Multiquark Hadrons - the Next Frontier of QCD

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DESY, Hamburg

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CEPC Workshop, IHEP, Beijing

- Experimental Evidence for Multiquark states X, Y, Z and P_c
- Models for X, Y, Z Mesons
- The Diquark model of Tetraquarks
- Mass Spectrum of the low-lying S and P Wave Tetraquark States
- A New Look at the excited Ω_c and the Y States in the Diquark Model
- Doubly Heavy Tetraquarks - Prospects at a Mega-Z Factory
- The Pentaquarks $\mathbb{P}^{\pm}(4380)$ and $\mathbb{P}^{\pm}(4450)$ in the Diquark Model
- Summary

$X(3872)$ - the poster Child of the X, Y, Z Mesons

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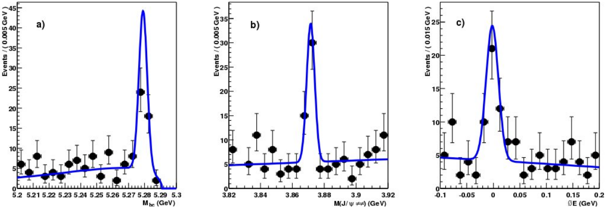
Observation of a Narrow Charmoniumlike State in Exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ Decays

S.-K. Choi,[5] S. L. Olsen,[6] K. Abe,[7] T. Abe,[7] I. Adachi,[7] Byoung Sup Ahn,[14] H. Aihara,[43] K. Akai,[7] M. Akatsu,[20] M. Akemoto,[7] Y. Asano,[48] T. Aso,[47] V. Aulchenko,[1] T. Aushev,[11] A. M. Bakich,[38] Y. Ban,[31] S. Banerjee,[39] A. Bondar,[1] A. Bozek,[25] M. Bračko,[18,12] J. Brodzicka,[25] T. E. Browder,[6] P. Chang,[24] Y. Chao,[24] K.-F. Chen,[24] B. G. Cheon,[37] R. Chistov,[11] Y. Choi,[37] Y. K. Choi,[37] M. Danilov,[11] L. Y. Dong,[9] A. Drutskoy,[11] S. Eidelman,[1] V. Eiges,[11] J. Flanagan,[7] C. Fukunaga,[45] K. Furukawa,[7] N. Gabyshev,[7] T. Gershon,[7] B. Golob,[17,12] H. Guler,[6] R. Guo,[22] C. Hagner,[50] F. Handa,[42] T. Hara,[29] N. C. Hastings,[7] H. Hayashii,[21] M. Hazumi,[7] L. Hinz,[16] Y. Hoshi,[41] W.-S. Hou,[24] Y. B. Hsiung,[24,*] H.-C. Huang,[24] T. Iijima,[20] K. Inami,[20] A. Ishikawa,[20] R. Itoh,[7] M. Iwasaki,[43] Y. Iwasaki,[7] J. H. Kang,[52] S. U. Kataoka,[21] N. Katayama,[7] H. Kawai,[2] T. Kawasaki,[27] H. Kichimi,[7] E. Kikutani,[7] H. J. Kim,[52] Hyunwoo Kim,[14] J. H. Kim,[37] S. K. Kim,[36] K. Kinoshita,[3] H. Koiso,[7] P. Koppenburg,[7] S. Korpar,[18,12] P. Križan,[17,12] P. Krokovny,[1] S. Kumar,[30] A. Kuzmin,[1] J. S. Lange,[4,33] G. Leder,[10] S. H. Lee,[36] T. Lesiak,[25] S.-W. Lin,[24] D. Liventsev,[11] J. MacNaughton,[10] G. Majumder,[39] F. Mandl,[10] D. Marlow,[32] T. Matsumoto,[45] S. Michizono,[7] T. Mimashi,[7] W. Mitaroff,[10] K. Miyabayashi,[21] H. Miyake,[29] D. Mohapatra,[50] G. R. Moloney,[19] T. Nagamine,[42] Y. Nagasaka,[8] T. Nakadaira,[43] T. T. Nakamura,[7] M. Nakao,[7] Z. Natkaniec,[25] S. Nishida,[7] O. Nitoh,[46] T. Nozaki,[7] S. Ogawa,[40] Y. Ogawa,[7] K. Ohmi,[7] Y. Ohnishi,[7] T. Ohshima,[20] N. Ohuchi,[7] K. Oide,[7] T. Okabe,[20] S. Okuno,[13] W. Ostrowicz,[25] H. Ozaki,[7] H. Palka,[25] H. Park,[15] N. Parslow,[38] L. E. Piilonen,[50] H. Sagawa,[7] S. Saitoh,[7] Y. Sakai,[7] T. R. Sarangi,[49] M. Satapathy,[49] A. Satpathy,[7,3] O. Schneider,[16] A. J. Schwartz,[3] S. Semenov,[11] K. Senyo,[20] R. Seuster,[6] M. E. Sevior,[19] H. Shibuya,[40] T. Shidara,[7] B. Shwartz,[1] V. Sidorov,[1] N. Soni,[30] S. Stanič,[48,†] M. Starič,[12] A. Sugiyama,[34] T. Sumiyoshi,[45] S. Suzuki,[51] F. Takasaki,[7] K. Tamai,[7] N. Tamura,[27] M. Tanaka,[7] M. Tawada,[7] G. N. Taylor,[19] Y. Teramoto,[28] T. Tomura,[43] K. Trabelsi,[6] T. Tsukamoto,[7] S. Uehara,[7] K. Ueno,[24] Y. Unno,[2] S. Uno,[7] G. Varner,[6] K. E. Varvell,[38] C. C. Wang,[24] C. H. Wang,[23] J. G. Wang,[50] Y. Watanabe,[44] E. Won,[14] B. D. Yabsley,[50] Y. Yamada,[7] A. Yamaguchi,[42] Y. Yamashita,[26] H. Yanai,[27] Heyoung Yang,[36] J. Ying,[31] M. Yoshida,[7] C. C. Zhang,[9] Z. P. Zhang,[35] and D. Žontar[17,12]

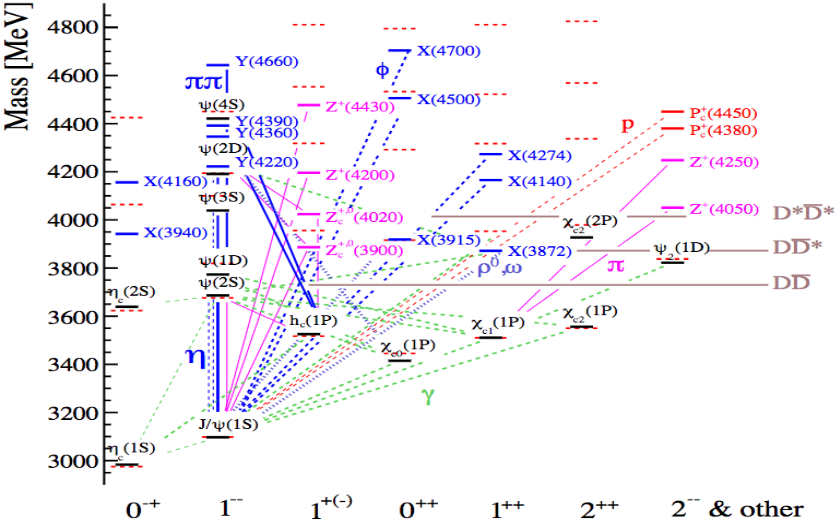
(Belle Collaboration)



- Discovery Mode : $B \to J/\psi\pi^{+}\pi^{-}K$
- $M = 3872.0 \pm 0.6 \pm 0.5$ MeV
- $\Gamma < 2.3$ MeV
- $J^{PC} = 1^{++}$ [LHCb] [PRL110, 22201(2013)]

