

# Recent Development in Perturbative QCD for LHC

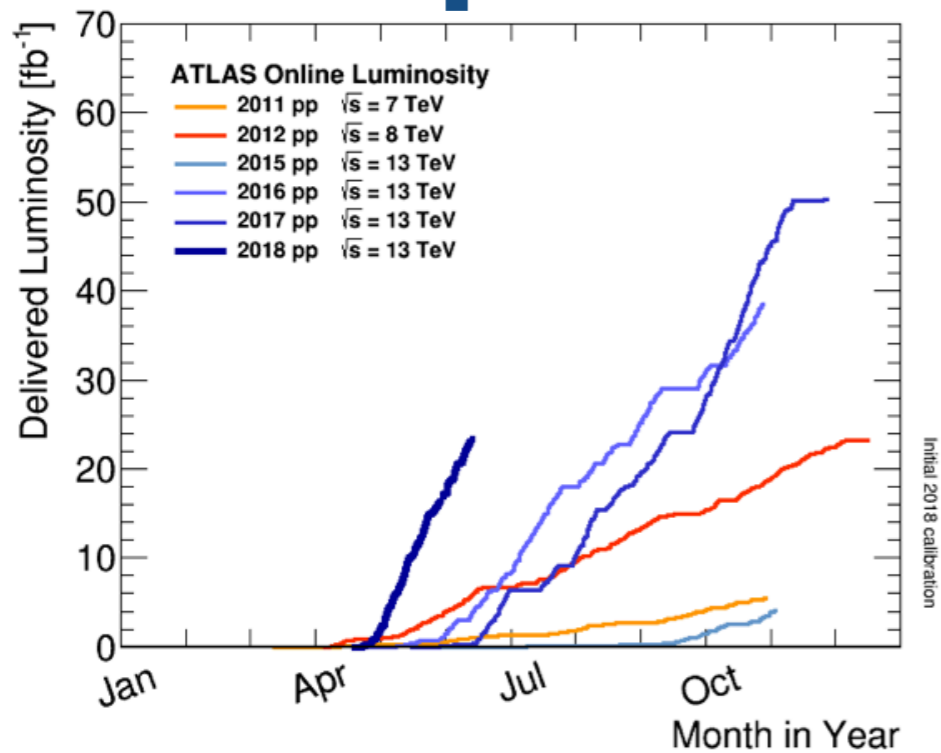
Hua Xing Zhu (朱华星)

Zhejiang University

中国物理学会高能分会年会  
上海交大和李政道研究所, 6月23日, 2018



# pQCD in the LHC era

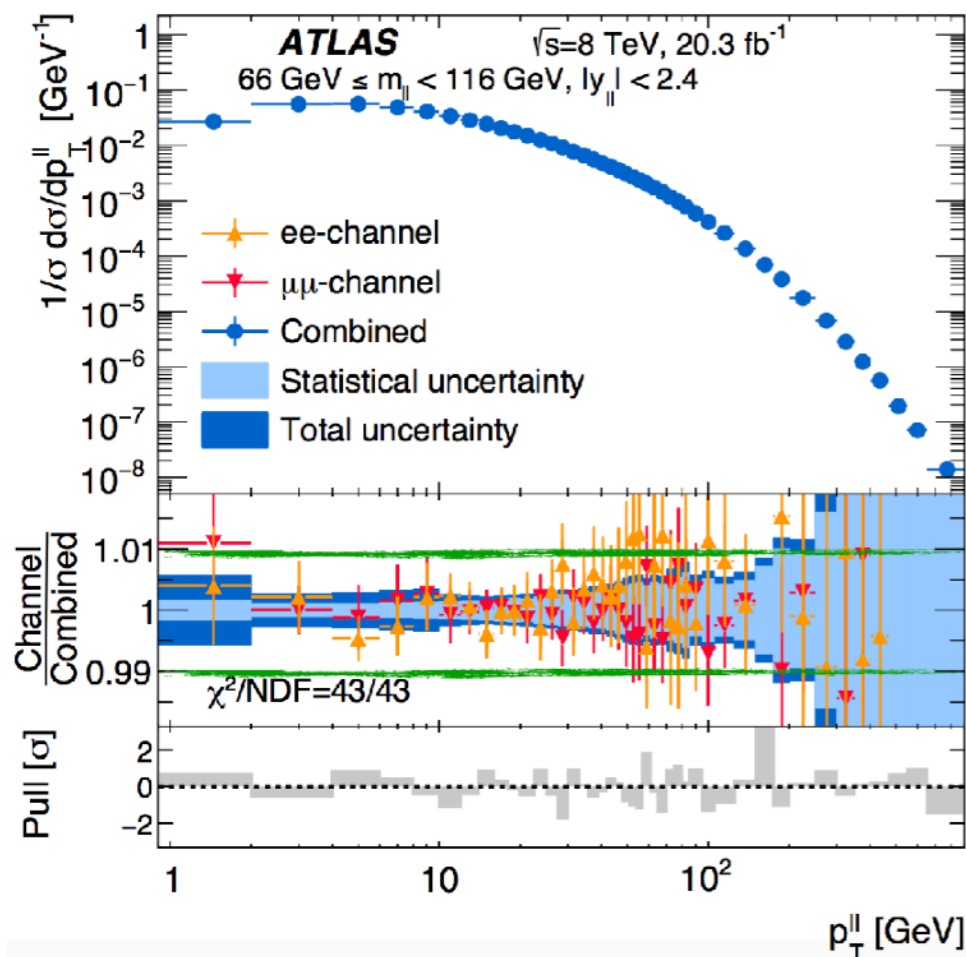


## Today

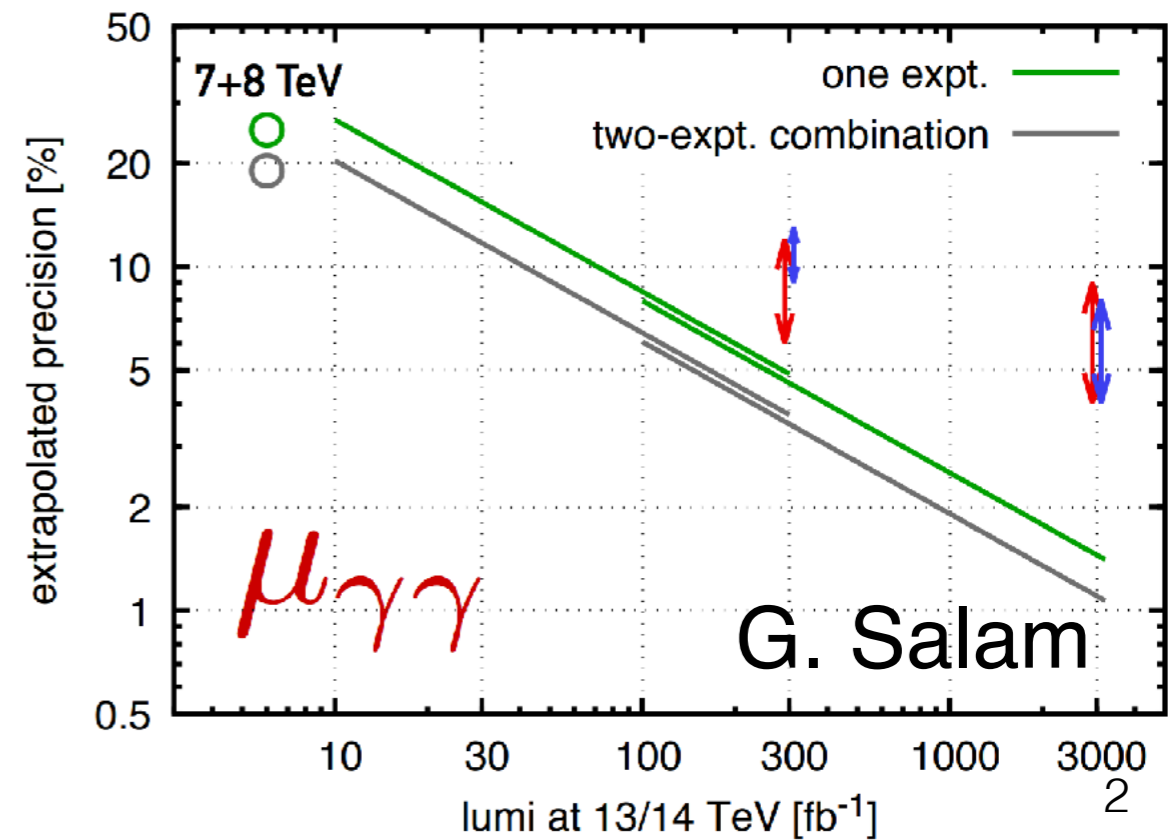
- **20  $\text{fb}^{-1}$  @ 8 TeV**
- **$\sim 110 \text{ fb}^{-1}$  @ 13 TeV**

## Future

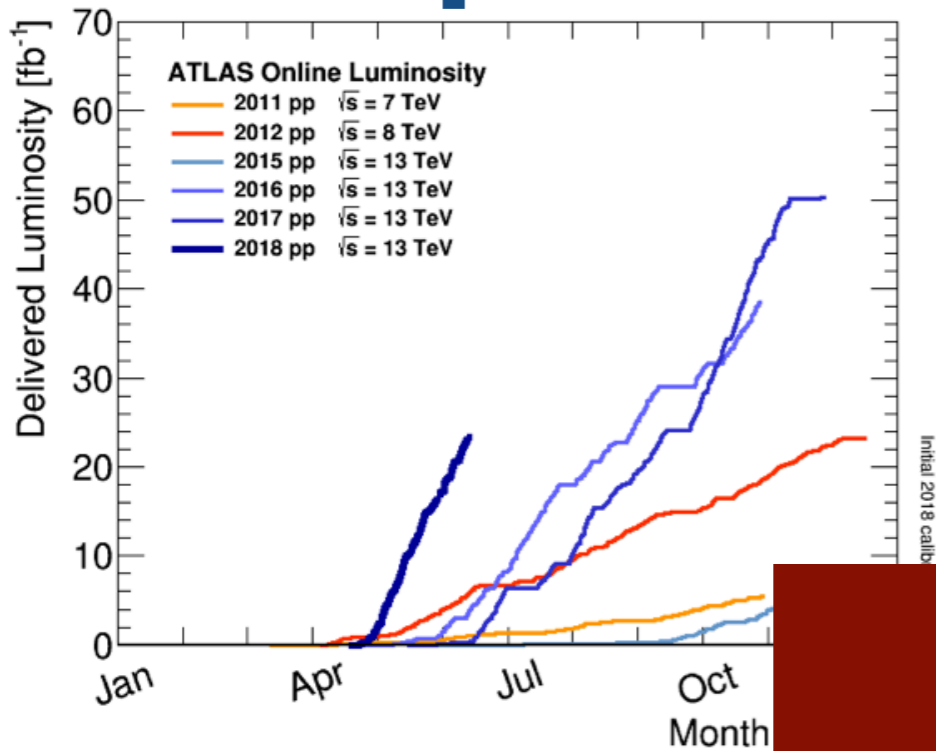
- **2023: 300  $\text{fb}^{-1}$**
- **2035: 3000  $\text{fb}^{-1}$  @ 14 TeV**



$\pm 1\%$



# pQCD in the LHC era



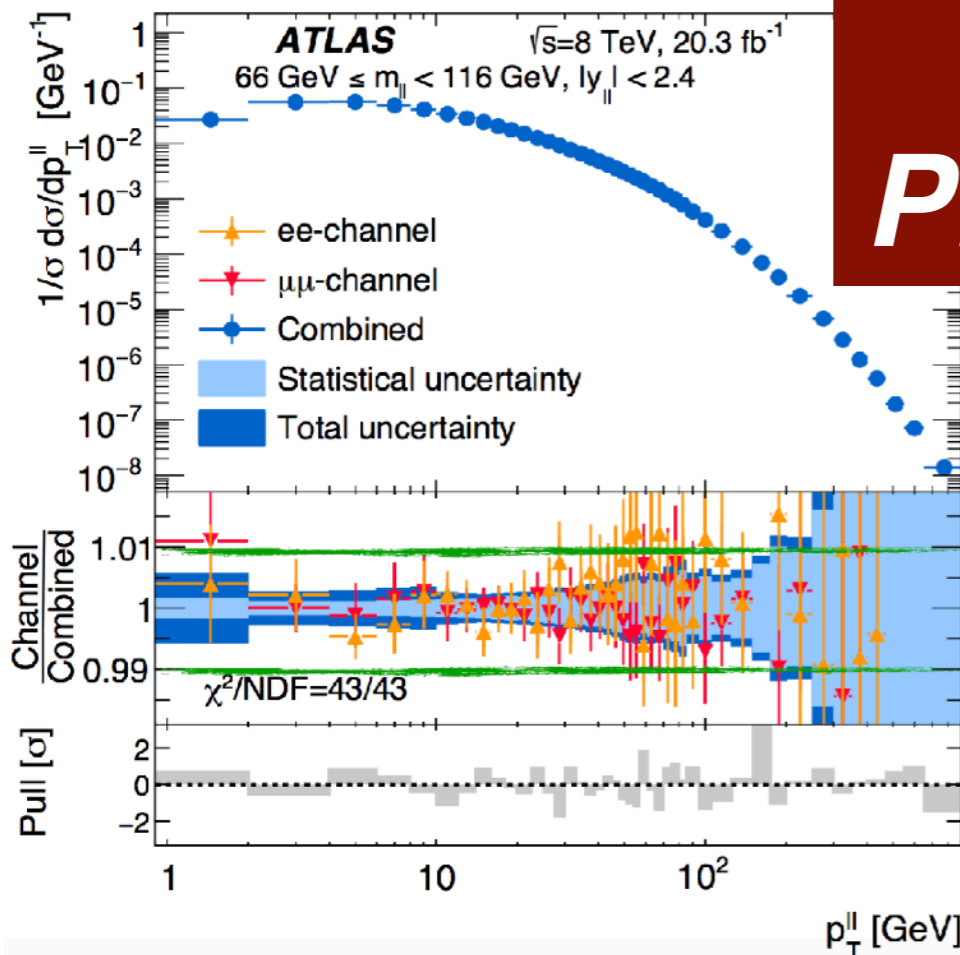
**Today**

- **20 fb<sup>-1</sup> @ 8 TeV**
- **~110 fb<sup>-1</sup> @ 13 TeV**

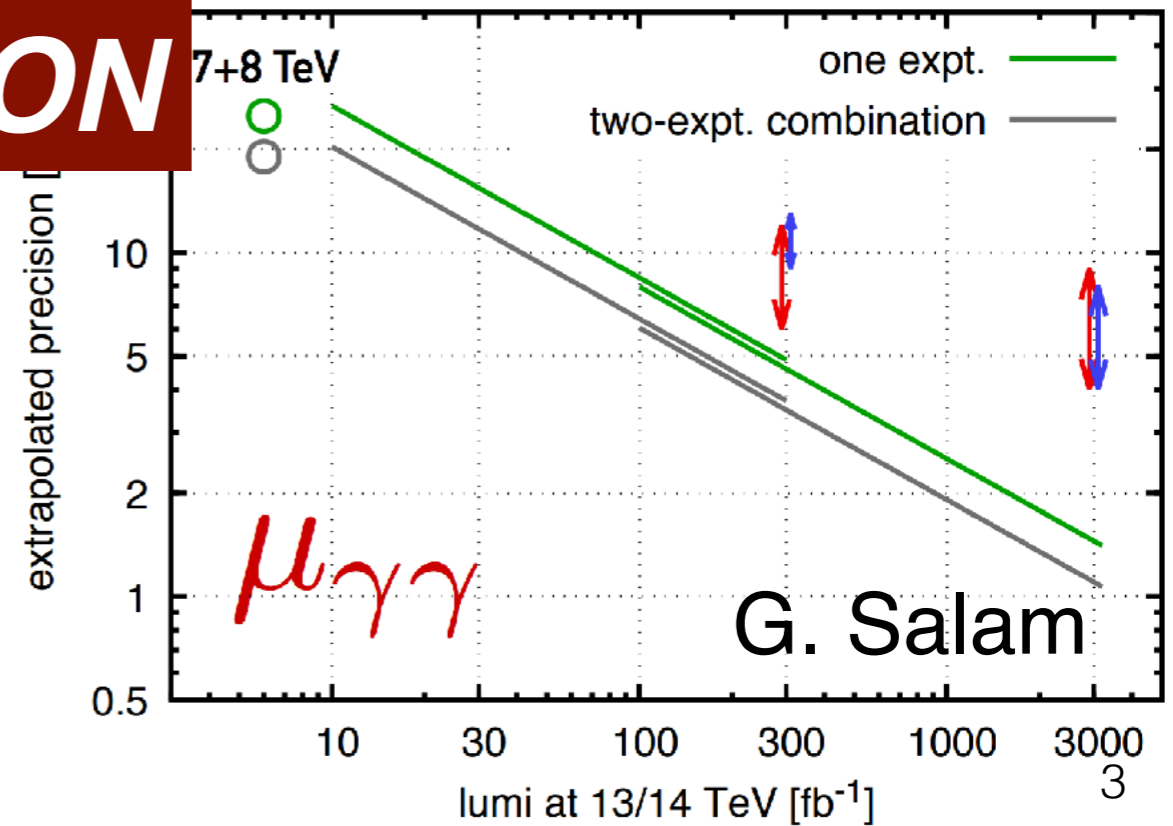
**Future**

- 2023: 300 fb<sup>-1</sup>**
- 2035: 3000 fb<sup>-1</sup> @ 14 TeV**

**AGE OF PRECISION**



$\pm 1\%$



# Why precision QCD

NP Signal?

QCD LO 10%

➔

QCD LO + NLO

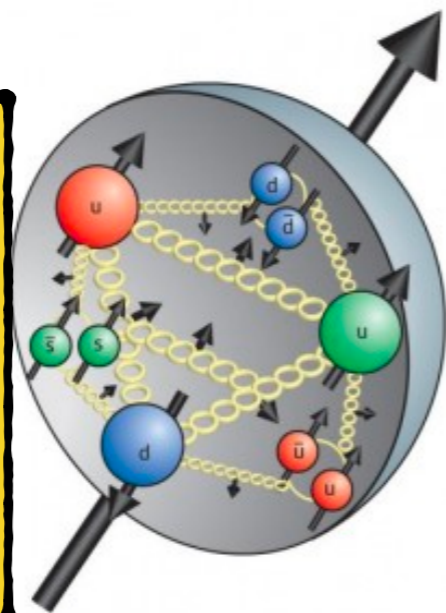
**Precision for Discovery**

Precision knowledge of background is crucial in declaring discovering

See also H. Zhang's talk

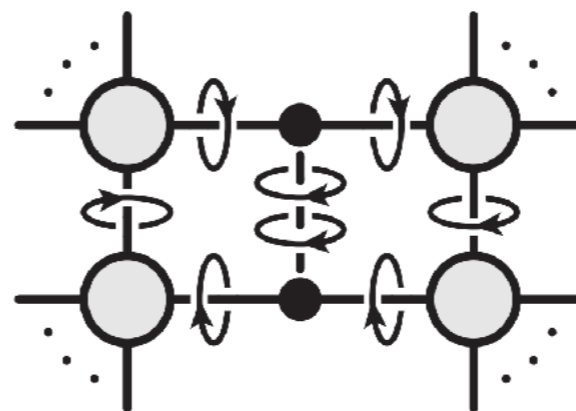
## Precision for understanding QCD

- Structure of proton
- $\alpha_s$  determination
- splitting/fragmentation dynamics
- power corrections
- light-ray operator
- .....



See also B.W. Xiao's talk

## Precision $\Leftrightarrow$ beauty in gauge theory

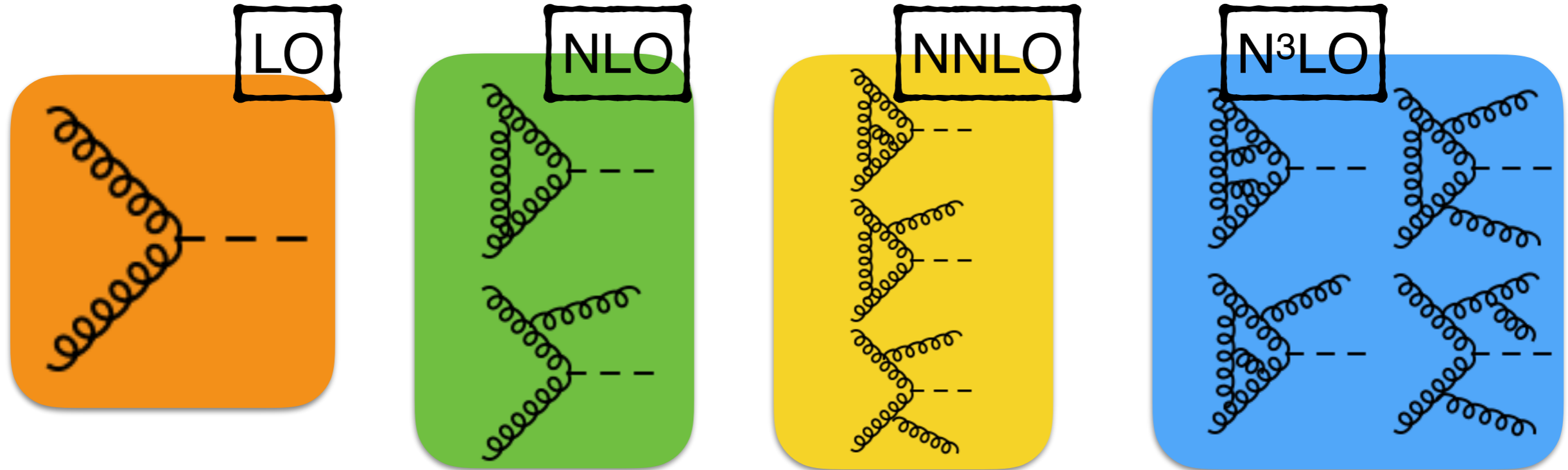


- Polylog & elliptic polylog
- iterative integral
- generalized unitarity
- scattering equatoin
- integrability
- amplituhedron
- positive geometry
- .....

