November 9, 2017
Workshop on Electroweak and Flavor Physics at CEPC @ IHEP

Georgi-Machacek Model Beyond Tree Level

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CWC, AL Kuo, K Yagyu, PLB 774 (2017) 119 [1707.04176]
CWC, AL Kuo, K Yagyu, [1712,xxxxx]

Overview

- Motivations
- Georgi-Machacek (GM) model
- Renormalization and radiative corrections
- Numerical results
- Summary

Higgs Physics Program

- Higgs mechanism in SM offers an elegant and minimal way to give mass to weak bosons and charged fermions:

W, Z mass through gauge interactions

fermion mass through Yukawa interactions

$$\mathcal{L}_\Phi \supset |\mathcal{D}_\mu \Phi|^2 + \mu^2 |\Phi|^2 - \lambda |\Phi|^4 - Y \overline{\psi_L} \Phi \psi_R + \text{H.c.}$$

self interaction

- It also features in a self interaction that plays an important role in electroweak phase transition in the early Universe.

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self interaction

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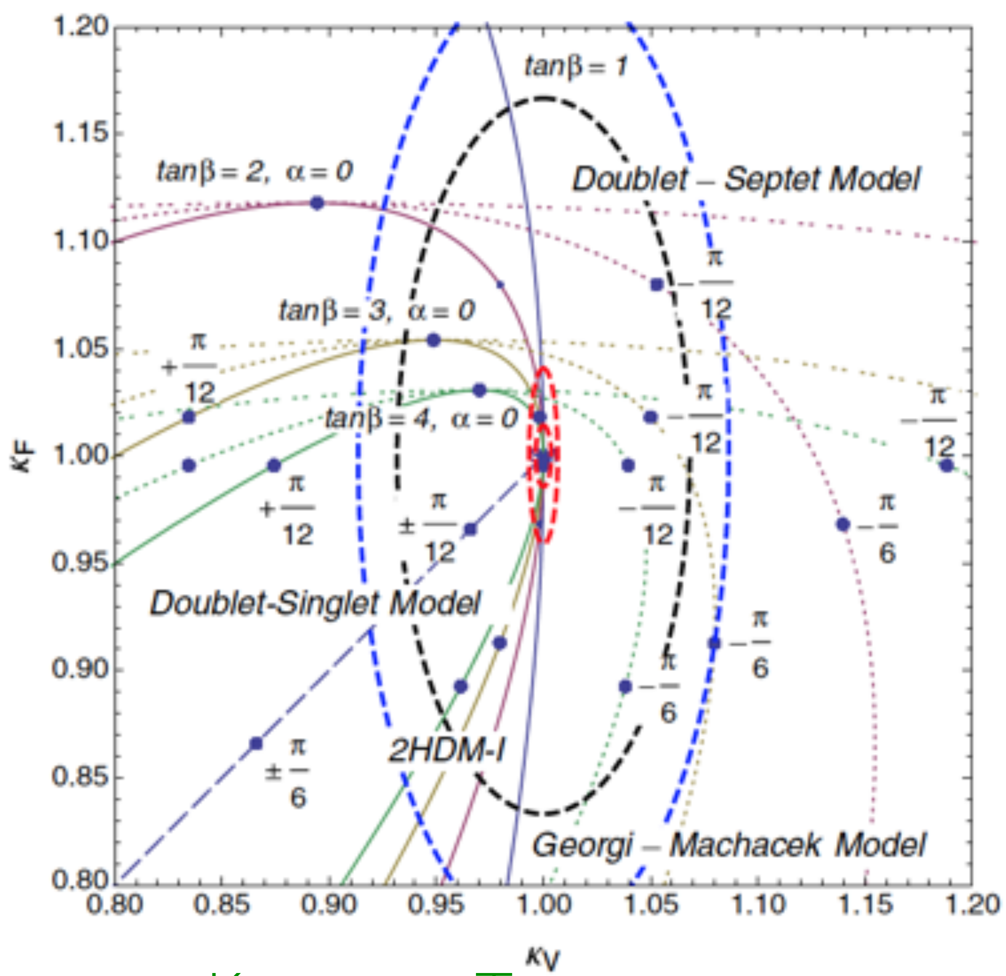
- In the post-Higgs era, it has become an important program in particle physics to determine all its interactions with SM particles (including itself).

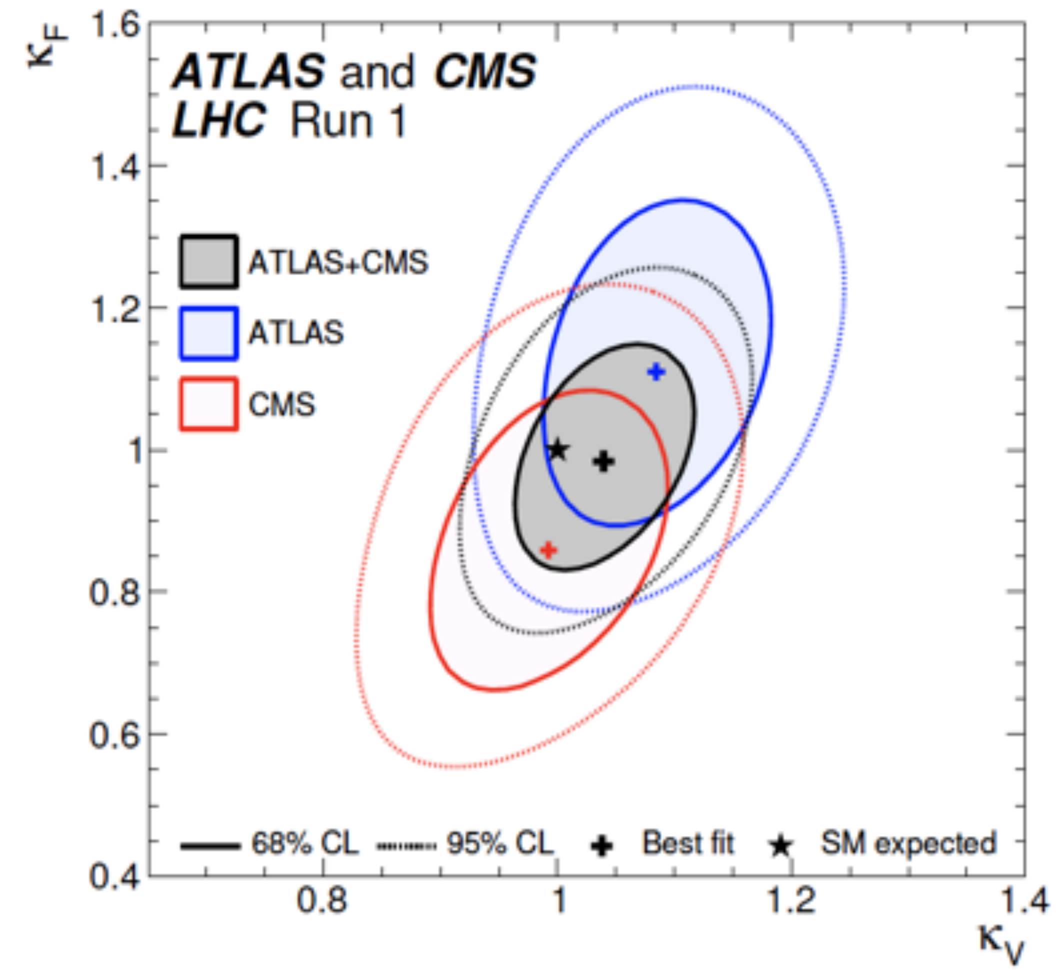
Higgs Physics Program

- Fingerprinting and global fits of Higgs couplings (assuming universal scaling factors $\kappa_{F,V}$) from LHC Run-I
  ➠ quite consistent with SM ($\kappa_V > 1$?)



HSM: long dashed
2HDM: solid
GM: dotted
Septet: thick dashed

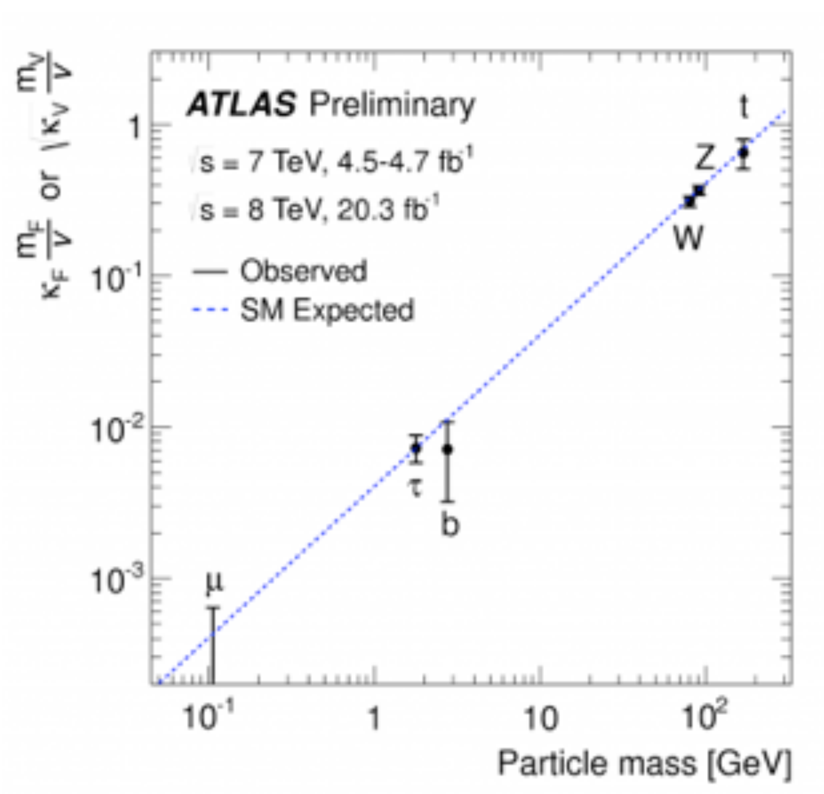
Kanemura, Tsumura, Yagyu, and Yokoya 2014

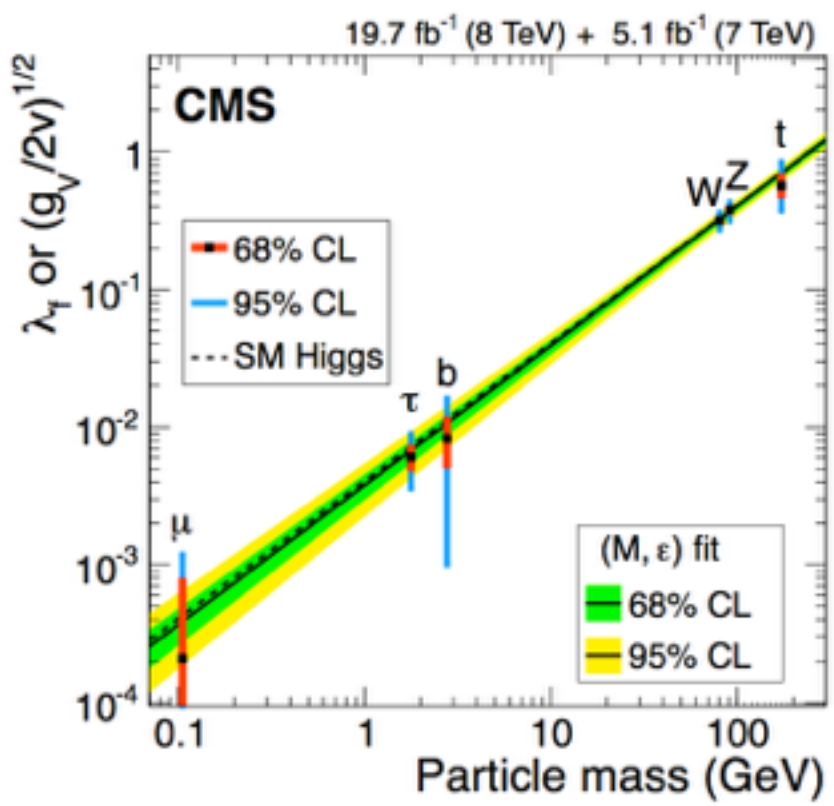


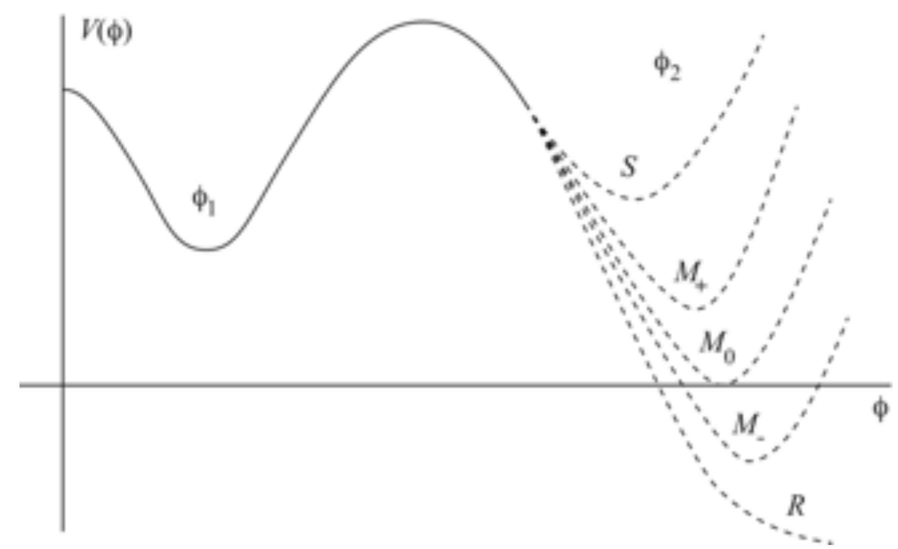
circa 2016 summer

Higgs Physics Program

- Precision coupling measurements are required!
  - ➠ any tiny deviations from SM expectations?
  - ➠ hint of a non-standard structure in the Higgs sector?
  - ➠ an extended Higgs sector?
  - ➠ how EWSB exactly happens?







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- Other than usual symmetries, no guiding principles in constructing the scalar sector:
  - ➠ representations of scalar bosons
  - ➠ numbers of scalar bosons
  - ➠ extra symmetries (continuous/discrete)
  - ➠ required by new physics
    (neutrino mass, DM, EWBG, SUSY, etc)

cf. 3 generations of fermions and 3 gauge interactions

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Study of predictions and constraints of models with an extended Higgs sector

Higgs Extensions

- Higgs extensions are subject to a stringent constraint

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1.00040 \pm 0.00024$$

PDG 2014

- In models with an extended Higgs sector, at tree level

$$\rho_{\rm tree} = \frac{\sum_i v_i^2 \left[T_i(T_i+1) - Y_i^2\right]}{\sum_i 2Y_i^2 v_i^2}$$

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- If only one new $SU(2)_L$ rep is added to the SM, $\rho_{\rm tree} = 1$ gives the following possibilities:
  (0,0) – real singlet, ➠ interacting mainly with h_{SM}
  (1/2,1/2) – doublet, ➠ a popular choice (e.g., 2HDM)
  (3,2) – septet,
  (25/2, 15/2), (48,28), (361/2,209/2), etc.

