"Decay constants" of doubly heavy baryons in QCD sum rules

Zhen-Xing Zhao
Shanghai Jiao Tong University
2017. 12

CLHCP-2017@Nanjing University

arXiv:1711.10289
Xiao-Hui Hu, Yue-Long Shen, Wei Wang and Zhen-Xing Zhao

Outline

- Introduction
- QCD sum rules study
- Numerical results
- Summary and outlook

Introduction

Selected for a Viewpoint in *Physics*
PRL **119,** 112001 (2017) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2017

Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.*[*]
(LHCb Collaboration)

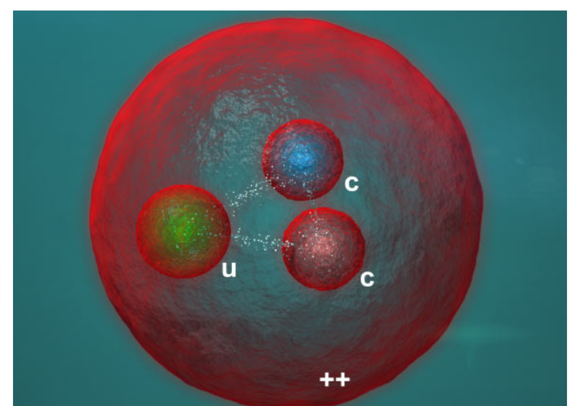
(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)

A highly significant structure is observed in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum, where the Λ_c^+ baryon is reconstructed in the decay mode $pK^-\pi^+$. The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon Ξ_{cc}^{++}. The difference between the masses of the Ξ_{cc}^{++} and Λ_c^+ states is measured to be $1334.94 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.})\ \text{MeV}/c^2$, and the Ξ_{cc}^{++} mass is then determined to be $3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}/c^2$, where the last uncertainty is due to the limited knowledge of the Λ_c^+ mass. The state is observed in a sample of proton-proton collision data collected by the LHCb experiment at a center-of-mass energy of 13 TeV, corresponding to an integrated luminosity of 1.7 fb^{-1}, and confirmed in an additional sample of data collected at 8 TeV.

DOI: 10.1103/PhysRevLett.119.112001

Properties

- Mass
- Lifetime
- Transition form factors
- Decay modes
- …



Quantum numbers and quark content

Baryon	Quark Content	s_h^π	J^P	Baryon	Quark Content	s_h^π	J^P
Ξ_{cc}	$\{cc\}q$	1^+	$1/2^+$	Ξ_{bb}	$\{bb\}q$	1^+	$1/2^+$
Ξ_{cc}^*	$\{cc\}q$	1^+	$3/2^+$	Ξ_{bb}^*	$\{bb\}q$	1^+	$3/2^+$
Ω_{cc}	$\{cc\}s$	1^+	$1/2^+$	Ω_{bb}	$\{bb\}s$	1^+	$1/2^+$
Ω_{cc}^*	$\{cc\}s$	1^+	$3/2^+$	Ω_{bb}^*	$\{bb\}s$	1^+	$3/2^+$
Ξ_{bc}'	$\{bc\}q$	0^+	$1/2^+$	Ω_{bc}'	$\{bc\}s$	0^+	$1/2^+$
Ξ_{bc}	$\{bc\}q$	1^+	$1/2^+$	Ω_{bc}	$\{bc\}s$	1^+	$1/2^+$
Ξ_{bc}^*	$\{bc\}q$	1^+	$3/2^+$	Ω_{bc}^*	$\{bc\}s$	1^+	$3/2^+$

Doubly charmed baryon spectrum ☺

- Many models have been applied to calculate masses: quark models or QCD sum rules