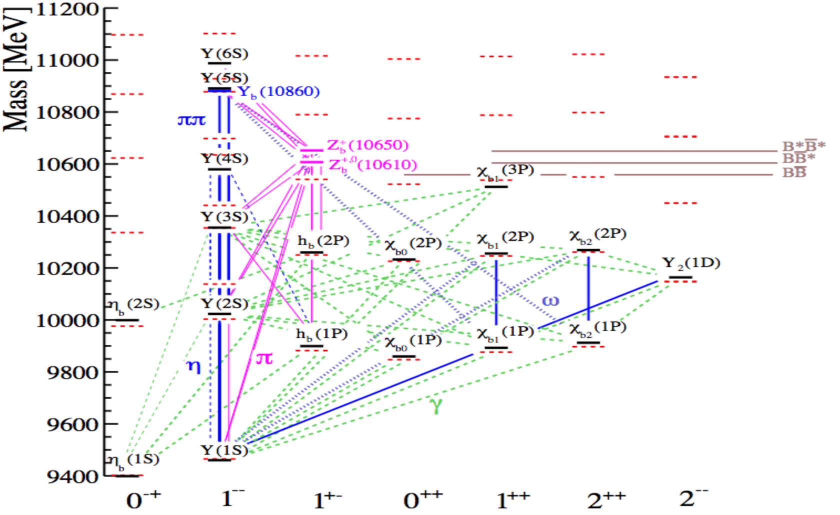
X, Y, Z, P_c and Charmonium States

[S.L. Olsen, T. Skwarnicki, D. Zieminska, arxiv: 1708.04012]



Bottomonium and Bottomonium-like States

[S.L. Olsen, T. Skwarnicki, D. Zieminska, arxiv: 1708.04012]

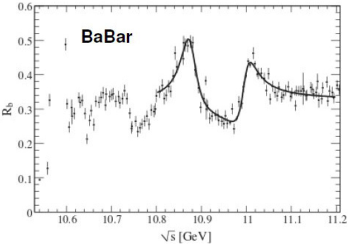


Enigmatic $\Upsilon(5S)$ Decays!

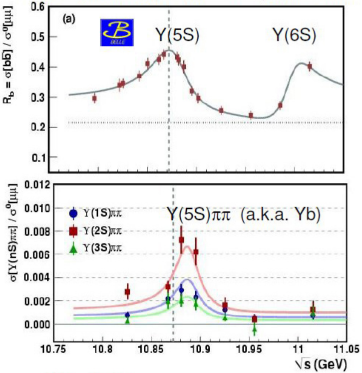
PRL 100, 112001 **(2008)**
21.7 fb^{-1} at 10.580 GeV

$\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$	$0.59 \pm 0.04 \pm 0.09$
$\Upsilon(5S) \to \Upsilon(2S)\pi^+\pi^-$	$0.85 \pm 0.07 \pm 0.16$
$\Upsilon(5S) \to \Upsilon(3S)\pi^+\pi^-$	$0.52^{+0.20}_{-0.17} \pm 0.10$
$\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$	0.0060
$\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$	0.0009
$\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$	0.0019

PRL 102, 012001 **(2009)**



PRD 89, 091106 **(2010)** ~1 fb^{-1}/point SCAN



Belle 2010

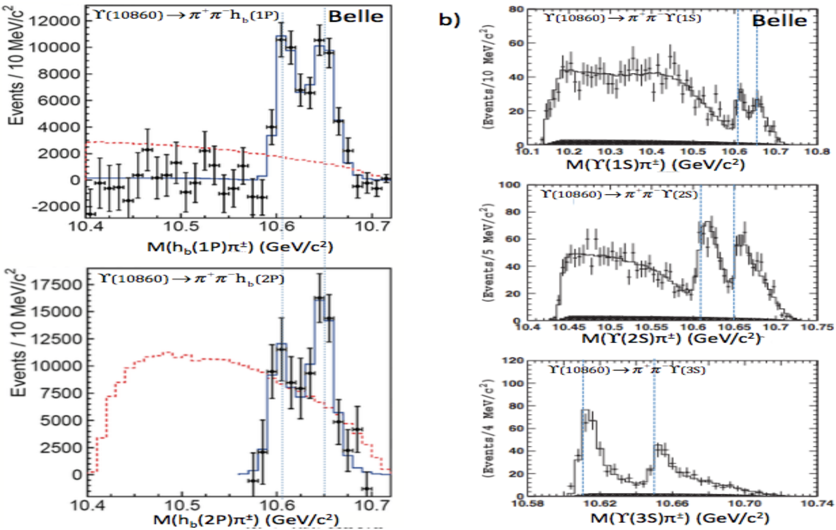
M(5S)$b\bar{b}$ = **10869 ± 2 MeV**
M(5S)$\pi\pi$ = **10888.4 ± 2.7 ± 1.2 MeV**
M(5S) - M(5S)$\pi\pi$ = -9 ± 4 MeV

- Is there a $Y_b(10890)$ close to $\Upsilon(5S)$? If yes, what is it??

[AA, Hambrock, Ishtiaq Ahmed, Jamil Aslam, PLB 684 (2010) 28]

The charged $Z_b^{\pm}(10610)$ and $Z_b^{\pm}(10650)$ states

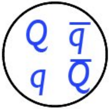
[Belle, PRL 108, 122001 (2012)]



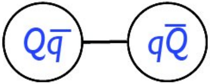
Models for *XYZ* Mesons

Quarkonium Tetraquarks

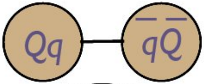
- compact tetraquark



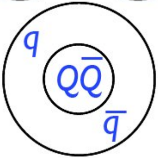
- meson molecule



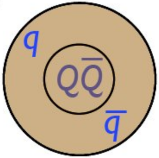
- diquark-onium



- hadro-quarkonium

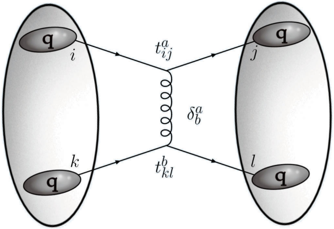


- quarkonium adjoint meson



Diquarks: Color Representation

- One gluon exchange model [Jaffe,Phys.Rept.(2005)]



Color factor determines binding:
Negative sign → Attractive

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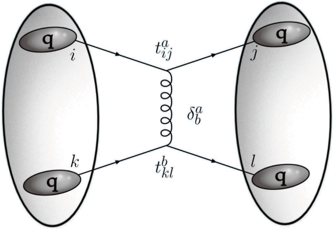

↳ Color factor determines binding:
Negative sign → Attractive

- Quarks in diquark transform as:

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

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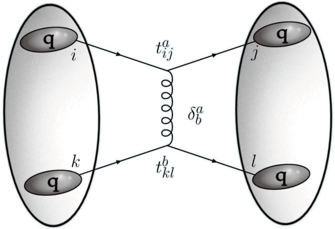

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- qq bound state color factor:

$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{\mathbf{3}}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } \mathbf{6}}$$

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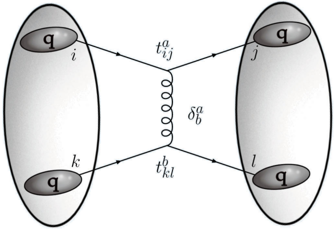

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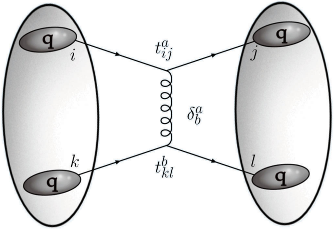

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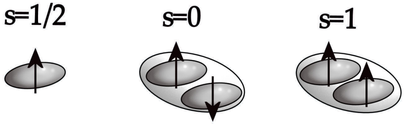

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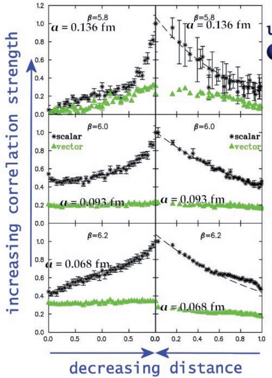
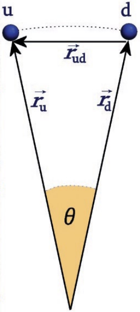
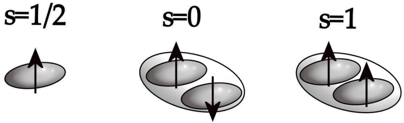


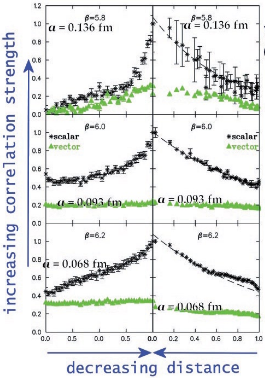


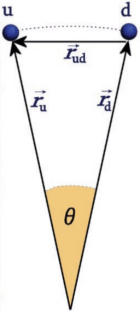


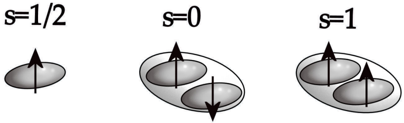
Lattice simulations for light quarks
[Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Calculation of 2 quark correlation strength
- Decreasing distance
  ↪ Increasing strength for "good" diquarks
- Diquark size $\mathcal{O}(1\text{fm})$

