$B \to \pi\pi$ form factors from LCSRs

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November 9, 2017 , IHEP, Beijing, EW and Flavor@CEPC

Outline

Background and Significance

Overview for QCD Light-cone Sum Rule

$B \to \pi\pi$ form factors from LCSRs

- With B meson DAs
- With dipion DAs

Conclusion and Prospect

[S. Cheng, A. Khodjamirian and J. Virto, JHEP05(2017)157]
[S. Cheng, A. Khodjamirian and J. Virto, Phys.Rev.D96(2017)051901(R)]
[C. Hambrock and A, Khodjamirian, Nucl.Phys.B905(2016)373]

Background

- SM works beautifully, but an effective theory valid up to some scale;
- BSM are needed to explained the large $\not{C}\not{P}$ matter antimatter asymmetry;
- † Direct search for new particles;
- † Comparison of precise measurements with prediction from SM.
- † Precision study:
  beta decay, (1983, UA1,UA2, W boson);
  kaon physics, (1974, BNL/SLAC, charm quark;
  1977, Fermilab E288, beauty quark);
  1987, ARGUS, large $\Delta m_{B_d,B_s}$ (1994, CDF/D0, top quark);
  $A_{CP}(t,f)$ in B decays (irreducible phase in CKM paradigm);
  . . .

Significance

Interests [S. Faller, T. F, A. K, T. M and Danny v Dyk, Phys.Rev.D89(2014)014015]

- Quark level: $b \to u$ is induced by tree-level W-exchange in SM, weighted by $|V_{ub}|$ (rare decay).
- ~~Tensions of $|V_{ub}|$~~: [BABAR, Phys.Rev.D95(2017)072001, talks in FPCP2017, Prague] CKM data analysis is 15% smaller than inclusive B decays; $B \to \pi l \bar{\nu}_l$, is 2σ smaller than, $B \to X_u l \bar{\nu}_l$; $\Longrightarrow$ Unitarity of CKM or NP?
- Traditional $B \to \rho$ form factors, single ρ meson with narrow width: width effect ? nonresonance background?
- We need enlarge the set of exclusive processes ($B \to \pi\pi l \bar{\nu}_l / B_{l4}$).

Phenomenological Applications

- A competitive determination of the $|V_{ub}|$ with accurate FFs;
- FCNC $B \to \pi\pi l^+ l^-$, $B \to \pi\pi\pi$.

LCSRs Overview [P. Colangelo and A. Khodjamirian, arXiv:hep-ph/0010175]

- QCD sum rules approach: twofold way of treating correlation fucntion.
- Mutually versions for different types of hadronic matrix elements: LCSRs(hadronic form factors), 2pSRs(decay constants).

LCSRs

- Non-local correlation function, putting the external meson on-shell, expanding on LC to get LCDAs;
- Quark level: evaluated with OPE LCDAs,
  Hadron level: with intermediate interpolating, convolution of two decoupled quark current by dispersion relation;
- Quark-hadron duality: equate hadron dispersion integral to the OPE calculation (threshold s_0);
- Borel transformation: mitigate the harassment of ultraviolet subtraction scheme from the OPE side & suppress the contributions from higher excited and continuum states from the hadron aspect (M^2).

LCSRs for $\bar{B}^0 \to \pi^+\pi^0$ transition FFs

Some highlights:

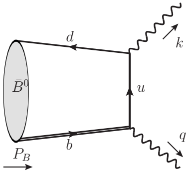
- For definiteness and conciseness, only the transition $\bar{B}^0 \to \pi^+\pi^0$ was concentrated, iso-vector dipion final state,
  FFs with iso-scalar dipion can be further studied.
- How much of the dominant intermediate ρ contribution to the FFs, accurate interpretation (10%) of the $B \to \pi\pi l\bar{\nu}_l$ measurements.
- ⧫ We are now at LO for preliminary attempts, NLO corrections from soft gluon, hard gluon exchanging correction will be added.
- ⧫ Our predictions are effected/constrained by hadronic inputs:
  dipion DAs (twist, strong phase) / B meson DAs (λ_B).

With B meson DAs

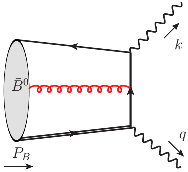
With B meson DAs

Correlation Function: [A. Khodjamirian, T. Mannel and Nils Offen, Phys.Rev.D75(2014)054013]

$$
\begin{aligned}
F_{\mu\nu}(k,q) &= i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0)\gamma_\nu(1-\gamma_5)b(0)\}|\bar{B}^0(q+k)\rangle \\
&\downarrow \quad \textit{Lorentz decomposition} \\
&\equiv \varepsilon_{\mu\nu\rho\sigma}q^\rho k^\sigma F_{(\varepsilon)}(k^2,q^2) + ig_{\mu\nu}F_{(g)}(k^2,q^2) + iq_\mu k_\nu F_{(qk)}(k^2,q^2) \\
&+ ik_\mu k_\nu F_{(kk)}(k^2,q^2) + iq_\mu q_\nu F_{(qq)}(k^2,q^2) + ik_\mu q_\nu F_{(kq)}(k^2,q^2) \qquad (1)
\end{aligned}
$$



(a)



(b)

With B meson DAs

- $(q+k)^2 = m_B^2$, When $\Lambda_{QCD}^2 \ll q^2 \ll m_B^2,\ |k^2| \gg \Lambda_{QCD}^2$,
  large virtuality of intrmediate u quark,
  OPE is applicable with DAs defined in HQET ($k \cdot x \sim 0$),
  q^2 is chosen from 0 to $10\mathrm{GeV}^2$.
- At LO, free propagator (u quark) + "Soft" gluon correction,
  heavy-light bilocal $q\bar{q}$ and $qG\bar{q}$ matrix (two- and three-particle DAs).
- In parallel, employ the hadronic dispersion relation in the k^2 respecting to the $\bar{d}\gamma_\mu u$ current interpolation.
- Take the invariant amplitude $F_\varepsilon(k^2, q^2)$ for example to outline.

