Hadronic input and observables in semileptonic and FCNC decays of B-mesons

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CKM matrix in Standard Model

- quark-flavour weak transitions in SM

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \sum_{U=u,c,t;D=d,s,b} \left(V_{UD} \bar{U}\gamma_\mu (1-\gamma_5) D W^\mu + V^*_{UD} \bar{D}(1-\gamma_5) U W^{\dagger\mu} \right)$$

- the unitary CKM matrix in Wolfenstein representation:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- a persistent tension between inclusive and exclusive $|V_{ub}|$ determinations;
- limited accuracy of V_{td} determination
- the task: to extend the set of processes where CKM parameters can be independently and accurately determined,

  an indirect way to reveal BSM physics

Problem to be addressed:

- CKM parameters from semileptonic and FCNC exclusive B-meson decays
  - quark-level transitions: $b \to u\ell^-\bar{\nu}_\ell$, $b \to s\ell^+\ell^-$, $b \to d\ell^+\ell^-$
  - described by H_{eff} absorbing short-distance physics of SM
  - apart from measured observables (widths, asymmetries) need hadronic matrix elements

- Hadronic input from continuum QCD-based methods: light-cone sum rules (LCSR), hadronic dispersion relations
  - $B \to \pi\ell\nu_\ell$, $B_s \to K\ell\nu_\ell$ and $|V_{ub}|$ determination
  - Nonlocal hadronic contributions to $B \to K\ell^+\ell^-$, $B \to \pi\ell^+\ell^-$ and alternative determination of Wolfenstein parameters A, η, ρ

Method of QCD Light-Cone Sum Rules

[I.I.Balitsky, V.M.Braun, A.V. Kolesnichenko (1986); V.L.Chernyak, I.Zhitnisky (1990)]

- many useful applications to heavy-flavour decays:
- $B \to P$ transition form factors:

$$\langle P(p)|\bar{q}\gamma^\mu b|B(p+q)\rangle = f^+_{BP}(q^2)\left[2p^\mu + \left(1 - \frac{m_B^2 - m_P^2}{q^2}\right)q^\mu\right] + \ldots$$

$$\langle P(p)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = \frac{i f^T_{BP}(q^2)}{m_B + m_P}\left[2q^2 p^\mu + \left(q^2 - \left(m_B^2 - m_P^2\right)\right)q^\mu\right],$$

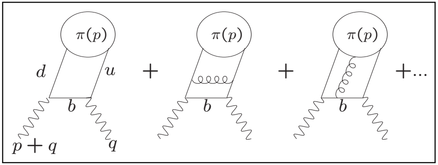
- correlation function of two quark currents,

between vacuum and P-meson state

$$\begin{aligned} F^\mu_{BP}(p,q) &= i\int d^4x e^{iqx}\langle P(p)|T\{\bar{q}_1(x)\Gamma^\mu b(x), (m_b + m_{q_2})\bar{b}(0)i\gamma_5 q_2(0)\}|0\rangle \\ &= \begin{cases} F_{BP}(q^2,(p+q)^2)p^\mu + \tilde{F}_{BP}(q^2,(p+q)^2)q^\mu, & \Gamma^\mu = \gamma^\mu, \\ F^T_{BP}(q^2,(p+q)^2)\left[q^2 p^\mu - (q\cdot p)q^\mu\right], & \Gamma^\mu = -i\sigma^{\mu\nu}q_\nu, \end{cases} \end{aligned}$$

- calculable at spacelike $(p+q)^2$, $q^2 \ll (m_B - m_P)^2$, finite m_b

LCSR for $B \to \pi$ form factors



$\Leftarrow$ the correlation function

calculated in terms of

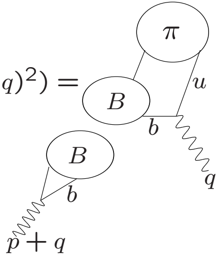
Operator Product Expansion

at $(p+q)^2, q^2 \ll m_b^2$

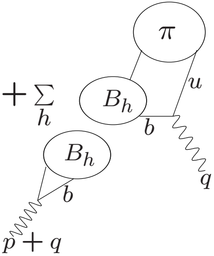
$\Downarrow$

hadronic dispersion relation }

$F(q^2,(p+q)^2) =$



$+ \sum_h$

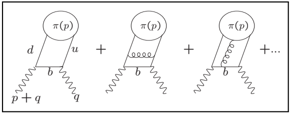


$f_B f^{+}_{B\pi}(q^2)$

$\sum_{B_h} \to duality\ (s_0^B)$

OPE calculation

- the correlation function $q^2 \ll m_b^2$



$$[F(q^2,(p+q)^2)]_{OPE} =$$

$$= \sum_{t=2,3,4,..} \int_0^1 \mathcal{D}u \; T^{(t)}(\alpha_s, m_b, m_q; q^2, (p+q)^2, u, \mu) \, \varphi_\pi^{(t)}(u,\mu)$$