Diquarks: Spin representation







Lattice simulations for light quarks [Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Binding for "good" spin 0 diquarks
- No binding for "bad" spin 1 diquarks

- Calculation of 2 quark correlation strength
- Decreasing distance
  ➥ Increasing strength for "good" diquarks
- Diquark size $\mathcal{O}$(1fm)

Spin decoupling in HQ-Limit

➥ "Bad" diquarks in b-sector might bind

Diquark Model of Tetra- and Pentaquarks

Diquarks and Anti-diquarks are the building blocks of Tetraquarks
Color representation: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive; $C_{\bar{3}} = 1/2\, C_3$

Interpolating diquark operators for the two spin-states of diquarks

$$\begin{aligned} \text{Scalar:} \quad & 0^+ \quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}(\bar{c}_c^{\beta}\gamma_5 q_i^{\gamma} - \bar{q}_{i_c}^{\beta}\gamma_5 c^{\gamma}) \\ \text{Axial-Vector:} \quad & 1^+ \quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}(\bar{c}_c^{\beta}\vec{\gamma} q_i^{\gamma} + \bar{q}_{i_c}^{\beta}\vec{\gamma} c^{\gamma}) \end{aligned}$$

α, β, γ: $SU(3)_C$ indices

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NR limit: States parametrized by Pauli matrices :

$$\begin{array}{rccl} \text{Scalar:} & 0^+ & \Gamma^0 & = \frac{\sigma_2}{\sqrt{2}} \\ \text{Axial-Vector:} & 1^+ & \vec{\Gamma} & = \frac{\sigma_2 \vec{\sigma}}{\sqrt{2}} \end{array}$$

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Diquark spin $s_{\mathcal{Q}}$ ➡ tetraquark total angular momentum J:

$$\left| Y_{[bq]} \right\rangle = \left| s_{\mathcal{Q}}, s_{\bar{\mathcal{Q}}};\, J \right\rangle$$

➥ Tetraquarks:

$$\begin{aligned} \left|0_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}};\, 0_J\right\rangle &= \Gamma^0 \otimes \Gamma^0 \\ \left|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\, 0_J\right\rangle &= \frac{1}{\sqrt{3}} \Gamma^i \otimes \Gamma_i \ldots \\ \left|0_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}};\, 1_J\right\rangle &= \Gamma^0 \otimes \Gamma^i \end{aligned}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2} L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}\left[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}\right] + b_Y \frac{S_{12}}{4}$$

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constituent mass

with

$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

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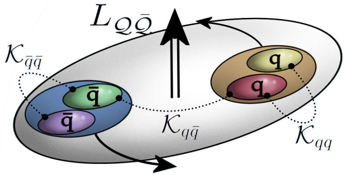
qq spin coupling $\quad$ $q\bar{q}$ spin coupling

with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$



$$= b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

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LS coupling *LL* coupling

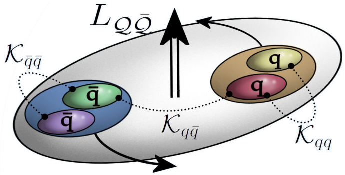
with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L})$$

$$H_{LL} = B_{\mathcal{Q}}\frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}}+1)}{2}$$



$$= b_Y\left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}L^2 + 2A_Q(L\cdot S) + 2\kappa_{qQ}[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}] + b_Y\frac{S_{12}}{4}$$

NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

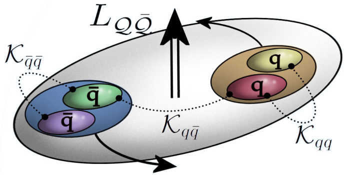
with

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$$H_{SS}^{(q\bar{q})} = 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}})$$

$$H_{SL} = 2A_{\mathcal{Q}}(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L})$$

$$H_{LL} = B_{\mathcal{Q}} \frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}(L_{\mathcal{Q}\bar{\mathcal{Q}}}+1)}{2}$$



$$H_T = b_Y \frac{S_{12}}{4} = b_Y \left[3(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{n})(\mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{n}) - (\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{S}_{\bar{\mathcal{Q}}})\right]; \quad (\mathbf{n} = \text{unit vector})$$

- In the following, assume $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X,Y,Z) = 2m_{\mathcal{Q}} + \frac{B_Q}{2}L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}\left[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}\right] + b_Y \frac{S_{12}}{4}$$

- In the $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$and $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$bases, the positive parity S-wave tetraquarks are listed below; $M_{00} = 2m_{\mathcal{Q}}$

Label	J^{PC}	$\lvert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$\lvert s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
X_0	0^{++}	$\lvert 0,0;0,0\rangle_0$	$\left(\lvert 0,0;0,0\rangle_0 + \sqrt{3}\lvert 1,1;0,0\rangle_0\right)/2$	$M_{00} - 3\kappa_{qQ}$
X_0'	0^{++}	$\lvert 1,1;0,0\rangle_0$	$\left(\sqrt{3}\lvert 0,0;0,0\rangle_0 - \lvert 1,1;0,0\rangle_0\right)/2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$\left(\lvert 1,0;1,0\rangle_1 + \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$\lvert 1,1;1,0\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$\left(\lvert 1,0;1,0\rangle_1 - \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$\left(\lvert 1,0;1,0\rangle_1 - \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$\lvert 1,1;1,0\rangle_1$	$\left(\lvert 1,0;1,0\rangle_1 + \lvert 0,1;1,0\rangle_1\right)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$\lvert 1,1;2,0\rangle_2$	$\lvert 1,1;2,0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters, $M_{00}(Q)$and κ_{qQ}, $Q = c, b$, hence very predictive
- Some of the states, such as X_0, X_0', X_2, still missing and are being searched for at the LHC

Parameters in the Mass Formula

	charmonium-like	bottomonium-like
M_{00} [MeV]	3957	10630
κ_{qQ} [MeV]	67	22.5

		charmonium-like		bottomonium-like	
Label	J^{PC}	State	Mass [MeV]	State	Mass [MeV]
X_0	0^{++}	—	3756	—	10562
X_0'	0^{++}	—	4024	—	10652
X_1	1^{++}	$X(3872)$	3890	—	10607
Z	1^{+-}	$Z_c^+(3900)$	3890	$Z_b^{+,0}(10610)$	10607
Z'	1^{+-}	$Z_c^+(4020)$	4024	$Z_b^+(10650)$	10652
X_2	2^{++}	—	4024	—	10652

A new look at the Y tetraquarks and the excited Ω_c states in the Diquark model

- Observation of 5 narrow excited Ω_c baryons in $\Omega_c \to \Xi_c^+ K^-$ [LHCb, PRL 118, 182001 (2017)]
- Measured masses (in MeV) [LHCb] and plausible J^P quantum numbers, assuming diquark model $\Omega_c(= css) = c[ss]$ [M. Karliner, J.L. Rosner, PR D95, 114012 (2017)]

$$
\begin{aligned}
M(\Omega_c(3000)) &= 3000.4 \pm 0.2 \pm 0.1;\ J^P = 1/2^- \\
M(\Omega_c(3050)) &= 3050.2 \pm 0.1 \pm 0.1;\ J^P = 1/2^- \\
M(\Omega_c(3066)) &= 3065.6 \pm 0.1 \pm 0.3;\ J^P = 3/2^- \\
M(\Omega_c(3090)) &= 3090.2 \pm 0.3 \pm 0.5;\ J^P = 3/2^- \\
M(\Omega_c(3119)) &= 3119.1 \pm 0.3 \pm 0.9;\ J^P = 5/2^-
\end{aligned}
$$

- For the P states, important to take into account the tensor couplings

$$
\begin{aligned}
H_{\text{eff}} &= m_c + m_{[ss]} + \kappa_{ss} S_s \cdot S_s + \frac{B_{\mathcal{Q}}}{2} L^2 + V_{SD}, \\
V_{SD} &= a_1 L \cdot S_{[ss]} + a_2 L \cdot S_c + b \frac{\langle S_{12} \rangle}{4} + c\, S_{[ss]} \cdot S_c
\end{aligned}
$$

