Measuring γ in a binned $D \to K\pi\pi\pi$ analysis

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Joint BESIII-LHCb workshop 2018

$D^0 \to K^{\mp}\pi^{\pm}\pi^{\pm}\pi^{\mp}$ decay modes

$D^0 \to K^-\pi^+\pi^+\pi^-$, largest contribution from:

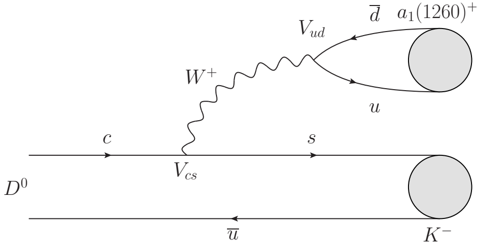


Diagram is $\mathcal{O}(1)$ in terms of CKM matrix element $\to$ Cabibbo favoured (CF). BR. $\sim 8\%$

$D^0 \to K^+\pi^-\pi^-\pi^+$, largest contribution from:

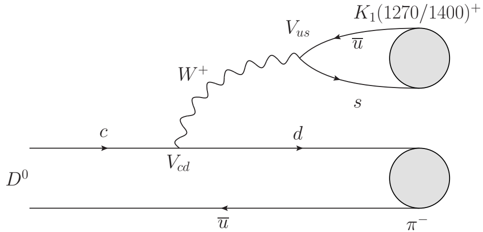
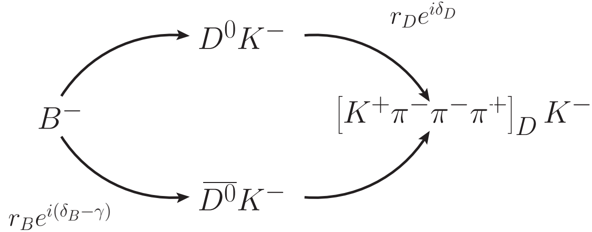


Diagram has two off-diagonal CKM elements $\to$ doubly-Cabibbo suppressed (DCS). BR. $\sim 2 \times 10^{-4}$

ADS method



- Both DCS and CF amplitudes contribute to $B^\mp \to [K^\pm\pi^\mp\pi^\mp\pi^\pm]_D K^\mp$ with differing weak phases.
- Phase-space integrated rate is

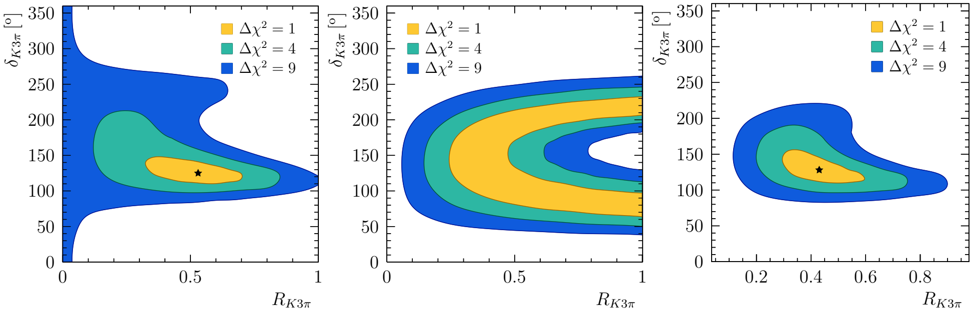
$$\Gamma(B^\mp \to \left[K^\pm\pi^\mp\pi^\mp\pi^\pm\right]_D K^\mp) \propto r_{K3\pi}^2 + r_B^2 + 2 r_B R_{K3\pi} r_{K3\pi} \cos(\delta_B \mp \gamma - \delta_{K3\pi})$$

- Where the coherence factor $R_{K3\pi}$ and $\delta_{K3\pi}$, the average strong-phase difference are defined by:

$$R_{K3\pi} e^{-i\delta_{K3\pi}} = \frac{\int \mathrm{d}\mathbf{x} \mathcal{A}_{D^0\to K^+\pi^-\pi^-\pi^+}(\mathbf{x}) \mathcal{A}^*_{\bar{D}^0\to K^+\pi^-\pi^-\pi^+}(\mathbf{x})}{A_{D^0\to K^+\pi^-\pi^-\pi^+} A_{\bar{D}^0\to K^+\pi^-\pi^-\pi^+}}$$