OPE Calculation

$$F^{\rm OPE}_{(\varepsilon)}(k^2, q^2) = f_B m_B \int_0^1 d\sigma \, \frac{\phi_+^B(\sigma m_B)}{(1-\sigma)(s-k^2)} + \cdots \quad (2)$$

$$s = s(\sigma, q^2) = \sigma m_B^2 - \sigma q^2/\bar{\sigma}, \qquad \bar{\sigma} \equiv 1 - \sigma \quad (3)$$

- σ : the momentum carried by the light quark in B.
  Ellipsis : subleading 3-particle DA contributions (soft gluon correction),
- Dispersion integral formula in k^2 for $F^{\rm OPE}_{(\varepsilon)}(k^2, q^2)$

$$F^{\rm OPE}_{(\varepsilon)}(k^2, q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \, \frac{{\rm Im} F^{\rm OPE}_{(\varepsilon)}(s, q^2)}{s - k^2} \quad (4)$$

$$\frac{1}{\pi} {\rm Im} F^{\rm OPE}_{(\varepsilon)}(s, q^2) = f_B m_B \left[\left(\frac{d\sigma}{ds} \right) \frac{\phi_+^B(\sigma m_B)}{(1-\sigma)} \right]_{\sigma(s)} + \cdots \quad (5)$$

Hadronic dispersion relation

$$F_{(\varepsilon)}(k^2, q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \, \frac{\mathrm{Im} F_{(\varepsilon)}(s, q^2)}{s - k^2} \tag{6}$$

$$\begin{aligned} 2\,\mathrm{Im} F_{\mu\nu}(k, q) = & \int d\tau_{2\pi} \langle 0 | \bar{d} \gamma_\mu u \, | \pi^+(k_1) \pi^0(k_2) \rangle \\ & \langle \pi^+(k_1) \pi^0(k_2) | \bar{u} \gamma_\nu (1 - \gamma_5) b | \bar{B}^0(q + k) \rangle + \cdots \end{aligned} \tag{7}$$

- unitarity relation: Inserting the complete set of state with quantum number of the $\bar{d}\gamma_\mu u$ current.
- Ellipsis : contributions from $4\pi, K\bar{K}, etc$.

Hadronic dispersion relation

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu d|0\rangle = -\sqrt{2}(k_1 - k_2)_\mu F_\pi(k^2), \quad (8)$$

$$i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle$$
$$= F_\perp(k^2, q^2, q\cdot \overline{k})\,\frac{2}{\sqrt{k^2}\sqrt{\lambda}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma + \cdots \quad (9)$$

- $k = k_1 + k_2$, $\bar{k} = k_1 - k_2$; Normalization $F_\pi^{\rm em}(0) = 1$;
  In the isospin symmetry limit $F_\pi(k^2) = F_\pi^{\rm em}(k^2)$,
- Källén function:
  $\lambda \equiv \lambda(m_B^2, q^2, k^2) = m_B^4 + q^4 + k^4 - 2(m_B^2 q^2 + m_B^2 k^2 + q^2 k^2)$
- $q\cdot\overline{k} = \frac{1}{2}\sqrt{\lambda}\;\beta_\pi(k^2)\cos\theta_\pi$ with $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$,
- θ_π : the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame,
- Ellipsis: other FFs with different kinematic definition.

Hadronic dispersion relation

Legendre expansion of $F_i(k^2, q^2, q\cdot\overline{k}), i = \perp, \|, t, 0$

$$F_{0,t}(k^2, q^2, q\cdot\bar{k}) = \sqrt{3}\, F_{0,t}^{(\ell=1)}(k^2, q^2)\, P_1^{(0)}(\cos\theta_\pi) + \cdots\,,$$
$$F_{\perp,\|}(k^2, q^2, q\cdot\bar{k}) = \sqrt{3}\, F_{\perp,\|}^{(\ell=1)}(k^2, q^2)\, \frac{P_1^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi} + \cdots\,, \qquad (10)$$

- (Associated) Legendre polynomials: $P_1^{(0)}(\cos\theta_\pi) = \cos\theta_\pi$ and $P_1^{(1)}(\cos\theta_\pi) = -\sin\theta_\pi$.
- Only the P-wave components survive $\Longleftarrow$ P-wave projector from F_π only with $J^P = 1^-$ contribution,.
- Different interpolated currents lead to different partial wave contributions.

Hadronic dispersion relation

The imaginary part

$$\frac{1}{\pi}\,\mathrm{Im}F_{(\varepsilon)}(s,q^2) = \frac{\sqrt{s}\,[\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}}\,F_\pi^\star(s)\,F_\perp^{(\ell=1)}(s,q^2) + \cdots \tag{11}$$

- We are studying on pion form factors at low $s \lesssim 1.0 - 1.5\ \mathrm{GeV}^2$, $4\pi, K\bar{K}, etc$ contributions are expected suppression.
  [S.Eidelman and S.Lukaszuk, Phys.Lett.B582(2004)27]
  [X.W. Kang, B. Kubis, C. Hanhart and U-G. Meissner, Phys.Rev.D89(2014)053015]

- Borel-transforming Eq.(6),
  Semi-local quark-hadron duality approximation (more general):

$$\int_{s_0^{2\pi}}^{\infty} ds\ e^{-s/M^2}\,\mathrm{Im}F_{(\varepsilon)}(s,q^2) = \int_{s_0^{2\pi}}^{\infty} ds\ e^{-s/M^2}\,\mathrm{Im}F_{(\varepsilon)}^{\mathrm{OPE}}(s,q^2) \tag{12}$$