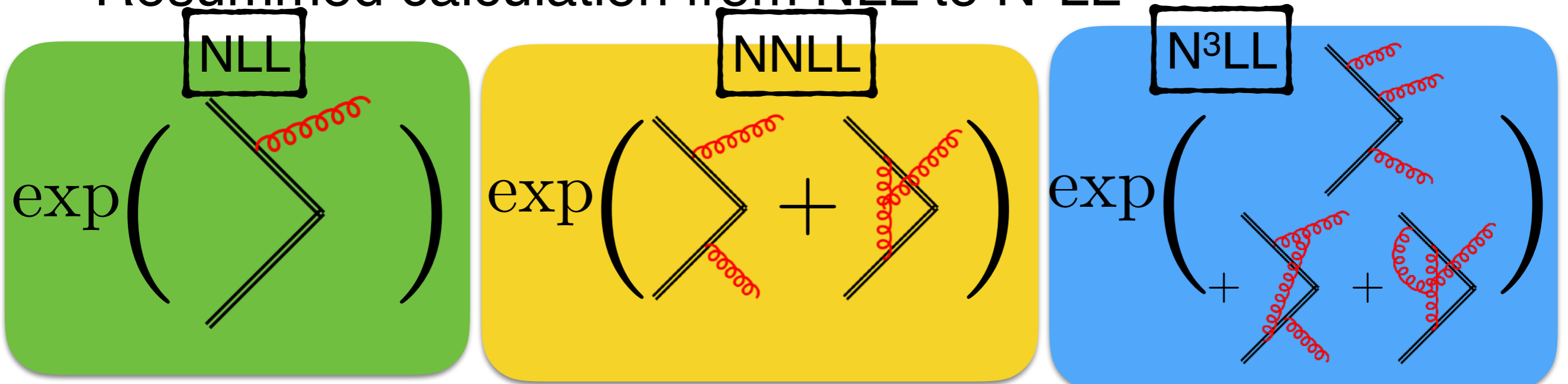
See also S. He and Y.Q. Ma's talk

# Anatomy of perturbative calculation

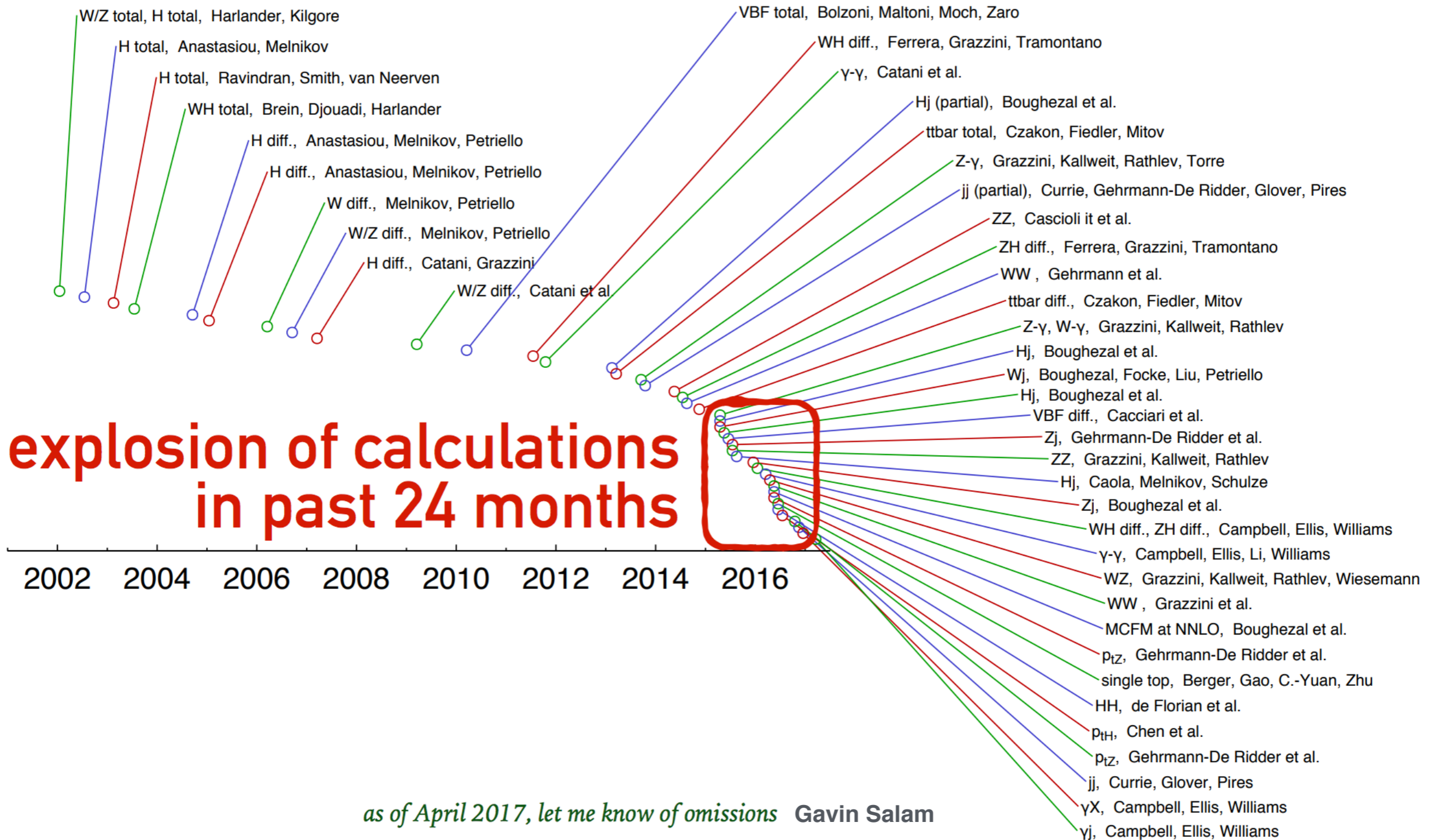
- Fixed-order calculation from LO to N<sup>3</sup>LO



- Resummed calculation from NLL to N<sup>3</sup>LL

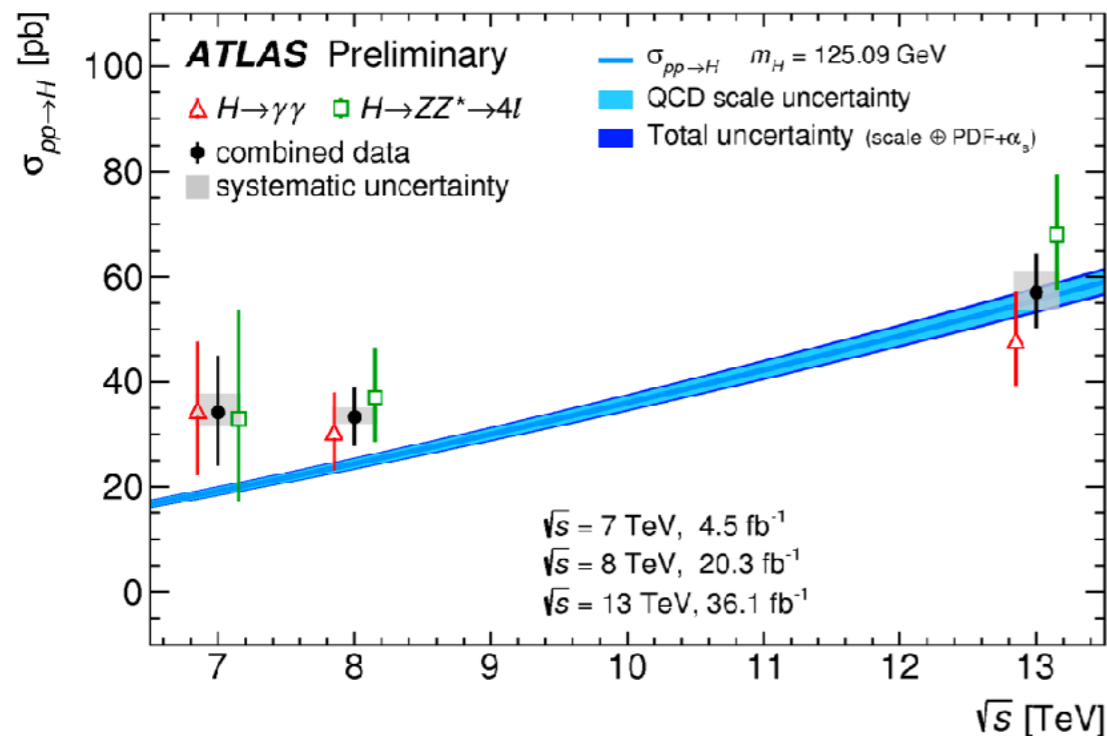


# Explosion of NNLO calculations

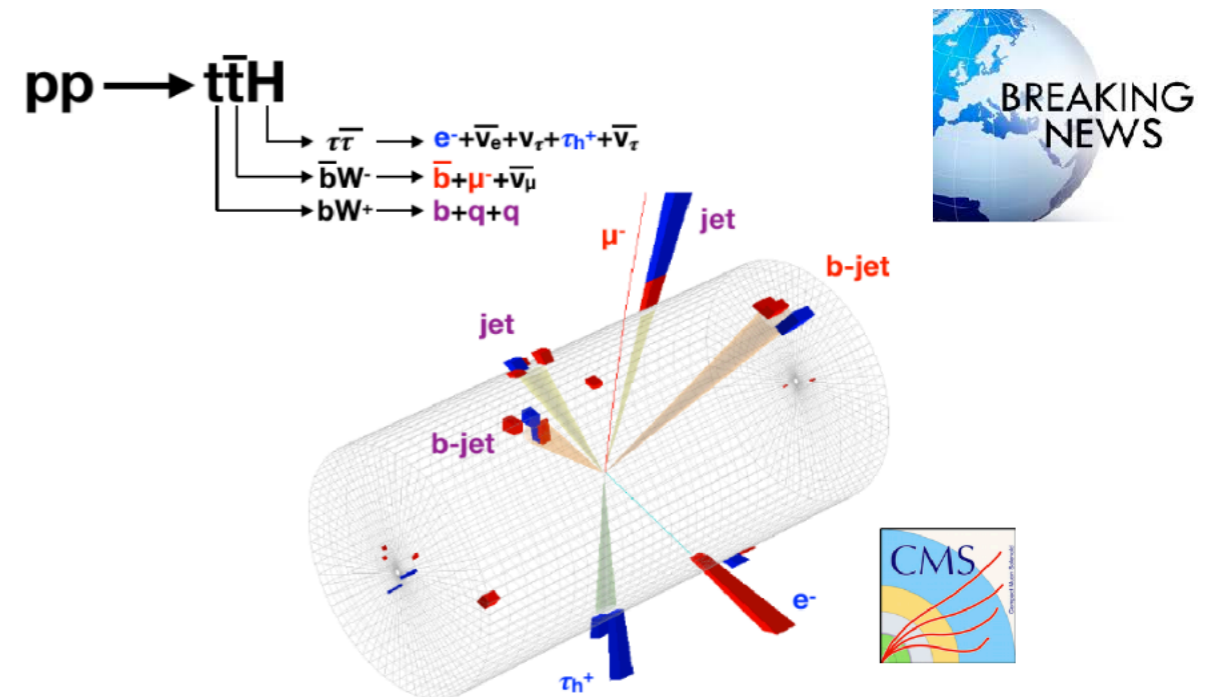


# The Higgs boson

- The discovery of the Higgs boson is just the beginning of a long journey
- **The only fundamental scalar in the SM:** mass origin, vacuum stability, portal to new physics, etc
- **$O(10^8)$  Higgs produced during the LHC life time:** a precision era await

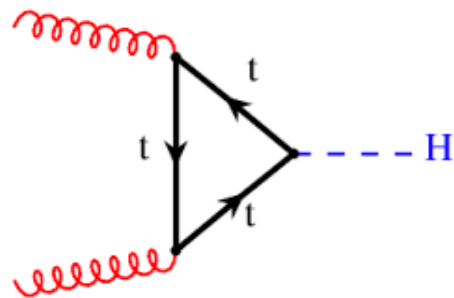


H inclusive cross section



# Inclusive cross section prediction

- **90%** of Higgs produced through **gg** fusion at the LHC
- Notorious **slowly convergent** perturbative series



$$\sigma_H = 16 \text{ pb} + 20.6 \text{ pb} + 9.79 \text{ pb} + ?$$

LO

$\delta$ NLO

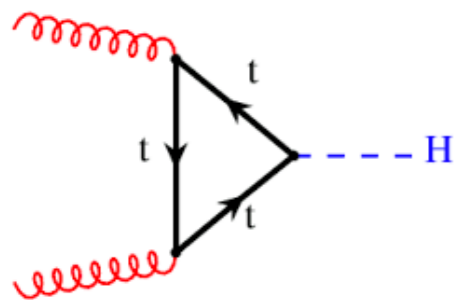
$\delta$ NNLO

$\delta$ N<sup>3</sup>LO



# Inclusive cross section prediction

- **90%** of Higgs produced through **gg** fusion at the LHC
- Notorious **slowly convergent** perturbative series



$$\sigma_H = 16 \text{ pb} + 20.6 \text{ pb} + 9.79 \text{ pb} + \mathbf{2 \text{ pb}}$$

LO

$\delta$ NLO

$\delta$ NNLO

$\delta$ N<sup>3</sup>LO

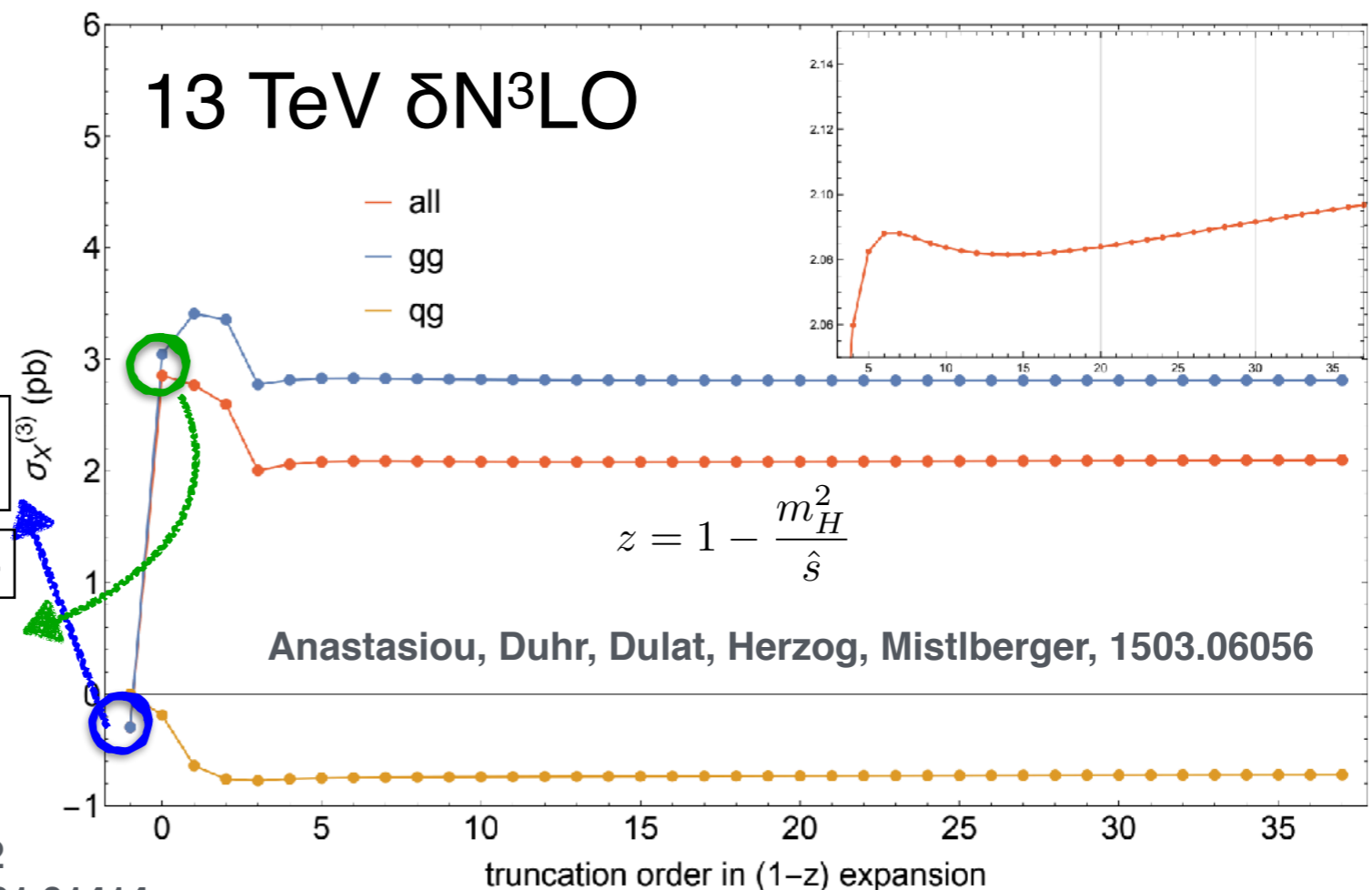
- Success application of **near threshold soft expansion**
- First N3LO hadronic cross section
- Crucial for precision Higgs program

Anastasiou, Duhr, Dulat, Furlan, et al., 1403.4616  
Y. Li, von Manteuffel, Scharbinger, HXZ, 1412.2771

Anastasiou, Duhr, Dulat, Furlan, et al., 1411.3584

Alternative approach:  
**principle of maximum conformality  
applied to Higgs production/decay**

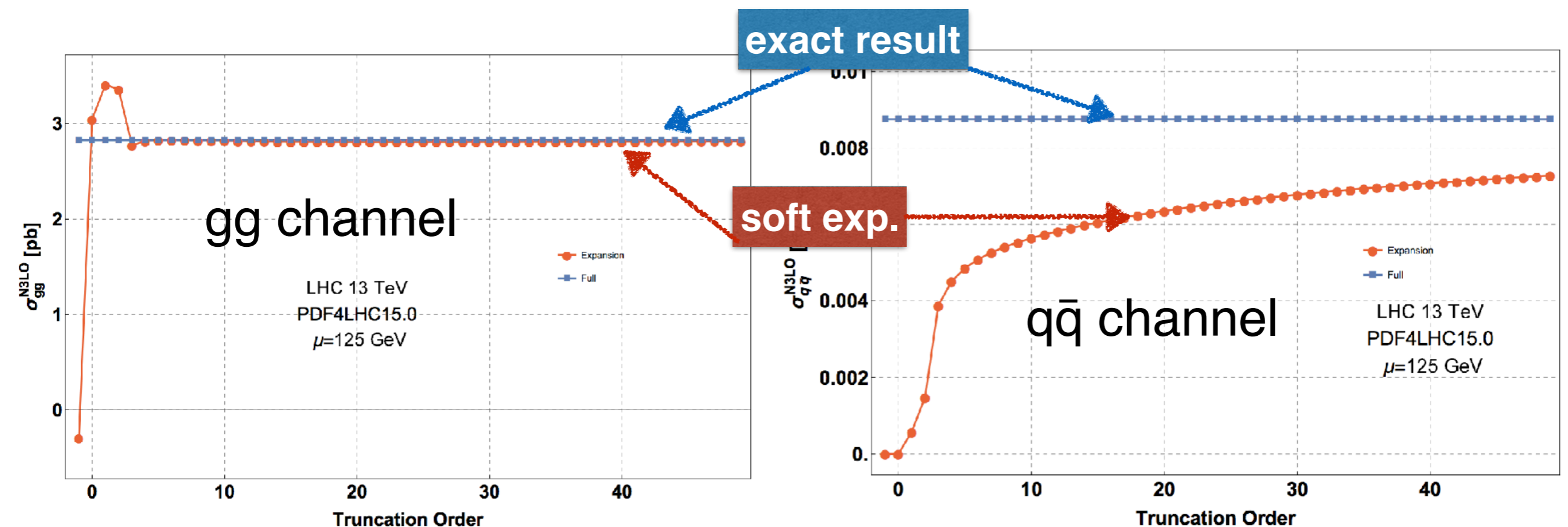
S.Q. Wang, X.G. Wu, Brodsky, Mojaza, 1605.02572  
J. Zeng, X.G. Wu, S. Bu, J.M. Shen, S.Q. Wang, 1801.01414



