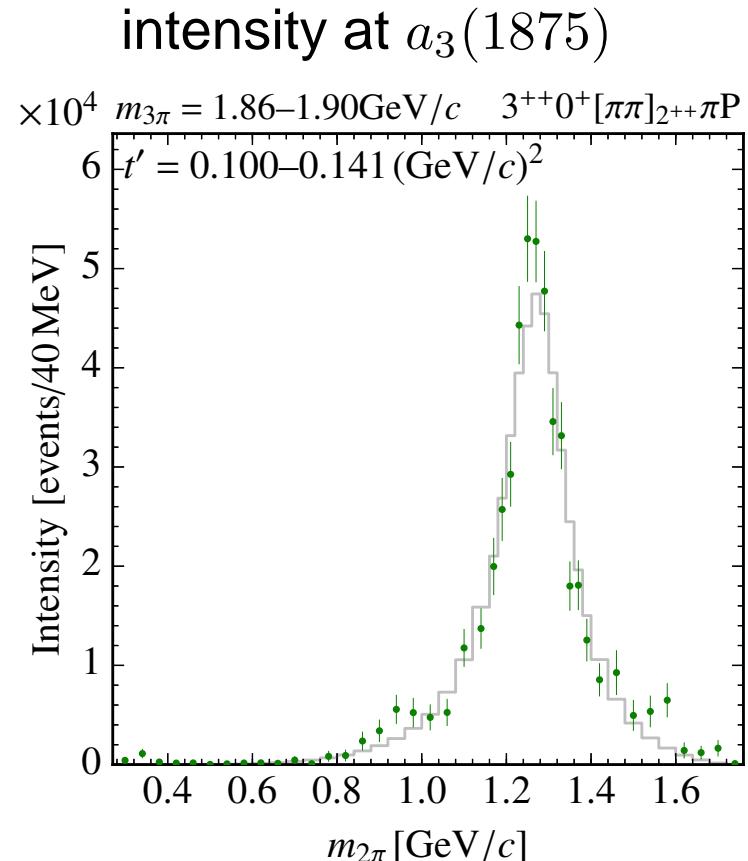
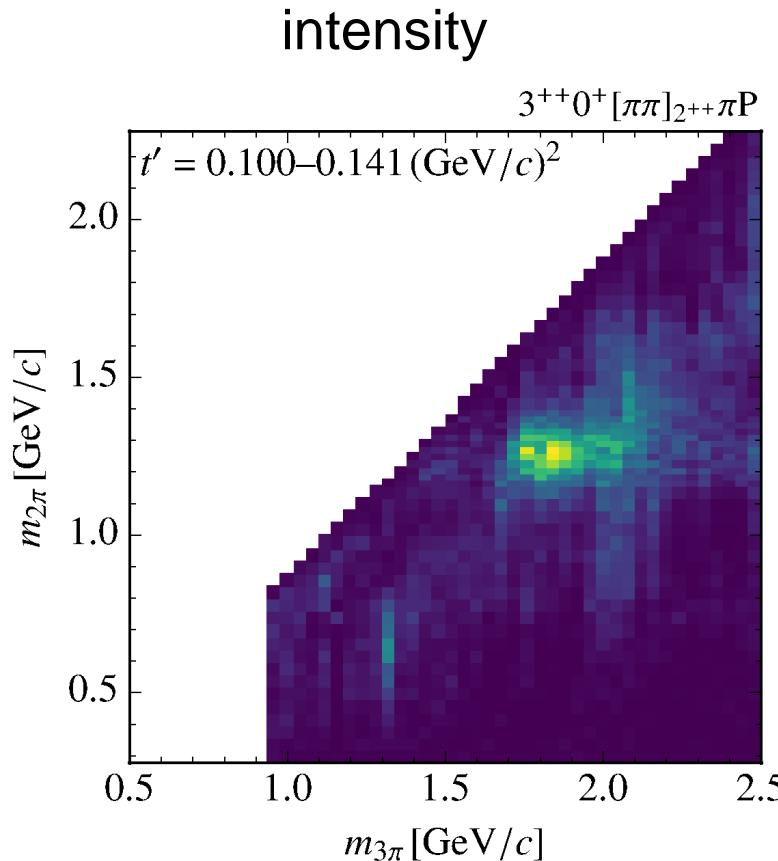


