# Improvement of the Simplest Little Higgs Model

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S.-P. He, **Y.-N. Mao**, C. Zhang, and S.-H. Zhu, arXiv: 1709.08929;

K. Cheung, S.-P. He, **Y.-N. Mao**, C. Zhang, and Y. Zhou, in preparation×2.

# I. INTRODUCTION

- In most composite models, the scalar fields are nonlinear realized;
- We should carefully check the normalization in the scalar sector and obtain exact Goldstone fields in such kind of models;
- The vertices may also be different comparing with the naively calculation results;
- In this talk, we take the simplest little Higgs (SLH) model as an example, to discuss the standard procedure above, and the improved properties of this model—differences from the previous results about $\eta$ vertices, which have already existed for over ten years. [W. Kilian, D. Rainwater, and J. Reuter, Phys. Rev. D71 (2005) 015008; etc.]

## II. A BRIEF REVIEW OF SLH MODEL

SLH model is one of the little Higgs models to solve the "little hierarchy" problem:

- Global Symmetry breaking $(\mathrm{SU}(3) \times \mathrm{U}(1))^2 \to (\mathrm{SU}(2) \times \mathrm{U}(1))^2$ at scale $f \gg v$;
- Gauge symmetry breaking $\mathrm{SU}(3) \times \mathrm{U}(1) \to \mathrm{SU}(2)_L \times \mathrm{U}(1) \to \mathrm{U}(1)_{\mathrm{em}}$;
- Ten Nambu-Goldstone boson are generated, in which eight are eaten by the massive gauge bosons, and two are left as physical scalars;
- One $(h)$ is a $0^+$ scalar, we treat it as the 125 GeV Higgs, the other $(\eta)$ is a $0^-$ scalar;
- The two scalar triplets $\Phi_{1,2}$ transformed as $(\mathbf{1}, \mathbf{3})$ and $(\mathbf{3}, \mathbf{1})$ respectively.

[D. E. Kaplan and M. Schmaltz, JHEP 0310 (2003), 039; etc.]

- The nonlinear realization of the scalar triplets:

$$\Phi_1 = \mathrm{e}^{\mathrm{i}\Theta'}\mathrm{e}^{\mathrm{i}t_\beta\Theta}\begin{pmatrix}\mathbf{0}_{1\times2}\\ f c_\beta\end{pmatrix}, \qquad \Phi_2 = \mathrm{e}^{\mathrm{i}\Theta'}\mathrm{e}^{-\mathrm{i}\Theta/t_\beta}\begin{pmatrix}\mathbf{0}_{1\times2}\\ f s_\beta\end{pmatrix};$$

with the definitions of the matrix fields

$$\Theta \equiv \frac{1}{f}\left(\frac{\eta\mathbb{I}_{3\times3}}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times2} & \phi\\ \phi^\dagger & 0\end{pmatrix}\right), \quad \text{and} \quad \Theta' \equiv \frac{1}{f}\left(\frac{G'\mathbb{I}_{3\times3}}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times2} & \varphi\\ \varphi^\dagger & 0\end{pmatrix}\right).$$

- $\eta$ is the pseudoscalar field; $\phi \equiv \left((v_h + h - \mathrm{i}G)/\sqrt{2}, G^-\right)^T$ is the usual Higgs doublet; $G'$ and $\varphi \equiv (y^0, x^-)^T$ are all eaten by the five heavy gauge bosons.

- The covariant derivative term $(D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2)$;

- $D_\mu \equiv \partial_\mu - \mathrm{i}\mathbb{G}_\mu$, where the gauge field matrix

$$\mathbb{G} = \frac{A^3}{2}\begin{pmatrix} 1 & & \\ & -1 & \\ & & \end{pmatrix} + \frac{A^8}{2\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} & W^+ & Y^0 \\ W^- & & X^- \\ \bar{Y}^0 & X^+ & \end{pmatrix} + \frac{t_W B}{3\sqrt{1-t_W^2/3}}\mathbb{I}$$

- $\theta_W$ is the electro-weak mixing angle, $W^\pm$ and $X^\pm$ are charged, $Y(\bar{Y}) = (Y_1 \pm \mathrm{i}Y^2)/\sqrt{2}$

- $A^3, A^8, B$ are linear combinations of $\gamma, Z, Z'$ at leading order of $(v/f)$;

- $Z, Z', Y^2$ have further mixing beyond LO of $(v/f)$.