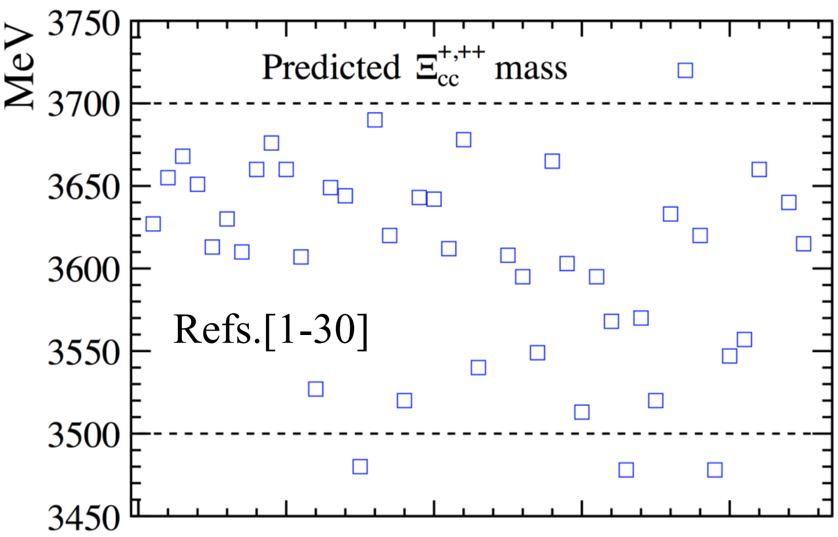
✓ Predicted Mass:

$m(\Xi_{cc}^{++}) \sim m(\Xi_{cc}^{+}) \sim (3.5-3.7)\mathrm{GeV}$

$m(\Omega_{cc}) \sim m(\Xi_{cc}^{+}) + 0.1\mathrm{GeV}$

✓ Mass splitting between

Ξ_{cc}^{++} and Ξ_{cc}^{+} is a few MeV



- Lattice QCD Calculation:

$m(\Xi_{cc}) \sim 3.6\mathrm{GeV}, m(\Omega_{cc}) \sim 3.7\mathrm{GeV}$

References can be found in 1703.09086

Lifetime ☹

- Large uncertainties: differ by a factor of 4~8
- $\tau(\Xi_{cc}^{++}) \gg \tau(\Xi_{cc}^{+}) \sim \tau(\Omega_{cc})$

literature	Ξ_{cc}^{++}	Ξ_{cc}^{+}	Ω_{cc}^{+}
Karliner, Rosner, 2014	185	53	
Chang, Li, Li, Wang, 2007	670	250	210
Kiselev, Likhoded,2002	460 ± 50	160 ± 50	270 ± 60
Kiselev, Likhoded, 1998	430 ± 100	110 ± 10	
Guberina, Melic, Stefancic, 1998	1550	220	250

Introduction

$$\langle 0|J_H|H(q,s)\rangle = \lambda_H u(q,s)$$

dimension-3

$$\langle 0|J_H|H(q,s)\rangle = f_H m_H^2 u(q,s)$$

dimension-1

- Lifetime
- Transition form factors

- Ξ_{cc}, Ω_{cc}
- Ξ_{bb}, Ω_{bb}
- Ξ_{bc}, Ω_{bc}

Applications of QCDSR

- Mass spectrum
- Decay constants
- Transition form factors
- Mixing matrix elements of K-meson and B-meson systems
- …

QCD sum rules study

The interpolating current

$$J_{\Xi_{QQ}} = \epsilon_{abc}\left(Q_a^T C\gamma^\mu Q_b\right)\gamma_\mu\gamma_5 q_c,$$
$$J_{\Omega_{QQ}} = \epsilon_{abc}\left(Q_a^T C\gamma^\mu Q_b\right)\gamma_\mu\gamma_5 s_c,$$

$$J_{\Xi_{bc}} = \frac{1}{\sqrt{2}}\epsilon_{abc}\left(b_a^T C\gamma^\mu c_b + c_a^T C\gamma^\mu b_b\right)\gamma_\mu\gamma_5 q_c,$$
$$J_{\Omega_{bc}} = \frac{1}{\sqrt{2}}\epsilon_{abc}\left(b_a^T C\gamma^\mu c_b + c_a^T C\gamma^\mu b_b\right)\gamma_\mu\gamma_5 s_c.$$