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- One can also choose to add a custodial symmetric rep (n,n) ($n \in \mathbb{N}$) under $(SU(2)_L, SU(2)_R)$ with vacuum alignment.

  ➠ generalized GM model — Logan, Rentala 2015

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- Simplest CP-conserving custodial Higgs models:
  - real Higgs singlet model (rHSM): Φ_{SM} + S
  - two Higgs doublet model (2HDM): Φ_{SM} + Φ'
  - GM model: Φ_{SM} + Δ

Cheng-Wei Chiang (NTU)

Georgi-Machacek Model

- The Higgs sector includes SM doublet field $\phi(2,1/2)$ and triplet fields $\chi(3,1)$ and $\xi(3,0)$

Georgi, Machacek 1985
Chanowitz, Golden 1985

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ \phi^{-} & \phi^{0} \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \chi^{0*} & \xi^{+} & \chi^{++} \\ \chi^{-} & \xi^{0} & \chi^{+} \\ \chi^{--} & \xi^{-} & \chi^{0} \end{pmatrix}$$

$SU(2)_L$ $SU(2)_R$

transformed under $SU(2)_L \times SU(2)_R$ as

$$\Phi \to U_L \Phi U_R^\dagger \text{ and } \Delta \to U_L \Delta U_R^\dagger \qquad [U_{L,R} = \exp(-i\theta^a_{L,R} T^a)]$$

and T^a being corresponding SU(2) generators.

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and T^a being corresponding SU(2) generators.

- Take $v_\chi = v_\xi \equiv v_\Delta$ (aligned VEV).

➠ $SU(2)_L \times SU(2)_R \to$ custodial $SU(2)_V$

➠ $\rho = 1$ at tree level

Georgi-Machacek Model

- The most general Higgs potential allowed by gauge and Lorentz symmetries and built in with custodial symmetry is

$$V(\Phi,\Delta) = \frac{1}{2}m_1^2\mathrm{tr}[\Phi^\dagger\Phi] + \frac{1}{2}m_2^2\mathrm{tr}[\Delta^\dagger\Delta] + \lambda_1(\mathrm{tr}[\Phi^\dagger\Phi])^2 + \lambda_2(\mathrm{tr}[\Delta^\dagger\Delta])^2$$
$$+ \lambda_3\mathrm{tr}[(\Delta^\dagger\Delta)^2] + \lambda_4\mathrm{tr}[\Phi^\dagger\Phi]\mathrm{tr}[\Delta^\dagger\Delta] + \lambda_5\mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right]\mathrm{tr}\left[\Delta^\dagger T^a\Delta T^b\right]$$
$$+ \mu_1\mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right](P^\dagger\Delta P)_{ab} + \mu_2\mathrm{tr}\left[\Delta^\dagger T^a\Delta T^b\right](P^\dagger\Delta P)_{ab}$$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -(\phi^+)^* & \phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} (\chi^0)^* & \xi^+ & \chi^{++} \\ -(\chi^+)^* & \xi^0 & \chi^+ \\ (\chi^{++})^* & -(\xi^+)^* & \chi^0 \end{pmatrix}, \quad P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}$$

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assume $m_1^2 < 0$, but $m_2^2 > 0$

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$$
\begin{aligned}
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&+ \mu_1 \mathrm{tr}\left[\Phi^\dagger\frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right](P^\dagger\Delta P)_{ab} + \mu_2 \mathrm{tr}\left[\Delta^\dagger T^a\Delta T^b\right](P^\dagger\Delta P)_{ab}
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- Decoupling limit: $m_2 \to \infty$

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- Decoupling limit: $m_2 \to \infty$

- v_Δ induced by v_Φ through μ_1

Higgs Spectrum

Higgs Spectrum

	1 complex triplet + 1 real triplet	1 complex doublet
$\boxed{SU(2)_L \otimes SU(2)_R}$	Δ: **3** $\otimes$ **3**	Φ: **2** $\otimes$ **2**

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$SU(2)_L \otimes SU(2)_R$	Δ: $\mathbf{3} \otimes \mathbf{3}$	Φ: $\mathbf{2} \otimes \mathbf{2}$
$SU(2)_V$	$\mathbf{5} \oplus \mathbf{3} \oplus \mathbf{1}$	$\mathbf{3} \oplus \mathbf{1}$

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$$H_5 \equiv \begin{bmatrix} H_5^{++} \\ H_5^{+} \\ H_5^{0} \\ H_5^{-} \\ H_5^{--} \end{bmatrix}_{m_{H5}}$$

CP-even

Higgs Spectrum

1 complex triplet + 1 real triplet

1 complex doublet

SU(2)$_L$ ⊗ SU(2)$_R$

Δ: 3 ⊗ 3

Φ: 2 ⊗ 2

SU(2)$_V$

5 ⊕ 3 ⊕ 1

3 ⊕ 1

mixing angle β

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m_{H3}

CP-odd

$$\Phi_3 \equiv \begin{bmatrix} w^{+} \\ z^{0} \\ w^{-} \end{bmatrix}$$

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$$H_1 \equiv [H_1^0]$$

m_{H1}

h (125 GeV)

m_h

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$$\tan\beta = \frac{v_\phi}{2\sqrt{2}v_\Delta}$$

$$v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} = (246\ \text{GeV})^2$$

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Φ: **2** $\otimes$ **2**

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gaugephobic

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h (125 GeV)

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5 physical parameters to scan

SU(2)V

5 ⊕ **3** ⊕ **1**

3 ⊕ **1**

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h (125 GeV)

mh

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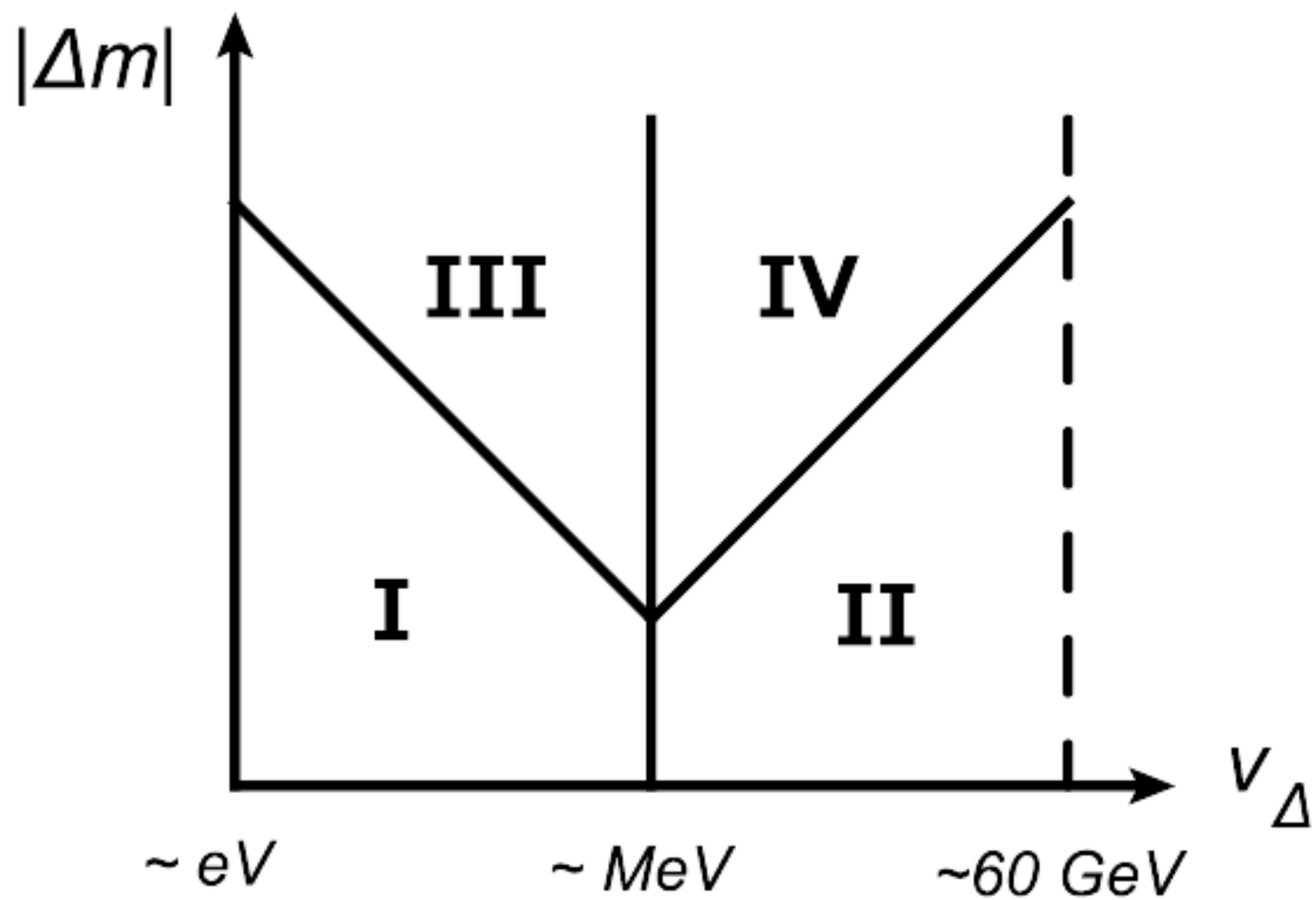
$$v^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} = (246\ \mathrm{GeV})^2$$

Features of GM Model

- Higgs triplet models have the following features:
  - type-II seesaw for Majorana neutrino mass, generated by the VEV of the new triplet scalar field;
  - existence of a doubly-charged Higgs boson, leading to like-sign LNV and possibly even LFV processes at tree level;
    ➠ a link between neutrino and LHC physics
  - SM-like Higgs possibly having stronger/weaker couplings with weak bosons;
  - existence of a $H_5^{\pm}W^{\mp}Z$ vertex at tree level through mixing and proportional to v_Δ (only loop-induced in models such as 2HDM);
  - GM model with custodial symmetry allowing a larger v_Δ.

Decay Pattern

CWC, Yagyu JHEP 2012



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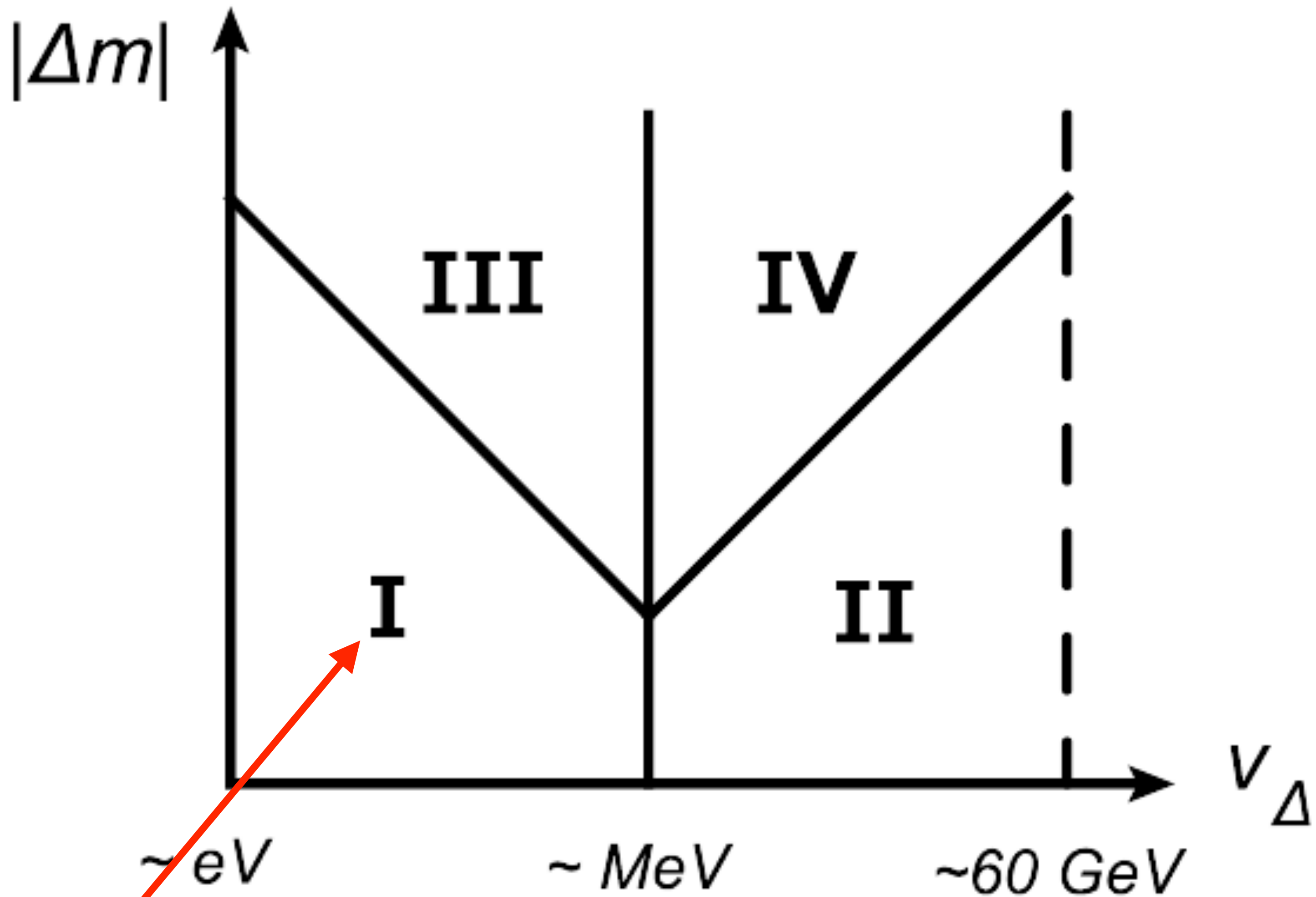

$$H_5^{++} \to \ell^+\ell^+ \ , \ H_5^+ \to \ell^+\nu \ , \ H_5^0 \to \nu\nu \ ,$$
$$H_3^+ \to \ell^+\nu \ , \ H_3^0 \to \nu\nu.$$