Analysis of the excited Ω_c states in the Diquark-Quark model

- $b\langle S_{12}\rangle/4$ represents the matrix element of the tensor interaction

$$\frac{S_{12}}{4} = Q(S_1, S_2) = 3(S_1 \cdot n)(S_2 \cdot n) - (S_1 \cdot S_2)$$

- $\boldsymbol{S_1 = S_{[ss]}}$ and $\boldsymbol{S_2 = S_c}$ are the spins of the diquark and the charm quark, respectively, $\vec{n} = \vec{r}/r$ is the unit vector along the radius vector
- The scalar operator above can be expressed as the convolution $3S_1^i S_2^j N_{ij}$

$$N_{ij} = n_i n_j - \frac{1}{3}\delta_{ij}$$

- Need matrix elements of this operator between the states with the same fixed value L of the angular momentum operator L, using an identity from Landau and Lifshitz :

$$\langle N_{ij}\rangle = a(L)(L_i L_j + L_j L_i - \frac{2}{3}\delta_{ij} L(L+1))$$

  where $a(L) = \frac{-1}{(2L-1)(2L+3)}$
- $\longrightarrow \langle Q(S_X, S_X)\rangle = -\frac{3}{5}\langle[2(L\cdot S_X)^2 + (L\cdot S_X) - \frac{4}{3}(S_X \cdot S_X)]\rangle$
  where $S_X = S_{[ss]}, S_c, S = S_{[ss]} + S_c$

Analysis of the excited Ω_c states in the Diquark-Quark model-contd.

- For $L = 1$, and $S_{[ss]} = 1$, all three terms are non-zero, as opposed to the charmonium case, and one has to calculate the matrix element

$$\langle L, S'; J | L \cdot S_X | L, S; J \rangle$$

$$\frac{\langle S_{12} \rangle}{2} = \langle 2Q(S_{[ss]}, S_c) \rangle = \langle Q(S,S) - Q(S_c, S_c) - Q(S_{[ss]}, S_{[ss]}) \rangle$$

- Tensor operator mixes the two $J = 1/2$, and the two $J = 3/2$ states

$$J = 1/2 : \quad \frac{1}{4} \langle S_{12} \rangle = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -1 \end{pmatrix}$$

$$J = 3/2 : \quad \frac{1}{4} \langle S_{12} \rangle = \begin{pmatrix} 0 & -\frac{1}{2\sqrt{5}} \\ -\frac{1}{2\sqrt{5}} & \frac{4}{5} \end{pmatrix}$$

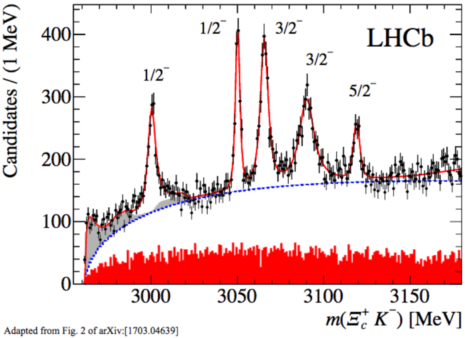
$$J = 5/2 : \quad \frac{1}{4} \langle S_{12} \rangle = -\frac{1}{5}$$

Analysis of the excited Ω_c states in the Diquark-Quark model- contd.

- Coeffs. determined from the masses of the J^P states (in MeV)

a_1	a_2	b	c	M_0
26.95	25.75	13.52	4.07	3079.94

$$M_0 \equiv m_c + m_{[ss]} + 2\kappa_{ss} + B_{\mathcal{Q}}$$



Adapted from Fig. 2 of arXiv:[1703.04639]

Analysis of the tetraquark Y states in the diquark model

$$
\begin{aligned}
H_{\text{eff}} &= 2m_{\mathcal{Q}} + \frac{B_{\mathcal{Q}}}{2}L^2 - 3\kappa_{cq} + 2a_Y L\cdot S + b_Y \frac{\langle S_{12}\rangle}{4} \\
&+ \kappa_{cq}\left[2(S_q\cdot S_c + S_{\bar{q}}\cdot S_{\bar{c}}) + 3\right]
\end{aligned}
$$

$$
\frac{1}{4}\langle S_{12}\rangle = \begin{pmatrix} 0 & 2/\sqrt{5} \\ 2/\sqrt{5} & -7/5 \end{pmatrix}
$$

- There are four $L = 1$ and one $L = 3$ tetraquark states with $J^{PC} = 1^{--}$
- Tensor couplings non-vanishing only for the states with $S_{\mathcal{Q}} = S_{\bar{\mathcal{Q}}} = 1$

P-wave ($J^{PC} = 1^{--}$) states

Label	J^{PC}	$\vert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	Mass
Y_1	1^{--}	$\vert 0,0;0,1\rangle_1$	$M_{00} - 3\kappa_{qQ} + B_Q \equiv \tilde{M}_{00}$
Y_2	1^{--}	$\left(\vert 1,0;1,1\rangle_1 + \vert 0,1;1,1\rangle_1\right)/\sqrt{2}$	$\tilde{M}_{00} + 2\kappa_{qQ} - 2A_Q$
Y_3	1^{--}	$\vert 1,1;0,1\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_+$
Y_4	1^{--}	$\vert 1,1;2,1\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_-$
Y_5	1^{--}	$\vert 1,1;2,3\rangle_1$	$M_{Y_2} + 2\kappa_{qQ} - 14A_Q + 5B_Q - 8/5b_Y$

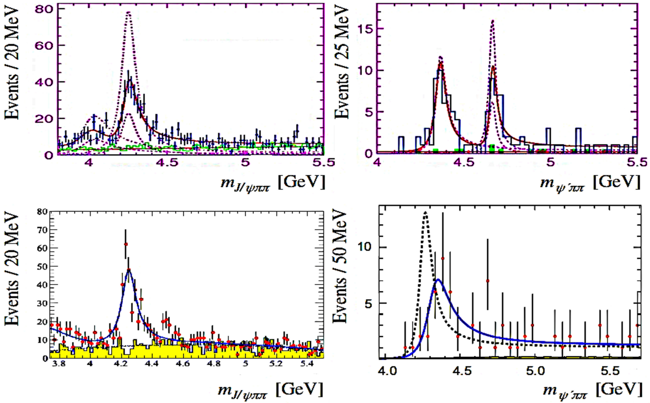
$$
E_\pm = \frac{1}{10}\left(-30A_Q - 7b_Y \mp \sqrt{3}\sqrt{300A_Q^2 + 140A_Q b_Y + 43b_Y^2}\right)
$$

Experimental situation with the tetraquark Y states rather confusing

- Summary of the Y states observed in Initial State Radiation (ISR) processes in e^+e^- annihilation [BaBaR, Belle, CLEO]

$$e^+e^- \to \gamma_{\text{ISR}}\ \ J/\psi\pi^+\pi^-;\ \ \gamma_{\text{ISR}}\ \ \psi'\pi^+\pi^-$$

$$\Longrightarrow\ Y(4008),\ Y(4260),\ Y(4360),\ Y(4660)$$

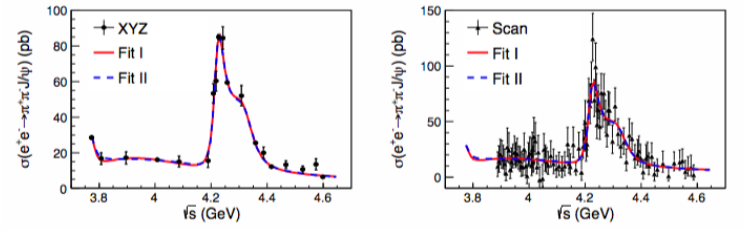


$e^+e^- \to J/\psi\pi^+\pi^-$ cross section at $\sqrt{s} = (3.77 - 4.60)$ GeV

(BESIII, PRL 118, 092001 (2017)

- Y(4008) is not confirmed; Y(4260) is split into 2 resonances: Y(4220) and Y(4320), with the Y(4220) probably the same as Y(4260)

Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6}$ $(\cdots)$
$\Gamma_{tot}(R_1)$	$476.9^{+78.4}_{-64.8}$ $(\cdots)$
$M(R_2)$	4222.0 ± 3.1 (4220.9 ± 2.9)
$\Gamma_{tot}(R_2)$	44.1 ± 4.3 (44.1 ± 3.8)
$M(R_3)$	4320.0 ± 10.4 (4326.8 ± 10.0)
$\Gamma_{tot}(R_3)$	$101.4^{+25.3}_{-19.7}$ $(98.2^{+25.4}_{-19.6})$

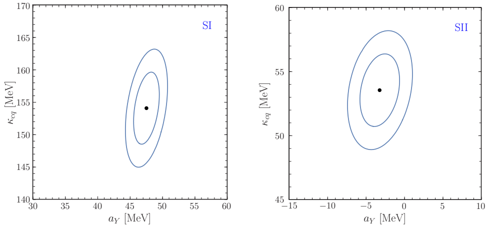


Two Experimental Scenarios for the Y States

[AA, L. Maiani, A. Borisov, I. Ahmed, A. Rehman, M.J. Aslam, A. Parkhomenko, A.D. Polosa, arxiv:1708.04650]

- SI (Based on CLEO, BaBaR, Belle): Y(4008), Y(4260), Y(4360), Y(4660)
- SII (BESIII, PRL 118, 092001 (2017): Y(4220), Y(4320), with Y(4390), Y(4660) the same as in SI

a_Y - κ_{cq} Correlations



- SII (based on BESIII data) is favored, with a_Y and κ_{cq} values similar to the Ω_c analysis

Correlations (Contd.)

