A fast and accurate method for perturbative resummation of pT dependent observables

arXiv:1710.00078, DK, Lee, Vaidya

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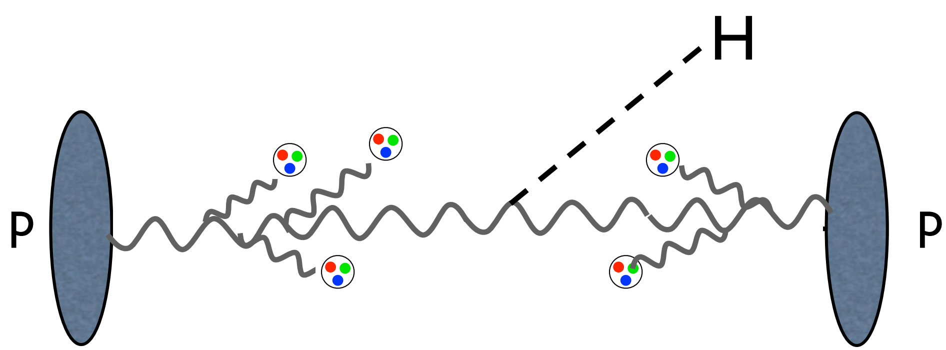
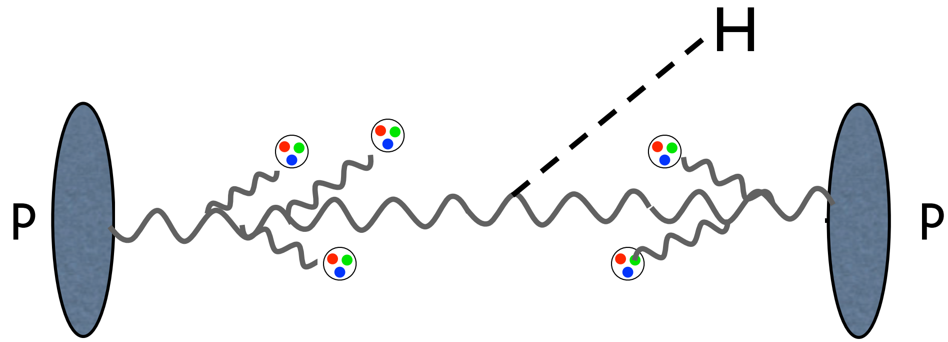
pT dependent observables

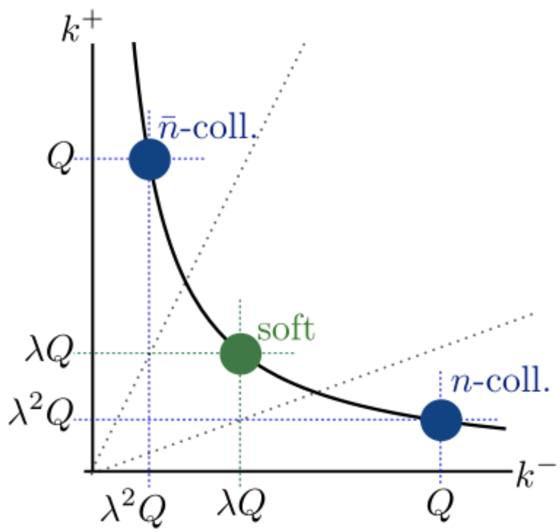
- small pT respect to beam or jet axis
  - Drell-Yan and Higgs production
  - final hadron in Semi-Inclusive DIS
  - Event shape broadening: pT around thrust axis



- Large logarithm: Log[pT/Q]
  Q is heavy boson mass or the collision energy

soft and collinear modes





	$(p-, p+, pT)$	μ_i, ν_i
$p_H \sim$	$(1,\ 1,\ 1)\,Q$	Q
$p_c \sim$	$(1,\ \lambda^2,\ \lambda)\,Q$	pT, Q
$p_s \sim$	$(\lambda,\ \lambda,\ \lambda)\,Q$	pT, pT

$\lambda \sim pT/Q << 1$, Q=higgs mass

hierarches in virtuality and in rapidity induce large logs:

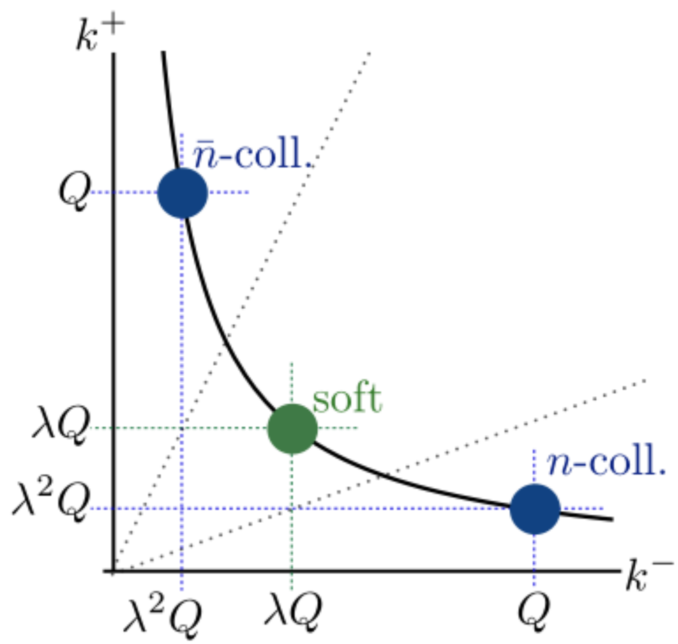
$\mathrm{Log}[\mu_{H/}\mu_L]$ and $\mathrm{Log}[\nu_{H/}\nu_L]$ and they should be resummed.

What to set for $\mu_{H,L}$ and $\nu_{H,L}$? looks trivial but NOT

Factorization in p space

- complicated convolutions in p-space:

$$\frac{d\sigma}{d^2q_T dy} = C_t^2(M_t^2,\mu)\sigma_0 H(Q^2;\mu)\int d^2\vec{q}_{Ts}d^2\vec{q}_{T1}d^2\vec{q}_{T2}\delta^2(\vec{q_T}-(\vec{q_{Ts}}+\vec{q}_{T1}+\vec{q}_{T2}))$$
$$\times S(\vec{q}_{Ts};\mu,\nu)f_1^{\perp}\left(\vec{q}_{T1},x_1,p^-;\mu,\nu\right)f_2^{\perp}\left(\vec{q}_{T2},x_2,p^+;\mu,\nu\right),$$



	(p-, p+, pT)	μ_i, ν_i
p_H~	(1, 1, 1) Q	Q
p_c~	(1, λ^2, λ)Q	pT, Q
p_s~	(λ, λ , λ)Q	pT, pT

λ~pT/Q<<1, Q=higgs mass

RGE in μ_i, ν_i resums large log: Log[$\mu_{H/}\mu_L$] and Log[$\nu_{H/}\nu_L$].

Factorization in p and b space

- complicated convolutions in p-space:

$$\frac{d\sigma}{d^2q_T dy} = C_t^2(M_t^2,\mu)\sigma_0 H(Q^2;\mu)\int d^2\vec{q}_{Ts}d^2\vec{q}_{T1}d^2\vec{q}_{T2}\delta^2(\vec{q_T}-(\vec{q_{Ts}}+\vec{q}_{T1}+\vec{q}_{T2}))$$
$$\times S(\vec{q}_{Ts};\mu,\nu)f_1^{\perp}\Big(\vec{q}_{T1},x_1,p^-;\mu,\nu\Big)f_2^{\perp}\Big(\vec{q}_{T2},x_2,p^+;\mu,\nu\Big)\,,$$

- Fourier transform $\widehat{f}(\vec{b}) \equiv \int \frac{d^2q_T}{(2\pi)^2}e^{i\vec{b}\cdot\vec{q_T}}f(\vec{q_T})$

- a simple product in b-space

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0\pi(2\pi)^2 C_t^2(M_t^2,\mu)H(Q^2,\mu)\int db\, b J_0(bq_T)$$
$$\times \widetilde{S}(b,\mu,\nu)\widetilde{f}_1^{\perp}(b,x_1,p^-;\mu,\nu)\widetilde{f}_2^{\perp}(b,x_2,p^+;\mu,\nu)\,,$$