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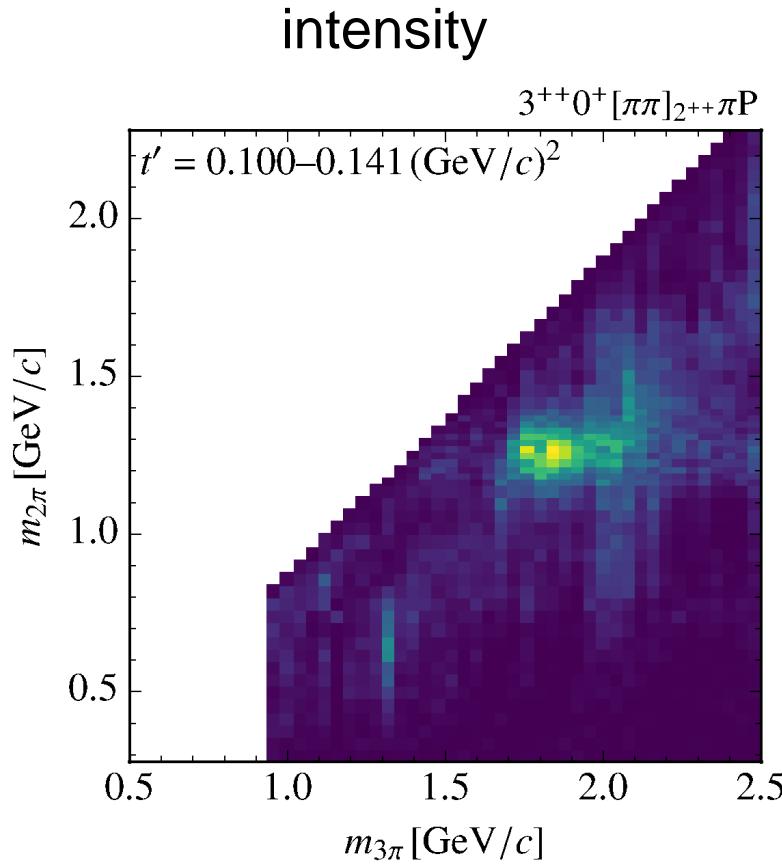


- example: 3π system with $J^{PC} = 3^{--}$
→ $\pi\pi$ subsystem with $J^{PC} = 2^{++}$ and π in P -wave
- peak = $a_3(1875) \rightarrow f_2(1270) + \pi$

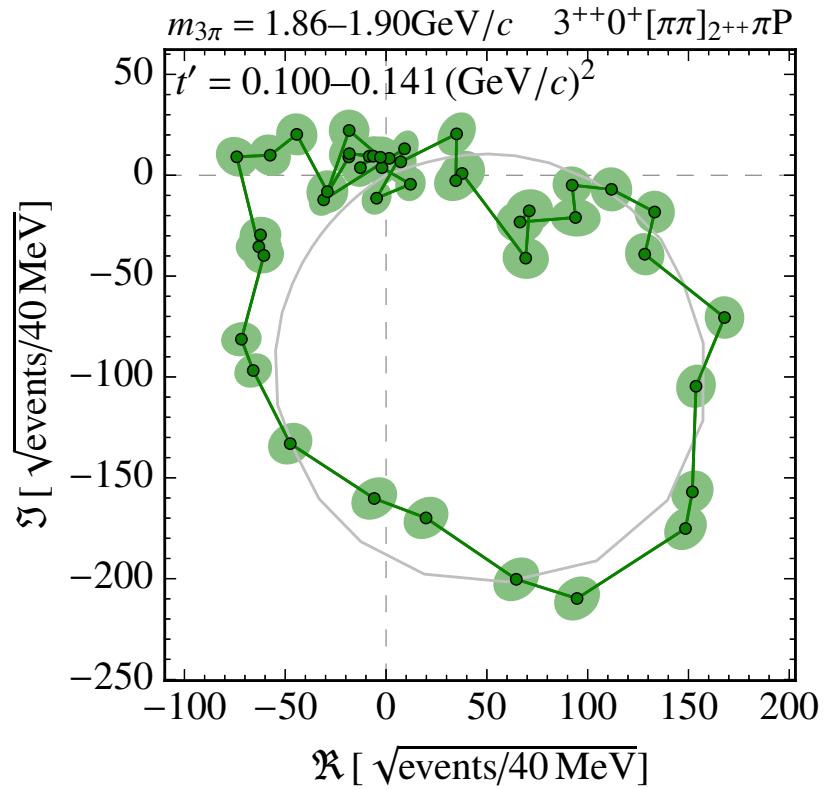
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$\bar{N}N$ interactions

