Project B8: *Relativistic Corrections to Exclusive $\chi_c + \gamma$ Production from e^+e^- annihilation*

Wen Chen[1]

in collaboration with

Nora Brambilla[2], Yu Jia[1], Vladyslav Shtabovenko[3], and Antonio Vairo[2]

[1]Institute of High Energy Physics, China

[2]Technische Universität München, Germany

[3]Zhe Jiang University, China

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Outline:

- Motivation and Previous Works
- NRQCD Factorization
- Strategies to Determine the NRQCD Matching Coefficients
- Results of SDCs
- Numerical Results
- Conclusion and Discussion

Motivation and Previous Works

- Heavy quarkonia probe all the energy regimes of QCD, and are thus an ideal, and to some extent, unique laboratory to test our understanding of QCD (both perturbative and nonperturbative aspects). (Brambilla and Vairo, RMP (2005), Brambilla, et. al, EPJC (2011)).
- NRQCD provides an elegant approach to separate relativistic physics of annihilation from the nonrelativistic physics of quarkonium structure. (Bodwin, Braaten, and Lepage, PRD (1995))
- Large radiation corrections are found for the production of double charmonia, which can remedy the large discrepancy between experiments and theories. (Zhang, Gao, and Chao, PRL (2006), Gong, and Wang, PRD (2008))
- For charmonia, $v^2 \approx 0.3, \alpha_s \approx 0.24$, thus relativistic correction is as significant as radiation correction.
- Cross section for $e^+e^- \to H_{c\bar{c}} + \gamma$ can be larger than that for $e^+e^- \to J/\psi + H$ by 2 orders, and thus could be observed at B-factory. (Chung, Lee, and Yu, PRD (2008))
- $\mathcal{O}(\alpha_s^0 v^0)$: Chung, Lee, and Yu, PRD 78, 074022 (2008);
- $\mathcal{O}(\alpha_s v^0)$: Sang, and Chen, PRD 81, 034028 (2010); Li, He, and Chao, PRD 80, 114014 (2009).
- $\mathcal{O}(\alpha_s^0 v^2)$: Li, Xu, Liu, and Zhang , JHEP 01, 022 (2014); Chao, He, Li, and Meng, arXive:1310.8597 (2013).
- $\mathcal{O}(\alpha_s v^2)$: Xu, Li, Liu, and Zhang, JHEP 10, 071 (2014);

velocity-scaling rules:

(BBL)

Operator	Estimate	Description
α_s	v	effective quark-gluon coupling constant
ψ	$(Mv)^{3/2}$	heavy-quark (annihilation) field
χ	$(Mv)^{3/2}$	heavy-antiquark (creation) field
D_t	Mv^2	gauge-covariant time derivative
$\mathbf{D}$	Mv	gauge-covariant spatial derivative
$g\mathbf{E}$	M^2v^3	chromoelectric field
$g\mathbf{B}$	M^2v^4	chromomagnetic field
$g\phi$ (in Coulomb gauge)	Mv^2	scalar potential
$g\mathbf{A}$ (in Coulomb gauge)	Mv^3	vector potential

$$i\chi^\dagger \boldsymbol{D}\cdot\boldsymbol{\sigma}\boldsymbol{D}^2\psi \sim g\chi^\dagger \boldsymbol{E}\cdot\boldsymbol{\sigma}\psi \sim v^6$$

NRQCD Factorization

effective Lagrangian:

$$\mathcal{L}_{2-f} = \psi^\dagger \left(\frac{\mathbf{D}^2}{2m} + \frac{\boldsymbol{\sigma}\cdot g\mathbf{B}}{2m} + \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \frac{i\boldsymbol{\sigma}\cdot[\mathbf{D}\times, g\mathbf{E}]}{8m^2} + \frac{\mathbf{D}^4}{8m^3} + \frac{\{\mathbf{D}^2, \boldsymbol{\sigma}\cdot g\mathbf{B}\}}{8m^3} \right)\psi$$
$$+ \chi^\dagger \left(-\frac{\mathbf{D}^2}{2m} - \frac{\boldsymbol{\sigma}\cdot g\mathbf{B}}{2m} + \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \frac{i\boldsymbol{\sigma}\cdot[\mathbf{D}\times, g\mathbf{E}]}{8m^2} - \frac{\mathbf{D}^4}{8m^3} + \frac{\{\mathbf{D}^2, \boldsymbol{\sigma}\cdot g\mathbf{B}\}}{8m^3} \right)\chi.$$

$$+ \sum_n \mathcal{O}_n(\Lambda)$$

n

four-ferion operators

$$\mathcal{O}_n(\Lambda) = \psi^\dagger \mathcal{K}_n' \chi \chi^\dagger \mathcal{K}_n \psi$$

factorization:

decay rate:

$$\Gamma(H) = \sum_n \frac{2\mathrm{Im}\, f_n(\Lambda)}{m^{d_n-4}} \langle H|\mathcal{O}_n(\Lambda)|H\rangle$$

$f_n(\Lambda)$: effective coupling constant

$\mathcal{O}_n(\Lambda)$: four-fermion interactions inclusive cross section:

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$$

$\mathrm{F_n}(\Lambda)$: short-distance coefficients (SDCs)

Perturbative & process-dependent

$\langle 0|\mathcal{O}_\mathrm{n}^\mathrm{H}(\Lambda)|0\rangle$: long-distance matrix elements (LDMEs)

Nonperturbative & process-independent

$$\mathcal{O}_n(\Lambda) = \sum_{\mathrm{pol}} \chi^\dagger \mathcal{K}_n \psi |H\rangle\langle H|\psi^\dagger \mathcal{K}_n' \chi$$

vacuum saturation:

$$\langle H|\mathcal{O}_n(\Lambda)|H\rangle \approx \langle H|\psi^\dagger \mathcal{K}_n'\chi|0\rangle\langle 0|\chi^\dagger\mathcal{K}_n\psi\,|H\rangle = \frac{1}{2J+1}\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$$

Factorization at amplitude level:

(for electromagnetic decay or exclusive electromagnetic production)

$$\mathcal{T} = \sum_n c_n \langle H|\psi^\dagger \mathcal{K}_n' \chi|0\rangle$$

Strategies to Determine the NRQCD Matching Coefficients

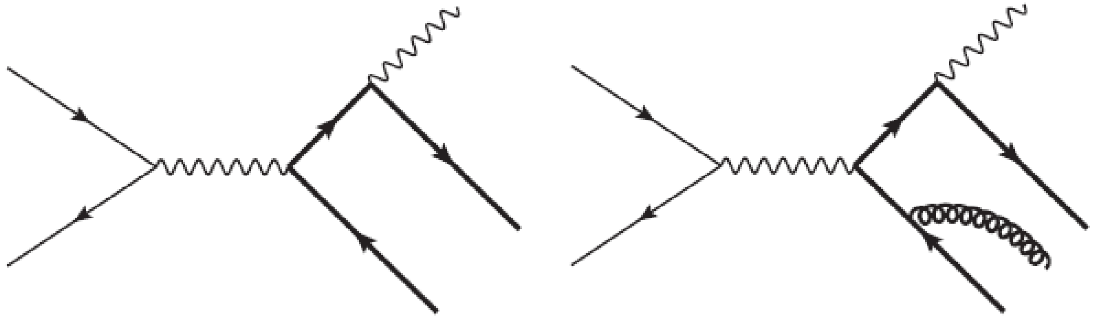


Figure 1. Representative QCD diagrams for the processes $e^+e^- \to Q\overline{Q} + \gamma$ and $e^+e^- \to Q\overline{Q}g + \gamma$ that are relevant for the matching to NRQCD. For the production of $Q\overline{Q}$ there are only two diagrams in total: the one displayed here and a second one where the photon is emitted from the heavy antiquark line. The production of $Q\overline{Q}g$ is described by six diagrams, where the gluon can be emitted before the photon or from a different heavy fermion line.

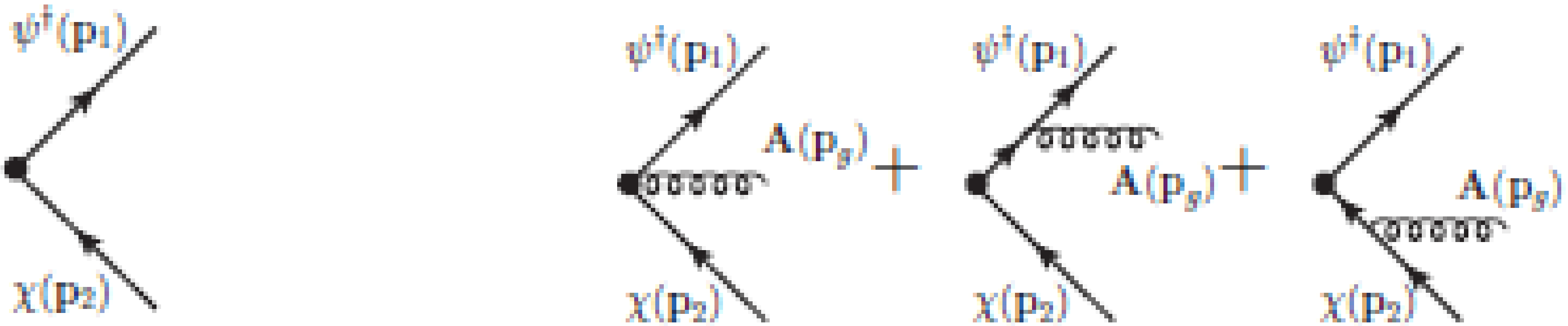


Figure 2. NRQCD diagrams for the process $e^+e^- \to Q\ \bar{Q} + \gamma$ (the first one) and $e^+e^- \to Q\ \bar{Q} + \gamma$ (the last three).

