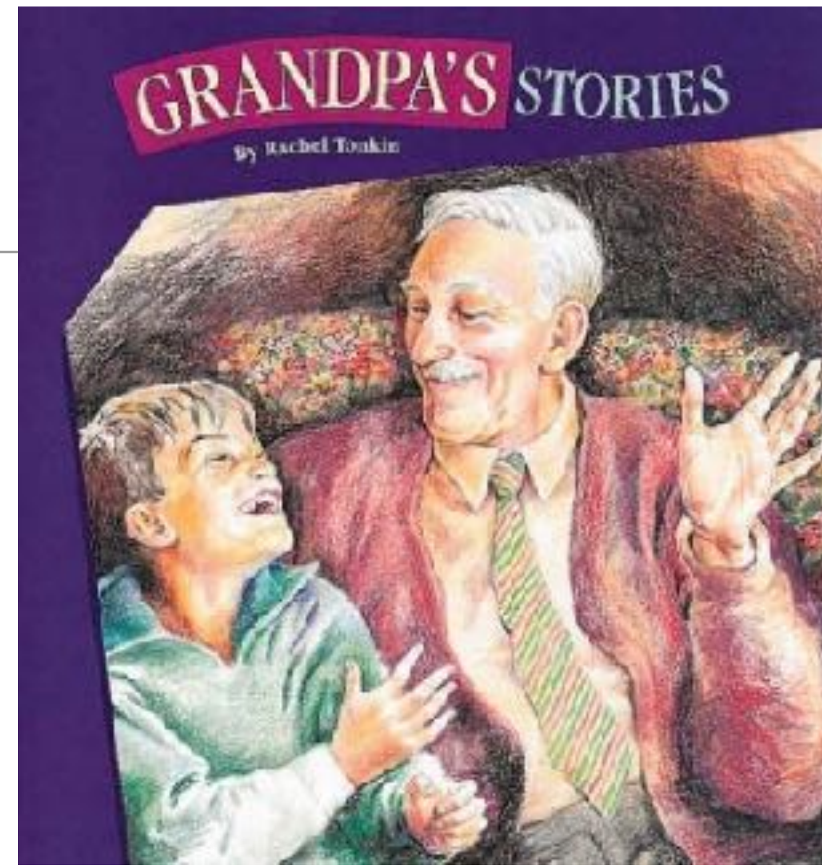
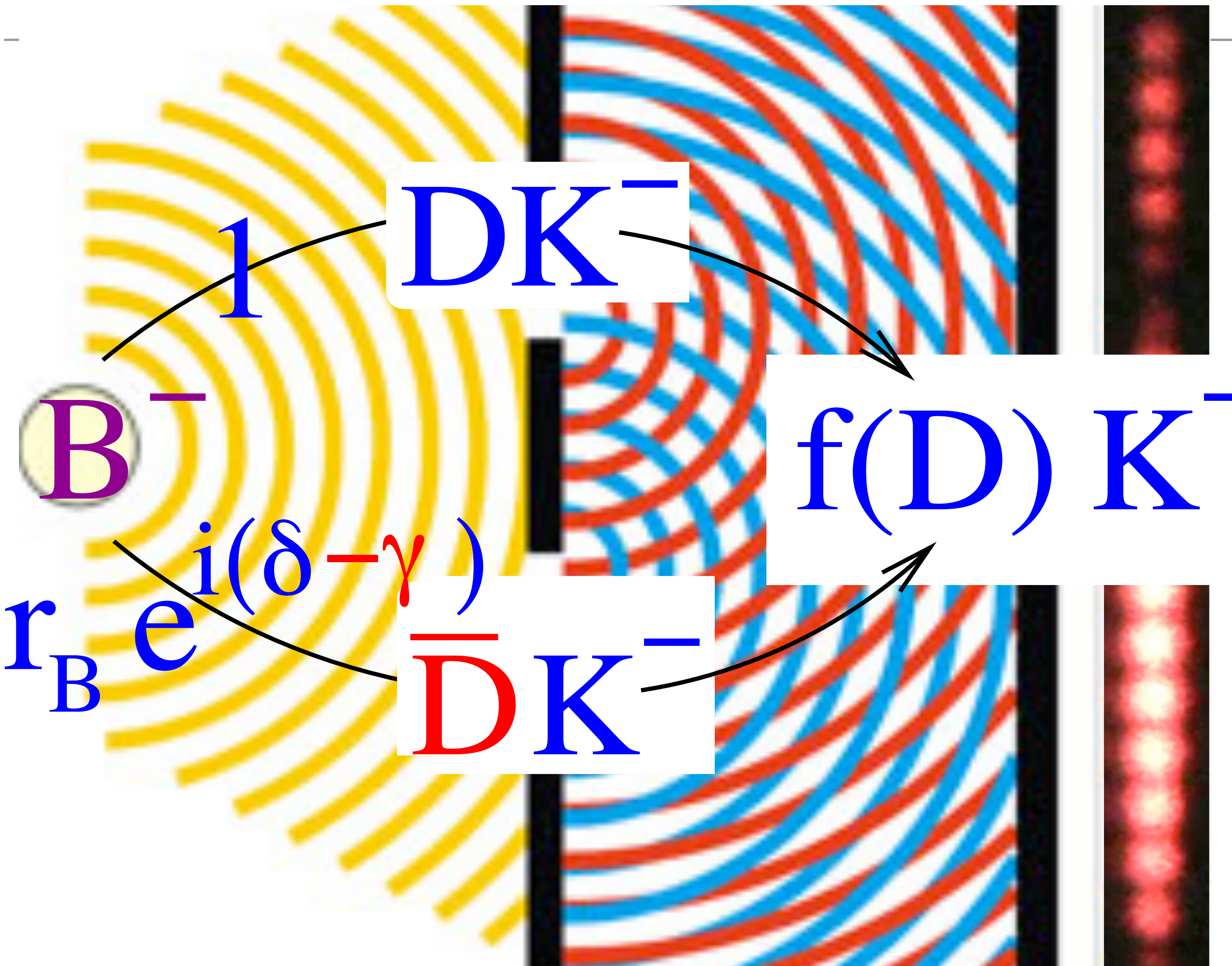
CLEO-c quantum correlated measurements

Jonas Rademacker
(LHCb, CLEO [when it existed], University of Bristol)



Measuring and γ with $B \to DK$



Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

GLW vs ADS

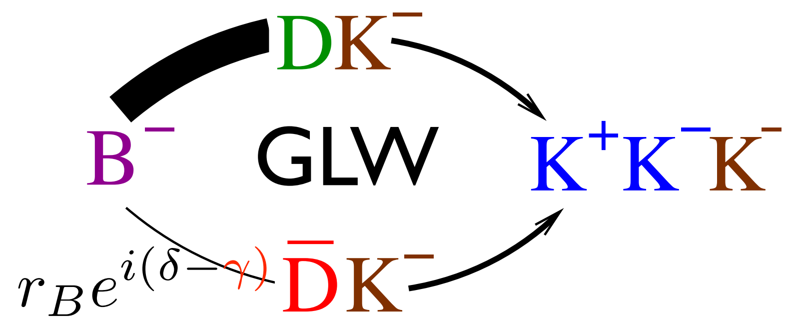
- Unfortunately,

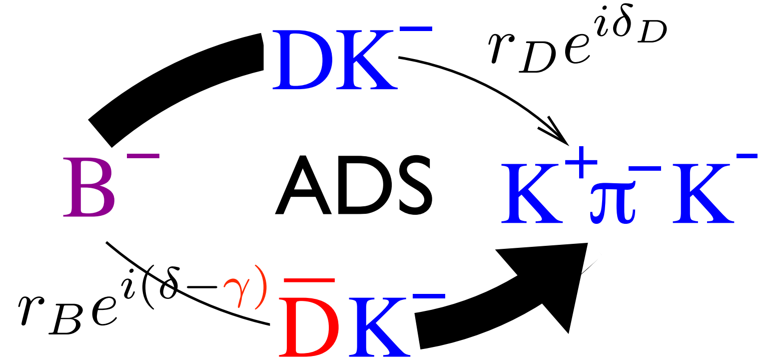
  A(B⁻→DK⁻) ≈ 0.1 $\overline{\text{A}}$(B⁻→DK⁻)

- The ADS trick: Instead of using a D decay to a CP eigenstate ("GLW" method), use a D decay where one decay path is doubly Cabibbo suppressed to balance the amplitudes.

- CLEO input: $|\delta| = (10^{+28+13}_{-53-0})^{\circ}$

  Phys.Rev. D86 (2012) 112001





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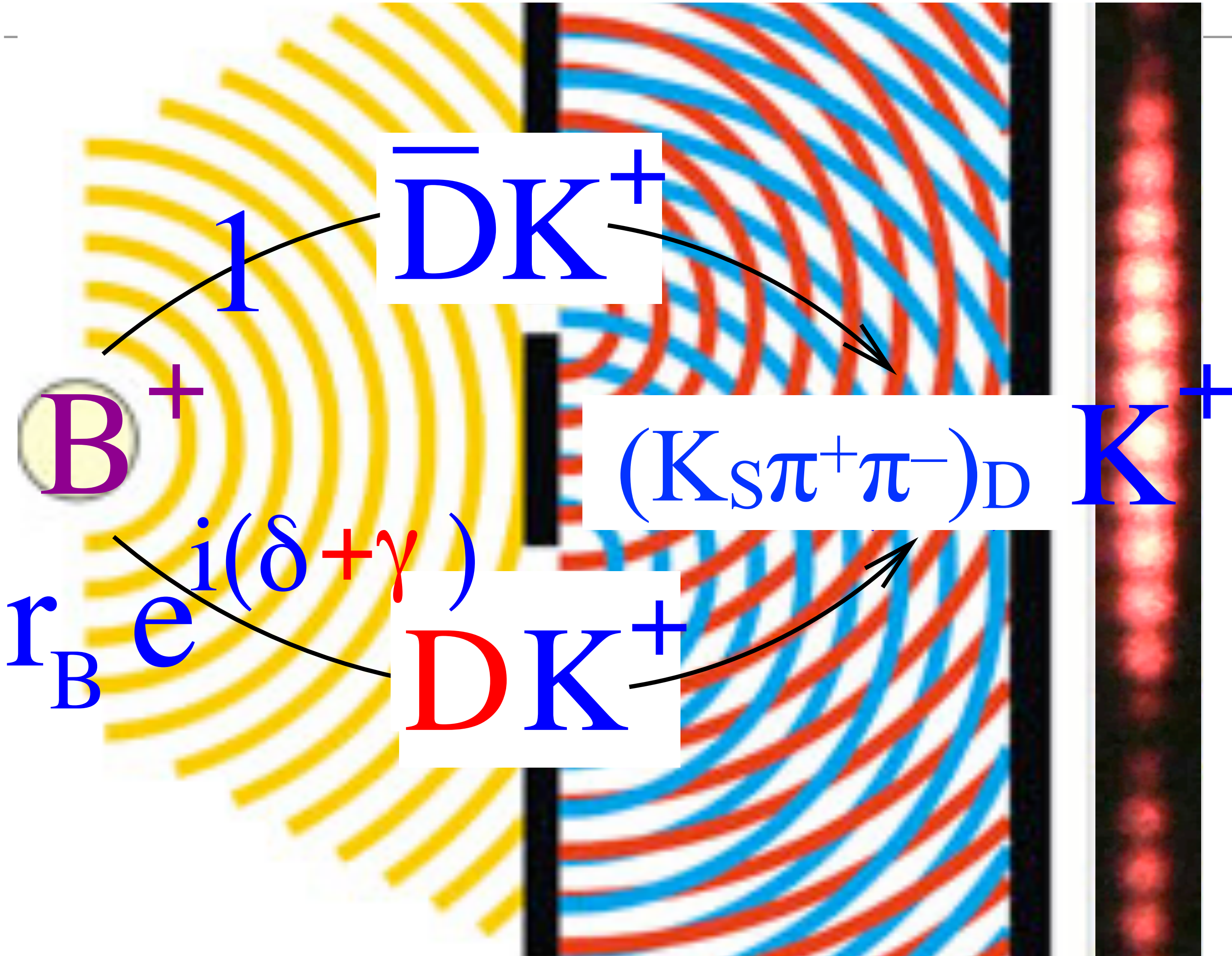

Model-uncertainty would eventually become limiting factor in these measurements … motivates model-independent approaches.

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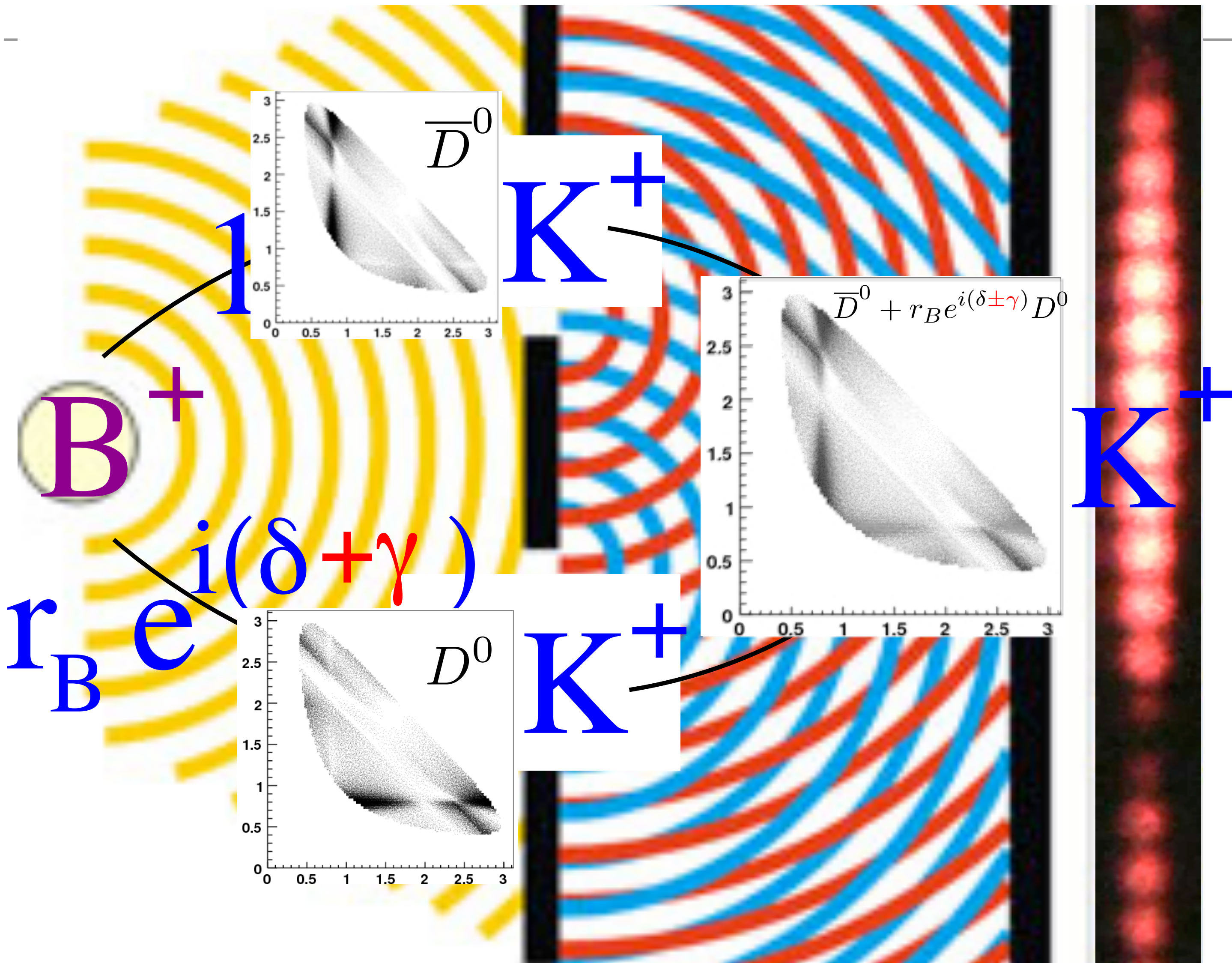
Measuring and γ with B→DK



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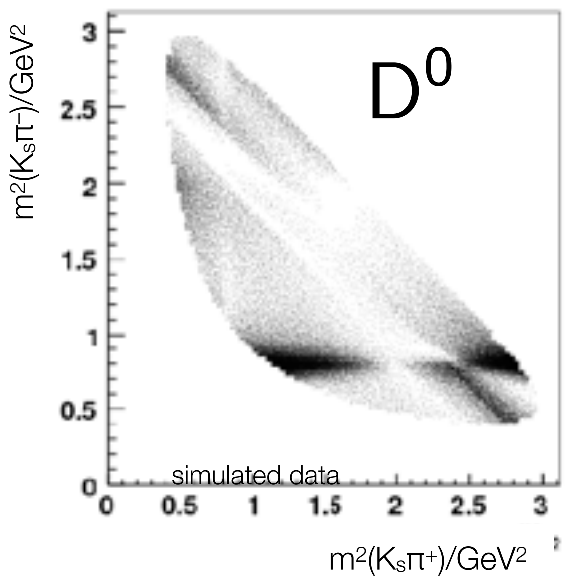
CP and flavour tagged D° (CLEO-c/BESIII)

ψ''

$\overline{D}_{flavour} \rightarrow K^+ e^- \nu_e$

$D_{flavour} \rightarrow K_S \pi^+ \pi^-$



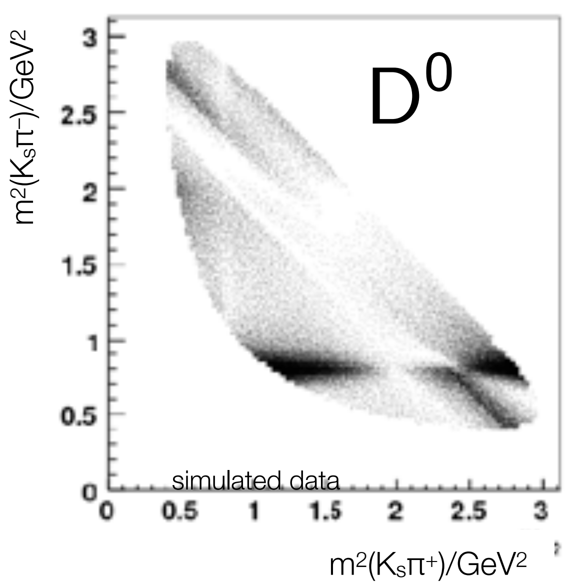
CP and flavour tagged D° (CLEO-c/BESIII)

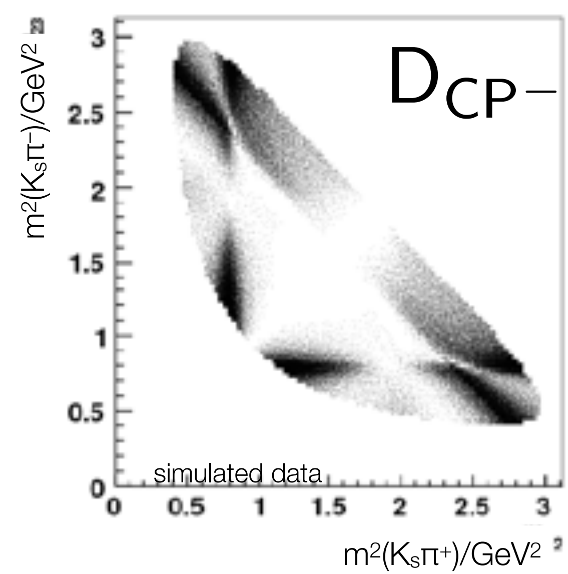
ψ'' → $\bar{D}_{flavour} \to K^+ e^- \nu_e$

ψ'' → $D_{flavour} \to K_s \pi^+ \pi^-$

ψ'' → $D_{CP+} \to KK$

ψ'' → $D_{CP-} \to K_s \pi^+ \pi^-$





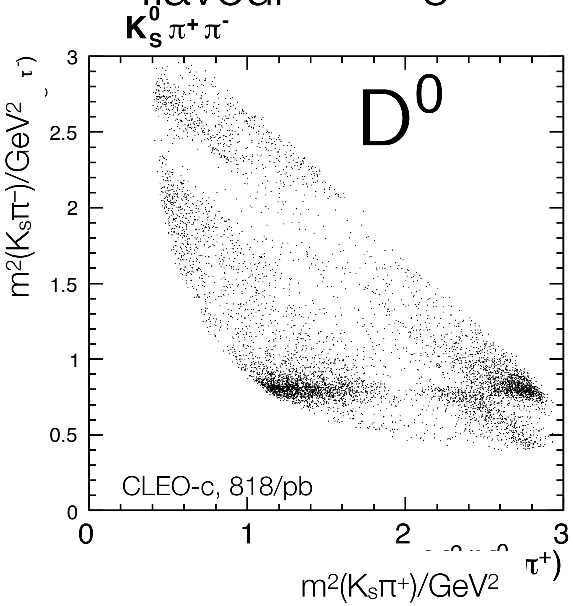
CP and flavour tagged D° at CLEO

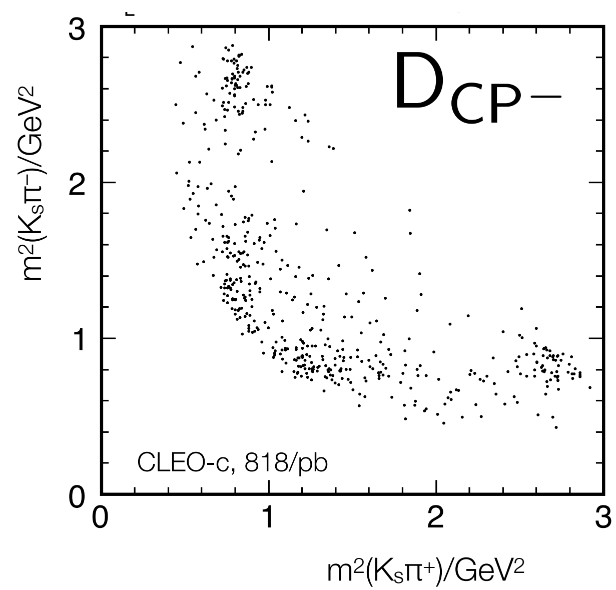
$\psi'' \to \bar{D}_{\text{flavour}} \to K^+ e^- \nu_e$

$\psi'' \to D_{\text{flavour}} \to K_s \pi^+ \pi^-$

$\psi'' \to D_{CP+} \to KK$

$\psi'' \to D_{CP-} \to K_s \pi^+ \pi^-$





CLEO-c: Phys. Rev. D80, 032002 (2009), updated in Phys.Rev. D82 (2010) 112006

CP-even fractions, measured with CLEO-c data

- CP-even fraction F_+ allows to use self-conjugate decays like $D \rightarrow \pi^+\pi^-\pi^0$, in $B^- \rightarrow DK^-$ like 2-body D decays.
- $$F_+ = \frac{N_{\mathrm{CP-even}}}{N_{\mathrm{CP-even}} + N_{\mathrm{CP-odd}}}$$

  $F_+ = 1$ for CP-even eigenstates, $F_+ = 0$ for CP odd ones.

$$\Gamma(B^\pm \to D(f_{\mathrm{self-con}})K^\pm) \propto 1 + r_B^2 + 2(2\boxed{F_+} - 1)\cos(\delta_B \pm \gamma)$$

(plus with some corrections for charm mixing)

PLB740, 1 (2015) PLB747, 9 (2015)

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- $$F_+ = \frac{N_{\mathrm{CP-even}}}{N_{\mathrm{CP-even}} + N_{\mathrm{CP-odd}}}$$

  number of events tagged as CP-even

  number of events tagged as CP-odd

  (assumes equal detection efficiencies)

  $F_+ = 1$ for CP-even eigenstates, $F_+ = 0$ for CP odd ones.

$$\Gamma(B^\pm \to D(f_{\mathrm{self-con}})K^\pm) \propto 1 + r_B^2 + 2(2\boxed{F_+} - 1)\cos(\delta_B \pm \gamma)$$

(plus with some corrections for charm mixing)

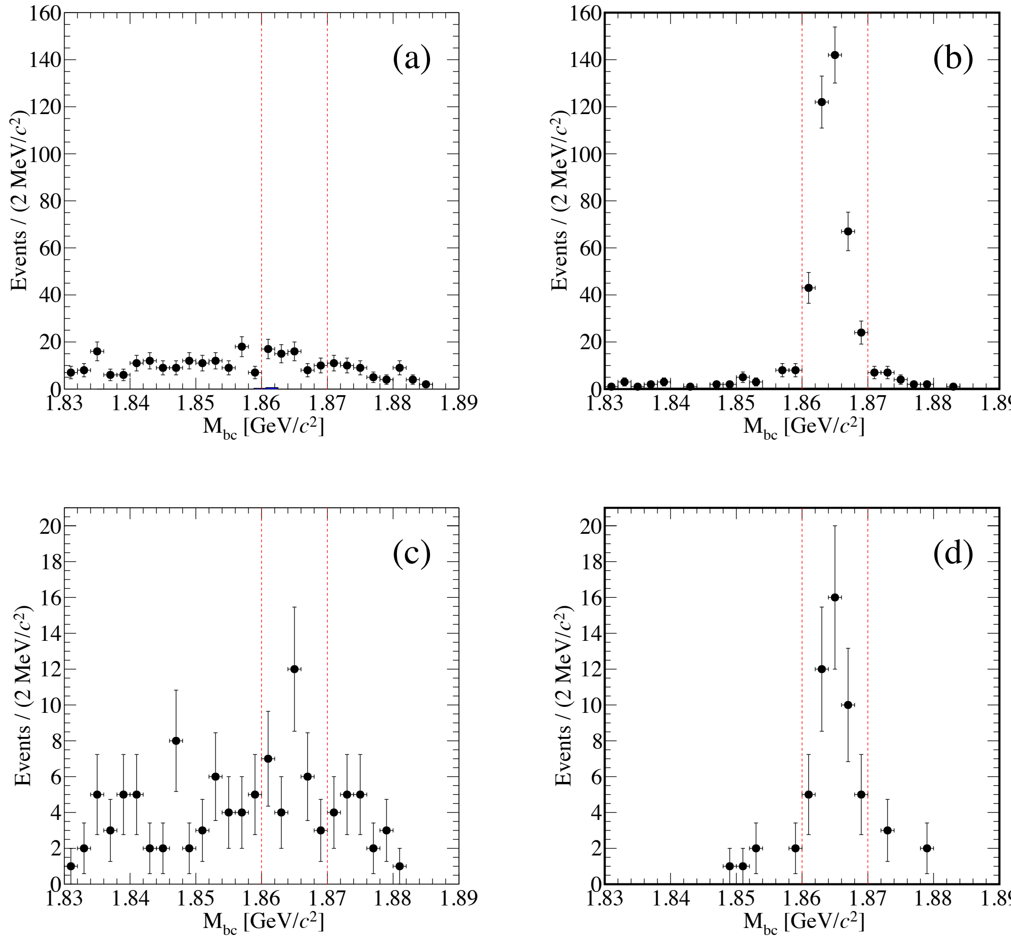
PLB740, 1 (2015) PLB747, 9 (2015)

CP-even fractions, measured with CLEO-c data

tagged as CP odd tagged as CP even

$D^0 \to \pi^+\pi^-\pi^0$

$D^0 \to K^+K^-\pi^0$



(does not include all tags)

PLB740, 1 (2015) PLB747, 9 (2015)

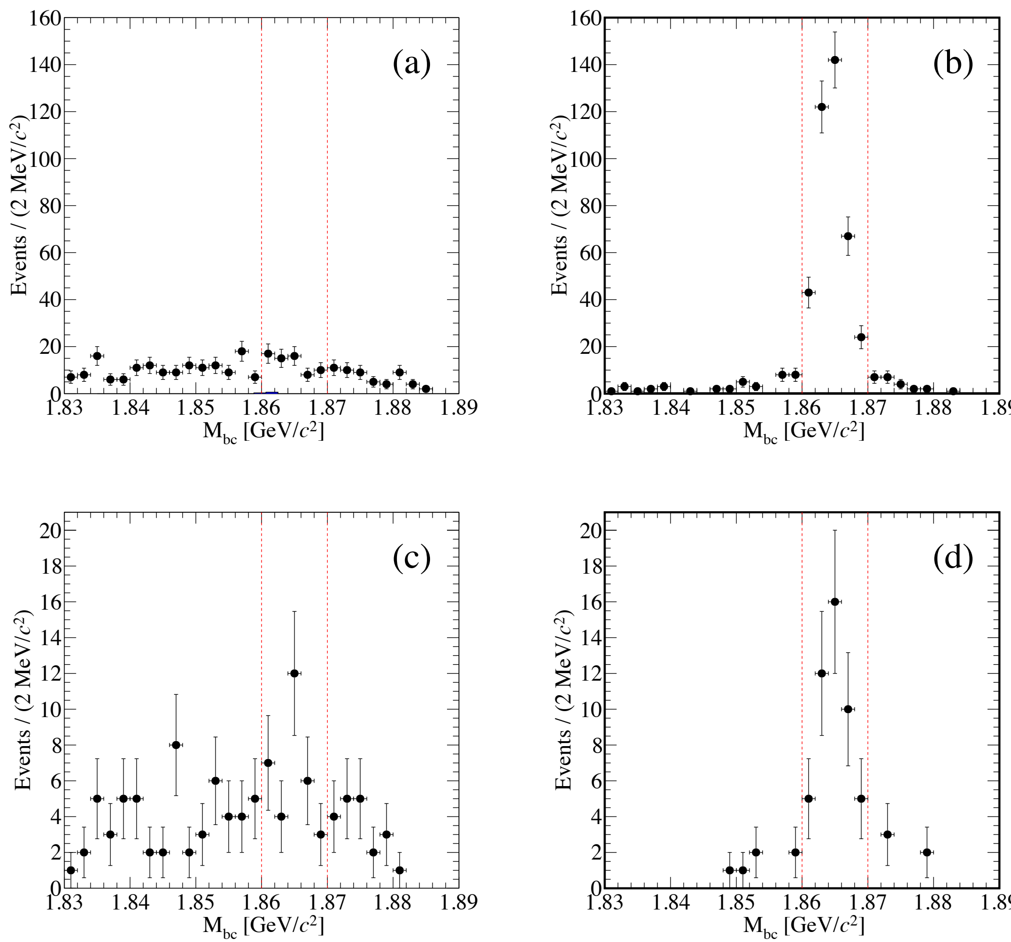
CP-even fractions, measured with CLEO-c data

tagged as CP odd | tagged as CP even

$D^0 \to \pi^+\pi^-\pi^0$

$F_+ = 0.973 \pm 0.017$

$D^0 \to K^+K^-\pi^0$



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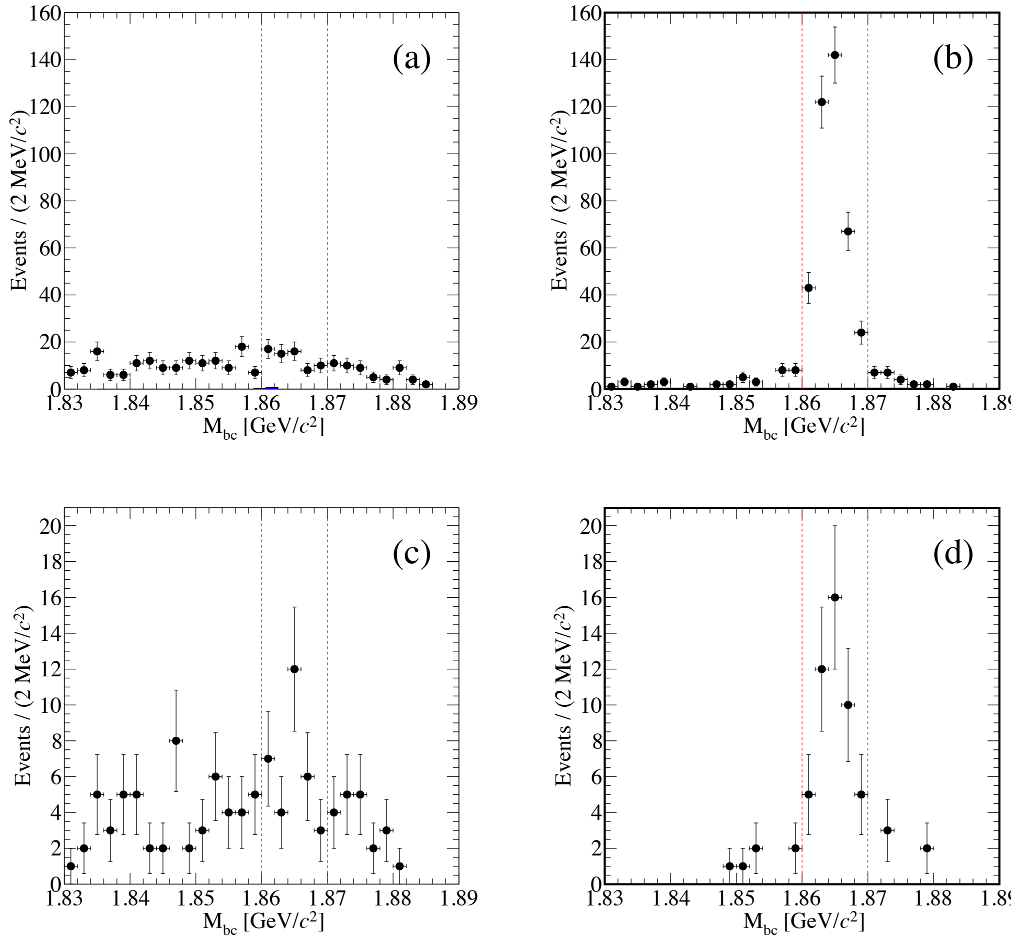