- $h$ can acquire its mass through loop corrections, and $\eta$ is still massless;

- We now don't consider the $\eta$ mass term $(-\mu^2\Phi_1^\dagger\Phi_2 + \text{H.c.})$;

- The Yukawa lagrangian with anomaly free embedding: F. del Águila, J. I. Illana, and M. D. Jenkins, JHEP 1103 (2011) 080; O. C. W. Kong, Report No. NCU-HEP-k009.

$$
\begin{aligned}
\mathcal{L}_y \;=\; & \mathrm{i}\lambda_N^j \bar{N}_{R,j}\Phi_2^\dagger L_j - \frac{\mathrm{i}\lambda_\ell^{jk}}{\Lambda}\bar{\ell}_{R,j}\det\left(\Phi_1, \Phi_2, L_k\right) \\
& +\mathrm{i}\left(\lambda_t^a \bar{u}_{R,3}^a \Phi_1^\dagger + \lambda_t^b \bar{u}_{R,3}^b \Phi_2^\dagger\right) Q_3 - \mathrm{i}\frac{\lambda_{b,j}}{\Lambda}\bar{d}_{R,j}\det\left(\Phi_1, \Phi_2, Q_3\right) \\
& +\mathrm{i}\left(\lambda_{d,n}^a \bar{d}_{R,n}^a \Phi_1^T + \lambda_{d,n}^b \bar{d}_{R,n}^b \Phi_2^T\right) Q_n - \mathrm{i}\frac{\lambda_u^{jk}}{\Lambda}\bar{u}_{R,j}\det\left(\Phi_1^*, \Phi_2^*, Q_k\right)
\end{aligned}
$$

- Fermion doublets are enlarged to triplets, $L = (\nu_L, \ell_L, \mathrm{i}N_L)^T$, $Q_1 = (d_L, -u_L, \mathrm{i}D_L)$, $Q_2 = (s_L, -c_L, \mathrm{i}S_L)$, $Q_3 = (t_L, b_L, \mathrm{i}T_L)$.

Calculate the useful $\eta$ couplings:

- If naively perform it with the degrees of freedom in the triplets $\Phi_{1,2}$, we will obtain
- The SM $\eta q\bar{q}$ vertices: $\mathcal{L} \supset \pm(m_q/f)(t_\beta - t_\beta^{-1})\bar{q}\gamma^5 q\eta$;
- The anti-symmetric $Zh\eta$ vertex: $\mathcal{L} \supset (m_Z/\sqrt{2}f)(t_\beta^{-1} - t_\beta)(h\partial_\mu\eta - \eta\partial_\mu h)Z^\mu$;
- Both gave a $\mathcal{O}(v/f)$ vertex which induced rich interesting phenomenology about $\eta$;
- However, the results are totally wrong since the naive treatments are careless about most two-point transitions: including the non-canonically normalized kinetic terms, gauge boson-Goldstone transitions, and gauge fixing terms;
- The formalism must be improved if we want to remove all the two-point transitions.

## III. IMPROVED FORMALISM OF THE SLH MODEL

The key points:

- The CP-odd scalar sector is not canonically normalized;
- We must find a basis to remove all cross-terms in the scalar kinetic part of the lagrangian, the two point function must have the form $\mathrm{i}\delta^2\Gamma/\delta S_a \delta S_b = -\mathrm{i}(p_a^\mu p_{b,\mu}\delta_{ab} - m_{ab})$;
- The gauge fixing term must cancel all the two-point transitions like $V_\mu \partial^\mu S$;
- No additional two-point transitions generated from these operations, for example, no additional cross-terms arising from the gauge boson kinetic parts;
- All sectors can be diagonalized together.

- First, calculate the non-canonically normalized scalar kinetic terms: $\mathcal{L} \supset (\mathbb{K}_{ij}/2)\partial_\mu G_i \partial^\mu G_j$ where $\mathbb{K}_{ij} \neq \delta_{ij}$, $G_i$ denotes one of $(\eta, G, G', y^2)$;

- Consider the linear space spanned by the four $G_i$, we need a new basis $S_i$, in which the fields are canonically normalized: $\mathcal{L} \supset (\delta_{ij}/2)\partial_\mu S_i \partial^\mu S_j$;

- Define the inner product $\langle S_i | S_j \rangle = \delta_{ij}$, it is easy to show $\langle G_i | G_j \rangle = (\mathbb{K}^{-1})_{ij}$;

- This relation is very useful in the following procedure.