B meson LCSRs for $F_\perp(s,q^2)$

$$
\begin{aligned}
& f_B m_B \left[\int_0^{\sigma_0^{2\pi}} d\sigma \; e^{-s(\sigma,q^2)/M^2} \; \frac{\phi_+^B(\sigma m_B)}{\bar{\sigma}} + m_B \, \Delta V^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right] \\
= \; & \int_{4m_\pi^2}^{s_0^{2\pi}} ds \; e^{-s/M^2} \frac{\sqrt{s}\,[\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}} \, F_\pi^\star(s) \, F_\perp^{(\ell=1)}(s,q^2) \;\; , \qquad (13)
\end{aligned}
$$

- $\sigma_0^{2\pi} m_B^2 - \sigma_0^{2\pi} q^2/\bar{\sigma}_0^{2\pi} \equiv s_0^{2\pi}$; $\quad \Delta V^{BV}$: three-particle DA contribution
- $F_{(\parallel,0)}(s,q^2)$ are deduced by the Axial-Vector weak curret.
- $F_{(t)}(s,q^2)$, correlation function with current $im_b \bar{u}(0)\gamma_5 b(0) \cdots$

B meson LCSRs for $F_\perp(s, q^2)$

Comments on Eq.(13)

- The s dependence in $F_\perp^{(\ell=1)}(s, q^2)$ is convoluted with $F_\pi^\star(s)$,
- † Reality condition:

$$\mathrm{Im}[\text{ r.h.s of Eq.(13)}] = 0. \tag{14}$$

- † Resonance model for $F_\perp^{(\ell=1)}$ should recover the $B \to \rho$ sum rules
- † Constraints $B \to \pi\pi$ FFs in a certain ansatz and/or (resonance) model;
- ♦ B meson LCDAs and timelike pion vector FF are the main inputs;
- ♦ Validity and consistency with an alternative method/ansatz/model;

Resonance ansatz for $F_\perp(s, q^2)$

$$F_\perp^{(\ell=1)}(s,q^2) \quad = \quad \frac{\sqrt{s}\sqrt{\lambda}}{\sqrt{3}} \sum_{R=\rho,\,\rho',\,\rho''} \frac{g_{R\pi\pi} V^{B\to R}(q^2) e^{i\phi_R(s)}}{(m_B+m_R)\left[m_R^2 - s - i\sqrt{s}\Gamma_R(s)\right]}, \quad (15)$$

- $\langle \pi^+(k_1)\pi^0(k_2)|R\rangle = g_{R\pi\pi}(k_1-k_2)^\alpha \epsilon_\alpha$;
- $\langle R^+(k)|\bar{u}\gamma_\nu b|\bar{B}^0(p)\rangle = \epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha} q^\beta k^\gamma \frac{2V^{B\to R}(q^2)}{m_B+m_R}$;
- strong phase $\phi_R(s)$: interactions?
- tacitly assumed to be same for all $F_{\perp,\parallel,t,0}^{(\ell=1)}(s,q^2)$;
- First ansatz: the simplest way without interaction,

$$\phi_R(s,q^2) \to \phi_R(s) \to -\mathrm{Arg}\left[\frac{F_\pi^\star(s)}{m_R^2 - s - i\sqrt{s}\Gamma_R(s)}\right]. \qquad (16)$$

Resonance ansatz for $F_\perp(s,q^2)$, go to narrow ρ appr.

Adopting $\rho \to \pi\pi$ width:

$$\Gamma_\rho(s) = \frac{g^2_{\rho\pi\pi}[\beta_\pi(s)]^3\sqrt{s}}{48\pi}\theta(s-4m_\pi^2) = \Gamma_\rho^{\rm tot}\left[\frac{\beta_\pi(s)}{\beta_\pi(m_\rho^2)}\right]^3\frac{\sqrt{s}}{m_\rho}\theta(s-4m_\pi^2), \tag{17}$$

$$F_\pi^\star(s) = \frac{f_\rho g_{\rho\pi\pi} m_\rho}{\sqrt{2}(m_\rho^2 - s + i\sqrt{s}\Gamma_\rho(s))}, \tag{18}$$

r.h.s of Eq.(13) $\Rightarrow$

$$\frac{2f_\rho m_\rho V^{B\to\rho}(q^2)}{(m_B+m_\rho)}\int_{4m_\pi^2}^{s_0^{2\pi}} ds\ e^{-s/M^2}\left(\frac{1}{\pi}\frac{\Gamma_\rho(s)\sqrt{s}}{(m_\rho^2-s)^2+s\Gamma_\rho^2(s)}\right)$$
$$\xrightarrow{\Gamma_\rho^{\rm tot}\to 0} \frac{2f_\rho m_\rho V^{B\to\rho}(q^2)}{(m_B+m_\rho)}e^{-m_\rho^2/M^2}. \tag{19}$$

Determining of LCSRs paras: M^2, $s_0^{2\pi}$

Borel parameter $M^2 = 1.25 \pm 0.25\mathrm{GeV}^2$:

[P. Colangelo and A. Khodjamirian, arXiv:hep-ph/0010175]

- Convergence(30%) of OPE is manifested by the relatively small three-parton DA contribution (power counting);
- Higher spectral density contribution does not exceed 40% of the total integral, (quark-hadron duality).

Cut threshold $s_0^{2\pi} = 1.5 \pm 0.1\mathrm{GeV}^2$

- A separate investigation with using vector $F_\pi(s)$ data from Belle
  [The Belle Collaboration, Phys.Rev.D78(2008)072006; Phys.Rev.D93(2016)032003,]
- Employing 2pSRs (SVZ) for the isospin-1 light-quark vector currents,
  [M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl.Phys.B147(1979)385]
  $\rho \to \pi\pi$, substitute $F_\pi(s)$ in the hadronic part.

