半单举正负电子湮灭过程中的三维碎裂函数

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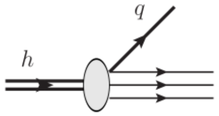
目录

- 研究背景
- $e^+e^- \to q\,\bar{q}$
  - 夸克纵向极化
  - 夸克-反夸克横向极化关联
- $e^+e^- \to h\,\bar{q}\,X$
  - 方位角不对称
  - 强子极化
- 唯象学研究
- 总结

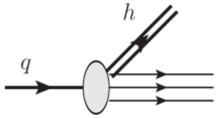
研究背景

- 部分子分布函数和碎裂函数是描述高能反应的重要物理量。

$f_1^{q/h}(x)$: 在强子 h 中找到动量分数为 x 的部分子 q 的几率。



$D_1^{h/q}(z)$: 在部分子q中找到动量分数为z的强子h 的几率。



- 若强子自旋不为0，需要考虑自旋相关的部分子函数。
- 若描述更精细的强子结构，需要考虑三维的部分子函数。

强子极化 / 夸克极化	U	L	T
U	f_1 数密度		$f_{1T}^{\perp}$ − Sivers
L		g_{1L} −	g_{1T} −
T	$h_1^{\perp}$ − Boer-Mulders	$h_{1L}^{\perp}$ −	h_{1T} − Transversity $h_{1T}^{\perp}$ −

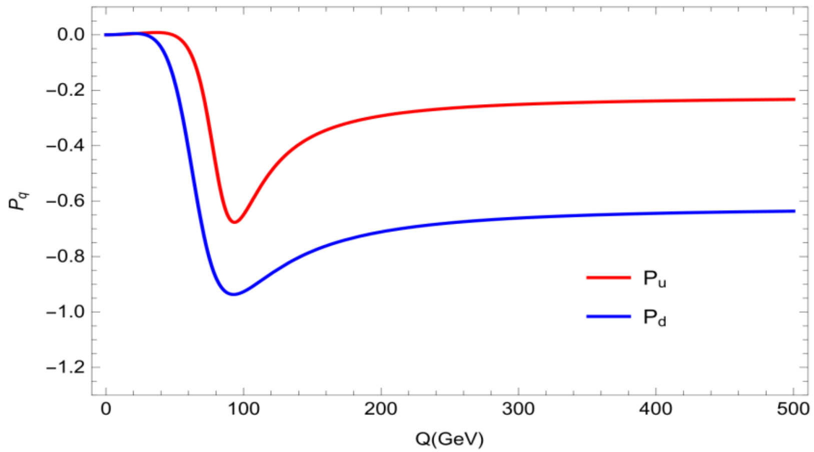
强子极化 / 夸克极化	U	L	T
U	D_1 数密度		$D_{1T}^{\perp}$ −
L		G_{1L} − 纵向极化	G_{1T} −
T	$H_1^{\perp}$ − Collins	$H_{1L}^{\perp}$ −	H_{1T} − 横向极化 $H_{1T}^{\perp}$ −

- e^+e^-湮灭过程是研究部分子碎裂函数最为干净的过程。

$e^+e^- \to q\,\bar{q}$

➢ 夸克纵向极化

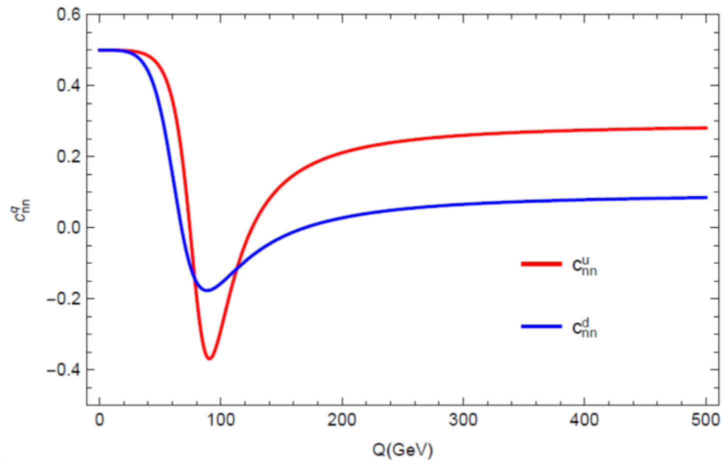
$$P_q(Q) = -\frac{\chi c_1^e c_3^q + \chi_{int}^q c_V^e c_A^q}{\chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q + e_q^2},$$