$J_0(x)$: Bessel function from Fourier kernel

RGE and scale setting

- Solution of RGE resums large logs

$$\widetilde{S}(b,\mu_L,\nu_H) \propto \exp\left[-\Gamma_0 \frac{\alpha(\mu_L)}{\pi} \ln\left(\frac{\nu_H}{\nu_L}\right) \ln(\mu_L b_0)\right]$$

$$b_0 = b e^{\gamma_E}/2$$

- high scales: $\mu_H, \nu_H \sim$ Higgs mass
- conventional b-space choice $\mu_L, \nu_L \sim 1/b$:
  but Landau pole $\alpha_s(1/b)$ and IR cutoff needed
- p-space $\mu_L, \nu_L \sim pT$:
  NO pole $\alpha_s(pT)$ and NO cutoff needed
  but singular in UV limit $\widetilde{S} \propto b_0^{-\Gamma_0}$

A prescription for the scales

$$\nu_L \to \nu_L^* = \nu_L(\mu_L b_0)^{-1+p} \qquad p = \frac{1}{2}\left[1 - \frac{\alpha_s(\mu_L)\beta_0}{2\pi}\ln\frac{\nu_H}{\nu_L}\right]$$

- Soft exponent: $-\dfrac{\alpha_s(\mu_L)}{4\pi} 4\Gamma_0 \ln(\mu_L b_0) \ln\dfrac{\nu_H}{\nu_L^*}$

$$\rightarrow \quad -4\Gamma_0 \frac{\alpha_s}{4\pi} \ln(\mu_L b_0)\left[\ln\left(\frac{\nu_H}{\nu_L}\right) + (1-p)\ln(\mu_L b_0)\right]$$

Not arbitrary and designed to give $O(\alpha_s^2)$ term

- Always Gaussian at higher orders

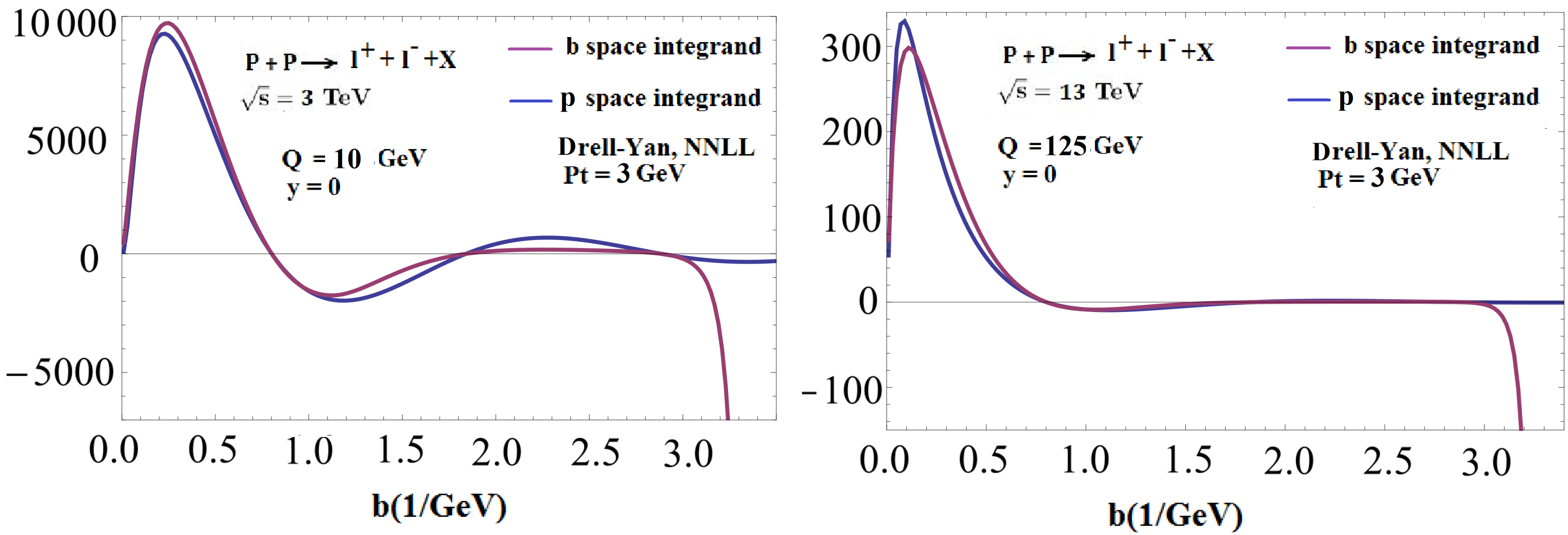
$$\ln\frac{\nu_H}{\nu_L}\sum_{n=0}^{\infty}\left(\frac{\alpha_s(\mu_L)}{4\pi}\right)^{n+1}\left(\mathbb{Z}_S\Gamma_n \ln\mu_L b_0 + \gamma_{RS}^n\right) \rightarrow -A\ln^2(\Omega b)$$