$D^0 \to \pi^+\pi^-\pi^0$ — $F_+ = 0.973 \pm 0.017$

$D^0 \to K^+K^-\pi^0$ — $F_+ = 0.732 \pm 0.055$

(does not include all tags)

PLB740, 1 (2015) PLB747, 9 (2015)

CP-even fractions, measured with CLEO-c data

tagged as CP odd | tagged as CP even

$D^0 \to \pi^+\pi^-\pi^0$

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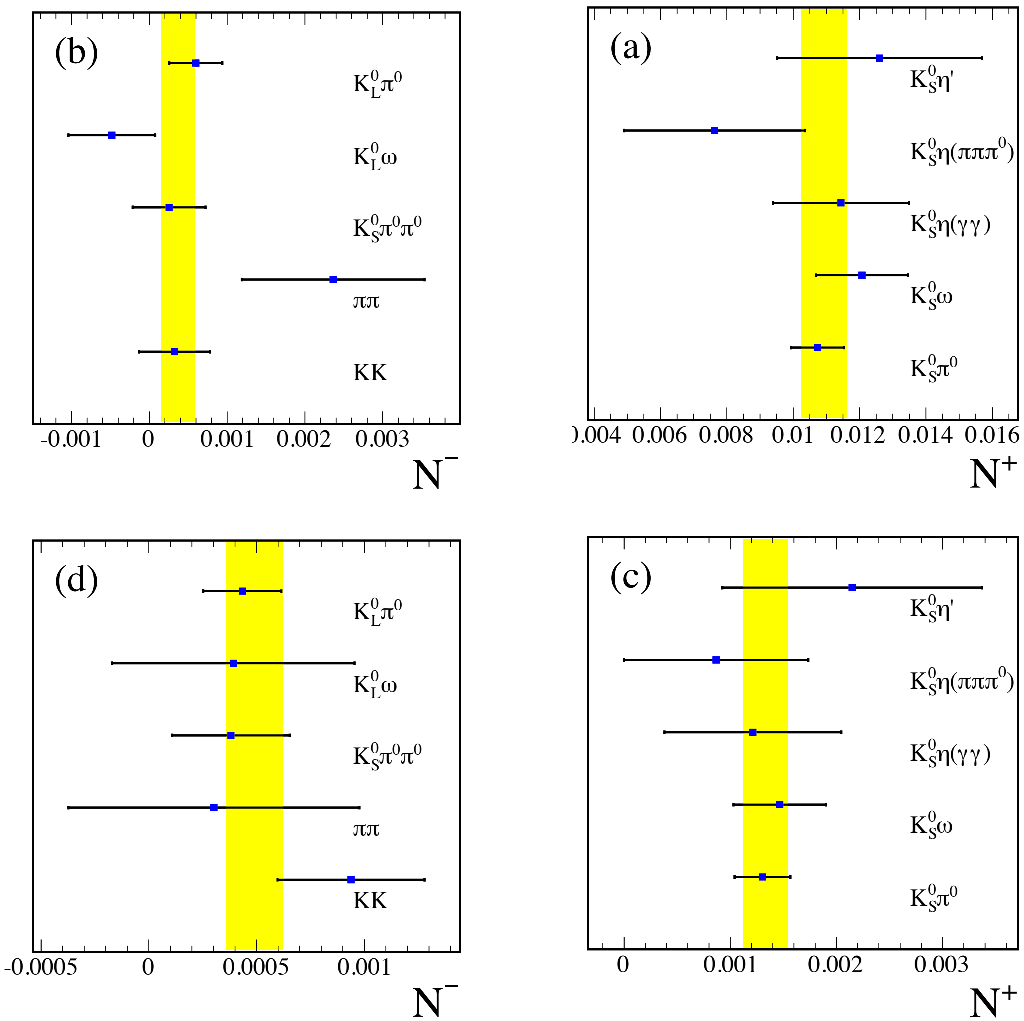

PLB740, 1 (2015) PLB747, 9 (2015)

CP-even fractions, measured with CLEO-c data

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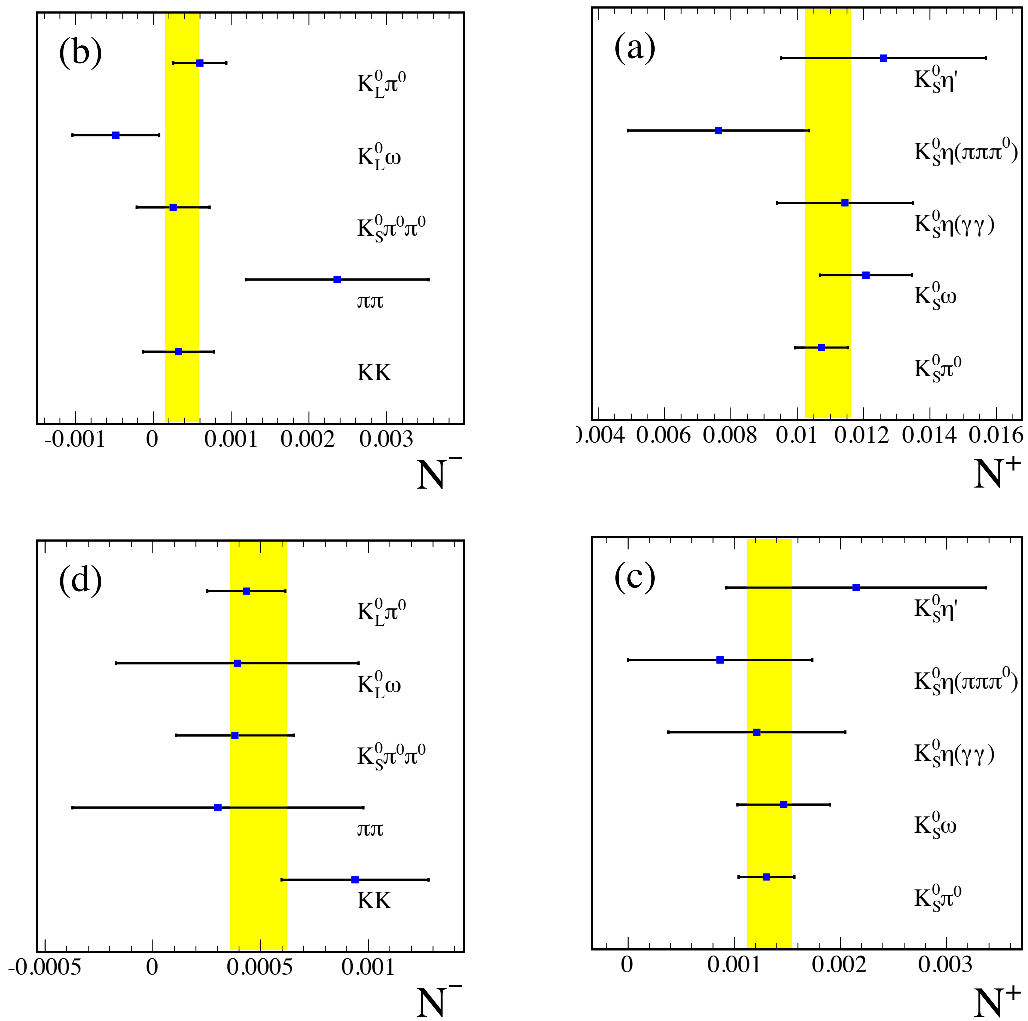


$F_+ = 0.973 \pm 0.017$

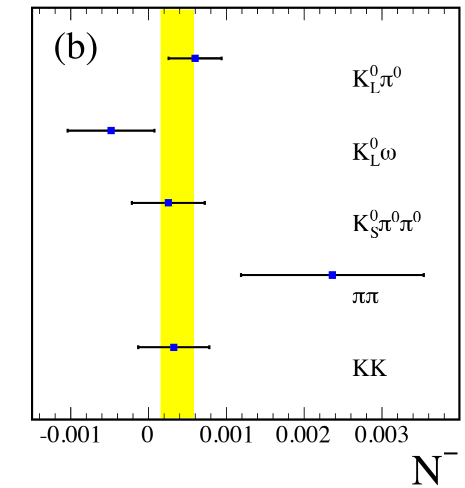
PLB740, 1 (2015) PLB747, 9 (2015)

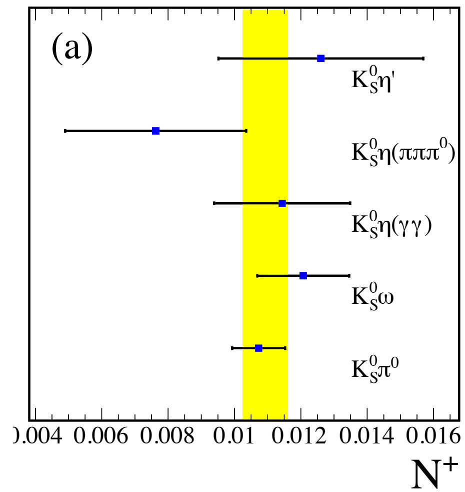
CP-even fractions, measured with CLEO-c data

tagged as CP odd

tagged as CP even

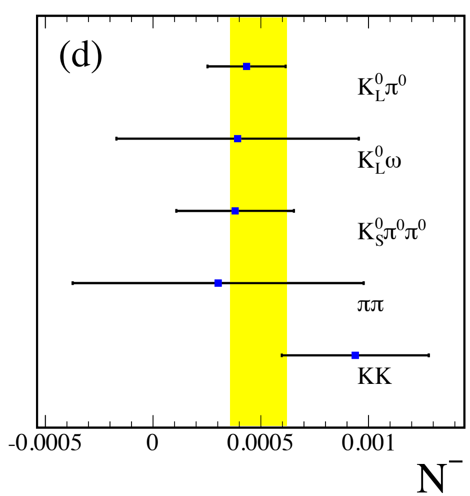
$D^{o} \rightarrow \pi^{+}\pi^{-}\pi^{o}$

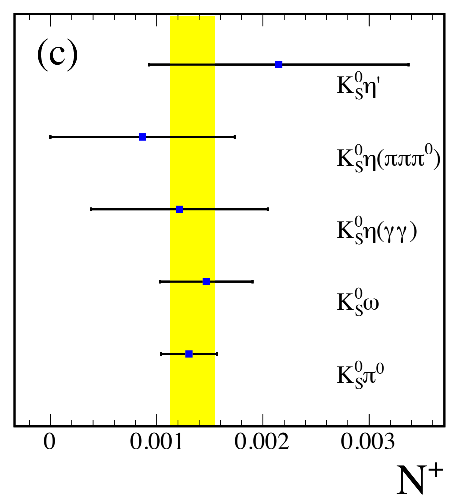




$F_+ = 0.973 \pm 0.017$

$D^{o} \rightarrow K^{+}K^{-}\pi^{o}$





$F_+ = 0.732 \pm 0.055$

PLB740, 1 (2015) PLB747, 9 (2015)

CP-even fractions, measured with CLEO-c data

- Self-tags (i.e. $D^0 \to \pi^+\pi^-\pi^0$ vs $D^0 \to \pi^+\pi^-\pi^0$) can also be used (rate $\propto 1 - (F_+ - 1)^2$)

- Mixed CP tags work as long as CP content is known. It is for example for the Dalitz bins of $D \to K_S\pi\pi$ and $D \to K_L\pi\pi$ from the ci, si measurements discussed later. (rate $\propto 1 - (F_+^{\mathrm{signal}} - 1)(F_+^{\mathrm{tag}} - 1)$)

	$\pi^+\pi^-\pi^+\pi^-$
K^+K^-	19.3 ± 6.3
$\pi^+\pi^-$	3.3 ± 8.2
$K^0_S\pi^0\pi^0$	18.6 ± 5.2
$K^0_L\pi^0$	49.2 ± 10.9
$K^0_L\omega$	22.0 ± 6.5
$K^0_S\pi^0$	112.8 ± 11.0
$K^0_S\omega$	41.0 ± 6.8
$K^0_S\eta(\gamma\gamma)$	18.8 ± 4.5
$K^0_S\eta(\pi^+\pi^-\pi^0)$	3.1 ± 2.7
$K^0_S\eta'$	9.3 ± 3.3
$K^0_S\pi^+\pi^-$	217.9 ± 16.8
$K^0_L\pi^+\pi^-$	485.0 ± 26.3
$\pi^+\pi^-\pi^+\pi^-$	41.0 ± 16.3

PLB747, 9 (2015)

CP-even fractions, measured with CLEO-c data

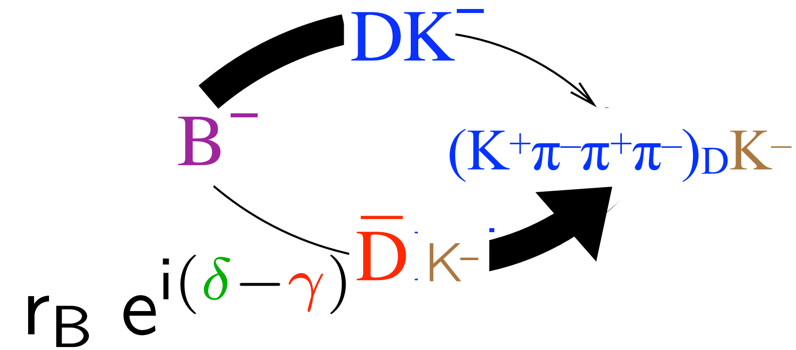
$$F_+ = \frac{N_{\mathrm{CP-even}}}{N_{\mathrm{CP-even}} + N_{\mathrm{CP-odd}}}$$

$$\Gamma(B^\pm \to D(f_{\mathrm{self-con}})K^\pm) \propto 1 + r_B^2 + 2(2\boxed{F_+} - 1)\cos(\delta_B \pm \gamma)$$

(plus with some corrections for charm mixing)

$F_+ (D^0 \to \pi^+\pi^-\pi^0)$	$= 0.973 \pm 0.017$	PLB740, 1 (2015) PLB747, 9 (2015)
$F_+ (D^0 \to K^+K^-\pi^0)$	$= 0.732 \pm 0.055$	
$F_+ (D^0 \to \pi^+\pi^-\pi^+\pi^-)$	$= 0.769 \pm 0.022$	JHEP 1801 (2018) 144
$F_+ (D^0 \to K_S\pi^-\pi^+\pi^0)$	$= 0.238 \pm 0.018$	JHEP 1801 (2018) 082

Coherence Factor Analysis of



- Treat K3π like two-body decay with single effective strong phase δ_D.

$$\Gamma\left(\mathrm{B}^- \to \left(\mathrm{K}^+ 3\pi\right)_{\mathrm{D}} \mathrm{K}^-\right) \propto r_B^2 + \left(r_D^{K3\pi}\right)^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos\left(\delta_B + \delta_D^{K3\pi} - \gamma\right)$$

$$r_B = \left|\frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)}\right| \quad r_D = \left|\frac{A(D^0 \to K^+\pi^-\pi^+\pi^-)}{A(\overline{D}^0 \to K^+\pi^-\pi^+\pi^-)}\right|$$

- $$Re^{-i\delta_D} = \frac{1}{\mathcal{A}_\Omega \mathcal{B}_\Omega} \int_\Omega \langle f_{\mathbf{p}}|\hat{H}|D^0\rangle \langle f_{\mathbf{p}}|\hat{H}|\overline{D}^0\rangle^* \left|\frac{\partial^n \phi}{\partial(p_1 \ldots p_n)}\right| \mathrm{d}^n p.$$

$$\mathcal{A}_\Omega \equiv \sqrt{\int_\Omega |\langle f_{\mathbf{p}}|\hat{H}|D^0\rangle|^2 \left|\frac{\partial^n \phi}{\partial(p_1 \ldots p_n)}\right| \mathrm{d}^n p}, \quad \mathcal{B}_\Omega \equiv \sqrt{\int_\Omega |\langle f_{\mathbf{p}}|\hat{H}|\overline{D}^0\rangle|^2 \left|\frac{\partial^n \phi}{\partial(p_1 \ldots p_n)}\right| \mathrm{d}^n p}$$

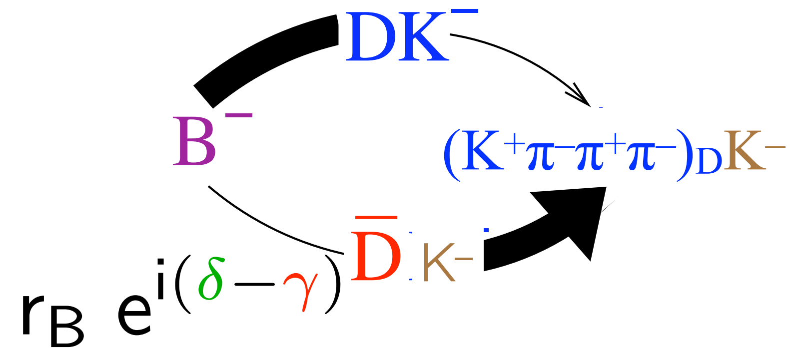
- Coherence factor R < 1;

R_{K3pi} is the coherence factor introduced by Atwood & Soni, PRD68 (2003) 033003
Also interesting in relation to charm mixing, see
- Bondar, Poluektov, Vorobiev: PRD 82 (2010) 034033,
- Malde & Wilkinson PLB701 (2011) 353-356,
- Malde, Thomas & Wilkinson PRD91 (2015) no.9, 094032,
- Harnew & JR: JHEP 1503 (2015) 169, PLB 728 (2014) 296
- LHCb: PRL116 (2016) no.24, 241801
- Evans et al: PLB 757 (2016) 520

CLEO-c input theory: Atwood, Soni: Phys.Rev. D68 (2003) 033003

CLEO-c data: Phys.Rev. D80 (2009) 031105, PLB 731 (2014) 197, Phys.Lett. B757 (2016) 520-527

Coherence Factor Analysis of



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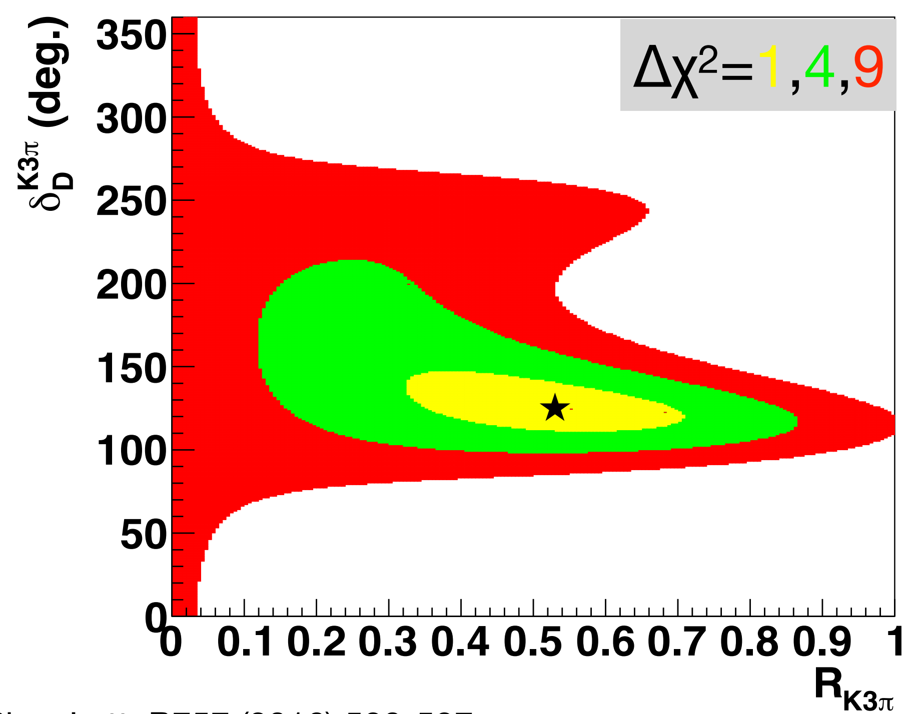
Double Tag Rate	Sensitive To
$K^\pm\pi^\mp\pi^+\pi^-$ vs. $K^\pm\pi^\mp\pi^+\pi^-$	$(R_{K3\pi})^2$
$K^\pm\pi^\mp\pi^+\pi^-$ vs. CP	$R_{K3\pi}\cos(\delta^{K3\pi})$
$K^\pm\pi^\mp\pi^+\pi^-$ vs. $K^\pm\pi^\mp$	$R_{K3\pi}\cos(\delta^{K\pi} - \delta^{K3\pi})$



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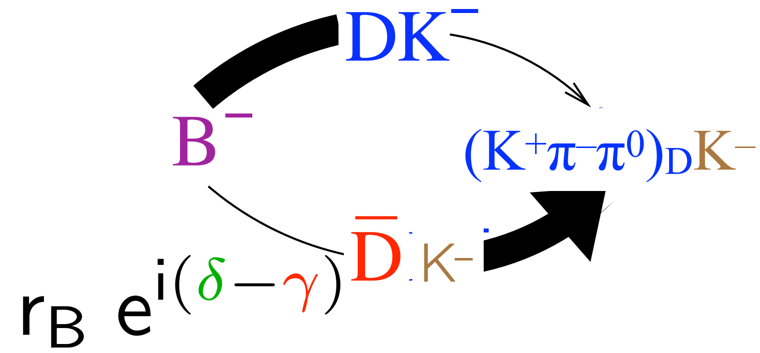

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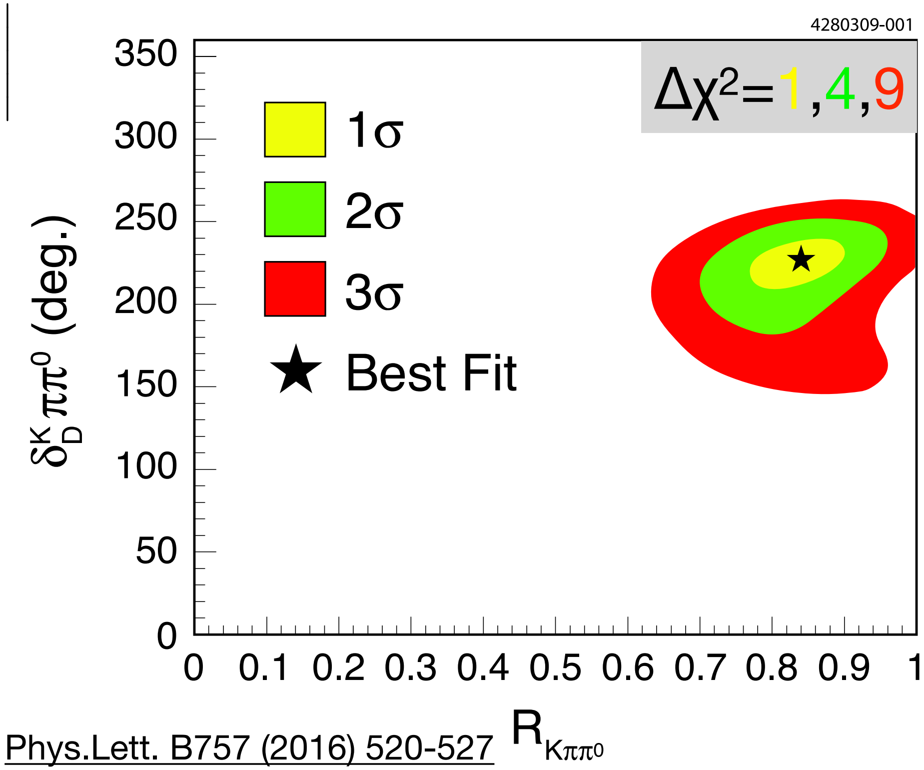

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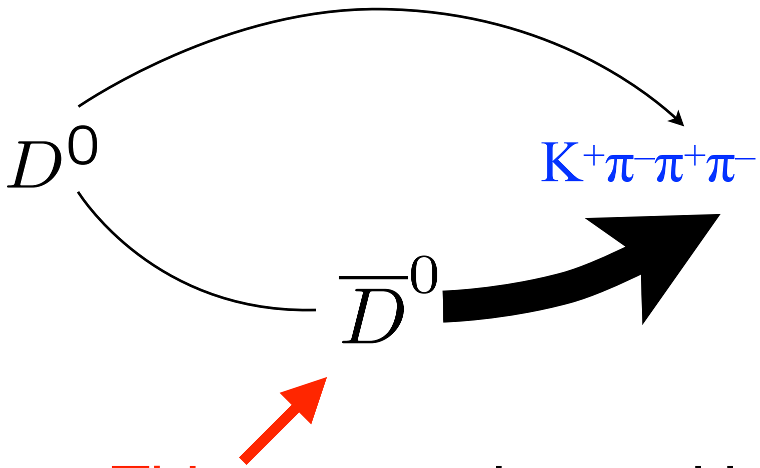
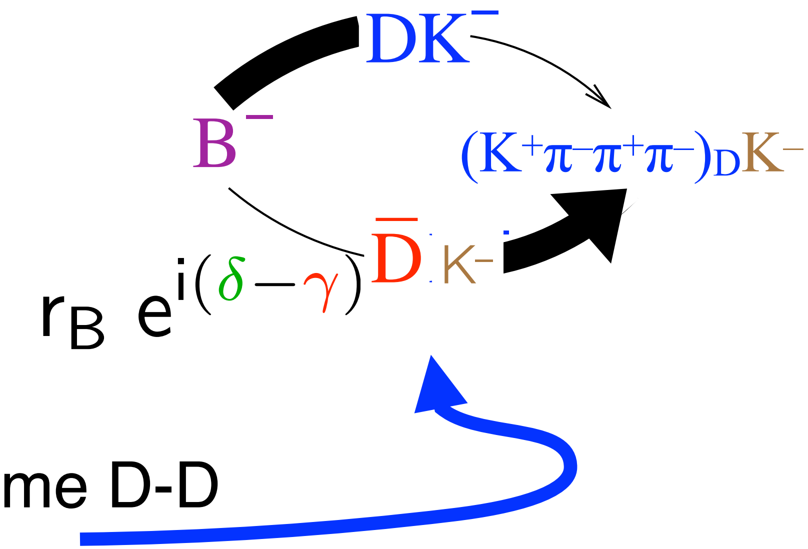
D° Mixing as input to γ from $B^{\pm} \to DK^{\pm}$





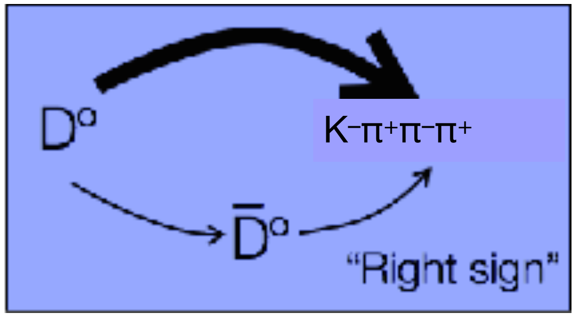
This process is sensitive to the same D-D interference effects that pollute this measurement.

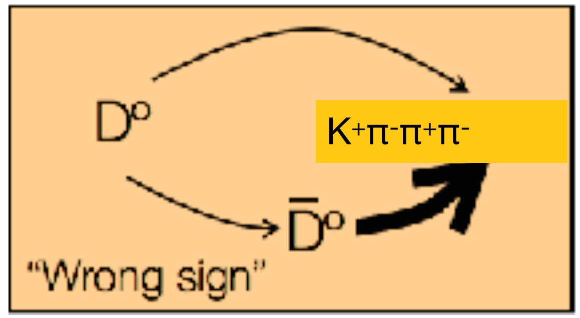
Can use charm mixing to constrain charm amplitude parameters in γ measurements - see previous talk.

PLB728 (2014) 296-302
JHEP 1503 (2015) 169

LHCb PRL 116 (2016) no.24, 241801

Charm mixing in D→Kπππ

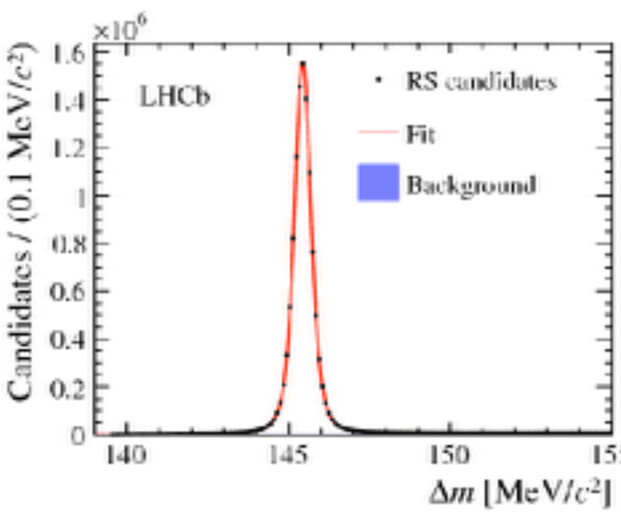


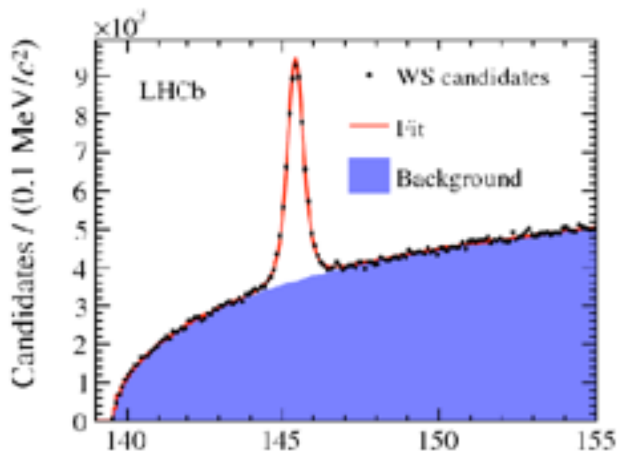


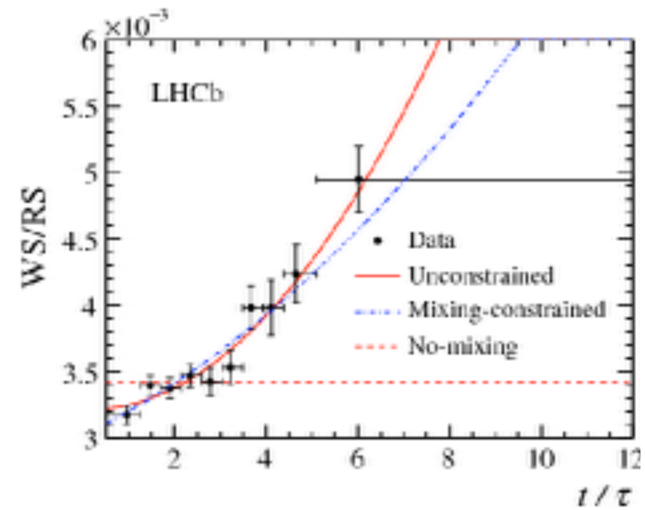
$$\frac{\Gamma(D^0 \to K^-\pi^+\pi^-\pi^+)}{\Gamma(D^0 \to K^+\pi^-\pi^+\pi^-)}$$

11M RS events

43k WS events



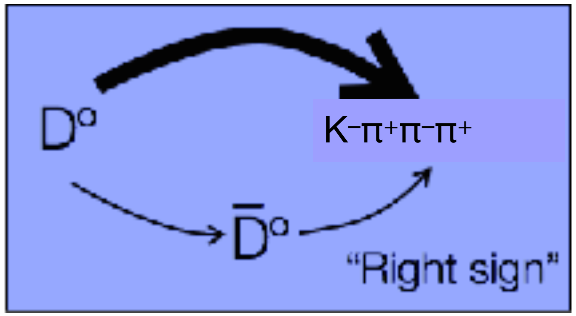




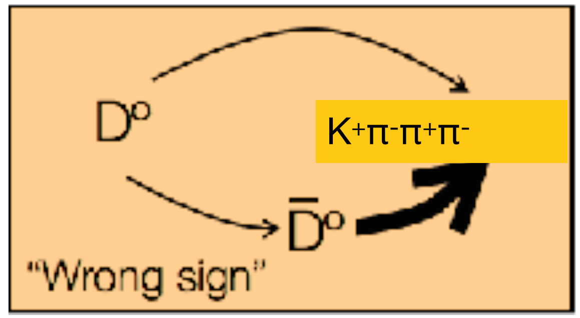
First >5σ observation of charm mixing in a single channel other than D→Kπ

LHCb PRL 116 (2016) no.24, 241801

Charm mixing in D→Kπππ

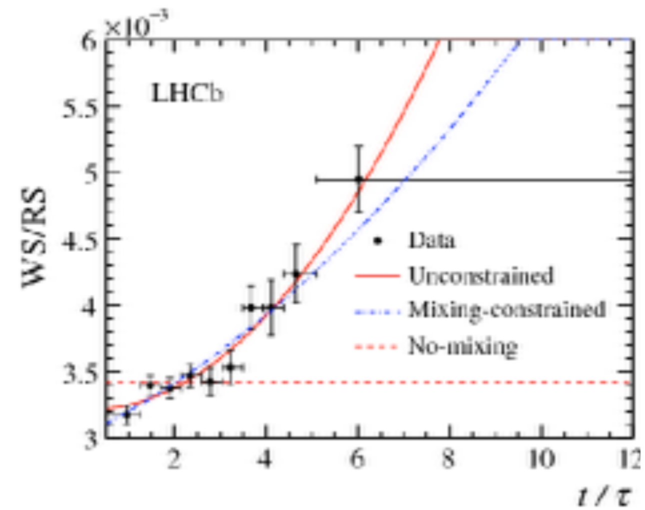


11M RS events



43k WS events

$$\frac{\Gamma(D^0 \to K^-\pi^+\pi^-\pi^+)}{\Gamma(D^0 \to K^+\pi^-\pi^+\pi^-)}$$



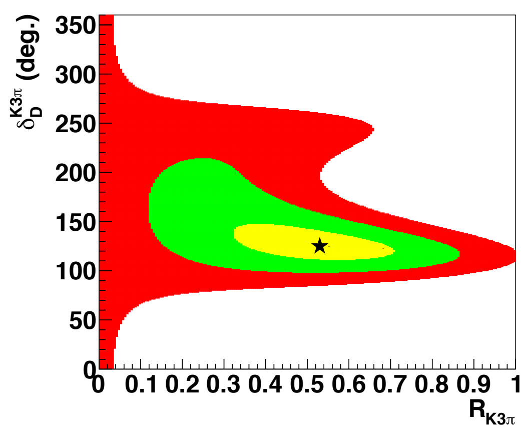
First >5σ observation of charm mixing in a single channel other than D→Kπ

Charm input to γ from CLEO-c and LHCb mixing measurements

Use interference effects in charm as input to γ

$$\Gamma\left(B^- \to \left(K^+ 3\pi\right)_D K^-\right) \propto r_B^2 + \left(r_D^{K3\pi}\right)^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos\left(\delta_B + \delta_D^{K3\pi} - \gamma\right)$$

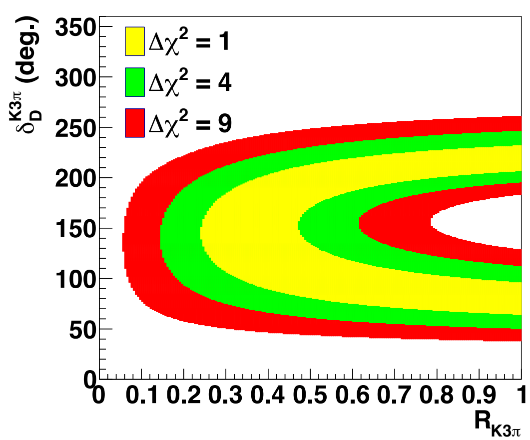
from D-$\bar{\text{D}}$ superpositions at CLEO-c



Phys.Lett. B757 (2016) 520-527

+

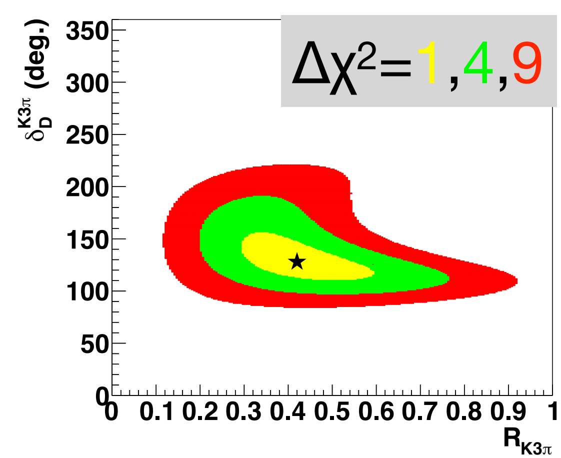
Input from charm mixing (LHCb)



PRL 116 (2016) no.24, 241801

=

Combination:CLEO-c and mixing.