# Beyond soft expansion: analytic

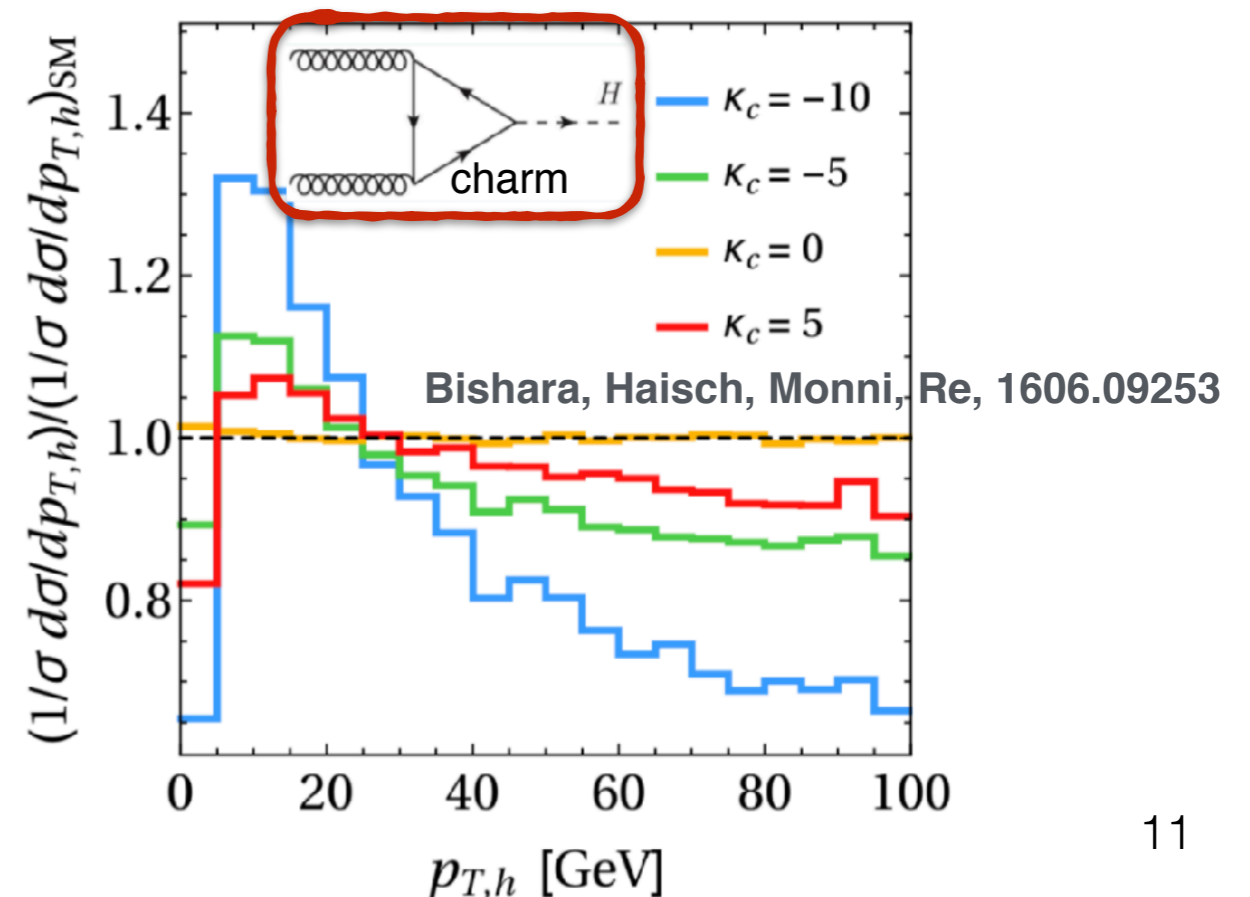
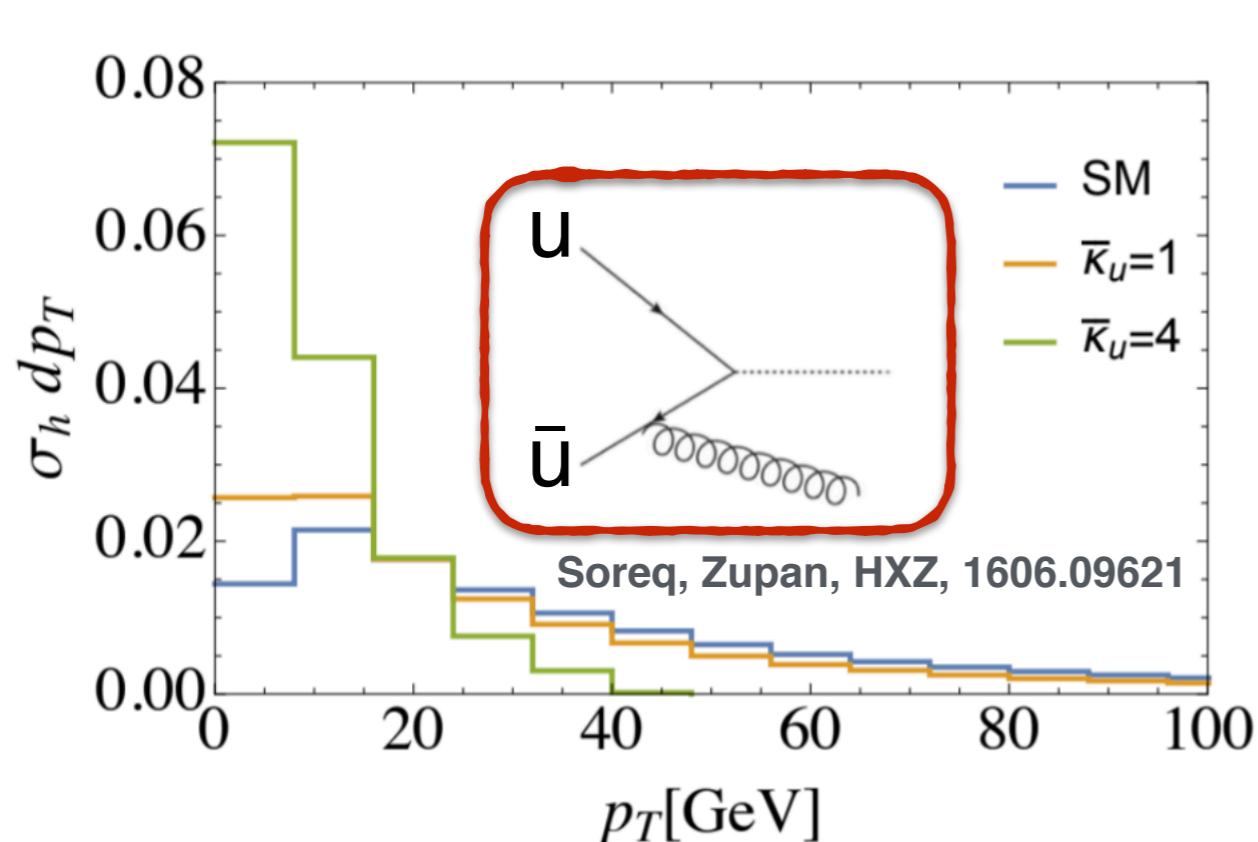
Mistlberger, 1802.00833

- Full analytic expression recently obtained
- $O(1000)$  Feynman integrals, **elliptic integrals** involved
- **First analytic hadronic cross section with elliptic integrals**
- Excellent convergence for gg channel, much worse for qq
- Results will facilitate  $N^3\text{LO}$  calculation for **Drell-Yan**



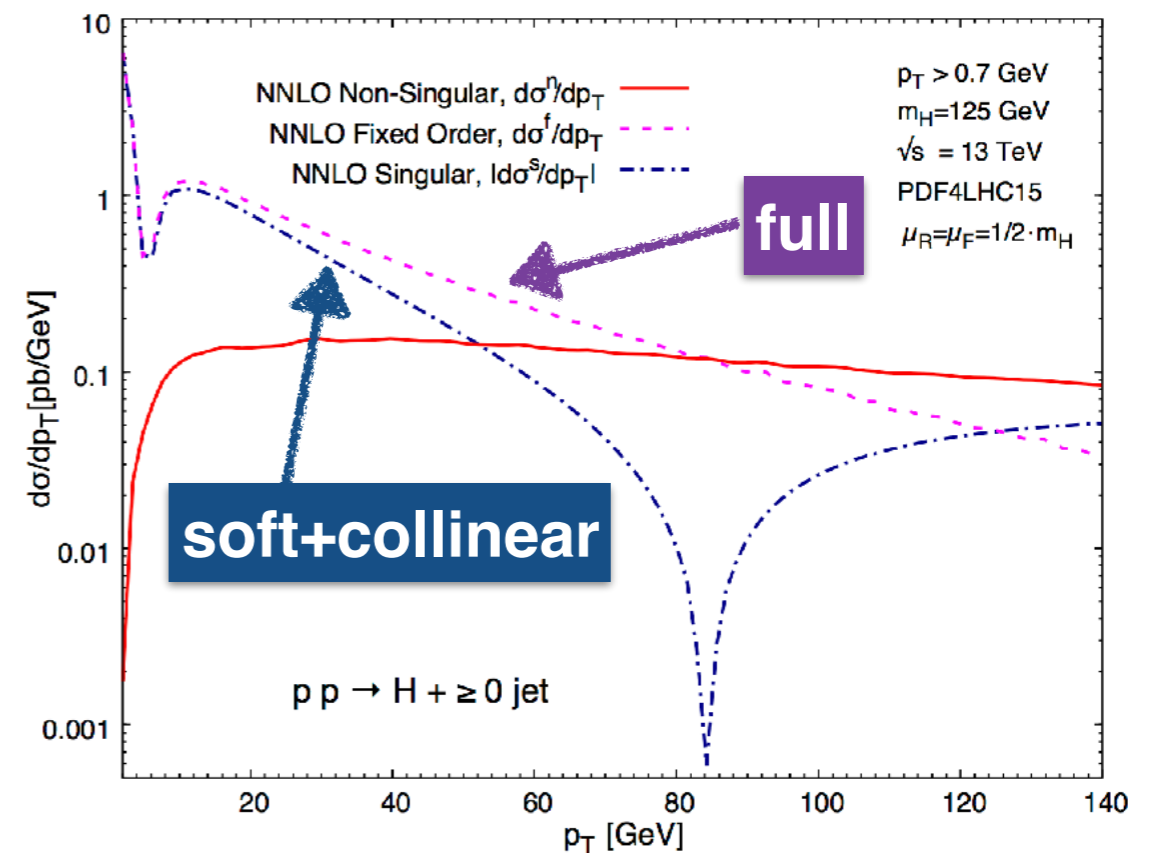
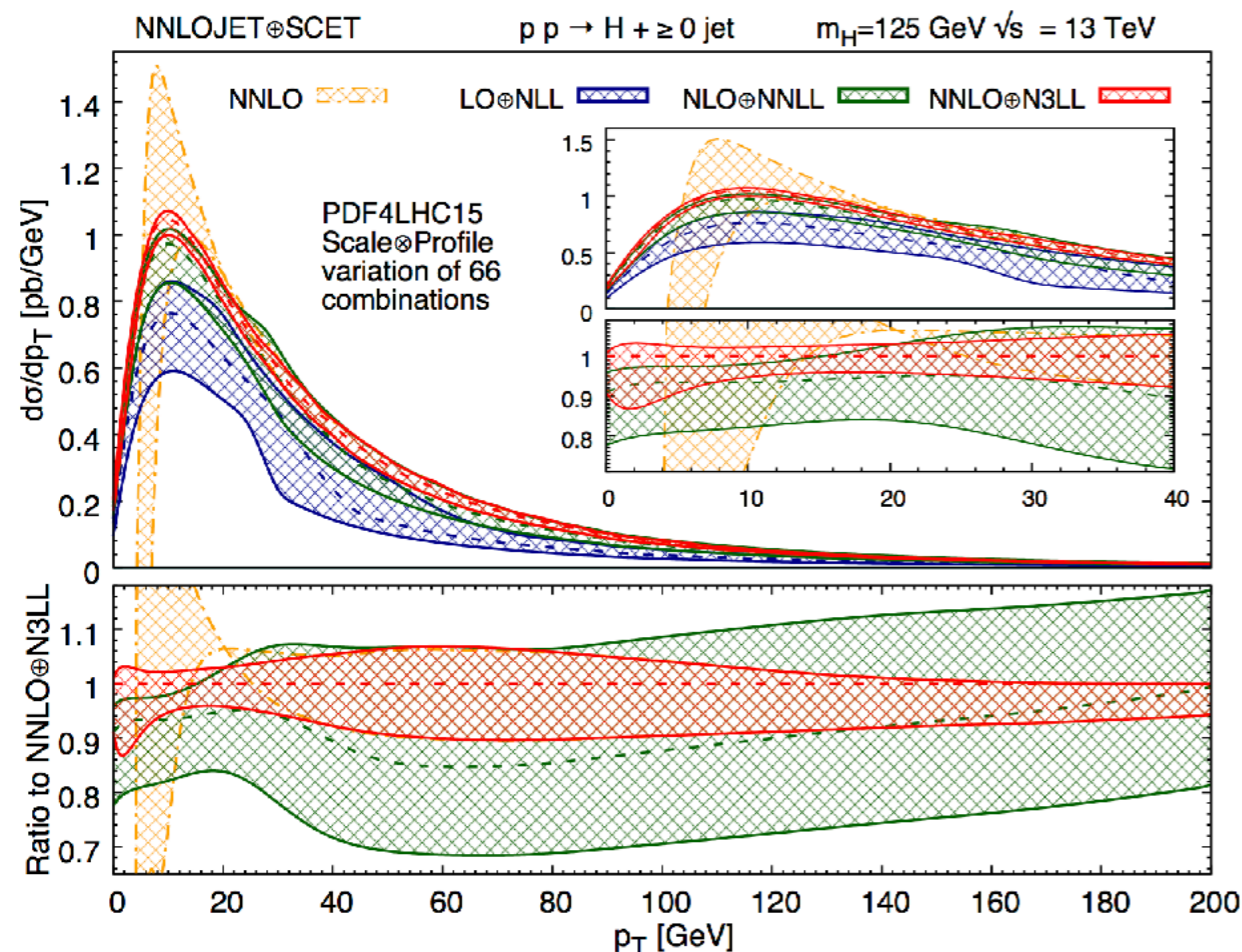
# Higgs differential distribution

- 80fb<sup>-1</sup> run-2 data analyses, about **4 million Higgs**, precision become possible even for differential dist.
- Differential distribution (p<sub>T</sub>, jet-bin Xsec,...) allows more detailed scrutiny of Higgs properties
- One example: **light quark Yukawa through Higgs p<sub>T</sub>**



# Higgs $p_T$ spectrum prediction

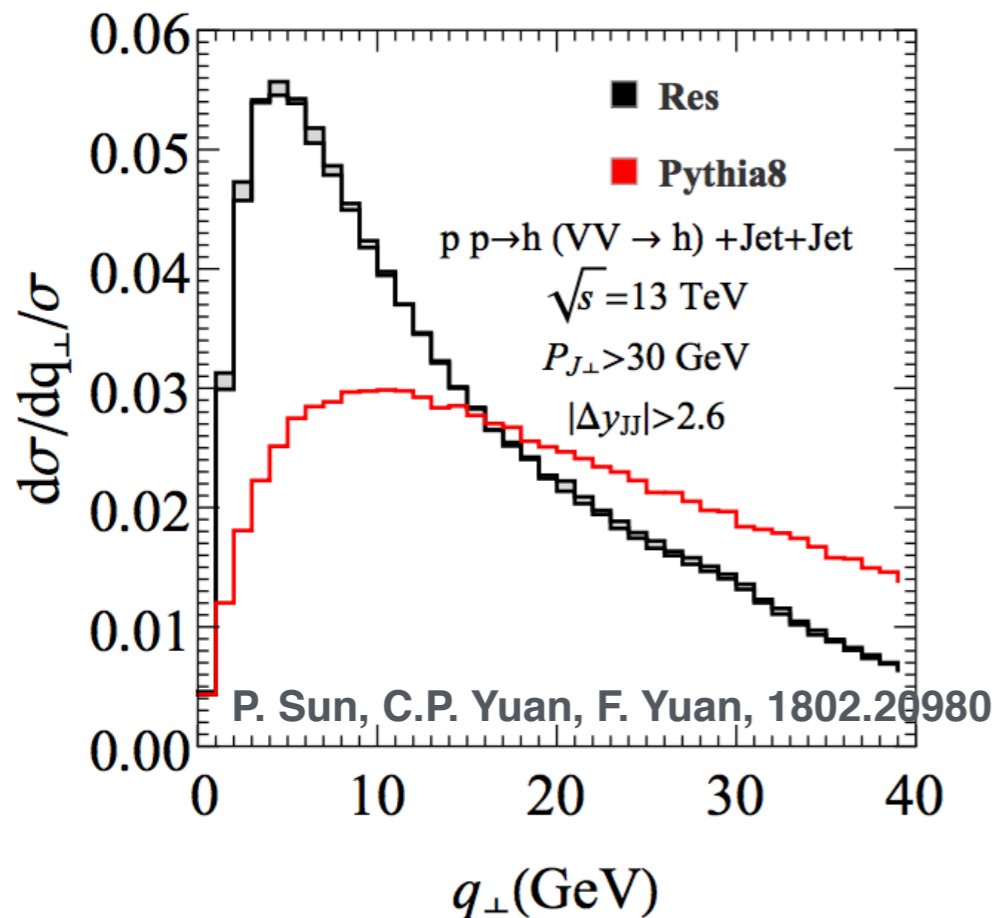
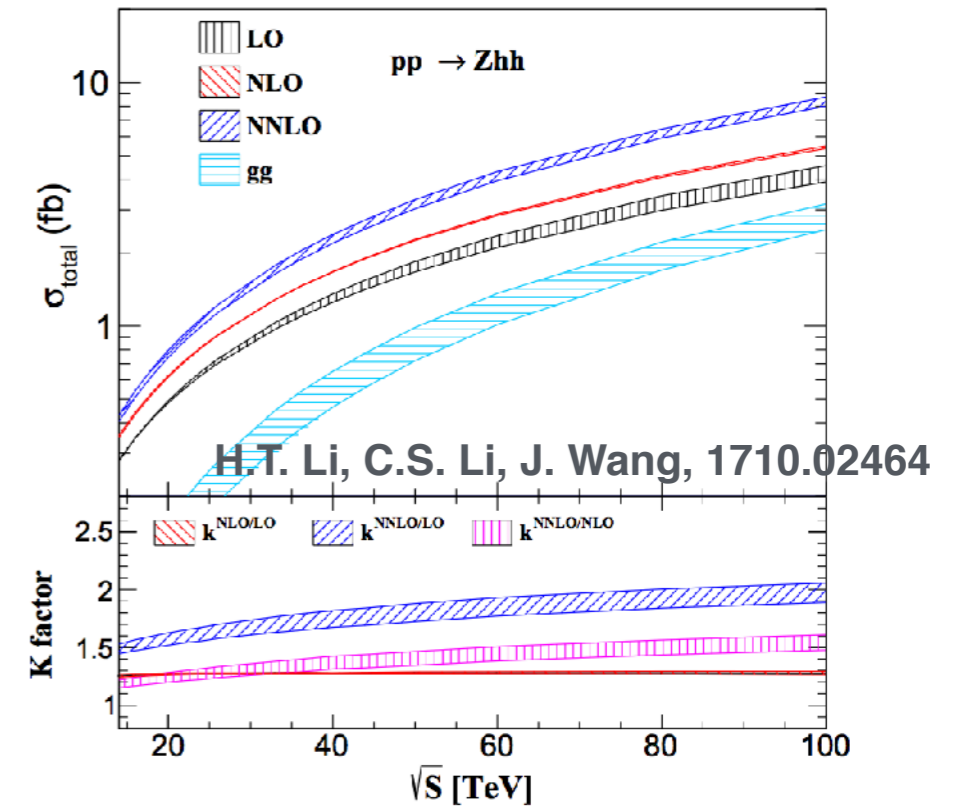
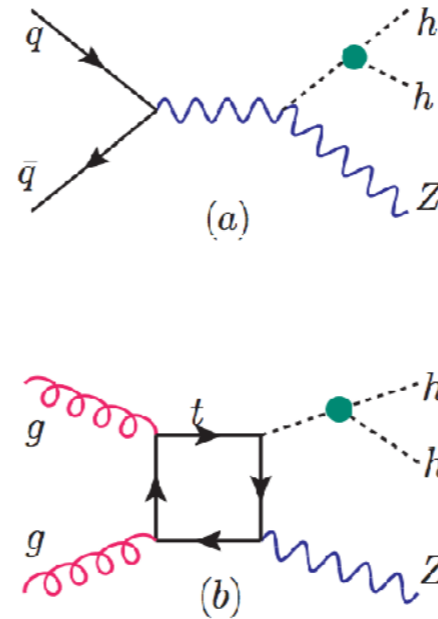
- Differential  $p_T$  distribution for Higgs exhibits **large logarithms**  $\ln^k(p_T)/p_T$
- Several different approaches for their resummation
  - Collins-Soper-Sterman formula
  - TMD factorization (Ji-Ma-Yuan, hep-ph/0404183)
  - Soft-Collinear Effective Theory



- Recently first  $p_T$  resummation calculation at **NNLO + N3LL** with high resolution
- Many years of calculation efforts in fixed order, anomalous dimension, matching coefficients
- **Continuous reduction of theoretical uncertainties:** demonstrate power of pQCD

# Higgs associated production

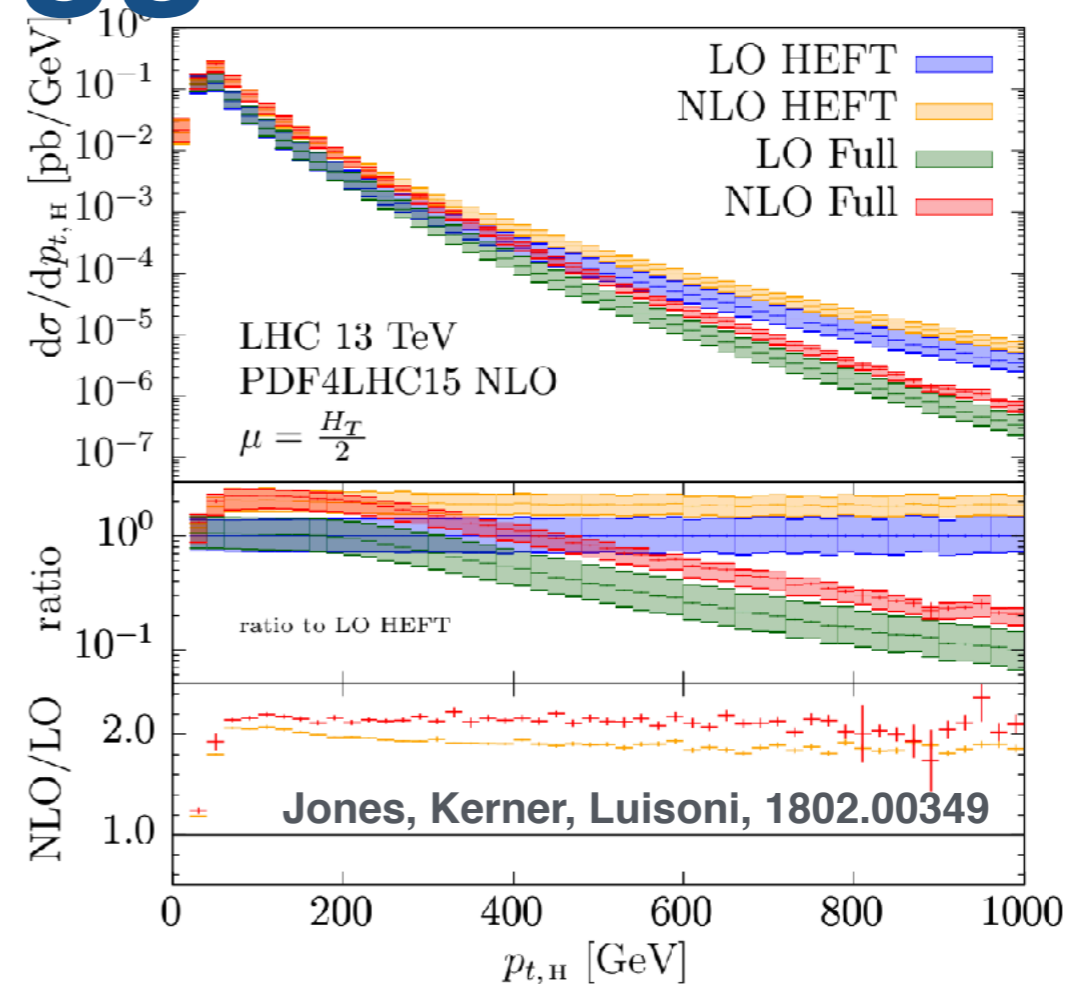
- Higgs pair production associated with Z
- Sensitive to Higgs trilinear coupling
- **Large K factor** at NNLO



- Higgs VBF production
- Resummation of  $q_T$  for **Higgs + di-jet system**
- **Significant difference** between resummation and Pythia
- Important to understand the origin of difference