calculation of QCD amplitude

amplitude with definite hadron spin → series expansion in v → spherical decomposition

explicit form of Dirac spinor:

$$u(p) = \sqrt{\frac{E(\mathbf{p},m)+m}{2E(\mathbf{p},m)}}\begin{pmatrix} \xi \\ \frac{\mathbf{p}\cdot\sigma}{E(\mathbf{p},m)+m}\xi \end{pmatrix} \qquad v(p) = \sqrt{\frac{E(\mathbf{p},m)+m}{2E(\mathbf{p},m)}}\begin{pmatrix} \frac{\mathbf{p}\cdot\sigma}{E(\mathbf{p},m)+m}\eta \\ \eta \end{pmatrix}$$

$$\xi^{\dagger}\sigma\eta = \sqrt{2}\epsilon_J$$

polarization vector of hadron

scaling:

quark (antiquark) velocity $q \sim v$

gluon velocity $k \sim v/v^2$

spherical decomposition:

Coope, and Snider JMP 11, 1003 (1970)

$$\mathrm{a}^i\mathrm{b}^j\boldsymbol{\sigma}^i\mathrm{q}^j \to \begin{cases} \frac{1}{3}\delta^{ij}\mathrm{a}^i\mathrm{b}^j(\boldsymbol{\sigma}\cdot\mathrm{q}) & \text{for } J=0 \\ \mathrm{a}^i\mathrm{b}^j\frac{\boldsymbol{\sigma}^i\mathrm{q}^j-\boldsymbol{\sigma}^j\mathrm{q}^i}{2} & \text{for } J=1 \\ \mathrm{a}^i\mathrm{b}^j\left(\frac{\boldsymbol{\sigma}^i\mathrm{q}^j+\boldsymbol{\sigma}^j\mathrm{q}^i}{2}-\frac{1}{3}\delta^{ij}(\boldsymbol{\sigma}\cdot\mathrm{q})\right) & \text{for } J=2 \end{cases}$$

matching

$$\mathcal{A}^{J=0}_{\text{pert. NRQCD}} = \frac{c_0^{J=0}}{m}\langle H|\psi^\dagger\chi|0\rangle + \frac{c_1^{J=0}}{m^2}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma}\right)\chi|0\rangle$$
$$+\frac{c_2^{J=0}}{m^3}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi|0\rangle + \frac{c_3^{J=0}}{m^4}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma}\right)\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi|0\rangle$$
$$+\frac{d_0^{J=0}}{m^3}\langle H|\psi^\dagger g\mathbf{B}\cdot\boldsymbol{\sigma}\chi|0\rangle + \frac{d_1^{J=0}}{m^3}\langle H|\psi^\dagger g\mathbf{E}\cdot\boldsymbol{\sigma}\chi|0\rangle$$

$$\mathcal{A}^{J=1}_{\text{pert. NRQCD}} = \frac{(c_1^{J=1})^i}{m^2}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma}\right)^i\chi|0\rangle$$
$$+\frac{(c_3^{J=1})^i}{m^4}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma}\right)^i\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi|0\rangle + \frac{(d_0^{J=1})^i}{m^3}\langle H|\psi^\dagger(g\mathbf{B}\times\boldsymbol{\sigma})^i\chi|0\rangle$$
$$+\frac{(d_1^{J=1})^i}{m^3}\langle H|\psi^\dagger(g\mathbf{E}\times\boldsymbol{\sigma})^i\chi|0\rangle$$

$$\mathcal{A}^{J=2}_{\text{pert. NRQCD}} = \frac{(c_1^{J=2})^{ij}}{m^2}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)}\right)\chi|0\rangle + \frac{(c_2^{J=2})^{ij}}{m^3}\langle H|\psi^\dagger\left(-\frac{i}{2}\right)^2\overleftrightarrow{\mathbf{D}}^{(i}\overleftrightarrow{\mathbf{D}}^{j)}\chi|0\rangle$$
$$+\frac{(c_3^{J=2})^{ij}}{m^4}\langle H|\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)}\right)\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi|0\rangle$$
$$+\frac{(d_0^{J=2})^{ij}}{m^3}\langle H|\psi^\dagger g\mathbf{B}^{(i}\boldsymbol{\sigma}^{j)}\chi|0\rangle + \frac{(d_1^{J=2})^{ij}}{m^3}\langle H|\psi^\dagger g\mathbf{E}^{(i}\boldsymbol{\sigma}^{j)}\chi|0\rangle$$

cross section

$$\sigma(e^+e^- \to \chi_{c_0} + \gamma) = \frac{F_1(^3P_0)}{3m^4} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|\chi_{c0}\rangle \langle \chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\chi|0\rangle$$
$$+ \frac{G_1(^3P_0)}{6m^6} \left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|\chi_{c0}\rangle \langle \chi_{c0}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.} \right)$$
$$+ \frac{iT_8(^3P_0)}{3m^5} \left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma})\psi|\chi_{c0}\rangle \langle \chi_{c0}|\psi^\dagger(g\mathbf{E} \cdot \boldsymbol{\sigma})\chi|0\rangle + \text{h.c.} \right)$$
$$\equiv \frac{F_1(^3P_0)}{m^4} \langle 0|\mathcal{O}_1(^3P_0)|0\rangle + \frac{G_1(^3P_0)}{m^6} \langle 0|\mathcal{P}_1(^3P_0)|0\rangle + \frac{T_8(^3P_0)}{m^5} \langle 0|\mathcal{T}_8(^3P_0)|0\rangle \,,$$