Phys.Lett. B757 (2016) 520-527

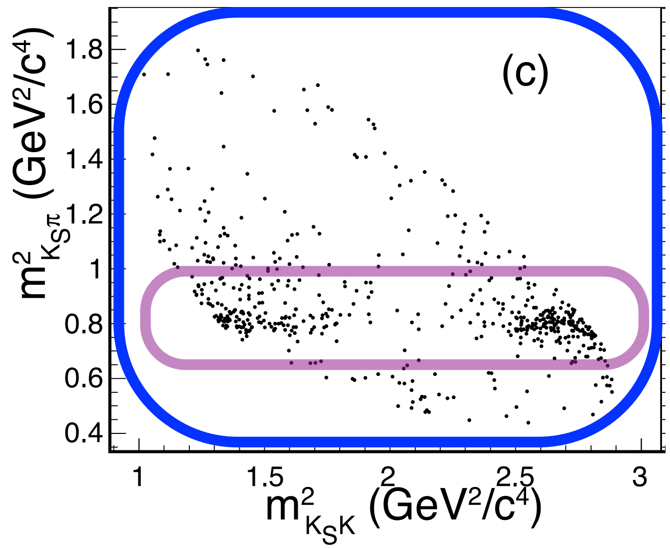
CLEO-c input theory: Atwood, Soni: Phys.Rev. D68 (2003) 033003
CLEO-c input: Phys.Rev.D80:031105,2009, update
mixing/gamma theory: JHEP 1503 (2015) 169 Phys.Lett. B728 (2014) 296-302

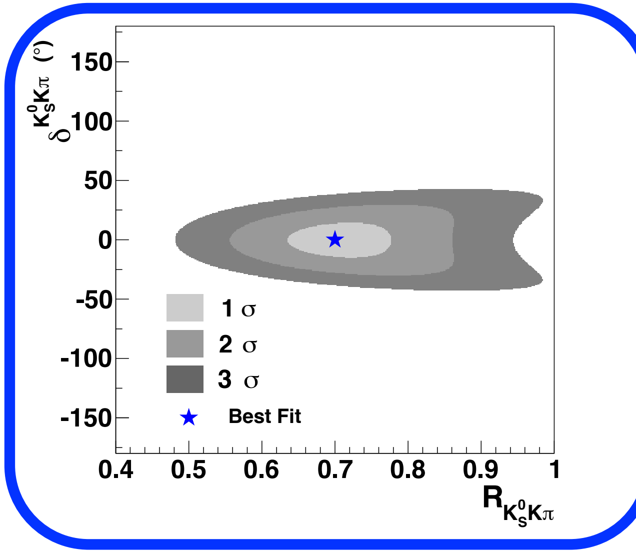
Input from the charm threshold for $D^o \to K_S K \pi$

CLEO-c Phys.Rev. D85 092016 (2012)

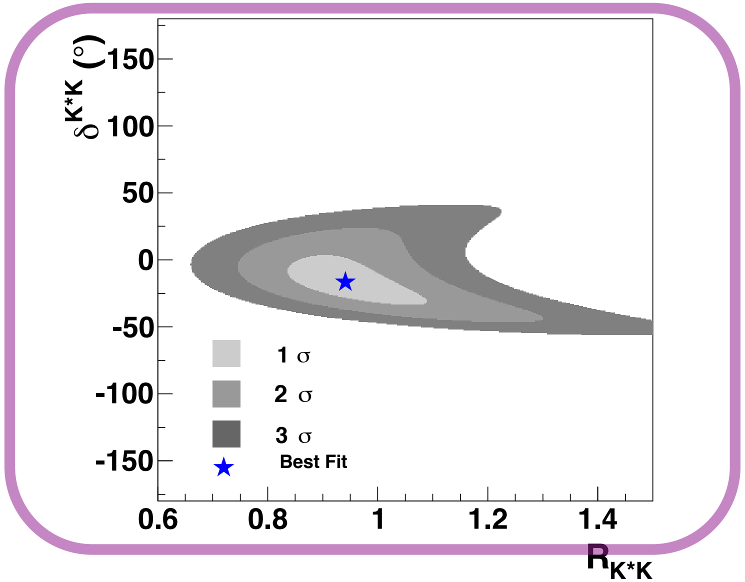
- Similar input as for $K_S\pi\pi$, K_SKK, different nomenclature:

- Typically measured in one single bin across Dalitz space, but analyses with several bins (where statistics allow) could increase sensitivity.





Dalitz plot shows CF decay for illustration. Sensitivity to mixing comes from the DCS decay, and CLEO-c's sensitivity to R_D, δ_D from well-defined superpositions of D^o and D^obar accessible at the charm threshold

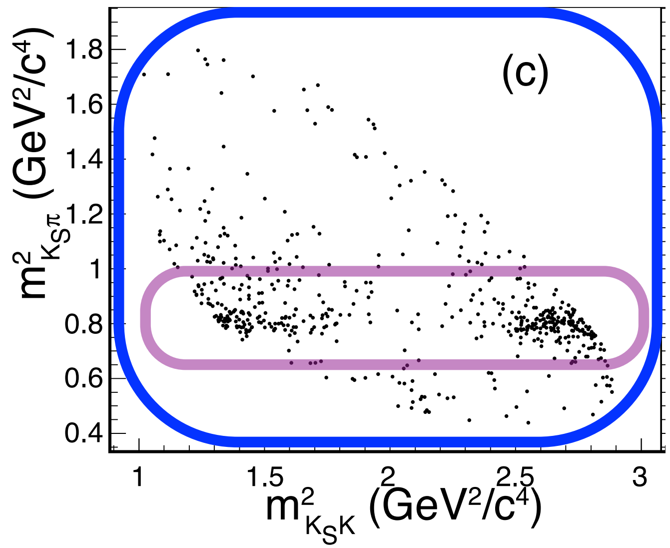


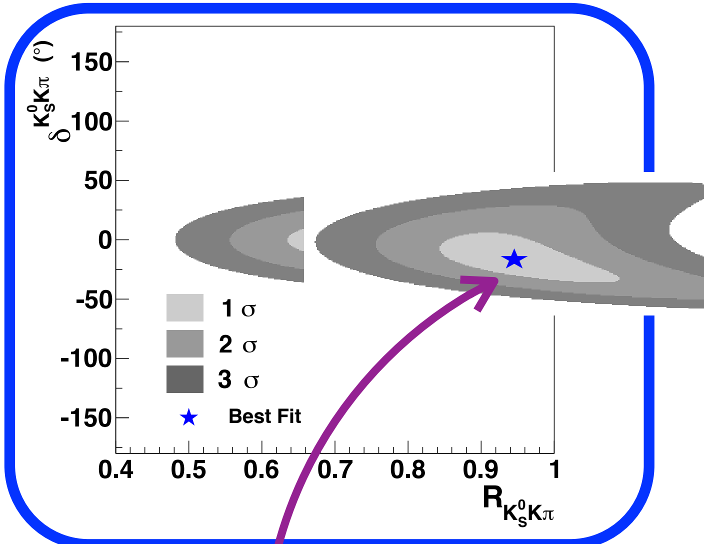
Input from the charm threshold for $D^o \to K_S K\pi$

CLEO-c Phys.Rev. D85 092016 (2012)

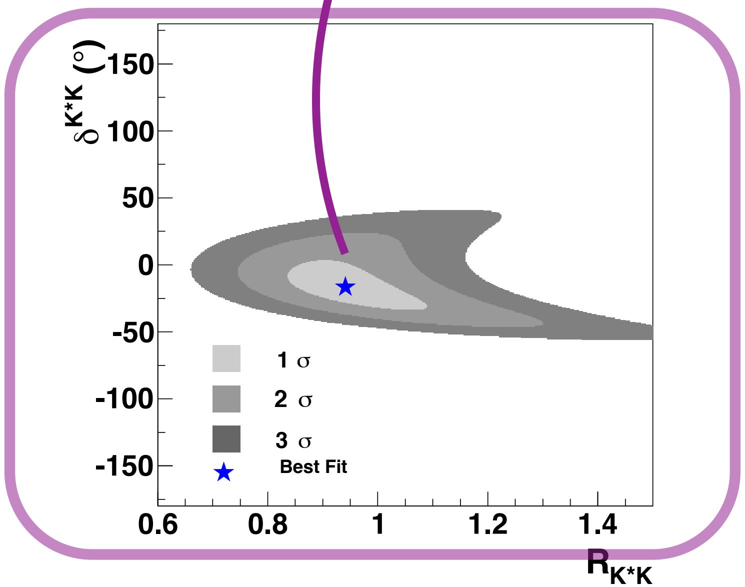
- Similar input as for $K_S\pi\pi$, K_SKK, different nomenclature:

- Typically measured in one single bin across Dalitz space, but analyses with several bins (where statistics allow) could increase sensitivity.

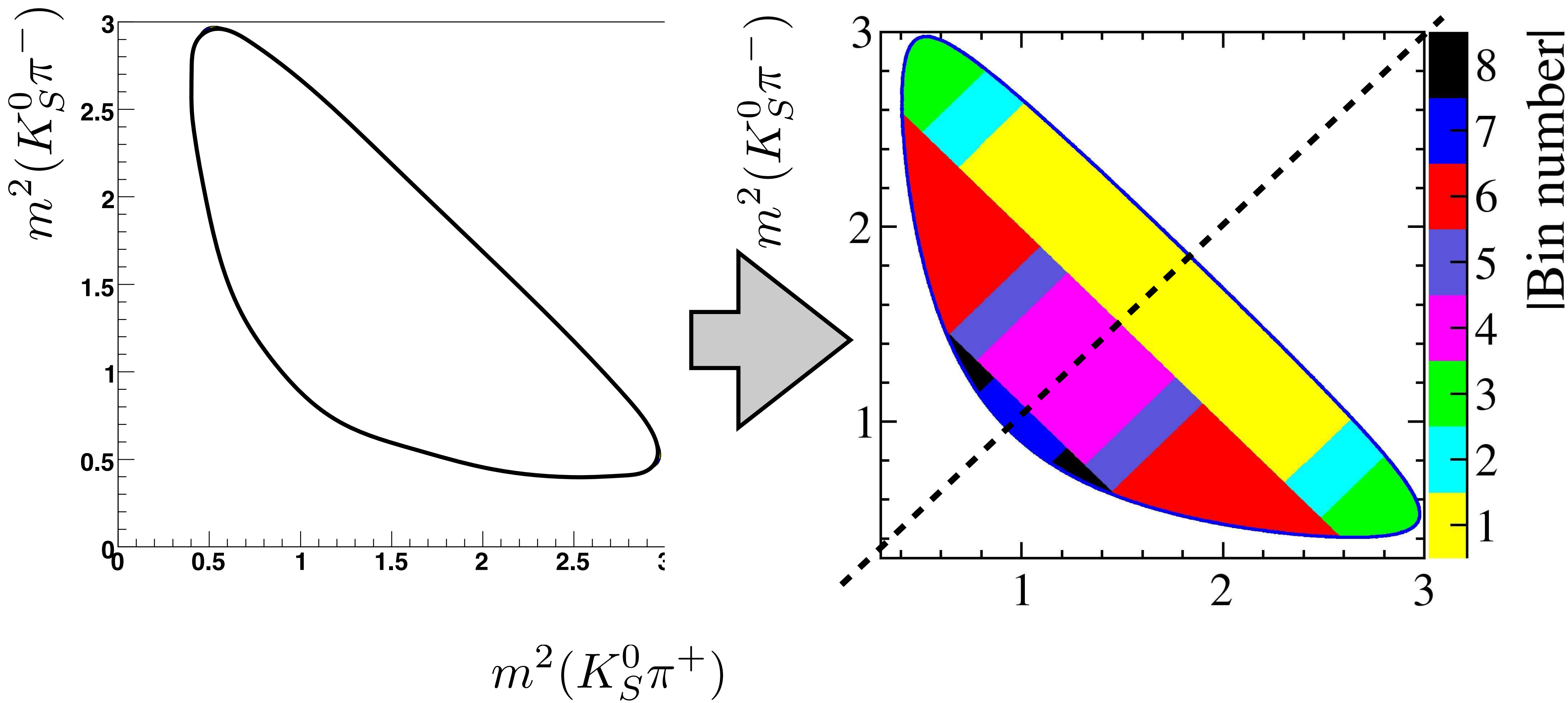




Dalitz plot shows CF decay for illustration. Sensitivity to mixing comes from the DCS decay, and CLEO-c's sensitivity to R_D, δ_D from well-defined superpositions of D^o and D^obar accessible at the charm threshold



Winning by binning



Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Model independent, binned γ fit

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binned decay rate:

$$\Gamma\left(B^{\pm} \to D(K_s\pi^+\pi^-)K^{\pm}\right)_i =$$

$$\mathcal{T}_i + r_B^2\mathcal{T}_{-i} + 2r_B\sqrt{\mathcal{T}_i\mathcal{T}_{-i}}\left\{c_i \cos\left(\delta \pm \gamma\right) + s_i \sin\left(\delta \pm \gamma\right)\right\}$$

$\mathcal{T}_i$ known from flavour-specifc D decays

(weighted) average of $\cos(\delta_D)$ and $\sin(\delta_D)$ over bin i, where δ_D = phase difference between D→Ksππ and Dbar→Ksππ

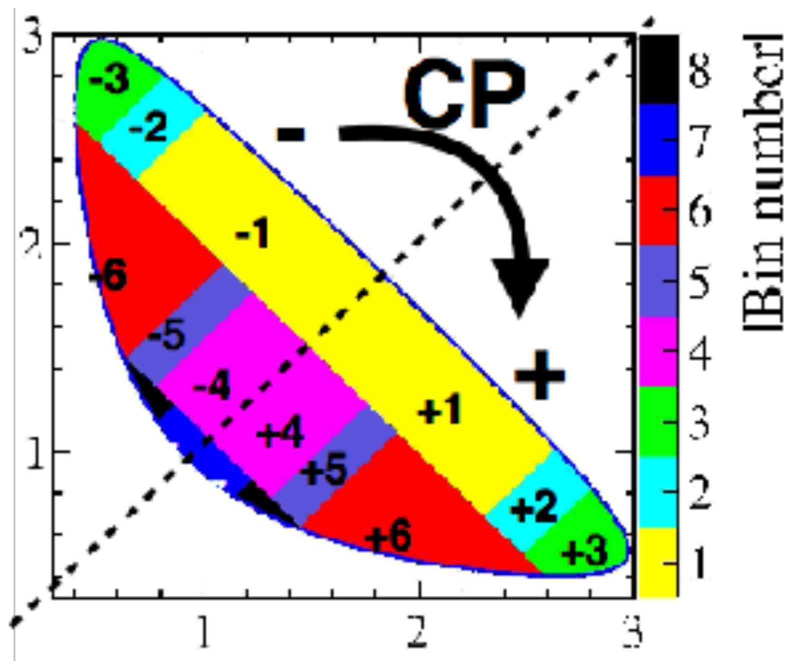
- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$

- Note: $$F_+ = \frac{1}{2}\left(1 + 2c_1\sqrt{\mathcal{T}_1\mathcal{T}_{-1}}\right)$$

$$Re^{-i\delta_D} = c_i + is_i$$

- Distribution sensitive to c_i, s_i, r_B, δ and γ.

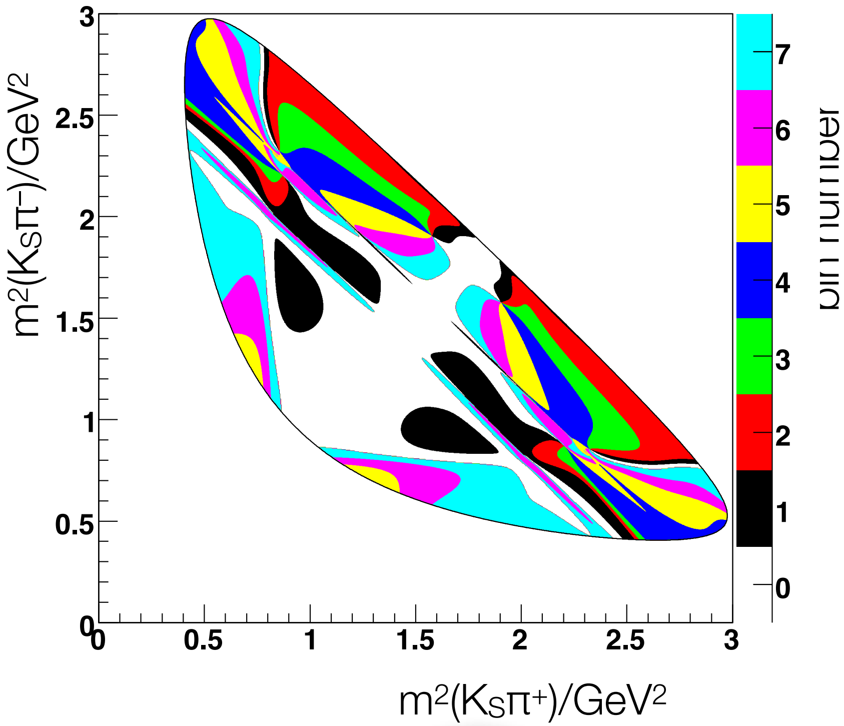
- c_i, s_i, measured at charm threshold.



Optimal binning

- Sensitivity proportional to ~ $\sqrt{c_i^2 + s_i^2}$
- Best γ sensitivity if phase difference δ_D is as constant as possible over each bin[1].
- Plot shows CLEO-c's 8 bins, uniform in δ_D, (based on BaBar isobar model*).
- Choice of model will not bias result, but a bad model would reduce the statistical precision.

Binning at CLEO-c based on BaBar model*



[1] Bondar, Poluektov hep-ph/0703267v1 (2007)

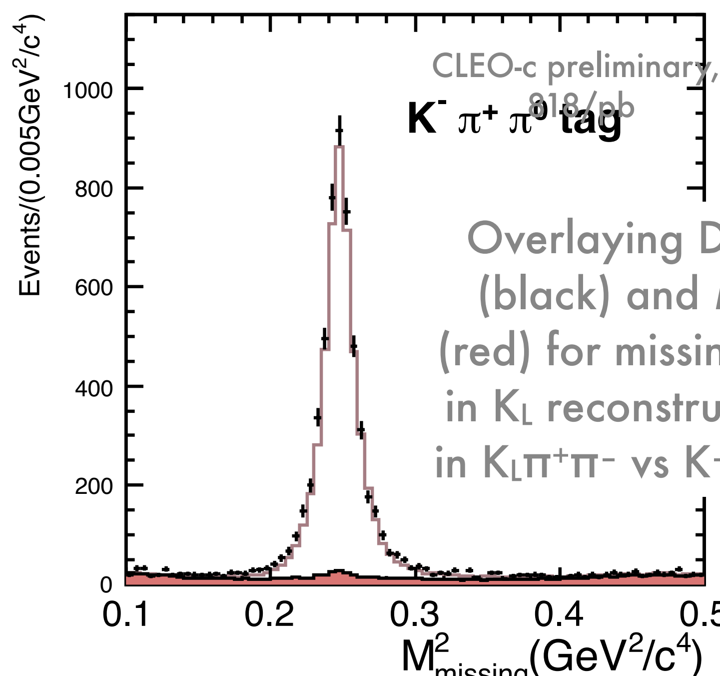
*model = BaBar PRL 95 (2005) 121802

CP-even $K_L\pi\pi$ ≈ CP-odd $K_S\pi\pi$

$$K_S^0 = (K^0 + \bar{K}^0)/\sqrt{2}$$
$$K_L^0 = (K^0 - \bar{K}^0)/\sqrt{2}.$$

$$-\mathrm{A}\left(\mathrm{D}^0 \to \mathrm{K}_\mathrm{L}^0\pi^+\pi^-\right) = \underbrace{\mathrm{A}\left(\mathrm{D}^0 \to \mathrm{K}_\mathrm{S}^0\pi^+\pi^-\right)}_{\text{CF+DCS}} \underbrace{-\sqrt{2}\mathrm{A}\left(\mathrm{D}^0 \to \mathrm{K}_\text{flavour}^0\pi^+\pi^-\right)}_{\text{DCS}}$$

- Charm threshold's clean environment allows the reconstruction of K_L from kinematic constraints.
- More than doubles statistics.
- There is price to pay: A $O(\tan^2\theta_C)$ model-dependent correction. Carefully evaluated (small) systematic uncertainty.



Overlaying Data (black) and MC (red) for missing M^2 in K_L reconstruction in $K_L\pi^+\pi^-$ vs $K^-\pi^+\pi^0$

arXiv:0903.1681v1 [hep-ex]

Table of tags

Mode	DT	
	$K_S^0\pi^+\pi^-$	$K_L^0\pi^+\pi^-$
Flavor tags		
$K^-\pi^+$	1444	2857
$K^-\pi^+\pi^0$	2759	5133
$K^-\pi^+\pi^+\pi^-$	2240	4100
$K^-e^+\nu$	1191	
CP-even tags		
K^+K^-	124	357
$\pi^+\pi^-$	61	184
$K_S^0\pi^0\pi^0$	56	
$K_L^0\pi^0$	237	
$K_L^0\eta(\gamma\gamma)$		
$K_L^0\eta(\pi^+\pi^-\pi^0)$		
$K_L^0\omega$		
$K_L^0\eta'$		
CP-odd tags		
$K_S^0\pi^0$	189	288
$K_S^0\eta(\gamma\gamma)$	39	43
$K_S^0\eta(\pi^+\pi^-\pi^0)$		
$K_S^0\omega$	83	
$K_S^0\eta'$		
$K_L^0\pi^0\pi^0$		
$K_S^0\pi^+\pi^-$	473	1201
$K_L^0\pi^+\pi^-$		
$K_S^0K^+K^-$		

(S/B between 10 and 100, depending on tag mode).

Table of tags

Mode		DT
	$K^0_S\pi^+\pi^-$	$K^0_L\pi^+\pi^-$
Flavor tags		
$K^-\pi^+$	1444	2857
$K^-\pi^+\pi^0$	2759	5133
$K^-\pi^+\pi^+\pi^-$	2240	4100
$K^-e^+\nu$	1191	
CP-even tags		
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$K^0_S\omega$	83	
$K^0_S\eta'$		
$K^0_L\pi^0\pi^0$		
$K^0_S\pi^+\pi^-$	473	1201
$K^0_L\pi^+\pi^-$		
$K^0_S K^+K^-$		

CP tags:

$$2|A_{CP}|^2 = |A \pm \bar{A}|^2 = |A|^2 + |\bar{A}|^2 \pm |A||\bar{A}|\cos\delta$$

sensitive to c_i

(S/B between 10 and 100, depending on tag mode).

Table of tags

Mode	DT $K^0_S\pi^+\pi^-$	$K^0_L\pi^+\pi^-$
Flavor tags		
$K^-\pi^+$	1444	2857
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$K^0_L\omega$		
$K^0_L\eta'$		
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$K^0_S\pi^0$	189	288
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$K^0_S\omega$	83	
$K^0_S\eta'$		
$K^0_L\pi^0\pi^0$		
$K^0_S\pi^+\pi^-$	473	1201
$K^0_L\pi^+\pi^-$		
$K^0_S K^+K^-$		

CP tags:

$$2|A_{CP}|^2 = |A \pm \bar{A}|^2 = |A|^2 + |\bar{A}|^2 \pm |A||\bar{A}|\cos\delta$$

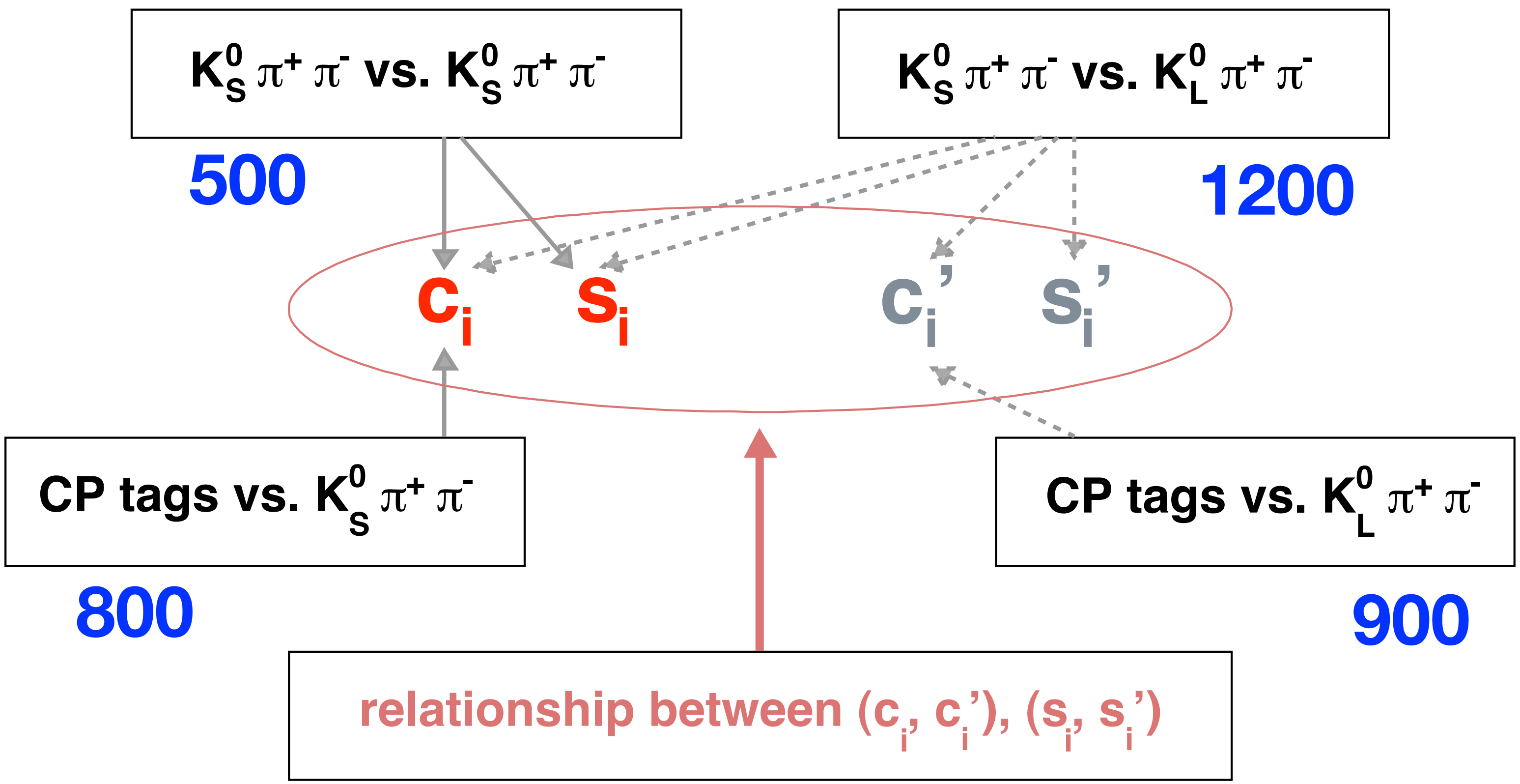
sensitive to c_i

(S/B between 10 and 100, depending on tag mode).

$K_S\pi\pi$ vs $K_{S,L}\pi\pi$ also sensitive to s_i.