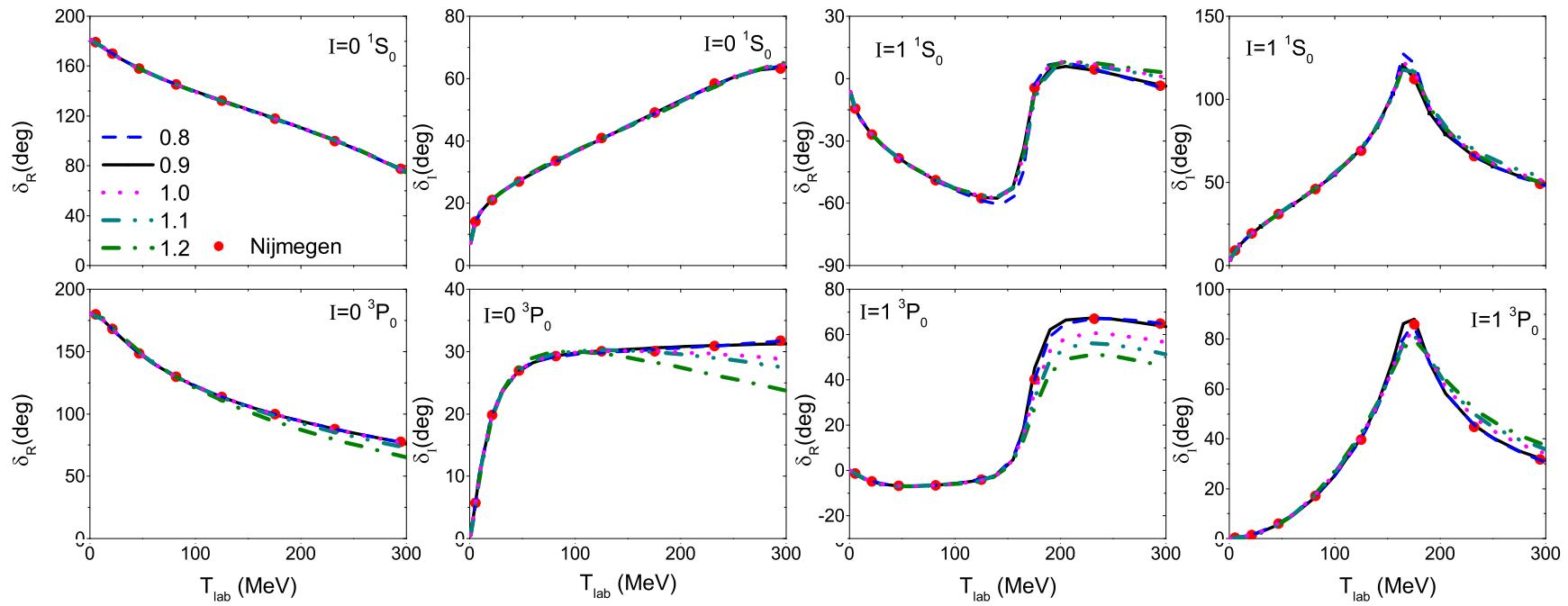
- renewed interested in **$\bar{N}N$ FSI**: $J/\psi \rightarrow V^0 \bar{p}p$, $e^+e^- \rightarrow \bar{p}p \dots$
- theoretical development:
 - ▷ chiral potential at **N^3LO** $\leftrightarrow NN$ potentials (G-parity related)
 - ▷ (imaginary) **annihilation potential** constrained by unitarity
 - ▷ sufficiently high order to consider **observables** (\leftrightarrow PWAs)