$$\sigma(e^+e^- \to \chi_{c1} + \gamma) = \frac{F_1(^3P_1)}{2m^4} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\psi|\chi_{c1}\rangle \cdot \langle \chi_{c1}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\chi|0\rangle$$
$$+ \frac{G_1(^3P_1)}{4m^6} \left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\psi|\chi_{c1}\rangle \cdot \langle \chi_{c1}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.} \right)$$
$$+ \frac{iT_8(^3P_1)}{2m^5} \left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}} \times \boldsymbol{\sigma})\psi|\chi_{c1}\rangle \cdot \langle \chi_{c1}|\psi^\dagger(g\mathbf{E} \times \boldsymbol{\sigma})\chi|0\rangle + \text{h.c.} \right)$$

$$\equiv \frac{F_1(^3P_1)}{m^4} \langle 0|\mathcal{O}_1(^3P_1)|0\rangle + \frac{G_1(^3P_1)}{m^6} \langle 0|\mathcal{P}_1(^3P_1)|0\rangle + \frac{T_8(^3P_1)}{m^5} \langle 0|\mathcal{T}_8(^3P_1)|0\rangle$$

$$\sigma(e^+e^- \to \chi_{c2} + \gamma) = \frac{F_1(^3P_2)}{m^4} \langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\psi|\chi_{c2}\rangle \langle \chi_{c2}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\chi|0\rangle$$
$$+ \frac{G_1(^3P_2)}{2m^6} \left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\psi|\chi_{c2}\rangle \langle \chi_{c2}|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle + \text{h.c.} \right)$$
$$+ \frac{iT_8(^3P_2)}{m^5} \left(\langle 0|\chi^\dagger(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}^{(i}\boldsymbol{\sigma}^{j)})\psi|\chi_{c2}\rangle \langle \chi_{c2}|\psi^\dagger(g\mathbf{E}^{(i}\boldsymbol{\sigma}^{j)})\chi|0\rangle + \text{h.c.} \right)$$
$$\equiv \frac{F_1(^3P_2)}{m^4} \langle 0|\mathcal{O}_1(^3P_2)|0\rangle + \frac{G_1(^3P_2)}{m^6} \langle 0|\mathcal{P}_1(^3P_2)|0\rangle + \frac{T_8(^3P_2)}{m^5} \langle 0|\mathcal{T}_8(^3P_2)|0\rangle \,,$$

Results of SDCs

SDCs in the rest frame of hadron

$$c_{0,R}^{J=0} = -\lambda(\mathbf{V}\cdot(\hat{\mathbf{k}}_R \times \varepsilon^*_{\gamma,R})),$$
$$c_{1,R}^{J=0} = \frac{i}{3}(1-2a)\lambda(\mathbf{V}\cdot\varepsilon^*_{\gamma,R}),$$
$$c_{2,R}^{J=0} = \frac{2}{3}\lambda(\mathbf{V}\cdot(\hat{\mathbf{k}}_R \times \varepsilon^*_{\gamma,R})),$$
$$c_{3,R}^{J=0} = -\frac{i}{30}(9-16a)\,\lambda(\mathbf{V}\cdot\varepsilon^*_{\gamma,R}),$$
$$d_{0,R}^{J=0} = -(1+a)\lambda(\mathbf{V}\cdot(\hat{\mathbf{k}}_R \times \varepsilon^*_{\gamma,R})),$$
$$d_{1,R}^{J=0} = \frac{1}{6}(1+6a+4a^2)\lambda(\mathbf{V}\cdot\varepsilon^*_{\gamma,R}),$$

$$(c_{1,R}^{J=1})^i = \frac{i}{2}\lambda\left(2(1+a)(\mathbf{V}\times\varepsilon^*_{\gamma,R})^i - (\mathbf{V}\cdot\hat{\mathbf{k}}_R)(\hat{\mathbf{k}}_R\times\varepsilon^*_{\gamma,R})^i\right),$$
$$(c_{3,R}^{J=1})^i = -\frac{i}{10}\lambda\left((8+3a)(\mathbf{V}\times\varepsilon^*_{\gamma,R})^i - 4(\mathbf{V}\cdot\hat{\mathbf{k}}_R)(\hat{\mathbf{k}}_R\times\varepsilon^*_{\gamma,R})^i\right),$$
$$(d_{0,R}^{J=1})^i = \frac{1}{2}\lambda\left(2(1+a)(\mathbf{V}\cdot\varepsilon^*_{\gamma,R})\hat{\mathbf{k}}_R^i - (1+2a)(\mathbf{V}\cdot\hat{\mathbf{k}}_R)\varepsilon_R^{*i}\right),$$
$$(d_{1,R}^{J=1})^i = -\frac{1}{2}\lambda\left((2+3a+2a^2)(\mathbf{V}\times\varepsilon^*_{\gamma,R})^i - (1+a)(\mathbf{V}\cdot\hat{\mathbf{k}}_R)(\hat{\mathbf{k}}_R\times\varepsilon^*_{\gamma,R})^i\right),$$