The correlation function

$$\Pi(q) = i\int d^4x e^{iq\cdot x}\langle 0|T[J(x),\bar{J}(0)]|0\rangle,$$

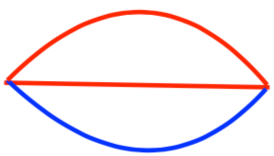
QCD sum rules study

Hadronic level

$$\Pi(q) \;=\; \lambda_H^2 \frac{\not{q} + m_H}{m_H^2 - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2},$$

only 1/2⁺ are considered

OPE level



$$\Pi(q) = \not{q}\Pi_1(q^2) + \Pi_2(q^2)$$

$$\Pi_i(q^2) \;=\; \int_{(m_Q+m_{Q'})^2}^{\infty} ds \frac{\rho_i(s)}{s - q^2}, \quad i = 1, 2,$$

$$\rho_i(s) \;=\; \frac{1}{\pi}\mathrm{Im}\Pi_i^{\mathrm{OPE}}(s)$$

Sum rules

$$\lambda_H^2 e^{-m_H^2/M^2} \;=\; \int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_1(s) e^{-s/M^2},$$

$$\lambda_H^2 m_H e^{-m_H^2/M^2} \;=\; \int_{(m_Q+m_{Q'})^2}^{s_0} ds \rho_2(s) e^{-s/M^2}.$$

QCD sum rules study

$$\rho_1^{\rm pert}(s) = \frac{6}{(2\pi)^4}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta}\Big([\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2]^2 + (1-\alpha-\beta)m_Q m_{Q'}[\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2]\Big), \quad (21)$$

$$\rho_1(s) = \rho_1^{\rm pert}(s)+\frac{\langle g_s^2G^2\rangle}{72}\left(m_Q^2\frac{\partial^3}{(\partial m_Q^2)^3}+m_{Q'}^2\frac{\partial^3}{(\partial m_{Q'}^2)^3}\right)\rho_1^{\rm pert}(s) \quad (22)$$

$$+\frac{4m_Q m_{Q'}\langle g_s^2G^2\rangle}{(4\pi)^4}\left(\frac{\partial^2}{(\partial m_Q^2)^2}+\frac{\partial^2}{(\partial m_{Q'}^2)^2}\right)\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta}(1-\alpha-\beta)(\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2)$$

$$+\frac{2\langle g_s^2G^2\rangle}{(4\pi)^4}\left(\frac{\partial}{\partial m_Q^2}+\frac{\partial}{\partial m_{Q'}^2}\right)\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{\rm min}}^{1-\alpha}d\beta(3\alpha m_Q^2+3\beta m_{Q'}^2-m_Q m_{Q'}-4\alpha\beta s) \quad (23)$$

$$\rho_2(s) = -\frac{\langle\bar{q}q\rangle}{2\pi^2}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha(3\alpha(1-\alpha)s-2\alpha m_Q^2-2(1-\alpha)m_{Q'}^2+2m_Q m_{Q'})$$

$$-\frac{\langle\bar{q}g_s\sigma Gq\rangle}{8\pi^2}\left(1+\frac{s}{M^2}\right)A(s)-\frac{2\langle\bar{q}g_s\sigma Gq\rangle}{(4\pi)^2}\Big((\alpha_{\rm max}-\alpha_{\rm min}) + \frac{1}{2s(\alpha_{\rm max}-\alpha_{\rm min})}[\alpha_{\rm max}(1-\alpha_{\rm max})s+\alpha_{\rm min}(1-\alpha_{\rm min})s+4m_Q m_{Q'}]\Big), \quad (24)$$

QCD Sum rules with the negative parity baryon

D. Jido, N. Kodama and M. Oka, Phys. Rev. D 54, 4532 (1996)

Hadronic level

$$\Pi(q) = \lambda_+^2 \frac{\not{q} + m_+}{m_+^2 - q^2} + \lambda_-^2 \frac{\not{q} - m_-}{m_-^2 - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2},$$

$1/2^+$ $1/2^-$

$$\frac{1}{\pi}\mathrm{Im}\Pi(q) = \lambda_+^2(\not{q} + m_+)\delta(q^2 - m_+^2) + \lambda_-^2(\not{q} - m_-)\delta(q^2 - m_-^2) + \cdots$$