$\Uparrow$ $\Uparrow$

{diagrams with b-propagator} $\otimes$ {pion Distribution Amplitudes}

- pion DA's, polynomial expansion:

$$\varphi_\pi^{(t)}(u,\mu) = f_\pi^{(t)}(\mu)\{C_0(u) + \sum_{n=1} a_n^{(t)}(\mu) C_n(u)\}$$

- accuracy of OPE
  - precision of the input: m_b, m_q, α_s, $f_\pi^{(t)}(\mu_0)$, $a_n^{(t)}(\mu_0)$
  - truncation level: $O(\alpha_s)$, $t \leq 4(6)$, $n \leq 4$
  - variable scales: μ, $(p+q)^2 \to M^2 \sim m_b \chi$, $m_b \gg \chi \gg \Lambda_{QCD}$

Hadronic dispersion relation

- based on analyticity $\oplus$ unitarity in QFT

$$[F(q^2,(p+q)^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2}$$

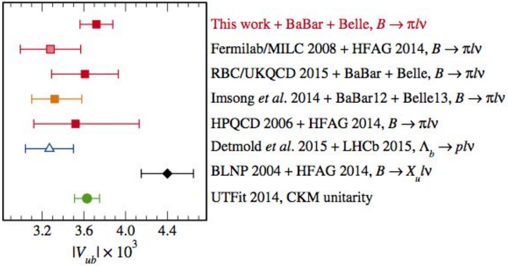
$$\Uparrow$$

- quark-hadron "semilocal" duality

$$\boxed{\int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2} = \int\limits_{s_0}^{\infty} ds\, \frac{[\mathrm{Im}F(q^2,s)]_{OPE}}{s-(p+q)^2}}$$

- accuracy:
  - f_B calculated from 2-point QCD SR
  - variable scale: $(p+q)^2 \to M^2 \sim m_b\chi \to$ optimal interval of M^2
  - duality approximation, s_0 (determined by calculating m_B^2)

$|V_{ub}|$ determination from $B \to \pi \ell \nu_\ell$



from [J. A. Bailey *et al.* [Fermilab Lattice and MILC Collaborations], arXiv:1503.07839 [hep-lat].]

Update of LCSRs for $B \to K$, $B_s \to K$ form factors

[AK, A.Rusov, JHEP 08 (2017) 112]

- $|V_{ub}|$ from $B_s \to K\ell\nu_\ell$ decay measurable by LHCb

$$\Delta\zeta_{B_sK}\,[0,q_0^2] \equiv \frac{G_F^2}{24\pi^3}\int\limits_0^{q_0^2} dq^2\, p_{B_sK}^3 |f^+_{B_sK}(q^2)|^2$$

$$= \frac{1}{|V_{ub}|^2\tau_{B_s}}\int\limits_0^{q_0^2} dq^2\, \frac{dB(\bar{B}_s \to K^+\ell\bar{\nu}_\ell)}{dq^2}\,,$$

- the numerical result

$$\Delta\zeta_{B_sK}\,(0, 12\,\mathrm{GeV}^2) = 7.03^{+0.67}_{-0.63}\ \mathrm{ps}^{-1}$$

- for comparison, from $B \to \pi$ form factor

$$\Delta\zeta_{B\pi}\,(0, 12\,\mathrm{GeV}^2) = 5.30^{+0.67}_{-0.61}\ \mathrm{ps}^{-1}$$