- Second, calculate the gauge boson-scalar two point transition: they come from the VEVs of $\Phi_{1,2}$ and can be parameterized as $V_p^\mu \mathbb{F}_{pi} \partial_\mu G_i$, where $\mathbb{F}$ is a $4 \times 3$ matrix;

- It must be canceled by the gauge fixing terms thus $\mathcal{L}_{\text{G.F.}} \supset (\partial_\mu V_p^\mu)\mathbb{F}_{pi} G_i$;

- Third, try to find the exact Goldstone fields, define another basis $\bar{G}_p = \mathbb{F}_{pi} G_i$, we have the inner product $\langle \eta | \bar{G}_p \rangle = 0$ and $\langle \bar{G}_p | \bar{G}_q \rangle = (\mathbb{M}_V^2)_{pq}$, where $\mathbb{M}_V^2$ is the mass matrix in the basis $(Z, Z', Y^2)$;

- We can use a matrix $\mathbb{R}$ to diagonalize $\mathbb{M}_V^2$ as $(\mathbb{R}\mathbb{M}_V^2\mathbb{R}^T)_{pq} = m_p^2 \delta_{pq}$;

- It is natural to define $\tilde{G}_p = \mathbb{R}_{pq}\bar{G}_q / m_p = (\mathbb{R}\mathbb{F})_{pi} G_i / m_p$ thus $\langle \tilde{G}_p | \tilde{G}_q \rangle = \delta_{pq}$.

- Now we have a canonically normalized basis $(\eta/\sqrt{(\mathbb{K}^{-1})_{11}}, \tilde{G}_p)$;

- The two-point transition becomes $m_p \tilde{V}_p^\mu \partial_\mu \tilde{G}_p$ with $\tilde{V}_p^\mu = \mathbb{R}_{pq} V_q$; thus we choose the gauge fixing term $\mathcal{L}_{\text{G.F.}} = -\sum_p (1/2\xi_p)(\partial_\mu \tilde{V}_p^\mu - \xi_p m_p \tilde{G}_p)^2$;

- $\tilde{G}_p$ is the corresponding Goldstone eaten by $\tilde{V}_p$ with its mass $\sqrt{\xi_p} m_p$;

- Comparing with the naive treatment: $\eta$ mass eigenstate is proportional to $\eta$, however, all the three original Goldstone degrees of freedom $(G, G', y^2)$ contain $\eta$ component;

- When we consider the interactions including $\eta$, we must consider the same interaction with $(G, G', y^2)$ together from the original lagrangian.

$\eta$ vertices calculation:

- Divide $\mathbb{F}$ into two parts: $\mathbb{F} = (\tilde{f}, \tilde{\mathbb{F}})$, where $\tilde{f}_p = \mathbb{F}_{p1}$ is a $1\times 3$ vector, whose components are the coefficients of $V^p_\mu \partial^\mu \eta$ transition;
- Thus in $G_j$, the coefficient of $\eta$ component is $(\tilde{\mathbb{F}}^{-1}\tilde{f})_j$;
- Assuming the coefficient of an interaction term including a CP-odd scalar is $c_{G_i}$;
- The physical coefficient including $\eta$ is $\tilde{c}_\eta = \sqrt{(\mathbb{K}^{-1})_{11}} \left(c_\eta - (\tilde{\mathbb{F}}^{-1}\tilde{f})_j c_j\right)$;
- Any vertex should be calculated following the procedure above;
- The method can be generalized into other nonlinear realized models.

Updated results including $\eta$:

- $Zh\eta$: $\mathcal{L} \supset m_Z/(2\sqrt{2}c_W^2 t_{2\beta} v)(v/f)^3(h\partial_\mu\eta - \eta\partial_\mu h)Z^\mu$;
- The tree-level vertex appear at $\mathcal{O}(v^3/f^3)$, gauge boson mixing is also important;
- $\eta q\bar{q}$: $\mathcal{L} \supset \sum_{q=t,d,s} \pm(im_q/\sqrt{2}f s_{2\beta})(c_{2\beta} + c_{2\theta_R})\bar{q}\gamma^5 q\eta$, where we choose "$-$" for $t$, "$+$" for $d$ and $s$, $\theta_R$ is the right-handed mixing angle between SM and the corresponding additional quarks;
- No tree level $\eta f\bar{f}$ couplings for $f = u, c, b, \nu, \ell$;
- These updated results would significantly modify the phenomenology of $\eta$.