Taking $\vec{q} = 0$

$$\frac{1}{\pi}\mathrm{Im}\Pi(q_0) = \gamma_0 A(q_0) + B(q_0) + \cdots,$$

$$A(q_0) = \frac{1}{2}[\lambda_+^2 \delta(q_0 - m_+) + \lambda_-^2 \delta(q_0 - m_-)],$$

$$B(q_0) = \frac{1}{2}[\lambda_+^2 \delta(q_0 - m_+) - \lambda_-^2 \delta(q_0 - m_-)].$$

$$\mathrm{A} + \mathrm{B} \Rightarrow \lambda_+$$

QCD sum rules study

OPE level

Taking $\vec{q} = 0$

$$\frac{1}{\pi}\mathrm{Im}\Pi(q_0) = \gamma_0\rho^A(q_0) + \rho^B(q_0) + \cdots$$

$$\int_\Delta^{\sqrt{s_0}} dq_0 \times \exp\left[-q_0^2/M^2\right]$$

weight function

A + B

$\rho^A + \rho^B$

$$\lambda_+^2 \exp\left[-\frac{m_+^2}{M^2}\right] = \int_\Delta^{\sqrt{s_0}} dq_0(\rho^A(q_0) + \rho^B(q_0)) \exp\left[-\frac{q_0^2}{M^2}\right]$$

New sum rules

Numerical results

Numerical results

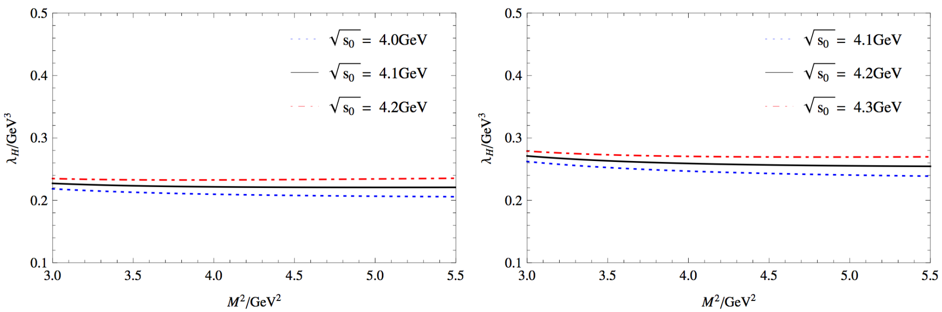


FIG. 1: The dependence on M^2 of the decay constants of Ξ_{cc} and Ω_{cc} at the scale $\mu = 2.1\,\mathrm{GeV}$. The continuum threshold are taken as $\sqrt{s_0} = 4.0 \sim 4.2$ GeV and $\sqrt{s_0} = 4.1 \sim 4.3$ GeV, respectively.

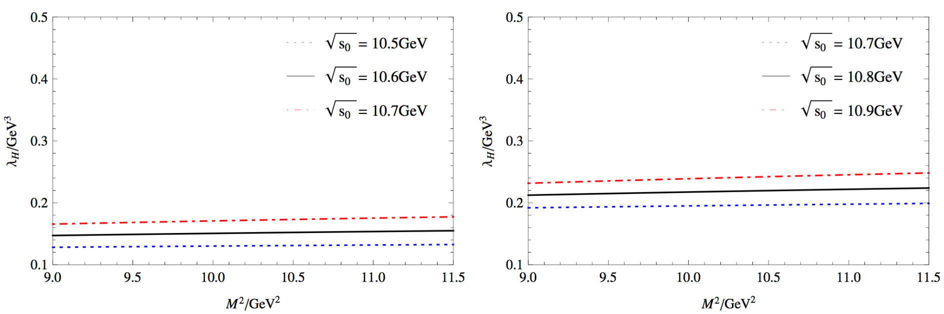


FIG. 2: The dependence on M^2 of the decay constant of Ξ_{bb} and Ω_{bb} at the scale $\mu = 2.1\,\mathrm{GeV}$. The continuum threshold is taken as $\sqrt{s_0} = 10.5 \sim 10.7$ GeV and $\sqrt{s_0} = 10.7 \sim 10.9$ GeV, respectively.