Comparison with lattice QCD

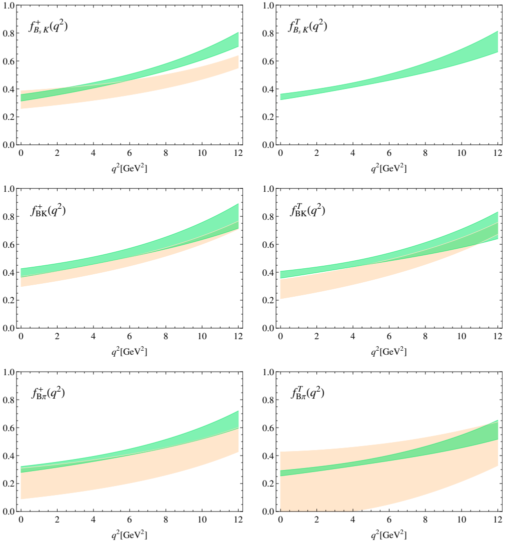
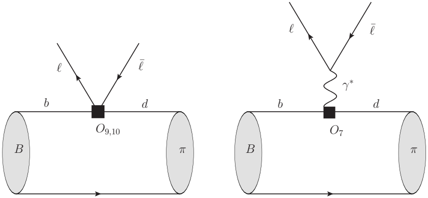


Figure 1. The vector (tensor) form factors of $B_s \to K$, $B \to K$ and $B \to \pi$ transitions calculated from LCSRs including estimated parametrical uncertainties are shown on the upper, middle and lower left (right) panels, respectively, with the dark-shaded (green) bands. Extrapolations of the lattice QCD results for $B_s \to K$ [Fermilab-MILC (2014)], $B \to K$ [HPQCD] and $B \to \pi$ [Fermilab=MILC (2015)] form factors are shown with the light-shaded (orange) bands.

FCNC decays: example of $B \to \pi\ell\ell$

[C. Hambrock, A. K. , A. Rusov, PRD **92** (2015) [arXiv:1506.07760 [hep-ph]]],

- dominant contribuitons , only form factors needed



- "background" contributions e.g. the weak interaction operators $O_{1,2}$ combined with e.m. emission of a lepton pair,
- defined via nonlocal hadronic matrix elements

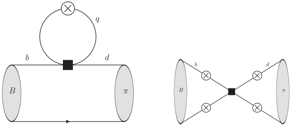
$$\mathcal{H}^{(p)}(q^2)\left[(p\cdot q)q_\mu - q^2 p_\mu\right] = i\int d^4x e^{iqx}\langle\pi(p)|\mathrm{T}\Big\{j_\mu^{\rm em}(x), \Big[C_1\mathcal{O}_1^p(0) + C_2\mathcal{O}_2^p(0) + \sum_{k=3-6,8g} C_k\mathcal{O}_k(0)\Big]\Big\}|B(p+q)\rangle, \quad (p=u,c),$$

How do we obtain $\mathcal{H}^{(u,c)}(q^2)$

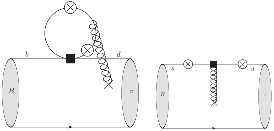
the method invented earlier for $B \to K\ell\ell$

[A. K., T. Mannel and Y. M. Wang, JHEP **1302** (2013) 010 [arXiv:1211.0234 [hep-ph]]]

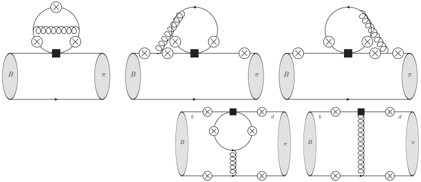
- calculate $\mathcal{H}^{(u,c)}(q^2 < 0)$ at $|q^2| \gg \Lambda^2_{QCD}$



  - LO diagrams: factorizable loop, weak annihilation



  - soft-gluon nonfactorizable contributions (LCSR with B-meson DA)
    [A.K., T. Mannel, A. A. Pivovarov and Y.-M. Wang, JHEP **1009** (2010) 089 [arXiv:1006.4945 [hep-ph]]].



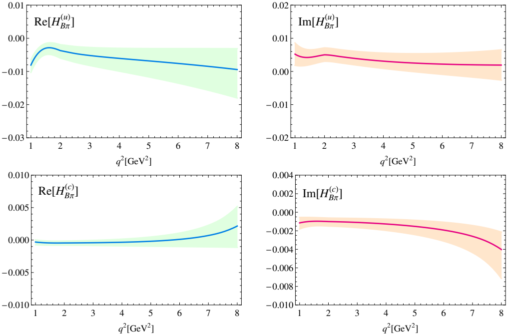
  - NLO (hard-gluon) contributions from QCD factorization (at $q^2 < 0$)
    M. Beneke, T. Feldmann, D. Seidel, (2001)

two-loop diagrams taken from

H. H. Asatryan, C. Greub and M. Walker,(2002)

How do we obtain $\mathcal{H}^{(u,c)}(q^2)$

- $\mathcal{H}^{(u,c)}(q^2 > 0)$ obtained matching the OPE result at $q^2 < 0$ to the hadronic dispersion relation, continuing to $q^2 > 0$
- including $V = \rho, \omega, \phi, J/\psi, \psi(2S)$ resonances;
- inputs: decay constants of V's, $B \to V\pi$ nonleptonic amplitudes
- the result valid at large recoil $q^2 \ll m_b^2$, up to charmonium region