- Fixing $\kappa_{cq} = 67$ MeV(from the S states); fitted the two scenarios $\Longrightarrow$ clear preference for SII, with the following parameters (in MeV)

Scenario	M_{00}	a_Y	b_Y	$\chi^2_{\rm min}/{\rm n.d.f.}$
SI	4321 ± 79	2 ± 41	-141 ± 63	$12.8/1$
SII	4421 ± 6	22 ± 3	-136 ± 6	$1.3/1$

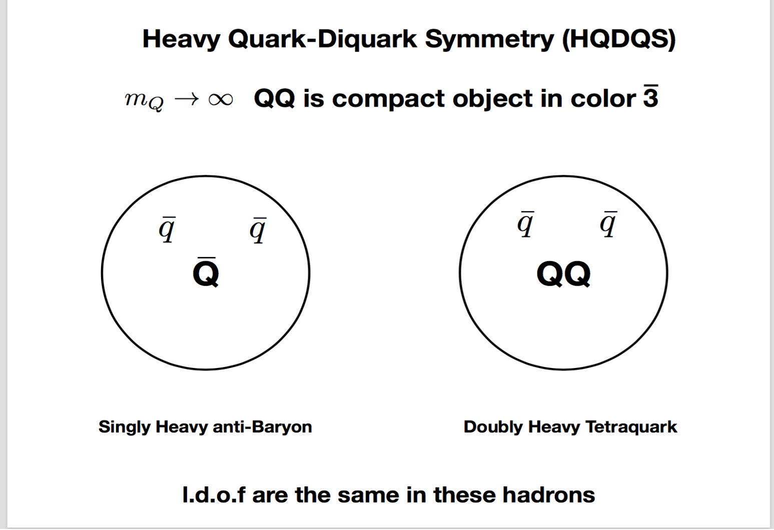
- SII: $M_{00} \equiv 2m_{\mathcal{Q}} + B_{\mathcal{Q}} \Longrightarrow B_{\mathcal{Q}} = 442$ MeV
- Comparable to the orbital angular momentum excitation energy in charmonia

$$B_{\mathcal{Q}}(c\bar{c}) = M(h_c) - \frac{1}{4}\left[3M(J/\psi) + M(\eta_c)\right] = 457 \text{ MeV}$$

- κ_{cq} and a_Y for Y states similar to the ones in (X, Z) and Ω_c
- Precise data on the Y-states is needed to confirm or refute the diquark picture

Stable Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[See Eichten, Mehen, Karliner @ Quarkonium 2017; S-Q. Luo et al., EPJC 77:709 (2017)]



Doubly Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[E. Eichten, C. Quigg, arxiv:1707.09575]

Heavy quark symmetry relations

- In the heavy limit, the color of the core Q_iQ_j is $\bar{3}$ the same as a $\bar{Q}_x$. Hence in leading order of $\mathcal{M}^{-1}$ the light degrees of freedom have the same dynamics in the two systems leading to the following relations

$$
\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y)\ .
\end{aligned}
$$

- Finite mass corrections for all the states in these relations:

$$\delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j}_\ell}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}}$$

Stable Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[E. Eichten, C. Quigg, arxiv:1707.09575]

- $T(bb[\bar{u}\bar{d}])$ and $T(bb[\bar{q}\bar{s}])$ are stable against strong decay
- Will decay weakly by charged current interactions

Expectations for ground-state tetraquark masses

State	J^P	$m(Q_iQ_j\bar{q}_k\bar{q}_l)$	Decay Channel	Q [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	3978	D^+D^{*0} 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	4156	$D^+D_s^{*-}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	4146, 4167, 4210	D^+D^0, D^+D^{*0} 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	7229	B^-D^+/B^0D^0 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	7406	B_sD 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	7439	B^*D/BD^* 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	7272	B^*D/BD^* 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	7445	DB_s^* 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	7461, 7472, 7493	BD/B^*D 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	10482	$B^-\bar{B}^{*0}$ 10603	**−121**
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	10643	$\bar{B}\bar{B}_s^*/\bar{B}_s\bar{B}^*$ 10695/10691	**−48**
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	10674, 10681, 10695	B^-B^0, B^-B^{*0} 10559, 10603	115, 78, 136

Stable Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$, $(QQ = cc, cb, bb)$

[T. Mehen @Quarkonium 2017]

Stable Doubly Heavy Tetraquarks

quark model — M. Karliner, J. Rosner, arXiv:1707.07666

$T_{bb\bar{u}\bar{d}}$ $\quad J^P = 1^+ \quad I = 0 \quad 10389 \pm 12\,\mathrm{MeV}$

$215\,\mathrm{MeV}$ below $B\bar{B}^*$ threshold, stable to strong interaction

heavy quark symmetry — E. Eichten, C. Quigg, arXiv:1707.09575

$10468\,\mathrm{MeV}$ $\quad 135\,\mathrm{MeV}$ below threshold

lattice QCD

$189 \pm 10\,\mathrm{MeV}$ below threshold — A. Francis, et. al., PRL **118**, 142001 (2017)

$60^{+30}_{-38}\,\mathrm{MeV}$ below threshold — P. Bicudo, et. al., PRD **95**, 142001 (2017)

no analogous prediction for $T_{cc\bar{q}\bar{q}}, T_{bc\bar{q}\bar{q}}$ tetraquarks

molecular $T_{cc} = DD^*$ — D. Janc, M. Rosina, Few Body Syst. **35** (2004) 175

arguments for stability in heavy quark limit

J.-P. Ader, J.-M. Richard, P. Taxil, PRD **25** (1982) 2370

Prospects of Stable Tetraquarks at a Tera-Z Factory

- LEP : $\mathcal{B}(Z \to b\bar{b}) = (15.12 \pm 0.05)\%$
  $\mathcal{B}(Z \to b\bar{b}b\bar{b}) = (3.6 \pm 1.3) \times 10^{-4}$
- at a Tera-Z factory (CPEC, CERN), anticipate 10^{10} Zs (or more)
- Great opportunity for a Tera-Z Factory to do precision b-baryon and $b\bar{b}/bb$ multiquark physics
- $T^{\{bb\}}_{[\bar{u}\bar{d}]} \equiv \{bb\}[\bar{u}\bar{d}]$ with $J^P = 1^+$ E. Eichten @ Quarkonium-2017
  bound by 121 MeV (77 MeV below $B^-\bar{B}^0\gamma$ threshold)
- Decay modes of $T^{\{bb\}}_{[\bar{u}\bar{d}]}(10482)^-$
  - $T^{\{bb\}}_{[\bar{u}\bar{d}]}(10482)^- \to B^- D^+ \pi^-;\ \bar{B}^0 D^0 \pi^-$
  - $T^{\{bb\}}_{[\bar{u}\bar{d}]}(10482)^- \to \bar{p}\, \Xi^0_b\, J/\psi$
  - $T^{\{bb\}}_{[\bar{u}\bar{d}]}(10482)^- \to \bar{p}\, \Xi^0_{bc};\ \bar{\Lambda}_c \Omega^0_{bc}$
- In two-body hadronic decays, unobserved doubly heavy baryons Ξ^0_{bc} and Ω^0_{bc} are produced