A, $\Omega(\mu_L, \nu_L, \mu_H, \nu_H, \ldots)$ are constant of *b*.

Back to p-space

- Essential integral $$I_b^k \equiv \int_0^\infty db\, b J_0(bq_T)\ln^k(\mu_L b_0) e^{-A\ln^2 \Omega b}$$

- Integrands at small pT~3 GeV for p-space (ours) VS b-space scale choices



Landau pole is absent with our scheme.

Back to p-space

- Essential integral in b-space

$$I_b^k \equiv \int_0^\infty db\, b J_0(bq_T) \ln^k(\mu_L b_0) e^{-A \ln^2 \Omega b}$$

- done after *numerical* integration over b*!* *but a rapid* oscillation with *b* and slow convergence.
- Can we do it analytically? *With* Mellin-Barnes rep. b integral becomes *t*=*c*+*ix* integral.

$$J_0(z) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} \left(\frac{1}{2} z\right)^{2t}$$

$$\downarrow$$

$$-\frac{\sin(\pi t)}{\pi} \Gamma(-t)^2$$

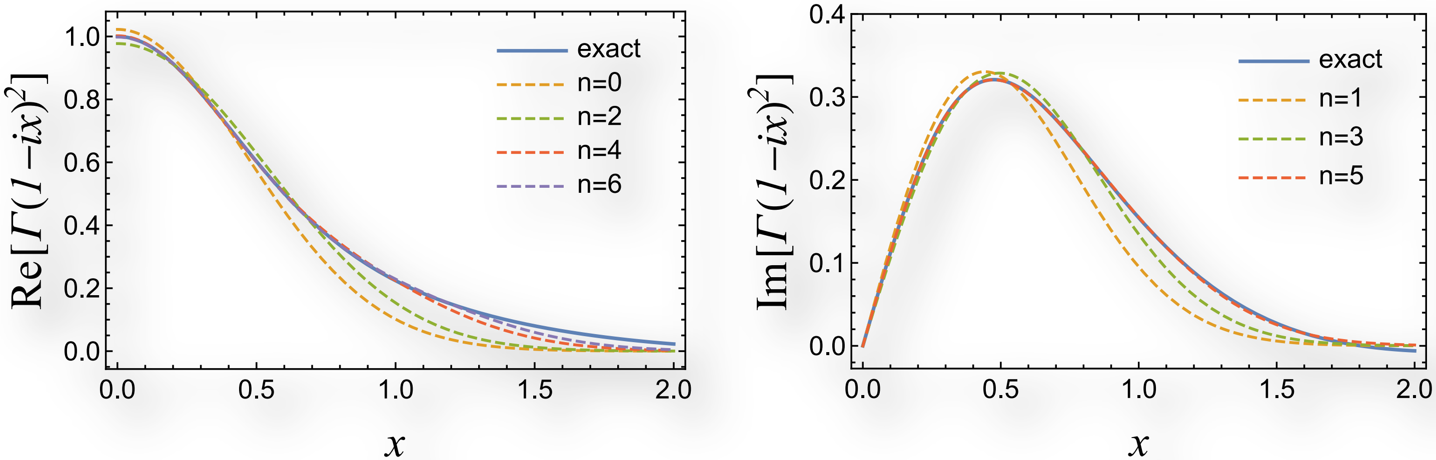
Semi-analytic method

- Series expansion with Hermite polynomial

$$\Gamma(1-ix)^2 = e^{-a_0 x^2}\sum_{n=0}^{\infty} c_{2n} H_{2n}(\alpha x) + \frac{i\gamma_E}{\beta} e^{-b_0 x^2}\sum_{n=0}^{\infty} c_{2n+1} H_{2n+1}(\beta x)$$