Anatomy of $B \to P\ell\ell$ amplitude

- The decay amplitude can be represented in the following form:

$$A(\bar{B} \to P\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} \Bigg\{ \left[\lambda_t^{(q)} f_{BP}^+(q^2) c_{BP}(q^2) + \lambda_u^{(q)} d_{BP}(q^2)\right] \left(\bar{\ell}\gamma^\mu \ell\right) p_\mu$$

$$+\lambda_t^{(q)} C_{10} f_{BP}^+(q^2) \left(\bar{\ell}\gamma^\mu\gamma_5 \ell\right) p_\mu \Bigg\},$$

- Combination of CKM parameters:

$\lambda_p^{(q)} = V_{pb} V_{pq}^* \qquad (p = u, c, t;\, q = d, s)$, $m_\ell = 0$,

- unitarity of the CKM matrix, fixing $\lambda_c^{(q)} = -(\lambda_t^{(q)} + \lambda_u^{(q)})$.
- compact notation:

$$c_{BP}(q^2) = C_9 + \frac{2(m_b + m_q)}{m_B + m_P} C_7^{\rm eff} \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} + 16\pi^2 \frac{\mathcal{H}_{BP}^{(c)}(q^2)}{f_{BP}^+(q^2)},$$

$$d_{BP}(q^2) = 16\pi^2 \left(\mathcal{H}_{BP}^{(c)}(q^2) - \mathcal{H}_{BP}^{(u)}(q^2)\right).$$

$$\delta_{BP}(q^2) = {\rm Arg}(d_{BP}(q^2)) - {\rm Arg}(c_{BP}(q^2)).$$

Observables in $B \to P\ell^+\ell^-$

- Binned branching fraction:

$$\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) \equiv \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \frac{dB(\bar{B} \to P\ell^+\ell^-)}{dq^2} \,,$$

- in terms of the hadronic input:

$$\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(q)}|^2}{192\pi^5} \Big\{ \mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2 \mathcal{D}_{BP}[q_1^2, q_2^2]$$

$$+ 2\kappa_q \Big(\cos\xi_q \, \mathcal{C}_{BP}[q_1^2, q_2^2] - \sin\xi_q \, \mathcal{S}_{BP}[q_1^2, q_2^2] \Big) \Big\} \tau_B \,,$$

- the ratio of CKM matrix elements is parametrized in terms of its module and phase:

$$\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} = \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \equiv \kappa_q \, e^{i\xi_q}, \quad (q = d, s) \,, \qquad (2)$$

we use the following notation for the phase-space weighted and integrated parts of the decay amplitude squared:

$$\mathcal{F}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int\limits_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 |f_{BP}^+(q^2)|^2 \Big(\big|c_{BP}(q^2)\big|^2 + |C_{10}|^2 \Big) ,$$

$$\mathcal{D}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int\limits_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \big|d_{BP}(q^2)\big|^2 ,$$

$$\mathcal{C}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int\limits_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \big|f_{BP}^+(q^2) c_{BP}(q^2) d_{BP}(q^2)\big| \cos\delta_{BP}(q^2) ,$$

$$\mathcal{S}_{BP}[q_1^2, q_2^2] = \frac{1}{q_2^2 - q_1^2} \int\limits_{q_1^2}^{q_2^2} dq^2 \, p_{BP}^3 \big|f_{BP}^+(q^2) c_{BP}(q^2) d_{BP}(q^2)\big| \sin\delta_{BP}(q^2) .$$

Final form of the observables

- *CP*-averaged branching fraction:

$$\mathcal{B}_{BP}[q_1^2, q_2^2] \equiv \frac{1}{2}\Big(\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) + \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])\Big)$$

$$= \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(q)}|^2}{192\pi^5}\left\{\mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2\,\mathcal{D}_{BP}[q_1^2, q_2^2] + 2\kappa_q \cos\xi_q\,\mathcal{C}_{BP}[q_1^2, q_2^2]\right\}\tau_B,$$