- 低能区, $P_u \approx P_d \approx 0$;
- $Q = M_Z, P_u \approx 0.6717, P_d \approx 0.9362.$

➢ 夸克-反夸克横向极化关联

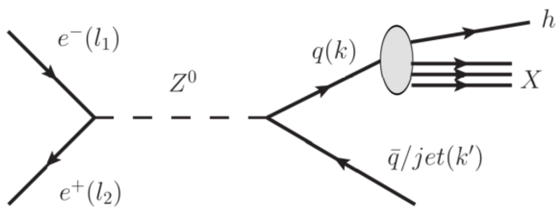
$$c_{nn}^q(Q) = \frac{\chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q + e_q^2}{2[\chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q + e_q^2]},$$



- 低能区, $c_{nn}^u \approx c_{nn}^d \approx 0.5$;
- $Q \approx M_Z$, $c_{nn}^u \approx -0.3683, c_{nn}^d \approx -0.1763$.

$e^+e^- \to h\,\bar{q}\,X$

1. $e^+e^- \to h\bar{q}X$ 微分截面



$$\frac{E_p d\sigma}{d^3p d^2k'_\perp} = \frac{\alpha^2\chi}{4\pi^2 s^3} L_{\mu\nu}(l_1,l_2) W^{\mu\nu}(q,p,S,k'_\perp).$$

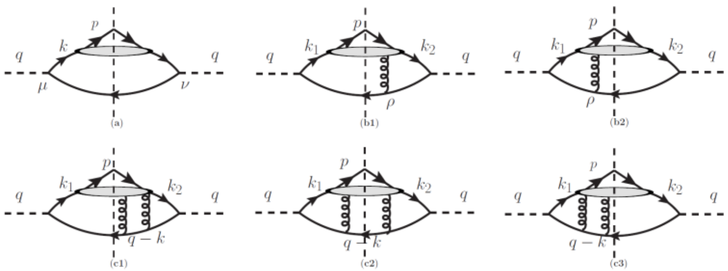
- 轻子张量：$L_{\mu\nu}(l_1,l_2) = c_1^e\left(l_{1\mu}\, l_{2\nu} + l_{1\nu}\, l_{2\mu} - g_{\mu\nu} l_1 \cdot l_2\right) + ic_3^e \varepsilon_{\mu\nu l_1 l_2}.$
- 强子张量：$W^{\mu\nu} = \int \frac{d^4y}{2\pi} e^{-iqy} \sum_X \langle 0|J^\nu(y)|p,S,k'_\perp;X\rangle\langle p,S,k'_\perp;X|J^\mu(0)|0\rangle.$

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\hat{h}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}],$$

$$\tilde{W}^{(1,L)}_{\mu\nu} = -\frac{1}{4(p\cdot q)}\mathrm{Tr}[\hat{h}^{(1)\rho}_{\mu\nu}\hat{\Xi}^{(1)}_{\rho}],$$

$$\tilde{W}^{(2,L)}_{\mu\nu} = \frac{1}{4(p\cdot q)^2}\mathrm{Tr}[\hat{N}^{(2)\rho\sigma}_{\mu\nu}\hat{\Xi}^{(2)}_{\rho\sigma}],$$

$$\tilde{W}^{(2,M)}_{\mu\nu} = \frac{1}{4(p\cdot q)^2}\mathrm{Tr}[\hat{h}^{(2)\rho\sigma}_{\mu\nu}\hat{\Xi}^{(2,M)}_{\rho\sigma}].$$