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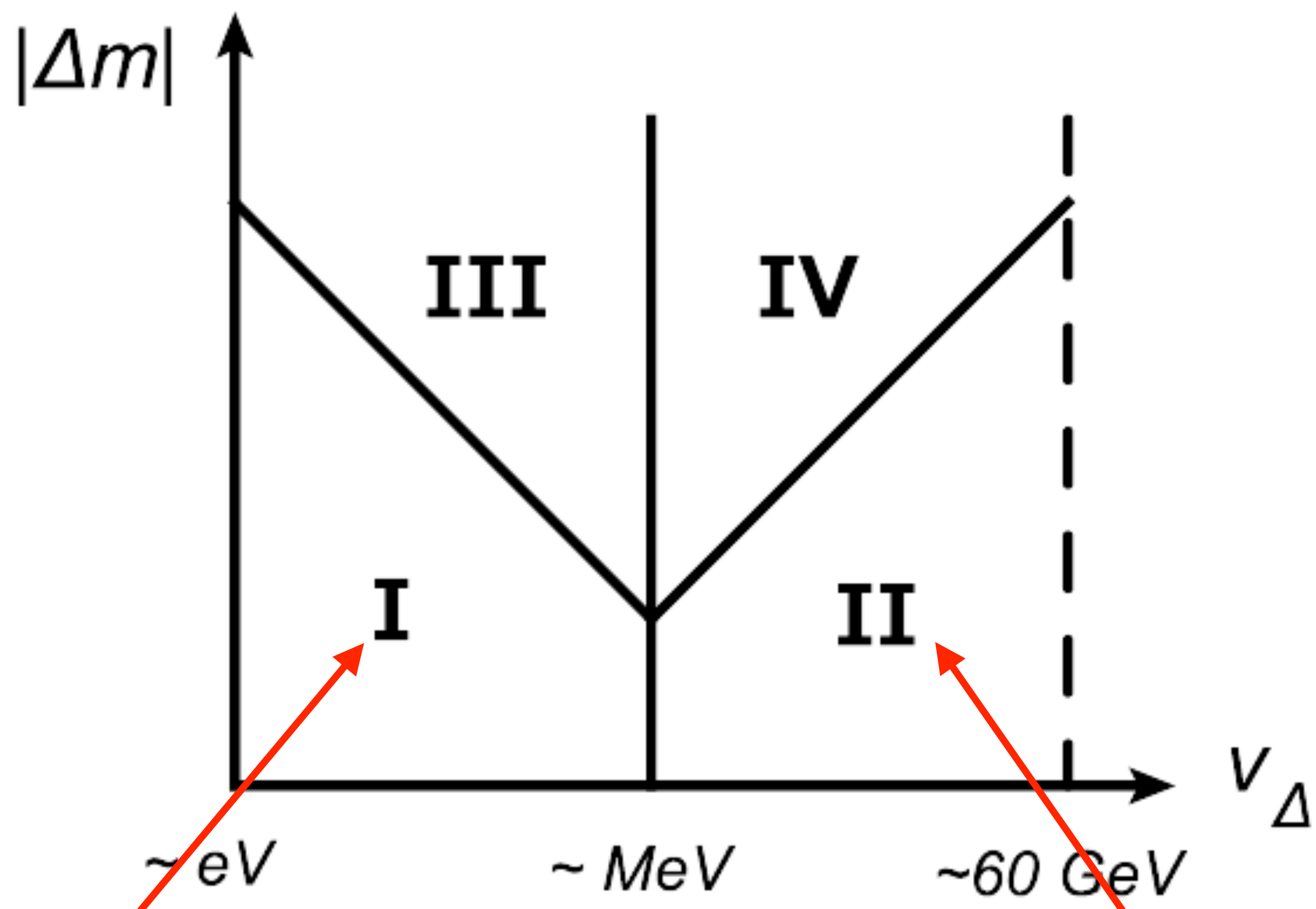

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$H_5^{++} \to W^+W^+ \ , \ \ H_5^+ \to W^+Z \ , \ \ H_5^0 \to W^+W^-/ZZ \ ,$
$H_3^+ \to \tau^+\nu/c\bar{s}/t\bar{b} \ , \ \ H_3^0 \to b\bar{b}.$

Decay Pattern

CWC, Yagyu JHEP 2012

$\Delta m > 0: \ (m_{H_3} > m_{H_5})$

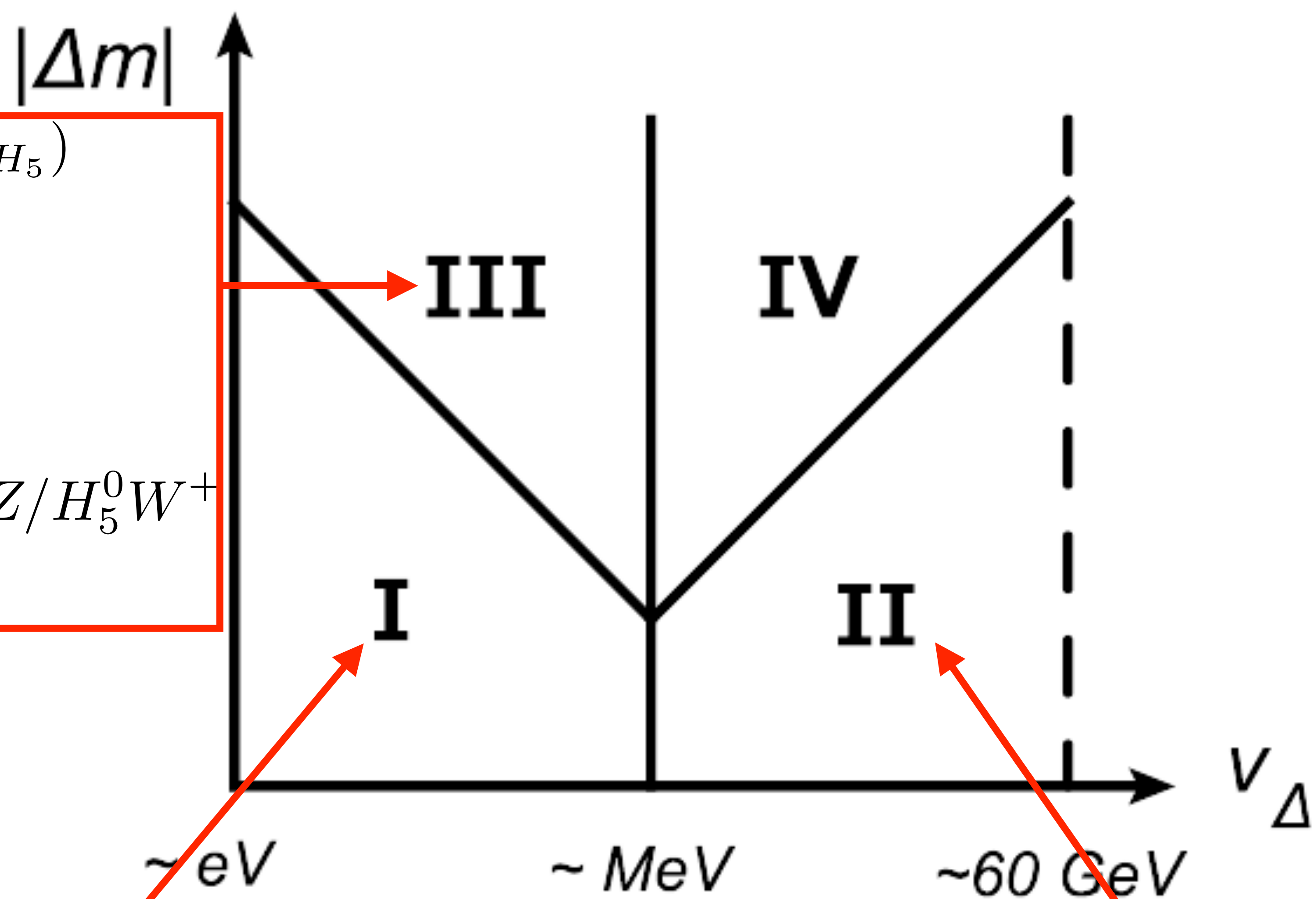
$H_5^{++} \to \ell^+ \ell^+$

$H_5^+ \to \ell^+ \nu$

$H_5^0 \to \nu\nu$

$H_3^+ \to H_5^{++} W^- / H_5^+ Z / H_5^0 W^+$

$H_3^0 \to H_5^{\pm} W^{\mp} / H_5^0 Z$



$H_5^{++} \to \ell^+ \ell^+ \ , \ H_5^+ \to \ell^+ \nu \ , \ H_5^0 \to \nu\nu \ ,$
$H_3^+ \to \ell^+ \nu \ , \ H_3^0 \to \nu\nu .$

$H_5^{++} \to W^+ W^+ \ , \ H_5^+ \to W^+ Z \ , \ H_5^0 \to W^+ W^- / ZZ \ ,$
$H_3^+ \to \tau^+ \nu / c\bar{s} / t\bar{b} \ , \ H_3^0 \to b\bar{b} .$

Decay Pattern

CWC, Yagyu JHEP 2012

$\Delta m > 0: \ (m_{H_3} > m_{H_5})$

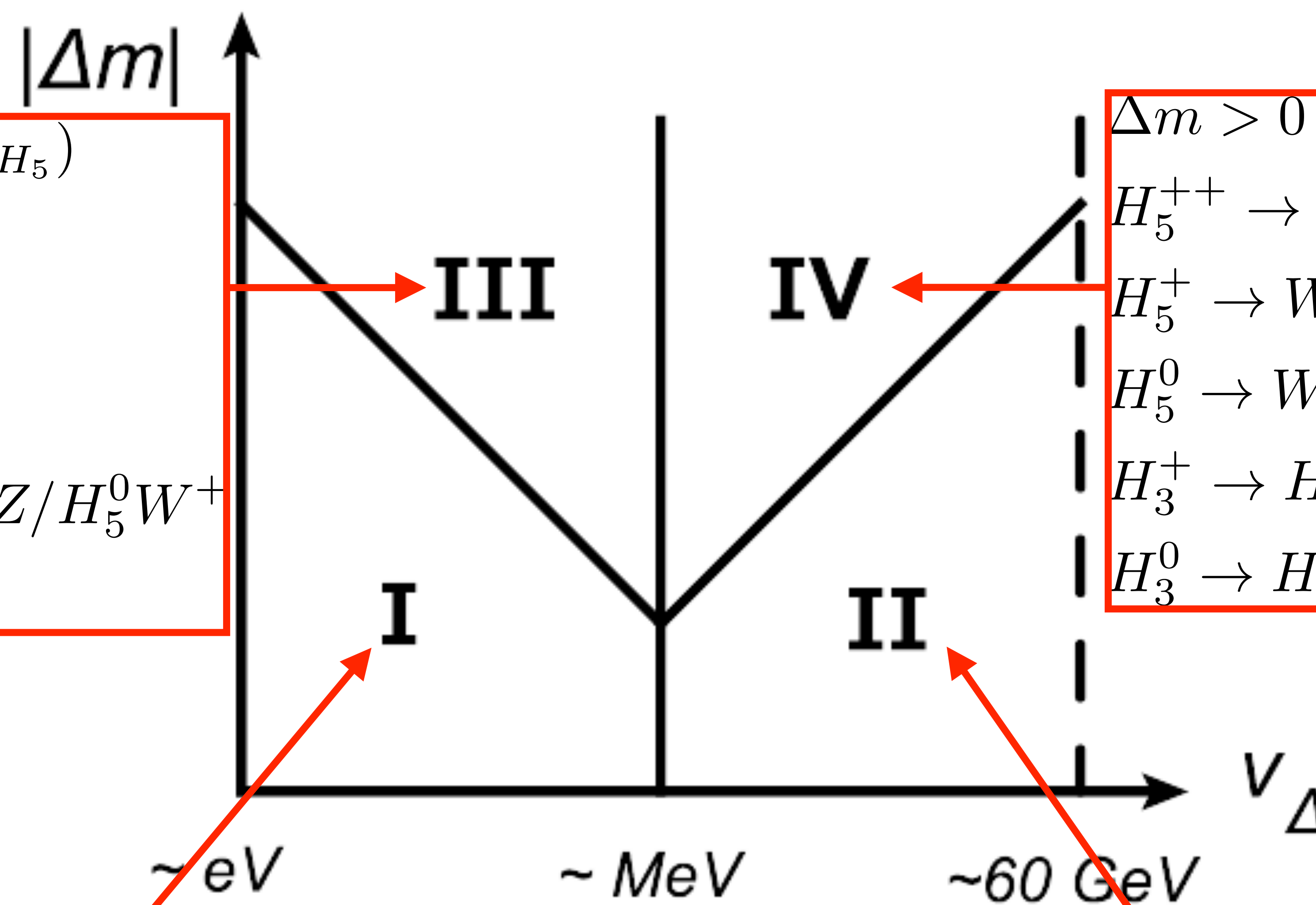
$H_5^{++} \to \ell^+\ell^+$

$H_5^+ \to \ell^+\nu$

$H_5^0 \to \nu\nu$

$H_3^+ \to H_5^{++}W^- / H_5^+ Z / H_5^0 W^+$

$H_3^0 \to H_5^{\pm}W^{\mp} / H_5^0 Z$



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$H_5^{++} \to \ell^+\ell^+ \ , \quad H_5^+ \to \ell^+\nu \ , \quad H_5^0 \to \nu\nu \ ,$
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$H_5^{++} \to W^+W^+ \ , \quad H_5^+ \to W^+ Z \ , \quad H_5^0 \to W^+W^- / ZZ \ ,$
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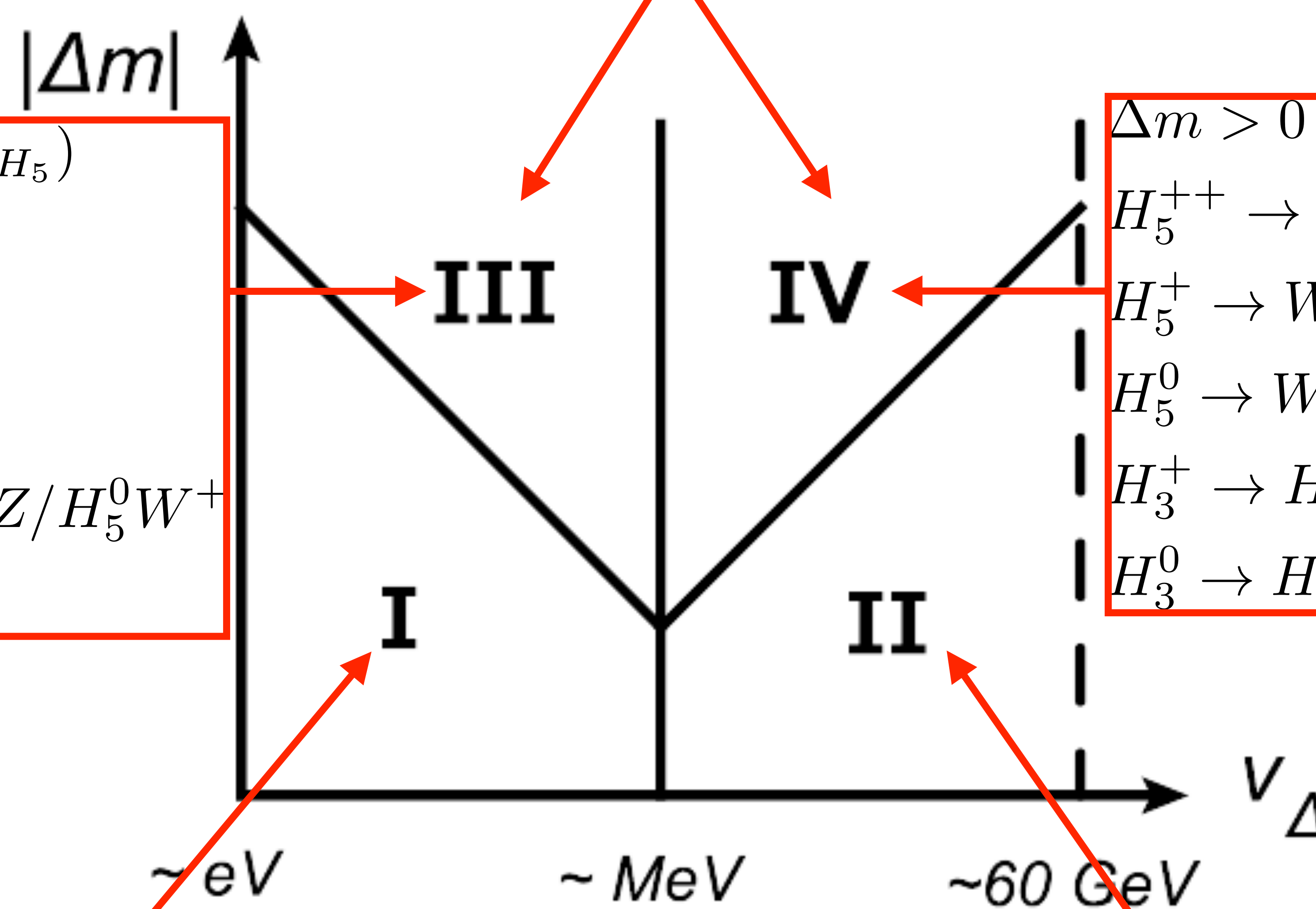
Decay Pattern