Prospects of Stable Tetraquarks at a Tera-Z Factory

- $T^{\{bb\}}_{[\bar{u}\bar{s}]} \equiv \{bb\}[\bar{u}\bar{s}]$ and $T^{\{bb\}}_{[\bar{d}\bar{s}]} \equiv \{bb\}[\bar{d}\bar{s}]$ with $J^P = 1^+$

bound by 48 MeV (3 MeV below $B\bar{B}_s\gamma$ threshold)

- Decay modes of $T^{\{bb\}}_{[\bar{u}\bar{s}]}(10643)^-$
  - $T^{\{bb\}}_{[\bar{u}\bar{s}]}(10643)^- \to B^- D_s^+ \pi^-;\ \bar{B}_s^0 D^0 \pi^-$
  - $T^{\{bb\}}_{[\bar{u}\bar{s}]}(10643)^- \to \bar{\Sigma}^- \, \Xi_b^0 \, J/\psi;\ \bar{\Lambda}^0 \, \Xi_b^- \, J/\psi$
  - $T^{\{bb\}}_{[\bar{u}\bar{s}]}(10643)^- \to \bar{\Sigma}^- \, \Xi_{bc}^0;\ \bar{\Sigma}_c^- \Omega_{bc}^0$
- Decay modes of $T^{\{bb\}}_{[\bar{d}\bar{s}]}(10643)^0$
  - $T^{\{bb\}}_{[\bar{d}\bar{s}]}(10643)^0 \to \bar{B}^0 D_s^+ \pi^-;\ \bar{B}_s^0 D^+ \pi^-$
  - $T^{\{bb\}}_{[\bar{d}\bar{s}]}(10643)^0 \to \left(\bar{\Lambda}^0, \bar{\Sigma}^0\right) \Xi_b^0 \, J/\psi;\ \bar{\Sigma}^+ \, \Xi_b^- \, J/\psi$
  - $T^{\{bb\}}_{[\bar{d}\bar{s}]}(10643)^0 \to \left(\bar{\Lambda}^0, \bar{\Sigma}^0\right) \Xi_{bc}^0;\ \bar{\Sigma}_c^0 \Omega_{bc}^0$
- In two-body hadronic decays, unobserved doubly heavy baryons Ξ_{bc}^0 and Ω_{bc}^0 are produced

The Pentaquarks $P_c^+(4380)$ and $P_c^+(4450)$ as resonant $J/\psi p$ states

- Discovery Channel (LHC; $\sqrt{s} = 7$ & 8 TeV; $\int Ldt = 3$ fb^{-1})

$$pp \to b\bar{b} \to \Lambda_b X; \quad \Lambda_b \to K^- J/\psi p$$

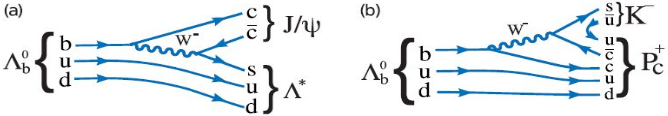


Figure 1: Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

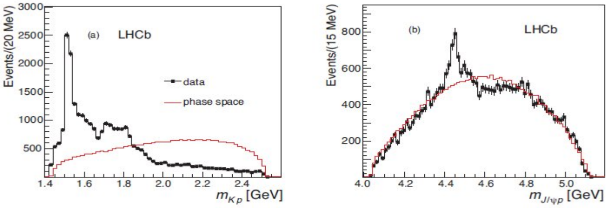


Figure 2: Invariant mass of (a) K^-p and (b) $J/\psi p$ combinations from $\Lambda_b^0 \to J/\psi K^- p$ decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.

Summary of the LHCb Pentaquark Measurements

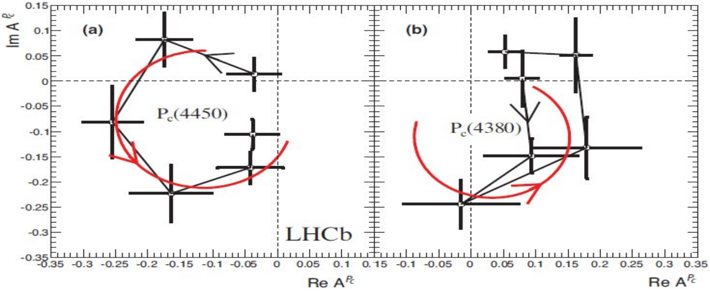
- Higher mass state (statistical significance 12σ)

$$M = 4449.8 \pm 1.7 \pm 2.5 \;\; \text{MeV}; \;\; \Gamma = 39 \pm 5 \pm 19 \;\; \text{MeV}$$

- Lower mass state (statistical significance 9σ)

$$M = 4380 \pm 8 \pm 29 \;\; \text{MeV}; \;\; \Gamma = 205 \pm 18 \pm 86 \;\; \text{MeV}$$

- Fitted Values of the real and imaginary parts of the amplitudes



- For $P_c^+(4450)$, fit shows a phase change in amplitudes consistent with a resonance

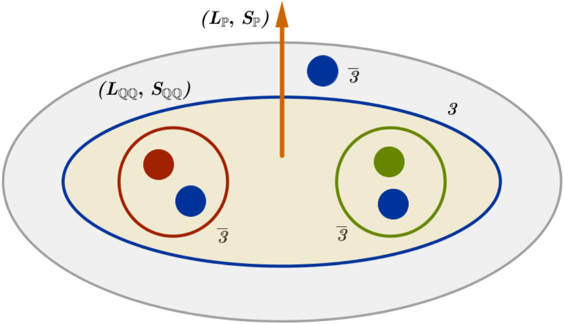
Summary of the LHCb Pentaquark Measurements (Contd.)

Possible J^P assignments and the energies of the nearby thresholds

	$P_c(4380)^+$	$P_c(4450)^+$
Mass	$4380 \pm 8 \pm 29$	$4449.8 \pm 1.7 \pm 2.5$
Width	$205 \pm 18 \pm 86$	$35 \pm 5 \pm 19$
Assignment 1	$3/2^-$	$5/2^+$
Assignment 2	$3/2^+$	$5/2^-$
Assignment 3	$5/2^+$	$3/2^-$
$\Sigma_c^{*+} \bar{D}^0$	4382.3 ± 2.4	
$\chi_{c1} p$		4448.93 ± 0.07
$\Lambda_c^{*+} \bar{D}^0$		4457.09 ± 0.35
$\Sigma_c^+ \bar{D}^{*0}$		4459.9 ± 0.5
$\Sigma_c^+ \bar{D}^0 \pi^0$		4452.7 ± 0.5

Effective Hamiltonian for Pentaquarks

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]



Diquark – Diquark – Antiquark Model of Pentaquarks

$$H_{\text{eff}}(\mathbb{P}) = H_{\text{eff}}([\mathcal{Q}\mathcal{Q}]) + m_{\bar{c}} + \kappa_{\bar{c}[\mathcal{Q}\mathcal{Q}]}(s_{\bar{c}} \cdot S_{[\mathcal{Q}\mathcal{Q}]}) - 2a_{\mathbb{P}}(L_{\mathbb{P}} \cdot S_{\mathbb{P}}) + \frac{B_{\mathbb{P}}}{2}\langle L_{\mathbb{P}}{}^2\rangle$$

- $S_{[\mathcal{Q}\mathcal{Q}]}$ is the spin of the tetraquark; $s_{\bar{c}}$ is the spin of the $\bar{c}$
  $L_{\mathbb{P}}$ and $S_{\mathbb{P}}$ are the orbital angular momentum and spin of the pentaquark, respectively

Pentaquarks in the diquark model [Maiani et al., arxiv:1507.04980]

- $\Lambda_b(bud) \to \mathbb{P}^+ K^-$ decaying according to $\mathbb{P}^+ \to J/\Psi + p$
- $\mathbb{P}^+$ carry a unit of baryonic number and have the valence quarks

$$\mathbb{P}^+ = \bar{c}cuud$$

- Assume the assignments

$$\mathbb{P}^+(3/2^-) = \left\{\bar{c}\,[cq]_{s=1}[q'q'']_{s=1}, L=0\right\}$$
$$\mathbb{P}^+(5/2^+) = \left\{\bar{c}\,[cq]_{s=1}[q'q'']_{s=0}, L=1\right\}$$

- Mass difference:

  • Level spacing for $\Delta L = 1$ in light baryons; $\Lambda(1405) - \Lambda(1116) \sim 290$ MeV

  • Light-light diquark mass difference for $\Delta S = 1$:
  $[qq']_{s=1} - [qq']_{s=0} = \Sigma_c(2455) - \Lambda_c(2286) \simeq 170$ MeV

- Orbital gap $\mathbb{P}^+(3/2^-) - \mathbb{P}^+(5/2^+)$ is thereby reduced to 120 MeV, more or less in agreement with data, 70 MeV

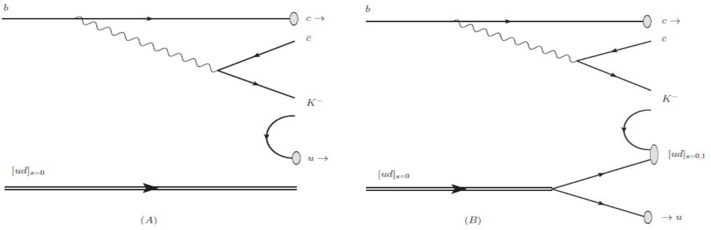
Pentaquark production mechanisms in $\Lambda_b^0 \to K^- J/\psi p$

- Two possible mechanisms are proposed by Maiani et al.