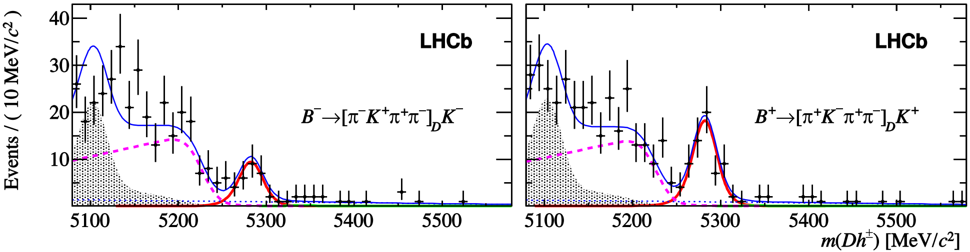
- So $0 \leqslant R_{K3\pi} \leqslant 1$

Coherence Factor and associated hadronic parameters



- Quantum correlated observables associated with $e^+e^- \to \psi(3770) \to D^0\overline{D}^0$ have direct sensitivity to coherence factor and strong-phase differences.
- Measured at CLEO-c using the ‘double-tag’ method [1] (left).
- Also some sensitivity in $D^0\overline{D}^0$ mixing, as shown by LHCb [2] (centre).
- Right: Combination of CLEO-c and LHCb mixing results.

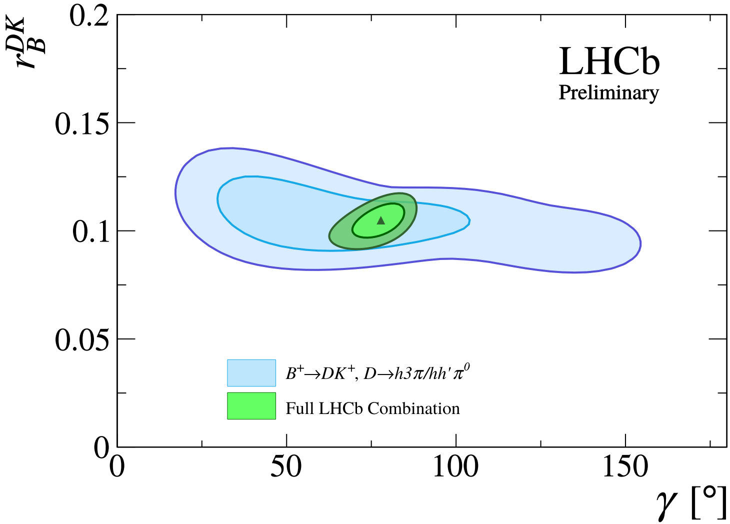
Integrated *CP* asymmetry (Run I)



- Phase-space integrated asymmetries and charge-average ratios measured by LHCb in Run I [3].
- Different diagrams are roughly the same order, therefore can generate large *CP* asymmetries, in this case $\sim 40\%$.

Constraints on γ

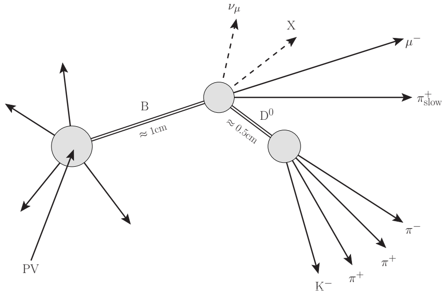
Take CP asymmetries, add measurement of $R_{K3\pi}, \delta_{K3\pi}, r_{K3\pi}$. Useful for constraining γ, but does not provide a stand-alone measurement. Combine with other D (and B) decay modes to measure γ [4]



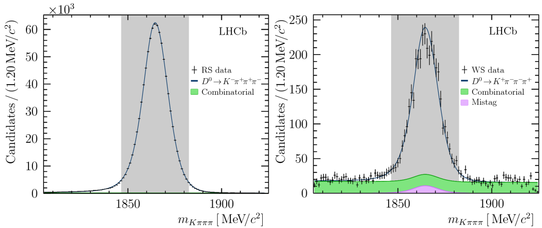
Binned method

- Potential gains if variation in amplitudes across the phase-space of the D^0 decay is understood.
- In principle, could do an amplitude analysis to extract γ.
- **BUT:** Model-dependent uncertainties associated with amplitude analyses are notoriously difficult to estimate reliably.
- Instead, use models to develop binning schemes with optimal sensitivity to γ.
- Then in these bins, measure CP asymmetries with LHCb and hadronic parameters at charm threshold and in mixing at LHCb.
- Gives a model-independent method to measure γ with a single D^0 decay mode.
- Will very briefly introduce the amplitude models.