CWC, Yagyu JHEP 2012

$\Delta m < 0 \quad (m_{H_5} > m_{H_3})$

$H_5^{++} \to H_3^+ W^+ \ , \ H_5^+ \to H_3^+ Z / H_3^0 W^+ \ , \ H_5^0 \to H_3^\pm W^\mp / H_3^0 Z$

$H_3^+ \to H_1^0 W^+ \ , \ H_3^0 \to H_1^0 Z$



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Custodial Higgs Models

- Couplings of h modified by exotic Higgs fields due to their EW charges, and mixing with Φ_{SM}.
- At tree level, their hVV couplings satisfy

$$\kappa_W = \kappa_Z$$

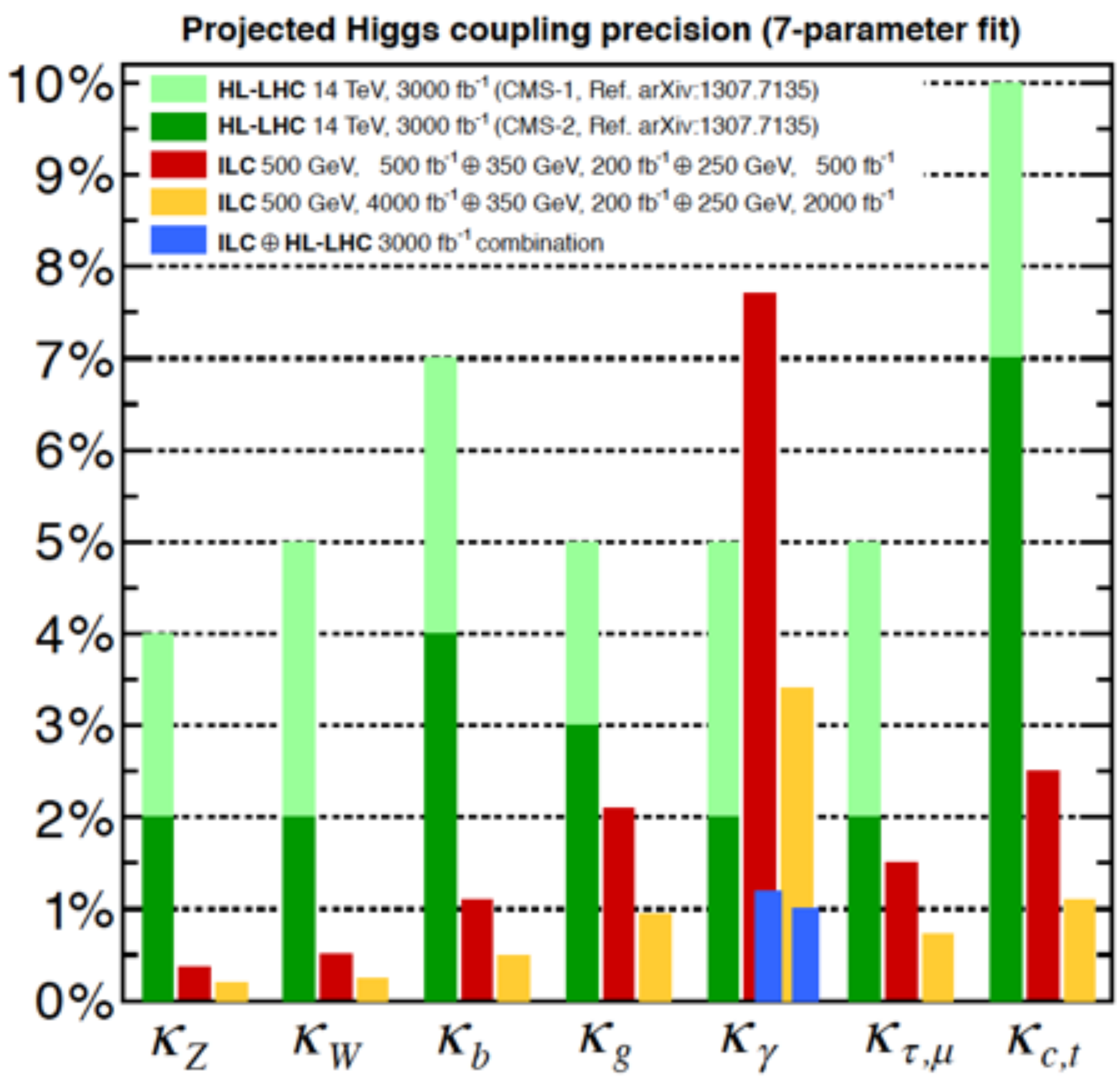
	rHSM	2HDM	GM model
$\Phi^{\rm new}$	S	$H, A, H^\pm$	$H_1, (H_3^0, H_3^\pm), (H_5^0, H_5^\pm, H_5^{\pm\pm})$
parameters	$m_H, \alpha, m_S,$	$m_H, m_A, m_{H^\pm},$	$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha,$
$(m_h, v) = (125, 246)$ GeV	$\mu_S, \lambda_{\Phi S}, \lambda_S$	$\mu, \alpha, \tan\beta = v_2/v_1$	$\tan\beta = v_\Phi/(2\sqrt{2}v_\Delta)$
$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\rm SM}$	$\cos\alpha$	$\sin(\beta-\alpha)$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$

always ≤ 1 through mixing

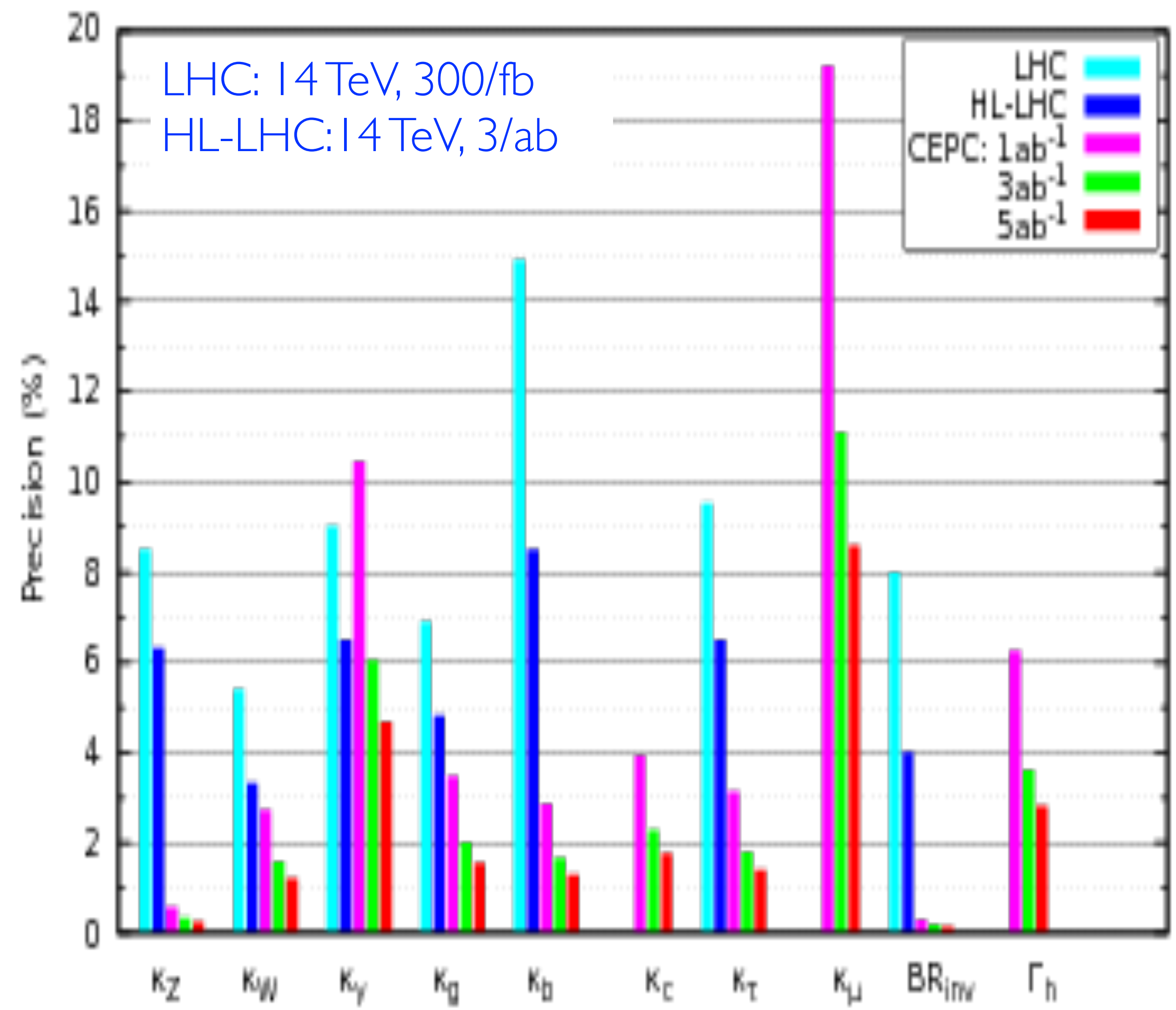
group factor that makes it possible for the entire factor to be > 1 (mixing required)

Expected Coupling Precision

- All Higgs couplings will be determined by HL-LHC, ILC or CEPC to O(1) or sub percent level.
  ➠ need to know radiative corrections



Fujii et al 2015



Ge, He, Xiao 2016

Radiative Corrections

- Radiative corrections can lead to at least two effects:
  - changes in the magnitudes of various couplings
  - deviations from tree-level relations among couplings

due to various custodial symmetry breaking parameters (couplings, masses).

	rHSM	2HDM	GM model
$\Phi^{\rm new}$	S	$H, A, H^{\pm}$	$H_1, (H_3^0, H_3^{\pm}), (H_5^0, H_5^{\pm}, H_5^{\pm\pm})$
parameters	$m_H, \alpha, m_S,$	$m_H, m_A, m_{H^{\pm}},$	$m_{H_1}, m_{H_3}, m_{H_5}, \mu_1, \mu_2, \alpha,$
$(m_h, v) = (125, 246)$ GeV	$\mu_S, \lambda_{\Phi S}, \lambda_S$	$\mu, \alpha, \tan\beta = v_2/v_1$	$\tan\beta = v_\Phi/(2\sqrt{2}v_\Delta)$
$\kappa_{W,Z} = g_{hVV}/g_{hVV}^{\rm SM}$	$\cos\alpha$	$\sin(\beta-\alpha)$	$\sin\beta\cos\alpha - \sqrt{\frac{8}{3}}\cos\beta\sin\alpha$
$\delta\kappa_V$	$-\sin\alpha\delta\alpha$	$\cos(\beta-\alpha)(\delta\beta-\delta\alpha)$	$\frac{\partial\kappa_V}{\partial\alpha}\delta\alpha + \frac{\partial\kappa_V}{\partial\beta}\delta\beta + \frac{\partial\kappa_V}{\partial\rho}\delta\rho$

Renormalization of GM Model

- Independent counter terms in the model:

gauge sector

$$m_W^2 \to m_W^2 + \delta m_W^2 \ ,$$

$$m_Z^2 \to m_Z^2 + \delta m_Z^2 \ ,$$

$$\alpha_{\rm em} \to \alpha_{\rm em} + \delta\alpha_{\rm em} \ ,$$

$$B_\mu \to \left(1 + \frac{1}{2}\delta Z_B\right) B_\mu \ ,$$

$$W_\mu^a \to \left(1 + \frac{1}{2}\delta Z_W\right) W_\mu^a \ .$$

scalar sector

$$m_X^2 \to m_X^2 + \delta m_X^2 \ ,$$

$$(X = H_5, H_3, H_1, h)$$

$$\mu_i \to \mu_i + \delta\mu_i \ , (i = 1, 2)$$

$$v_\Delta \to v_\Delta + \delta v_\Delta \ ,$$

$$\nu \to 0 + \delta\nu \ , \ (\nu = v_\xi - v_\chi)$$

$$\alpha \to \alpha + \delta\alpha \ .$$

- Most calculations are standard though tedious, but there are a few subtleties…

Renormalization Conditions

- In addition to the renormalization conditions in the SM to get physical G_F, m_Z, and α_{EM}, the GM model allows one additional condition, which we take to make

$$\alpha_{\rm em} T = \frac{\Pi^{\rm 1PI}_{ZZ}(0) - \Pi^{\rm 1PI}_{ZZ}(0)\big|_{\rm SM}}{m_Z^2} - \frac{\Pi^{\rm 1PI}_{WW}(0) - \Pi^{\rm 1PI}_{WW}(0)\big|_{\rm SM}}{m_W^2}$$

$$+\delta\rho$$

$$\delta\rho = \frac{8 v_\Delta \delta\nu}{v^2}$$

equal to 0 or its experimental value.