Numerically for $F_{\perp}(s, q^2)$ with B DAs

Sum rules Eq.(13), in a compact version

$$
\begin{aligned}
I_V^{\rm OPE}(q^2, M^2, s_0^{2\pi}) &= \sum_R X_V^R V^R(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \left[F_\pi^\star(s) D^R(s) e^{i\phi_R(s)}\right] \\
&\equiv \sum_R X_V^R V^R(q^2) \mathcal{I}^R(M^2, s_0^{2\pi}), \qquad (20)
\end{aligned}
$$

- $\mathcal{I}_V^{\rm OPE}(q^2, M^2, s_0^{2\pi})$ is the l.h.s,
- $X_V^R = 1/(m_B + m_R)$, $V^R(q^2)$ is $z-$series formula.
  [A. Khodjamirian, T. Mannel, A.A. Pivovarov and Y.M. Wang, JHEP 09 (2010) 089]
- Notice the large hierarchy of $|\mathcal{I}^R(1.0, 1.5)|$

$$
\text{Abs}\{\mathcal{I}^\rho,\ \mathcal{I}^{\rho'},\ \mathcal{I}^{\rho''}\} \cdot 10^2 = \{2.6,\ 0.43,\ 0.25\}, \qquad (21)
$$

  will generate large uncertainties in parameters fitting.
- Second ansatz to fix the relative weight of different R to $F(s, q^2)$.

Numerically with B DAs

Model-0: single-ρ with width

	$V^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$A_0^{B\rho}(0)$
narrow-ρ	0.34	0.26	0.21	0.30
$F_\pi^{(\rho)}$	0.36 ± 0.17	0.27 ± 0.13	0.22 ± 0.15	0.30 ± 0.06
F_π	0.41 ± 0.11	0.31 ± 0.08	0.25 ± 0.10	0.34 ± 0.04
ρ-DAs	0.33 ± 0.03	0.26 ± 0.03	0.23 ± 0.04	0.36 ± 0.04

- The finite width of ρ does not impact the $B \to \rho$ form factors significant, but the higher resonances in F_π does $(15\% - 20\%)$.

Numerically with B DAs

Model-I: assess ρ' contribution

- Assume the $B \to \rho$ FFs are well determined from the LCSRs with $\rho-$meson DAs, $g_{\rho\pi\pi} = 5.94 \leftarrow$ Eq.(17)

R	$g_{R\pi\pi} V^{BR}(0)$	$g_{R\pi\pi} A_1^{BR}(0)$	$g_{R\pi\pi} A_2^{BR}(0)$	$g_{R\pi\pi} A_0^{BR}(0)$,
ρ	2.0 ± 0.2	1.6 ± 0.2	1.4 ± 0.2	2.1 ± 0.2
ρ'	3.0 ± 2.5	1.5 ± 1.4	1.0 ± 2.2	-0.3 ± 0.4

- Large uncertainties generate because of the suppressed sensitivity in the ρ' region, a quite appreciable ρ' contribution.

Numerically with B DAs

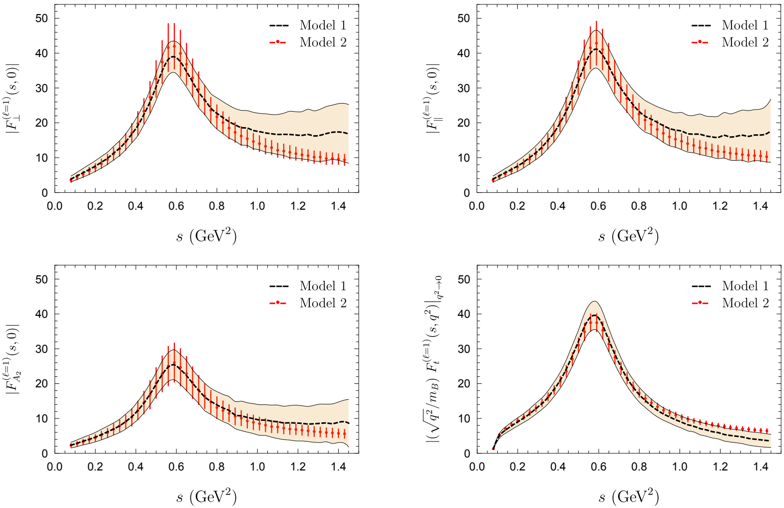
Model-II: $\rho + \rho' + \rho''$

- Assume the relative size of contributions from R is the same as in F_π

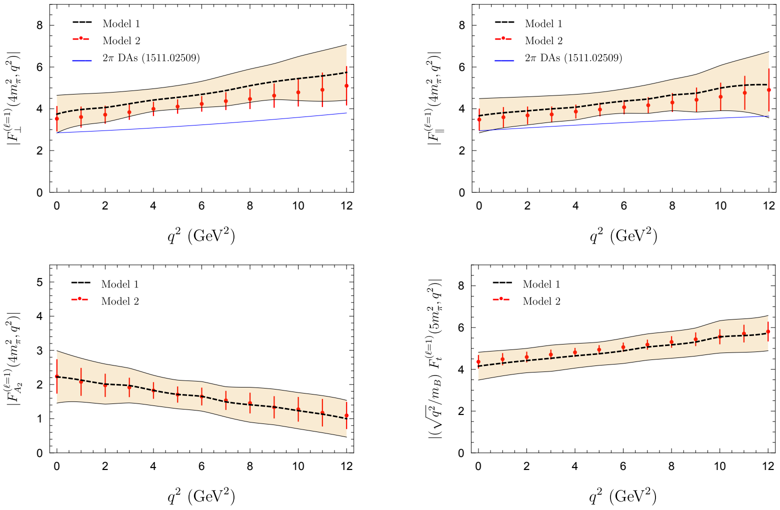
R	$g_{R\pi\pi} V^{BR}(0)$	$g_{R\pi\pi} A_1^{BR}(0)$	$g_{R\pi\pi} A_2^{BR}(0)$	$g_{R\pi\pi} A_0^{BR}(0)$
ρ	2.4 ± 0.4	1.8 ± 0.3	1.5 ± 0.3	1.9 ± 0.1
ρ'	0.35 ± 0.06	0.27 ± 0.04	0.22 ± 0.05	0.29 ± 0.02
ρ''	0.09 ± 0.01	0.07 ± 0.01	0.05 ± 0.01	0.07 ± 0.01

- Uncertainties decrease, ρ, ρ', ρ'' contributions are in the strong hierarchy .