$$H_0(x) = 1 \qquad H_2(x) = 4x^2 - 2 \qquad a_0 = 2\gamma_E^2 + \frac{\pi^2}{6} \qquad \alpha^2 - a_0 = 4$$

$$c_0 = 1.02248\,, \qquad c_2 = 0.02254\,, \qquad c_4 = 0.00206\,, \qquad c_6 = 3.42\times 10^{-5}$$



- It is independent of variables $\mu_{L,H}$, $\nu_{L,H}$, qT and results are analytic func. of those variables.

Final expression

$$\boxed{\begin{aligned}\frac{d\sigma}{dq_T^2dy} &= \frac{1}{2}\sigma_0 C_t^2(M_t^2,\mu_T)H(Q^2,\mu_H)U(\mu_L,\mu_H,\mu_T)C(\mu_L,\nu_L,\nu_H)\\ &\times\sum_{n=0}^{\infty}\sum_{k=0}^{2n}\Big(\frac{\alpha_s(\mu_L)}{4\pi}\Big)^n\widetilde{F}_k^{(n)}(x_1,x_2,Q;\mu_L,\nu_L,\nu_H)I_b^k(q_T;\mu_L,\nu_L,\nu_H;\alpha,a_0;\beta,b_0)\,.\end{aligned}}$$

$$I_b^0 = \frac{2}{\pi q_T^2}\sum_{n=0}^{\infty}\mathrm{Im}\Big\{c_{2n}\mathcal{H}_{2n}(\alpha,a_0)+\frac{i\gamma_E}{\beta}c_{2n+1}\mathcal{H}_{2n+1}(\beta,b_0)\Big\}$$

- Integral of Hermite against the Gaussian func.

$$\mathcal{H}_0 = e^{-\frac{A}{1+a_0A}(L-i\pi/2)^2}\frac{1}{\sqrt{1+a_0A}} \qquad \mathcal{H}_1 = -\frac{2z_0\alpha}{1+a_0A}\mathcal{H}_0$$

$$z_0 = A(\pi/2+iL)$$

$$L = \ln\left(\frac{2\Omega}{q_T}\right)$$

- coefficients of Log in S, f functions

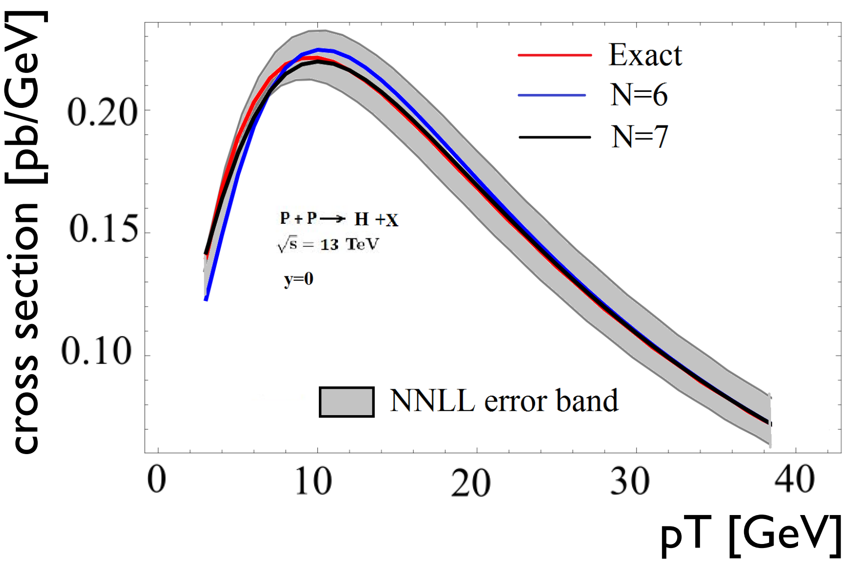
$$\widetilde{F}_0^{(0)} = f_i(x_1,\mu_L)f_{\bar{i}}(x_2,\mu_L)\,,$$