- Use on-shell conditions to fix other counter terms.

Gauge Dependence

- Observe gauge dependence in mixed 2-point (H_3-G) functions.
  ➠ fixed by use of pinch technique in physical f f → f f processes due to pinch terms extracted from vertex corrections and box diagrams
- Unlike 2HDM, one needs to sum up H_3-G, H_3-H_3, and G-G diagrams in the GM model to observe the gauge dependence cancellation.
  ➠ due to gauge dependence in 2-point diagrams involving H_5 bosons and pinch terms do not help, as H_5's are fermiophobic

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  Cornwall 1982, 1989
  Papavassiliou 1990
  Degrassi, Sirlin 1992
  Papavassiliou, Pilaftsis 1998

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  ➠ due to gauge dependence in 2-point diagrams involving H_5 bosons and pinch terms do not help, as H_5's are fermiophobic

hVV Couplings

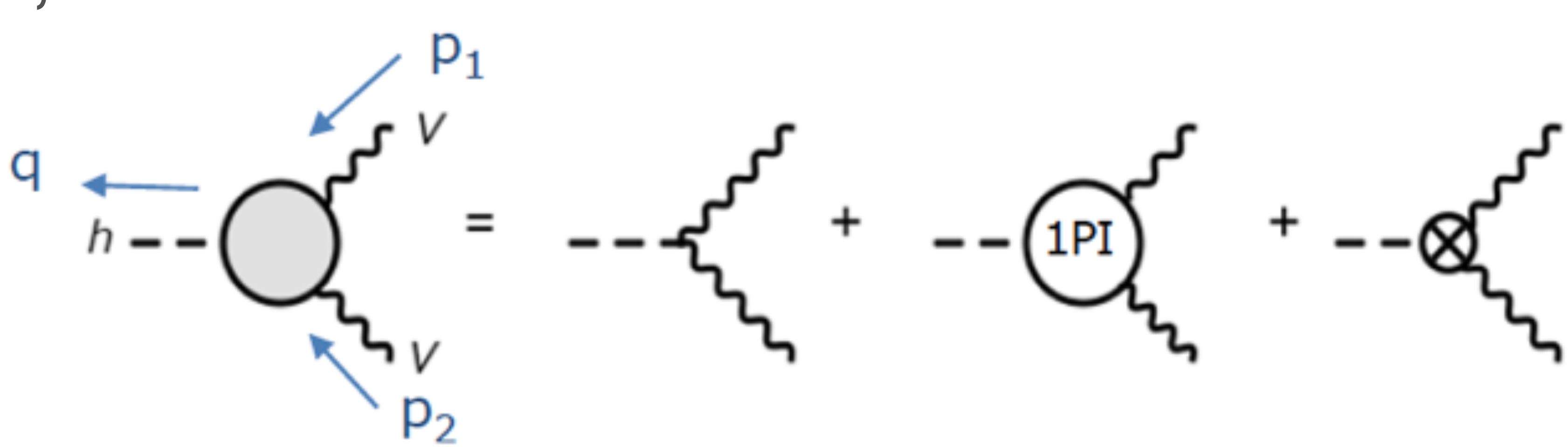
- In general, the renormalized $hV_\mu V_\nu$ vertices can be decomposed as

$$\hat{\Gamma}^{\mu\nu} = \hat{\Gamma}_1 g^{\mu\nu} + \underbrace{\hat{\Gamma}_2 p_1^\mu p_2^\nu + \hat{\Gamma}_3 \epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}}_{\text{loop induced terms}}$$

where

$$\hat{\Gamma}_1 = \frac{2m_V^2}{v}\kappa_V + \Gamma^{1\mathrm{PI}} + \delta\Gamma$$

has contributions from the tree-level coupling, 1PI diagrams, and counter terms.



κ_Z AND κ_W

- hVV scaling factors at 1-loop (symbols with a hat) with momentum dependence are defined as:

$$\hat{\kappa}_V(p^2) \equiv \frac{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{\rm NP}}{\hat{\Gamma}_1(m_V^2, p^2, m_h^2)_{\rm SM}}$$

- Radiative corrections in SM:

$$\frac{g_{hVV}^{1-\rm loop}}{g_{hVV}^{\rm tree}} \simeq \begin{cases} -1.2 \;\; (+1.0)\ \% \;\; (hZZ)\ , \\ +0.4 \;\; (+1.3)\ \% \;\; (hWW)\ , \end{cases}$$

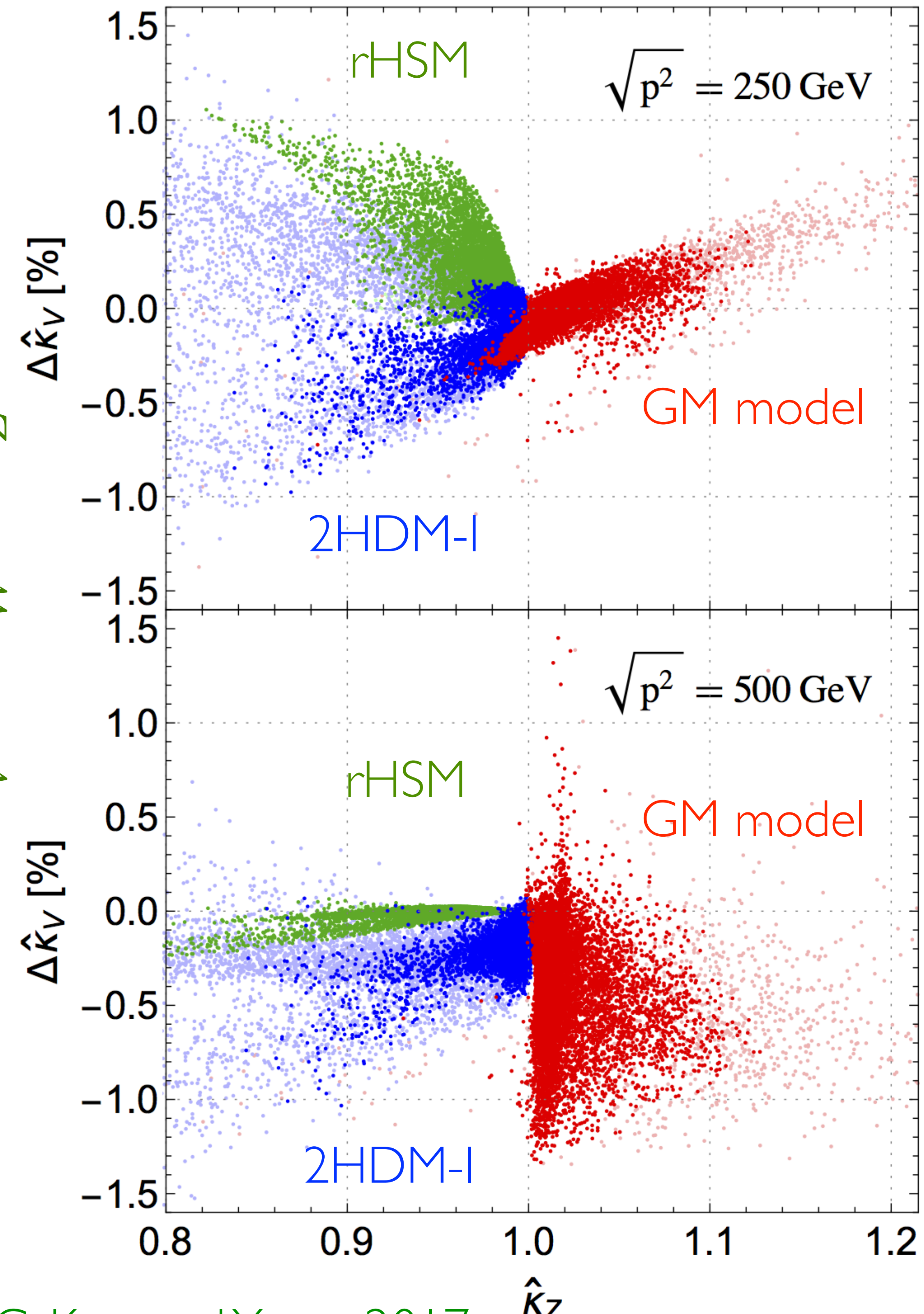
$$\text{for } \sqrt{p^2} = 250\ (500)\ \text{GeV}$$

1-Loop Results

- Same parameter sets in the two plots, only different in E_{cm}.

- Lighter dots satisfy theoretical constraints (unitarity, stability, perturbativity, and oblique parameters [S and T]).
- **Darker** dots further satisfy Higgs data from LHC Run-I.

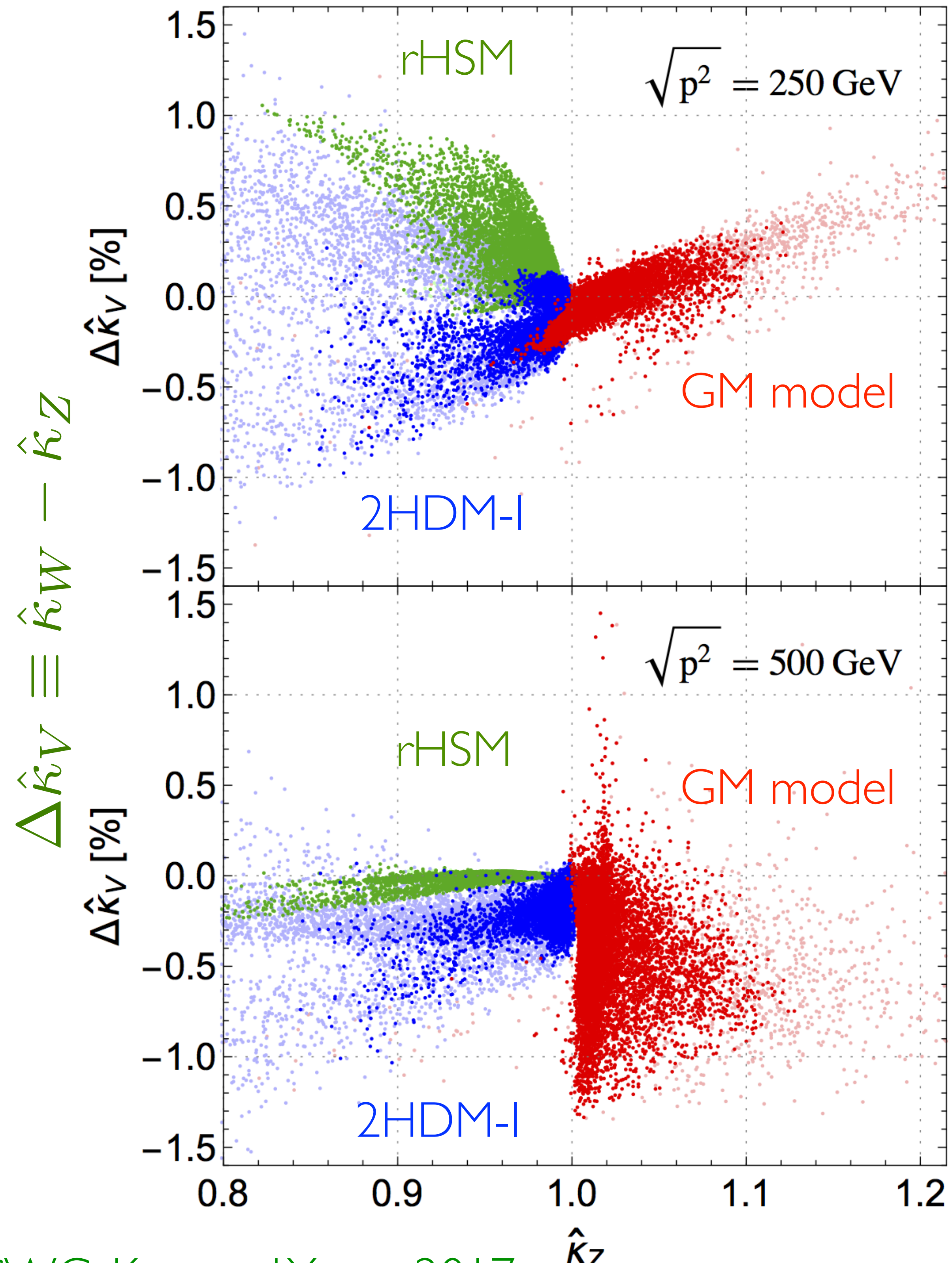
$\Delta\hat{\kappa}_V \equiv \hat{\kappa}_W - \hat{\kappa}_Z$



CWC, Kuo, and Yagyu 2017

1-Loop Results

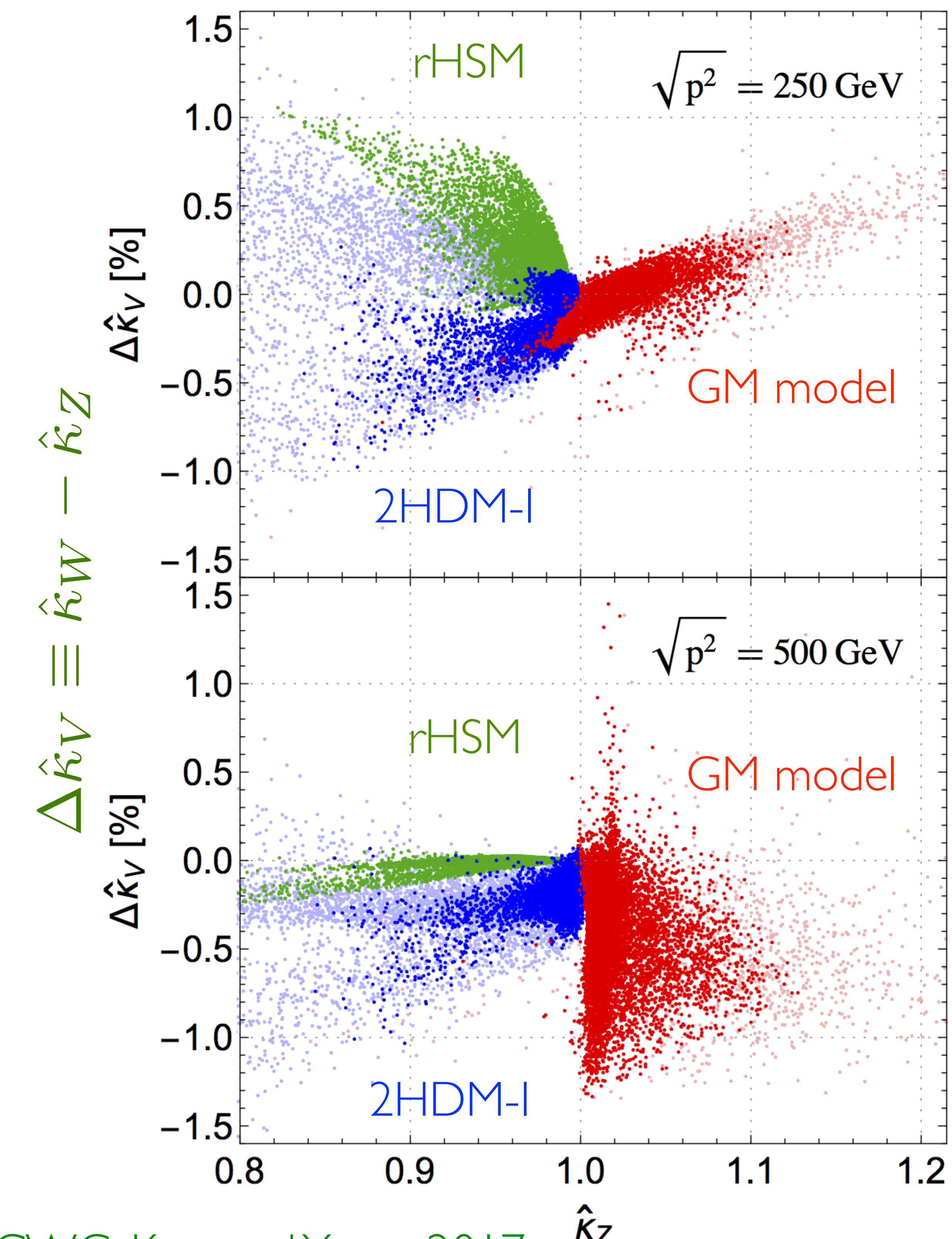
- Other types of 2HDM are expected to have a similar result as 2HDM-I.
- Green dots change little after imposing the Higgs data, while 2HDM and GM dots shrink significantly.
- GM prefers $\kappa_Z \in [0.88, 1.12]$, while the others have $\kappa_Z \in [0.8, 1.0]$.
- Variation in $\Delta\kappa_V$ is small but could be measurable.