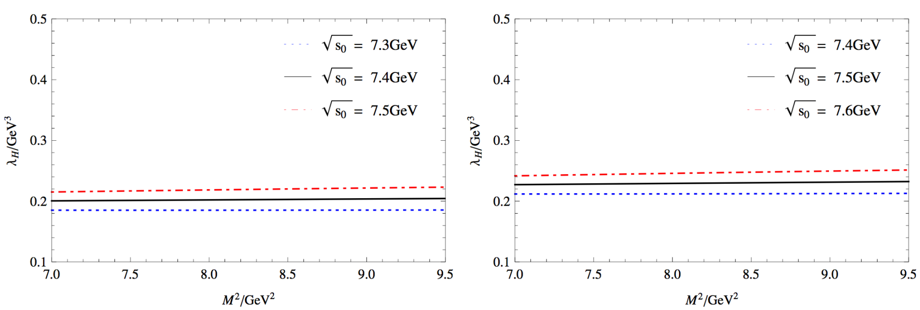


FIG. 3: The dependence on M^2 of the $\lambda_{\Xi_{bc}}$ and $\lambda_{\Omega_{bc}}$ at the scale $\mu = 2.1\,\mathrm{GeV}$. In the left and right panel, the continuum threshold are taken as $\sqrt{s_0} = 7.3 \sim 7.5$ GeV and $\sqrt{s_0} = 7.4 \sim 7.6$ GeV, respectively.

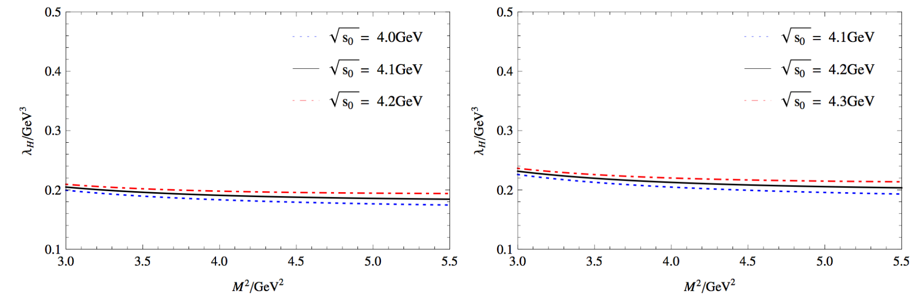


FIG. 5: The dependence on M^2 of the decay constant of Ξ_{cc} and Ω_{cc} at the scale $\mu = 2.1\,\mathrm{GeV}$. In the left and right panel, the continuum thresholds are taken as $\sqrt{s_0} = 4.0 \sim 4.2$ GeV and $\sqrt{s_0} = 4.1 \sim 4.3$ GeV, respectively.

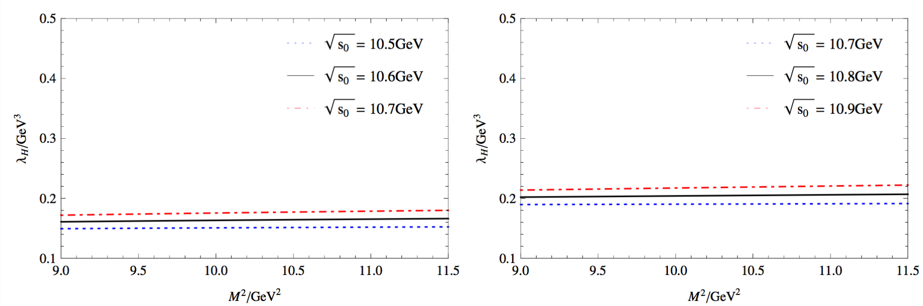


FIG. 6: The dependence on M^2 of the decay constant of Ξ_{bb} and Ω_{bb} at the scale $\mu = 2.1\,\mathrm{GeV}$. The continuum thresholds are taken as $\sqrt{s_0} = 10.5 \sim 10.7$ GeV and $\sqrt{s_0} = 10.7 \sim 10.9$ GeV for the left and

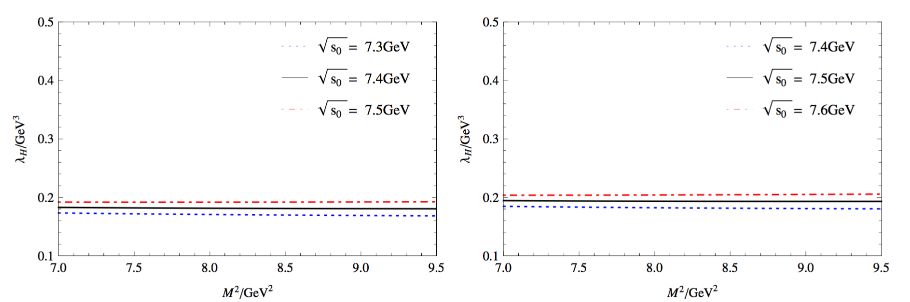


FIG. 7: The dependence on M^2 of the decay constant of Ξ_{bc} and Ω_{bc} at the scale $\mu = 2.1\,\mathrm{GeV}$. The continuum thresholds are taken as $\sqrt{s_0} = 7.3 \sim 7.5$ GeV and $\sqrt{s_0} = 7.4 \sim 7.6$ GeV for the left and right panel, respectively.