## IV. PARAMETER CONSTRAINTS ON SLH MODEL

- The constraints mainly come from: experimental—LHC direct search, theoretical—scalar potential analysis, Goldstone scattering unitarity bounds, etc;

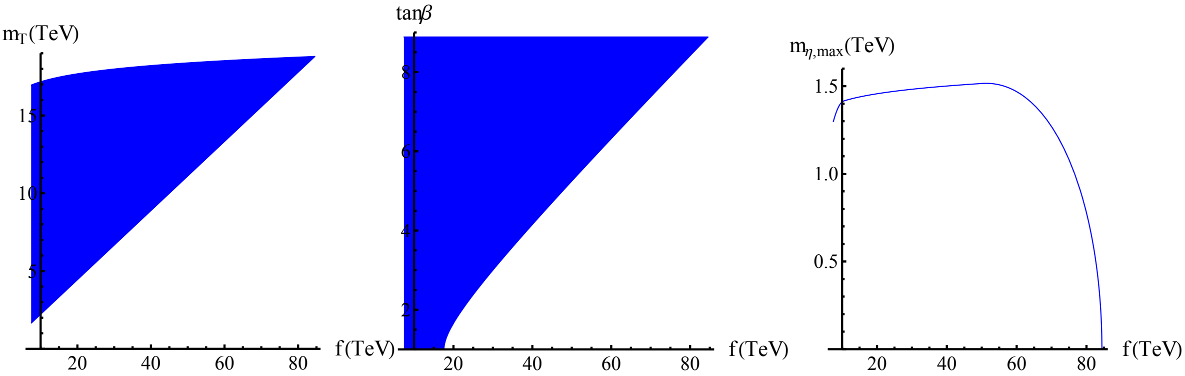
- Lower bound on $f$: LHC direct $Z'$ searches [ATLAS Collaboration, JHEP 1710 (2017), 182] showed that $f \gtrsim 7.5$ TeV [Y.-N. Mao, arXiv: 1703.10123];

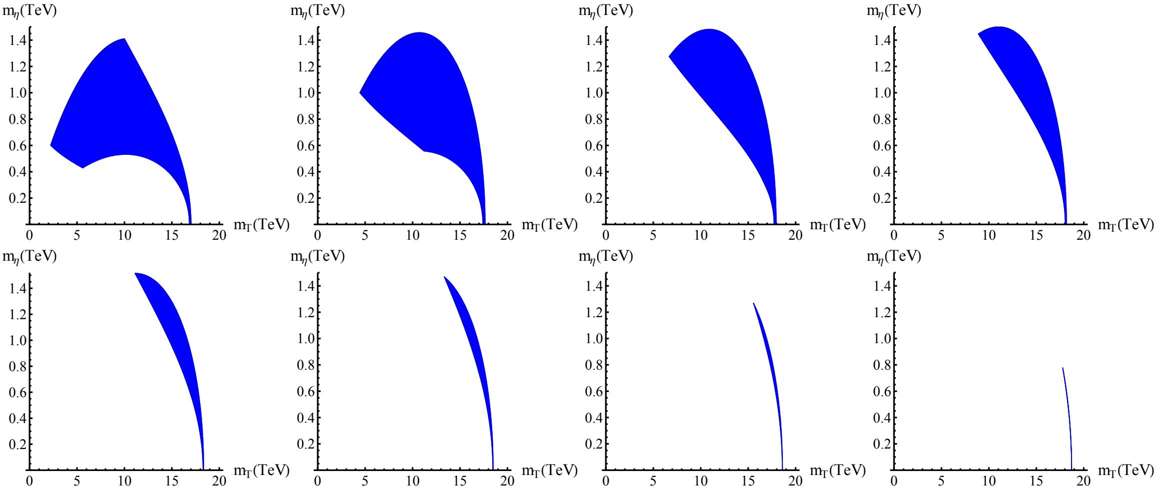
- The result has already become the strictest one, comparing with the electro-weak precision test constraint [J. Reuter and M. Tonini, JHEP 1302 (2013), 077];

- Scalar potential analysis show: $s_{2\beta} < m_T/f$ and $m_T < 14.7(f/v)^{4.24\times10^{-2}}$ TeV;

- Assuming $t_\beta \geq 1$, unitarity bound sets the cut-off scale $F = \sqrt{8\pi}c_\beta f$.