	$K^0_S\pi^+\pi^-$	$K^0_L\pi^+\pi^-$
$K^0_S\pi^+\pi^-$	473	1201

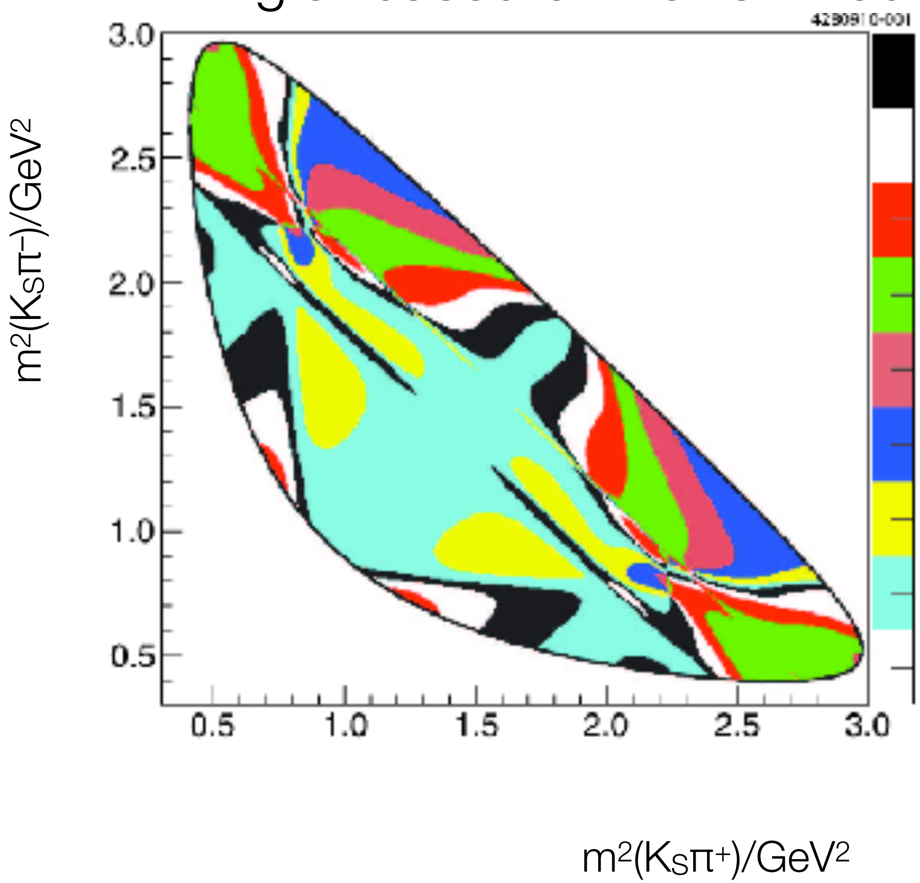
Ingredients of combined fit

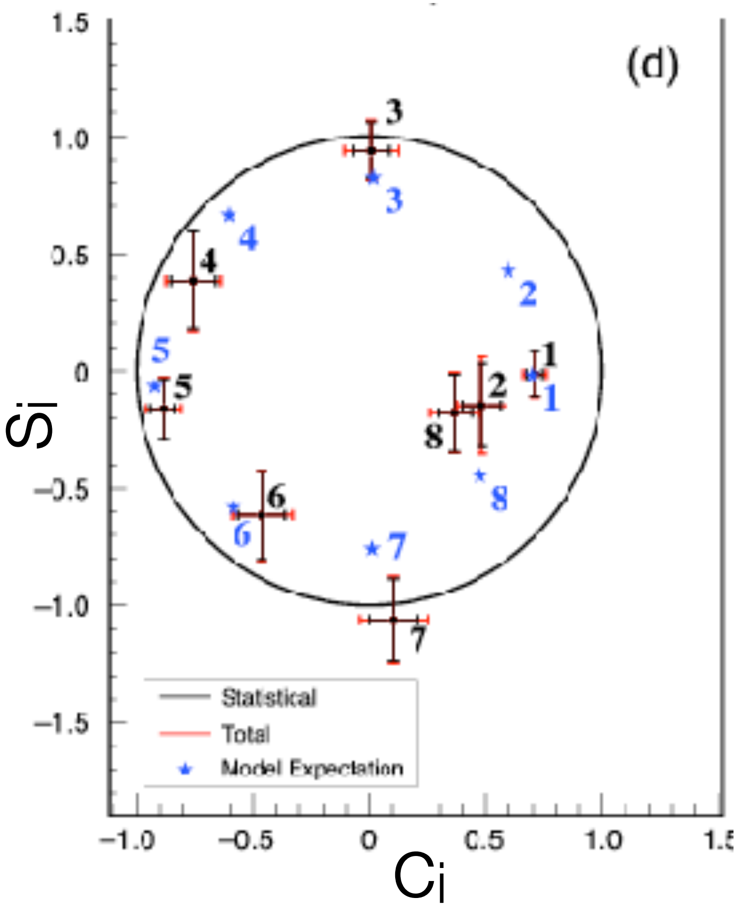


Notation: c_i, s_i from $K_S\pi\pi$. c_i', s_i' from $K_L\pi\pi$. $\Delta c_i \equiv c_i - c_i'$, $\Delta s_i \equiv s_i - s_i'$

$K_S\pi\pi$ ci, si at CLEO-c

Binning at based on BaBar model*





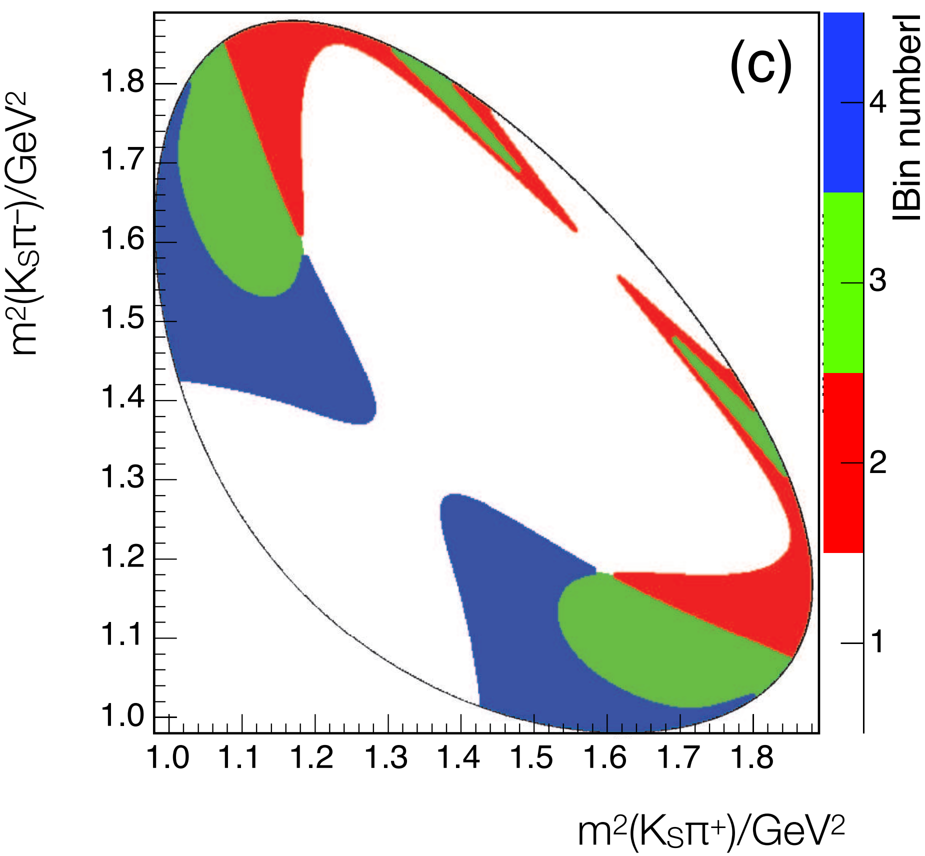
Phys.Rev. D82 (2010) 112006

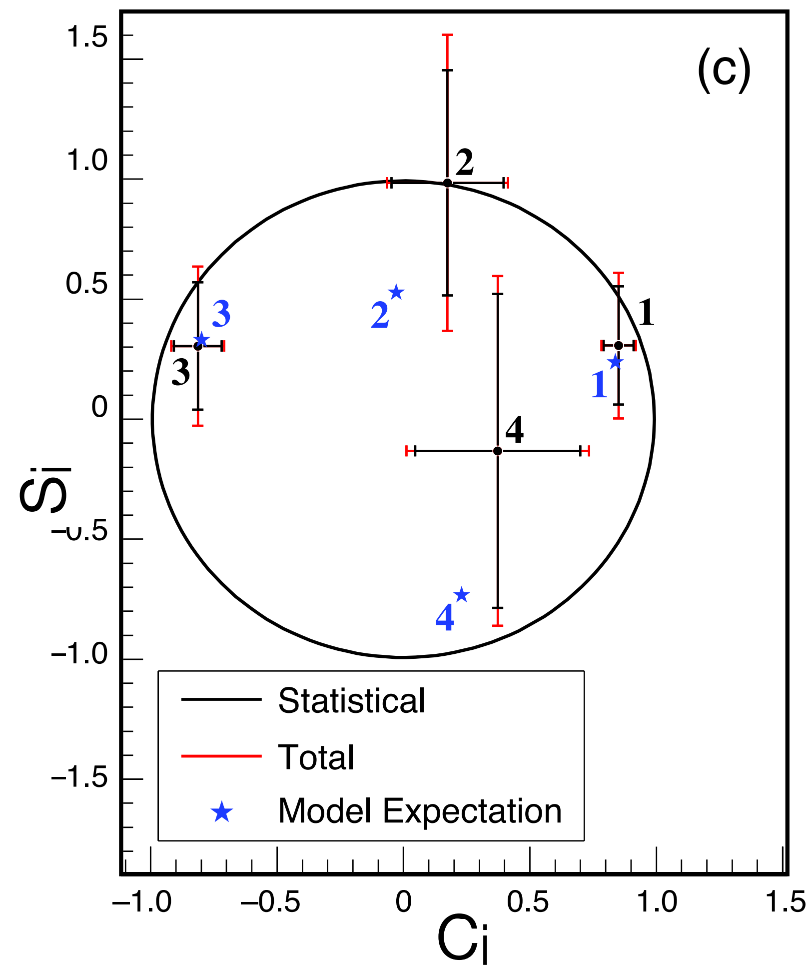
*model = BaBar PRL 95 (2005) 121802

[1] Bondar, Poluektov hep-ph/0703267v1 (2007)

$K_S KK$ ci, si at CLEO-c

Binning at based on BaBar model*





*model = PRL 105, 121801 (2010)

[1] Bondar, Poluektov hep-ph/0703267v1 (2007)

Phys.Rev. D82 (2010) 112006

CLEO-c binned s_i, c_i used for:

- γ with $B^{\pm}\to(K_S\pi\pi)_D K$ (also: $B^{\pm}\to(K_S KK)_D K$)

  LHCb: JHEP 10 (2014) 097
  LHCb: Phys. Lett. B718 (2012) 43
  BELLE: arXiv:1106.4046 (2011)

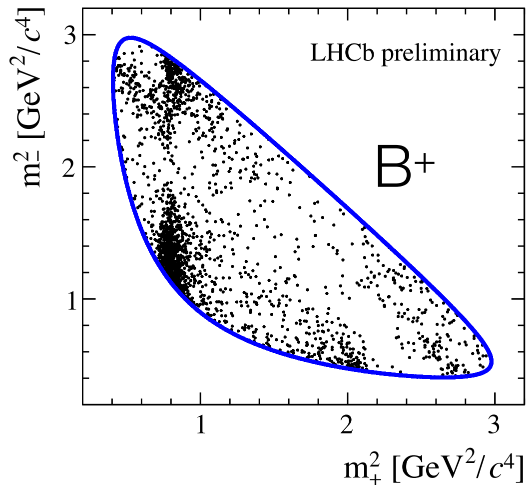
- Charm mixing (x, y) in $D^o\to K_S\pi\pi$

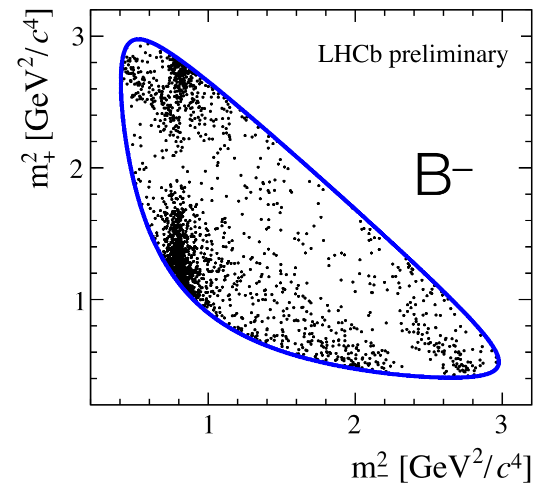
  LHCb: JHEP 1604 (2016) 033

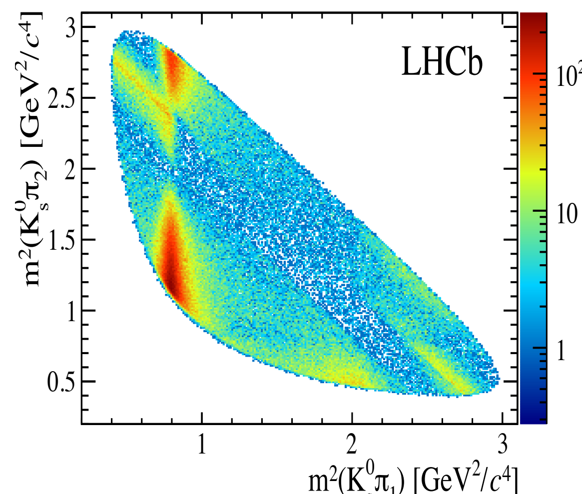
  $x=(-0.86\pm0.53\pm0.17)\times10^{-2}$
  $y=(+0.03\pm0.46\pm0.13)\times10^{-2}$

- $\sin(2\beta)$ in $B^o\to D^{(*)o}(\pi^o, \eta, \eta', \omega)$, $D\to K_S\pi\pi$

  BELLE: PRD94 (2016) no.5, 052004



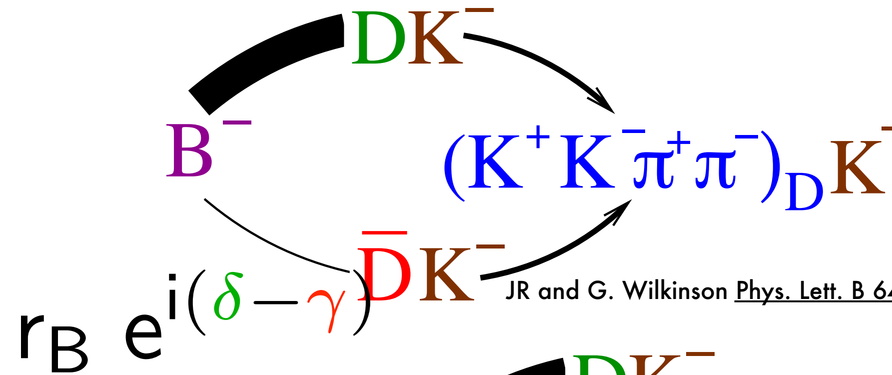




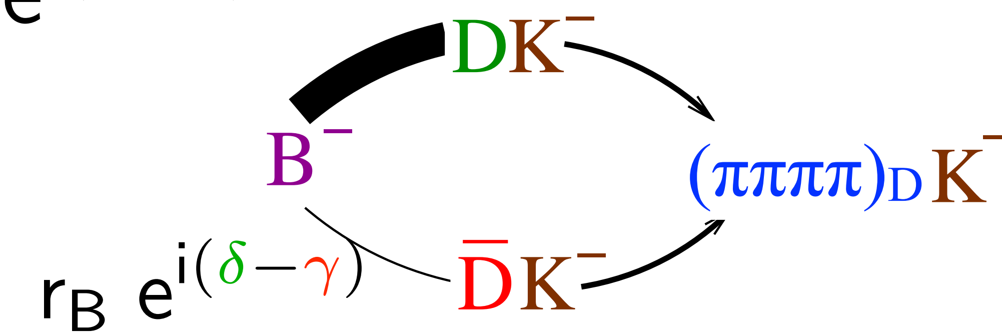
CLEO-c input:: Phys. Rev. D 82 112006.
Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).
Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)
BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

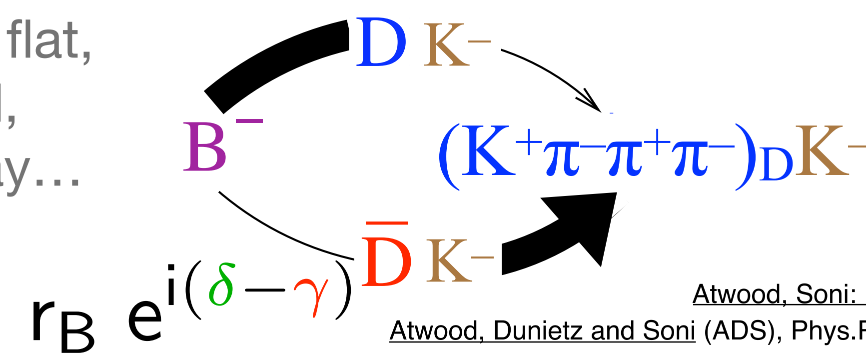
γ with 4-body decays

- All charged final states good for LHCb
- Simulation studies indicate 4-body amplitude analyses could add significantly to γ sensitivity at LHCb. So far they enter only phase-space integrated.
- Tricky… “Dalitz Plot” becomes 5-dimensional, phase space not flat, spin factors more complicated, good models even further away…



JR and G. Wilkinson Phys. Lett. B 647 (2007) 400-405

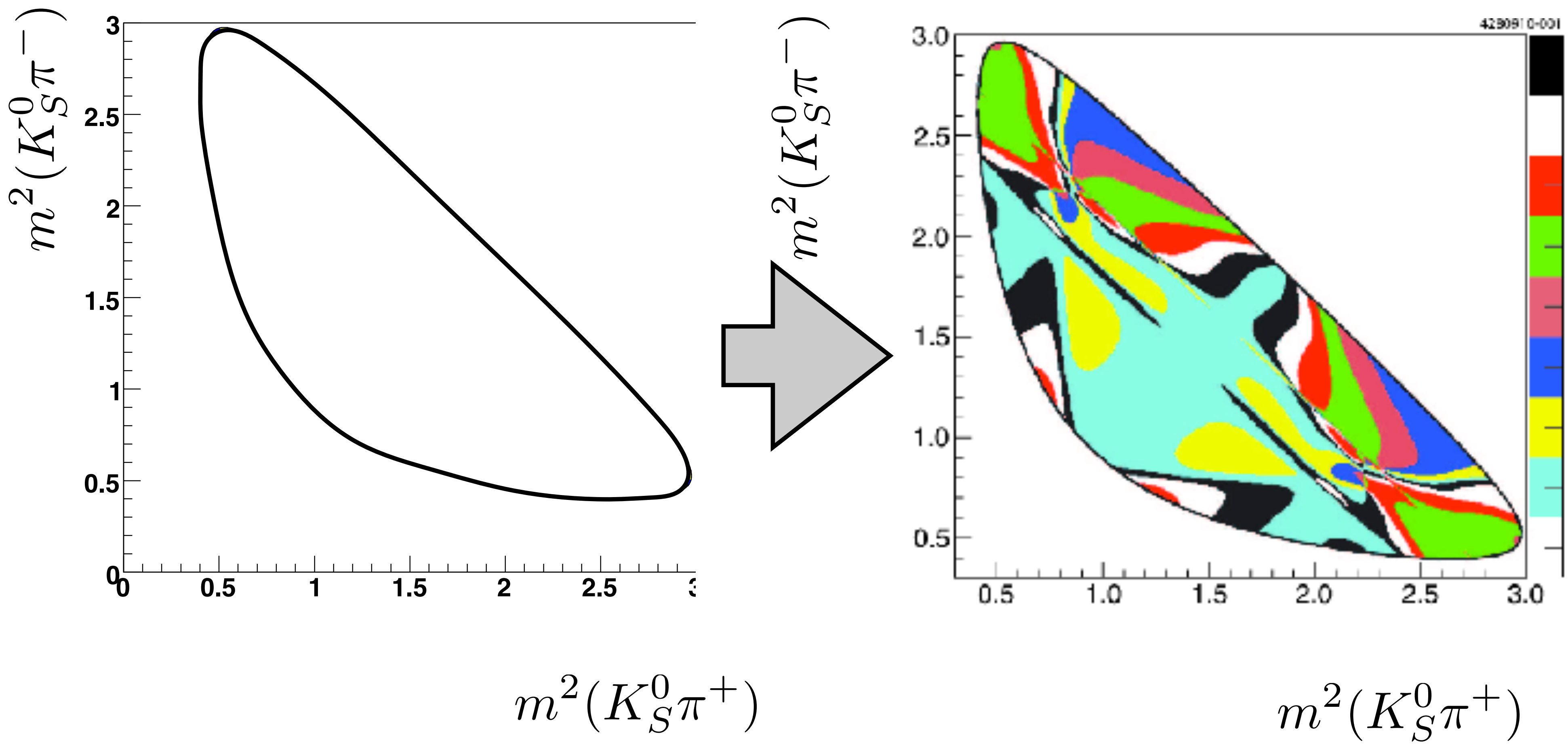




Atwood, Soni: Phys.Rev. D68 (2003) 033003

Atwood, Dunietz and Soni (ADS), Phys.Rev.Lett. 78 (1997) 3257-3260

Winning by binning in 2 dimensions…



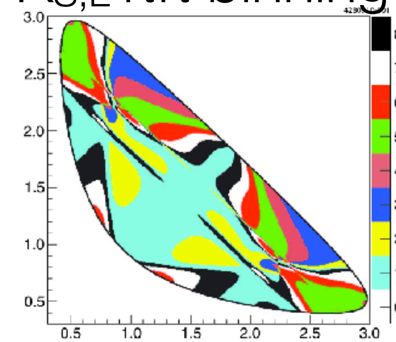
Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

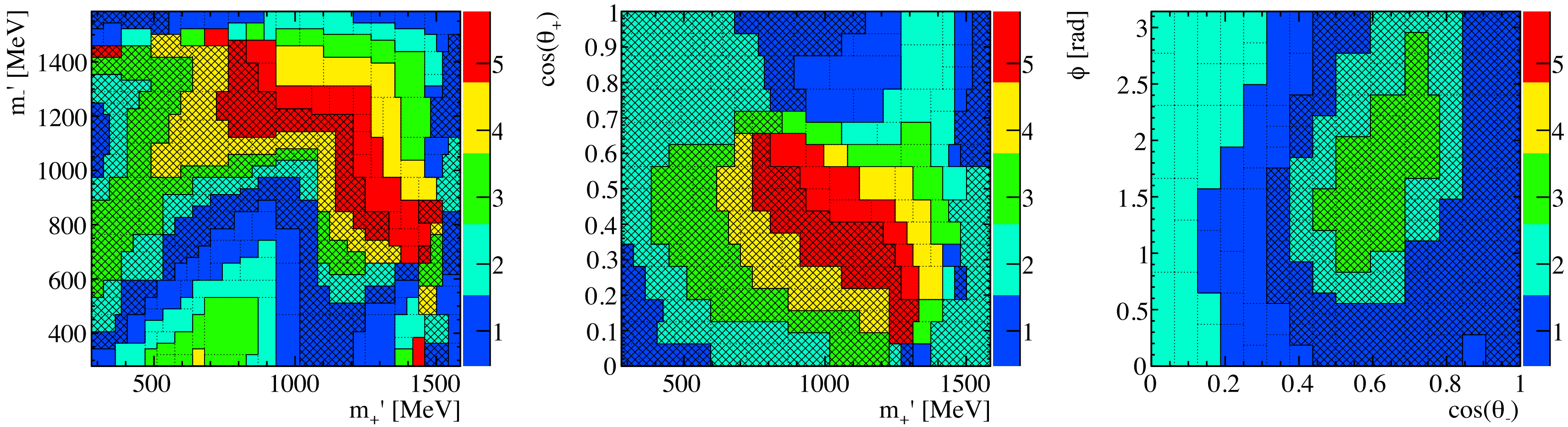
Winning by binning in 5 dimensions…

JHEP 1801 (2018) 144

Binning based on phase difference between D^o and $\bar{D}^o$ amplitudes going to same point in phase space, like optimised binning for $K_{S,L}\,\pi\pi$.
This approach requires a model.
Examples of 2-D slices through 5-D phase space based on $D\rightarrow\pi\pi\pi\pi$ amplitude model in JHEP 1705 (2017) 143.

$K_{S,L}\,\pi\pi$ binning





"The full binning schemes used in this paper are provided in both ASCII and Root format as supplementary material."

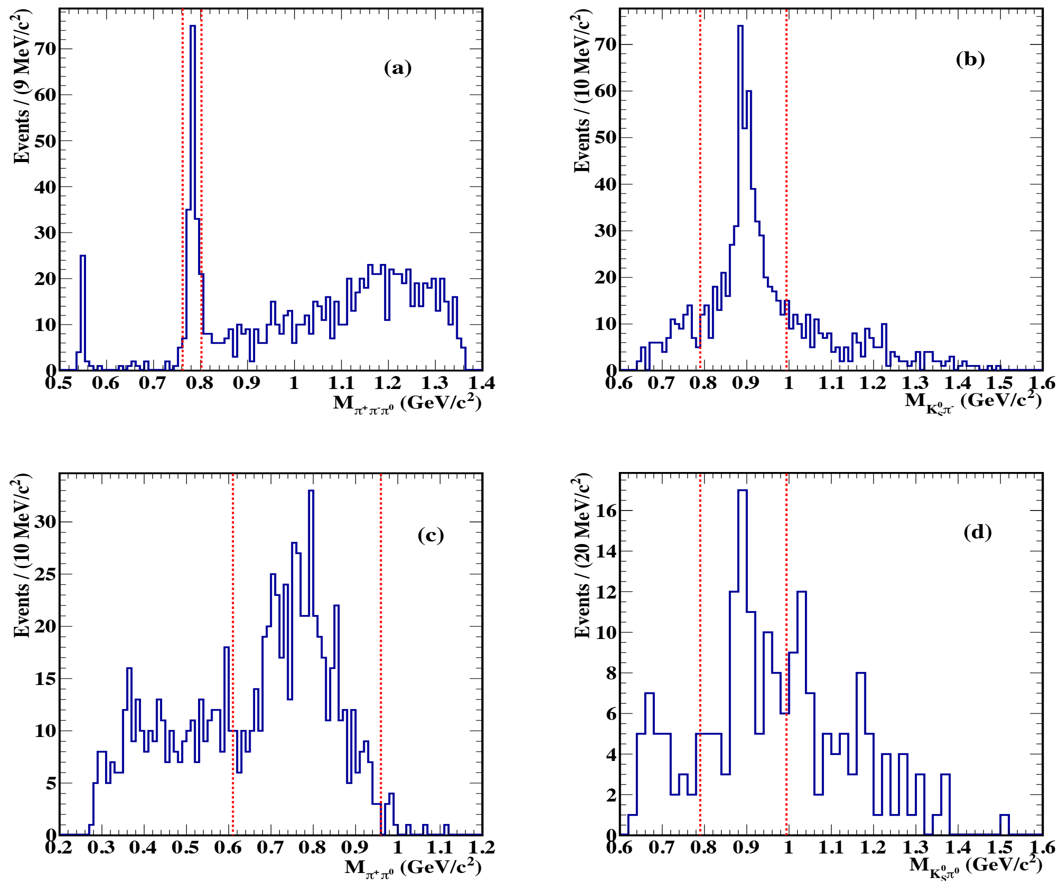
Communicating the binning "cleanly" (not via model) is key to its practical usability

Winning by binning in 5 dimensions…

JHEP 1801 (2018) 082

$D^0 \to K_S \pi^- \pi^+ \pi^0$ c_i, s_i

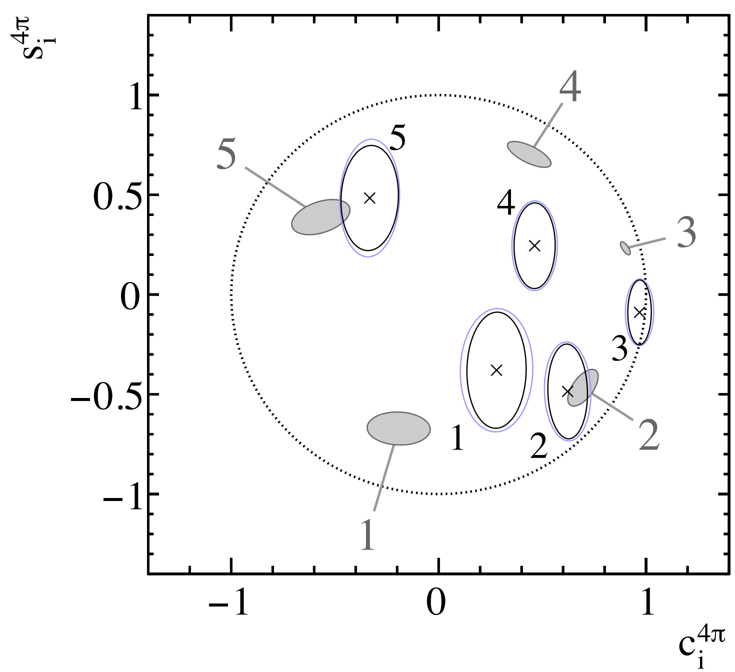
No amplitude model ⇒ binning around resonances:



Results for 4-body c_i, s_i.

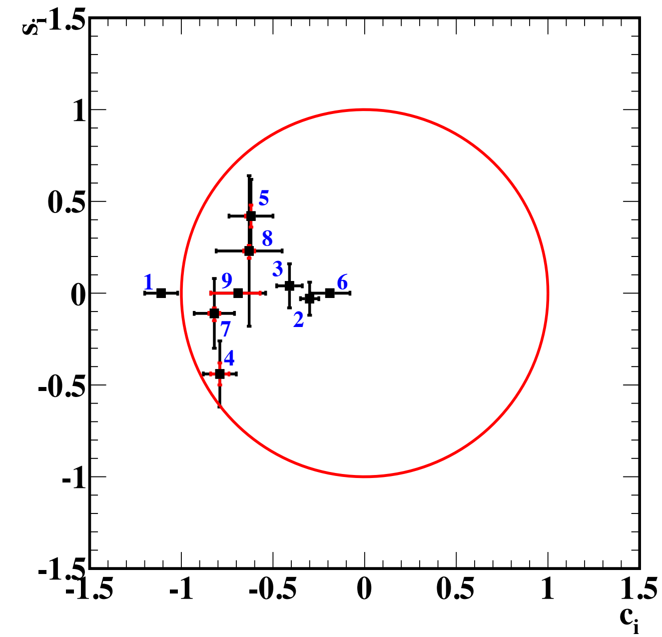
$D^0 \to \pi^+\pi^-\pi^+\pi^-$

(measurements × and model expectation ⬭)



JHEP 1801 (2018) 144

$D^0 \to K_S\pi^-\pi^+\pi^0$



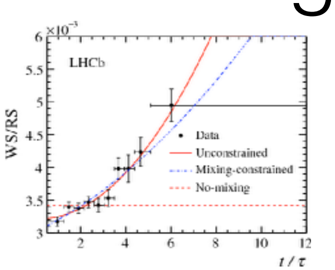
JHEP 1801 (2018) 082

Binned B±→DK±, D→K3π.

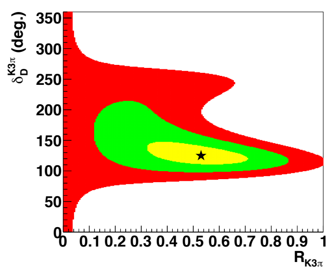
JHEP 1503 (2015) 169

Projected precision on γ from B±→DK±, D→K3π for LHCb run II statistics, different charm input scenarios, model-informed binning.

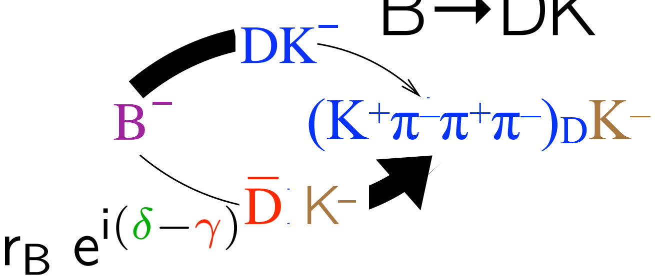
D mixing



ψ(3770)



B→DK



LHCb scenario	D^0 mix?	charm	threshold?	$\sigma(\gamma)$ [°]
run I				16
run II	Y	CLEO	binned	12
upgr				7.8
run I				12
run II	Y	BESIII	binned	8.6
upgr				4.1

Uses "made-up" amplitude model for feasibility study. Not necessary anymore: LHCb arXiv:1712.09320 (2018) provides amplitude model needed for optimal binning.

Conclusion

- Many ways to exploit quantum correlations - many only realised recently

- “Classic” c_i s_i with $K_S\pi\pi$, 2-3 × stats with $K_L\pi\pi$
  Do the same with $K_{S,L}KK$

- Many new measurements (latest results from 2018)
  - 4-body modes $KK\pi\pi$, $\pi\pi\pi\pi$, $K\pi\pi\pi$
    (binned $c_i + i\, s_i = R_i\, e^{-i\delta_i}$ and phase-space integrated)
  - F_+ in $\pi\pi\pi\pi$, $\pi\pi\pi^o$, $KK\pi^o$
  - Combination with mixing.