Numerically with B DAs ($s-$dpendence)



Numerically with B DAs (q^2-dependence)



Numerically with B DAs (Intermediate conclusion)

- At large recoil region $q^2 = 0$, evolution on dipion mass k^2 shape peaks around ρ mass.
- At low dipion mass $k^2 = 4m_\pi^2/0.1\mathrm{GeV}^2$, evolutions of FFs on q^2 are gentle with the acceptable uncertainties.
- Radially excited resonant states in our model contribute $\sim 20\%$.
- We can also fit out $B \to \rho', \rho''$ FFs if we know exactly how large of the couplings $g_{\rho'\pi\pi}, g_{\rho''\pi\pi}$.
- Interactions between different resonances $\phi_R(s, q^2)$ should be included.

With dipion DAs

With dipion DAs

Correlation Function: [C. Hambrock and A. Khodjamirian, Nucl.Phys.B905(2016)373]

$$F_\mu(k_1, k_2, q) = i\int d^4x e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle$$

$$\downarrow \quad \textit{Lorentz decomposition}$$

$$\equiv \quad \varepsilon_{\mu\nu\rho\sigma}q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}, \tag{22}$$

- $j_\mu^{V-A}(x) \equiv \bar{u}(x)\gamma_\mu(1-\gamma_5)b(x), \qquad j_5(0) \equiv im_b\bar{b}(0)\gamma_5 d(0);$
- Kinematics: $k = k_1 + k_2, \quad \bar{k} = k_1 - k_2, \quad p = k + q$
  Four independent invariant variables: $p^2, q^2, k^2, q\cdot k$;
- $p^2, q^2 \ll m_b^2$, to guarantee the validity of OPE near the LC ($x^2 \sim 0$),
- $k^2 \lesssim 1\mathrm{GeV}^2 \ll m_b^2$, to avoid generic $\mathcal{O}(k^2x^2)$ terms in LC expansion.

OPE Calculation with dipion DAs

Dipion DAs, [M.V. Polyakov, Nucl.Phys.B555(1999)231]

$$\begin{aligned}
&\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle \\
=\ & -\sqrt{2}k_\mu \int_0^1 du e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\zeta,k^2), \qquad (23)\\
&\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle \\
=\ & 2\sqrt{2}\frac{k_{1\mu}k_{2\nu}-k_{1\nu}k_{2\mu}}{2\zeta-1}\int_0^1 du e^{iu(k\cdot x)}\Phi_\perp^{I=1}(u,\zeta,k^2), \qquad (24)
\end{aligned}$$

- Chiral-even and -odd LC expansion respectively with gauge factor $[x,0]$.
- u quark carries longitudinal momentum faction,
  $2q\cdot\bar{k}$ determines the LC momentum distribution carried by two pions.
- Normalization condition:
  $\int_0^1 \Phi_\parallel^{I=1}(u,\zeta,k^2)=(2\zeta-1)F_\pi^{em}(k^2)$, $\int_0^1 \Phi_\perp^{I=1}(u,\zeta,k^2)=(2\zeta-1)F_\pi^{t}(k^2)$.
  $F_\pi^{em}(0)=1$ and $F_\pi^t(0)=1/f_{2\pi}^\perp$.
- Higher twist proportional to $1,\gamma_\mu\gamma_5$ are neglected here,
  γ_5 vanishes because of P-parity conservation.

Dipion DAs: [Nucl.Phys.B555(1999)231, M.V. Polyakov]

LO and twist-2 appro.

- Dipion DAs are presented in double expansion of Legendre and Gegenbauer polynomials: $C_l^{1/2}(2\zeta-1)$ & $C_n^{3/2}(2u-1)$; Partial wave & eigenfunction of evolution equation:

$$\Phi_{\perp/\parallel}(u,\zeta,k^2)=\frac{6u(1-u)}{f_{2\pi}^{\perp}/1}\sum_{n=0,2,\cdots}^{\infty}\sum_{l=1,3,\cdots}^{n+1}B_{nl}^{\perp/\parallel}(k^2)C_n^{3/2}(2u-1)C_l^{1/2}(2\zeta-1),$$

$$C_l^{1/2}(2\zeta-1)=\beta_\pi P_l^{(0)}\left(\frac{2\zeta-1}{\beta_\pi}\right), \tag{25}$$

- $B_{nl}^{\perp/\parallel}(k^2)$: renormalizable coefficients, $B_{01}^{\perp/\parallel}(0)=1$, $B_{01}^{\parallel}(k^2)=F_\pi^{em}(k^2)$.
- $n\geq 2$ at low k^2 determine the non-asymptotic part of DAs, decrease logarithmically at large scale.
- With truncating at a given n_{max}, l is restricted to $n_{max}+1$.
- unitarity relation, $B_{nl}^{\perp}(k^2)$ are complex functions at $k^2>4m_\pi^2$.