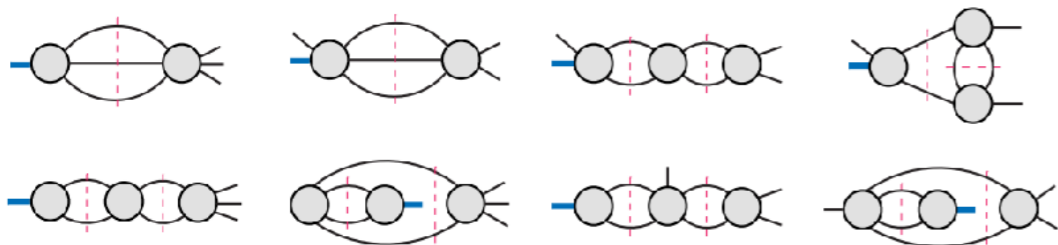
# Beyond Higgs EFT

- Higgs + jet production at NLO with **full top mass dependence**
- Numerical calculation of two-loop diagrams
  - **Quasi Monte-Carlo** for Feynman Integrals [Z. Li, J. Wang, Q.S. Yan, X.R. Zhao, 1508.02512]
- Full mass corrections become important for  $p_T > 200$  GeV



$$O_1 = H \text{Tr}(G_\mu^\nu G_\nu^\rho G_\rho^\mu),$$

$$O_2 = H \text{Tr}(D_\rho G_{\mu\nu} D^\rho G^{\mu\nu}),$$



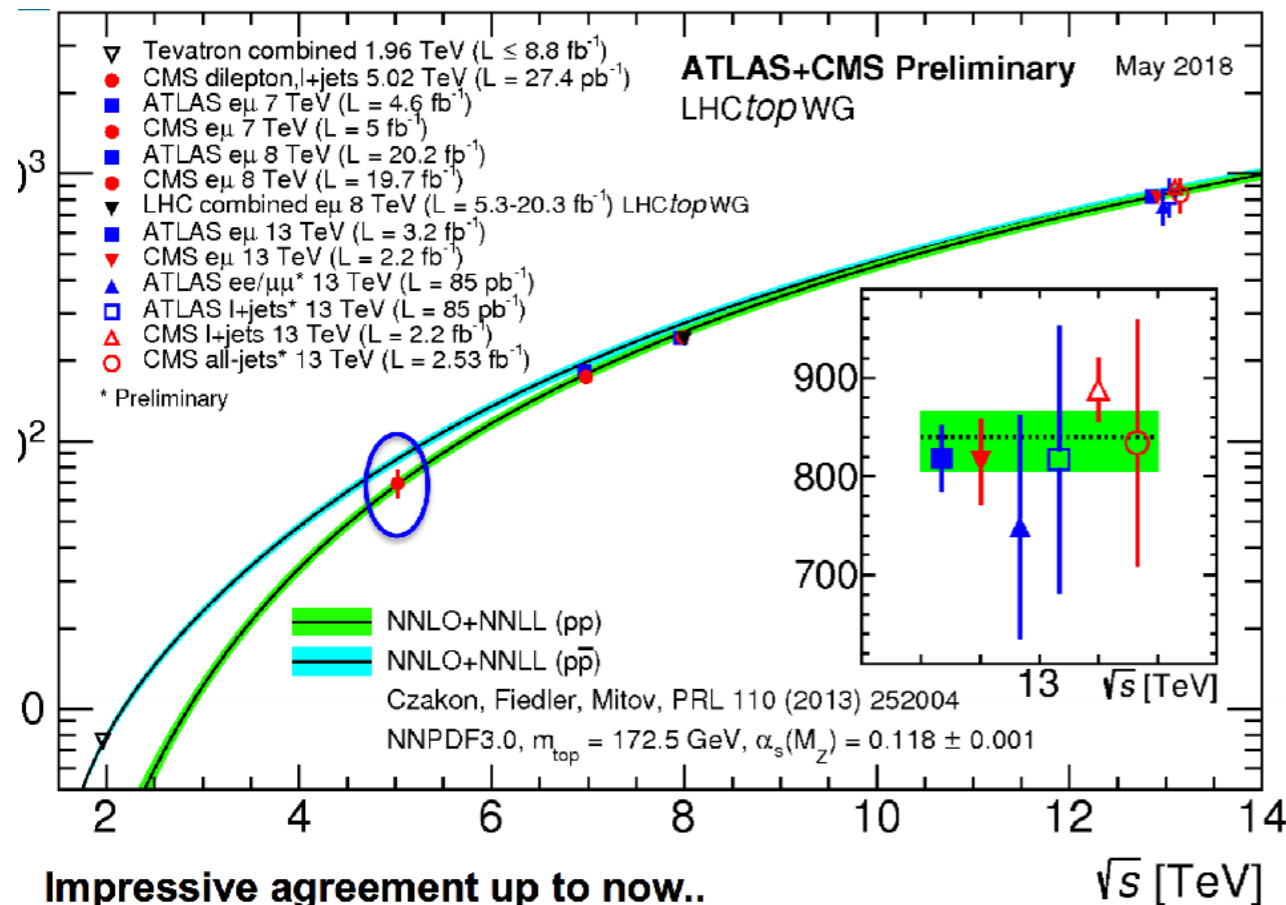
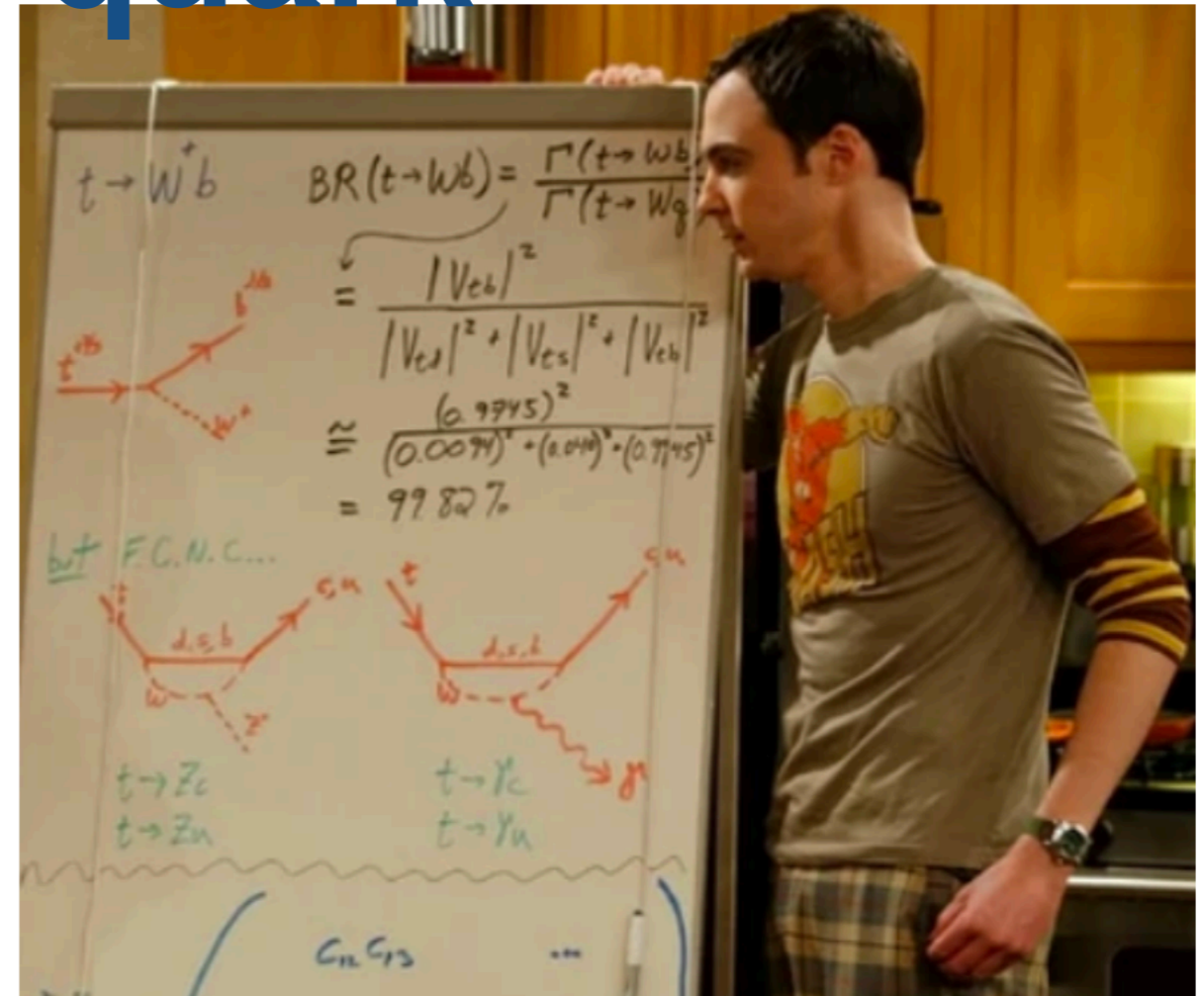
$$\Omega_{\mathcal{O}_{1;4}}^{(2)} = -\frac{3}{2} \text{Li}_4(u) + \frac{3}{4} \text{Li}_4\left(-\frac{uw}{v}\right) - \frac{3}{2} \log(w) \text{Li}_3\left(-\frac{u}{v}\right) + \frac{\log^2(u)}{32} [\log^2(u) + \log^2(v) + \log^2(w) - 4 \log(v) \log(w)]$$

$$+ \frac{\zeta_2}{8} [5 \log^2(u) - 2 \log(v) \log(w)] - \frac{1}{4} \zeta_4 - \frac{1}{2} \zeta_3 \log(-q^2) + \text{perms}(u, v, w). \quad (23)$$

- Mass effects can be partially taken into account by including higher dim. operator
- **Two-loop form factor** calculated for EFT operator up to dimension 7
- Maximal transcendental part resemble N=4 SYM; lower transcendental part also in compact form
- Important ingredient for NNLO H+jet with dim. 7 operator

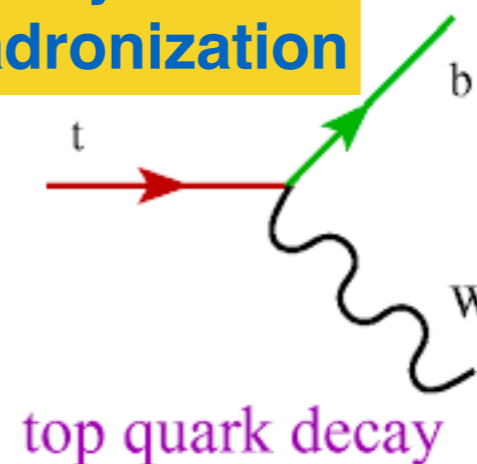
# The top quark

- The **heaviest** particle in the SM
- **Largest contribution** to fine tuning problem in the SM
- **Top mass** and the fate of the universe
- **Abundantly produced** at the LHC: ideal arena for precision
- Frequently appear as **background** in new physics searches



lifetime of  $\approx 0.5 \times 10^{-24}$  s

**decay before hadronization**



NLO QCD:

C.S. Li, Oakes, T.C. Yuan, 1991

NNLO QCD + NLO EW:

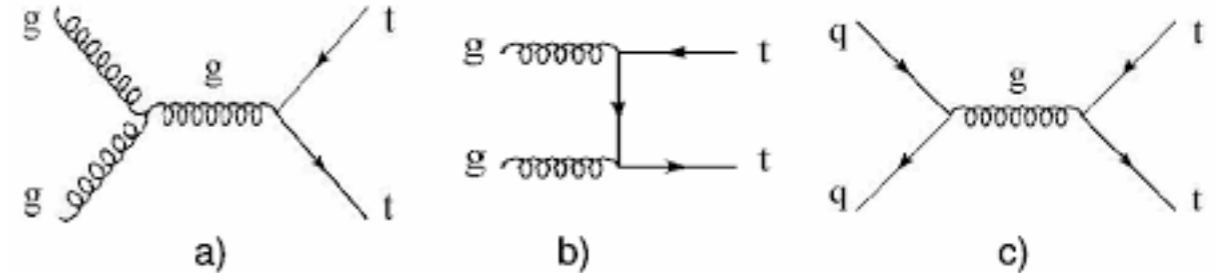
J. Gao, C.S. Li, HXZ, 1210.2808  
Brucherseifer, Caola, Melnikov, 1301.7133

NLO FCNC decay:

J.J. Zhang, C.S. Li, J. Gao, H. Zhang, Z. Li, C.-P. Yuan, T.C. Yuan, 0810.3889  
Durieux, Maltoni, C. Zhang, 1412.7166

# Boosted $t\bar{t}$ production

- Resummation for (boosted)  $t\bar{t}$  at **NNLO + NNLL'**
- Combine **fixed order, threshold resummation, and high energy top mass logs resummation**
- Optimal kinematical scale choice



NLO spin correlation:

Bernreuther, Brandenburg, Z.G. Si, Uwer, hep-ph/0107086

NLO QCD production + decay:

Bernreuther, Brandenburg, Z.G. Si, Uwer, hep-ph/0403035

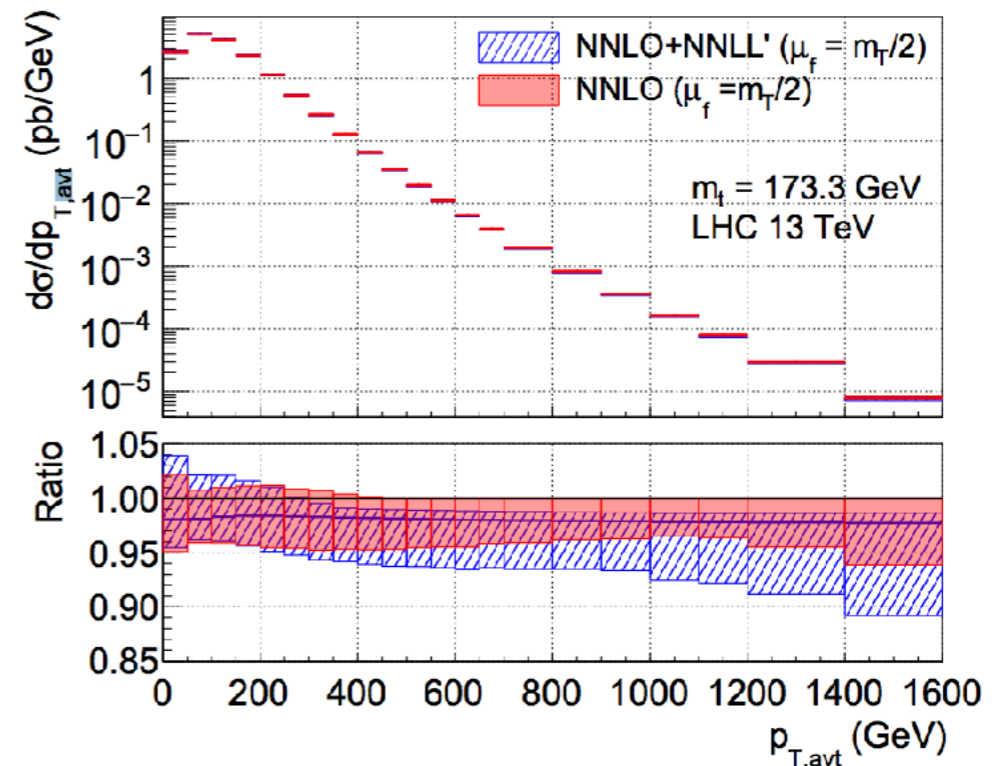
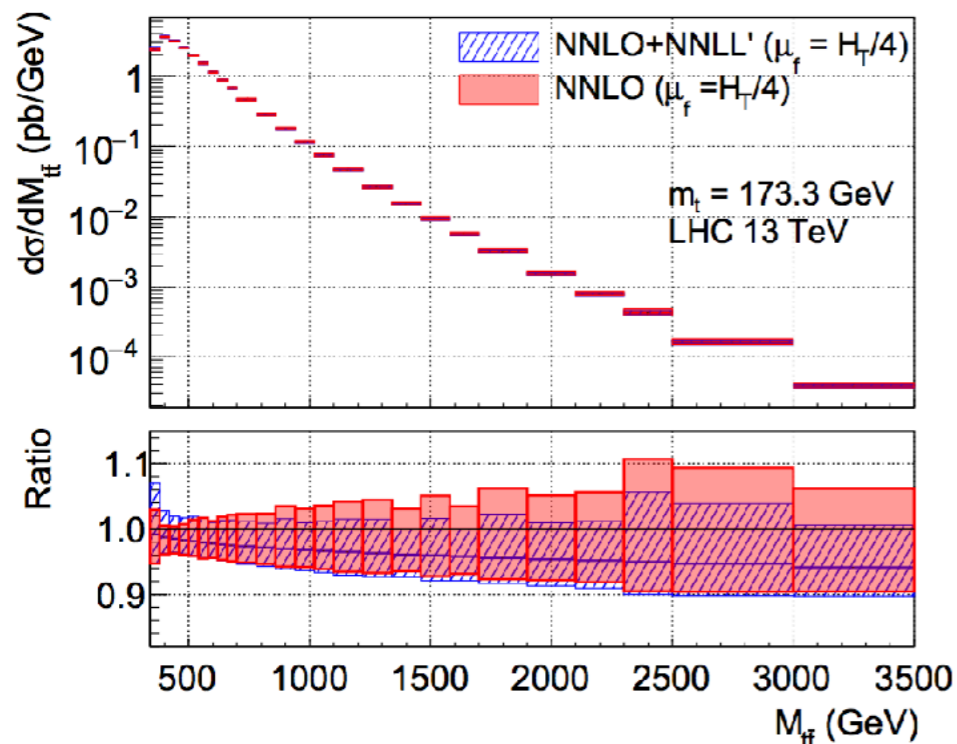
NLO EW:

Bernreuther, Fuecker, Z.G. Si, hep-ph/0610334

NNLO QCD:

Czakon, Fiedler, Mitov, 1303.6254

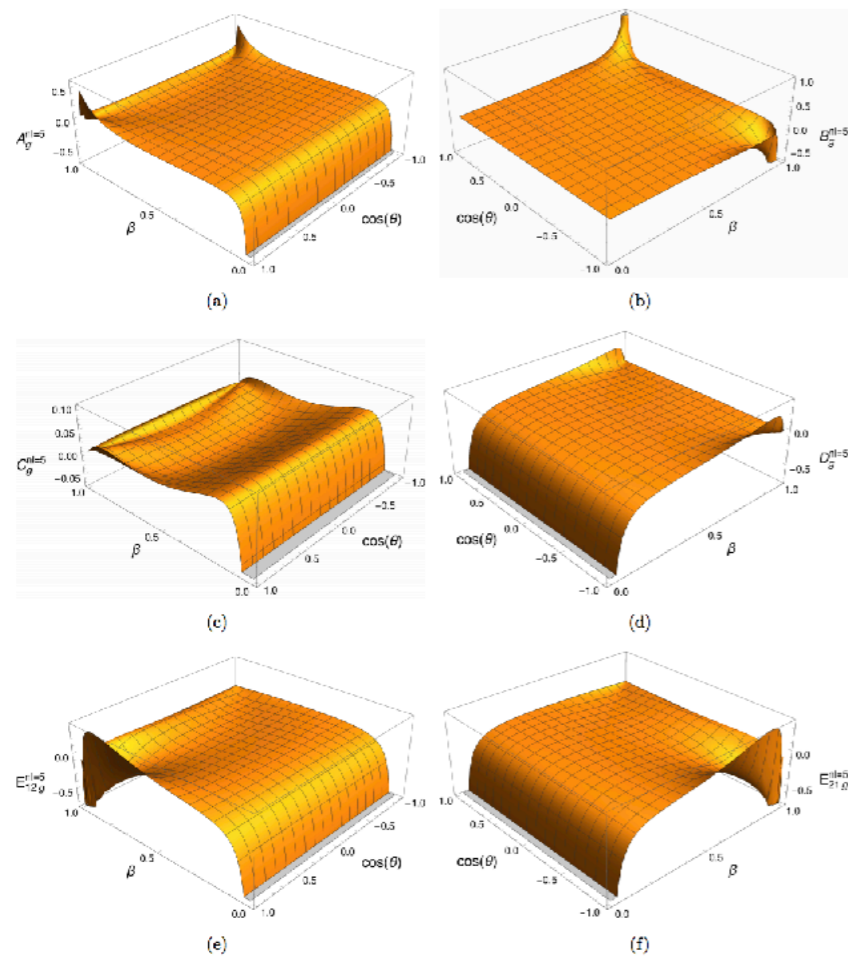
.....