$\hat{h}_{\mu\nu}$, 可微扰计算的硬部分;
$\hat{\Xi}^{(j)}$, 不可微扰计算的关联函数。

$e^+e^- \to h\,\bar{q}\,X$

2. 关联函数分解

- $\hat{\Xi}^{(0)} = \Xi^{(0)}_{\alpha}\gamma^{\alpha} + \tilde{\Xi}^{(0)}_{\alpha}\,\gamma^5\gamma^{\alpha},$

$$z\Xi^{(0)}_{\alpha} = p_{\alpha}(D_1 + S_{LL}D_{1LL}) - M\varepsilon_{\perp\rho\alpha}S^{\rho}D_T + \frac{M^2}{p^+}n_{\alpha}(D_3 + S_{LL}D_{3LL}) + \cdots$$

$$z\tilde{\Xi}^{(0)}_{\alpha} = -p_{\alpha}\lambda_h G_{1L} - MS_{T\alpha}D_T - \frac{M^2}{p^+}n_{\alpha}\lambda_h G_{3L} + \cdots$$

- $\hat{\Xi}^{(1)}_{\rho} = \Xi^{(1)}_{\rho\alpha}\gamma^{\alpha} + \tilde{\Xi}^{(1)}_{\rho\alpha}\,\gamma^5\gamma^{\alpha},$

$$z\Xi^{(1)}_{\rho\alpha} = -p_{\alpha}\mathrm{M}\varepsilon_{\perp\rho\beta}S^{\beta}_{\perp}D_{dT} + \mathrm{M}^2\mathrm{g}_{\perp\rho\alpha}\mathrm{D}_{3\mathrm{d}} + \mathrm{i}\lambda_h M^2\varepsilon_{\perp\rho\alpha}D_{3dL} + \cdots$$

$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = ip_{\alpha}\mathrm{M}S_{T\rho}G_{dT} + iM^2\varepsilon_{\perp\rho\alpha}G_{3d} + \lambda_h M^2 g_{\perp\rho\alpha}G_{3dL} + \cdots$$

- $\hat{\Xi}^{(2)}_{\rho\sigma} = \Xi^{(2)}_{\rho\sigma\alpha}\gamma^{\alpha} + \tilde{\Xi}^{(2)}_{\rho\sigma\alpha}\,\gamma^5\gamma^{\alpha},$

$$z\Xi^{(2)}_{\rho\sigma\alpha} = p_{\alpha}\left[\mathrm{M}^2\mathrm{g}_{\perp\rho\sigma}(\mathrm{D}_{3\mathrm{dd}}+\mathrm{S}_{\mathrm{LL}}\mathrm{D}_{3\mathrm{ddLL}}) + i\lambda_h M^2\varepsilon_{\perp\rho\alpha}D_{3ddL}\right] + \cdots$$

$$z\tilde{\Xi}^{(2)}_{\rho\sigma\alpha} = p_{\alpha}\left[i\mathrm{M}^2\varepsilon_{\perp\rho\sigma}(\mathrm{G}_{3\mathrm{dd}} + \mathrm{S}_{\mathrm{LL}}\mathrm{G}_{3\mathrm{ddLL}}) + \lambda_h M^2 g_{\perp\rho\alpha}G_{3ddL}\right] + \cdots$$