LHCb models of $D^0 \to K^{\mp}\pi^{\pm}\pi^{\pm}\pi^{\mp}$

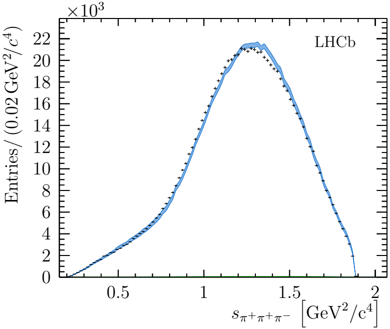


Reconstruct $B \to D^*(2010)^+ \left[D^0\pi^+\right] \mu^- X$ as a clean source of D^0 decays. Charge of 'slow' pion and muon relative to kaon is used to infer D^0 flavour at production.



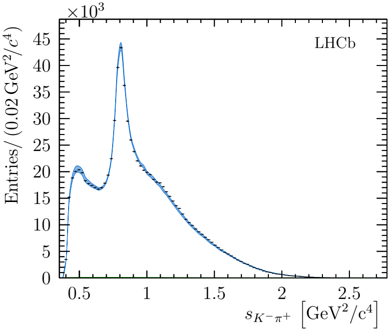
- Shown: D^0 mass peaks for 'Right Sign' (RS) and 'Wrong Sign' (WS).
- RS sample has $\sim 900,000$ candidates @ $> 99.9\%$ purity, WS has $\sim 3000@80\%$ purity.
- First ever amplitude analysis of the DCS amplitude.
- Submitted to EPJ-C, available on the arXiv [5].

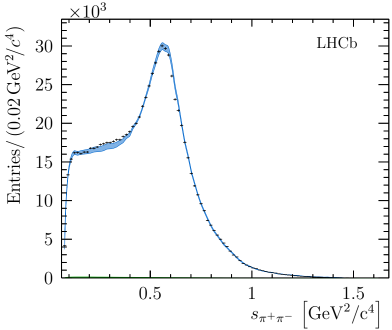
$D^0 \to K^-\pi^+\pi^+\pi^-$ amplitude model



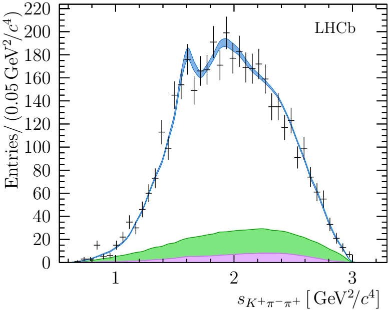
Largest contributions from:

- $D^0 \to a_1(1260)^+ K^- \sim 40\%$
- $D^0 \to \overline{K}^*(892)^0 \rho(770)^0 \sim 20\%$
- $D^0 \to [K^-\pi^+]^{L=0} [\pi^+\pi^-]^{L=0} \sim 20\%$





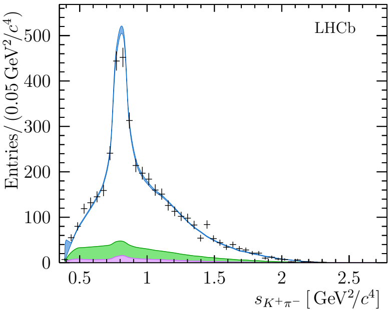
$D^0 \to K^+\pi^-\pi^-\pi^+$ amplitude model

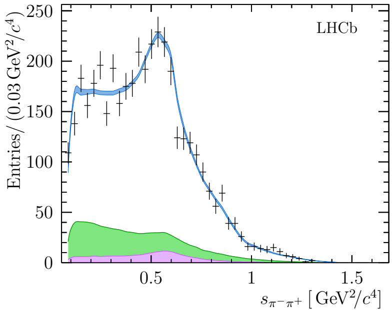


Largest contributions from:

- $D^0 \to K_1(1270/1400)^+\pi^- \sim 40\%$
- $D^0 \to K^*(892)^0\rho(770)^0 \sim 20\%$
- $D^0 \to [K^+\pi^-]^{L=0}[\pi^+\pi^-]^{L=0} \sim 20\%$

Roughly 20% background contribution shown by shaded area.