Numerical results

TABLE III: Decay constants λ_H (in units of GeV^3) and f_H (in units of 10^{-3} GeV) for the doubly heavy baryons at the scale $\mu = 2.1$ GeV and $\mu = 1$ GeV: $\lambda_H = f_H m_H^2$. The first and second errors come from the uncertainties of $\sqrt{s_0}$ and M^2 respectively. The sum rule in Eq. (19) has been used.

Baryon	$\lambda_H(\text{GeV}^3)[\mu = 2.1\ \text{GeV}]$	$f_H(10^{-3}\ \text{GeV})[\mu = 2.1\ \text{GeV}]$	$\lambda_H(\text{GeV}^3)[\mu = 1\ \text{GeV}]$	$f_H(10^{-3}\ \text{GeV})[\mu = 1\ \text{GeV}]$
Ξ_{cc}	$0.221 \pm 0.012 \pm 0.002$	$16.9 \pm 1.0 \pm 0.2$	$0.196 \pm 0.011 \pm 0.002$	$15.0 \pm 0.8 \pm 0.1$
Ω_{cc}	$0.258 \pm 0.013 \pm 0.006$	$18.4 \pm 0.9 \pm 0.4$	$0.229 \pm 0.012 \pm 0.005$	$16.4 \pm 0.8 \pm 0.4$
Ξ_{bb}	$0.151 \pm 0.021 \pm 0.002$	$1.47 \pm 0.20 \pm 0.02$	$0.134 \pm 0.018 \pm 0.002$	$1.31 \pm 0.18 \pm 0.02$
Ω_{bb}	$0.219 \pm 0.023 \pm 0.004$	$2.07 \pm 0.22 \pm 0.03$	$0.194 \pm 0.020 \pm 0.003$	$1.84 \pm 0.19 \pm 0.03$
Ξ_{bc}	$0.202 \pm 0.017 \pm 0.001$	$4.20 \pm 0.36 \pm 0.02$	$0.180 \pm 0.015 \pm 0.001$	$3.73 \pm 0.32 \pm 0.02$
Ω_{bc}	$0.230 \pm 0.018 \pm 0.002$	$4.69 \pm 0.36 \pm 0.03$	$0.204 \pm 0.016 \pm 0.001$	$4.16 \pm 0.32 \pm 0.03$

TABLE IV: "Decay constants" λ_H (in units of GeV^3) and f_H (in units of 10^{-3} GeV) for the doubly heavy baryons at the scale $\mu = 2.1$ GeV and $\mu = 1$ GeV: $\lambda_H = f_H m_H^2$. The first and second errors come from the uncertainties of $\sqrt{s_0}$ and M^2 respectively. The sum rule in Eq. (34) has been used.

Baryon	$\lambda_H(\text{GeV}^3)[\mu = 2.1\ \text{GeV}]$	$f_H(10^{-3}\ \text{GeV})[\mu = 2.1\ \text{GeV}]$	$\lambda_H(\text{GeV}^3)[\mu = 1\ \text{GeV}]$	$f_H(10^{-3}\ \text{GeV})[\mu = 1\ \text{GeV}]$
Ξ_{cc}	$0.189 \pm 0.008 \pm 0.007$	$14.4 \pm 0.6 \pm 0.5$	$0.168 \pm 0.007 \pm 0.006$	$12.8 \pm 0.5 \pm 0.5$
Ω_{cc}	$0.210 \pm 0.008 \pm 0.009$	$15.0 \pm 0.6 \pm 0.7$	$0.186 \pm 0.008 \pm 0.008$	$13.3 \pm 0.5 \pm 0.6$
Ξ_{bb}	$0.164 \pm 0.013 \pm 0.002$	$1.59 \pm 0.12 \pm 0.02$	$0.145 \pm 0.011 \pm 0.002$	$1.41 \pm 0.11 \pm 0.01$
Ω_{bb}	$0.204 \pm 0.014 \pm 0.001$	$1.94 \pm 0.13 \pm 0.01$	$0.181 \pm 0.012 \pm 0.001$	$1.72 \pm 0.12 \pm 0.01$
Ξ_{bc}	$0.181 \pm 0.011 \pm 0.001$	$3.76 \pm 0.23 \pm 0.02$	$0.161 \pm 0.010 \pm 0.001$	$3.33 \pm 0.20 \pm 0.02$
Ω_{bc}	$0.193 \pm 0.011 \pm 0.001$	$3.94 \pm 0.23 \pm 0.01$	$0.171 \pm 0.010 \pm 0.000$	$3.50 \pm 0.21 \pm 0.01$