- direct *CP*-asymmetry:

$$\mathcal{A}_{BP}[q_1^2, q_2^2] = \frac{\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) - \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])}{\mathcal{B}(\bar{B} \to P\ell^+\ell^-[q_1^2, q_2^2]) + \mathcal{B}(B \to \bar{P}\ell^+\ell^-[q_1^2, q_2^2])}$$

$$= \frac{-2\kappa_q \sin\xi_q\,\mathcal{S}_{BP}[q_1^2, q_2^2]}{\mathcal{F}_{BP}[q_1^2, q_2^2] + \kappa_q^2\,\mathcal{D}_{BP}[q_1^2, q_2^2] + 2\kappa_q \cos\xi_q\,\mathcal{C}_{BP}[q_1^2, q_2^2]}\,.$$

Various channels

- $B \to K\ell^+\ell^-$ we neglect $\lambda_u^{(s)}$, hence, $\kappa_s = 0$, vanishing *CP*-asymmetry

$$\mathcal{B}_{BK}[q_1^2, q_2^2] = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(s)}|^2}{192\pi^5} \mathcal{F}_{BK}[q_1^2, q_2^2] \tau_B,$$

- $B^- \to \pi^-\ell^+\ell^-$

$$\mathcal{B}_{B\pi}[q_1^2, q_2^2] = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(d)}|^2}{192\pi^5} \Big\{ \mathcal{F}_{B\pi}[q_1^2, q_2^2] + \kappa_d^2\, \mathcal{D}_{B\pi}[q_1^2, q_2^2] + 2\kappa_d \cos\xi_d\, \mathcal{C}_{B\pi}[q_1^2, q_2^2] \Big\} \tau_B\,,$$

$$\mathcal{A}_{B\pi}[q_1^2, q_2^2] = \frac{-2\kappa_d\, \sin\xi_d\, \mathcal{S}_{B\pi}[q_1^2, q_2^2]}{\mathcal{F}_{B\pi}[q_1^2, q_2^2] + \kappa_d^2\, \mathcal{D}_{B\pi}[q_1^2, q_2^2] + 2\kappa_d\, \cos\xi_d\, \mathcal{C}_{B\pi}[q_1^2, q_2^2]}\,,$$

- Byproduct: $\mathcal{B}_{B_sK}[q_1^2, q_2^2]$ and $\mathcal{A}_{B_sK}[q_1^2, q_2^2]$

Expressing CKM factors via Wolfenstein parameters

$$\lambda_t^{(s)} = -A\lambda^2\,,$$

$$\left|\frac{\lambda_t^{(d)}}{\lambda_t^{(s)}}\right| = \left|\frac{V_{td}}{V_{ts}}\right| = \lambda\sqrt{(1-\rho)^2+\eta^2},$$

$$\frac{\lambda_u^{(d)}}{\lambda_t^{(d)}} = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} \equiv \kappa_d e^{i\xi_d} = \left(1-\frac{\lambda^2}{2}\right)\frac{\rho(1-\rho)-\eta^2-i\eta}{(1-\rho)^2+\eta^2},$$

so that

$$\kappa_d = \left(1-\frac{\lambda^2}{2}\right)\frac{\sqrt{(\rho(1-\rho)-\eta^2)^2+\eta^2}}{(1-\rho)^2+\eta^2},$$

$$\sin\xi_d = \frac{-\eta}{\sqrt{(\rho(1-\rho)-\eta^2)^2+\eta^2}},\quad \cos\xi_d = \frac{\rho(1-\rho)-\eta^2}{\sqrt{(\rho(1-\rho)-\eta^2)^2+\eta^2}},$$

- we neglect very small $O(\lambda^4)$ corrections to these eqs
- λ, precisely determined from the global CKM fit used as an input.