  • In the first, b -quark spin is shared between the K^- ,and the $\bar{c}$ and $[cu]$ components, the final $[ud]$ diquark has spin-0 , Fig. A

  • In the second, the $[ud]$ diquark is formed from the original d quark, and the u quark from the vacuum $u\bar{u}$; angular momentum is shared among all components, and the diquark $[ud]$ may have both spins, $s = 0, 1$, Fig. B

- Which of the two diagrams dominate is a dynamical question; semileptonic decays of Λ_b hint that the mechanism in Fig. B is dynamically suppressed



Heavy quark symmetry and observed pentaquarks

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]

Selection rules from the the data on $b \to c$ baryonic decays and HQS

$$P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 1, J^P = \frac{5}{2}^+\}$$ **Favored**

$$P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_{\mathcal{P}} = 0, J^P = \frac{3}{2}^-\}$$ **Disfavored**

$\Longrightarrow$ $\frac{3}{2}^-$ **state may require a different interpretation**.

$m[\Lambda_c^+(2625); J^P = \frac{3}{2}^-] - m[\Lambda_c^+(2286); J^P = \frac{1}{2}^+] \simeq 341$ MeV $\Longrightarrow$ the mass of $J^P = 3/2^-$ state to be about 4110 MeV.

In **diquark-diquark-antiquark** spectrum, $\frac{3}{2}^-$ state is favored by HQS,

$$\{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_{\mathcal{P}} = 0, J^P = \frac{3}{2}^-\},$$

Third state anticipated in 4110-4130 MeV range. A renewed fit of the LHCb data by allowing a third resonance is called for.

Weak decays of the b-baryons into pentaquark states

$$\mathcal{A} = \left\langle \mathcal{P}\mathcal{M} \left| H_{\text{eff}}^{W} \right| \mathcal{B} \right\rangle, \text{ with } H_{\text{eff}}^{W} = \frac{4G_F}{\sqrt{2}} \left[V_{cb} V_{cq}^{*} (c_1 O_1^{(q)} + c_2 O_2^{(q)}) \right]$$

H_{eff}^{W} inducing the Cabibbo-allowed $\Delta I = 0, \Delta S = -1$ transition $b \to c\bar{c}s$, and the Cabibbo-suppressed $\Delta S = 0$ transition $b \to c\bar{c}d$.

$$O_1^{(q)} = (\bar{q}_\alpha c_\beta)_{V-A} (\bar{c}_\alpha b_\beta)_{V-A} \text{ and } O_2^{(q)} = (\bar{q}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A}$$

$\mathcal{B}_{ij}\ (\bar{3}) = \Lambda_b^0(udb),\ \Xi_b^0(usb),\ \Xi_b^-(dsb)$, $\quad \mathcal{C}_{ij}\ (6) = \Sigma_b^-(ddb),\ \Sigma_b^0(udb),\ \Sigma_b^+(uub)),\ \Xi_b'(dsb),\ \Xi_b'^0(usb),\ \Omega_b^-(ssb)$

$$\mathcal{M}_i^j = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \quad \mathcal{P}_i^j\ (J^P) = \begin{pmatrix} \frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_{\Sigma^+} & P_p \\ P_{\Sigma^-} & -\frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_n \\ P_{\Xi^-} & P_{\Xi^0} & -\frac{P_\Lambda}{\sqrt{6}} \end{pmatrix}.$$

A decuplet $\mathcal{P}_{ijk}$: $\mathcal{P}_{111} = P_{\Delta_{10}^{++}}, \mathcal{P}_{112} = P_{\Delta_{10}^{+}}/\sqrt{3}, \mathcal{P}_{122} = P_{\Delta_{10}^{0}}/\sqrt{3}, \mathcal{P}_{222} = P_{\Delta_{10}^{-}}, \mathcal{P}_{113} = P_{\Sigma_{10}^{+}}/\sqrt{3}, \mathcal{P}_{123} = P_{\Sigma_{10}^{0}}/\sqrt{6}, \mathcal{P}_{223} = P_{\Sigma_{10}^{-}}/\sqrt{3}, \mathcal{P}_{133} = P_{\Xi_{10}^{0}}/\sqrt{3}, \mathcal{P}_{233} = P_{\Xi_{10}^{-}}/\sqrt{3}$ and $\mathcal{P}_{333} = P_{\Omega_{10}^{-}}$.

◇ **Calculating the decay amplitudes is a formidable challenge.**
◇ **$SU(3)_F$ symmetry relations provided useful guide for pentaquark searches,** Li *et al.* [arXiv:1507.08252]

$SU(3)$ based analysis of $\Lambda_b \to \mathbb{P}^+ K^- \to (J/\psi\, p) K^-$

- With respect to flavor $SU(3)$, $\Lambda_b(bud) \sim \bar{3}$, and is isosinglet $I = 0$
- The weak non-leptonic Hamiltonian for $b \to c\bar{c}s$ decays is:

$$H_W^{(3)}(\Delta I = 0, \Delta S = -1)$$

- With M a nonet of $SU(3)$ light mesons, $\langle \mathbb{P}, M | H_W | \Lambda_b \rangle$ requires $\mathbb{P} + M$ to be in $8 \oplus 1$ representation
- Recalling the $SU(3)$ group multiplication rule

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27$$
$$8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35$$

the decay $\langle \mathbb{P}, M | H_W | \Lambda_b \rangle$ can be realized with $\mathbb{P}$ in either an octet (8) or a decuplet (10)

- The discovery channel $\Lambda_b \to \mathbb{P}^+ K^- \to J/\psi p K^-$ corresponds to $\mathbb{P}$ in an octet (8)

Weak decays with $\mathbb{P}$ in Decuplet representation

- Decays involving the decuplet (10) pentaquarks may also occur, if the light diquark pair having spin-0 $[ud]_{s=0}$ in Λ_b gets broken to produce a spin-1 light diquark $[ud]_{s=1}$

$$\Lambda_b \to \pi \mathbb{P}_{10}^{(S=-1)} \to \pi(J/\psi \Sigma(1385))$$

$$\Lambda_b \to K^+ \mathbb{P}_{10}^{(S=-2)} \to K^+(J/\psi \Xi^-(1530))$$

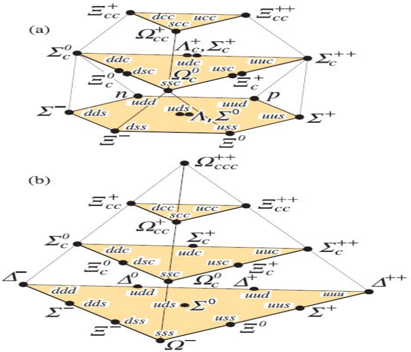
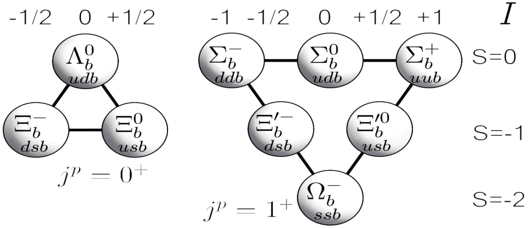


Figure 15.4: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

Weak decays with $\mathbb{P}$ in Decuplet representation - Contd.