Numerical results

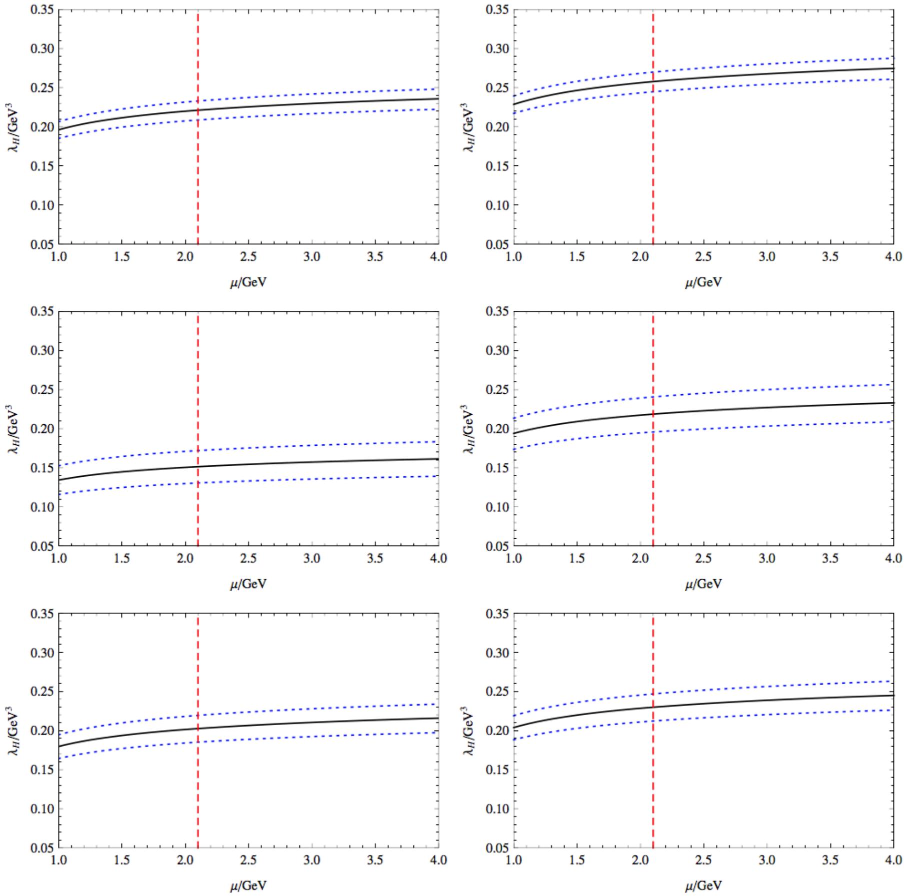


FIG. 4: The scale dependence (see Eq. (38)) of the "decay constants" for Ξ_{cc}, Ω_{cc}(the top two figures), Ξ_{bb}, Ω_{bb}(the middle two figures), Ξ_{bc} and Ω_{bc}(the bottom two figures) with the scale μ ranging from 1 GeV to 4 GeV. The solid line corresponds to $\sqrt{s_0} = 4.1$ GeV for Ξ_{cc}, $\sqrt{s_0} = 4.2$ GeV for Ω_{cc}, $\sqrt{s_0} = 10.6$ GeV for Ξ_{bb}, $\sqrt{s_0} = 10.8$ GeV for Ω_{bb}, $\sqrt{s_0} = 7.4$ GeV for Ξ_{bc}, $\sqrt{s_0} = 7.5$ GeV for Ω_{bc}, respectively. The dotted curves are obtained by varying the $\sqrt{s_0}$ by 0.1 GeV. The vertical line corresponds to the scale $\mu = 2.1$ GeV. The sum rule in Eq. (19) is considered.