- Complexities include communicating results, conventions, binning schemes etc between different groups (… although we might not need bins).

the end

CP-even $K_L\pi\pi \approx$ CP-odd $K_S\pi\pi$

unfortunately only "≈", not quite "="

$$K_S^0 = (K^0 + \bar{K}^0)/\sqrt{2}$$
$$K_L^0 = (K^0 - \bar{K}^0)/\sqrt{2}.$$

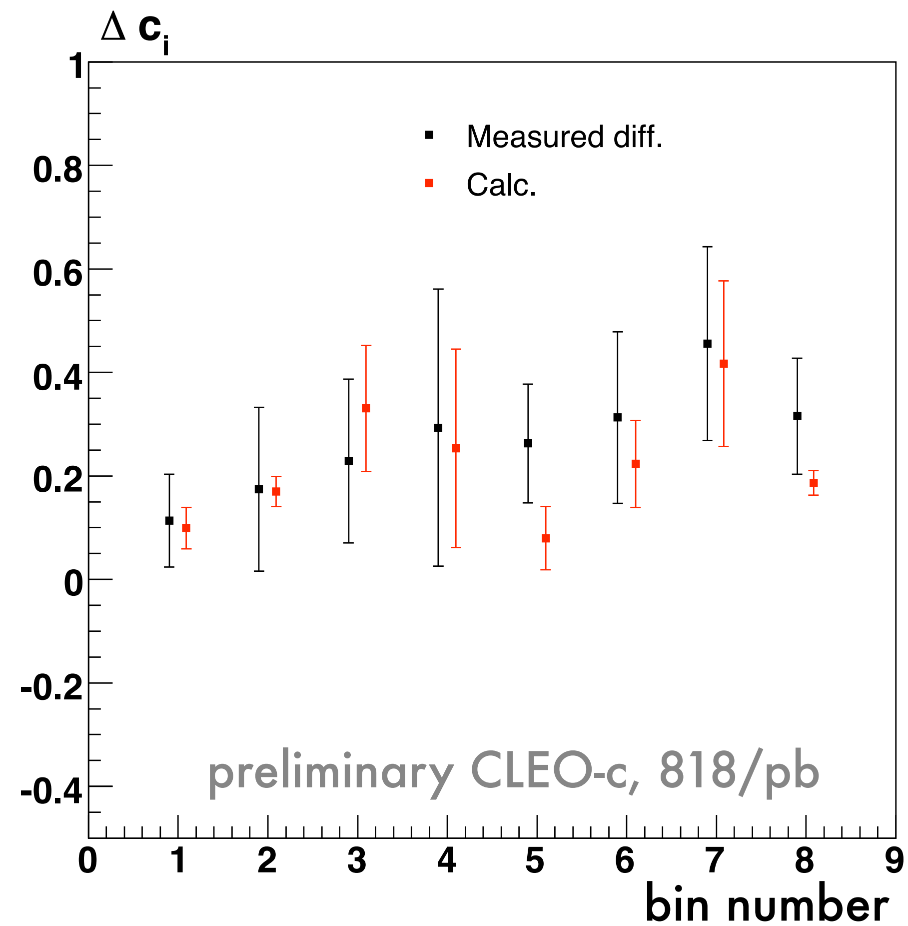
$$-A\left(D^0 \to K_L^0\pi^+\pi^-\right) = \underbrace{A\left(D^0 \to K_S^0\pi^+\pi^-\right)}_{\text{CF+DCS}} - \underbrace{\sqrt{2}A\left(D^0 \to K^0_{\text{flavour}}\pi^+\pi^-\right)}_{\text{DCS}}$$

- Using $K_L\pi\pi$ significantly enhances statistics.
- However, need a correction $O(\tan^2\theta_C)$. Model dependence enters as an uncertainty on something that is in itself a small-ish correction.
- Notation: c_i, s_i from $K_S\pi\pi$. c_i', s_i' from $K_L\pi\pi$.

$\Delta c_i \equiv c_i - c_i'$, $\Delta s_i \equiv s_i - s_i'$

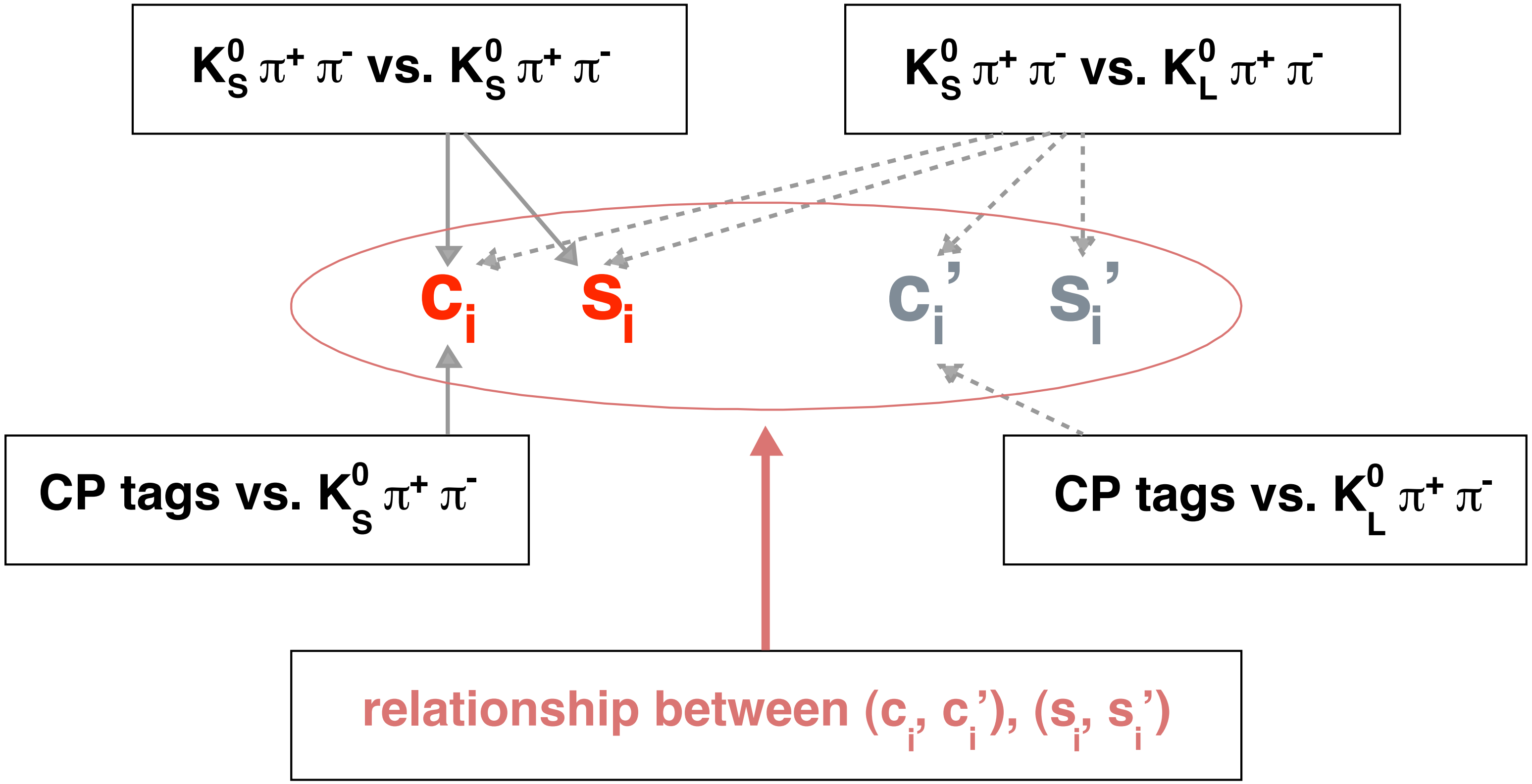
CP-even $K_L\pi\pi \approx$ CP-odd $K_S\pi\pi$

- Δc_i from 0.1 – 0.4, $\sigma_{sys}(\Delta c_i)$ from 0.06 – 0.19
- $|\Delta s_i|$ from 0.02 – 0.13, $\sigma_{sys}(\Delta s_i)$ from 0.07 to 0.14
- Plot compares estimated Δc_i with separate measurements in CP-tagged $K_S\pi\pi$, and $K_L\pi\pi$ events.



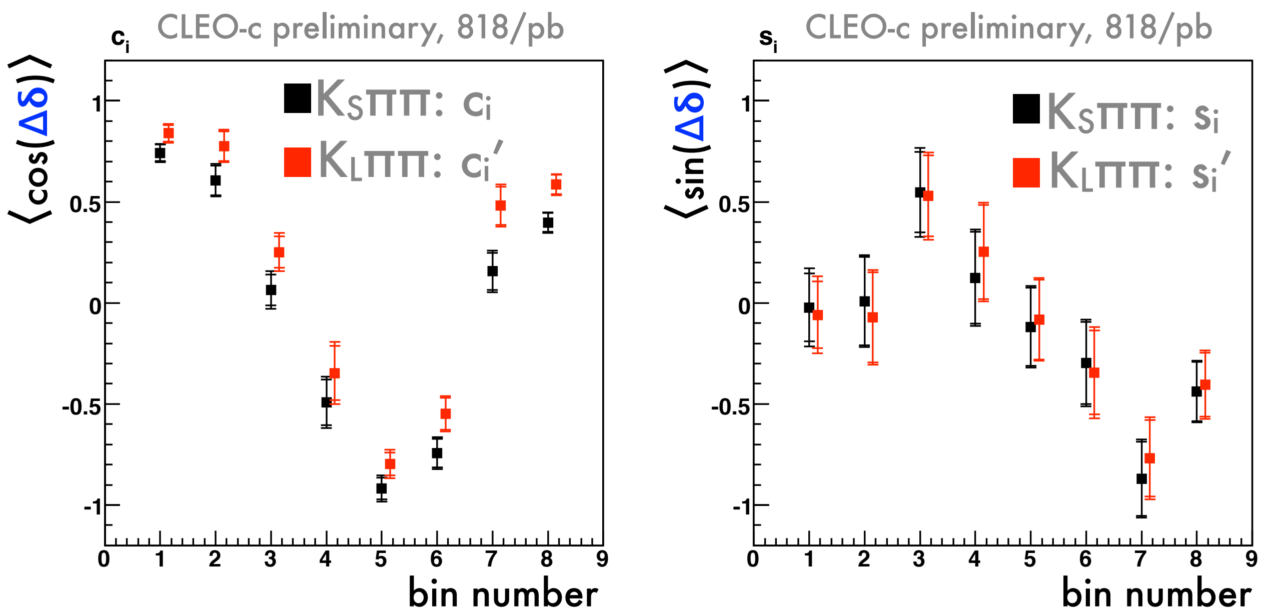
red: model predictions
black: CP-tagged data

Ingredients of combined fit



Notation: c_i, s_i from $K_S\pi\pi$. c_i', s_i' from $K_L\pi\pi$. $\Delta c_i \equiv c_i - c_i'$, $\Delta s_i \equiv s_i - s_i'$

The Result

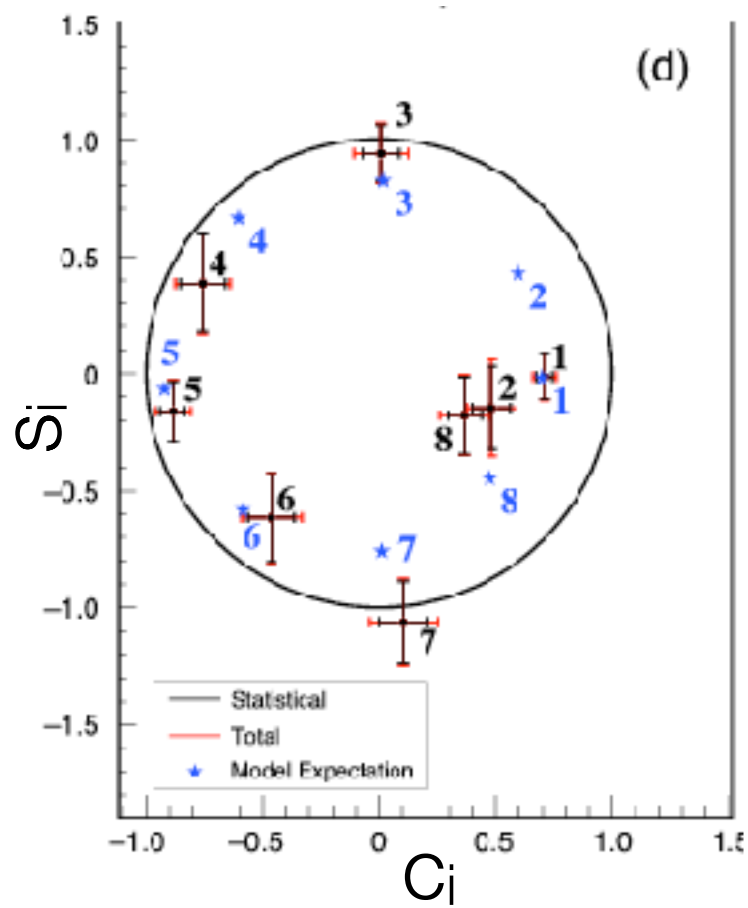


- Results of combined fit in terms of c_i, s_i in $K_S\pi\pi$ and c_i', s_i' in $K_L\pi\pi$.
- Each series of results (black / red) contains full information from both $K_S\pi\pi$ and $K_L\pi\pi$ data, related by Δc_i,

- Fit errors (include $\sigma_{statistical} \oplus$ errors on Δc_i, Δs_i constraints): c_i: 0.04-0.11 c_i': 0.04-0.14 s_i: 0.15-0.23 s_i': 0.16-0.23
- Systematic errors: c_i: 0.02-0.06 c_i': 0.02-0.07 s_i: 0.04-0.10 s_i': 0.06-0.10

CLEO-c results

- 818/fb at CLEO-c
- 20k $D^o \to K_{S,L}\pi^+\pi^-$ flavour tagged events
- 1.6 k CP-tagged $D^o \to K_{S,L}\pi^+\pi^-$ events (for c_i extraction)
- 1.7k $K_{L,S}\pi\pi$ vs $K_S\pi\pi$ (for c_i *and* s_i extraction)
- S/B between 10 and 100, depending on tag mode.



Phys.Rev. D82 (2010) 112006

Table of yields

More $K_L\pi\pi$ than $K_S\pi\pi$

Mode	DT yields			
	$K_S^0\pi^+\pi^-$	$K_L^0\pi^+\pi^-$	$K_S^0K^+K^-$	$K_L^0K^+K^-$
Flavor tags				
$K^-\pi^+$	1444	2857	168	302
$K^-\pi^+\pi^0$	2759	5133	330	585
$K^-\pi^+\pi^+\pi^-$	2240	4100	248	287
$K^-e^+\nu$	1191		100	
CP-even tags				
K^+K^-	124	357	12	32
$\pi^+\pi^-$	61	184	4	13
$K_S^0\pi^0\pi^0$	56		7	14
$K_L^0\pi^0$	237		17	
$K_L^0\eta(\gamma\gamma)$			4	
$K_L^0\eta(\pi^+\pi^-\pi^0)$			1	
$K_L^0\omega$			4	
$K_L^0\eta'$			1	
CP-odd tags				
$K_S^0\pi^0$	189	288	18	43
$K_S^0\eta(\gamma\gamma)$	39	43	4	6
$K_S^0\eta(\pi^+\pi^-\pi^0)$			2	1
$K_S^0\omega$	83		14	10
$K_S^0\eta'$			3	4
$K_L^0\pi^0\pi^0$			5	
$K_S^0\pi^+\pi^-$	473	1201	56	126
$K_L^0\pi^+\pi^-$			140	
$K_S^0K^+K^-$			4	9

	$K_S^0\pi^+\pi^-$	$K_L^0\pi^+\pi^-$
Flavor tags		
$K^-\pi^+$	1444	2857
$K^-\pi^+\pi^0$	2759	5133
$K^-\pi^+\pi^+\pi^-$	2240	4100
$K^-e^+\nu$	1191	
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$\pi^+\pi^-$	61	184
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$K_L^0\pi^0$	237	

Table of tags

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$K_L^0\pi^+\pi^-$			140	
$K_S^0K^+K^-$			4	9

CP tags:

$$2|A_{CP}|^2 = |A \pm \bar{A}|^2 = |A|^2 + |\bar{A}|^2 \pm |A||\bar{A}|\cos\delta$$

sensitive to c_i

Table of yields

Mode	DT yields			
	$K_S^0\pi^+\pi^-$	$K_L^0\pi^+\pi^-$	$K_S^0K^+K^-$	$K_L^0K^+K^-$
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$K_S^0K^+K^-$			4	9

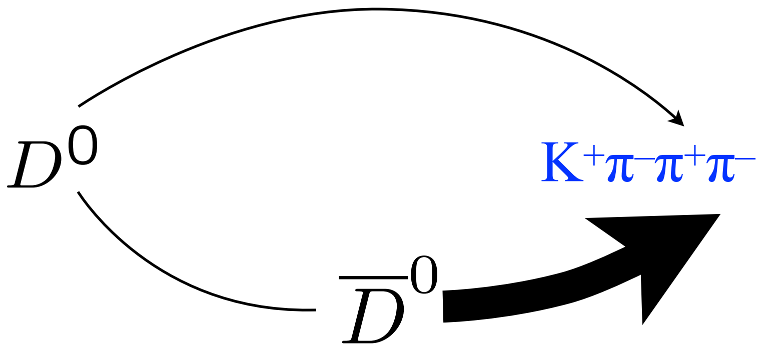
yields in $K_S\pi\pi$ vs $K_S\pi\pi$ bins i,j

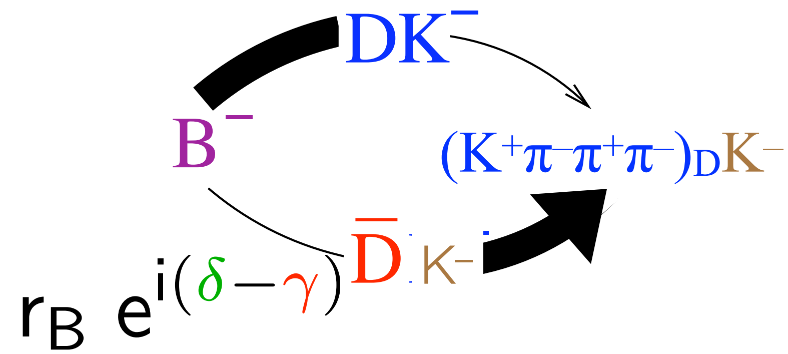
$$M_{ij} = h_{\mathrm{corr}}\left(K_iK_{-j} + K_{-i}K_j - 2\sqrt{K_iK_{-j}K_{-i}K_j}\,(c_ic_j + s_is_j)\right)$$

crucial for sensitivity to s_i

	$K_S^0\pi^+\pi^-$	$K_L^0\pi^+\pi^-$
$K_S^0\pi^+\pi^-$	473	1201

D^0 Mixing as input to γ from $B^\pm \to DK^\pm$





This process is sensitive to the same D-D interference effects that pollute this measurement.

Can use charm mixing to constrain charm amplitude parameters in γ measurements - see previous talk.

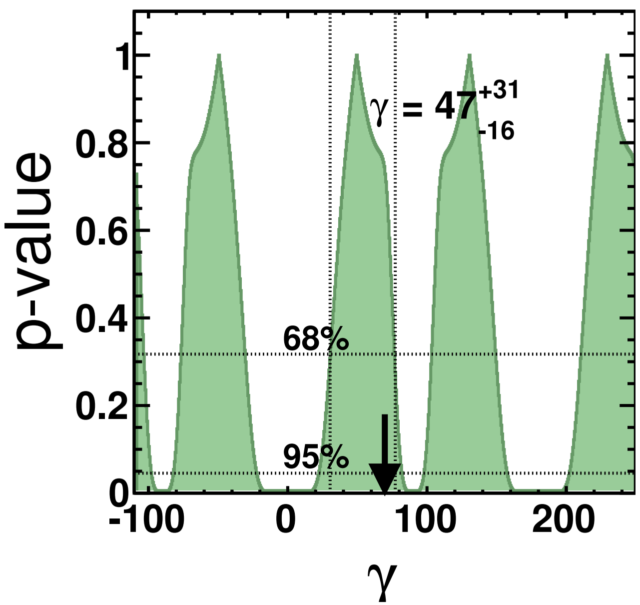
PLB728 (2014) 296-302
JHEP 1503 (2015) 169

Winning with binning & threshold + mixing combination, $B^{\pm}\rightarrow DK^{\pm}$, $D\rightarrow K3\pi$.

JHEP 1503 (2015) 169

Projected precision on γ from $B^{\pm}\rightarrow DK^{\pm}$, $D\rightarrow K3\pi$ for LHCb run II statistics, different charm input scenarios, model-informed binning.

D mixing (binned)
alone



Uses "made-up" amplitude model for feasibility study. Not necessary anymore: LHCb arXiv: 1712.09320 (2018) provides amplitude model needed for optimal binning.

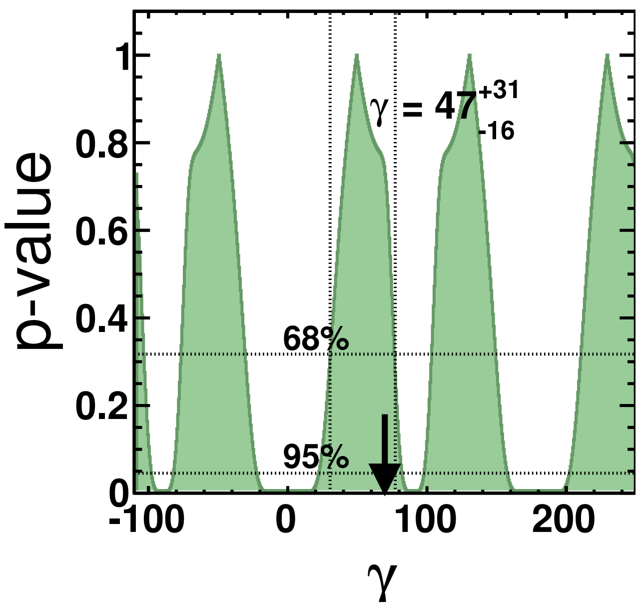
Winning with binning & threshold + mixing combination, $B^\pm \to DK^\pm$, $D \to K3\pi$.

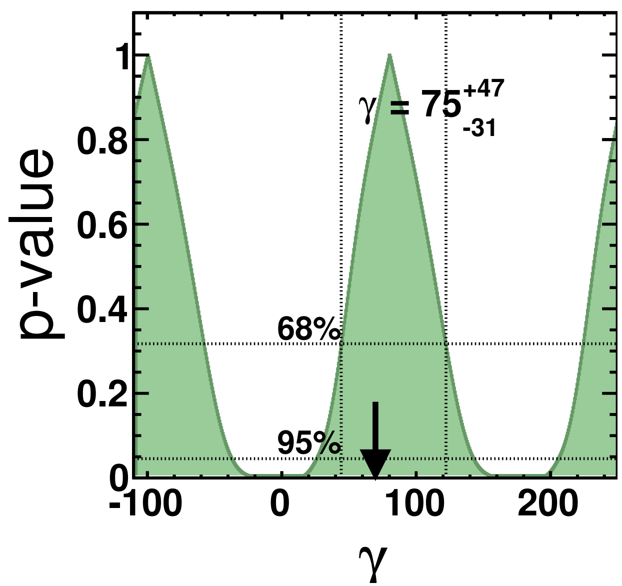
JHEP 1503 (2015) 169

Projected precision on γ from $B^\pm \to DK^\pm$, $D \to K3\pi$ for LHCb run II statistics, different charm input scenarios, model-informed binning.

D mixing (binned) alone

BES III binned alone





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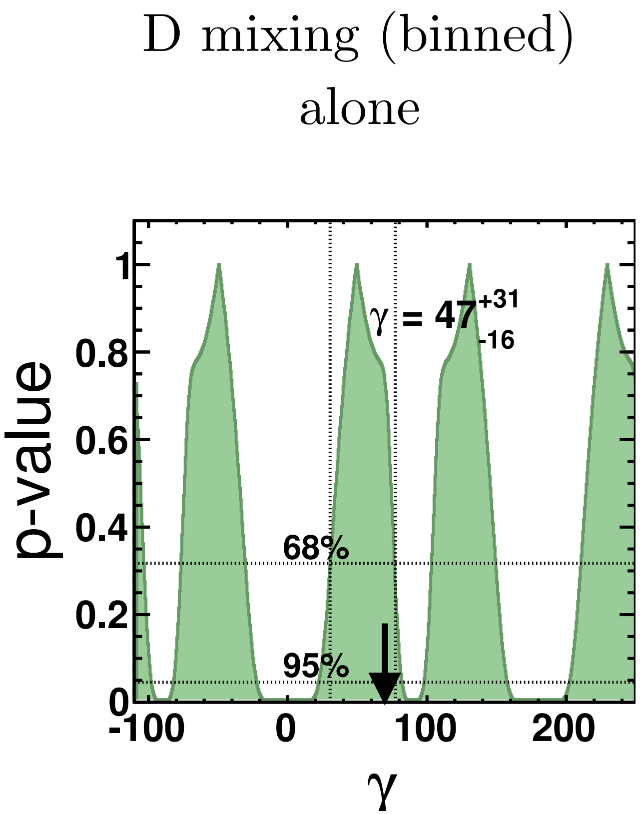
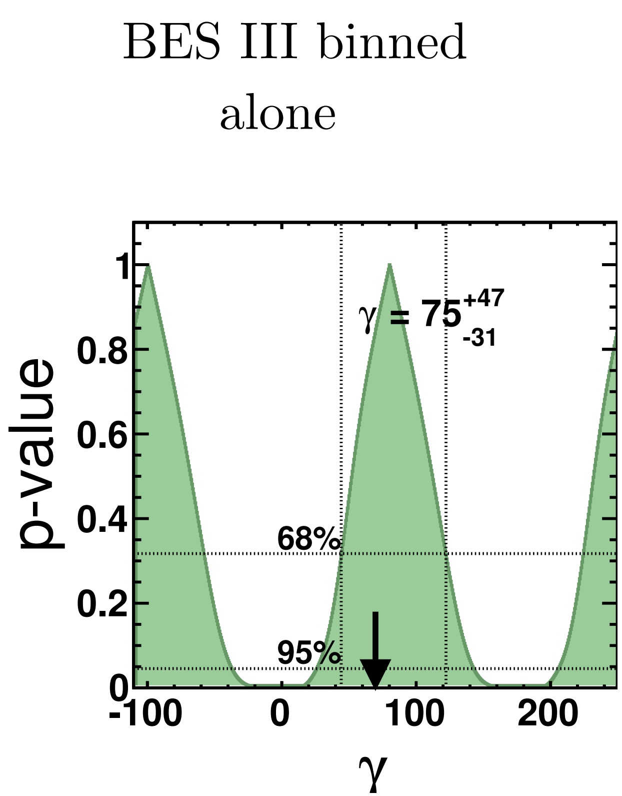
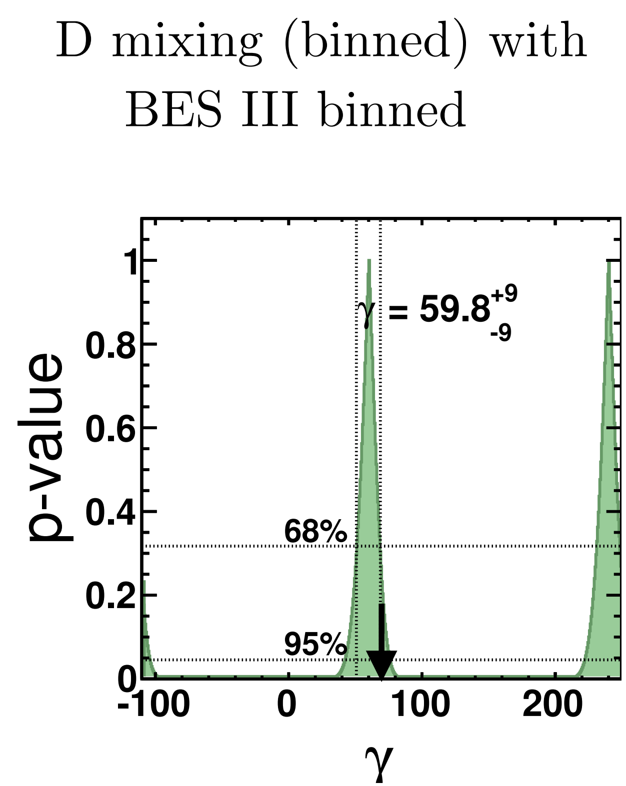
Winning with binning & threshold + mixing combination, $B^{\pm}\to DK^{\pm}$, $D\to K3\pi$.

JHEP 1503 (2015) 169

Projected precision on γ from $B^{\pm}\to DK^{\pm}$, $D\to K3\pi$ for LHCb run II statistics, different charm input scenarios, model-informed binning.







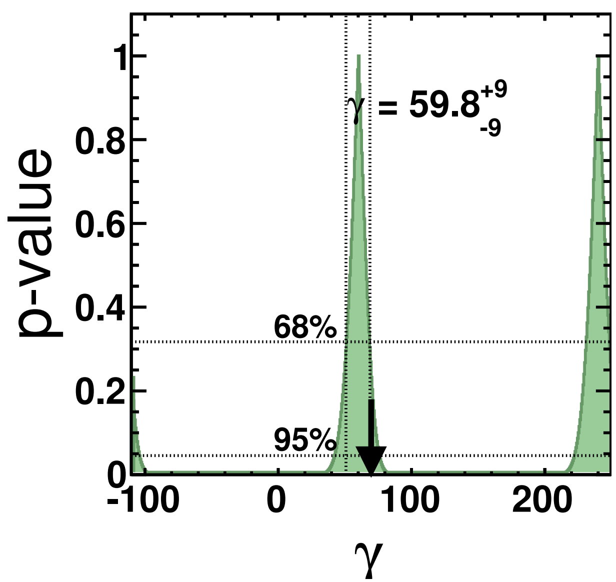
Uses "made-up" amplitude model for feasibility study. Not necessary anymore: LHCb arXiv: 1712.09320 (2018) provides amplitude model needed for optimal binning.

Binned $B^{\pm} \to DK^{\pm}$, $D \to K3\pi$.

JHEP 1503 (2015) 169

Projected precision on γ from $B^{\pm} \to DK^{\pm}$, $D \to K3\pi$ for LHCb run II statistics, different charm input scenarios, model-informed binning.