OPE Calculation with dipion DAs

At twist-2 accuracy

- $p^2 = p^2 - s + s \to s$.
- $s = s(u) = (m_b^2 - \bar{u}q^2 + u\bar{u}k^2)/u$.
- $F^{(A,k)}(s, q^2, k^2, \zeta)$:
  $q \cdot \bar{k}$ generate a cut at the real axis to avoid imaginary part,
  $(\sqrt{q^2} - \sqrt{k^2})^2 < p^2 < (\sqrt{q^2} + \sqrt{k^2})^2$,
  non physical intermediate state, a typical kinematic singularity.
- After Borel trans., this cut is enhanced respecting to the b-quark spectral.
- New correlator to touch this amplitude: PS current ($j_\mu^{V-A} \to j_5$).
- Multiplying q_μ on Eq.(9), solid at leading power precision.

Hadronic dispersion relation with dipion DAs

$$F_\mu(q,k_1,k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2 - p^2} + \cdots\,, \quad (26)$$

$$\Pi_5(p^2,q^2,k^2,\zeta) = \frac{\sqrt{q^2}F_t(q^2,k^2,\zeta) f_B m_B^2}{m_B^2 - p^2} + \ldots\,, \quad (27)$$

results at leading twist:

$$\frac{F_\perp(q^2,k^2,\zeta)}{\sqrt{k^2}\sqrt{\lambda_B}} = -\frac{m_b}{\sqrt{2}f_B m_B^2(2\zeta-1)} \int_{u_0}^1 \frac{du}{u} e^{\frac{-s+m_B^2}{M^2}} \Phi_\perp(u,\zeta,k^2), \quad (28)$$

$$\frac{F_\parallel(q^2,k^2,\zeta)}{\sqrt{k^2}} = -\frac{m_b}{\sqrt{2}f_B m_B^2(2\zeta-1)} \int_{u_0}^1 \frac{du}{u^2} e^{\frac{-s+m_B^2}{M^2}} (m_b^2 - q^2 + u^2k^2)\Phi_\perp(u,\zeta,k^2), \quad (29)$$

$$\sqrt{q^2}F_t(q^2,k^2,\zeta) = -\frac{m_b^2}{\sqrt{2}f_B m_B^2} \int_{u_0}^1 \frac{du}{u^2} e^{\frac{-s+m_B^2}{M^2}} \left(m_b^2 - q^2 + k^2u^2\right)\Phi_\parallel^{I=1}(u,\zeta,k^2)\,, \quad (30)$$

$$\sqrt{q^2}F_0(q^2,k^2,\zeta) = \frac{1}{m_B^2 - q^2 - k^2}\left[\sqrt{\lambda_B}\sqrt{q^2}F_t(q^2,k^2,\zeta) + 2\sqrt{k^2}q^2(2\zeta-1)F_\parallel(q^2,k^2,\zeta)\right]. \quad (31)$$

- $\langle \bar{B}^0(p)|\bar{b}im_b\gamma_5 d|0\rangle = f_B m_B^2$, $2\zeta - 1 = \beta_\pi cos\theta_\pi$.

Dipion LCSRs for $F_{\perp,\parallel}(s,q^2)$:

[C. Hambrock and A. Khodjamirian, Nucl.Phys.B905(2016)373]

[S. Cheng, A. Khodjamirian and J. Virto, Phys.Rev.D96(2017)051901(R)]

LO and twist-2 appro.

$$F_{\perp}^{(l)} = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^{\perp}}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{\frac{m_B^2}{M^2}}\sum_{n=0,2,\cdots}\sum_{l'=1,3}^{n+1} I_{ll'}B_{nl'}^{\perp}(k^2)J_n^{\perp}(q^2,k^2,M^2,s_0^B), \quad (32)$$

$$F_{\parallel}^{(l)} = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^{\perp}}\frac{m_b^3}{m_B^2 f_B}e^{\frac{m_B^2}{M^2}}\sum_{n=0,2,\cdots}\sum_{l'=1,3}^{n+1} I_{ll'}B_{nl'}^{\perp}(k^2)J_n^{\parallel}(q^2,k^2,M^2,s_0^B), \quad (33)$$

$$\sqrt{q^2}F_t^{(l)}(k^2,q^2) = -\frac{6m_b^2}{\sqrt{2}f_B m_B^2}\frac{\beta_\pi(k^2)}{\sqrt{2l+1}}\exp\left(\frac{m_B^2-s}{M^2}\right)$$

$$\times\sum_{n=l-1,l+1,\cdots}^{\infty} B_{nl}^{\parallel}(k^2)\int_{u_0}^{1}\frac{du}{u}\bar{u}\,(m_b^2-q^2+u^2k^2)\,C_n^{3/2}(u-\bar{u}), \quad (34)$$

$$I_{ll'} \equiv -\frac{\sqrt{2l+1}(l-1!)}{2(l+1)!}\int_{-1}^{1}\frac{dz}{z}\sqrt{1-z^2}P_l^{(1)}(z)P_{l'}^{(0)}(z), \quad (35)$$

$$J_n^{\perp}(q^2,k^2,M^2,s_0^B) = \int_{u_0}^{1}du\,e^{\frac{-s}{M^2}}6(1-u)C_n^{3/2}(2u-1), \quad (36)$$

$$J_n^{\parallel}(q^2,k^2,M^2,s_0^B) = \int_{u_0}^{1}\frac{du}{u}e^{\frac{-s}{M^2}}6(1-u)C_n^{3/2}(2u-1)\left(1-\frac{q^2-u^2k^2}{m_b^2}\right), \quad (37)$$

Dipion LCSRs for $F_{\perp,\parallel}(s, q^2)$:

Input paras

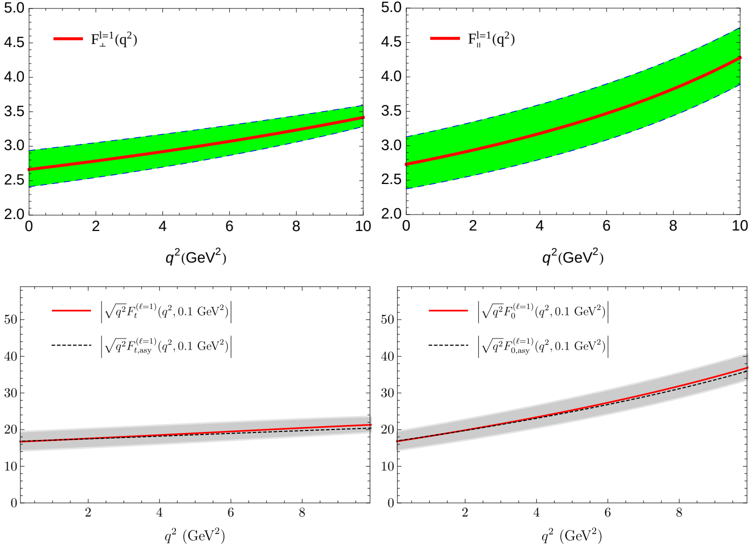
- $I_{ll'} = 0$ when $l > l'$, $I_{11} = 1/\sqrt{3}, I_{13} = -1/\sqrt{3}, I_{15} = 4/(5\sqrt{3})$;
  $l = 1$, asymptotic DAs, partial $P-$wave term remains in the FFs.
- The complexity of $B_{nl}^{\perp}(k^2)$, only predicable at $k^2 = 4m_\pi^2$ within the instanton model of QCD vacuum, WITHOT phase.
- Short-distance part of the correlator: $\mu = 3\,\mathrm{GeV}$ without NLO correction.
- Two-point QCD sum rules' prediction for $f_B = 207^{+9}_{-17}\mathrm{MeV}$.
- $M^2 = 16.0 \pm 4.0\mathrm{GeV}^2$ corresponding to $s_0^B = 37.5 \pm 2.5\mathrm{GeV}^2$.

◆ How large of $P-$wave contribution to $B \to \pi\pi$ FFs (higher partial wave) ?

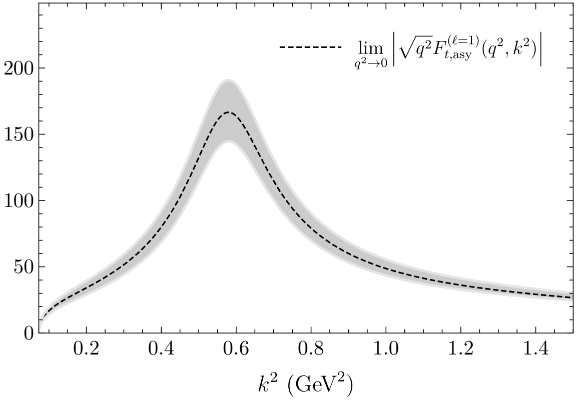
◆ How much ρ contained in $P-$wave $B \to \pi\pi$ FFs ? Resonance model.

◆ ρ meson: $a_2^{\perp} = 0.2 \pm 0.1, \quad a_{n>2} = 0, \quad f_\rho^{\perp} = 160 \pm 10\mathrm{MeV}$.

Numerically with dipion DAs $F_{\perp,\parallel}(q^2, k^2 = 4m_\pi^2/0.1\mathrm{GeV}^2)$



Numerically with dipion DAs $F_{t,0}(q^2 = 0, k^2)$



Numerically with dipion DAs (Intermediate conclusion)

- At LO and twist-2 approximation.
- $F_{\perp,\parallel}(q^2,k^2)$:
  At $k^2 = 4m_\pi^2$, $F_{\perp,\parallel}(q^2)$ has a smooth evolution;
  high partial waves give tiny contribution $\sim 2\%$.
- $\sqrt{q^2}F_{t,0}(q^2,k^2)$:
  At $k^2 = 0.1\mathrm{GeV}^2$, $\sqrt{q^2}F_{t,0}(q^2)$ is one order larger than $F_{\perp,\parallel}(q^2)$;
  high partial waves $< 10\%$.
  At $q^2 = 0\mathrm{GeV}^2$, $\sqrt{q^2}F_t(k^2) = \sqrt{q^2}F_0(k^2)$;
  $\sim 10\%$ contribution from ρ', ρ'' and NR background;
  high partial wave $< 10\%$.

Conclusion

Comparison of LCSRs with B DAS and dipion DAs

- They give similar plots for $B \to \pi^+\pi^0$ FFs, at the same order .
- They both predict sizable contribution (10% order) from $\rho', \rho'', \cdots$ and/or NR background for $P-$wave $B \to \pi^+\pi^0$ FFs.
- B meson LCSRs does not suggest any higher partial wave contribution. For LCSRs with dipion DAs, it exist but tiny.
- B LCSRs is more availiable (k^2, q^2, ϕ), but rely on the resonance model. Dipion LCSRs is limited by the poor knowledge of dipion DAs.

Prospect

- Further improvements for this approach:
  - Improving the accuracy of λ_B.
  - Gathering more data on time-like pion FF: $\mathcal{A}\&\phi$.
  - Forwarding to the NLO correction on the OPE side.
- Dipion DAs: sub-leading twist, s evolution, strong phase.
- Considering the iso-scalar $\pi\pi$ system (f_0, etc.,)
- Calculating $B \to$ Scalar FFs in B meson LCSRs.
- Extending the approach to $B_{(s)} \to K\pi, KK$ FFs.

Prospect @ CEPC

Why CEPC ?