$e^+e^- \to h\,\bar{q}\,X$

3. 强子张量

$$
\begin{aligned}
W_{t4\mu\nu} = \frac{4M^2}{z(p\cdot q)}\Bigg\{ & \frac{(zq-2p)_\mu(zq-2p)_\nu}{z^2(p\cdot q)}\Big[c_1^q\Big(D_3 - \frac{\varepsilon_\perp^{kS}}{M}D_{3T}^\perp + S_{LL}D_{3LL} + \frac{k_\perp\cdot S_{LT}}{M}D_{3LT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{3TT}^\perp\Big) \\
& + c_3^q\Big(\lambda_h G_{3L} - \frac{k_\perp\cdot S_T}{M}G_{3T}^\perp + \frac{\varepsilon_\perp^{kS_{LT}}}{M}G_{3LT}^\perp + \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{3TT}^\perp\Big)\Big] \\
& + \frac{k_{\perp\langle\mu}k_{\perp\nu\rangle}}{M^2}\Big[c_1^q\mathrm{Re}\Big(D_{-3d}^\perp + \frac{\varepsilon_\perp^{kS}}{M}D_{-3dT}^{\perp 2} + S_{LL}D_{-3dLL}^\perp + \frac{k_\perp\cdot S_{LT}}{M}D_{-3dLT}^{\perp 2} + \frac{S_{TT}^{kk}}{M^2}D_{-3dTT}^{\perp 2}\Big) \\
& + c_3^q\mathrm{Im}\Big(\lambda_h D_{+3dL}^\perp + \frac{k_\perp\cdot S_T}{M}D_{+3dT}^{\perp 4} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{+3dLT}^{\perp 4} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{+3dTT}^{\perp 4}\Big)\Big] \\
& + \frac{k_{\perp\{\mu}\tilde{k}_{\perp\nu\}}}{2M^2}\Big[c_1^q\mathrm{Re}\Big(\lambda_h D_{+3dL}^\perp + \frac{k_\perp\cdot S_T}{M}D_{+3dT}^{\perp 4} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{+3dLT}^{\perp 4} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{+3dTT}^{\perp 4}\Big) \\
& - c_3^q\mathrm{Im}\Big(D_{-3d}^\perp + \frac{\varepsilon_\perp^{kS}}{M}D_{-3dT}^{\perp 2} + S_{LL}D_{-3dLL}^\perp + \frac{k_\perp\cdot S_{LT}}{M}D_{-3dLT}^{\perp 2} + \frac{S_{TT}^{kk}}{M^2}D_{-3dTT}^{\perp 2}\Big)\Big] \\
& + (c_1^q g_{\perp\mu\nu} + ic_3^q\varepsilon_{\perp\mu\nu})\mathrm{Re}\Big(D_{-3dd} - \frac{\varepsilon_\perp^{kS}}{M}D_{-3ddT}^\perp + S_{LL}D_{-3ddLL} + \frac{k_\perp\cdot S_{LT}}{M}D_{-3ddLT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{-3ddTT}^\perp\Big) \\
& + (c_3^q g_{\perp\mu\nu} + ic_1^q\varepsilon_{\perp\mu\nu})\mathrm{Re}\Big(\lambda_h D_{-3ddL} - \frac{k_\perp\cdot S_T}{M}D_{-3ddT}^{\perp 3} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{-3ddLT}^{\perp 3} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{-3ddTT}^{\perp 3}\Big)\Bigg\}.
\end{aligned}
$$

$e^+e^- \to h\,\bar{q}\,X$

4. 方位角不对称

利用定义, 例如 $\langle sin\varphi\rangle_U = \frac{\int sin\varphi\, d\hat{\sigma} d\varphi}{\int d\hat{\sigma} d\varphi}$, 以非极化为例，可得4种方位角不对称，分别为

$$\langle \cos\varphi\rangle_U = -2k_{\perp M}\kappa_M \frac{D(y)T_2^q(y)D^\perp}{T_0^q(y)zD_1}, \quad \langle \cos 2\varphi\rangle_U = -\frac{1}{2}k_{\perp M}^2\kappa_M^2 \frac{C(y)c_1^e c_1^q\,\mathrm{Re}D_{-3d}^\perp}{T_0^q(y)zD_1},$$

$$\langle \sin\varphi\rangle_U = -2k_{\perp M}\kappa_M \frac{D(y)T_3^q(y)G^\perp}{T_0^q(y)zD_1}, \quad \langle \sin 2\varphi\rangle_U = -\frac{1}{2}k_{\perp M}^2\kappa_M^2 \frac{C(y)c_1^e c_3^q\,\mathrm{Im}D_{-3d}^\perp}{T_0^q(y)zD_1}.$$

- $\langle cos\varphi\rangle_U$ 和 $\langle sin\varphi\rangle_U$ 只有扭度-3碎裂函数贡献, $\langle cos2\varphi\rangle_U$和 $\langle sin2\varphi\rangle_U$ 只有扭度-4碎裂函数贡献。
- $\langle cos\varphi\rangle_U$ 和 $\langle cos2\varphi\rangle_U$ 是宇称守恒项, $\langle sin\varphi\rangle_U$ 和 $\langle sin2\varphi\rangle_U$ 是宇称破坏项。