$$(c_{1,R}^{J=2})^{ij} = -\frac{i}{2}\lambda\left(2a(\mathbf{V}^i\varepsilon_R^{*j} + i\leftrightarrow j) + (\mathbf{V}\cdot\hat{\mathbf{k}}_R)(\hat{\mathbf{k}}_R^i\varepsilon_R^{*j} + i\leftrightarrow j) - 2(\mathbf{V}\cdot\varepsilon^*_{\gamma,R})\hat{\mathbf{k}}_R^i\hat{\mathbf{k}}_R^j\right),$$
$$(c_{2,R}^{J=2})^{ij} = -\lambda(\mathbf{V}\cdot(\hat{\mathbf{k}}_R\times\varepsilon^*_{\gamma,R}))\,\hat{\mathbf{k}}_R^i\hat{\mathbf{k}}_R^j,$$
$$(c_{3,R}^{J=2})^{ij} = \frac{i}{10}\lambda\left(5a(\mathbf{V}^i\varepsilon_R^{*j} + i\leftrightarrow j) + 5(\mathbf{V}\cdot\hat{\mathbf{k}}_R)(\hat{\mathbf{k}}_R^i\varepsilon_R^{*j} + i\leftrightarrow j) - 6(\mathbf{V}\cdot\varepsilon^*_{\gamma,R})\hat{\mathbf{k}}_R^i\hat{\mathbf{k}}_R^j\right),$$
$$(d_{1,R}^{J=2})^{ij} = \frac{1}{2}\lambda\left((1+a)(\mathbf{V}\cdot\hat{\mathbf{k}}_R)(\hat{\mathbf{k}}_R^i\varepsilon_R^{*j} + i\leftrightarrow j) + a(1+2a)(\mathbf{V}^i\varepsilon_R^{*j} + i\leftrightarrow j) - 2(1+a)(\mathbf{V}\cdot\hat{\mathbf{k}}_R)\hat{\mathbf{k}}_R^i\hat{\mathbf{k}}_R^j\right)$$

with $a \equiv \frac{m}{|\mathbf{k}_R|}$, $V^i \equiv (\delta^{ij} - \frac{\mathbf{k}_R^i\mathbf{k}_R^j}{\sqrt{s+\mathbf{k}_R^2}})\boldsymbol{L}_R^j$ and $\lambda \equiv e^2e_Q^2/s$.

SDCs in the laboratory frame

$$c_0^{J=0} = -\lambda(\boldsymbol{L}\cdot(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)),\qquad c_3^{J=0} = -\frac{i}{30}(9-24r+35r^2)\bar{r}^2\lambda(\boldsymbol{L}\cdot\varepsilon_\gamma^*),$$

$$c_1^{J=0} = \frac{i}{3}(1-3r)\bar{r}\lambda(\boldsymbol{L}\cdot\varepsilon_\gamma^*),\qquad d_0^{J=0} = -\bar{r}\lambda(\boldsymbol{L}\cdot(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)),$$

$$c_2^{J=0} = \frac{2}{3}\lambda(\boldsymbol{L}\cdot(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)),\qquad d_1^{J=0} = \frac{1}{6}(1+3r)\bar{r}\lambda(\boldsymbol{L}\cdot\varepsilon_\gamma^*),$$

$$(c_1^{J=1})^i = -i\lambda\bar{r}\left((1-\sqrt{r})(\boldsymbol{L}\cdot\hat{\mathrm{k}})(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)^i - (\boldsymbol{L}\times\varepsilon_\gamma^*)^i\right),$$

$$(c_3^{J=1})^i = \frac{i}{10}\lambda\bar{r}^2\left((1-\sqrt{r})^2(8+13\sqrt{r})(\boldsymbol{L}\cdot\hat{\mathrm{k}})(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)^i - 2(4-9r)(\boldsymbol{L}\times\varepsilon_\gamma^*)^i\right),$$

$$(d_0^{J=1})^i = -\lambda\bar{r}\left(\sqrt{r}\,\varepsilon_\gamma^{*i}(\boldsymbol{L}\cdot\hat{\mathrm{k}}) - \hat{\mathrm{k}}^i(\boldsymbol{L}\cdot\varepsilon_\gamma^*)\right),$$

$$(d_1^{J=1})^i = \frac{1}{2}\lambda\bar{r}\left((2-\sqrt{r})(\boldsymbol{L}\cdot\hat{\mathrm{k}})(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)^i - 2(\boldsymbol{L}\times\varepsilon_\gamma^*)^i\right),$$

$$(c_1^{J=2})^{ij} = -i\lambda\bar{r}\left((1-\sqrt{r})\sqrt{r}(\boldsymbol{L}\cdot\hat{\mathrm{k}})(\hat{\mathrm{k}}^i\varepsilon_\gamma^{*j}+i\leftrightarrow j) + r(\boldsymbol{L}^i\varepsilon_\gamma^{*j}+i\leftrightarrow j)\ -(1-r)(\boldsymbol{L}\cdot\varepsilon_\gamma^*)\hat{\mathrm{k}}^i\hat{\mathrm{k}}^j\right),$$

$$(c_2^{J=2})^{ij} = -\lambda(\boldsymbol{L}\cdot(\hat{\mathrm{k}}\times\varepsilon_\gamma^*)\,\hat{\mathrm{k}}^i\hat{\mathrm{k}}^j),$$