CWC, Kuo, and Yagyu 2017

1-Loop Results

- It is possible to discriminate among the rHSM, 2HDMs and GM model once the precision reaches the sub-% level.
- 250-GeV ILC is better than 500-GeV in distinguishing rHSM and 2HDM-I.



CWC, Kuo, and Yagyu 2017

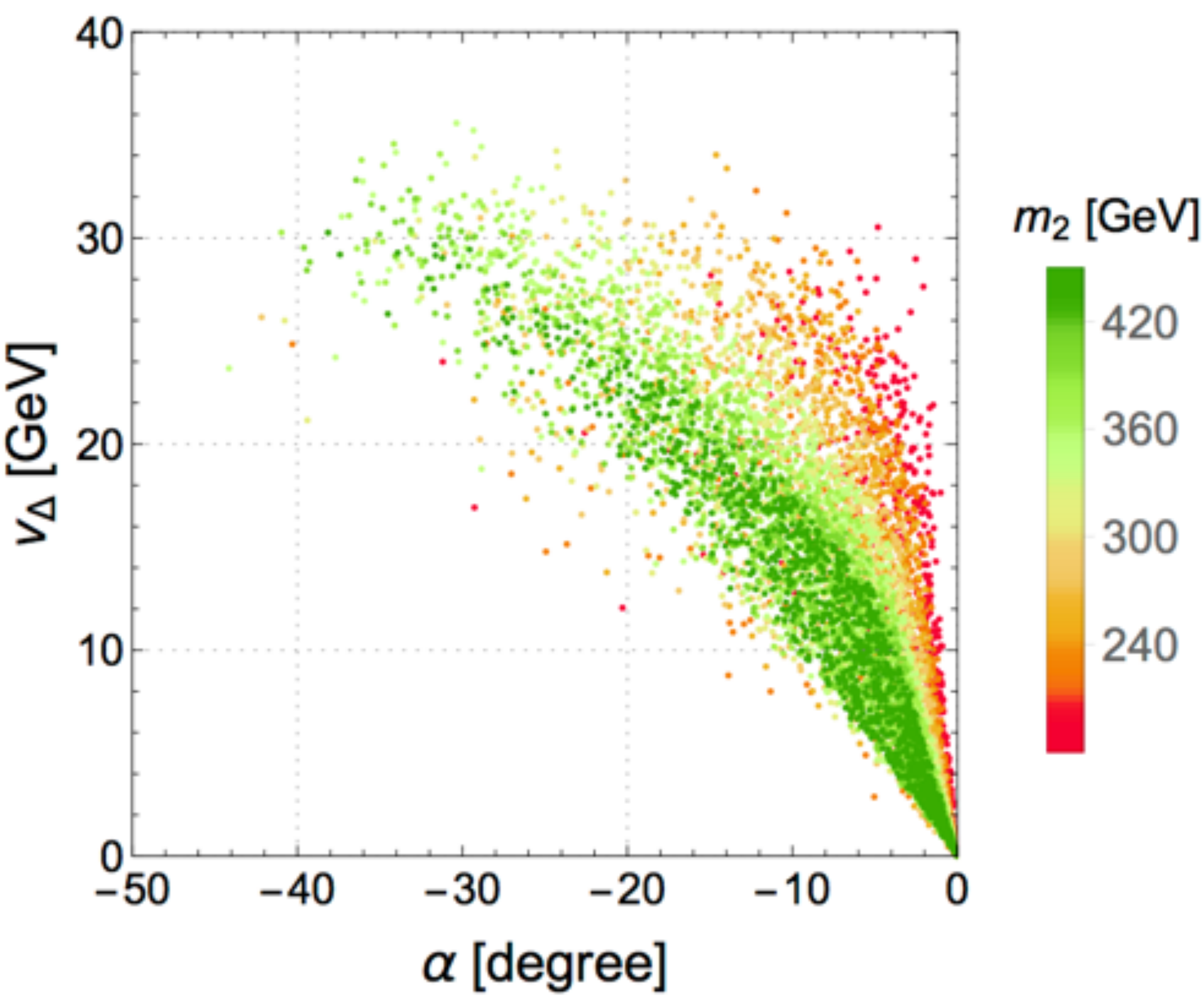
Allowed Space in GM Model

Preliminary

- Theoretical constraints (unitarity, stability, perturbativity, and oblique parameters).
- Higgs signal strengths from LHC Run-I.

scanned parameter ranges

$$-650 \leq \mu_1 \leq 0 \text{ GeV}$$
$$-400 \leq \mu_2 \leq 50 \text{ GeV}$$
$$180 \leq m_2 \leq 450 \text{ GeV}$$
$$-0.628 \leq \lambda_2 \leq 1.57$$
$$-1.57 \leq \lambda_3 \leq 1.88$$
$$-2.09 \leq \lambda_4 \leq 2.09$$
$$-8.38 \leq \lambda_5 \leq 8.38$$

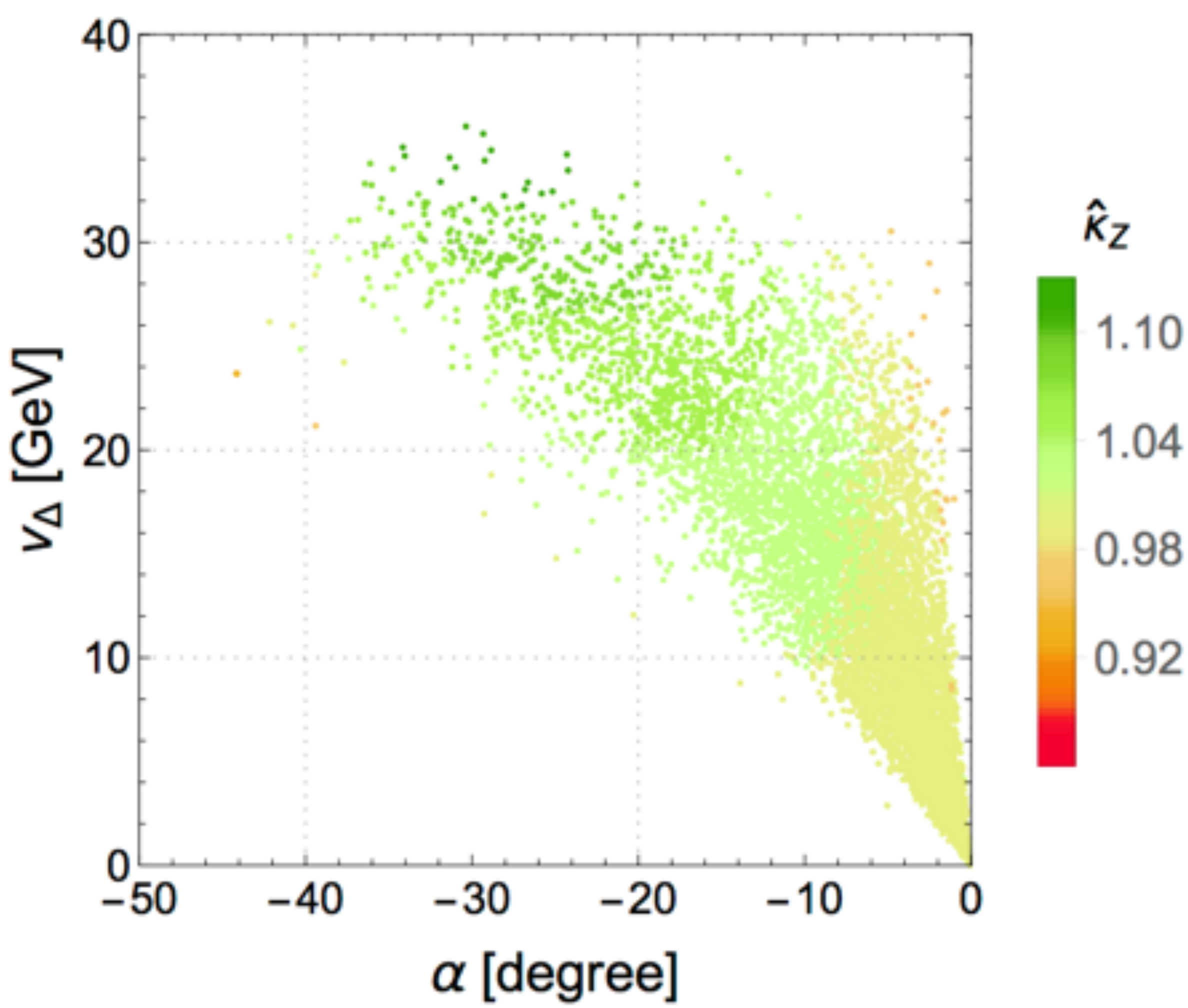


Scaling Factors in GM Model

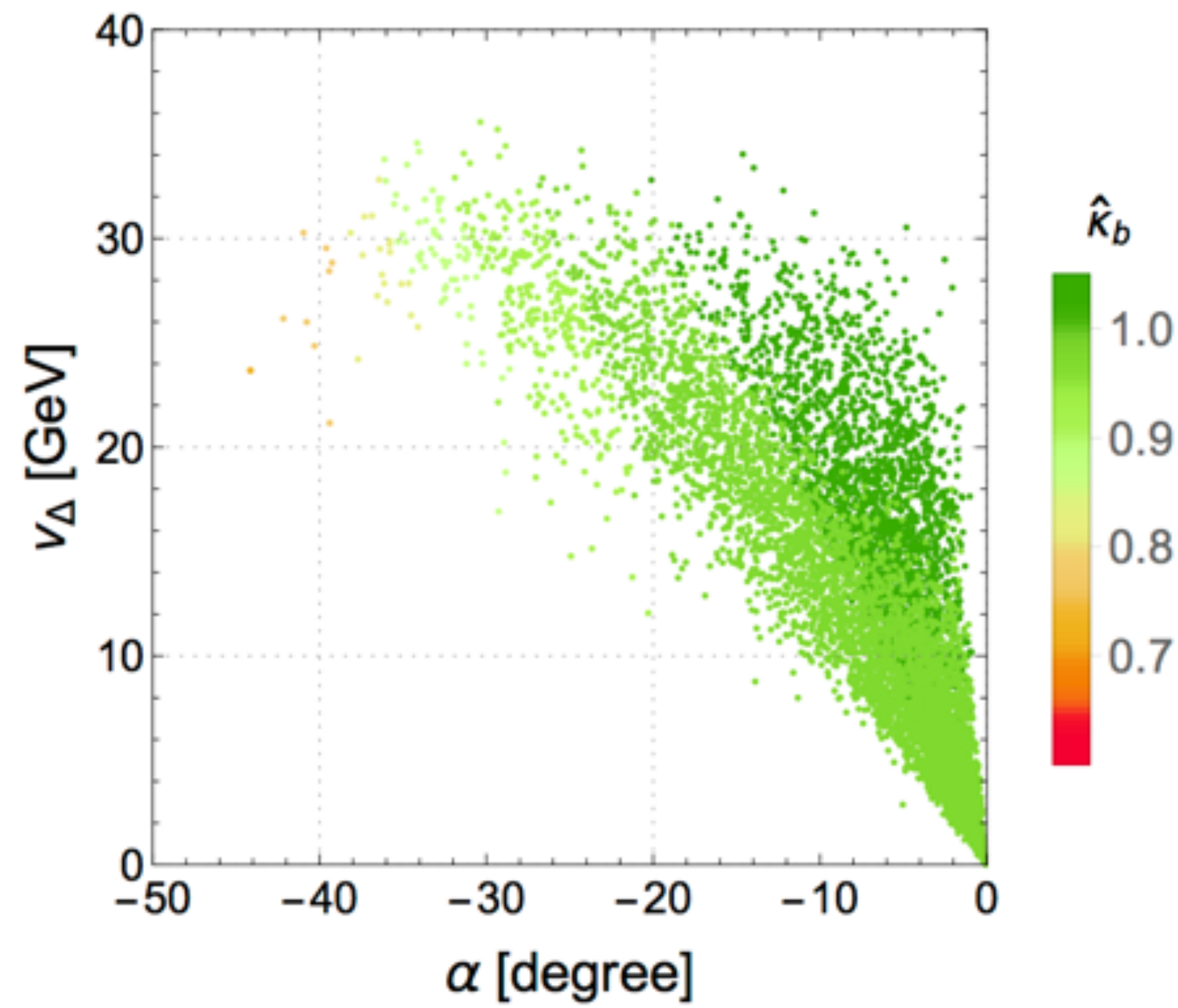
Preliminary

$$\hat{\kappa}_V(p_2^2) \equiv \frac{\hat{\Gamma}^1_{hVV}(m_V^2, p_2^2, m_h^2)_{\rm GM}}{\hat{\Gamma}^1_{hVV}(m_V^2, p_2^2, m_h^2)_{\rm SM}}$$

$$\hat{\kappa}_{b,\tau} \equiv \frac{\hat{\Gamma}^1_{hb\bar{b},h\tau\tau}(m_{b,\tau}^2, m_{b,\tau}^2, m_h^2)_{\rm GM}}{\hat{\Gamma}^1_{hb\bar{b},h\tau\tau}(m_{b,\tau}^2, m_{b,\tau}^2, m_h^2)_{\rm SM}}$$