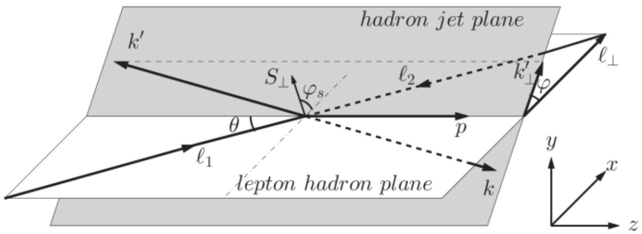
$e^+e^- \to h\,\bar{q}\,X$

5. 强子极化

纵向极化：

$$\langle\lambda_h\rangle = -\frac{2}{3}\frac{T_1^q(y)G_{1L}}{T_0^q(y)D_1}\left(1+\alpha_U\kappa_M^2-\alpha_L\kappa_M^2\right),$$

$$\langle S_{LL}\rangle = \frac{1}{2}\frac{T_0^q(y)D_{1LL}}{T_0^q(y)D_1}\left(1+\alpha_U\kappa_M^2-\alpha_{LL}\kappa_M^2\right).$$



横向极化-xy：

$$\langle S_T^x\rangle = \frac{8}{3}\kappa_M\frac{D(y)T_3^q(y)G_T}{T_0^q(y)zD_1},$$

$$\langle S_T^y\rangle = -\frac{8}{3}\kappa_M\frac{D(y)T_2^q(y)D_T}{T_0^q(y)zD_1},$$

$$\langle S_{TT}^{xx}\rangle = -\frac{1}{3}k_{\perp M}^4\kappa_M^2\frac{C(y)c_1^e c_1^q\mathrm{Re}\left(D_{-3dTT}^{\perp 2}-D_{+3dTT}^{\perp 4}\right)}{T_0^q(y)zD_1},$$

$$\langle S_{TT}^{xy}\rangle = -\frac{1}{3}k_{\perp M}^4\kappa_M^2\frac{C(y)c_1^e c_3^q\mathrm{Im}\left(D_{-3dTT}^{\perp 2}-D_{+3dTT}^{\perp 4}\right)}{T_0^q(y)zD_1}.$$

1. $\langle\lambda_h\rangle$ 和 $\langle S_{LL}\rangle$ 既有领头扭度贡献又有扭度-4贡献, 扭度-4贡献吸收到了扭度-4修正因子中, α_i。
2. $\langle S_T\rangle$ 和 $\langle S_{LT}\rangle$ 只有扭度-3碎裂函数贡献，$\langle S_{TT}\rangle$ 只有扭度-4碎裂函数贡献。

$e^+e^- \to h\,\bar{q}\,X$

5. 强子极化

强子极化-nt

$$\langle S_T^n\rangle = +\frac{2}{3}k_{\perp M}\frac{T_0^q(y)D_{1T}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_T^n\kappa_M^2),\qquad \alpha_T^n = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddT}^\perp - C(y)c_1^e c_1^q\, D_{3T}^\perp}{z^2T_0^q(y)D_{1T}^\perp},$$

$$\langle S_T^t\rangle = -\frac{2}{3}k_{\perp M}\frac{T_1^q(y)G_{1T}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_T^t\kappa_M^2),\qquad \alpha_T^t = 4\frac{zT_1^q(y)\mathrm{Re}D_{-3ddT}^{\perp 3} + C(y)c_1^e c_3^q\, G_{3T}^\perp}{z^2T_1^q(y)G_{1T}^\perp},$$

$$\langle S_{LT}^n\rangle = -\frac{2}{3}k_{\perp M}\frac{T_1^q(y)G_{1LT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{LT}^n\kappa_M^2),\qquad \alpha_{LT}^n = -4\frac{zT_1^q(y)\mathrm{Re}D_{-3ddLT}^{\perp 3} + C(y)c_1^e c_3^q\, G_{3LT}^\perp}{z^2T_1^q(y)G_{1LT}^\perp},$$