- Planning instantaneous luminosity $\mathcal{L} = 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ at Z pole, will collect $1ab^{-1}$ data and generate $\bar{b}b \sim 10^9$, comparable with LHCb $(14\mathrm{TeV}, 1\mathrm{ab}^{-1})$, but can not compete with Belle-II $(10.86\mathrm{GeV}, 50\mathrm{ab}^{-1})$.
- Belle-II will do excellent job for B_u, B_d cases and improve the triangle measurement by a factor $3-10$, need more time for CPV in B_s case, time-dependent analysis will not be done.
- LHCb (update) greatly improve the sensitivities in B_s, B_c decays into charged final states.
- CEPC project at Z pole, high statistics, good study of both charged and neutral decays of $B_s(B_c)$, strongly boosted of $B_{d,s}$ in the rest frame of Z pole ($2600\mu\mathrm{m}$, $290\mu\mathrm{m}$ at Belle-II), more precise measurement for mixing and CPV.

Prospect @ CEPC

Flavor at CEPC

- Rare decays $B \to \gamma X_{d,s}, l^+ l^- X_{d,s}$: $B \to K^* l_1^+ l_2^-$
- Very rare decays $B_{d,s} \to \mu^+ \mu^-$
- B baryon decays
  $\Lambda_b \to p\pi^-, p\pi^-\pi^0, pKK, \Lambda K^-, \quad pK^-, pK^-\pi^0, pK_s\pi^-, etc.,$
  $\Xi_b \to \Lambda^0\pi^-, \Lambda^0\pi^-\pi^0, \Lambda^0 KK, \quad \Lambda^0 K^-, \Lambda^0 K^-\pi^0, \Lambda^0 \bar{K}^0\pi^-, etc.,$
  $\Omega_b^- \to \Xi^0\pi^-, \Omega_- K^0, etc.,$
- Charm sector. $D_0 - \bar{D}_0$-oscillation, CPV $\cdots$
- τ decay

The End, Thanks.

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The pion timelike form factor in

$$F_\pi(s) = \frac{BW_\rho^{GS}(s) + |\beta| e^{i\phi_\beta} BW_{\rho'}^{GS}(s) + |\gamma| e^{i\phi_\gamma} BW_{\rho''}^{GS}(s)}{1 + |\beta| e^{i\phi_\beta} + |\gamma| e^{i\phi_\gamma}} , \quad (38)$$

$$BW_R^{GS}(s) = \frac{m_R^2 + m_R \Gamma_R d}{m_R^2 - s + f(s) - i\sqrt{s}\, \Gamma_R(s)} , \quad (39)$$

Parameters measured in $\tau^- \to \pi^- \pi^0 \nu_\tau$

Resonance	m_R(MeV)	Γ_R(MeV)	Weight factor
ρ	$774.6 \pm 0.2 \pm 0.5$	$148.1 \pm 0.4 \pm 1.7$	1.0
ρ'	$1446 \pm 7 \pm 28$	$434 \pm 16 \pm 60$	$\lvert\beta\rvert = 0.15 \pm 0.05^{+0.15}_{-0.04}$ $\phi_\beta = 202 \pm 4^{+41}_{-8}$
ρ''	$1728 \pm 17 \pm 89$	$164 \pm 21^{+80}_{-26}$	$\lvert\gamma\rvert = 0.037 \pm 0.006^{+0.065}_{-0.009}$ $\phi_\gamma = 24 \pm 9^{+118}_{-28}$

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$z-$parameterization for $B \to R$ decay FFs

$$\begin{aligned} \mathcal{F}^{B\to R}(q^2) &= \frac{\mathcal{F}^{B\to R}(0)}{1-q^2/m_{\mathcal{F}}^2}\left\{1+b_{\mathcal{F}}^R\left[z(q^2)-z(0)+\frac{1}{2}(z(q^2)^2-z(0)^2)\right]+\cdots\right\} \\ &\equiv \frac{\kappa_{\mathcal{F}}^R+\eta_{\mathcal{F}}^R\zeta_R(q^2)}{1-q^2/m_{\mathcal{F}}^2}, \quad \mathcal{F}=V, A_0, A_1, A_2, \end{aligned} \tag{40}$$

$$\tag{41}$$

$$\begin{aligned} &z(q^2)=\frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}, \\ &t_\pm \equiv (m_B \pm m_R)^2, \quad t_0=t_+(1-\sqrt{1-t_-/t_+}), \\ &\kappa_{\mathcal{F}}^R \equiv g_{R\pi\pi}\mathcal{F}^{B\to R}(0), \quad \eta_{\mathcal{F}}^R \equiv g_{R\pi\pi}\mathcal{F}^{B\to R}(0)b_V^R, \end{aligned} \tag{42}$$

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Orthogonality of Legendra Polynomials

$$\int_{-1}^{1} dx P_l^n(x) P_k^n(x) = \frac{2}{2l+1} \frac{(l+n)!}{(l-n)!} \delta_{kl}, \tag{43}$$

$$\int_{-1}^{1} dx \frac{P_l^m(x) P_l^n(x)}{1-x^2} = \frac{(l+m)!}{m(l-m)!} \delta_{mn}, \quad m = n \neq 0, \tag{44}$$

$$P_0^0(x) = 1, \quad P_0^1(x) = x, \quad P_1^1(x) = \sqrt{1-x^2}, \quad \cdots \tag{45}$$

Poisson Kernel

$$\eta_y(x) = \frac{1}{\pi} \frac{y}{x^2+y^2} = \int_{-\infty}^{\infty} d\zeta e^{2\pi i \zeta x - |y\zeta|} \tag{46}$$

$$\int_{-\infty}^{\infty} dx e^{-2\pi i(\zeta_1 - \zeta_2)x} = \delta(\zeta_1 - \zeta_2), \qquad \int_{-\infty}^{\infty} dx e^{-2\pi i \zeta x} = 1, \tag{47}$$