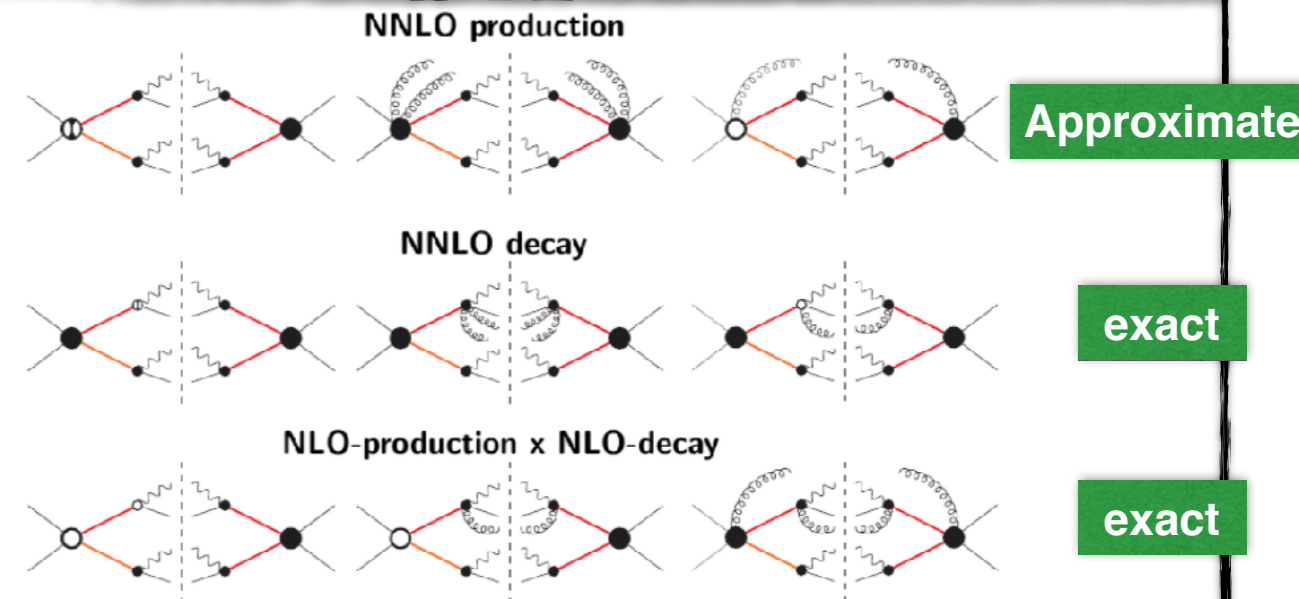


# Towards $t\bar{t}$ production and decay@NNLO

- **Two-loop polarized amplitudes** for  $t\bar{t}$  production
- Essential ingredient for NNLO production + decay
- **422 master integrals** with three scales, **elliptic integrals** involved
- Solved by numerical differential equation



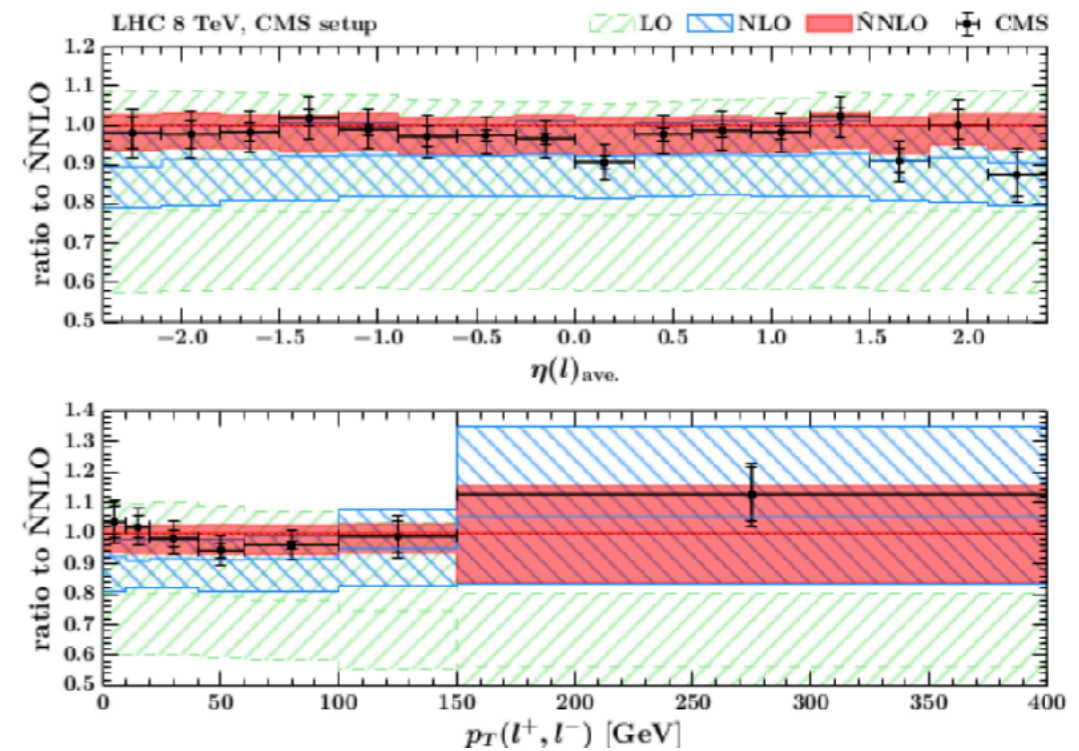
L. Chen, Czakon, Poncelet, 1712.08075



Approximate

exact

exact

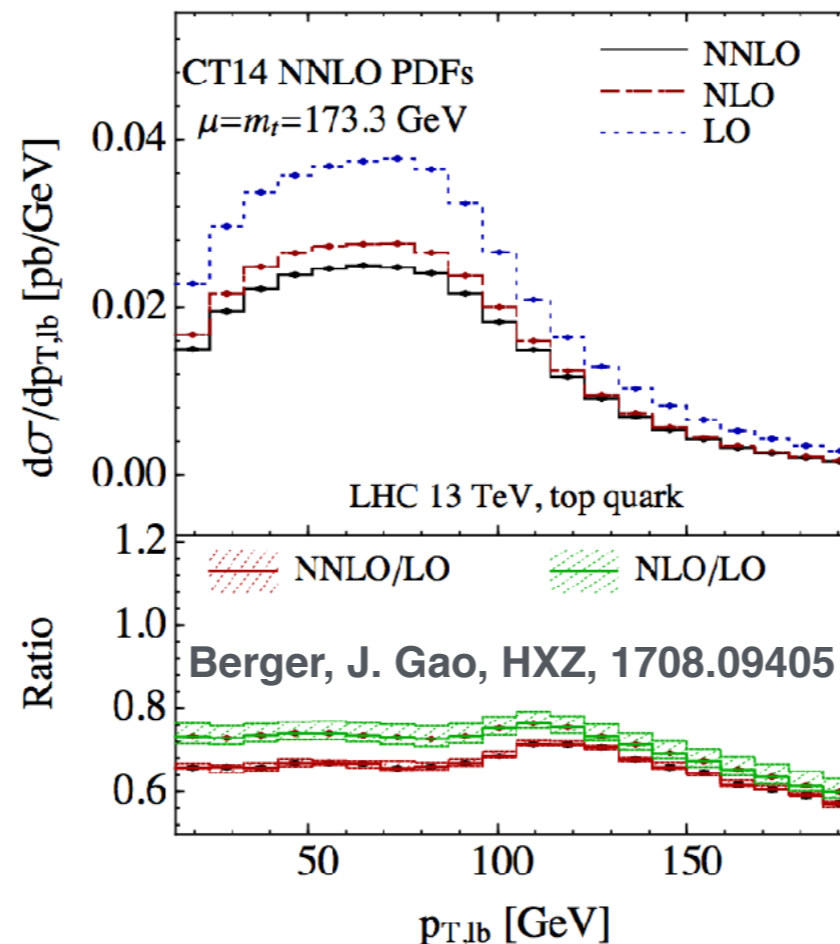
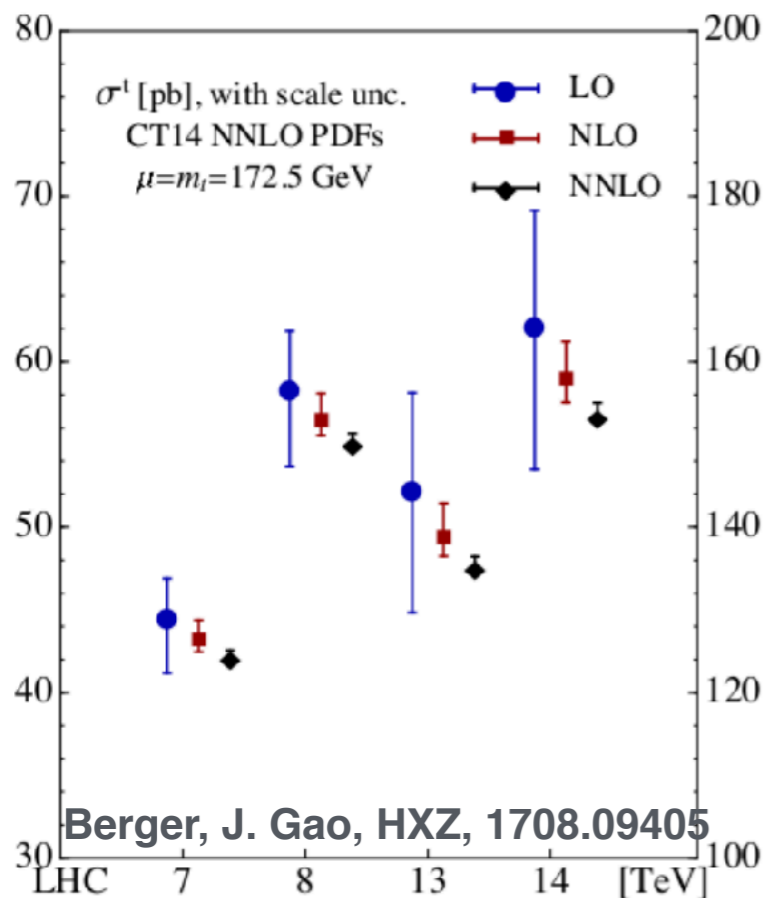
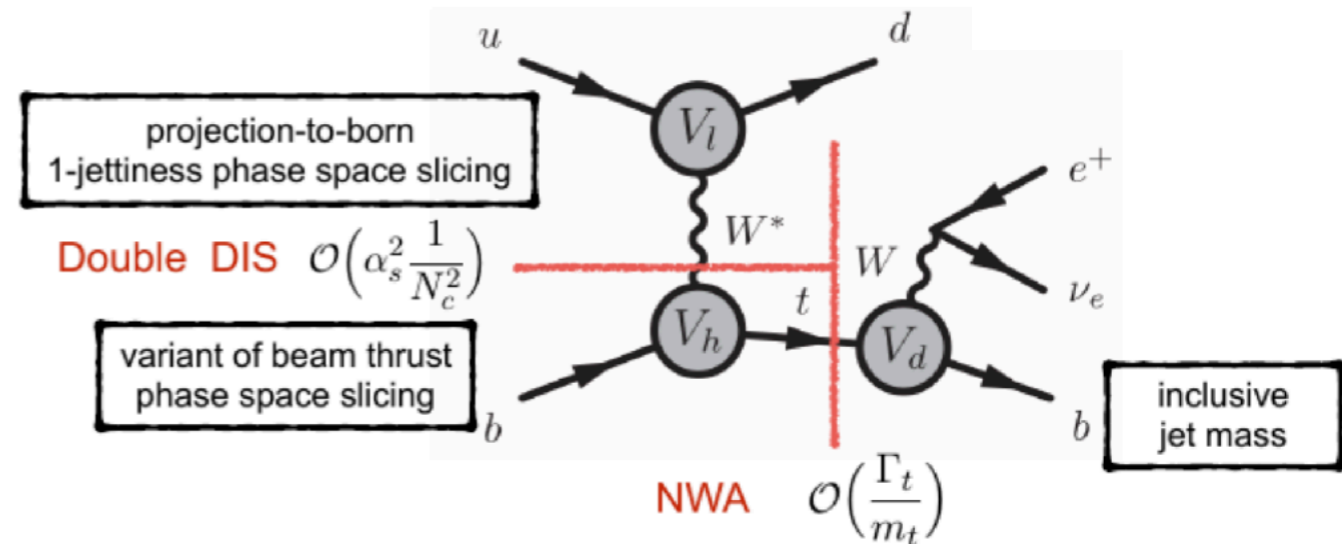


- **Approx. NNLO production + full NNLO decay**
- **Fully differential.** Significant corrections to fiducial distribution

J. Gao, Papanastasiou, 1705.08903

# Single top production and decay

- Sensitive to CKM element  $V_{tb}$
- Large Xsec ( $\sim 210\text{pb}@13\text{TeV}$ )
- Large background from  $t\bar{t}$ . Severe phase space cut
- Need accurate prediction for **fiducial distribution**



NLO QCD prod. + decay:  
Q.H. Cao, Schwienhorst, Benitez,  
Brock, C.P. Yuan, hep-ph/0504230

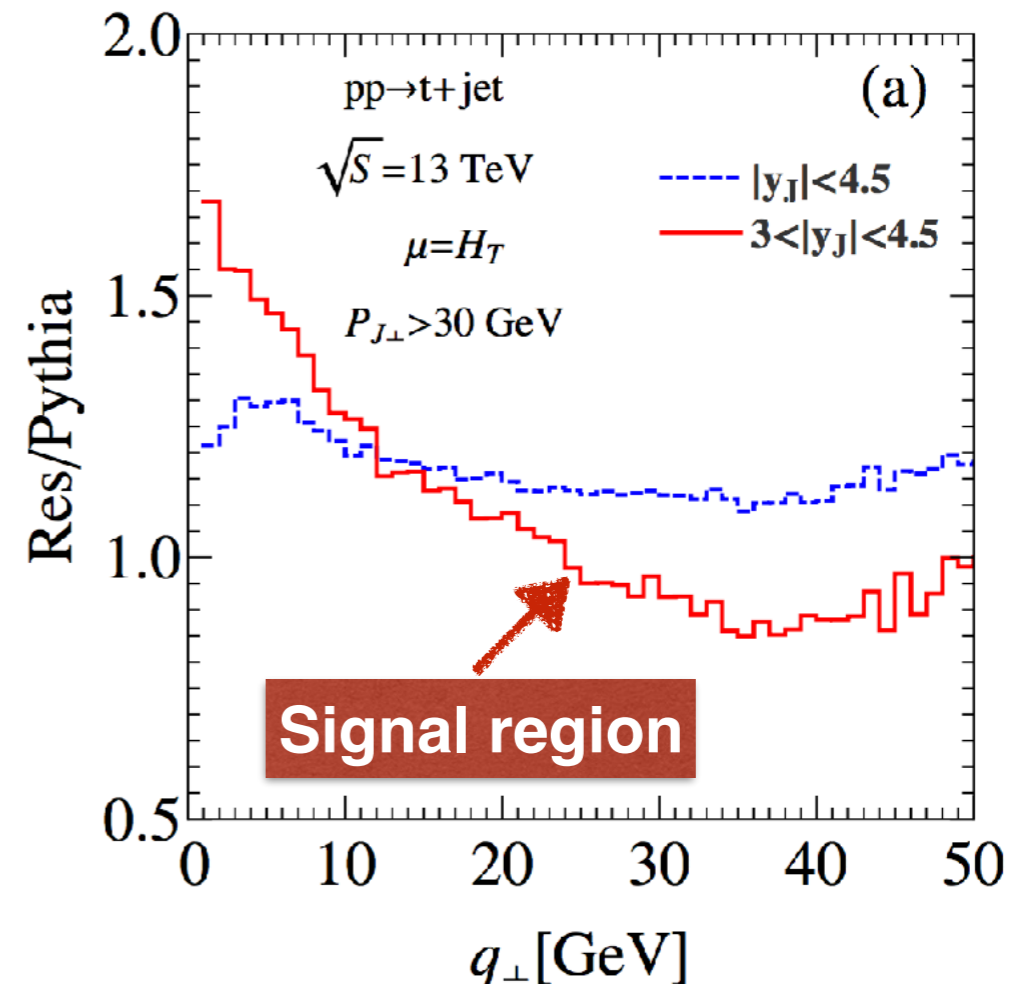
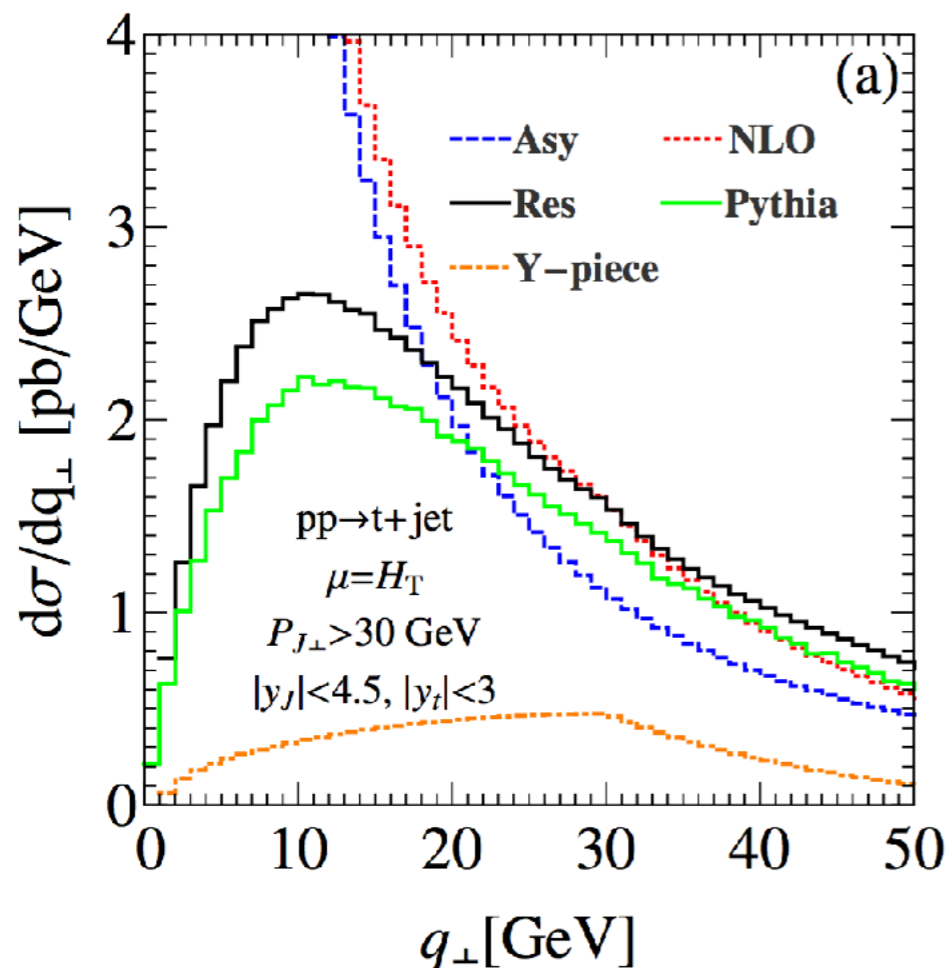
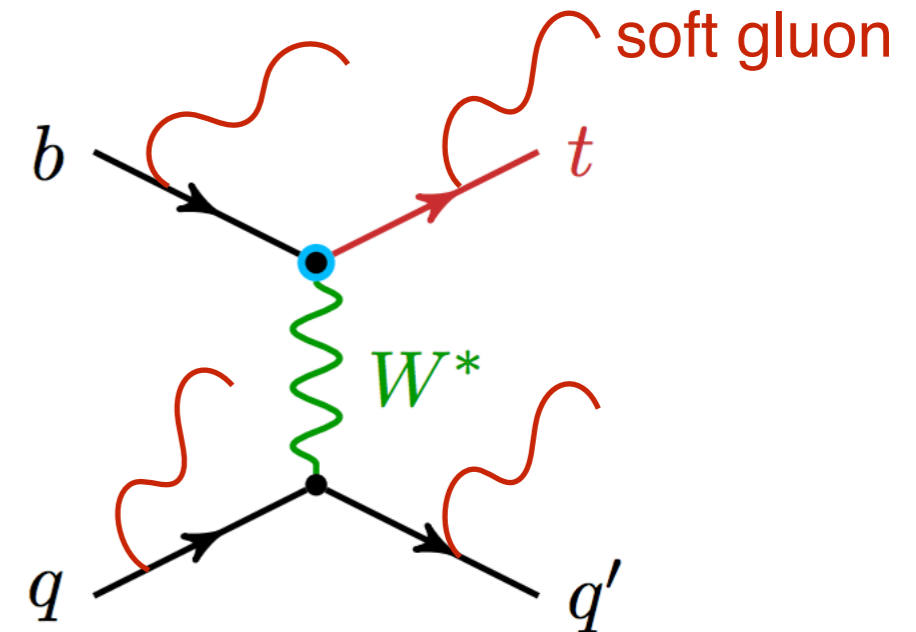
NNLO QCD production only:  
Brucherseifer, Caola, Melnikov,  
1404.7116

NNLO QCD prod. + decay:  
Berger, J. Gao, C.P. Yuan, HXZ,  
1606.08463  
Berger, J. Gao, HXZ, 1708.09405

**FIRST NNLO PROD. + DECAY  
CALCULATION FOR TOP !**

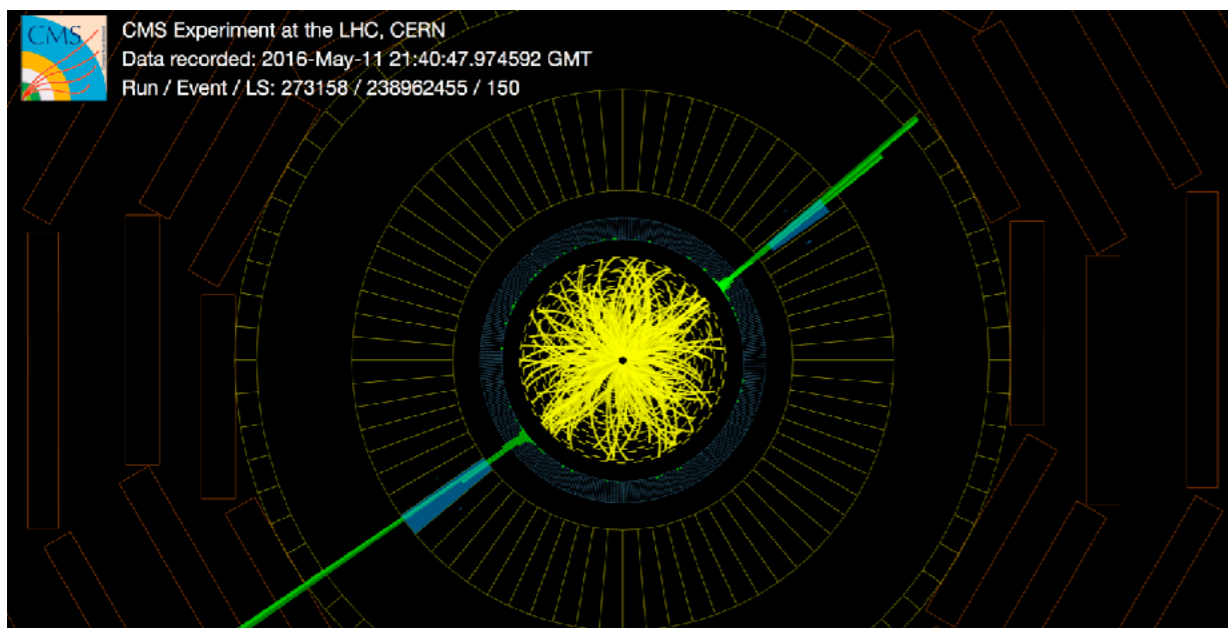
# Top + jet pT distribution

- The total transverse momentum of **top+jet** in single top is sensitive to ISR and FSR
- Resummation prediction can be used to **calibrate Monte-Carlo generator**
- **Substantial difference** with Pythia in the signal region ( $3 < |y_J| < 4.5$ )
- Must be careful when using event generator to calculate IR sensitive observable