Binned Measurement of γ

- Construct yield ratios in bins of D decay phase-space.

$$R^i_+ = \frac{N(B^+ \to [K^-\pi^+\pi^+\pi^-]_D K^+)}{N(B^+ \to [K^+\pi^-\pi^-\pi^+]_D K^+)}$$
$$R^i_- = \frac{N(B^- \to [K^+\pi^-\pi^-\pi^+]_D K^-)}{N(B^- \to [K^-\pi^+\pi^+\pi^-]_D K^-)}$$

- It may be preferable to measure asymmetries and charge-averaged ratios to simplify treatment of systematic uncertainties in practice.
- In terms of physics parameters:

$$R^i_\pm = r_B^2 + r_i^2 + 2 r_B r_i R_i \cos\left(\delta_B - \delta_i \mp \gamma\right),$$

where r_i, R_i, δ_i are the binned average amplitude ratio, coherence factor and average strong-phase difference.

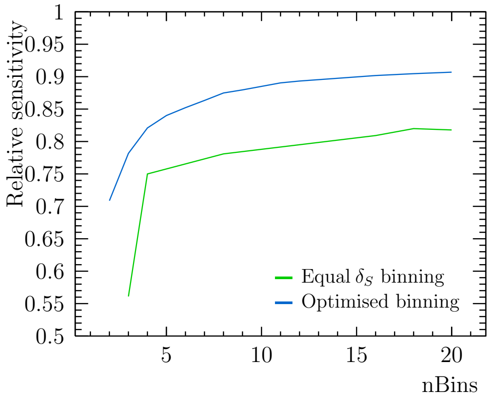
- Fit these ratios to extract γ, δ_B, r_B.

Binning schemes

- Phase space of four-body decays is five-dimensional: significantly less obvious what these should be than a three-body decay.
- Use the so-called 'helicity' coordinates.
- Divide phase-space into $\sim$ one million sub-bins: analogue of the 'pixels' of the $D^0 \to K^0_S \pi^+\pi^-$ binning.
- Approximately a $16 \times 16 \times 16 \times 16 \times 16$ mesh where each site has equal integrated phase-space density.
- *Equal δ_S binning*: Allocate sub-bins into small number of bins (4,8,16,...), grouped by equal strong-phase difference between WS and RS amplitudes.
- *Optimised binning*: Optimises sensitivity to γ, using the following algorithm:
  1. Start with equal δ_S binning.
  2. Loop over the sub-bins, changing the allocation of each sub-bin to maximise γ sensitivity.
  3. Repeat until the allocation of no sub-bin changes.

How well does it work?

- Define relative sensitivity as precision on γ from binned measurement compared to that of unbinned measurement with perfect knowledge of amplitude structure (unattainable).
- Unbinned sensitivity estimated using LHCb models and pseudo experiments.
- For LHCb Run I + II estimated to be $\sim 4^{o}$.
- With 8 bins, we can get to $\sim 90\%$ of that (or $\sim 4.6^{o}$).
- With 4 bins, $\sim 80\%$ (or $\sim 4.9^{o}$).

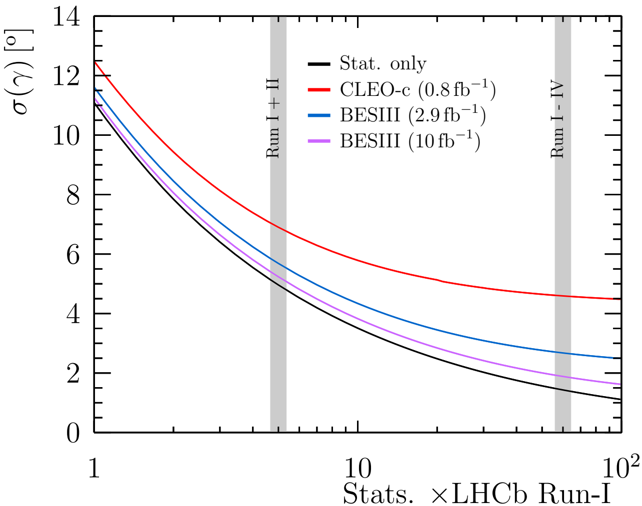


Hadronic uncertainties

- Can we measure, using Q.C. data and LHCb mixing measurements, the hadronic parameters in bins well enough?
- Rough estimates: take predicted central values of R_i, δ_i, assume uncertainties will be the same as for the global analysis, scaled by the statistics in bin.
- Repeat fits to binned yield ratios, floating hadronic parameters with Gaussian constraints $\rightarrow$ uncertainties automatically include hadronic part.