$$\widetilde{F}_0^{(1)} = f_i(x_1,\mu_L)f_{\bar{i}}(x_2,\mu_L)c_{\widetilde{S}}^1+[I_{ij}^{(1)}\otimes f_j(x_1,\mu_L)]f_{\bar{i}}(x_2,\mu_L)+f_i(x_1,\mu_L)[I_{\bar{i}j}^{(1)}\otimes f_j(x_2,\mu_L)]$$

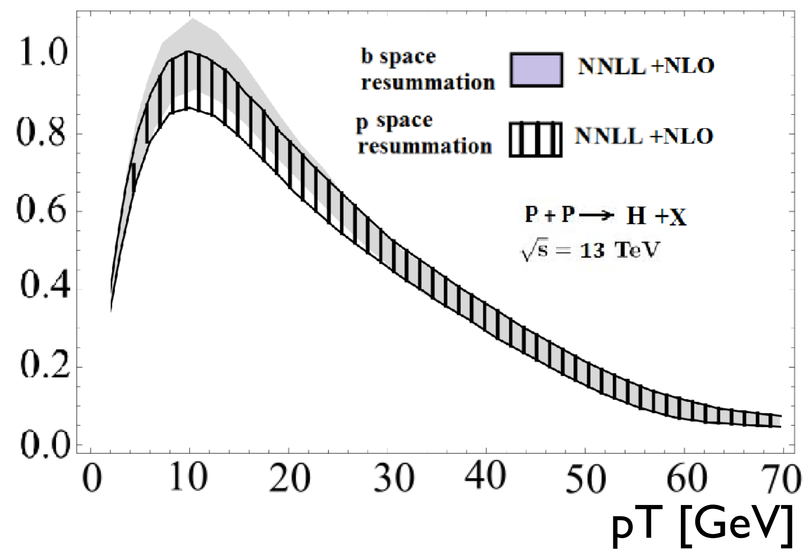
Final results

$$\frac{d\sigma}{dq_T^2 dy} = \frac{1}{2}\sigma_0 C_t^2(M_t^2, \mu_T) H(Q^2, \mu_H) U(\mu_L, \mu_H, \mu_T) C(\mu_L, \nu_L, \nu_H)$$
$$\times \sum_{n=0}^{\infty} \sum_{k=0}^{2n} \left(\frac{\alpha_s(\mu_L)}{4\pi}\right)^n \widetilde{F}_k^{(n)}(x_1, x_2, Q; \mu_L, \nu_L, \nu_H) I_b^k(q_T; \mu_L, \nu_L, \nu_H; \alpha, a_0; \beta, b_0)\,.$$

Convergence of series expansion



b- vs. p-space scheme



Summary

- **pT dependent observables**
  Higgs/Drell-Yan, SIDIS, Broadening

- **Higgs pT spectrum**
  focus on scale setting $\mu_L(pT)$, $\nu_L(b)$
  free from Landau pole and arbitrary cutoff
  semi-analytic (fast) result and systematic (accurate) expansion

Thank you!

High energy physics at Fudan U

- Remodelling since 2015 and 9 faculties at this moment
  2 seniors: Huangzhong Huang, Yongshi Wu,
  Yang Shen, Chuan Zheng, Zenghua Li, Xuguang Huang,
  New ones: Xiaolong Wang, Tao Luo, Daekyoung Kang



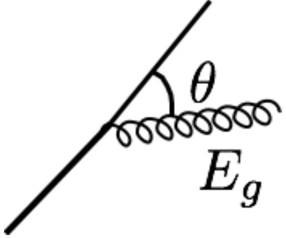
- 4 Postdoc's (Wanbing He, Subikash Choudhury, Long Ma, Weihu Ma)+ 2 Students (Yi Zhang ,Yu Hu)