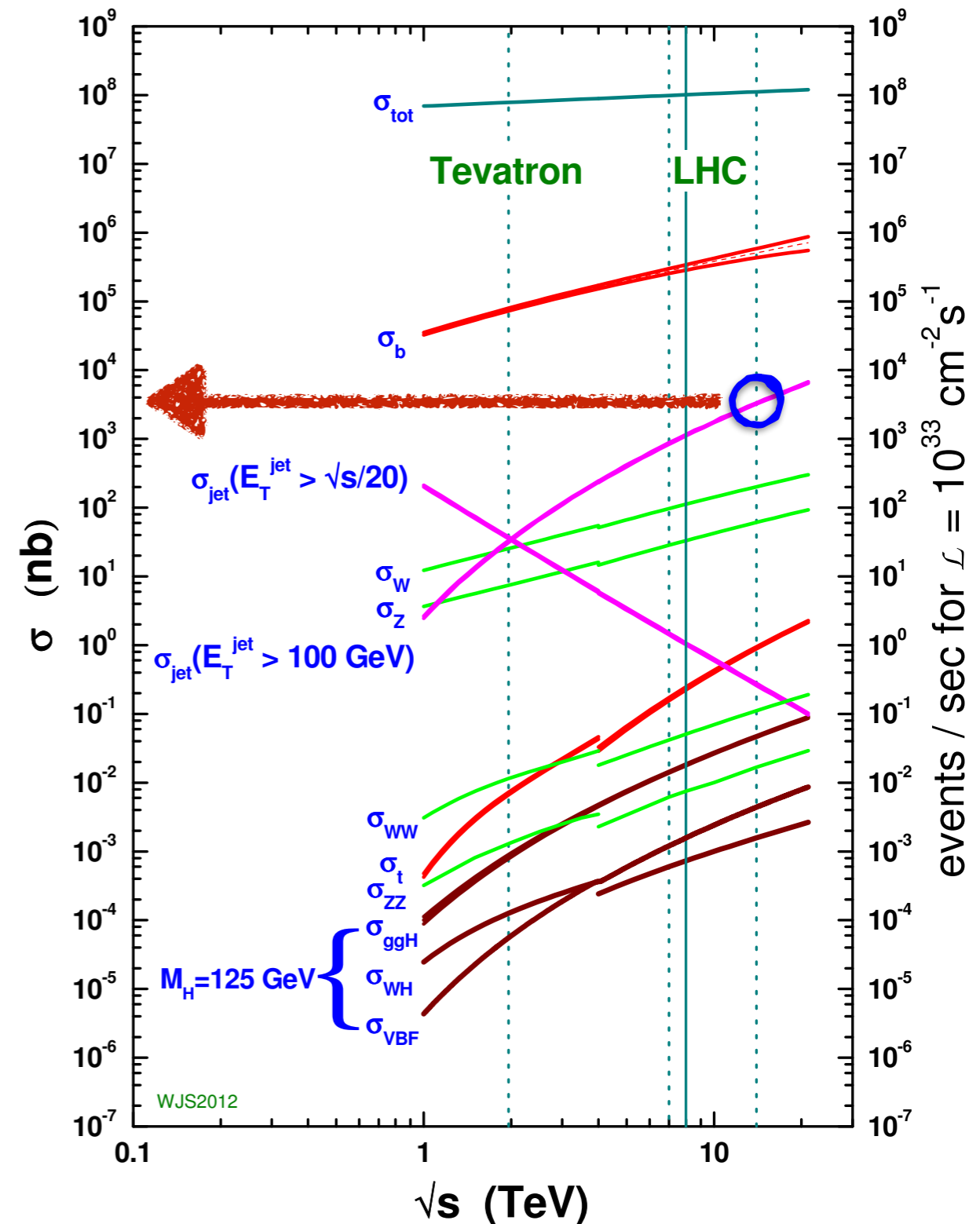
# Jets

- **Largest** Xsec in hard processes
- Important signal/background for new physics
- Precision  $\alpha_s$  extraction
- Sensitive to gluon PDF at large  $x$
- LHC cover **large kinematical range** in  $p_T$ : pressure test for pQCD

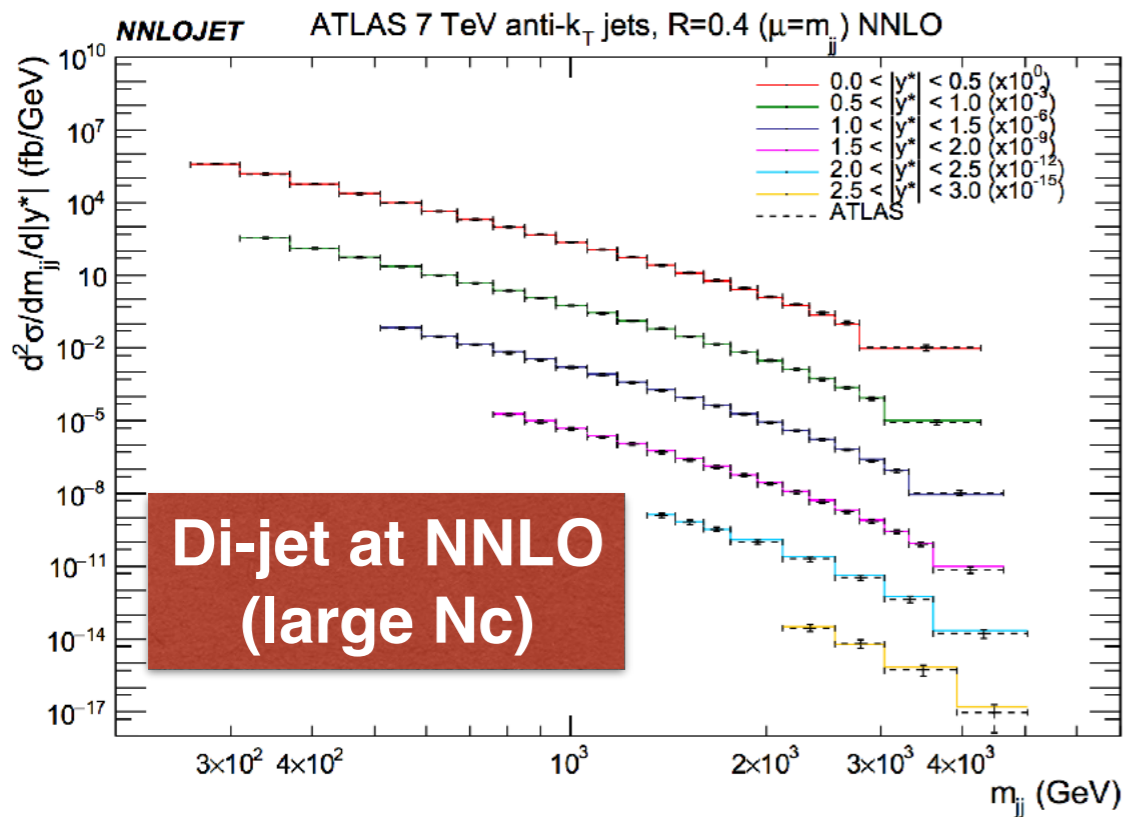


$$M_{JJ} = 7.7 \text{ TeV}$$

proton - (anti)proton cross sections



# Jet production



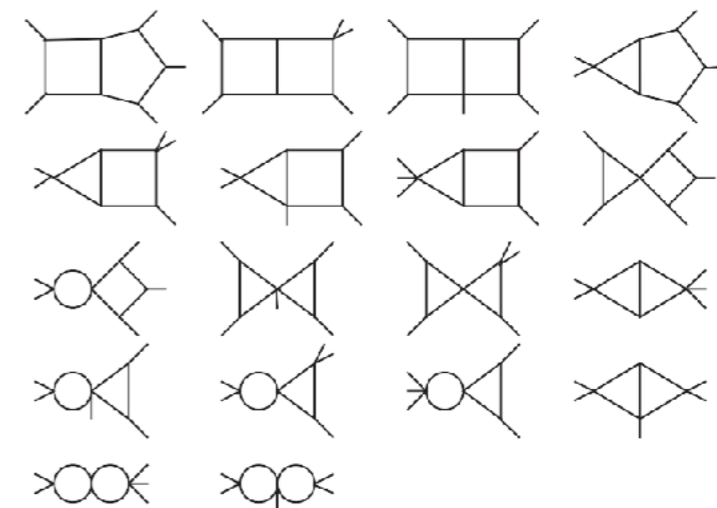
- Data span over **22 order of magnitude**
- The theoretical efforts towards NNLO results also span **over decades**
  - Two-loop virtual amplitudes: Smirnov, 1999; Tausk, 1999; Glover, et al, 2001-2003; Bern et al, 2002-2003
  - Infrared subtraction: Gehrmann-De Ridder, 2005-2013
  - Currently only leading color NNLO corrections available: Currie, Glover, Pires, 1611.01460
  - Phenomenology: Currie, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Pires, 1705.10271

- Towards **three-jet production** at NNLO: heroic efforts of many
- Generalized unitarity: Bern, Britto, Cachazo, Dixon, B. Feng, Kosower, Ita, ...
- integrand reduction: Badger, Frellesvig, Mastrolia, Ossola, Papadopoulos, Pittau, Peraro, Y. Zhang .....
- two-loop five-point planar integrals: Gehrmann, Henn, Lo Presti, Papadopoulos, Tommasini, Wever, ...
- Two-loop planar amplitudes: Abreu, Cordero, Badger, Boels, Brønnum-Hansen, Hartanto, H. Luo, Ita, Jaquier, Page, Peraro, M. Zeng, Y. Zhang, .....
- And many others

PHYSICAL REVIEW LETTERS **120**, 092001 (2018)

## First Look at Two-Loop Five-Gluon Scattering in QCD

Simon Badger,<sup>1,\*</sup> Christian Brønnum-Hansen,<sup>2</sup> Heribertus Bayu Hartanto,<sup>1</sup> and Tiziano Peraro<sup>3</sup>



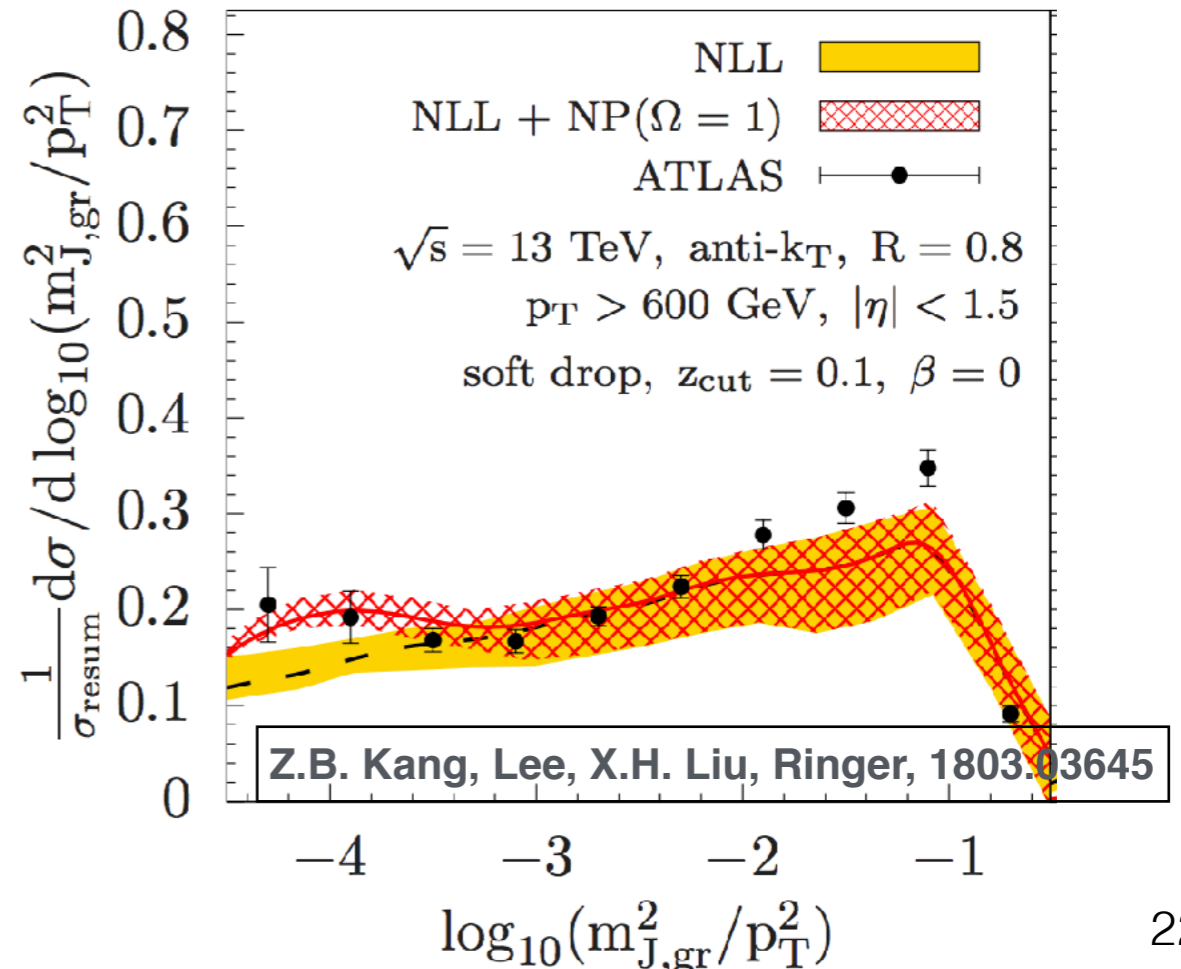
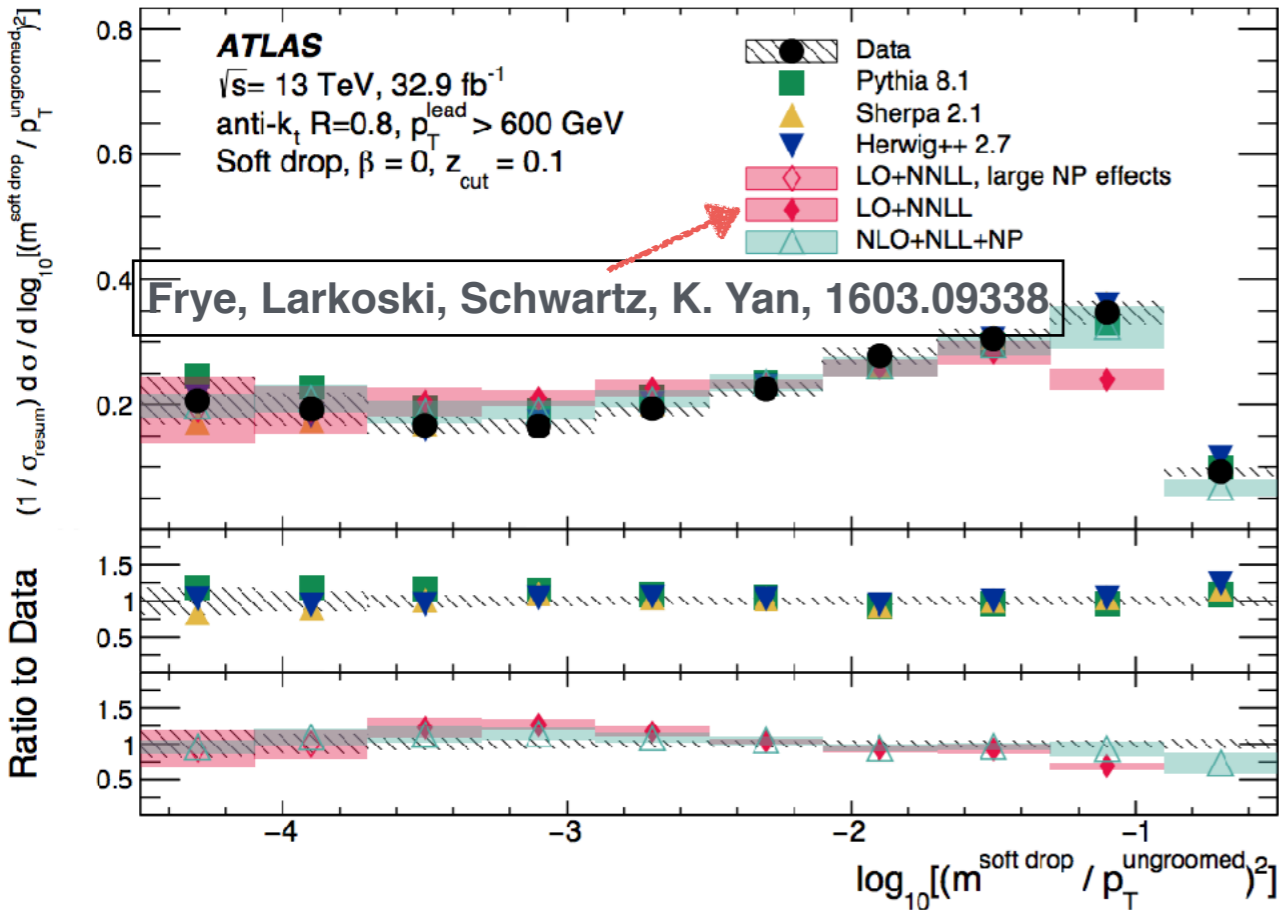
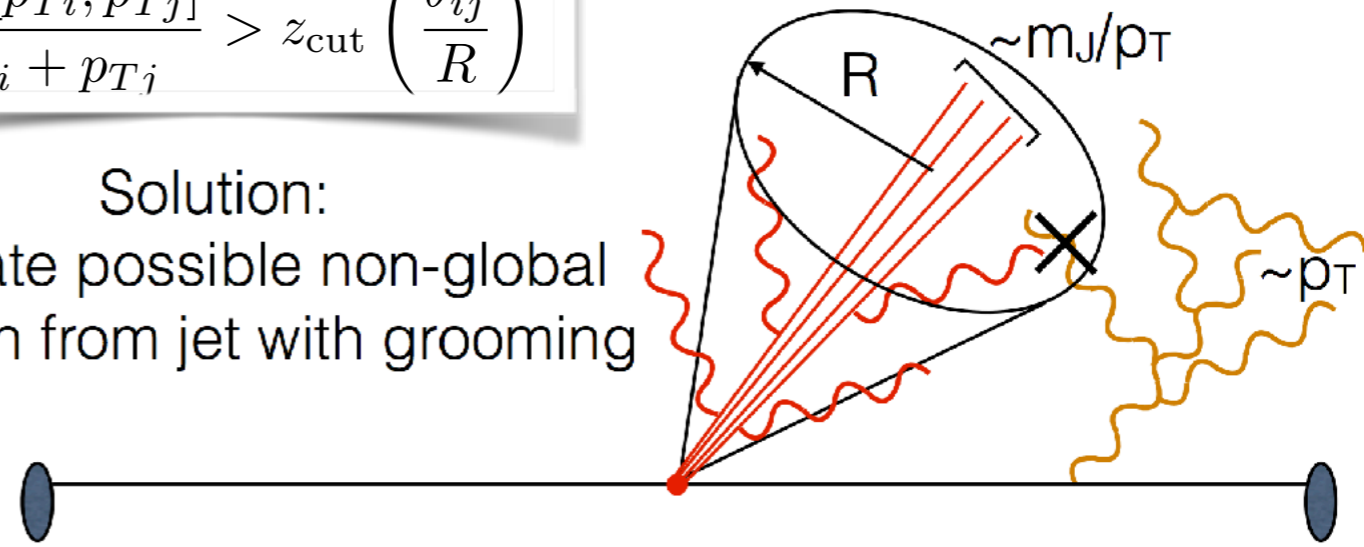
# Jet substructure at the LHC

- **Jet mass:** best studied jet substructure observable. Useful for **boosted particle tagging**
- Contamination by soft radiations outside the jet
- Jet Grooming: entering the precision era

$$\frac{\min[p_{Ti}, p_{Tj}]}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left( \frac{\theta_{ij}}{R} \right)^\beta$$

Solution:  
eliminate possible non-global radiation from jet with grooming

From Larkoski in LHCP 2018



# Jet vetos and Non-Global Logs

- Unlike new physics search, in precision QCD quite often experimentalists are **well ahead of theorists**

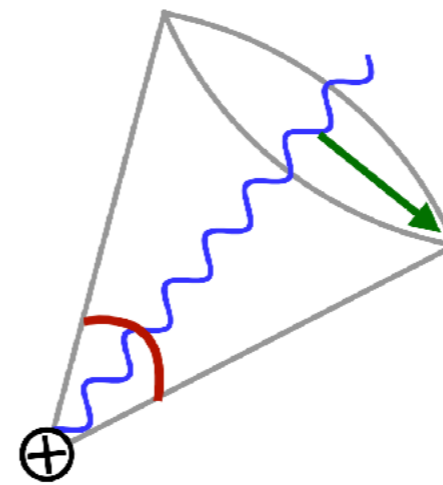
- Experiments use isolation criteria to define **isolated photons**

$$\sum_{\text{had} \in \mathcal{C}(R)} E_T^{\text{had}} \leq \epsilon_\gamma E_T^\gamma$$

- **Important in  $H \rightarrow \gamma\gamma$  measurements**

- When  $R$  and  $\epsilon_\gamma$  small, large logarithms (**Non-global logarithms**)

- LL resummation by solving **infinite dimension** evolution equation by Monte-Carlo method