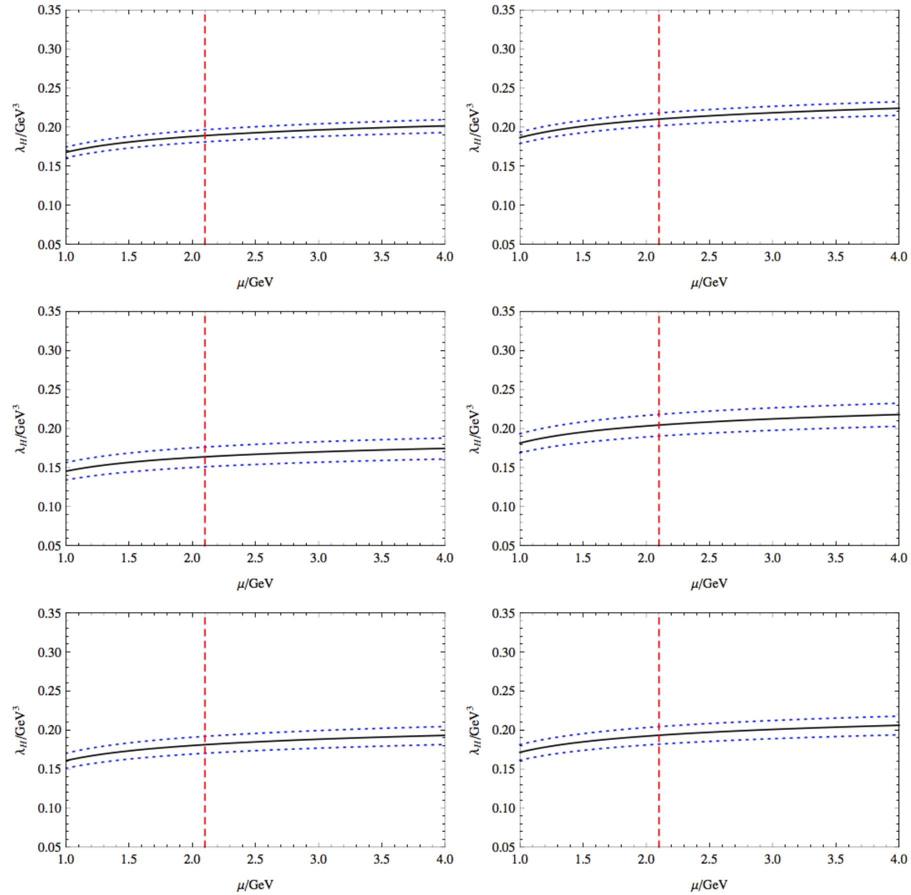


FIG. 8: Same as Figure 4 but for the sum rule in Eq. (34) is considered.

$$\lambda_H(\mu) = \lambda_H(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_{\lambda 0}}{2\beta_0}}$$

$$\frac{\gamma_{\lambda_0}}{2\beta_0} = \frac{-6}{25}$$

Numerical results

- It is necessary to point out that when including the contributions from the $1/2^-$ baryons the threshold parameter might be somewhat higher. In this analysis, we have approximately use the same values.

- Comparing the results in Table III and Table IV, one can see that the negative parity baryons do not provide significant modifications.

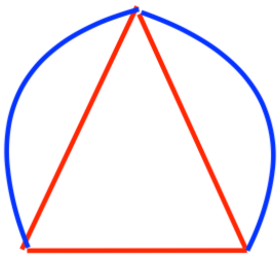
- From Table III and Table IV, we can see that, the decay constant of $\Omega_{QQ'}$ are slightly larger than that of $\Xi_{QQ'}$.

- The decay constants increases with the energy scale.

Summary and outlook

Summary and outlook

- QCD sum rules
- $\Xi_{cc}, \Omega_{cc}, \Xi_{bb}, \Omega_{bb}, \Xi_{bc}, \Omega_{bc}$
- Decay constants
- Positive parity and negative parity



- Lifetime
- …

Thank you for your attention!

backup

Subscript[\[Gamma], m0] = -4,
Subscript[\[Beta], 0] (f = 4) = 25/3,
Subscript[\[Gamma], m0]/(2 Subscript[\[Beta], 0]) = -6/25