- Apart from $\Lambda_b(bud)$, several b-baryons, such as $\Xi_b^0(usb)$, $\Xi_b^-(dsb)$ and $\Omega_b^-(ssb)$ undergo weak decays



- Examples of bottom-strange b-baryon in various charge combinations, respecting $\Delta I = 0$, $\Delta S = -1$ are:

$$\Xi_b^0(5794) \to K(J/\psi\Sigma(1385))$$

which corresponds to the formation of the pentaquarks with the spin configuration $(q, q' = u, d)$

$$\mathbb{P}_{10}(\bar{c}\,[cq]_{s=0,1}\,[q's]_{s=0,1})$$

Weak decays with $\mathbb{P}$ in Decuplet representation - Contd.

- The $s\bar{s}$ pair in Ω_b is in the symmetric (6) representation of flavor $SU(3)$ with spin 1; expected to produce decuplet Pentaquarks in association with a ϕ or a Kaon

$$\Omega_b(6049) \to \phi(J/\psi\,\Omega^-(1672))$$
$$\Omega_b(6049) \to K(J/\psi\,\Xi(1387))$$

- These correspond, respectively, to the formation of the following pentaquarks ($q = u, d$)

$$\mathbb{P}^-_{10}(\bar{c}\,[cs]_{s=0,1}\,[ss]_{s=1})$$
$$\mathbb{P}_{10}(\bar{c}\,[cq]_{s=0,1}\,[ss]_{s=1})$$

- These transitions are on firmer theoretical footings, as the initial $[ss]$ diquark in Ω_b is left unbroken; more transitions can be found relaxing this condition
- At a mega-Z factory, such as CPEC, the entire b-baryon multiplet will be measured through the process $Z \to b\bar{b}; b \to (\Lambda_b, \Xi_b, \Omega_b, ...)$, allowing to reconstruct a lot of pentaquarks in b-baryon decays

Estimates of the ratio of decay widths for $J^P = \frac{5}{2}^+$

[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2} K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2} K^-)$
$\Lambda_b \to P_p^{\{\Upsilon_2\}c_1} K^-$	1	$\Xi_b^- \to P_{\Sigma^-}^{\{\Upsilon_2\}c_2} \bar{K}^0$	2.07
$\Lambda_b \to P_n^{\{(\Upsilon_2\}c_1} \bar{K}^0$	1	$\Xi_b^0 \to P_{\Sigma^+}^{\{\Upsilon_2\}c_2} K^-$	2.07
$\Lambda_b \to P_{\Lambda^0}^{\{\Upsilon_2\}c_3} \eta'$	0.03	$\Lambda_b \to P_{\Lambda^0}^{\{\Upsilon_2\}c_3} \eta$	0.19
$\Xi_b^- \to P_{\Sigma^0}^{\{\Upsilon_2\}c_2} K^-$	1.04	$\Xi_b^- \to P_{\Lambda^0}^{\{\Upsilon_2\}c_2} K^-$	0.34
$\Omega_b^- \to P_{\Xi_{10}^-}^{\{\Upsilon_3\}c_5} \bar{K}^0$	0.14	$\Omega_b^- \to P_{\Xi_{10}^0}^{\{\Upsilon_3\}c_5} K^-$	0.14

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2} K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{5/2} K^-)$
$\Lambda_b \to P_p^{\{\Upsilon_2\}c_1} \pi^-$	**0.08**	$\Lambda_b \to P_n^{\{\Upsilon_2\}c_1} \pi^0$	0.04
$\Lambda_b \to P_n^{\{\Upsilon_2\}c_1} \eta$	0.01	$\Lambda_b \to P_n^{\{\Upsilon_2\}c_1} \eta'$	0
$\Xi_b^- \to P_{\Xi^-}^{\{\Upsilon_2\}c_4} K^0$	0.02	$\Xi_b^- \to P_{\Sigma^0}^{\{\Upsilon_2\}c_2} \pi^-$	0.08
$\Xi_b^- \to P_{\Sigma^-}^{\{\Upsilon_2\}c_2} \eta$	0.02	$\Xi_b^- \to P_{\Sigma^-}^{\{\Upsilon_2\}c_2} \eta'$	0.01
$\Xi_b^- \to P_{\Sigma^-}^{\{\Upsilon_2\}c_2} \pi^0$	0.08	$\Xi_b^0 \to P_{\Sigma^0}^{\{\Upsilon_2\}c_2} \pi^0$	0.04
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\ (\Upsilon_2)\}c_2} \eta$	0.01	$\Xi_b^0 \to P_{\Lambda^0}^{\{\Upsilon_2\}c_2} \eta'$	0.01
$\Xi_b^0 \to P_{\Lambda^0}^{\{\Upsilon_2\}c_2} \pi^0$	0.01	$\Omega_b^- \to P_{\Xi_{10}^-}^{\{\Upsilon_3\}c_5} \pi^0$	0.01
$\Omega_b^- \to P_{\Xi_{10}^0}^{\{\Upsilon_3\}c_5} \pi^-$	0.02		

- **We have used the pentaquark masses estimated in this work.**
- **$\Delta S = 0$ are suppressed by $|V_{cd}^*/V_{cs}^*|^2$ compared to $\Delta S = 1$.**

Estimates of the ratio of decay widths for $J^P = \frac{3}{2}^-$

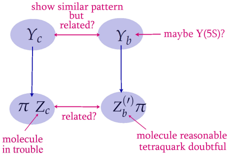
[Ahmed,Rehman,Aslam,AA, arxiv:1607.00987]

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$
$\Lambda_b \to P_p^{\{X_2\}_{c_1}} K^-$	1	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \bar{K}^0$	1.38
$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \bar{K}^0$	1	$\Xi_b^0 \to P_{\Sigma^+}^{\{X_2\}_{c_2}} K^-$	1.38
$\Lambda_b \to P_{\Lambda^0}^{\{X_2\}_{c_3}} \eta'$	0.17	$\Lambda_b \to P_{\Lambda^0}^{\{X_2\}_{c_3}} \eta$	0.22
$\Xi_b^- \to P_{\Sigma^0}^{\{X_2\}_{c_2}} K^-$	0.69	$\Xi_b^- \to P_{\Lambda^0}^{\{X_2\}_{c_2}} K^-$	0.23
$\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\}_{c_5}} \bar{K}^0$	0.24	$\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\}_{c_5}} K^-$	0.24

Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$	Decay Process	$\Gamma/\Gamma(\Lambda_b^0 \to P_p^{\{X_2\}_{c_1}} K^-)$
$\boldsymbol{\Lambda_b \to P_p^{\{X_2\}_{c_1}} \pi^-}$ [1]	**0.06**	$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \pi^0$	0.03
$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \eta$	0.01	$\Lambda_b \to P_n^{\{X_2\}_{c_1}} \eta'$	0.01
$\Xi_b^- \to P_{\Xi^-}^{\{X_2\}_{c_4}} K^0$	0.02	$\Xi_b^- \to P_{\Sigma^0}^{\{X_2\}_{c_2}} \pi^-$	0.03
$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \eta$	0.02	$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \eta'$	0.01
$\Xi_b^- \to P_{\Sigma^-}^{\{X_2\}_{c_2}} \pi^0$	0.04	$\Xi_b^0 \to P_{\Sigma^0}^{\{X_2\}_{c_2}} \pi^0$	0.02
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\}_{c_2}} \eta$	0	$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\}_{c_2}} \eta'$	0
$\Xi_b^0 \to P_{\Lambda^0}^{\{X_2\}_{c_2}} \pi^0$	0.01	$\Omega_b^- \to P_{\Xi_{10}^-}^{\{X_3\}_{c_5}} \pi^0$	0.01
$\Omega_b^- \to P_{\Xi_{10}^0}^{\{X_3\}_{c_5}} \pi^-$	0.02		

Summary

- A new facet of QCD is opened by the discovery of the exotic states X, Y, Z, $\mathbb{P}(4380)$, $\mathbb{P}(4450)$
- Important puzzles remain in the complex:



- What is the nature of $Y_c(4260)$? A tetraquark? or a $c\bar{c}g$ hybrid? Is $Y_c(4260)$split? How many P states are there? We do expect a tower of radial and orbital excited states in the diquark picture!
- A very rich spectrum of tetraquark and pentaquark states is anticipated, including the ones with a single c, or a single b quark, as well as those with multiple heavy quarks
- CEPC running as a Z factory will advance the multiquark sector of QCD decisively, in particular, the entire pentaquark spectrum with hidden charm discussed here can be measured from the production and decays of the b-baryons
- Also CEPC has great potential to discover doubly heavy tetraquarks, establishing diquarks as a building block in QCD
- We look forward to decisive experimental results from BESIII, Belle-II, LHC, and the CEPC