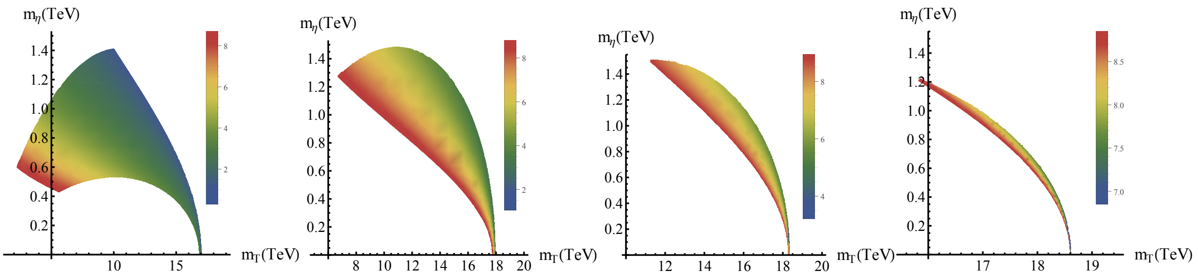
- All the appeared particles should appear below $F$, thus we can set several upper limit as: $f < 84.5$ TeV, $t_\beta < 8.9$, $m_T < 18.7$ TeV, $m_{Z'} < 47.4$ TeV, $m_\eta < 1.5$ TeV.






- Take $f = 10, 20, 30, 40, 50, 60, 70, 80$ TeV for the plots above in order.

- In this page, we show the $t_\beta(>1)$ distributions in the allowed regions as examples for $f = 10, 30, 50, 70$ TeV from left to right.



## V. A BRIEF INTRODUCTION TO $\eta$ PHENOMENOLOGY

| $m_\eta$ | Domain Decay Channels | Domain Production Channels |
|---|---|---|
| $< 2m_\pi$ | $\gamma\gamma$ | $e^+e^- \to (\gamma^*, \Upsilon, Z) \to \eta\gamma$ |
| $2m_\pi$-$m_\Upsilon$ | $\gamma\gamma, gg, d\bar{d}(\pi\pi), s\bar{s}(KK)$ | $e^+e^- \to (\gamma^*, \Upsilon, Z) \to \eta\gamma$ |
| $m_\Upsilon$-$m_Z$ | $\gamma\gamma, gg, b\bar{b}$ (loops) | $e^+e^- \to (Z^{(*)}, \gamma^*) \to \gamma(Z)\eta$, $pp \to \eta, \eta\eta, \eta h, \eta V.\eta g, \ldots$ |
| $m_Z$-$2m_t$ | $gg, \gamma\gamma, WW, ZZ$ (loops) | $e^+e^- \to \eta\gamma, \eta Z$, $pp \to \eta, \eta\eta, \eta h, \eta V, \eta g, \ldots$ |
| $> 2m_t$ | $t\bar{t}$ | $pp \to \eta, \eta\eta, \eta h, \eta V, \eta g, \ldots$ |

- Sorry I and my collaborators cannot finish the calculations before this workshop, so there are no more details about $\eta$ properties in this talk.

## VI. CONCLUSIONS AND DISCUSSIONS

- We accidentally noticed the mistakes in previous papers discussing the SLH model, thus we improved the formalism of the SLH model;

- With this example, we provide the procedure to treat the non-canonically normalized scalar sector and find the exact Goldstone fields for nonlinear realized models;

- Based on the improved formalism, we recalculated the vertices including $\eta$, and obtained the first correct $Zh\eta$ and $\eta f\bar{f}$ vertices in the SLH model (the wrong results have existed for over ten years);

- We also discussed the allowed parameter regions for the SLH model incidentally;

- According to the updated results, the phenomenology of $\eta$ must be renewed from head to foot, we have not finished them so we don't discuss more details here.

## Advertisement

- More collaborators are welcome on the $\eta$ phenomenology (but not limited on this topic), my email address maoyn@ihep.ac.cn; I am also in our workshop Weixin Group, you can find me through the icon “毛” with Parity-violation.

# Thank You