Physics Program in the group

- Experimental
  - RHIC-STAR in US (simulating early universe)
  - Belle II in Japan (CP symmetry violation in nature)
  - BES III in Beijing (charm quark physics)
  - Neutrino-less double beta decay (new physics signal)
- Theoretical
  - Collider pheno. including QCD jets, quarkonium
  - quark-gluon matter, compact stars
  - quantum gas at low temperatures
  - model for nuclear interaction

Backup

Log Log Log ...



probability of splitting $\sim \dfrac{1}{E_g(1-\cos\theta)}$ $\longrightarrow$ $\mathrm{Log}(E_g)\,\mathrm{Log}(\theta)$

soft and collinear enhancements

Perturbation threaten by large log : $\alpha_s\, L \sim 1$

$\mathrm{L} = \mathrm{Log}$

$$\sigma = 1 + \alpha_s + \alpha_s^2 + \cdots$$

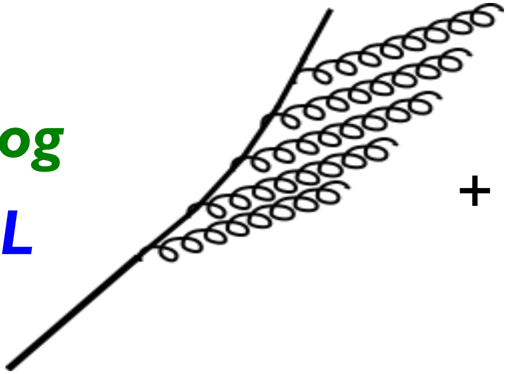
Resum large logs!