D mixing (binned) with
BES III binned



LHCb scenario	D^0 mix?	charm threshold?	$\sigma(\gamma)$ [°]
run I	Y	CLEO binned	16
run II			12
upgr			7.8
run I	Y	BESIII binned	12
run II			8.6
upgr			4.1

Uses “made-up” amplitude model for feasibility study. Not necessary anymore: LHCb arXiv: 1712.09320 (2018) provides amplitude model needed for optimal binning.

technique & 2011 data: Phys. Lett. B726 (2013) 151

2012 data: LHCb-CONF_2013-006)

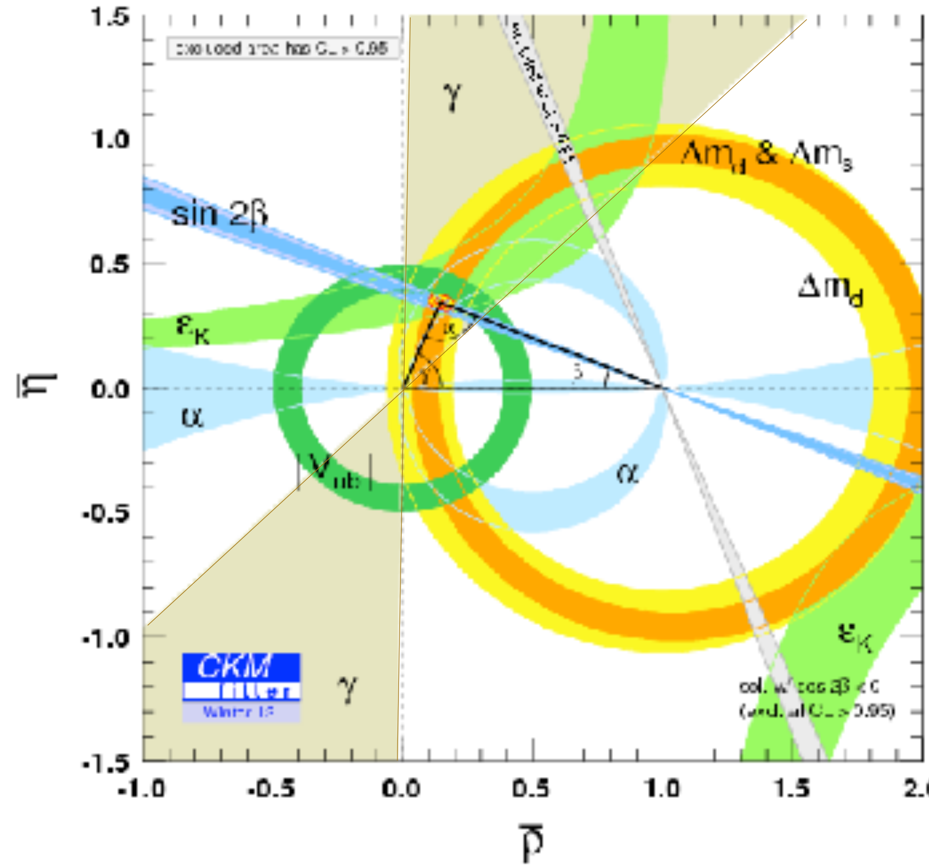
LHCb's γ combination 2013

- LHCb combines inputs from
  $B^{\pm} \to (hh')_D K^{\pm}$
  $B^{\pm} \to (K_S\pi\pi)_D K^{\pm}$
  $B^{\pm} \to (K_S KK)_D K^{\pm}$
  $B^{\pm} \to (K\pi\pi\pi)_D K^{\pm}$

- … and CLEO-c input for
  $D \to K_S\pi\pi$, $D \to K_S KK$, $D \to K\pi\pi\pi$

- Result: $\gamma = (67.2 \pm 12)^o$

World averages by CKM Fitter



WA 2012

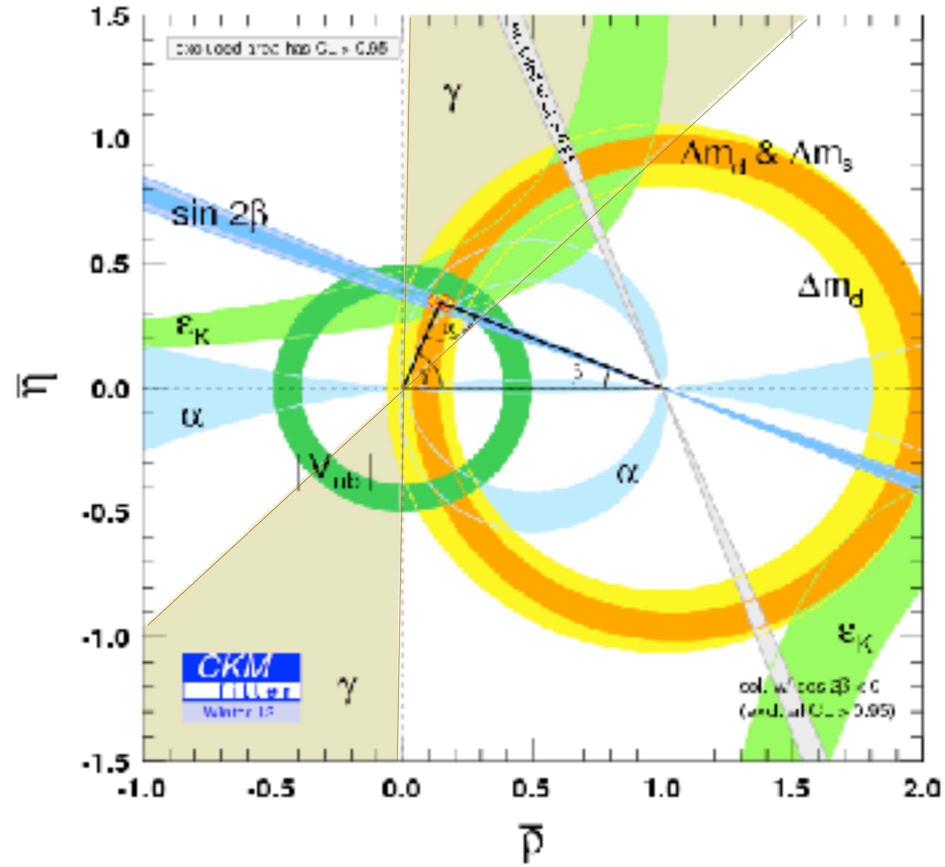
$$\gamma = 68^{\circ} \pm 12^{\circ}$$

LHCb's γ combination 2013

technique & 2011 data: Phys. Lett. B726 (2013) 151

2012 data: LHCb-CONF_2013-006)

World averages by CKM Fitter



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WA 2012

$$\gamma = 68^\circ \pm 12^\circ$$

LHCb 2013
γ=67±12°

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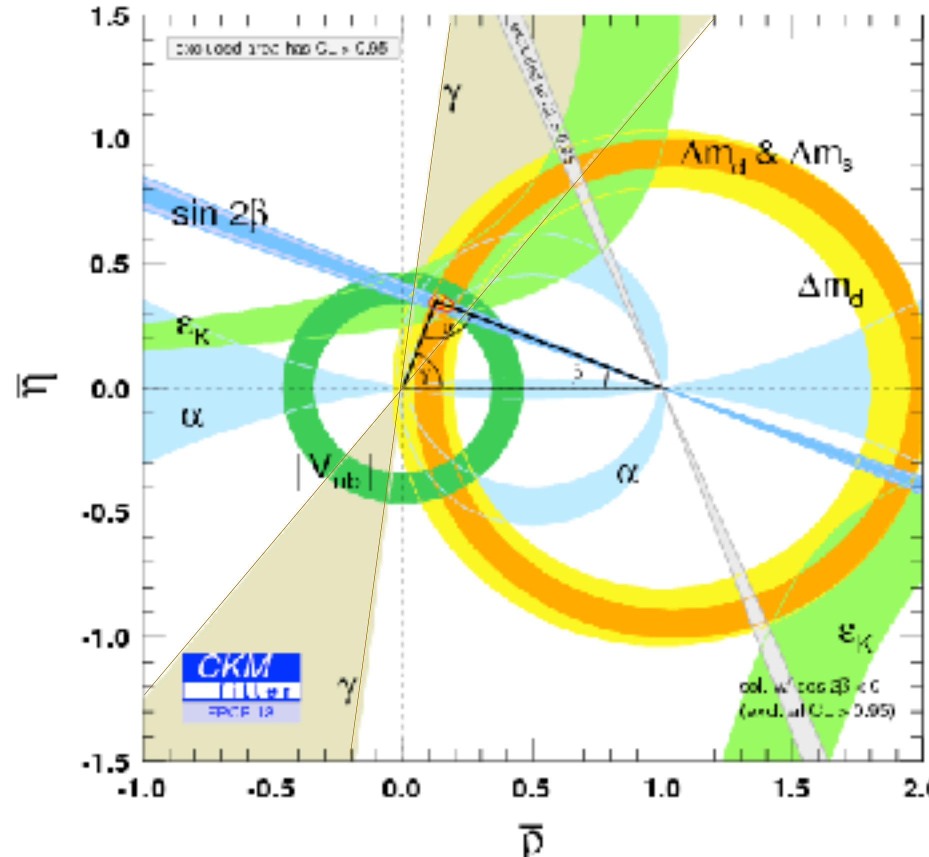

WA 2012

$$\gamma = 68^\circ \pm 12^\circ$$

LHCb 2013
γ=67±12°

WA 2013

$$\gamma = 68^\circ{}^{+8.0^\circ}_{-8.5^\circ}$$

LHCb-CONF-2017-004

LHCb's γ combination 2016 and 2017

JHEP 1612 (2016) 087

World averages by CKM Fitter

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  $B^\pm \to (K_S K\pi)_D K^\pm$, $B^\pm \to (K\pi\pi^0)_D K^\pm$,
  $B^\pm \to (K\pi\pi\pi)_D K^\pm$, $B^\pm \to (\pi\pi\pi\pi)_D K^\pm$
  $B^\pm \to D\pi^\pm$, $B^\pm \to DK^\pm\pi\pi$
  $B^0 \to DK^*$, $B^0 \to DK\pi$
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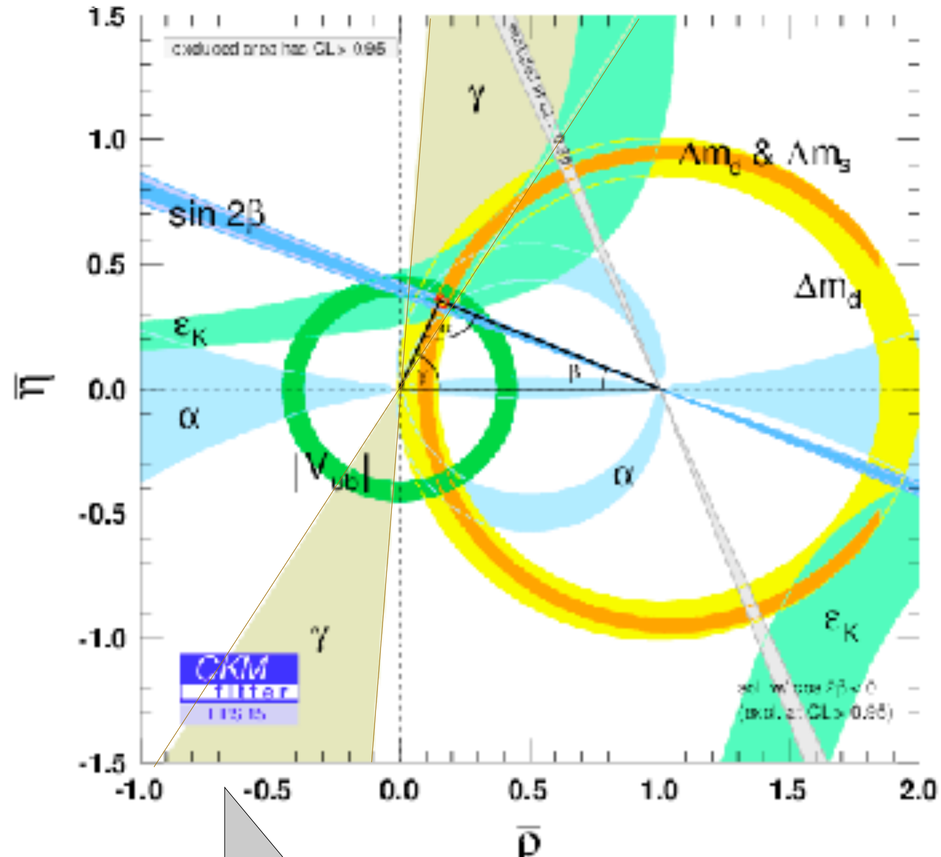
  $D \to \pi\pi\pi\pi$
- … and LHCb mixing in $D \to K\pi\pi\pi$
- Result: $\gamma = (76.8\,^{+5.1}_{-5.7})^\circ$



$\gamma = 68^\circ \pm 12^\circ$

WA 2012

LHCb 2013
γ=67±12°

$\gamma = 68^{\circ}\,^{+8.0^\circ}_{-8.5^\circ}$

WA 2013

LHCb 2016
γ=72°±7°

WA 2016

LHCb's γ combination 2016 and 2017

LHCb-CONF-2017-004

JHEP 1612 (2016) 087

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  $B^\pm \to (K_S K\pi)_D K^\pm$, $B^\pm \to (K\pi\pi^o)_D K^\pm$,
  $B^\pm \to (K\pi\pi\pi)_D K^\pm$, $B^\pm \to (\pi\pi\pi\pi)_D K^\pm$
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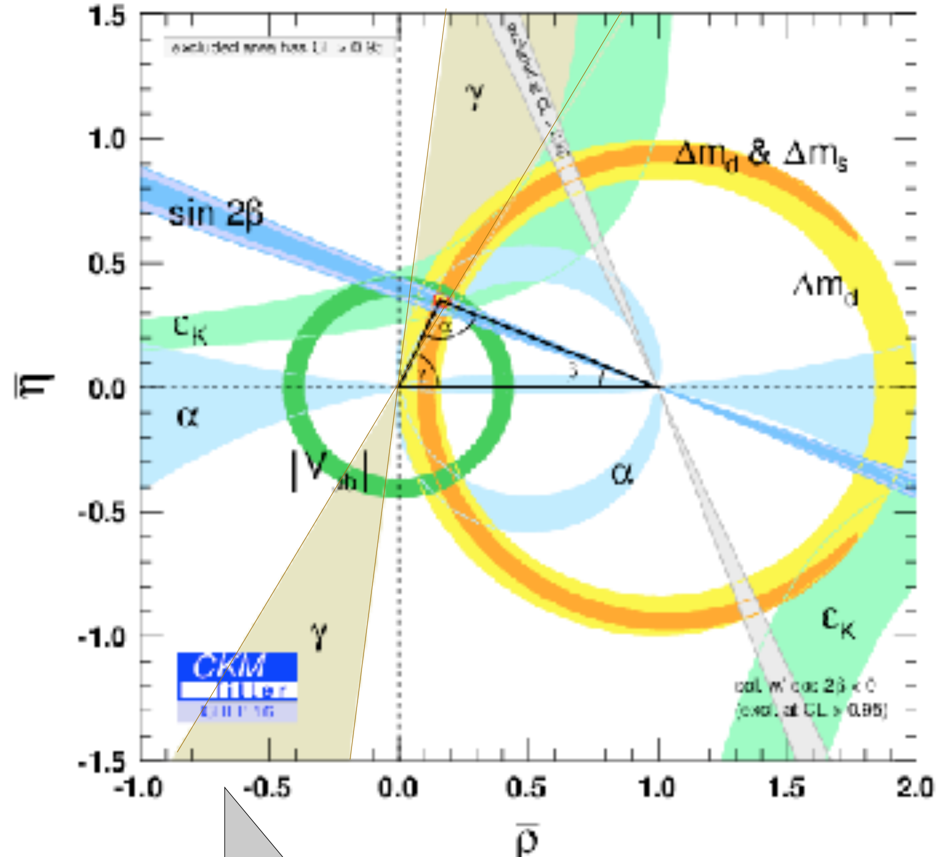

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LHCb 2013
γ=67±12°

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$72^{\circ}\,^{+5.4^\circ}_{-5.8^\circ}$

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LHCb-CONF-2017-004

JHEP 1612 (2016) 087

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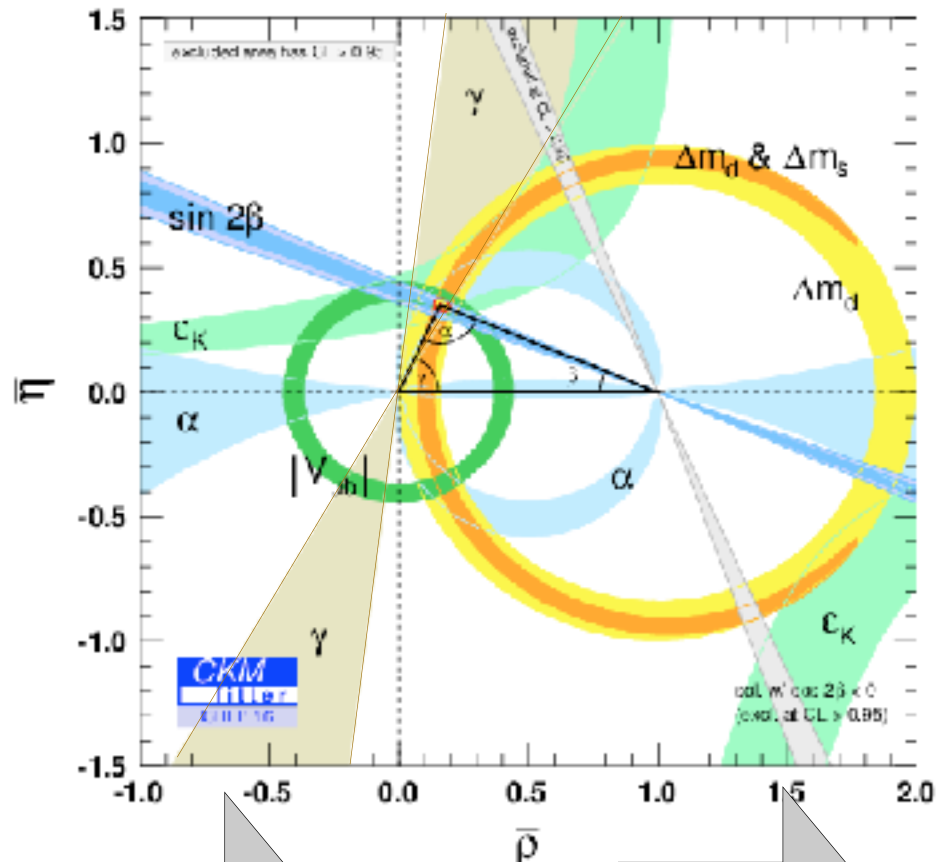


$\gamma = 68^\circ \pm 12^\circ$
WA 2012

LHCb 2013
γ=67±12°

$\gamma = 68^{\circ}\,^{+8.0^\circ}_{-8.5^\circ}$
WA 2013

LHCb 2016
γ=72°±7°

$72^{\circ}\,^{+5.4^\circ}_{-5.8^\circ}$
WA 2016

LHCb 2017
$\gamma = (76.8\,^{+5.1}_{-5.7})^\circ$

$\pi^+\pi^-\pi^+\pi^-$ Binning

- To describe a point in the $\pi^+\pi^-\pi^+\pi^-$ phase space we use:

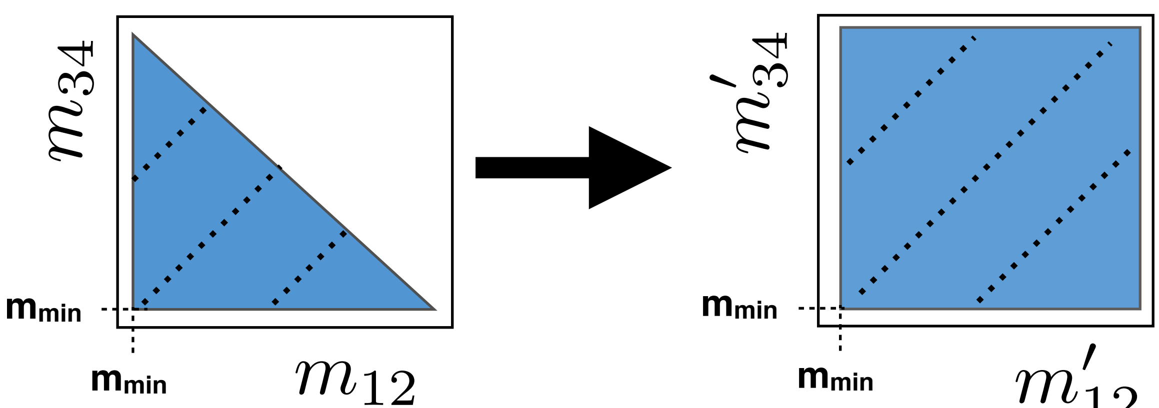
1 2 3 4

$$\mathbf{p} = (m'_{12}, m'_{34}, \cos\theta_{12}, \cos\theta_{34}, \phi)$$

Transformation ensures the phase space has rectangular boundaries

Helicity angle of the ij pair

Angle between the decay planes of 12 and 34



$$m'_{12} = m_{12} + \delta$$
$$m'_{34} = m_{34} + \delta$$
$$\delta = \min\{m_{12}, m_{34}\} - m_{\min}$$

$\pi^+\pi^-\pi^+\pi^-$ Binning

- This set of variables also has nice transformation properties under C and P

$$\mathbf{p} = (m'_{12}, m'_{34}, \cos\theta_{12}, \cos\theta_{34}, \phi)$$

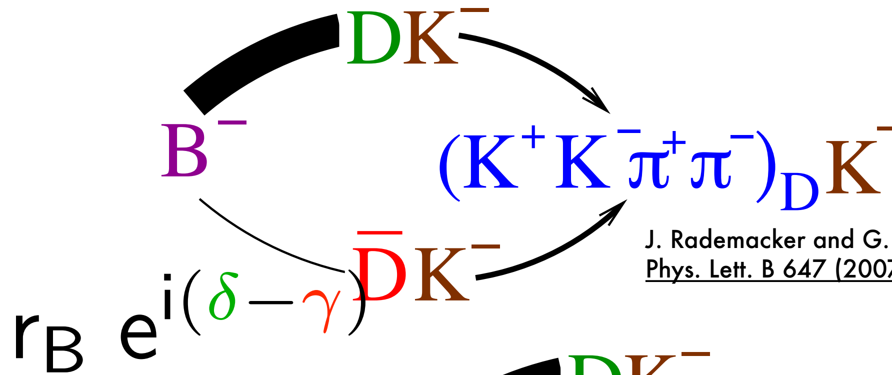
$$C : \mathbf{p} = (m'_{12}, m'_{34}, -\cos\theta_{12}, -\cos\theta_{34}, +\phi)$$
$$P : \mathbf{p} = (m'_{12}, m'_{34}, +\cos\theta_{12}, +\cos\theta_{34}, -\phi)$$

$$CP : \mathbf{p} = (m'_{12}, m'_{34}, -\cos\theta_{12}, -\cos\theta_{34}, -\phi)$$

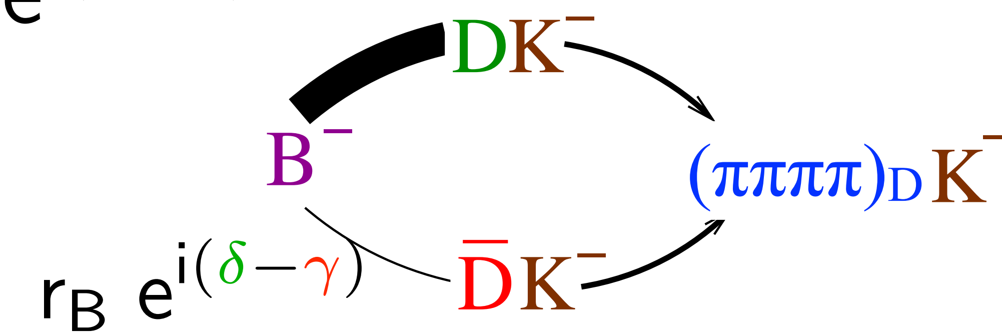
- This means the binning only has to be defined in $\phi > 0$ then can be reflected to get the remaining bins

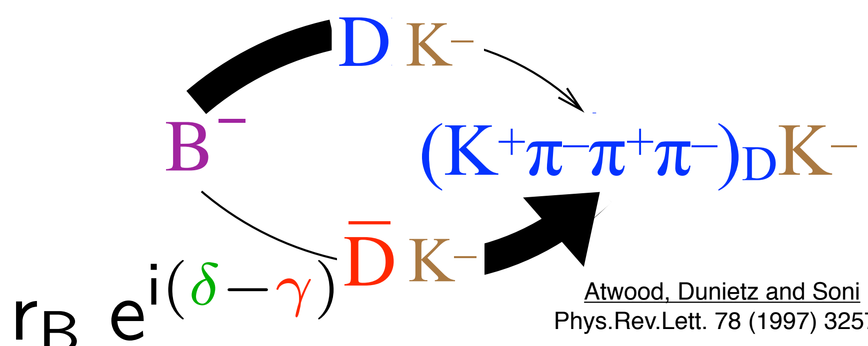
Exploiting the full 5-D phase space of 4-body D decays.

- Our studies indicate that 4-body have excellent sensitivity to γ.
- Challenging: 2-dimensional Dalitz plot → 5-dimensional phase space
- Next slides: a few first steps



J. Rademacker and G. Wilkinson
Phys. Lett. B 647 (2007) 400-405



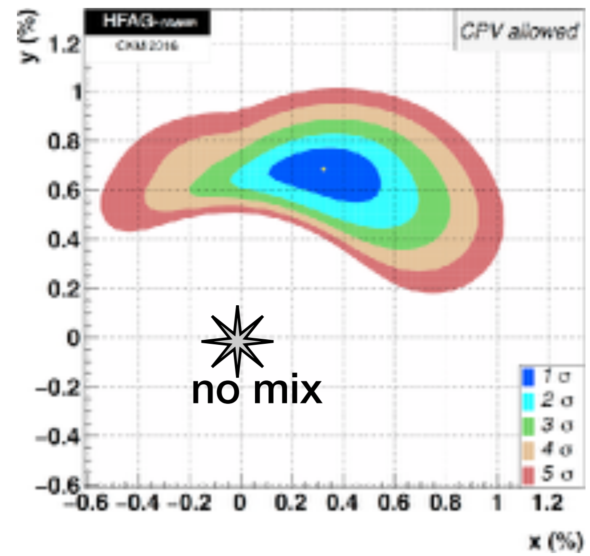


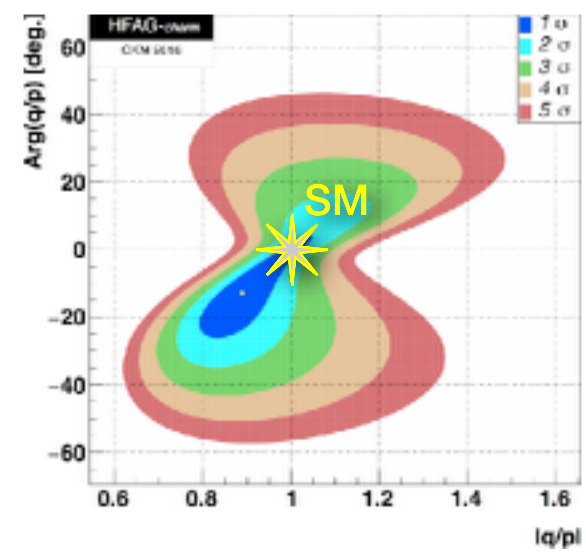
Phys. Lett. B 647 (2007) 400-405
Phys. Rev. D 80, 031105 (2009)
arXiv:1201.5716 [hep-ex] (submitted to PRD)

Atwood, Dunietz and Soni (ADS),
Phys.Rev.Lett. 78 (1997) 3257-3260
Atwood, Soni: Phys.Rev. D68 (2003) 033003

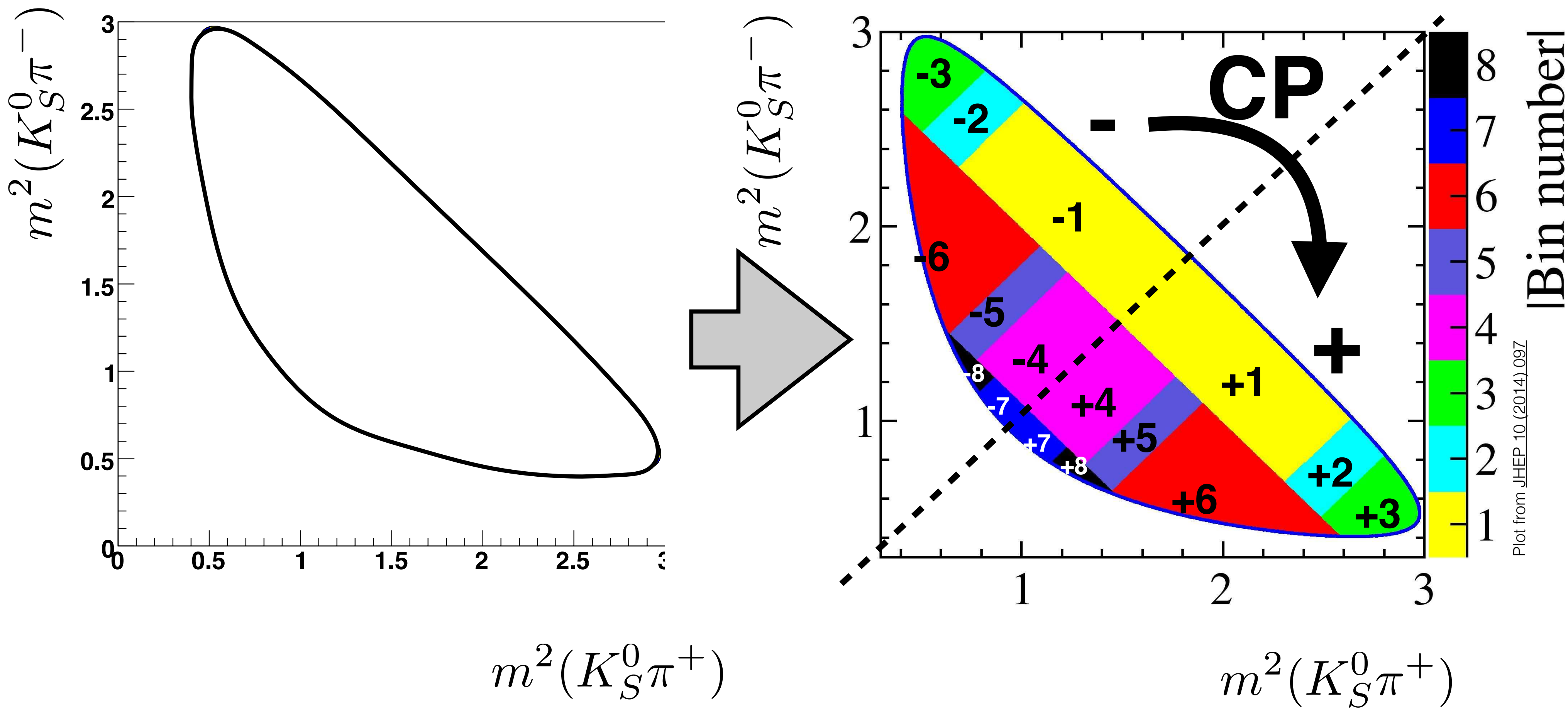
The parameters of the Charm System

- D mesons oscillate: $D^o \to \bar{D}^o \to \ldots$
- D mass eigenstates: $D_{1,2} = p|D\rangle \pm q|\bar{D}\rangle$
  - $x = \frac{\Delta m}{\bar{\Gamma}}$ ~mixing frequency
  - $y = \frac{\Delta \Gamma}{2\bar{\Gamma}}$ ~lifetime difference
  - CP-violation if $|p/q| \neq 1$ or CPV phase $\phi \neq 0$





Winning by binning…

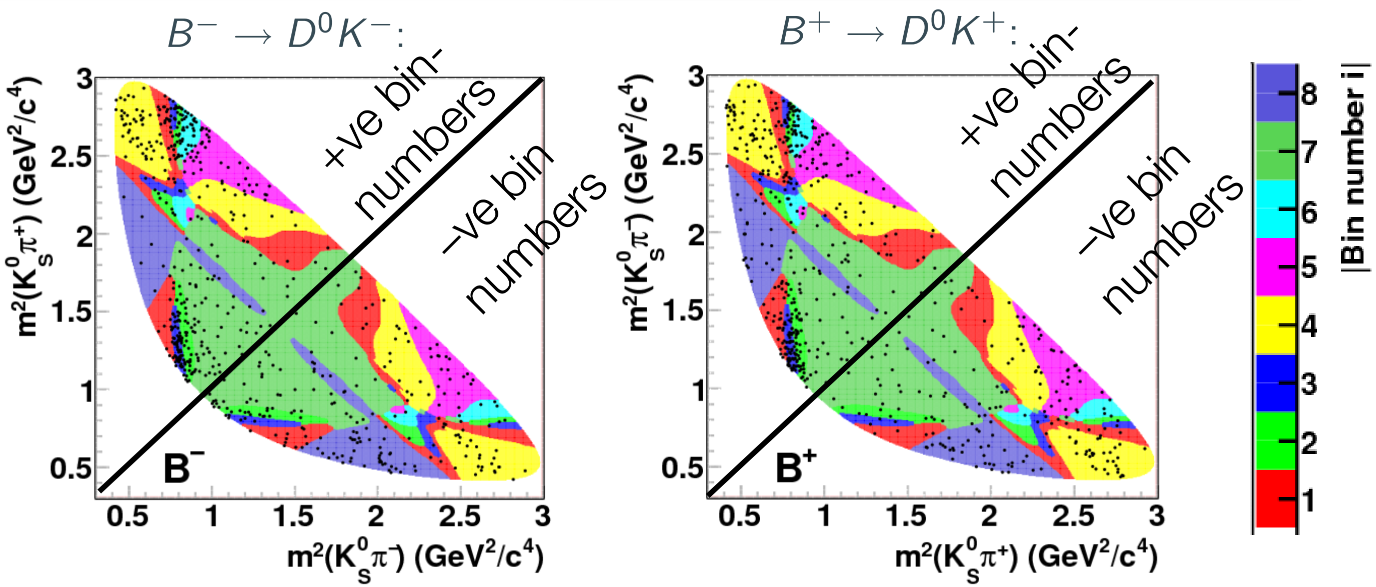


Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

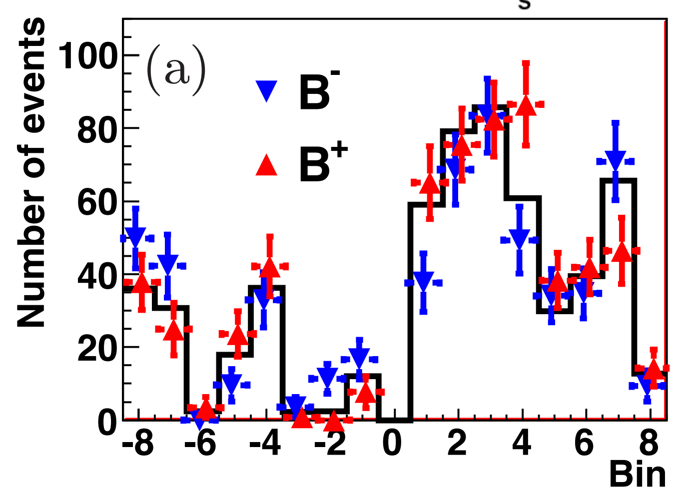
$D^0 \to \pi^+\pi^-\pi^+\pi^-$ Model

Decay channel	$\Re(a_i)$	$\Im(a_i)$	F_i (%)
$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+ \rho(770)^0\right]$	100.00 (fixed)	0.00 (fixed)	$38.1 \pm 2.3 \pm 3.2 \pm 1.7$
$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+ \sigma\right]$	$56.46 \pm 13.85 \pm 14.49$	$167.87 \pm 14.51 \pm 19.38$	$10.2 \pm 1.4 \pm 2.1 \pm 2.5$
$D^0 \to \pi^+ a_1(1260)^-$	$0.218 \pm 0.028 \pm 0.036$	$0.180 \pm 0.024 \pm 0.017$	-
$D^0 \to \pi^+ \left[a_1(1260)^- \to \pi^- \rho(770)^0\right]$	-	-	$3.1 \pm 0.6 \pm 0.5 \pm 0.9$
$D^0 \to \pi^+ \left[a_1(1260)^- \to \pi^- \sigma\right]$	-	-	$0.8 \pm 0.2 \pm 0.1 \pm 0.4$
$D^0 \to \pi^- \left[\pi(1300)^+ \to \pi^+ \sigma\right]$	$-15.11 \pm 3.08 \pm 9.44$	$19.80 \pm 3.54 \pm 5.90$	$6.8 \pm 0.9 \pm 1.5 \pm 3.1$
$D^0 \to \pi^+ \left[\pi(1300)^- \to \pi^- \sigma\right]$	$-6.48 \pm 2.39 \pm 6.08$	$15.19 \pm 2.62 \pm 7.52$	$3.0 \pm 0.6 \pm 2.0 \pm 2.0$
$D^0 \to \pi^- \left[a_1(1640)^+[D] \to \pi^+ \rho(770)^0\right]$	$-125.40 \pm 20.59 \pm 28.50$	$-10.89 \pm 15.07 \pm 13.75$	$4.2 \pm 0.6 \pm 0.9 \pm 1.8$
$D^0 \to \pi^- \left[a_1(1640)^+ \to \pi^+ \sigma\right]$	$77.57 \pm 21.59 \pm 31.24$	$-94.98 \pm 21.12 \pm 34.54$	$2.4 \pm 0.7 \pm 1.1 \pm 1.3$
$D^0 \to \pi^- \left[\pi_2(1670)^+ \to \pi^+ f_2(1270)\right]$	$-49.93 \pm 42.23 \pm 77.44$	$348.39 \pm 40.95 \pm 42.87$	$2.7 \pm 0.6 \pm 0.7 \pm 0.9$
$D^0 \to \pi^- \left[\pi_2(1670)^+ \to \pi^+ \sigma\right]$	$-51.35 \pm 22.21 \pm 15.18$	$-209.98 \pm 22.21 \pm 41.58$	$3.5 \pm 0.6 \pm 0.8 \pm 0.9$
$D^0 \to \sigma\, f_0(1370)$	$27.71 \pm 6.81 \pm 19.04$	$71.93 \pm 6.41 \pm 17.44$	$21.2 \pm 1.8 \pm 4.2 \pm 5.2$
$D^0 \to \sigma\, \rho(770)^0$	$41.99 \pm 4.19 \pm 4.42$	$-25.42 \pm 3.62 \pm 6.53$	$6.6 \pm 1.0 \pm 1.2 \pm 3.0$
$D^0[S] \to \rho(770)^0\, \rho(770)^0$	$2.37 \pm 1.24 \pm 2.00$	$8.89 \pm 1.35 \pm 1.83$	$2.4 \pm 0.7 \pm 1.1 \pm 1.0$
$D^0[P] \to \rho(770)^0\, \rho(770)^0$	$-2.51 \pm 1.33 \pm 1.46$	$-20.80 \pm 1.48 \pm 3.67$	$7.0 \pm 0.5 \pm 1.6 \pm 0.3$
$D^0[D] \to \rho(770)^0\, \rho(770)^0$	$-33.99 \pm 3.34 \pm 5.11$	$-7.64 \pm 2.62 \pm 4.77$	$8.2 \pm 1.0 \pm 1.7 \pm 3.5$
$D^0 \to f_2(1270)\, f_2(1270)$	$-34.47 \pm 21.71 \pm 22.46$	$-172.87 \pm 21.71 \pm 27.01$	$2.1 \pm 0.5 \pm 0.3 \pm 2.3$
Sum			$122.0 \pm 4.0 \pm 6.4 \pm 7.6$