Dai, Haidenbauer, Meißner 2017

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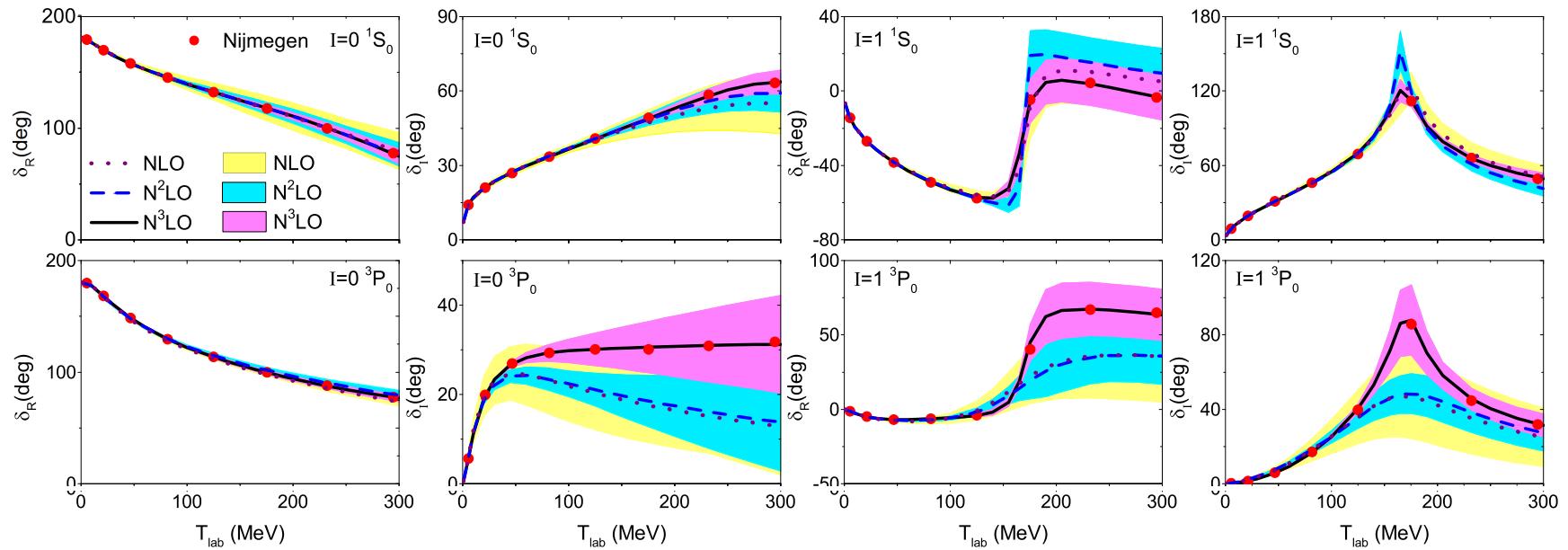


→ cutoff dependence small

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Dai, Haidenbauer, Meißner 2017



→ uncertainty assessed through chiral convergence

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Dai, Haidenbauer, Meißner 2017

- predictions for level shifts and widths of antiprotonic hydrogen:

	NLO	N^2LO	N^3LO	N^2LO [42]	Experiment
E_{1S_0} (eV)	-448	-446	-443	-436	-440(75) [98] -740(150) [97]
Γ_{1S_0} (eV)	1155	1183	1171	1174	1200(250) [98] 1600(400) [97]
E_{3S_1} (eV)	-742	-766	-770	-756	-785(35) [98] -850(42) [99]
Γ_{3S_1} (eV)	1106	1136	1161	1120	940(80) [98] 770(150) [99]