$$(c_3^{J=2})^{ij} = \frac{i}{10}\lambda\bar{r}^2\left(5\sqrt{r}(1+2\sqrt{r})(1-\sqrt{r})^2(\boldsymbol{L}\cdot\hat{\mathrm{k}})(\hat{\mathrm{k}}^i\varepsilon_\gamma^{*j}+i\leftrightarrow j)\ -10r^2(\boldsymbol{L}^i\varepsilon_\gamma^{*j}+i\leftrightarrow j) - 6(1-r)^2(\boldsymbol{L}\cdot\varepsilon_\gamma^*)\hat{\mathrm{k}}^i\hat{\mathrm{k}}^j\right),$$

$$(d_0^{J=2})^{ij} = 0,$$

$$(d_1^{J=2})^{ij} = \frac{1}{2}\lambda\bar{r}\left(\sqrt{r}(\boldsymbol{L}\cdot\hat{\mathrm{k}})(\hat{\mathrm{k}}^i\varepsilon_\gamma^{*j}+i\leftrightarrow j) - 2\hat{\mathrm{k}}^i\hat{\mathrm{k}}^j(\boldsymbol{L}\cdot\varepsilon_\gamma^*)\right),$$

SDCs in the rest frame of the hadron and those in the laboratory frame are related to each other through Lorentz boost combining with Gremm-Kapustin relations (Gremm, and Kapustin, PLB, 1997):

$$(M_{\chi_{c0}} - 2m)\langle\chi_{c0}|\, i\psi^\dagger \mathbf{D}\cdot\boldsymbol{\sigma}\chi\,|0\rangle \approx -\frac{1}{m}\langle\chi_{c0}|\, i\psi^\dagger \mathbf{D}\cdot\boldsymbol{\sigma}\mathbf{D}^2\chi\,|0\rangle + i\,\langle\chi_{c0}|\,\psi^\dagger g_s\mathbf{E}\cdot\boldsymbol{\sigma}\chi\,|0\rangle$$

$$(M_{\eta_c} - 2m)\langle\eta_c|\,\psi^\dagger\chi\,|0\rangle \approx -\frac{1}{m}\langle\eta_c|\,\psi^\dagger\mathbf{D}^2\chi\,|0\rangle - \frac{1}{m}\langle\eta_c|\,\psi^\dagger g_s\mathbf{B}\cdot\boldsymbol{\sigma}\chi\,|0\rangle$$

$$\begin{aligned}(M_{\chi_{c1}} - 2m)\langle\chi_{c1}|\,\psi^\dagger i(\mathbf{D}\times\boldsymbol{\sigma})^i\chi\,|0\rangle &\approx -\frac{1}{m}\langle\chi_{c1}|\,\psi^\dagger i(\mathbf{D}\times\boldsymbol{\sigma})^i\mathbf{D}^2\chi\,|0\rangle \\ &\quad + i\,\langle\chi_{c1}|\,\psi^\dagger g_s(\mathbf{E}\times\boldsymbol{\sigma})^i\chi\,|0\rangle\end{aligned}$$

$$(M_{\chi_{c2}} - 2m)\langle\chi_{c2}|\,\psi^\dagger iD^{(i}\sigma^{j)}\chi\,|0\rangle \approx -\frac{1}{m}\langle\chi_{c2}|\,\psi^\dagger iD^{(i}\sigma^{j)}\mathbf{D}^2\chi\,|0\rangle + i\,\langle\chi_{c2}|\,\psi^\dagger g_s E^{(i}\sigma^{j)}\chi\,|0\rangle$$

SDCs for cross section

$$F_1(^3P_0) = \frac{16\pi^2\alpha^3 e_Q^4(1-3r)^2 M_{\chi_{c0}}(s-M_{\chi_{c0}}^2)}{9s^3(1-r)^2},$$

$$G_1(^3P_0) = -\frac{16\pi^2\alpha^3 e_Q^4(1-3r)(9-24r+35r^2) M_{\chi_{c0}}(s-M_{\chi_{c0}}^2)}{45s^3(1-r)^3},$$

$$T_8(^3P_0) = \frac{8\pi^2\alpha^3 e_Q^4(1-9r^2) M_{\chi_{c0}}(s-M_{\chi_{c0}}^2)}{9s^3(1-r)^2},$$

$$F_1(^3P_1) = \frac{32\pi^2\alpha^3 e_Q^4(1+r) M_{\chi_{c1}}(s-M_{\chi_{c1}}^2)}{3s^3(1-r)^2},$$

$$G_1(^3P_1) = -\frac{32\pi^2\alpha^3 e_Q^4(8-15r-13r^2) M_{\chi_{c1}}(s-M_{\chi_{c1}}^2)}{15s^3(1-r)^3},$$

$$T_8(^3P_1) = -\frac{16\pi^2\alpha^3 e_Q^4(2+r) M_{\chi_{c1}}(s-M_{\chi_{c1}}^2)}{3s^3(1-r)^2},$$

$$F_1(^3P_2) = \frac{32\pi^2\alpha^3 e_Q^4(1+3r+6r^2) M_{\chi_{c2}}(s-M_{\chi_{c2}}^2)}{9s^3(1-r)^2},$$

$$G_1(^3P_2) = -\frac{16\pi^2\alpha^3 e_Q^4(2+3r)(3-3r-20r^2) M_{\chi_{c2}}(s-M_{\chi_{c2}}^2)}{45s^3(1-r)^3},$$

$$T_8(^3P_2) = -\frac{16\pi^2\alpha^3 e_Q^4(2+3r) M_{\chi_{c2}}(s-M_{\chi_{c2}}^2)}{9s^3(1-r)^2}.$$