	$\sigma(R_i)$	$\sigma(\delta_i)$
CLEO-c	0.30	45^{o}
BES III ($2.9\,\mathrm{fb}^{-1}$)	0.16	24^{o}
BES III ($10\,\mathrm{fb}^{-1}$)	0.08	12^{o}

Assumed precision on binned parameters (four bins), including constraints from Charm Mixing.



With Run I+II LHCb data set + current BES III data set, expect uncertainty of $\sim 5.5^{\text{o}}$ on γ. (four bins)

- Compare with determination of γ in decays $B^+ \to \left[K^0_S\pi^+\pi^-\right]_D K^+$, which measured $\gamma = 62^{+15}_{-14}$ in Run I [6] (current most precise single measurement).
- Scaling uncertainties by $1/\sqrt{5}$ to estimate uncertainties in Run I+II gives $\sigma(\gamma) = 6.7^{\text{o}}$

Conclusions and Outlook

- Possibility to measure γ with excellent precision using $B^{\pm} \rightarrow [K^{\mp}\pi^{\pm}\pi^{\pm}\pi^{\mp}]_D K^{\pm}$ decays.
- Model-independent determination requires models to inspire an optimal division of the D decay phase space.
- With $10\,\mathrm{fb}^{-1}$ of BES III data, measurement of γ will be limited by LHCb statistics until $\sim$ end of upgrade phase I.

T. Evans *et al.*, *Improved determination of the $D \to K^-\pi^+\pi^+\pi^-$ coherence factor and associated hadronic parameters from a combination of $e^+e^- \to \psi(3770) \to c\bar{c}$ and $pp \to c\bar{c}X$ data*, Phys. Lett. **B757** (2016) 520, arXiv:1602.07430, [Erratum: Phys. Lett. **B765** (2017) 402].

LHCb collaboration, R. Aaij *et al.*, *First observation of D^0 oscillations in $D^0 \to K^+\pi^-\pi^-\pi^+$ decays and measurement of the associated coherence parameters*, arXiv:1602.07224.

LHCb, R. Aaij *et al.*, *Measurement of CP observables in $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ with two- and four-body D decays*, Phys. Lett. **B760** (2016) 117, arXiv:1603.08993.

LHCb collaboration, *Update of the LHCb combination of the CKM angle γ using $B \to DK$ decays*, Tech. Rep. LHCb-CONF-2017-004, CERN, 2017.

LHCb, R. Aaij *et al.*, *Studies of the resonance structure in $D^0 \to K^\mp\pi^\pm\pi^\pm\pi^\mp$ decays*, arXiv:1712.08609.

LHCb collaboration, R. Aaij *et al.*, *Measurement of the CKM angle γ using $B^\pm \to DK^\pm$ with $D \to K^0_S\pi^+\pi^-$, $K^0_S K^+K^-$ decays*, JHEP **10** (2014) 097, arXiv:1408.2748.

Helicity coordinates

$$Q_K\phi, m_{K\pi}, m_{\pi\pi}, \cos(\theta_{\pi\pi}), \cos(\theta_{K\pi}) \quad (1)$$

- Where Q_K is the charge of the kaon (required so the coordinates are all CP even)
- ϕ is:

$$\phi = \tan^{-1}\left(\hat{\mathbf{n}}_{K^-\pi^+} \cdot \hat{\mathbf{n}}_{\pi^-\pi^+} / \frac{\mathbf{p}_{\pi^+} \cdot \hat{\mathbf{n}}_{K^-\pi^+}}{|\mathbf{p}_{\pi^+} \times \hat{\mathbf{p}}_{K^-\pi^+}|}\right),$$

  where all three vectors are evaluated in the D decay rest frame.
- $\cos(\theta_{\pi\pi})$ and $\cos(\theta_{K\pi})$ are helicity cosines (the angle between a pion(kaon) and the D meson in the rest frame of the $\pi\pi(\pi K)$ system.