First model-independent γ measurement (BELLE 2011)



$$\gamma = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$$
$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011$$
$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ,$$

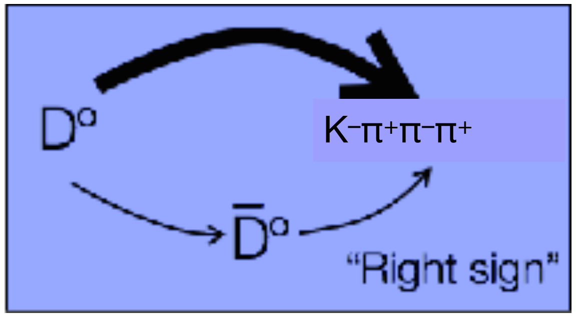


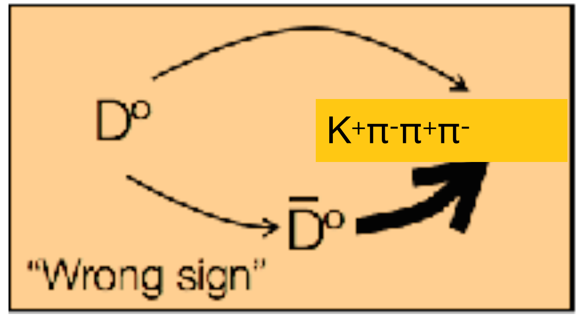
where the last uncertainty on γ of 4.3° the former model uncertainty of 8.9°

BELLE: **arXiv:1106.4046**. See also Anton Poluektov's talk at Moriond EW 2011 (from which I lifted several of the plots shown here): http://belle.kek.jp/belle/talks/moriondEW11/poluektov.pdf
CLEO-c input:Phys.Rev.D82:112006,2010.

LHCb PRL 116 (2016) no.24, 241801

Charm mixing in D→Kπππ

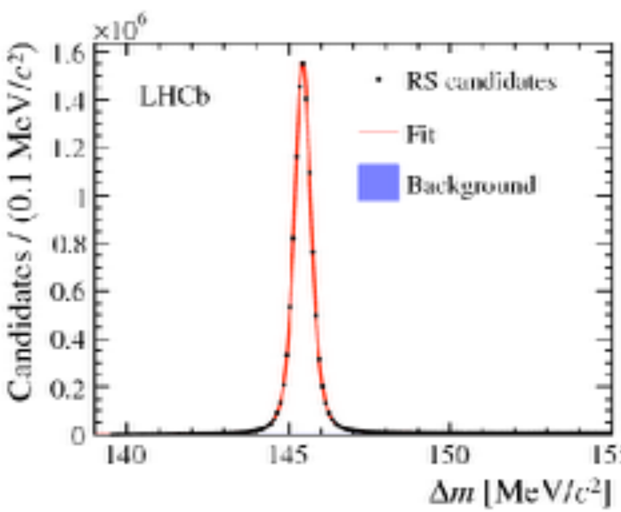


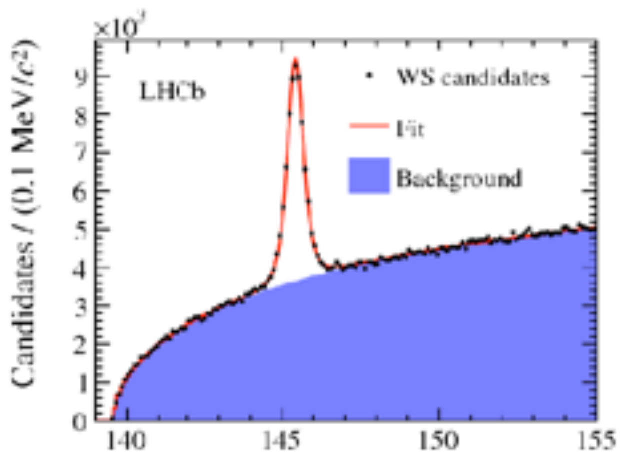


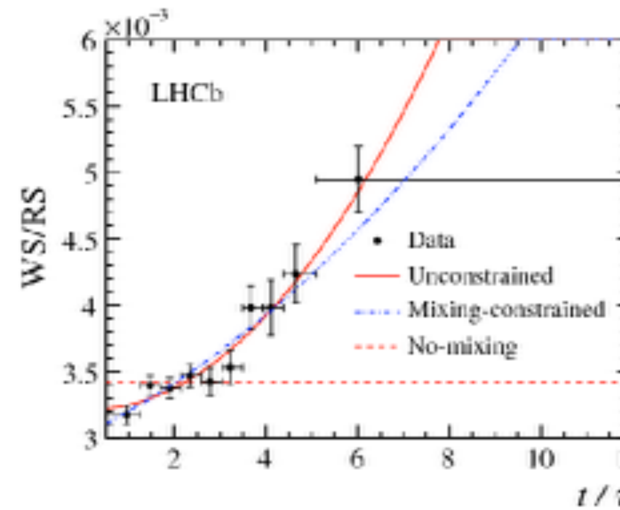
$$\frac{\Gamma(D^0 \to K^-\pi^+\pi^-\pi^+)}{\Gamma(D^0 \to K^+\pi^-\pi^+\pi^-)}$$

11M RS events

43k WS events



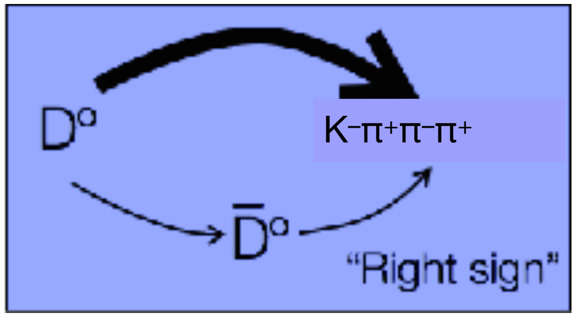




First >5σ observation of charm mixing in a single channel other than D→Kπ

LHCb PRL 116 (2016) no.24, 241801

Charm mixing in D→Kπππ



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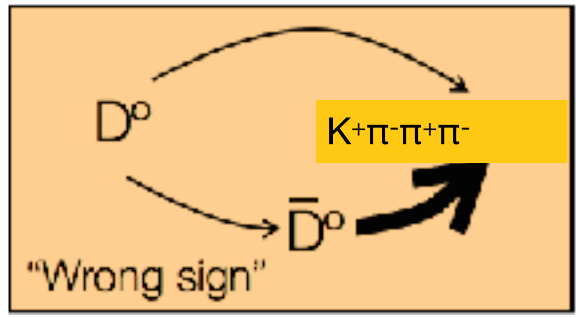
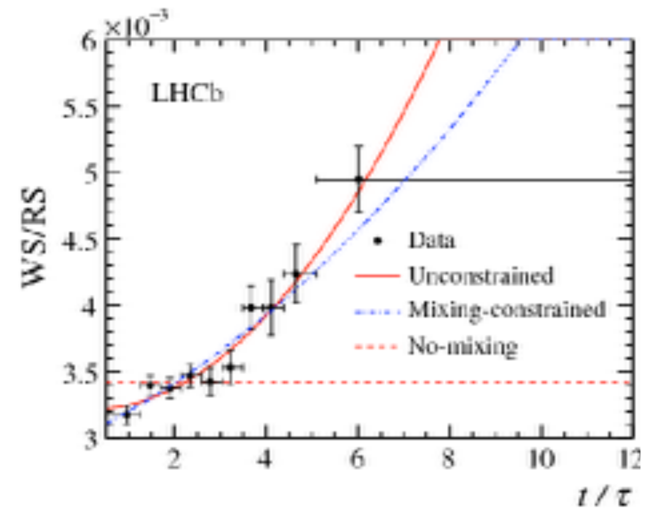


43k WS events

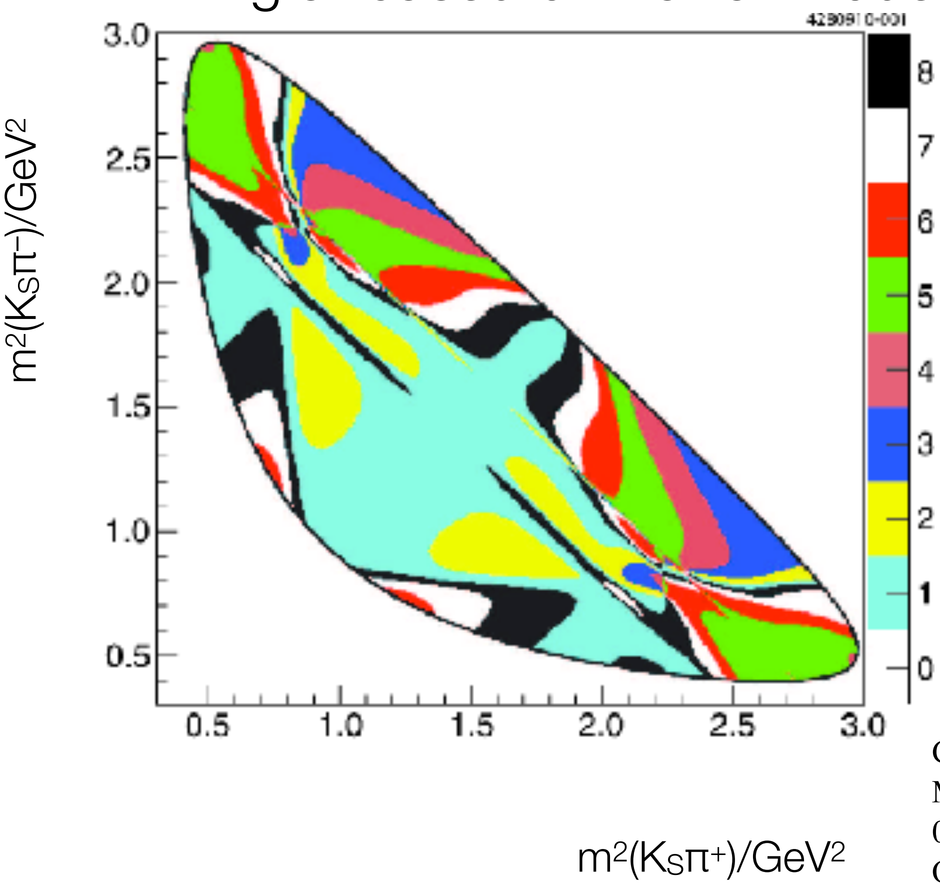
$$\frac{\Gamma(D^0 \to K^-\pi^+\pi^-\pi^+}{\Gamma(D^0 \to K^+\pi^-\pi^+\pi^-}$$



First >5σ observation of charm mixing in a single channel other than D→Kπ

Measurement of phases at CLEO-c

Binning at based on BaBar model*



Used by BELLE & LHCb for model-independent measurements of γ.

Looking forward to update by BESIII with much larger dataset.

*model = BaBar PRL 95 (2005) 121802

[1] Bondar, Poluektov hep-ph/0703267v1 (2007)

CLEO-c input:: Phys. Rev. D 82 112006

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)

BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

LHCb's latest model-independent γ measurement: JHEP 1410 (2014) 097

Multi-Generational Flavour Physics

Edward V. Brewer (1883 – 1971)

Multi-Generational Flavour Physics

Edward V. Brewer (1883 – 1971)

Regrettably, CLEO recently deceased - but her data live on.

Model-independent γ from $B^\pm \to (K_S\pi\pi)_D K$, $B^\pm \to (K_S KK)_D K$

- Binned, model-independent analysis using CLEO-c input.
  Phys. Rev. D 82 112006.

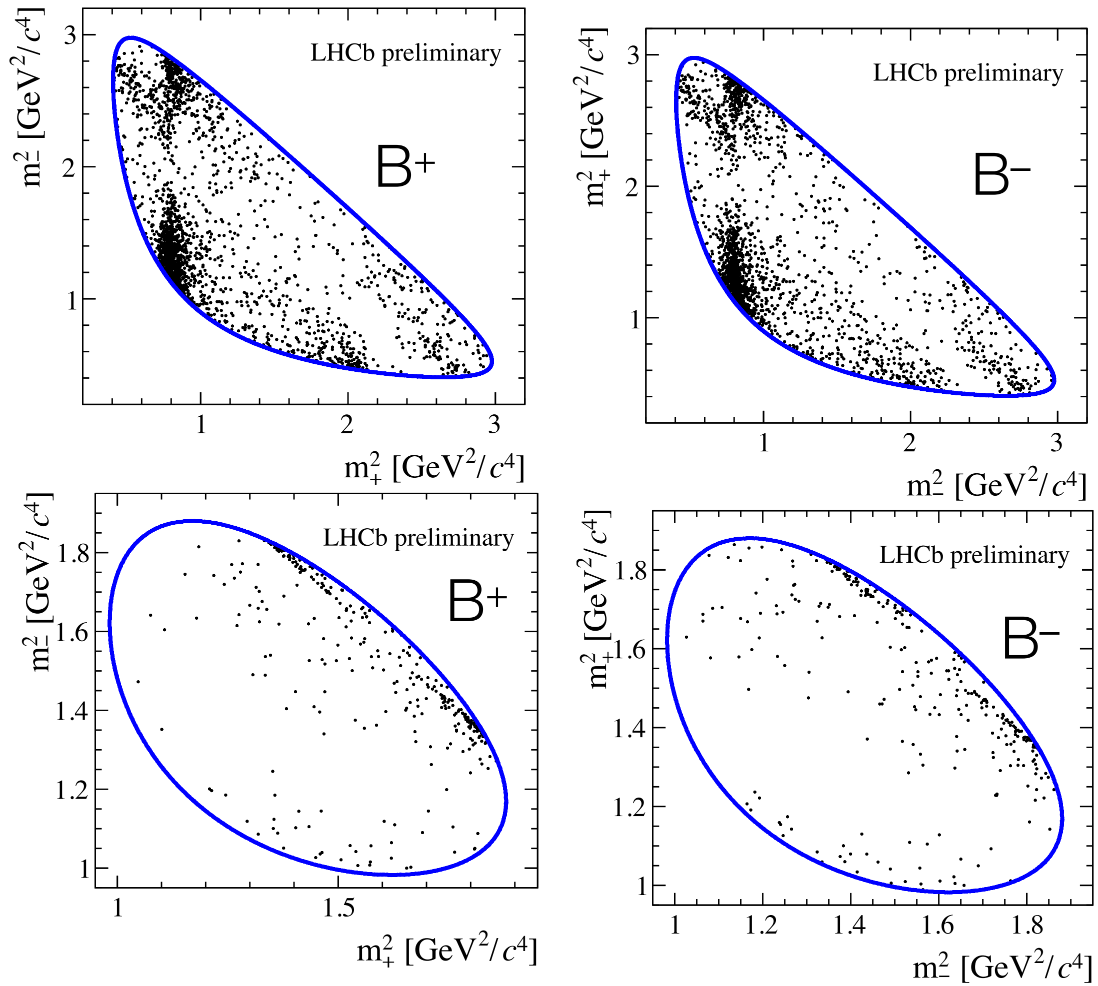
- Plots show LHCb data - the colours represent the bins, shaped to optimise sensitivity.

- Result of combined analysis (3fb^{-1}, 2011+2012, $K_S\pi\pi$ & $K_S KK$):

$$\gamma = (62\,^{+15}_{-14})^\circ$$

$$\delta_B = (134\,^{+14}_{-15})^\circ$$

$$r_B = 0.080\,^{+0.019}_{-0.021}$$



CLEO-c input:: Phys. Rev. D 82 112006.
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JHEP 10 (2014) 097
LHCb 2011 Result: Phys. Lett. B718 (2012) 43

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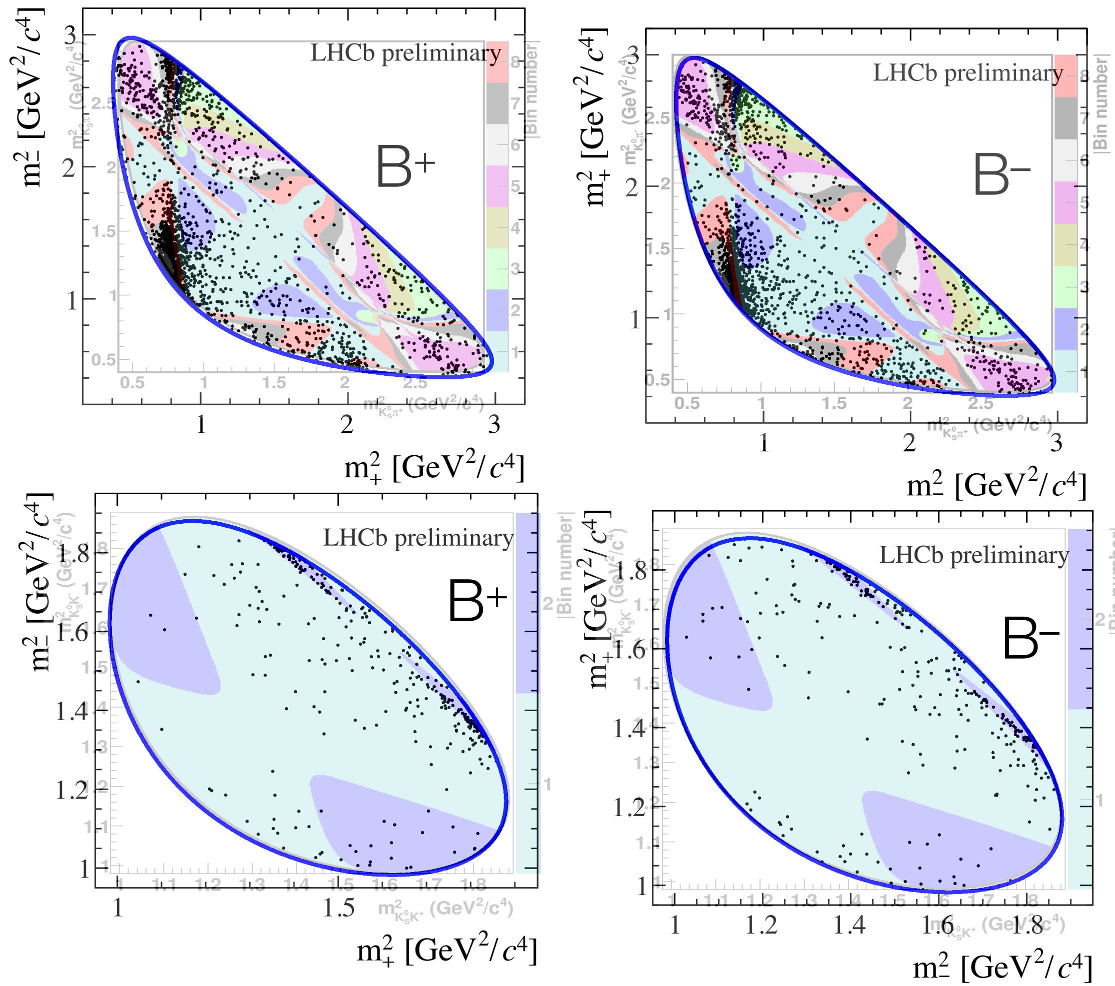

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Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).
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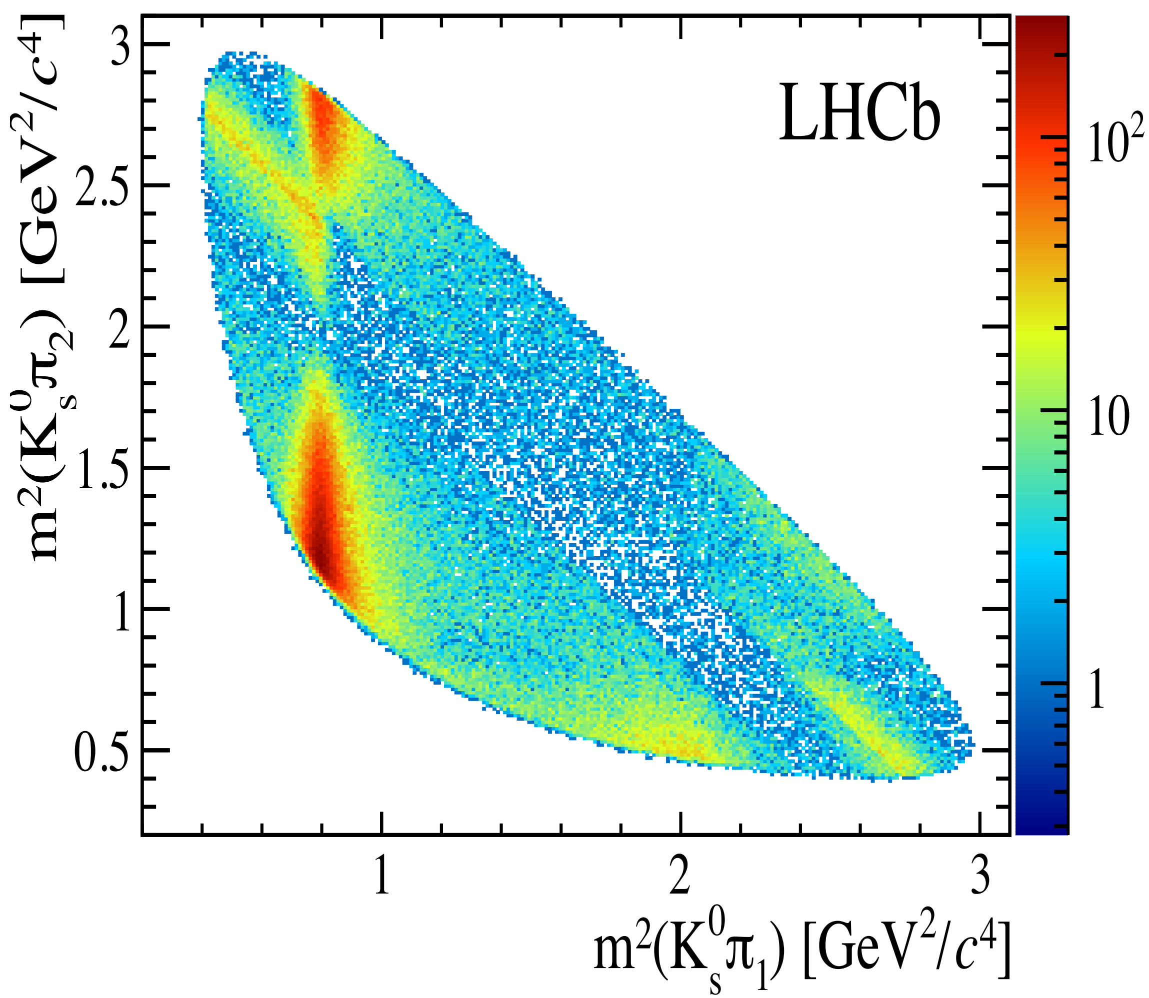
BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

Also input for model-independent charm mixing



LHCb: JHEP 1604 (2016) 033

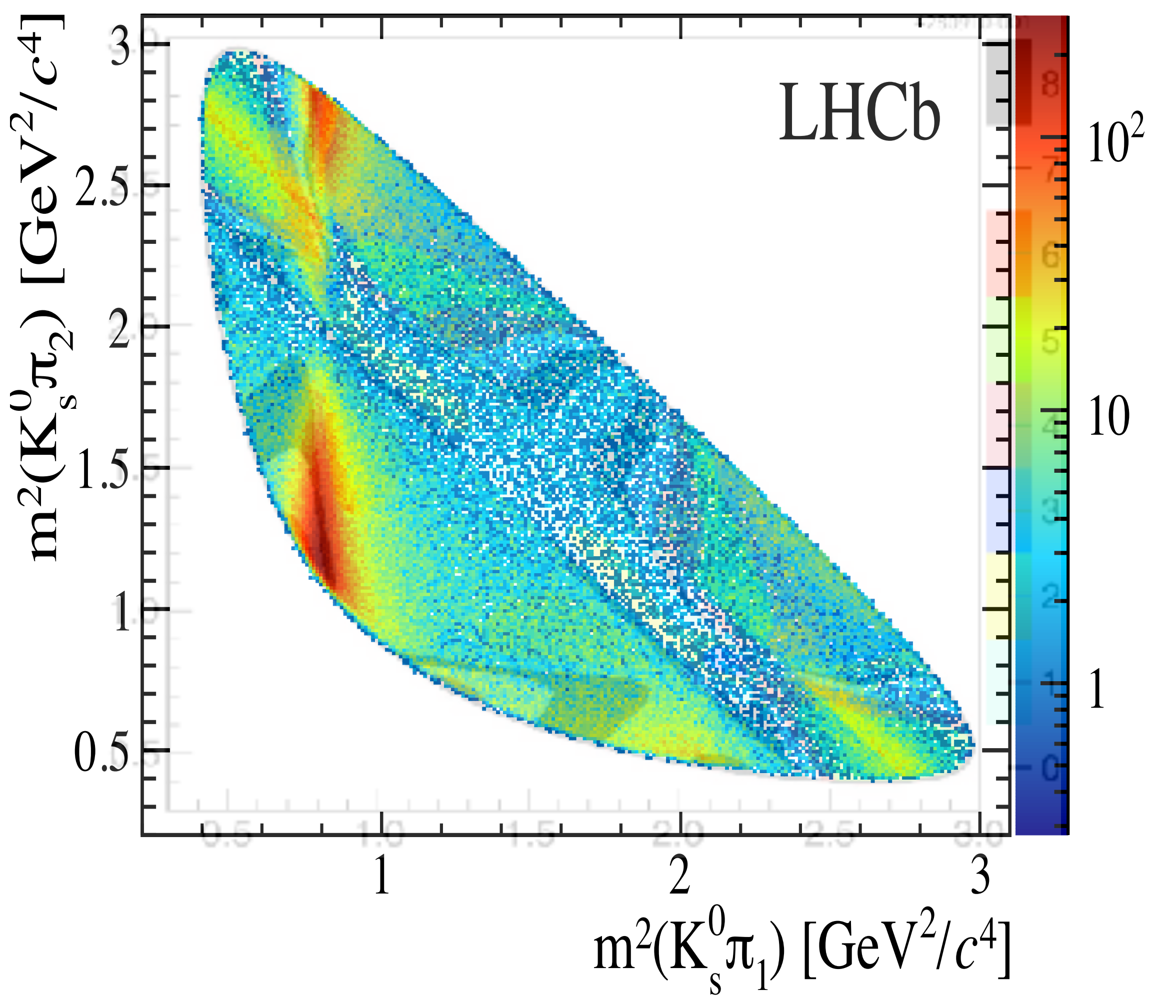
- First model-independent 3-body mixing, uses CLEO input
- 1/fb (only a fraction of Run 1)
- So far, only mixing, no CPV

  x=(–0.86±0.53±0.17)×10^{-2}

  y=(+0.03±0.46±0.13)×10^{-2}

  (compatible with, but less precise than BaBar, BELLE model-dependent analyses)
- Looking forward to analysis with much more data (and perhaps BESIII input)

Also input for model-independent charm mixing



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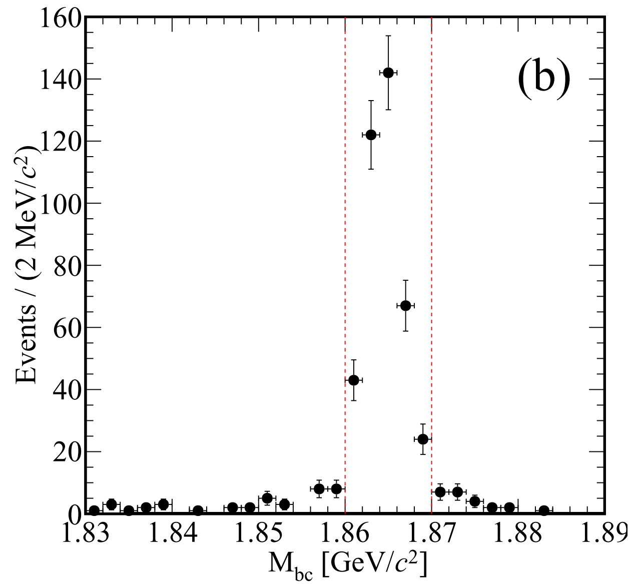
The CP content of $D^0 \to \pi^-\pi^+\pi^0$ measured at CLEO.