See D.Y. Shao's talk

$p_T^\gamma$

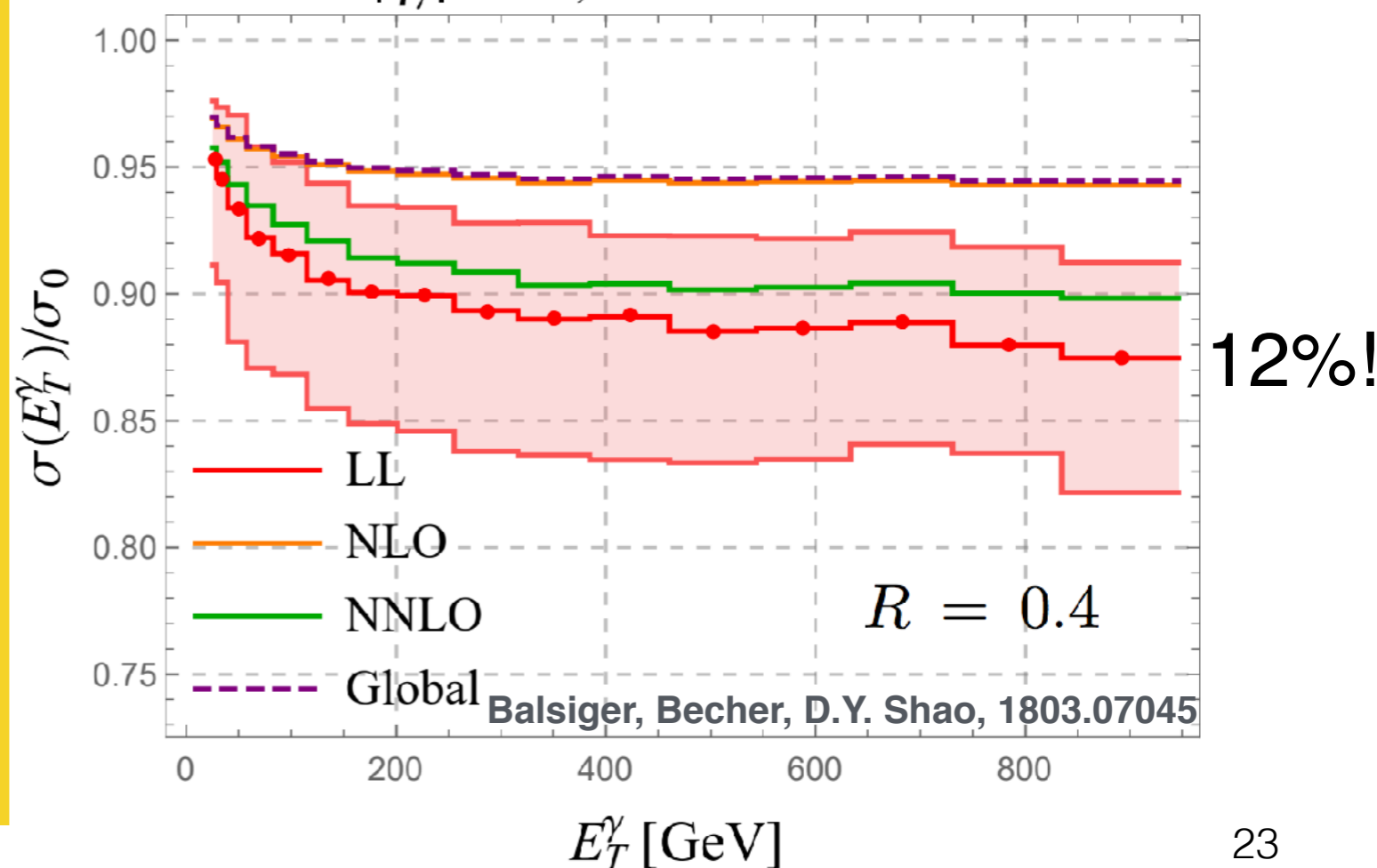
$R p_T^\gamma$

$\epsilon_\gamma p_T^\gamma$

Multiple scales problems

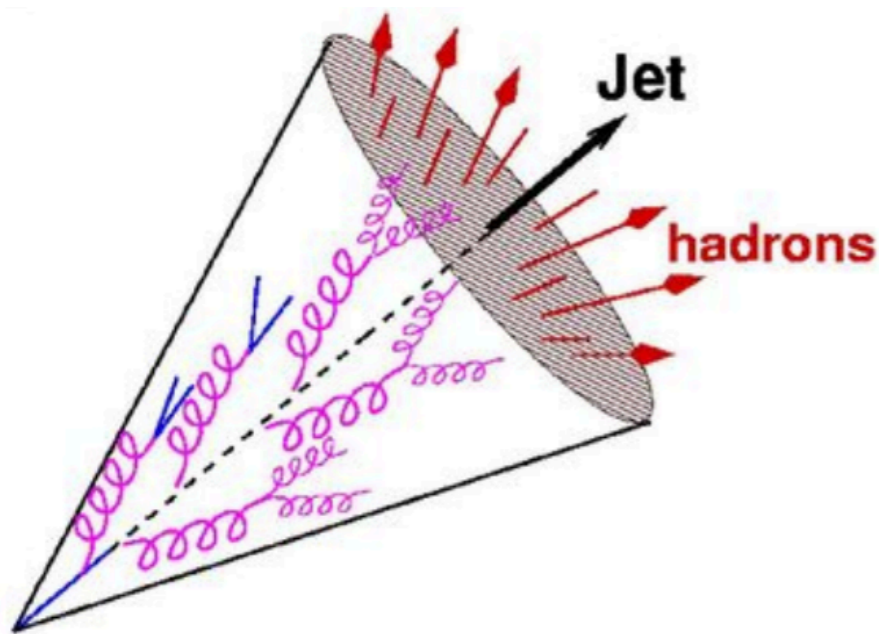
$$R^2 \times \alpha_s^n \ln^n \epsilon_\gamma \ln^{n-1} R$$

$|\eta_\gamma| < 0.6$ , ATLAS isolation

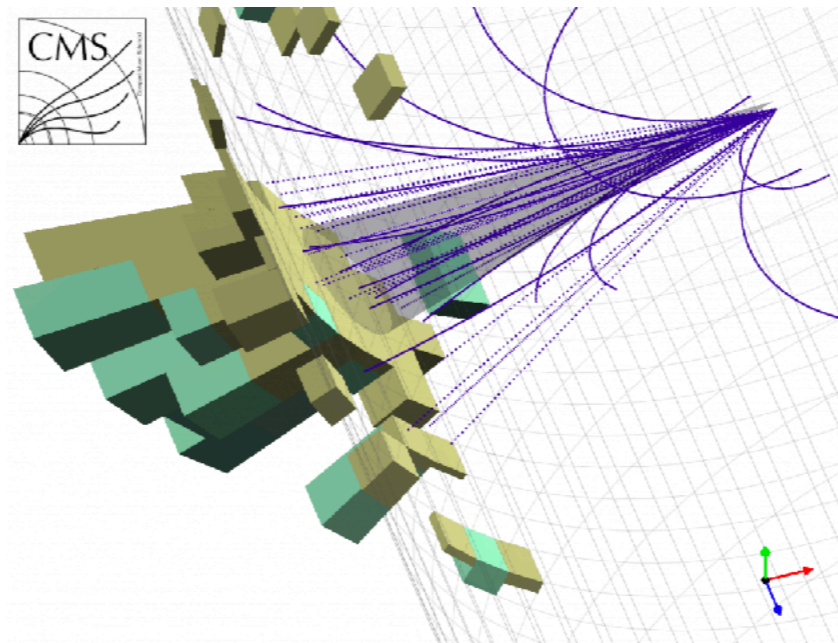


# Detailed look into “jets”

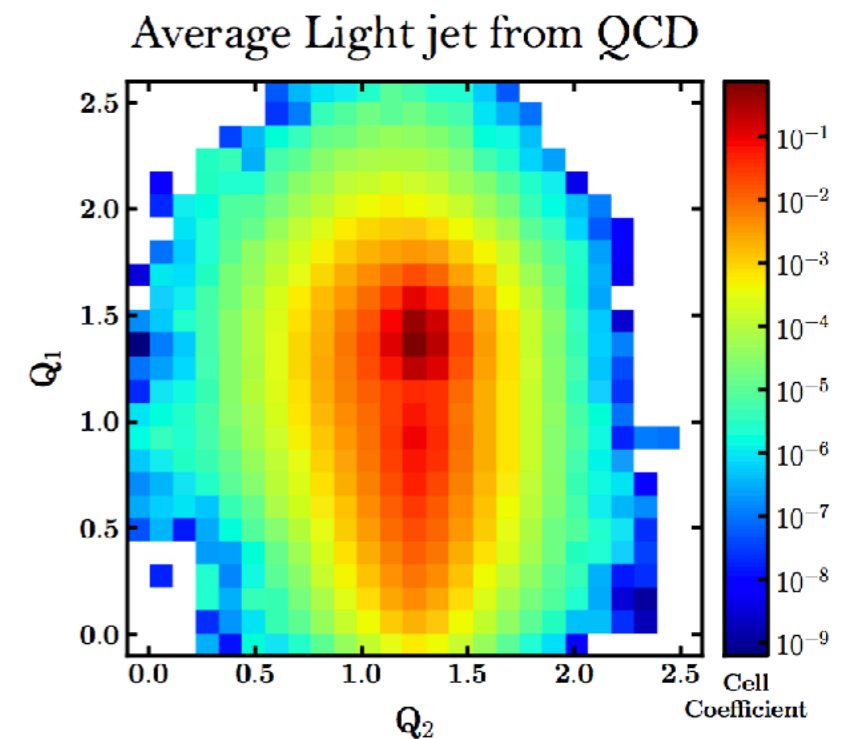
- **Huge amount of data sample will be available**
  - LHC  $O(10^{13})$  jet with  $p_T > 200\text{GeV}$
  - for CEPC at 250 GeV:  $\sim 1$  di-jet event per second



Feynman diagram



event display



energy deposition



- Current paradigm uses jet algorithm to define jet
- Example: the sequential clustering algorithm

As usual, one introduces distances  $d_{ij}$  between entities (particles, pseudojets)  $i$  and  $j$  and  $d_{iB}$  between entity  $i$  and the beam (B). The (inclusive) clustering proceeds by identifying the smallest of the distances and if it is a  $d_{ij}$  recombining entities  $i$  and  $j$ , while if it is  $d_{iB}$  calling  $i$  a jet and removing it from the list of entities. The distances are recalculated and the procedure repeated until no entities are left.

Cacciari, Salam, 0802.1189

- While jet algorithm is extremely useful in practice, it obscures the connection between physical observable and fundamental objects in quantum field theory

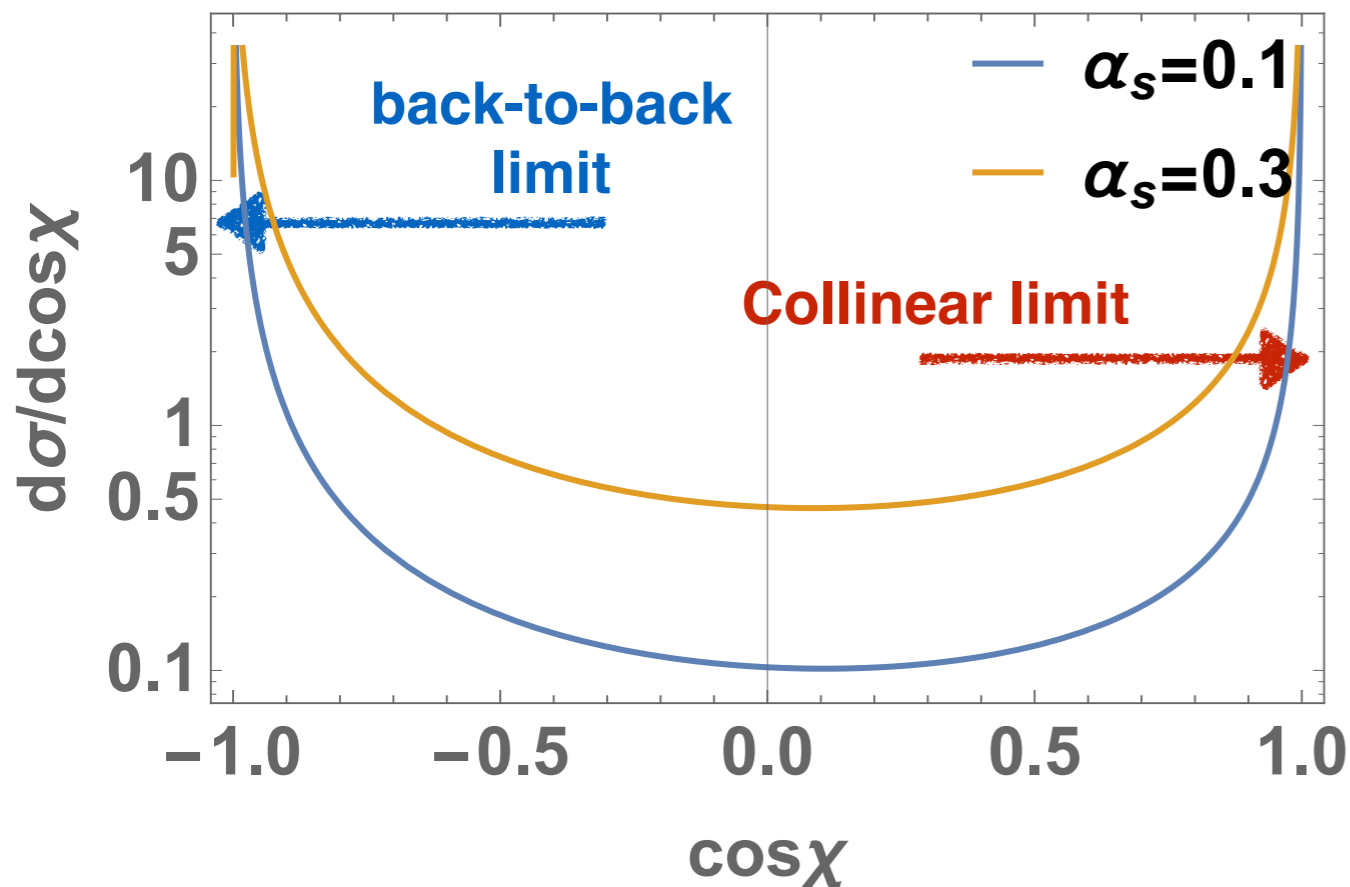
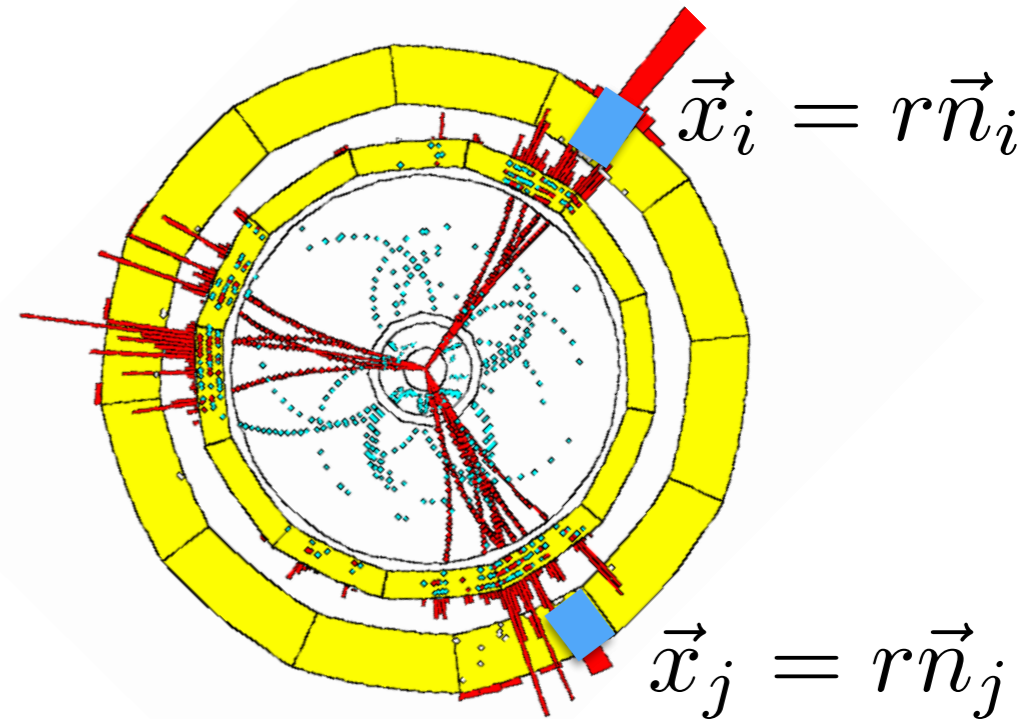
**Is there a physical observable that measures the jetty nature of QCD, while admits simple field theory interpretation?**

# Energy-Energy Correlation (EEC)

- Energy correlation of a pair of detector calorimeter at fixed angle  $\chi$ , sum over all pairs

Basham, Brown, Ellis, Love, 1978

$$\frac{1}{\sigma} \frac{d\Sigma_{\text{EEC}}(\chi)}{d \cos \chi} = \frac{1}{\Delta\chi N_{\text{events}}} \sum_{N_{\text{events}}} \sum_{ij} \frac{E_i E_j}{E^2}$$



- In the weak coupling limit, the di-jet nature of the event appear as **peak at the end point**
- As coupling becomes strong, the distribution is **flatten**

# Field theoretic definition for EEC

- EEC defined as (light-cone) **four point correlation function**

Hofmann, Maldacena, 0803.1467

Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 1309.0769

$$\text{EEC} \sim \underbrace{\int d^4x e^{iqx}}_{\text{Fourier}} \underbrace{\int_0^\infty dt_1 dt_2 \lim_{r_i \rightarrow \infty} r_1^2 r_2^2}_{\text{Detector limit}} \underbrace{\langle 0 | O^\dagger(x) T_{0\vec{n}_1}(x_1) T_{0\vec{n}_2}(x_2) O(0) | 0 \rangle_W}_{\text{Wightman correlation function}} \Big|_{x_i = (t, r\vec{n}_i)}$$

Energy-Momentum tensor

Operator that create QCD excitation

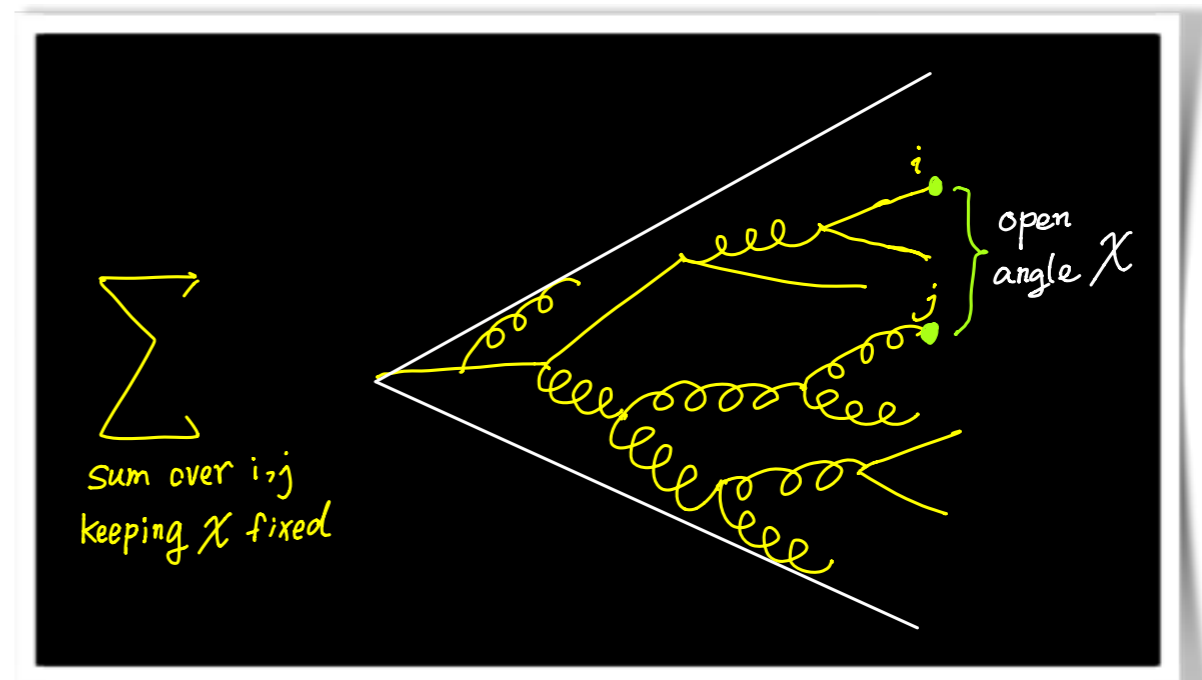
$O(x) = \bar{\psi} \gamma^\mu \psi(x)$

- In the collinear limit  $\chi \rightarrow 0$ , EEC measures energy correlation **within a jet**

- Can be studied by light-cone OPE

$$\langle \mathcal{E}(\vec{y}) \mathcal{E}(0) \rangle \sim \sum |y|^{-2 + (\tau_a - 2)} c_a \langle \mathcal{U}_a \dots \rangle$$

- $\mathcal{U}_a$  are spin 3, twist 2 operator



- Despite having nice field theoretic definition, the study for EEC is far from complete
  - Operator product expansion in the collinear limit
  - Non-perturbative power corrections
  - Correlation function definition for EEC and lattice calculation
  - EEC for precision  $\alpha_s$  extraction
  - .....