Determining Wolfenstein parameters from FCNC observables

- parameter A

$$A = \frac{(192\pi^5)^{1/2}}{G_F \alpha_{em} \lambda^2} \left(\frac{1}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \right)^{1/2} \left(\frac{\mathcal{B}_{BK}[q_1^2, q_2^2]}{\tau_B} \right)^{1/2} .$$

- parameter η

$$\eta = \frac{1}{2\lambda^2(1-\lambda^2/2)} \left(\frac{\mathcal{F}_{BK}[q_1^2, q_2^2]}{\mathcal{S}_{B\pi}[q_1^2, q_2^2]} \right) \left(\mathcal{A}_{B\pi}[q_1^2, q_2^2] \frac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} \right) .$$

- given η , the parameter ρ can be fitted/constrained from

$$\frac{\mathcal{B}_{B\pi}[q_1^2, q_2^2]}{\mathcal{B}_{BK}[q_1^2, q_2^2]} = \frac{\lambda^2}{\mathcal{F}_{BK}[q_1^2, q_2^2]} \Bigg(\left[(1-\rho)^2 + \eta^2\right] \mathcal{F}_{B\pi}[q_1^2, q_2^2] + \frac{\left[\rho(1-\rho) - \eta^2\right]^2 + \eta^2}{(1-\rho)^2 + \eta^2} \left(1 - \frac{\lambda^2}{2}\right)^2 \mathcal{D}_{B\pi}[q_1^2, q_2^2] + 2\left[\rho(1-\rho) - \eta^2\right] \left(1 - \frac{\lambda^2}{2}\right) \mathcal{C}_{B\pi}[q_1^2, q_2^2] \Bigg) .$$

Numerical results

- hadronic input for future determinations

Decay mode	$\mathcal{F}_{BP}[1.0, 6.0]$	$\mathcal{D}_{BP}[1.0, 6.0]$	$\mathcal{C}_{BP}[1.0, 6.0]$	$\mathcal{S}_{BP}[1.0, 6.0]$
$B^- \to K^- \ell^+ \ell^-$	$75.0^{+10.5}_{-9.7}$	—	—	—
$B^- \to \pi^- \ell^+ \ell^-$	$47.7^{+6.4}_{-5.9}$	$16.1^{+2.8}_{-10.1}$	$14.3^{+7.8}_{-5.8}$	$-9.8^{+7.1}_{-7.2}$
$\bar{B}_s \to K^0 \ell^+ \ell^-$	$61.0^{+7.0}_{-6.8}$	$7.8^{+3.4}_{-2.5}$	$-12.9^{+2.4}_{-2.2}$	$-3.4^{+1.1}_{-2.6}$

- substituting current global CKM fit results:

Decay mode	$B^- \to K^- \ell^+ \ell^-$	$B^- \to \pi^- \ell^+ \ell^-$	$\bar{B}_s \to K^0 \ell^+ \ell^-$
Measurement or calculation	$\mathcal{B}_{BK}[1.0\,, 6.0]$	$\mathcal{B}_{B\pi}[1.0\,, 6.0]$	$\mathcal{B}_{B_sK}[1.0, 6.0]$
Belle (2009)	$2.72\,^{+0.46}_{-0.42} \pm 0.16$	—	—
CDF (2011)	$2.58 \pm 0.36 \pm 0.16$	—	—
BaBar (2012)	$2.72\,^{+0.54}_{-0.48} \pm 0.06$	—	—
LHCb (2014,2015)	$2.42 \pm 0.7 \pm 0.12$	$0.091^{+0.021}_{-0.020} \pm 0.003$	—
HPQCD (2013)	3.62 ± 1.22	—	—
Fermilab/MILC (2015)	3.49 ± 0.62	0.096 ± 0.013	—
This work	$4.38^{+0.62}_{-0.57} \pm 0.28$	$0.131^{+0.023}_{-0.022} \pm 0.010$	$0.154^{+0.018}_{-0.017} \pm 0.011$

$$\mathcal{A}_{B\pi}[1.0, 6.0] = -0.15^{+0.11}_{-0.11}\,, \quad \mathcal{A}_{B_sK}[1.0, 6.0] = -0.04^{+0.01}_{-0.03}\,.$$

Summary

- LCSRs and related methods can provide form factors and nonlocal hadronic matrix elements for semileptonic and FCNC B-decays
- other channels of $b \to u$, $b \to s$, $b \to d$ exclusive transitions are also interesting
- determination of V_{td}/V_{ts} is not direct, as claimed in the literature, but can be done in the Wolfenstein form
- the relevance of these results for CEPC depends on the efficiency/statistics of B-mesons produced in the future detector and demands additional studies
- presumably, very rare exclusive FCNC channels e.g., $e^+e^- \to Z \to BK^{(*)}$