$$\langle S_{LT}^t\rangle = -\frac{2}{3}k_{\perp M}\frac{T_0^q(y)D_{1LT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{LT}^t\kappa_M^2),\qquad \alpha_{LT}^t = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddLT}^\perp - C(y)c_1^e c_1^q\, D_{3LT}^\perp}{z^2T_0^q(y)D_{1LT}^\perp},$$

$$\langle S_{TT}^{nn}\rangle = -\frac{2}{3}k_{\perp M}^2\frac{T_0^q(y)D_{1TT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{TT}^{nn}\kappa_M^2),\qquad \alpha_{TT}^{nn} = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3ddTT}^\perp - C(y)c_1^e c_1^q D_{3TT}^\perp}{z^2T_0^q(y)D_{1TT}^\perp},$$

$$\langle S_{TT}^{nt}\rangle = +\frac{2}{3}k_{\perp M}^2\frac{T_1^q(y)G_{1TT}^\perp}{T_0^q(y)D_1}(1+\alpha_U\kappa_M^2-\alpha_{TT}^{nt}\kappa_M^2),\qquad \alpha_{TT}^{nt} = -4\frac{zT_1^q(y)\mathrm{Re}D_{-3ddTT}^{\perp 3} + C(y)c_1^e c_3^q G_{3TT}^\perp}{z^2T_1^q(y)G_{1TT}^\perp}.$$

唯象学研究

1. 扭度-4贡献介绍

- 结构函数

$$zW_{U1} = c_1^e c_1^q (D_1 - 4\kappa_M^2 \mathrm{Re}D_{-3dd}/z),$$
$$zW_{U3} = 2c_3^e c_3^q (D_1 - 4\kappa_M^2 \mathrm{Re}D_{-3dd}/z)$$

- 方位角不对称

$$\langle \cos 2\varphi \rangle_U = -\frac{1}{2} k_{\perp M}^2 \kappa_M^2 \frac{C(y) c_1^e c_1^q \,\mathrm{Re}D_{-3d}^{\perp}}{T_0^q(y) z D_1},$$
$$\langle \sin 2\varphi \rangle_U = -\frac{1}{2} k_{\perp M}^2 \kappa_M^2 \frac{C(y) c_1^e c_3^q \,\mathrm{Im}D_{-3d}^{\perp}}{T_0^q(y) z D_1}.$$

- 强子极化

$$\langle \lambda_h \rangle = -\frac{2}{3} \frac{T_1^q(y) G_{1L}}{T_0^q(y) D_1} (1 + \alpha_U \kappa_M^2 - \alpha_L \kappa_M^2),$$
$$\langle S_{LL} \rangle = \frac{1}{2} \frac{T_0^q(y) D_{1LL}}{T_0^q(y) D_1} (1 + \alpha_U \kappa_M^2 - \alpha_{LL} \kappa_M^2).$$

$$\alpha_U = 4 \frac{z T_0^q(y) \mathrm{Re}D_{-3dd} - C(y) c_1^e c_1^q D_3}{z^2 T_0^q(y) D_1},$$
$$\alpha_L = 4 \frac{z T_1^q(y) \mathrm{Re}D_{-3ddL} + C(y) c_1^e c_3^q G_{3L}}{z^2 T_1^q(y) G_{1L}}.$$

唯象学研究

2. $g=0$ 近似

$g=0$: 忽略多胶子散射贡献，只保留横动量贡献。

$$\hat{\Xi}^{(1)}_{\rho}\Big|_{g=0} = -k_{\perp\rho}\,\hat{\Xi}^{(0)}\Big|_{g=0},$$

$$\hat{\Xi}^{(2,M)}_{\rho\sigma}\Big|_{g=0} = k_{\perp\rho}k_{\perp\sigma}\,\hat{\Xi}^{(0)}\Big|_{g=0},$$

$$\left(\hat{\Xi}^{(2)}_{\rho\sigma} + \gamma^0\hat{\Xi}^{(2)\dagger}_{\sigma\rho}\gamma^0\right)\Big|_{g=0} = z^2 k_{\perp\rho}k_{\perp\sigma}\frac{\partial\hat{\Xi}^{(0)}}{\partial z}\Big|_{g=0}.$$