Numerical Results

Input:

$$\langle 0|\mathcal{O}_1(^3P_J)|0\rangle = 0.107\,\mathrm{GeV}^5$$ PRD 52, 1726 (1995)

pole mass of c quark: $m_c = 1.44 \pm 0.03\mathrm{GeV}$

fine structure constant: $\alpha = 1/137$

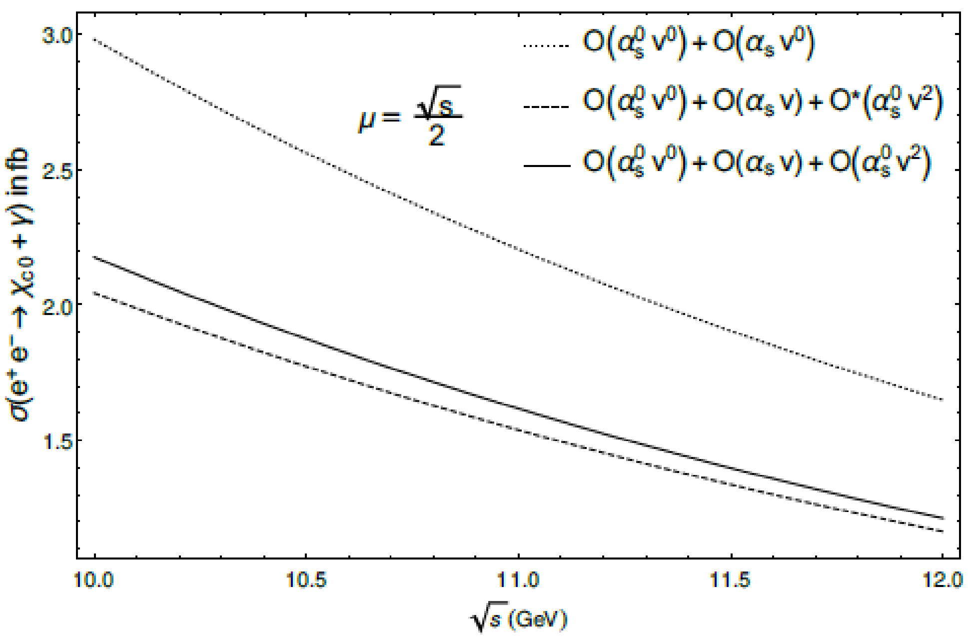
strong coupling constant: $\alpha_s(m_c) = 0.31$

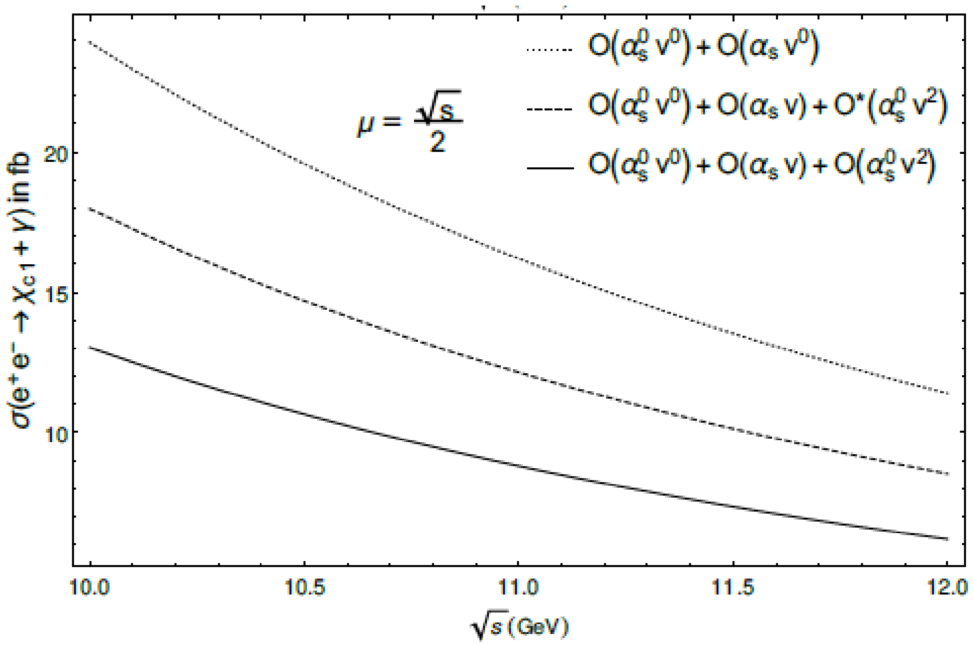
$$\langle 0|\mathcal{P}_1(^3P_0)|0\rangle = 0.046 \pm 0.005\,\mathrm{GeV}^7$$