Milestones

2016/2

- (✓) $B \rightarrow \pi\pi\ell\nu_\ell$ including D -waves MSc thesis Stephan Kürten 2017
- (✓) $D \rightarrow \pi K \ell \nu_\ell$ PhD thesis Johanna Daub 2017/18
- (✓) triangle singularities for Z_c / P_c Guo, Meißner, Nieves 2016
Bayar, Aceti, Guo, Oset 2016
- ✓ Coulomb / p - n mass difference in $\bar{N}N$ Dai, Haidenbauer, Meißner 2017
- (✓) amplitude fits of the $\pi\pi$ S-wave COMPASS

2017

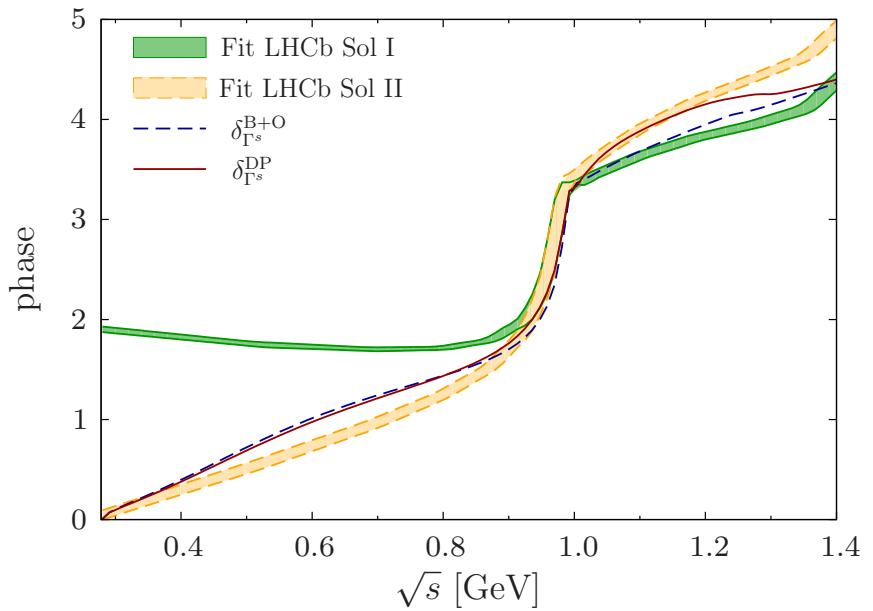
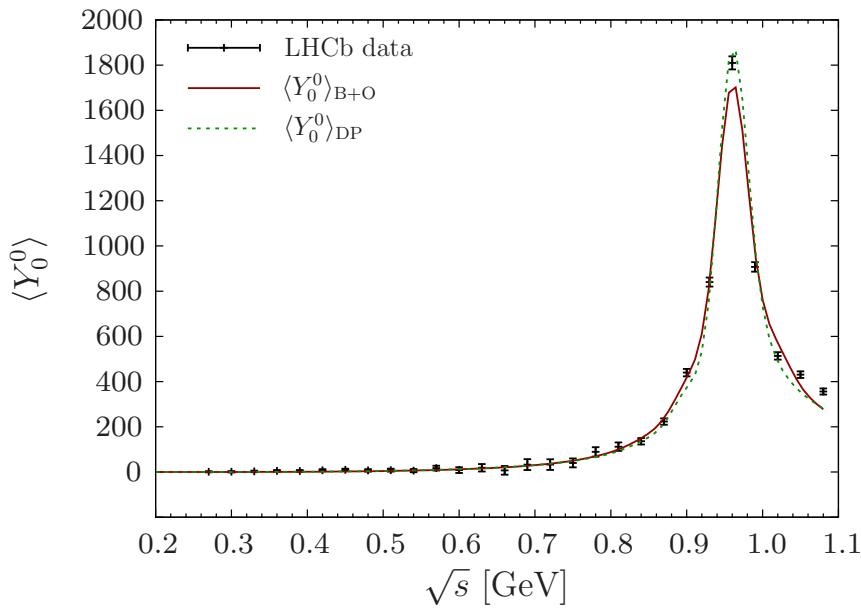
- (✓) $\pi\pi$ scalar form factors to higher energies Ropertz, Hanhart, BK
- (✓) $\eta' \rightarrow 3\pi$ Dalitz plot Isken, BK, Stoffer
- ✓ extension of $\bar{N}N$ to N^3LO Dai, Haidenbauer, Meißner 2017
- (✓) analysis of $K\pi\pi$ final state, extraction of πK S-wave COMPASS