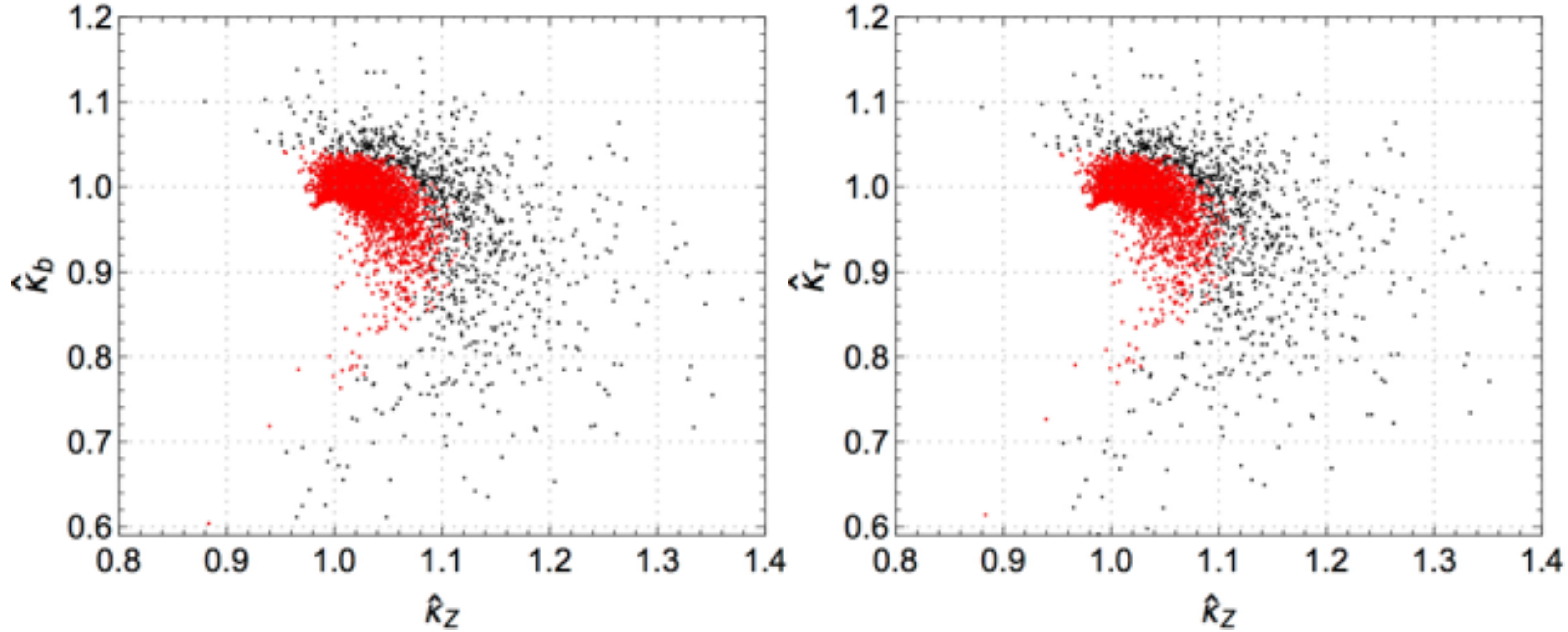
$0.88 \lesssim \hat{\kappa}_Z \lesssim 1.12$



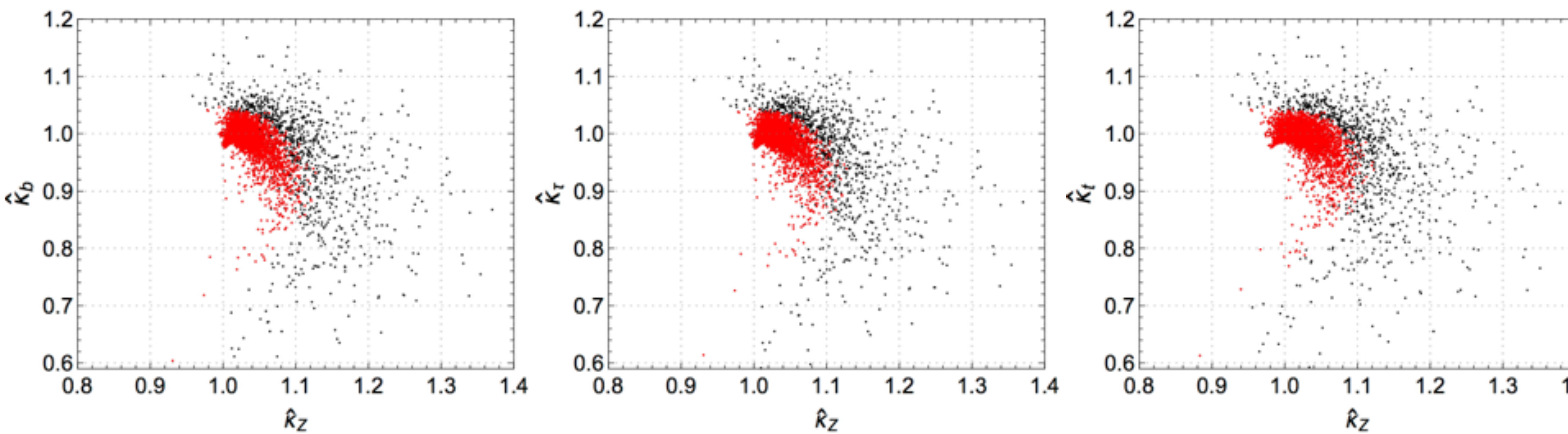
$0.60 \lesssim \hat{\kappa}_b \lesssim 1.05$

κ_f vs κ_Z

$$\sqrt{p_2^2} = 250\ \mathrm{GeV}$$



$$\sqrt{p_2^2} = 500\ \mathrm{GeV}$$

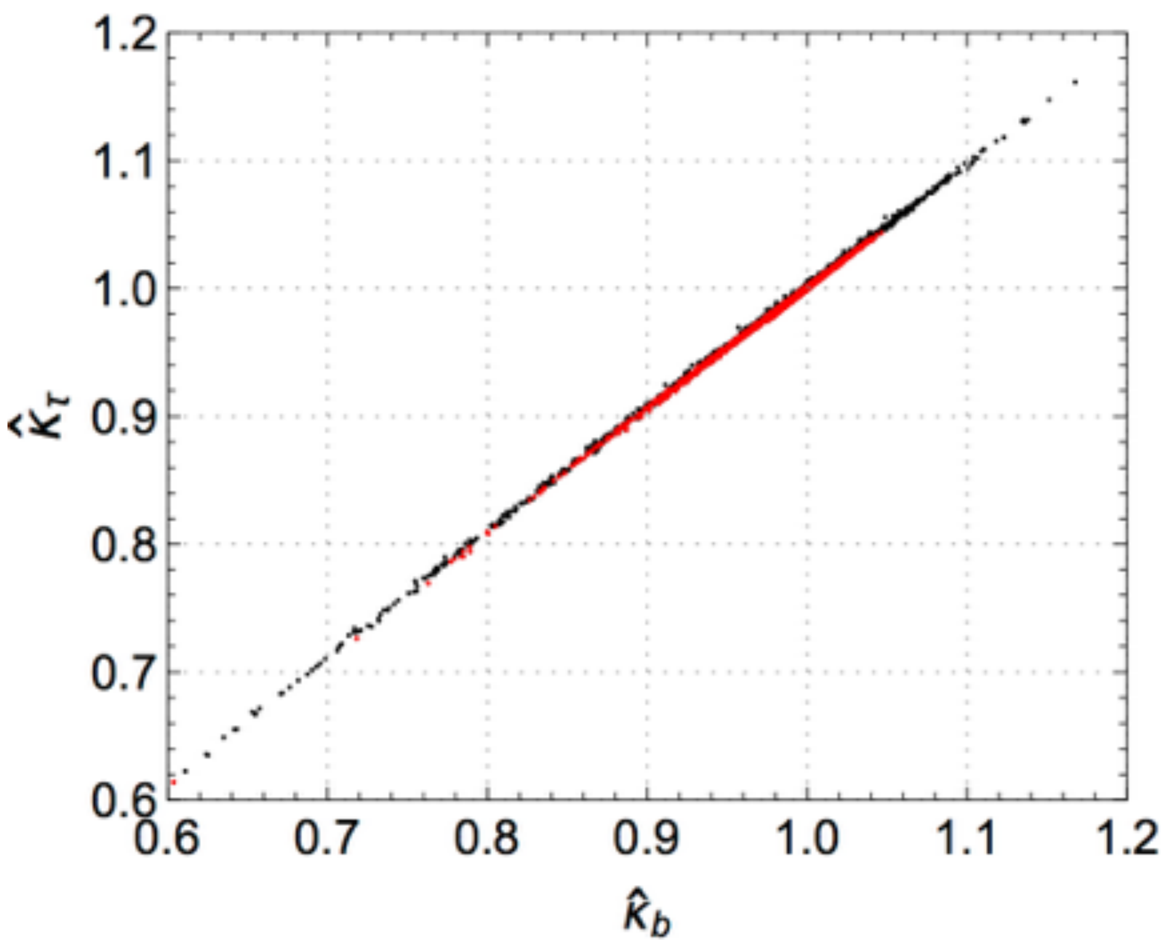


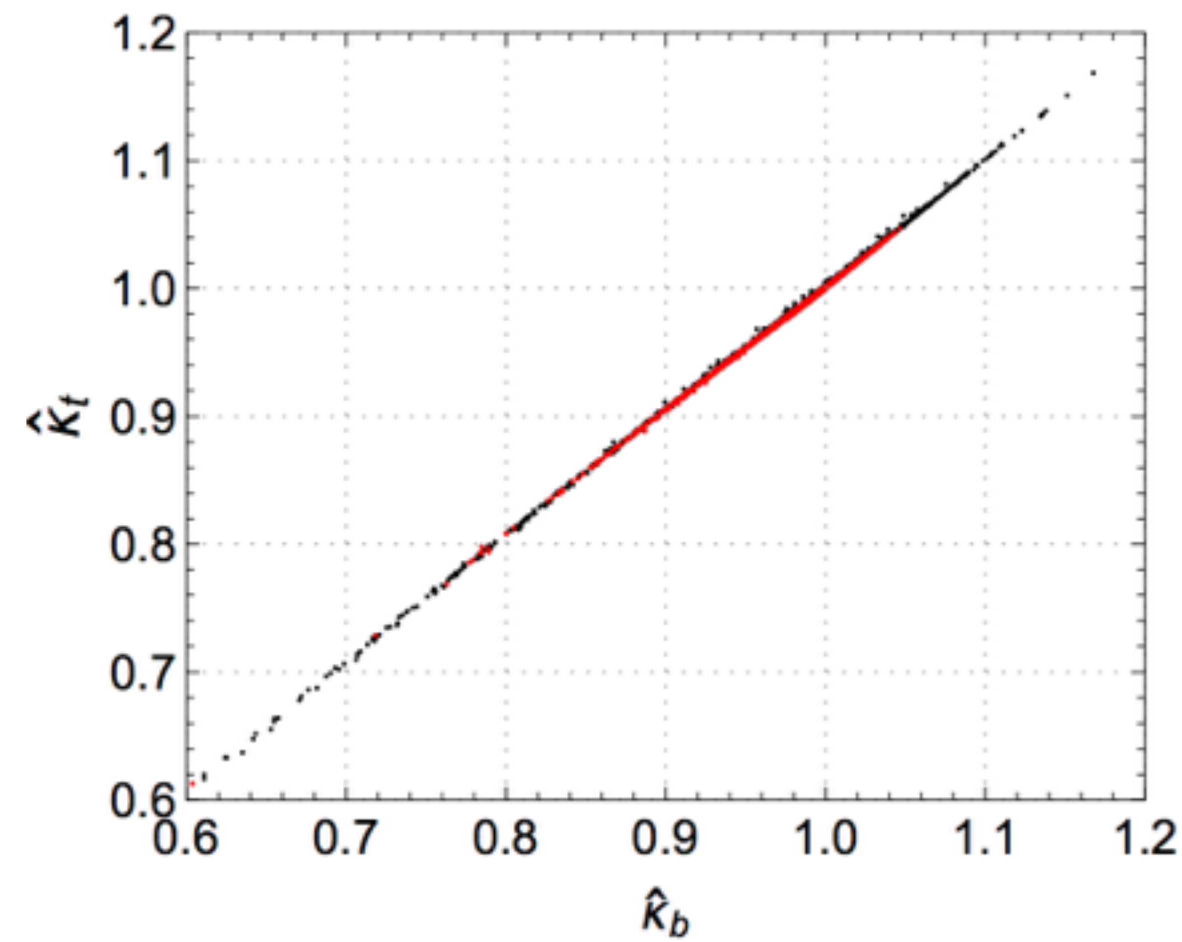
Preliminary

- very little changes in $\kappa_{t,b,\tau}$ and κ_Z in each row
- by definition, no p_2^2 dependence in $\kappa_{t,b,\tau}$