$$\langle 0|\mathcal{T}_8(^3P_0)|0\rangle = 0.018 \pm 0.004\,\mathrm{GeV}^6$$

	$\mathcal{O}(\alpha_s^0 v^0)$ and $\mathcal{O}(\alpha_s v^0)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}^*(\alpha_s^0 v^2)$	$\mathcal{O}(\alpha_s^0 v^0)$, $\mathcal{O}(\alpha_s v^0)$ and $\mathcal{O}(\alpha_s^0 v^2)$	$\frac{\sigma_8}{\sigma_1} - 1$
$\sigma(\chi_{c_0})$	$(2.49 \pm 0.20 \pm 0.06)$	$(1.72 \pm 0.14 \pm 0.06)$	$(1.82 \pm 0.14 \pm 0.06)$	5.6%
$\sigma(\chi_{c_1})$	$(18.8 \pm 1.15 \pm 1.22)$	$(14.1 \pm 0.82 \pm 1.22)$	$(10.2 \pm 1.20 \pm 1.22)$	−27.6%
$\sigma(\chi_{c_2})$	$(3.71 \pm 0.19 \pm 1.38)$	$(3.54 \pm 0.17 \pm 1.38)$	$(2.11 \pm 0.39 \pm 1.38)$	−40.4%

Table III. Estimates for $\sigma(e^+e^- \to \chi_{c_0}\gamma)$ (in fb) at $\sqrt{s} = 10.6\,\mathrm{GeV}$ with $\mu = \sqrt{s}/2$. The second columns shows results, where both $\langle 0|\mathcal{P}_1(^3P_0)|0\rangle$ and $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ are omitted. In the third column we include contributions from $\langle 0|\mathcal{P}_1(^3P_0)|0\rangle$ but not from $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$. In the fourth column all three LDMEs are included. The last column displays the change of the full cross section in percent, when $\langle 0|\mathcal{T}_8(^3P_0)|0\rangle$ is included (σ_8) or omitted (σ_1).





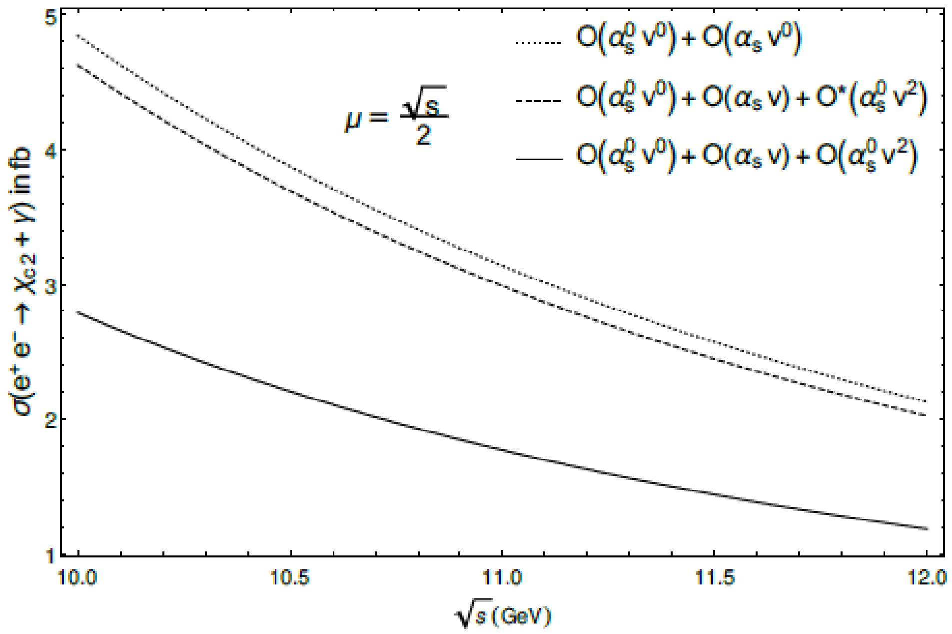


Figure 2. Cross sections for the production of χ_{cJ} in the energy region of Belle II. The dotted curve shows contributions only from the $\langle 0|\mathcal{O}_1(^3P_0)|0\rangle$ matrix element. The dashed curve includes also $\langle 0|\mathcal{P}_1(^3P_0)|0\rangle$, while the solid curve displays our final result with all the three contributions.

Conclusion and Discussion

In this work, we calculate the relativistic corrections to Exclusive $\chi_c + \gamma$ Production from e^+e^- annihilation, especially the contributions of operators with chromoelectric components, which are new. By explicit calculation, we show that the QCD amplitude can be exactly matched by the NRQCD amplitude, and the SDCs are indeed independent of the hadron state. The calculation is carried out in both the rest frame and the laboratory frame. The results are in good agreement reported results in literatures, except that our results for χ_{c2} disagree those in Li, et. al, JHEP (2014). Numerical calculations are also done in this work.

The End

Thanks for your attention!