Phys.Lett. B740 (2015) 1-7

$$F_+ \equiv \frac{N^+}{N^+ + N^-}$$

reconstructed D° $D^0 \to \pi^-\pi^+\pi^0$ mass for events tagged as…

CP-even



effectively a CP eigenstate

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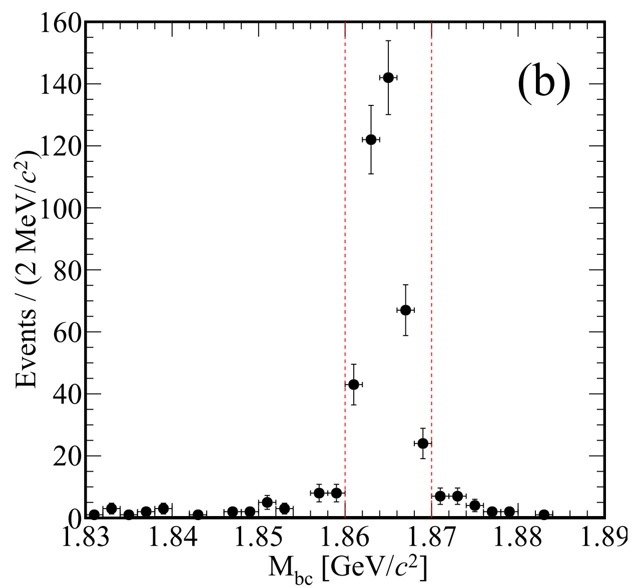
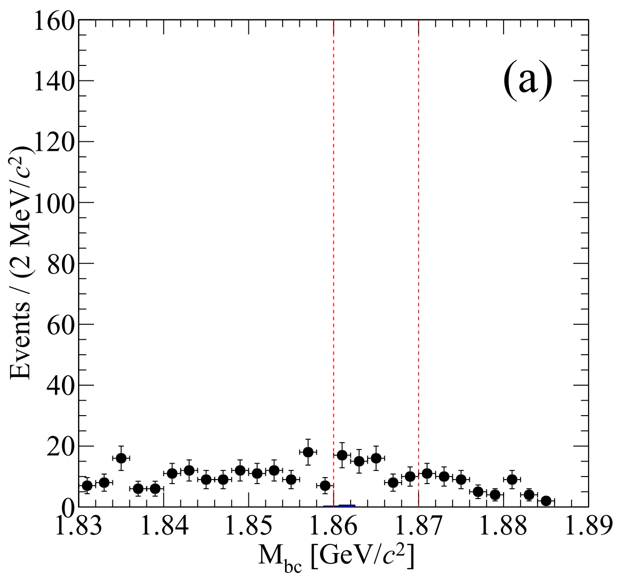


CP-odd



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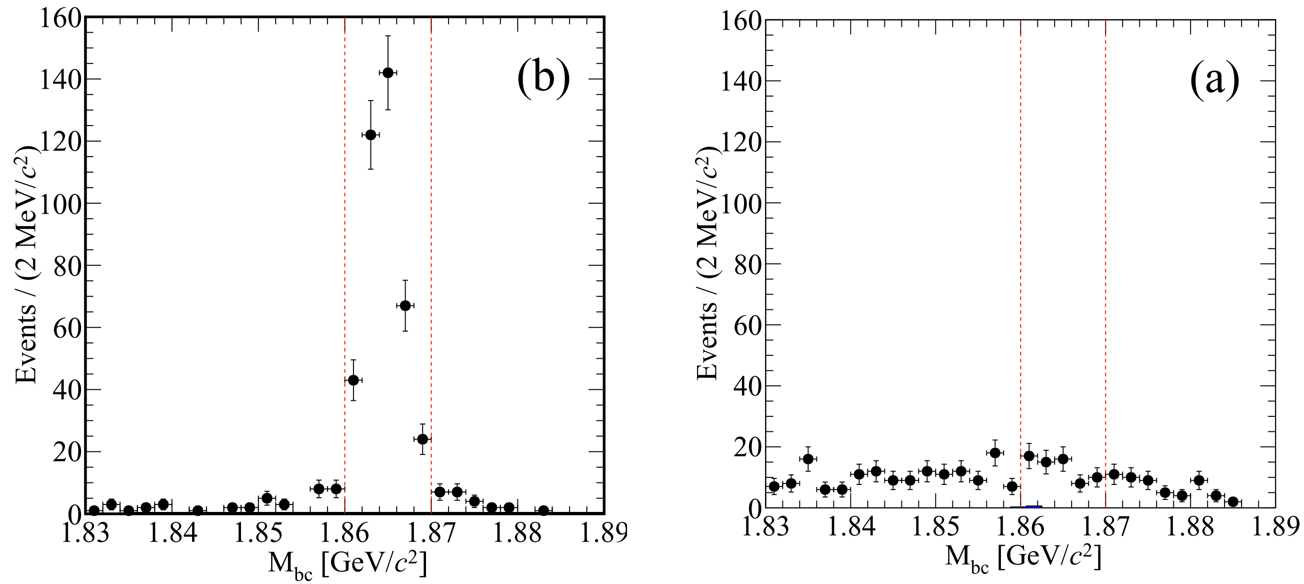

$$F_+(\pi^+\pi^-\pi^0) = 0.968 \pm 0.017 \pm 0.006$$

effectively a CP eigenstate

4pi model

Decay channel	$\Re(a_i)$	$\Im(a_i)$	$F_i\,(\%)$
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$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+\,\sigma\right]$	$56.46 \pm 13.85 \pm 14.49$	$167.87 \pm 14.51 \pm 19.38$	$10.2 \pm 1.4 \pm 2.1 \pm 2.5$
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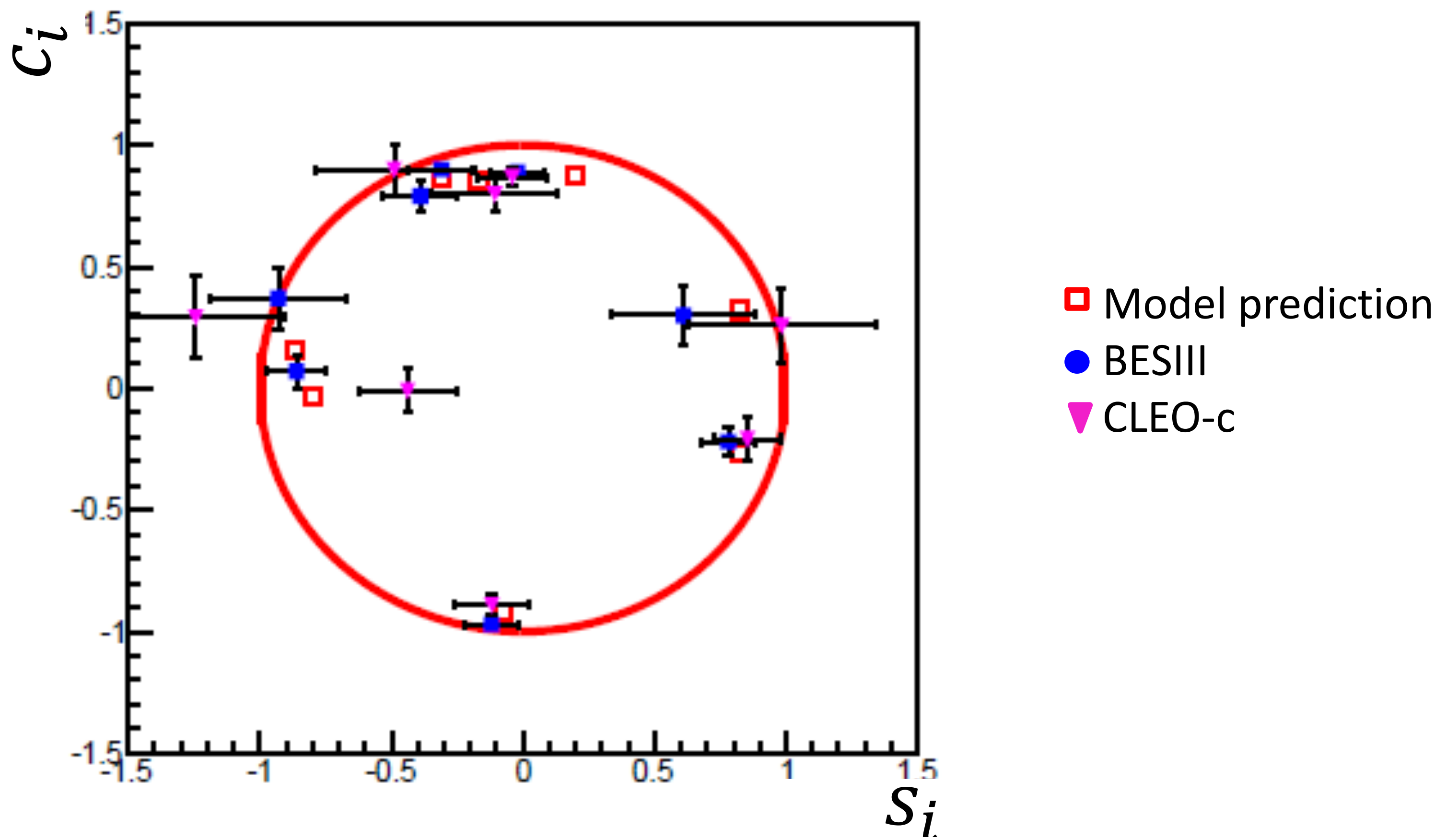
Sum			$122.0 \pm 4.0 \pm 6.4 \pm 7.6$

Measurement of $D^o \to K_S\pi\pi$ phases at BES III

BES III preliminary (see Dan Ambrose, APS 2014)

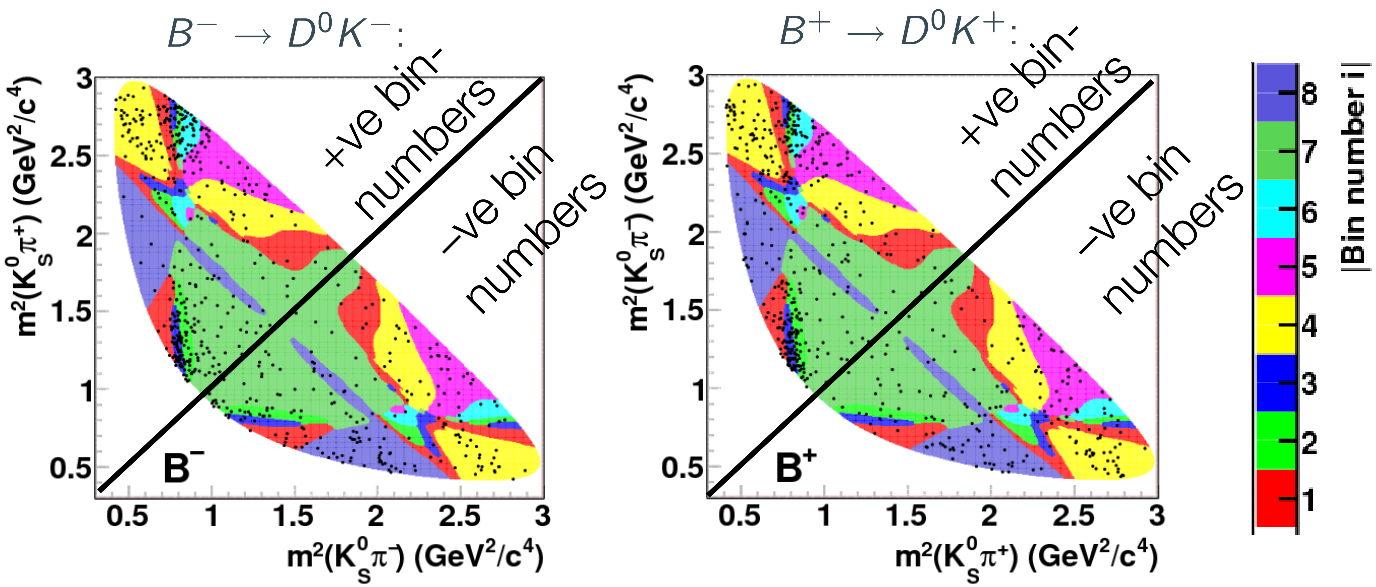


(different binning than prev slide)

Table of yields

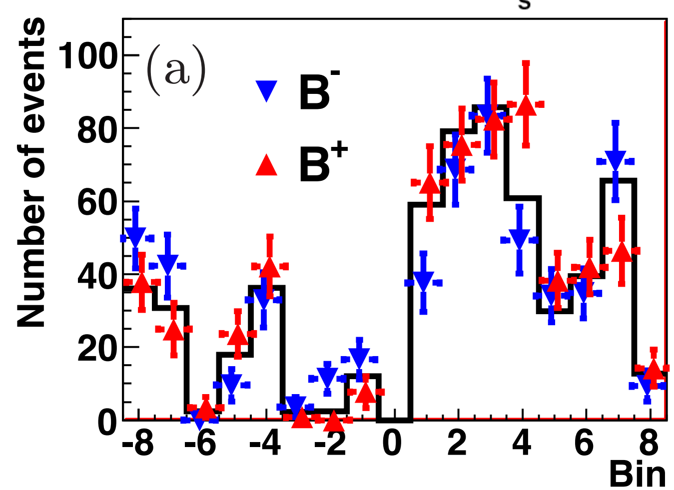
Mode	ST yield	DT yields			
		$K_S^0\pi^+\pi^-$	$K_L^0\pi^+\pi^-$	$K_S^0K^+K^-$	$K_L^0K^+K^-$
Flavor tags					
$K^-\pi^+$	144563 ± 403	1444	2857	168	302
$K^-\pi^+\pi^0$	258938 ± 581	2759	5133	330	585
$K^-\pi^+\pi^+\pi^-$	220831 ± 541	2240	4100	248	287
$K^-e^+\nu$		1191		100	
CP-even tags					
K^+K^-	13349 ± 128	124	357	12	32
$\pi^+\pi^-$	6177 ± 114	61	184	4	13
$K_S^0\pi^0\pi^0$	6838 ± 134	56		7	14
$K_L^0\pi^0$		237		17	
$K_L^0\eta(\gamma\gamma)$				4	
$K_L^0\eta(\pi^+\pi^-\pi^0)$				1	
$K_L^0\omega$				4	
$K_L^0\eta'$				1	
CP-odd tags					
$K_S^0\pi^0$	19753 ± 153	189	288	18	43
$K_S^0\eta(\gamma\gamma)$	2886 ± 71	39	43	4	6
$K_S^0\eta(\pi^+\pi^-\pi^0)$				2	1
$K_S^0\omega$	8830 ± 110	83		14	10
$K_S^0\eta'$				3	4
$K_L^0\pi^0\pi^0$				5	
$K_S^0\pi^+\pi^-$		473	1201	56	126
$K_L^0\pi^+\pi^-$				140	
$K_S^0K^+K^-$				4	9

First model-independent γ measurement (BELLE 2011)



$$\gamma = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$$
$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011$$
$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ,$$

where the last uncertainty on γ of 4.3° the former model uncertainty of 8.9°



BELLE: **arXiv:1106.4046**. See also Anton Poluektov's talk at Moriond EW 2011 (from which I lifted several of the plots shown here): http://belle.kek.jp/belle/talks/moriondEW11/poluektov.pdf
CLEO-c input:Phys.Rev.D82:112006,2010.

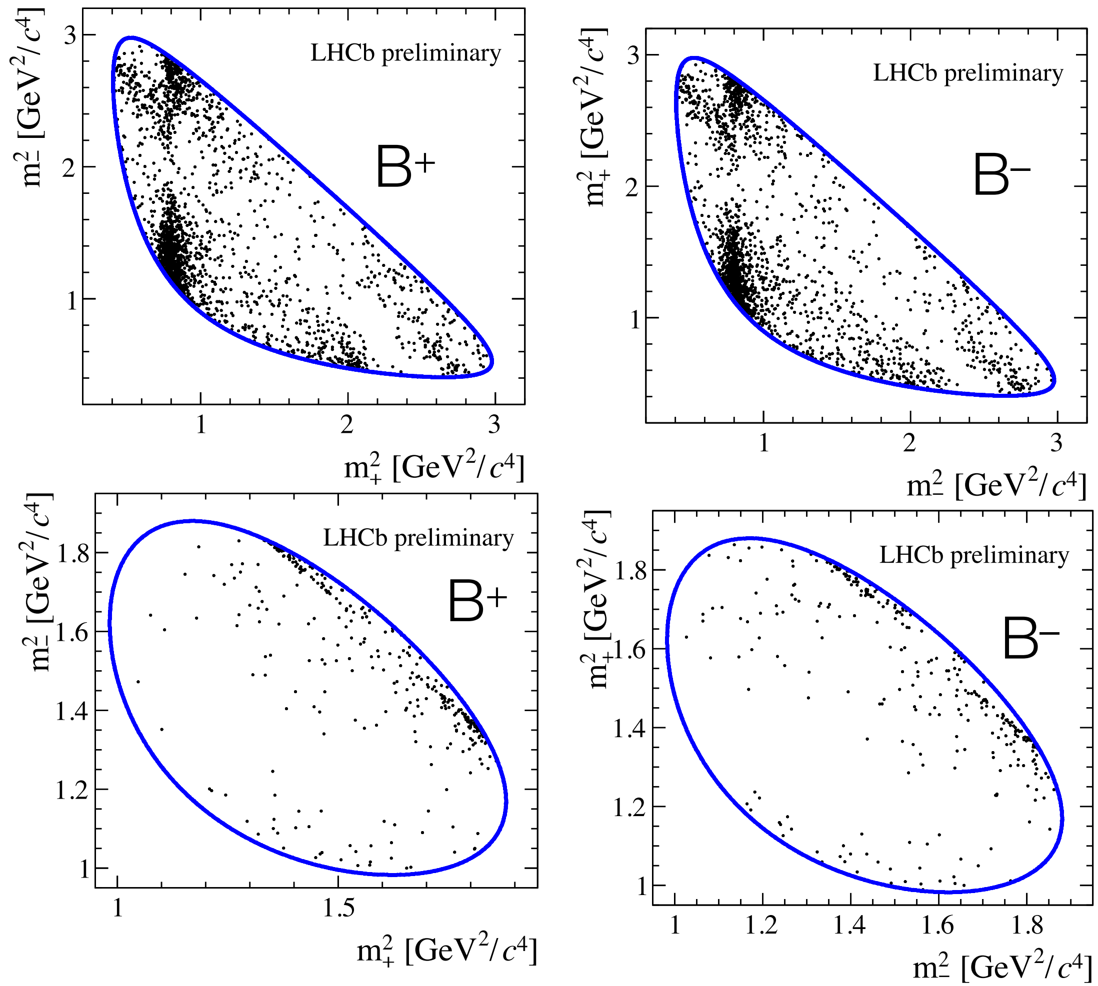
Model-independent γ from $B^\pm \to (K_S\pi\pi)_D K$, $B^\pm \to (K_S KK)_D K$

- Binned, model-independent analysis using CLEO-c input.
  Phys. Rev. D 82 112006.
- Plots show LHCb data - the colours represent the bins, shaped to optimise sensitivity.
- Run 1 analysis ($3\mathrm{fb}^{-1}$, $K_S\pi\pi$ & $K_S KK$):

$$\gamma = (62\,^{+15}_{-14})^\circ$$

$$\delta_B = (134\,^{+14}_{-15})^\circ$$

$$r_B = 0.080\,^{+0.019}_{-0.021}$$



CLEO-c input:: Phys. Rev. D 82 112006.
Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).
Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)
BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

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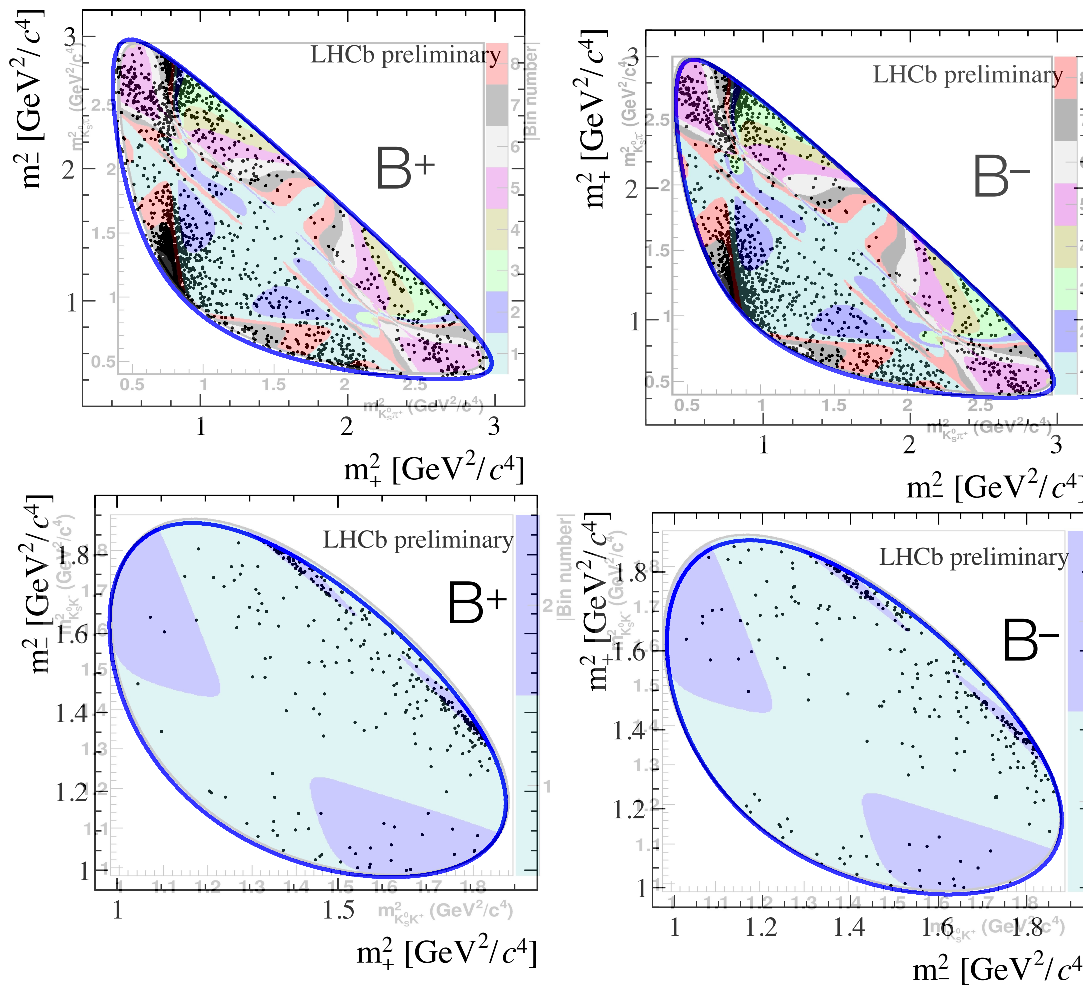

CLEO-c input:: Phys. Rev. D 82 112006.
Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).
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BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

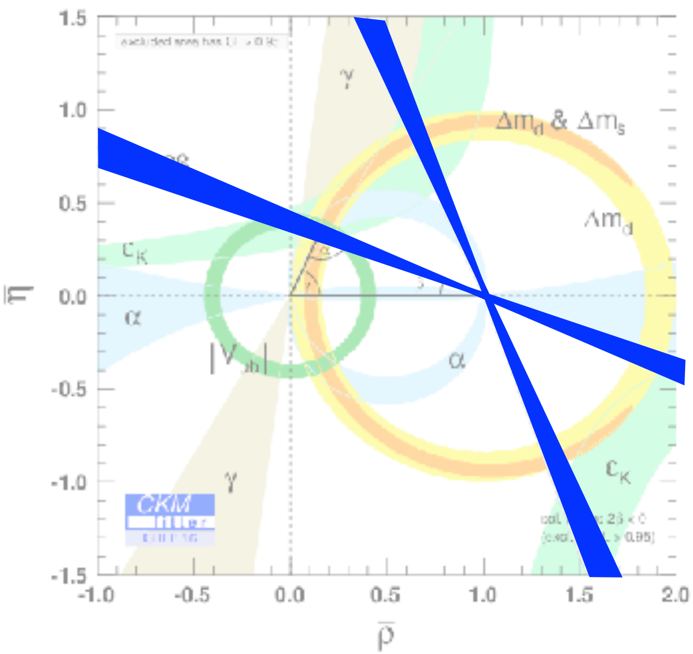
Binned time-dep $B^o \to D^{(*)o}(\pi^o, \eta, \eta', \omega)$, $D \to K_S\pi\pi$

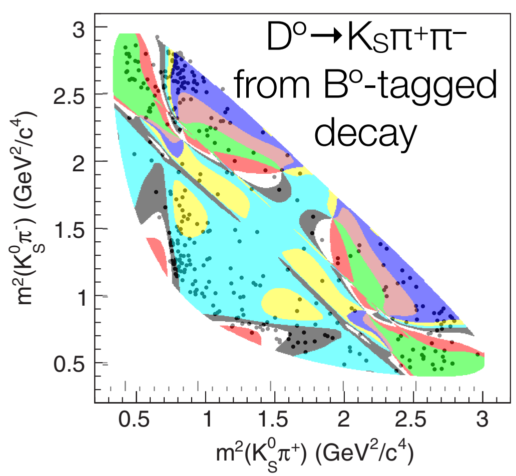
BELLE: PRD94 (2016) no.5, 052004

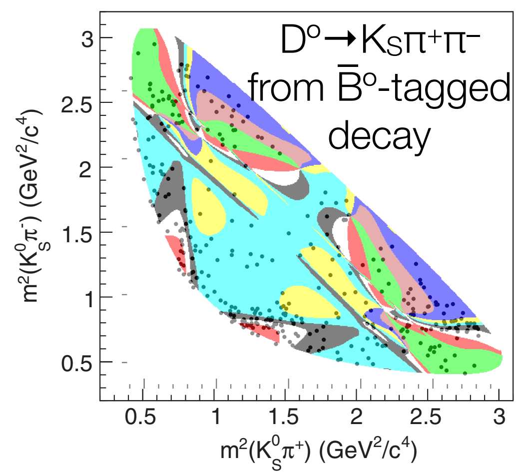
Result @ BELLE

$$\sin 2\varphi_1 = 0.43 \pm 0.27(\text{stat}) \pm 0.08(\text{syst})$$
$$\cos 2\varphi_1 = 1.06 \pm 0.33(\text{stat})^{+0.21}_{-0.15}(\text{syst})$$

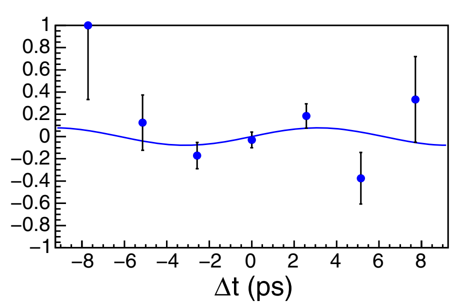
uncertainty due to (mainly stat) uncertainty in CLEO-c input:
0.03 (sin), +0.21–0.14 (cos)



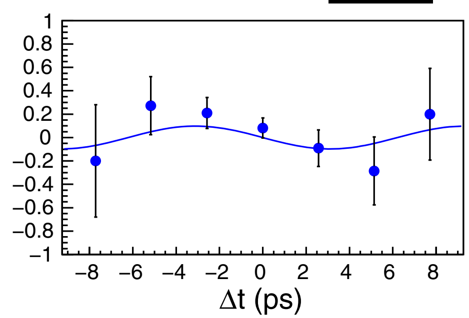




CP-asymmetry in **cyan** bin



CP-asymmetry in **green** bin



Previously (less data, model-dependent): BELLE PRL 97 (2006) 081801 BaBar: PRL 99 (2007) 231802

Binned time-dep $B^o \to D^{(*)o}(\pi^o, \eta, \eta', \omega)$, $D \to K_S\pi\pi$

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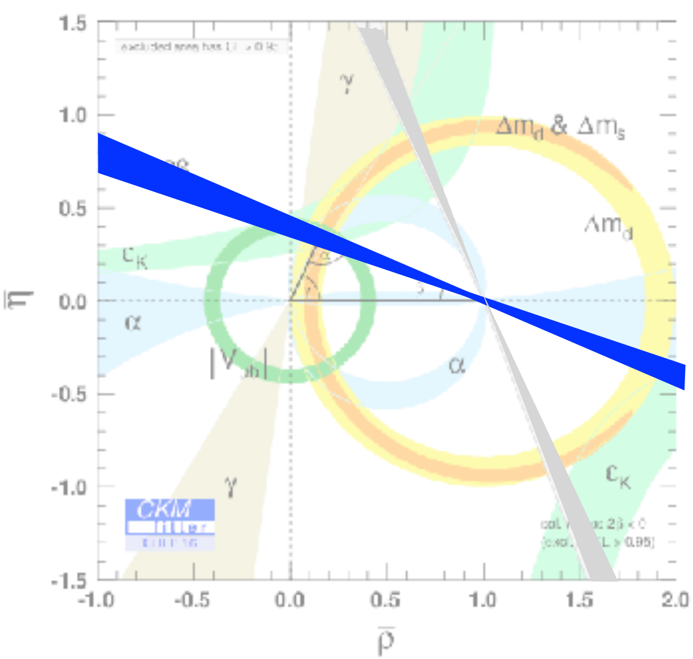
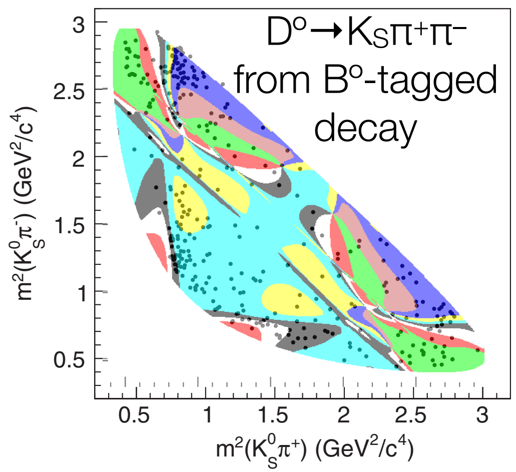
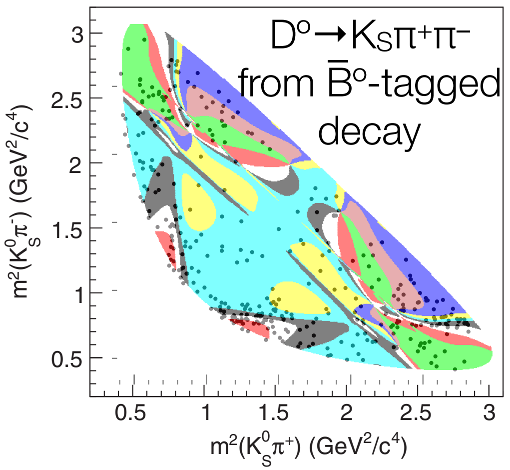
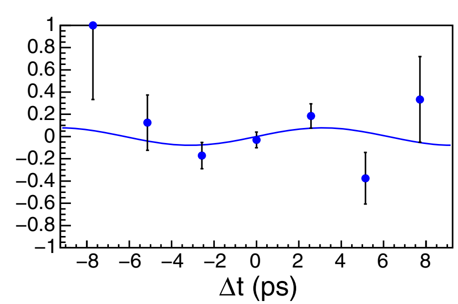


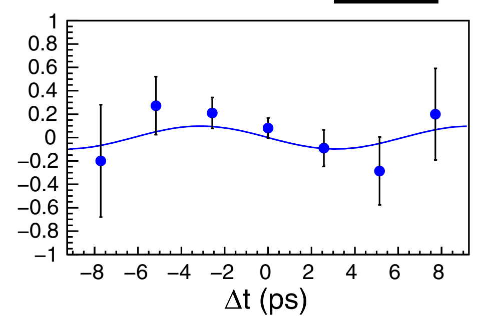




CP-asymmetry in **cyan** bin

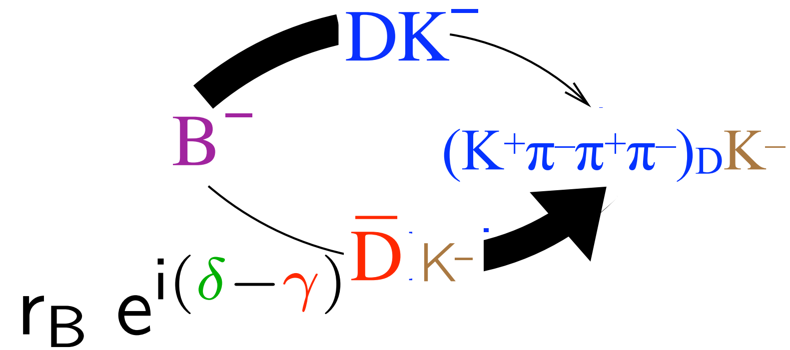


CP-asymmetry in **green** bin



Previously (less data, model-dependent): BELLE PRL 97 (2006) 081801 BaBar: PRL 99 (2007) 231802

Coherence Factor Analysis of



- Treat K3π like two-body decay with single effective strong phase δ_D.

$$\Gamma\left(B^- \to \left(K^+3\pi\right)_D K^-\right) \propto r_B^2 + \left(r_D^{K3\pi}\right)^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos\left(\delta_B + \delta_D^{K3\pi} - \gamma\right)$$

$$r_B = \left|\frac{A(B^- \to \overline{D}^0 K^-)}{A(B^- \to D^0 K^-)}\right| \qquad r_D = \left|\frac{A(D^0 \to K^+\pi^-\pi^+\pi^-)}{A(\overline{D}^0 \to K^+\pi^-\pi^+\pi^-)}\right|$$

Coherence factor R < 1;
larger R gives higher sensitivity to γ

- Conceptually the same as F+ and c_i, s_i (shown later), related through

$$Re^{-i\delta_D} = c_i + is_i$$

R_{K3pi} is the coherence factor introduced by Atwood & Soni, PRD68 (2003) 033003
Also interesting in relation to charm mixing, see

- Bondar, Poluektov, Vorobiev: PRD 82 (2010) 034033,
- Malde & Wilkinson PLB701 (2011) 353-356,
- Malde, Thomas & Wilkinson PRD91 (2015) no.9, 094032,
- Harnew & JR: JHEP 1503 (2015) 169, PLB 728 (2014) 296
- LHCb: PRL116 (2016) no.24, 241801
- Evans et al: PLB 757 (2016) 520