$$\log\sigma = \alpha_s L^2 + \alpha_s^2 L^3 + \cdots$$ ***Leading Log***

$$+\alpha_s L + \alpha_s^2 L^2 + \cdots$$ ***Next to LL***

$$+\alpha_s + \alpha_s^2 L + \cdots$$ ***NNLL***

$$+\alpha_s^2 + \cdots$$



+ other diagrams

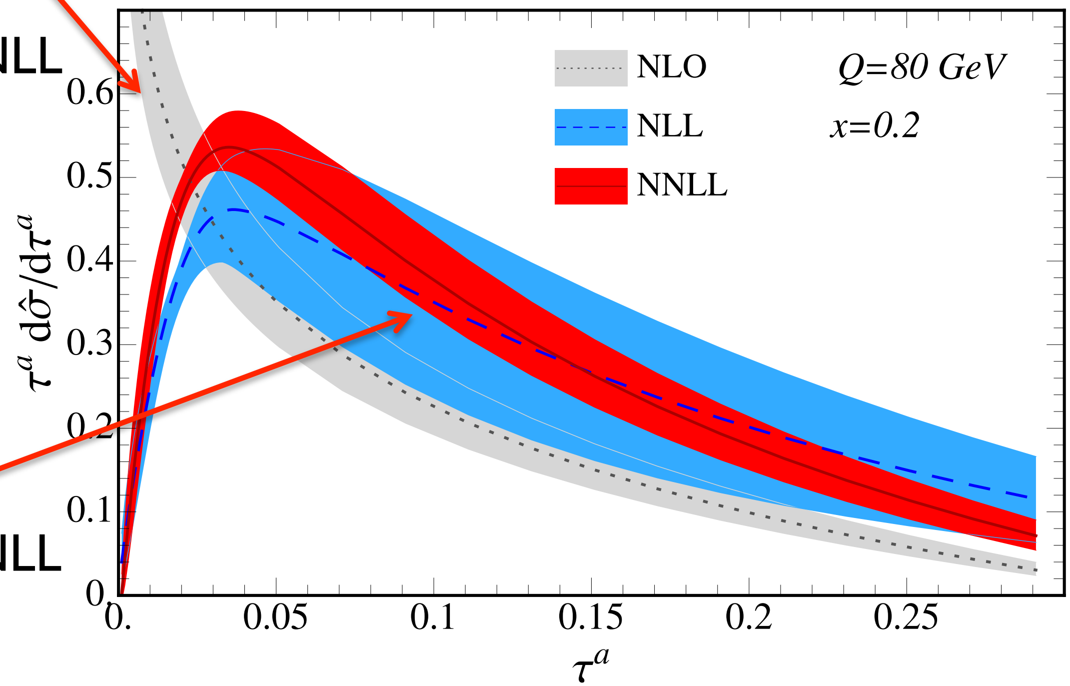
Large Log under control

Singular behavior due to divergence in local QFT

Renormalization leaves log of $\tau \sim \lambda$

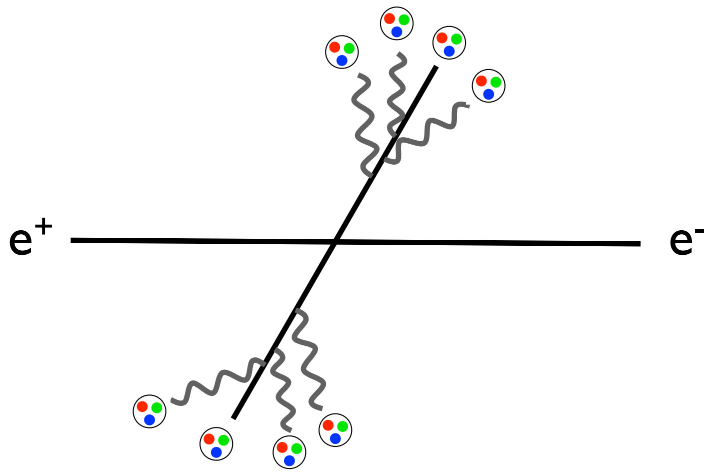
Log singularity in NLO

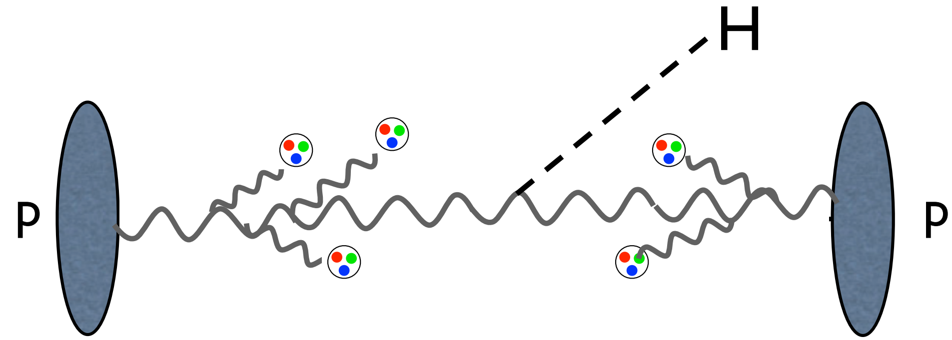
cured in NLL, NNLL



Good convergence from NLL to NNLL

SCET factorization for ee, *pp*





$$H_{ee} \times J \otimes J \otimes S_{ee} \qquad H_{pp} \times f \otimes f \otimes S_{pp}$$

Universal structure captured by EFT, Not easy in QCD!

H: q/g created at the short distance

J, f: final/initial radiation of coll. partons

S: radiation of soft partons

$p_H \sim (1, \;\; 1, \;\; 1\;)\, Q$

$p_c \sim (1, \;\; \lambda^2, \;\; \lambda\;) Q$

$p_s \sim (\lambda^n, \;\; \lambda^n, \lambda^n) Q$

$\lambda \sim pT/Q \ll 1$

Resummation by RG evolution

- RG equation

similar to H,B,S

$$\mu\frac{d}{d\mu}\widetilde{J}(\mu) = \gamma_J(\mu)\,\widetilde{J}(\mu) \quad \Rightarrow \quad \widetilde{J}(\mu) = \widetilde{J}(\mu_j)\boxed{e^{K(\mu_j,\mu)-\eta(\mu_j,\mu)\ln(\widetilde{\nu}\mu_j^2)}}$$

$$\boxed{K = L\sum_{k=1}^{\infty}(\alpha_s L)^k + \sum_{k=1}^{\infty}(\alpha_s L)^k + \cdots} \qquad L = \ln(\mu/\mu_J)$$

LL NLL

- Resum large logs
  - No large logs
    at its natural scale μ_i ~Q or, pT
  - *Evolution*
    from μ_i to common scale μ