2018

- (✓) study of FSI effects in $e^+e^- \rightarrow \bar{\Lambda}\Lambda$ Haidenbauer, Meißner 2016

Spares

$\bar{B}_s^0 \rightarrow J/\psi \pi^+ \pi^-$: fit results (S-wave)

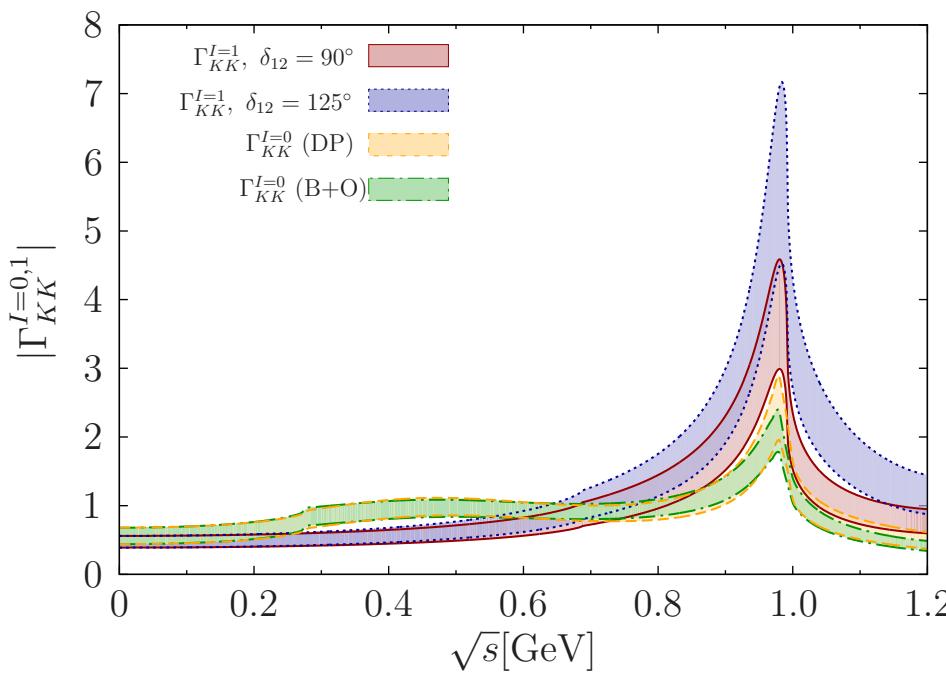


Daub, Hanhart, BK 2015

- 1 fit parameter, phase input varied ($\sqrt{s} \leq 1.05$ GeV)
- LHCb: 4–6 Breit–Wigner/Flatté parameters
- channel coupling allows to predict $\bar{B}_s^0 \rightarrow J/\psi K^+ K^-$ S-wave
→ ca. 1.1% background under the ϕ , agrees with LHCb

$\bar{B}_d^0 \rightarrow J/\psi(K^+K^- \text{ vs. } K^0\bar{K}^0)$

- naive picture: $\bar{d}d$ source \longrightarrow expect $K^0\bar{K}^0$ spectrum to dominate
 - ▷ $s = 0$: $\Gamma_K^{I=0}(0) \approx \Gamma_K^{I=1}(0) \approx 0.5$
 - ▷ constructive ($K^0\bar{K}^0$) / destructive (K^+K^-) interference
 - ▷ *not seen*



- prominent $f_0(980)$, but *more* prominent $a_0(980)$ structure \longrightarrow small $f_0:a_0$ ratio
- similar pole positions \longrightarrow conflict with both being $\bar{K}K$ molecules?

Albaladejo, Daub, Hanhart, BK, Moussallam 2017