K_f Correlations

Preliminary





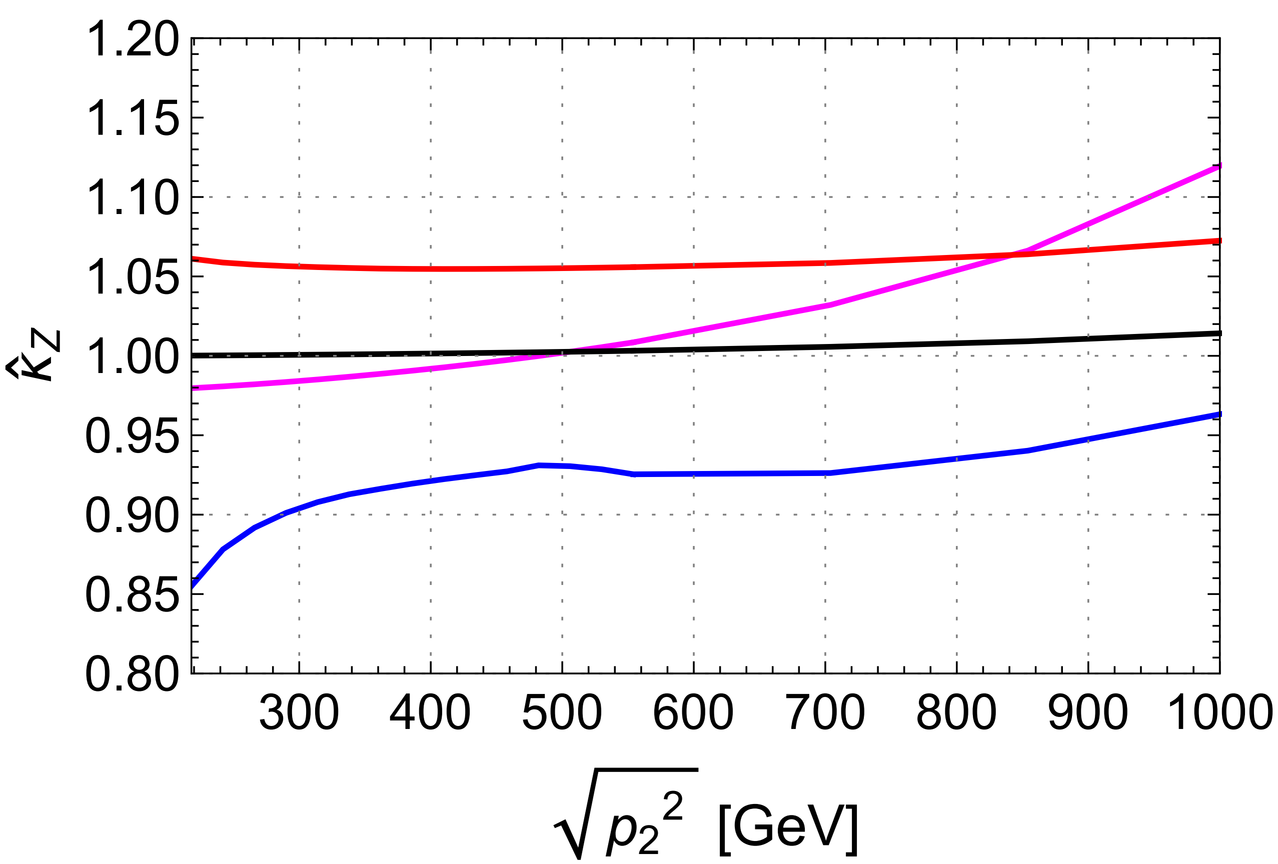
strong correlations among the hff scaling factors
with a range of ~ [0.6,1.2]

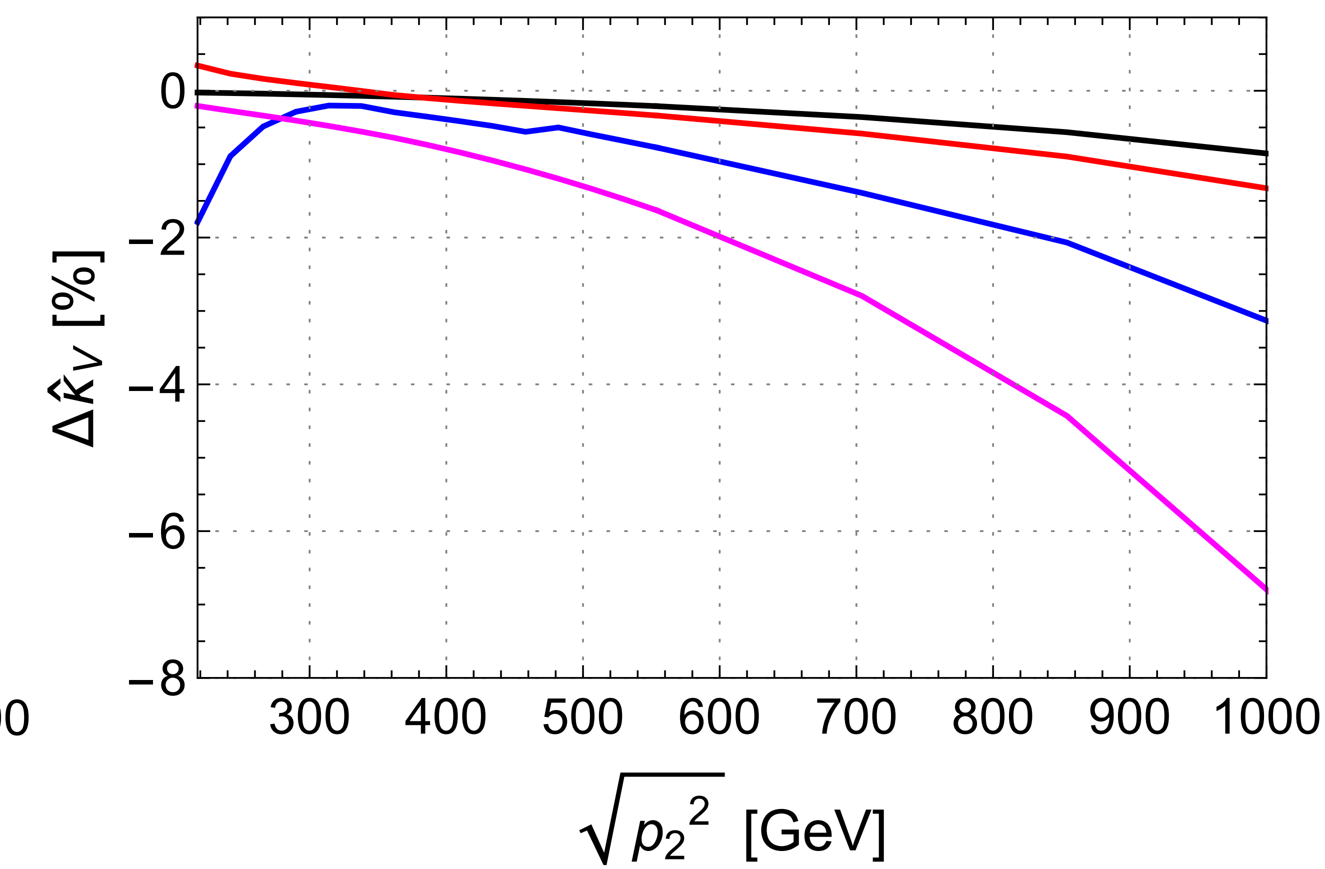
Momentum Dependence

Preliminary

- Examples of a few benchmark points:

v_Δ (GeV)	α	m_{H1} (GeV)	m_{H3} (GeV)	m_{H5} (GeV)	μ_1 (GeV)	μ_2 (GeV)
0.34	-0.27°	556	562	573	-7	-298
2.08	-0.56°	496	519	562	-34	-32
24.57	-17.21°	609	601	572	-614	-60
24.99	-51.68°	389	404	574	-179	-276





effects dominated by bosonic loops

Momentum Dependence

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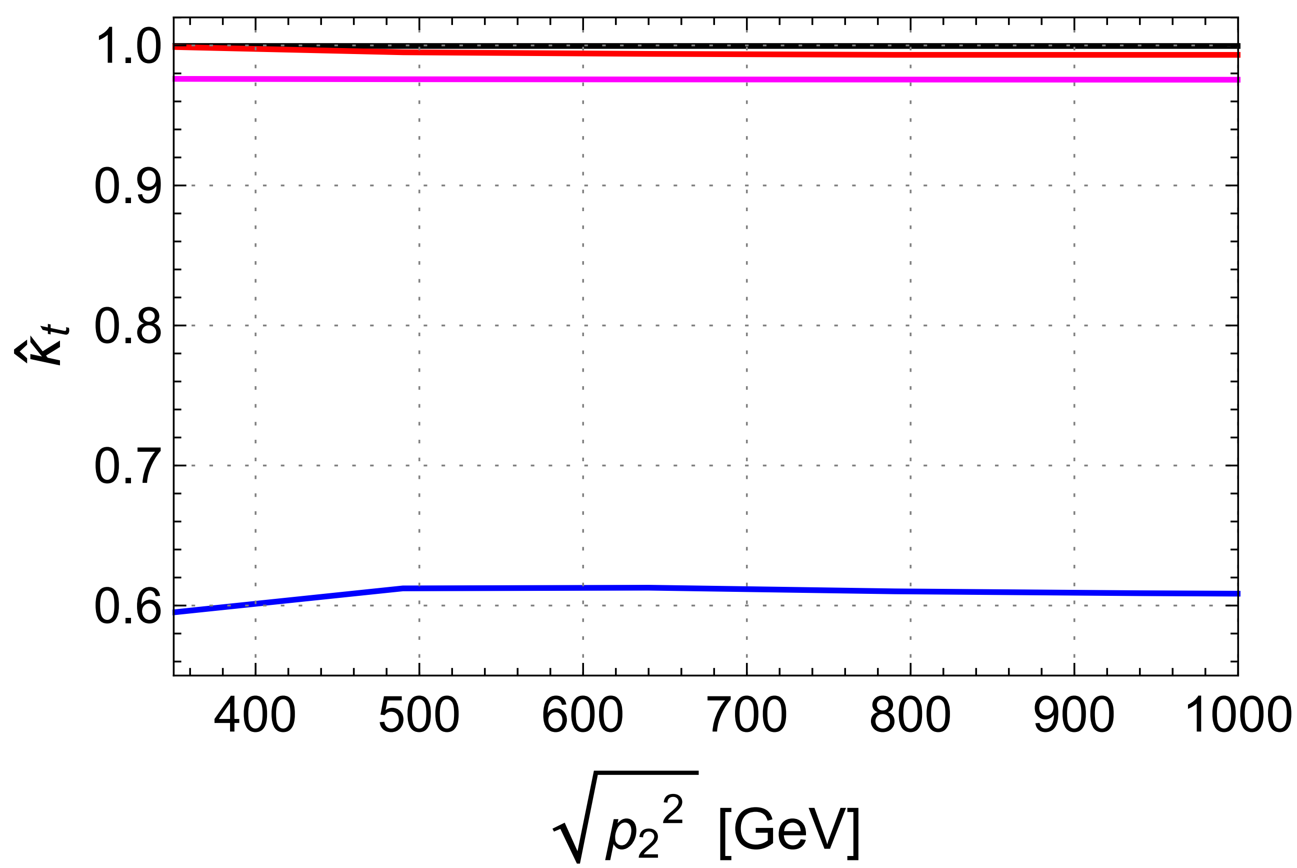


Summary

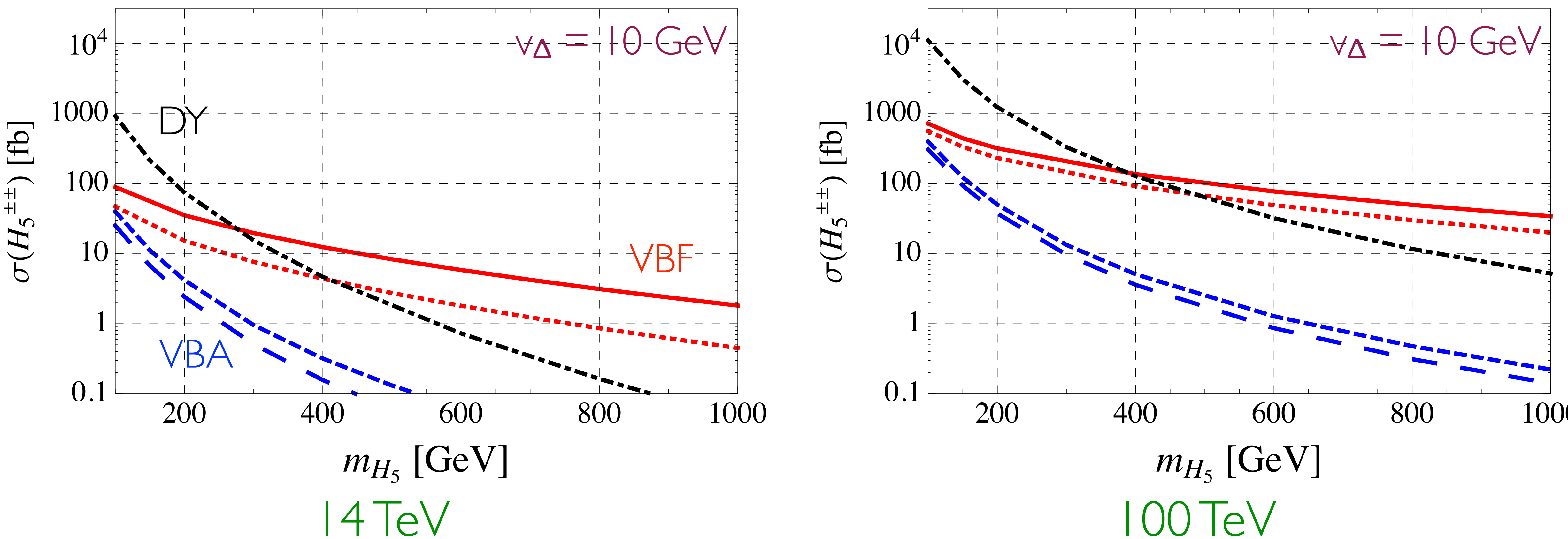
- We have been doing 1-loop radiative corrections to the Higgs couplings in the GM model.
- Theoretical (unitarity, stability, perturbativity, and oblique parameters) and experimental (Higgs signal strengths) constraints have been imposed to find viable parameter spaces.
- We have presented numerical results for the hVV couplings in the rHSM, 2HDM-I and GM models (all with custodial symmetry) at 250- and 500-GeV ILC, showing power to discriminate among the models.
- We have presented preliminary results of hff couplings in the GM model, showing their correlations among themselves and with hVV and the momentum dependence of these couplings.
- Radiative corrections to the hhh coupling are in progress.
- More phenomenological analyses will follow.

Backup Slides

Production for Large v_Δ

- For large v_Δ, $H^{\pm\pm}$ couples primarily to weak bosons.
- VBF as dominant production processes for sufficiently large v_Δ and sufficiently large $M_{H^{\pm\pm}}$.

CWC, Kuo, and Yamada JHEP 2016



- Upper curves for ++ and lower curves for −−.

an experimentally less explored scenario, and unique among simple Higgs models

Unitarity/Stability Bounds

- (Tree-level) perturbative unitarity bound Aoki, Kanemura 2008

$$|\,6\lambda_1 + 7\lambda_3 + 11\lambda_2\,| + \sqrt{(6\lambda_1 - 7\lambda_3 - 11\lambda_2)^2 + 36\lambda_4^2} < 4\pi \ ,$$

$$|\,\lambda_4 - \lambda_5\,| < 2\pi \ , \qquad |\,2\lambda_3 + \lambda_2\,| < \pi \ ,$$

$$|\,2\lambda_1 - \lambda_3 + 2\lambda_2\,| + \sqrt{(2\lambda_1 + \lambda_3 - 2\lambda_2)^2 + \lambda_5^2} < 4\pi \ .$$

- (Tree-level) vacuum stability bound Arhrib et al 2011

$$\lambda_1 > 0 \ , \quad \lambda_2 + \lambda_3 > 0 \ , \quad \lambda_2 + \frac{1}{2}\lambda_3 > 0 \ ,$$

$$-|\lambda_4| + 2\sqrt{\lambda_1(\lambda_2 + \lambda_3)} > 0 \ , \quad \lambda_4 - \frac{1}{4}|\lambda_5| + \sqrt{2\lambda_1(2\lambda_2 + \lambda_3)} > 0 \ .$$

- All λ's can be written in terms of physical parameters.