# Analytic NLO calculation for EEC

Dixon, M.X. Luo, Shtabovenko, T.Z. Yang, HXZ, 1801.03219

- Final result is simple enough to fit on (five) blackboards

$$\frac{1}{\sigma_0} \frac{d\Sigma}{d\cos\chi} = \frac{\kappa_s(\mu)}{2\pi} A(z) + \left(\frac{\kappa_s(\mu)}{2\pi}\right)^2 \left( \beta_0 \ln \frac{\mu}{Q} A(z) + B(z) \right) + \mathcal{O}(\alpha_s^3)$$

$$B(z) = C_F^2 B_{2c}(z) + C_F(C_A - 2C_F) B_{Nc}(z) + C_F N_f T_f B_{Nf}(z)$$

$$B_{2c}(z) = \frac{122400z^7 + 244800z^6 + 157060z^5 - 31000z^4 + 2064z^3 + 72305z^2 - 143577z + 63298}{1440(1-z)^2} - \frac{244800z^8 + 673200z^7 - 667280z^6 + 283140z^5 - 48122z^4 + 2716z^3 - 6201z^2 + 11309z - 9329z + 3007}{720(1-z)^2} - \frac{244800z^8 - 550800z^7 + 422480z^6 - 126100z^5 + 13052z^4 - 336z^3 + 17261z^2 - 38295z + 19938}{720(1-z)^4} + \frac{4z^7 + 10z^6 - 17z^5 + 25z^4 - 96z^3 + 296z^2 - 211z + 87}{24(1-z)^5} + \frac{-48900z^8 + 61200z^7 - 28480z^6 + 4040z^5 - 320z^4 - 160z^3 + 1126z^2 - 4726z + 3323}{120z^2} - \frac{1-11z}{48z^2} g_2^{(2)}(z)$$

$$B_{Nc}(z) = \frac{1}{120} \left[ \frac{110z^8 - 10z^7 + 97z^6 - 548z^5 + 143z^4}{480z^2} - \frac{120z^8 + 70z^7 - 52z^6 - 585z^5 - 281z^4 - 119z^3 + 33z^2}{60(1-z)^2} + \frac{640z^8 - 1920z^7 + 2196z^6 - 176z^5 + 318z^4 - 42z^3}{4(1-z)^2} + \frac{2z^7 - 3z^6 + 3z^5 - 2z^4 + 9z^3 - 7z^2 + 1}{12(1-z)^2} - \frac{(1-7z)(z-1)}{2(1-z)^2} g_1^{(1)}(z) - \frac{2z^2 - z^3 + 2z^2 + z^3}{4z^2} g_2^{(1)}(z) \right]$$

$$B_{Nf}(z) = \frac{5700z^8 - 11800z^7 + 7579z^6 - 175z^5 + 92z^4 + 1914z^3 - 7257z^2 + 9320}{720(1-z)^2} - \frac{[11700z^7 + 31800z^6 - 22180z^5 + 14754z^4 - 31035z^3 - 3725z^2 - 4571z + 1137z - 124(2z + 4880)] g_1^{(1)}(z)}{480(1-z)^2} - \frac{[29040z^8 - 61840z^7 + 412960z^6 - 138600z^5 + 18616z^4 - 712z^3 + 10771z^2 - 2502z + 11424] g_2^{(1)}(z)}{720(1-z)^2} + \frac{-11z^7 + 235z^6 - 184z^5 + 15z^4 - 140z^3 + 721z^2 - 702z + 119}{20(1-z)^2} g_3^{(1)}(z) - \frac{[17200z^8 - 259600z^7 - 14030z^6 - 2680z^5 - 340z^4 - 40z^3 + 315z^2 - 1481z + 1452] g_4^{(1)}(z)}{60z^2}$$

$$g_1^{(1)}(z) = \frac{110z^8 - 10z^7 + 97z^6 - 548z^5 + 143z^4}{480z^2} - \frac{120z^8 + 70z^7 - 52z^6 - 585z^5 - 281z^4 - 119z^3 + 33z^2}{60(1-z)^2} + \frac{640z^8 - 1920z^7 + 2196z^6 - 176z^5 + 318z^4 - 42z^3}{4(1-z)^2} + \frac{2z^7 - 3z^6 + 3z^5 - 2z^4 + 9z^3 - 7z^2 + 1}{12(1-z)^2} - \frac{(1-7z)(z-1)}{2(1-z)^2} g_1^{(1)}(z) - \frac{2z^2 - z^3 + 2z^2 + z^3}{4z^2} g_2^{(1)}(z)$$

$$g_2^{(1)}(z) = \frac{-720z^8 - 1440z^7 + 872z^6 - 582z^5 + 48z^4 + 182z^3 - 415z^2 + 2760}{144(1-z)^2} - \frac{7200z^8 - 19300z^7 + 193040z^6 - 77700z^5 + 10100z^4 - 100z^3 - 187z^2 + 329z - 48z + 189}{50(1-z)^2} + \frac{36000z^8 - 91000z^7 + 60520z^6 - 1650z^5 + 1190z^4 + 10z^3 + 429z^2 - 95z + 56}{10(1-z)^2} + \frac{-2z^8 + 18z^7 - 21z^6 + 9}{5(1-z)^2} g_3^{(1)}(z) - \frac{11000z^8 + 1200z^7 - 7840z^6 + 20z^5 + 20z^4 + 172z^3 - 72z^2 + 197z}{60z^2} g_4^{(1)}(z) + \frac{1-3z}{48z^2} g_5^{(1)}(z) + \frac{8z^2(11z^2+7z+7)}{60(1-z)^2} g_6^{(1)}(z) + 2(50z^2 - 100z^2 + 6(1-z)(1+z)) g_7^{(1)}(z)$$

**First analytic QCD event shape distribution!**

- In the collinear limit

$$\left. \frac{d\sigma}{d\chi} \right|_{\chi \rightarrow 0} = \frac{\alpha_s}{2\pi} \frac{3C_F}{4\chi} + \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{\chi} \left[ \left( \frac{25}{8} C_F^2 - \frac{107}{30} C_F C_A + \frac{53}{60} C_F n_f t_f \right) \ln \frac{\chi}{2} + C_F^2 \left( -2\zeta_3 + \frac{43}{36} \pi^2 - \frac{8263}{864} \right) + C_F C_A \left( \zeta_3 - \frac{25}{36} \pi^2 + \frac{17683}{1350} \right) - \frac{4913}{1800} C_F n_f t_f \right]$$

known from jet calculus

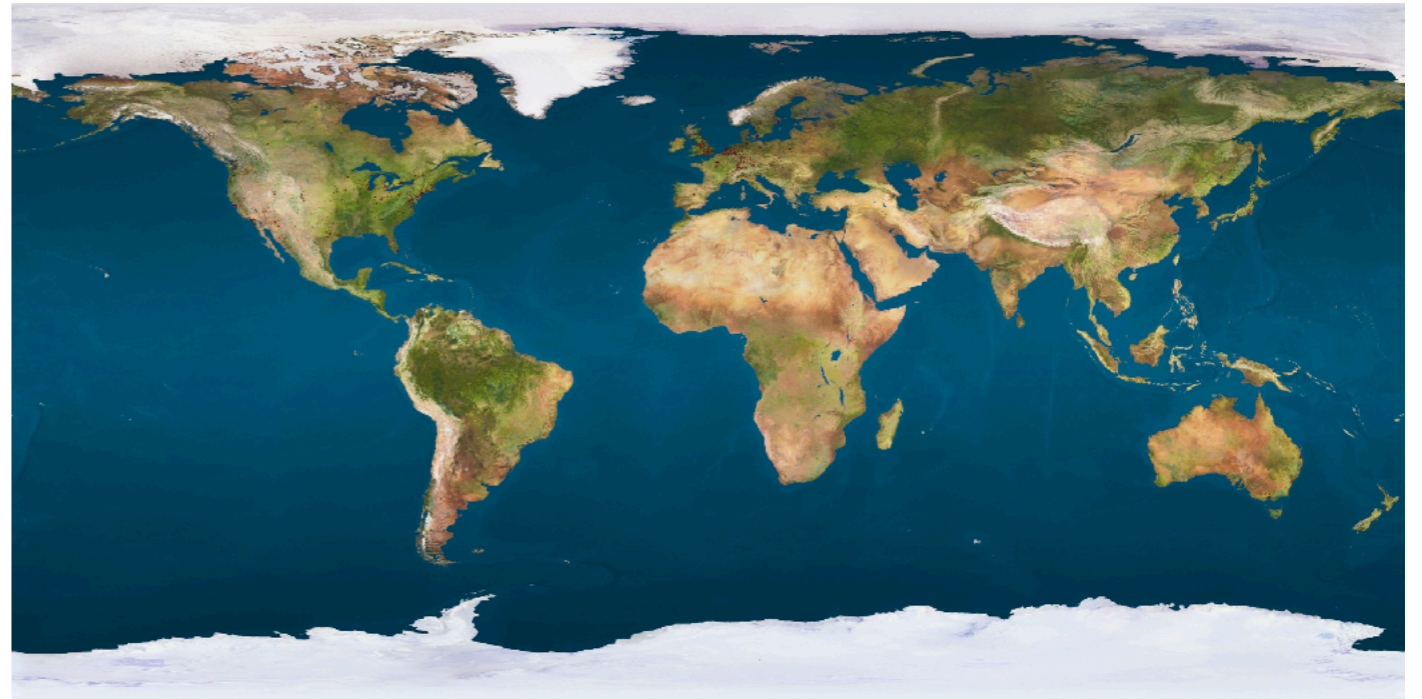
**New results!**

# Conclusion

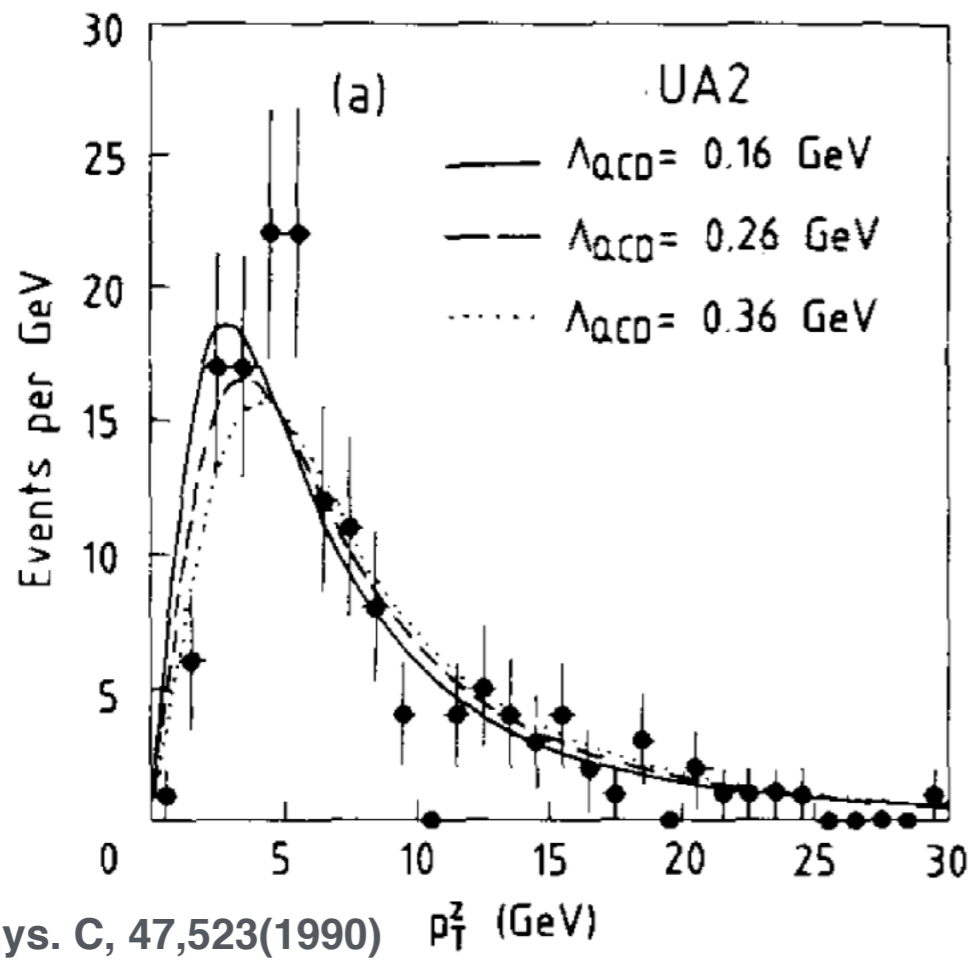
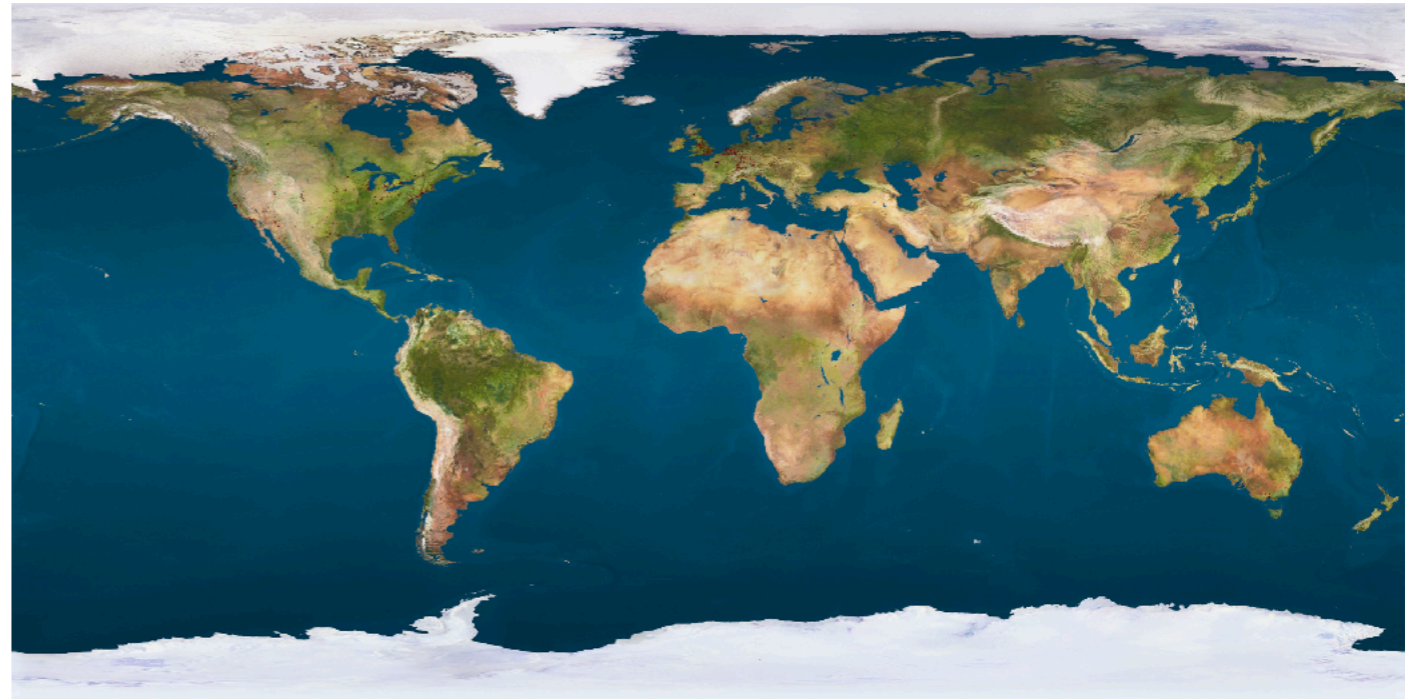
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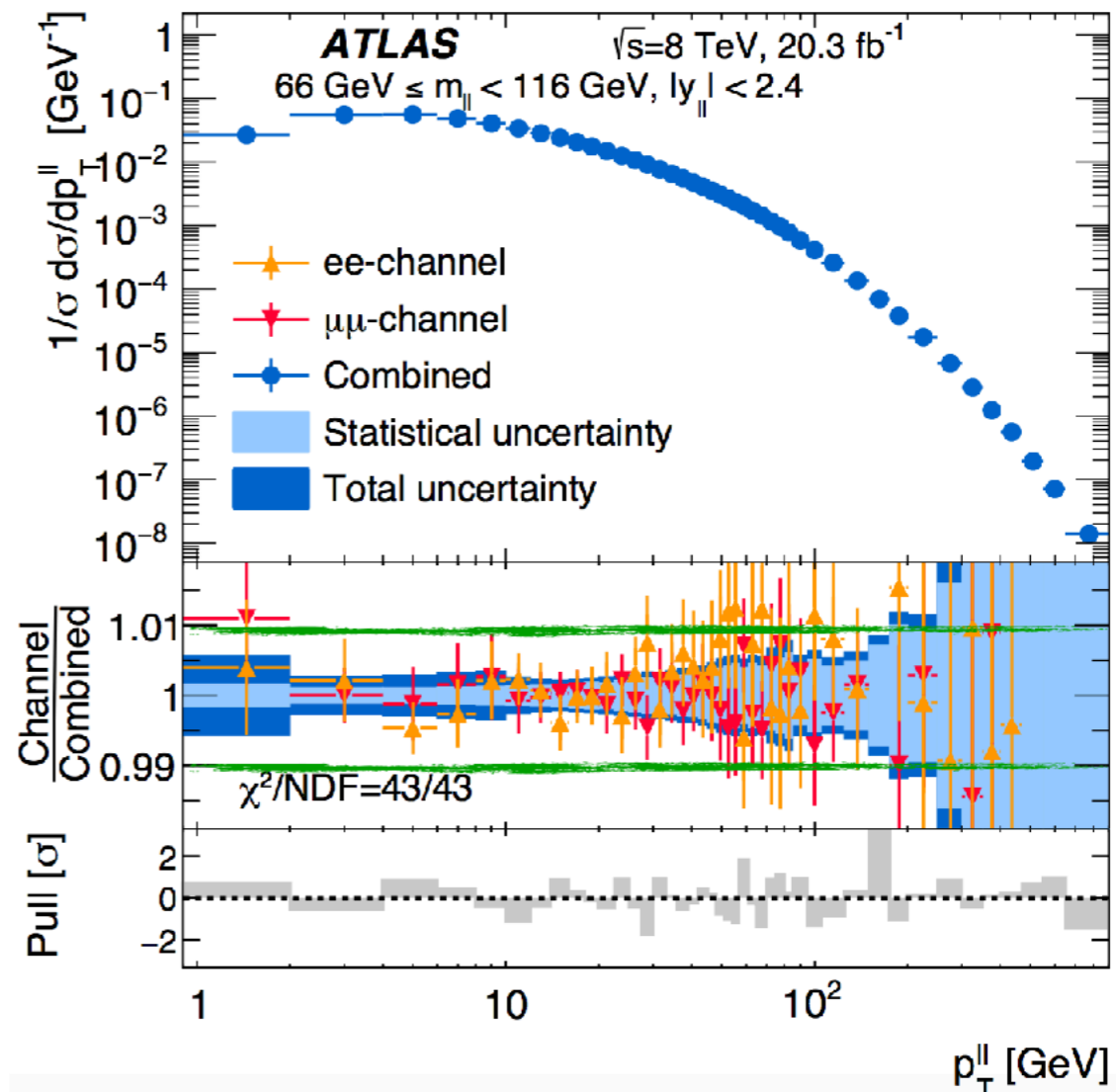
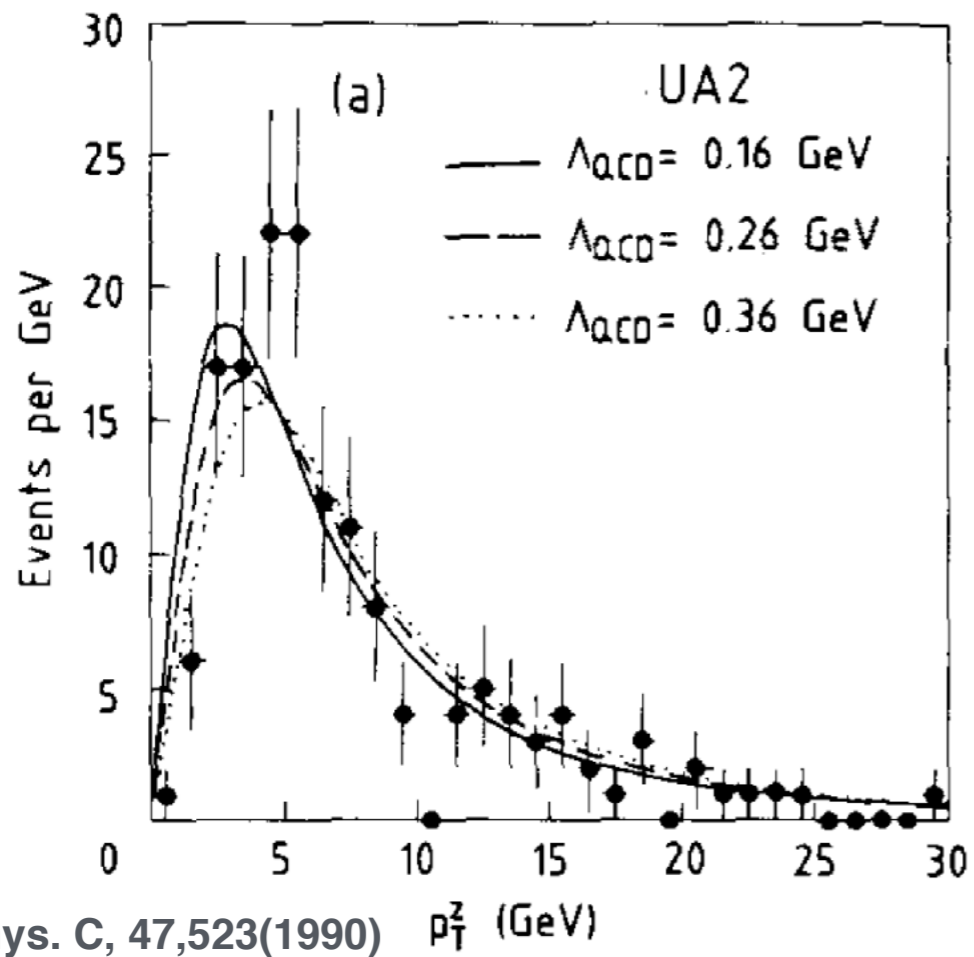
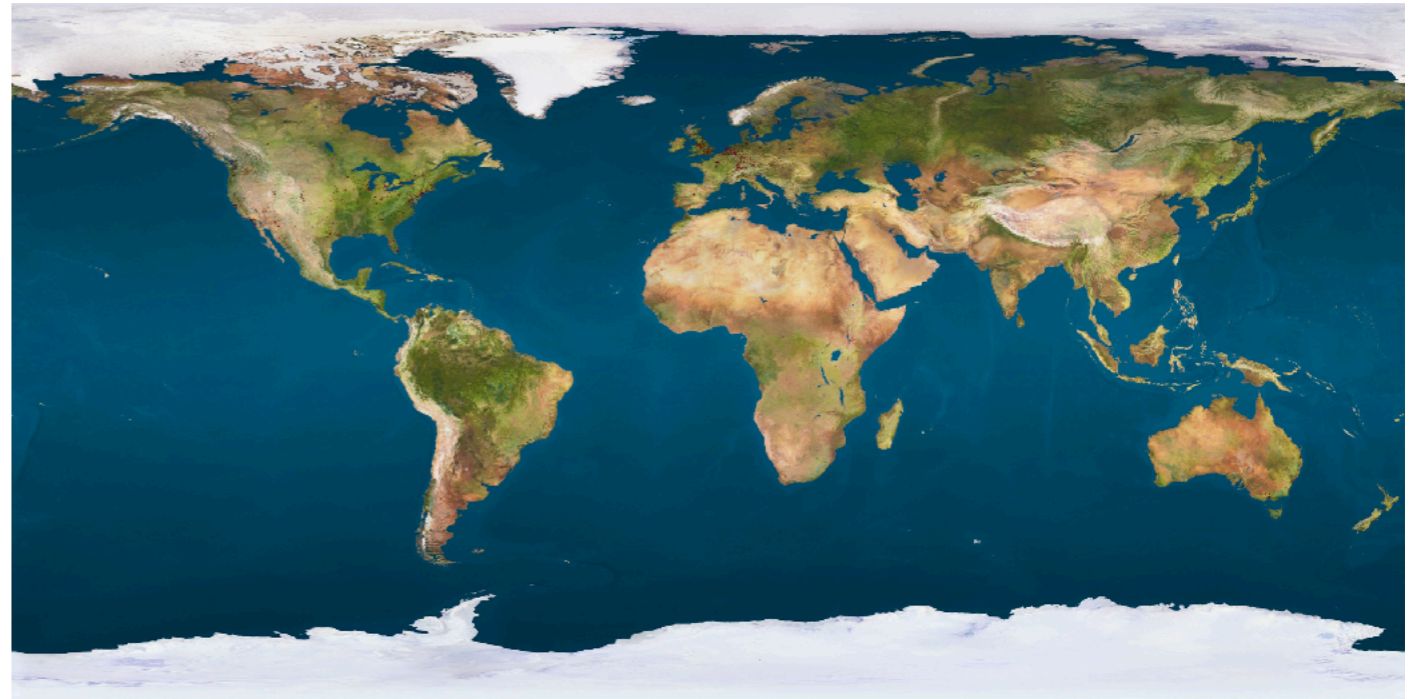


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**Thank you!**