$$D_{3d} = \frac{k_\perp^2}{2M^2}D_{3d}^\perp = \frac{1}{z}D_3 = -\frac{k_\perp^2}{2M^2}zD_1,$$

$$D_{3dLL} = \frac{k_\perp^2}{2M^2}D_{3dLL}^\perp = \frac{1}{z}D_{3LL} = -\frac{k_\perp^2}{2M^2}zD_{1LL},$$

$$G_{3dL} = i\frac{k_\perp^2}{2M^2}G_{3dL}^\perp = \frac{1}{z}G_{3L} = -\frac{k_\perp^2}{2M^2}zG_{1L},$$

$$\mathrm{Re}D_{3dd} = \frac{k_\perp^2}{2M^2}\mathrm{Re}D_{3dd}^\perp = z^2\frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}D_1,$$

$$\mathrm{Re}G_{3ddL} = \frac{k_\perp^2}{2M^2}\mathrm{Re}G_{3ddL}^\perp = -z^2\frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}G_{1L},$$

$$\mathrm{Re}D_{3ddLL} = \frac{k_\perp^2}{2M^2}\mathrm{Re}D_{3ddLL}^\perp = z^2\frac{k_\perp^2}{4M^2}\frac{\partial}{\partial z}D_{1LL},$$

唯象学研究

3. 扭度-4修正因子

$$\alpha_U = 4\frac{zT_0^q(y)\mathrm{Re}D_{-3dd} - C(y)c_1^e c_1^q D_3}{z^2 T_0^q(y) D_1}, \xrightarrow{g=0} \alpha_U \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y)D_1}{\partial \ln z} + \frac{2C(y)c_1^e c_1^q D_1}{T_0^q(y)D_1}\right].$$

$$\alpha_L \approx -k_{\perp M}^2\left[\frac{\partial \ln T_1^q(y)G_{1L}}{\partial \ln z} - \frac{2C(y)c_1^e c_3^q G_{1L}}{T_1^q(y)G_{1L}}\right], \qquad \alpha_{LT}^n \approx -k_{\perp M}^2\left[\frac{\partial \ln T_1^q(y)G_{1LT}^\perp}{\partial \ln z} - \frac{2C(y)c_1^e c_3^q G_{1LT}^\perp}{T_1^q(y)G_{1LT}^\perp}\right],$$

$$\alpha_{LL} \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y)D_{1LL}}{\partial \ln z} + \frac{2C(y)c_1^e c_1^q D_{1LL}}{T_0^q(y)D_{1LL}}\right], \quad \alpha_{LT}^t \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y)D_{1LT}^\perp}{\partial \ln z} + \frac{2C(y)c_1^e c_1^q D_{1LT}^\perp}{T_0^q(y)D_{1LT}^\perp}\right],$$

$$\alpha_T^n \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y)D_{1T}^\perp}{\partial \ln z} + \frac{2C(y)c_1^e c_1^q D_{1T}^\perp}{T_0^q(y)D_{1T}^\perp}\right], \qquad \alpha_{TT}^{nt} \approx -k_{\perp M}^2\left[\frac{\partial \ln T_1^q(y)G_{1TT}^\perp}{\partial \ln z} - \frac{2C(y)c_1^e c_3^q G_{1TT}^\perp}{T_1^q(y)G_{1TT}^\perp}\right],$$

$$\alpha_T^t \approx -k_{\perp M}^2\left[\frac{\partial \ln T_1^q(y)G_{1T}^\perp}{\partial \ln z} - \frac{2C(y)c_1^e c_3^q G_{1T}^\perp}{T_1^q(y)G_{1T}^\perp}\right], \qquad \alpha_{TT}^{nn} \approx -k_{\perp M}^2\left[\frac{\partial \ln T_0^q(y)D_{1TT}^\perp}{\partial \ln z} + \frac{2C(y)c_1^e c_1^q D_{1TT}^\perp}{T_0^q(y)D_{1TT}^\perp}\right].$$

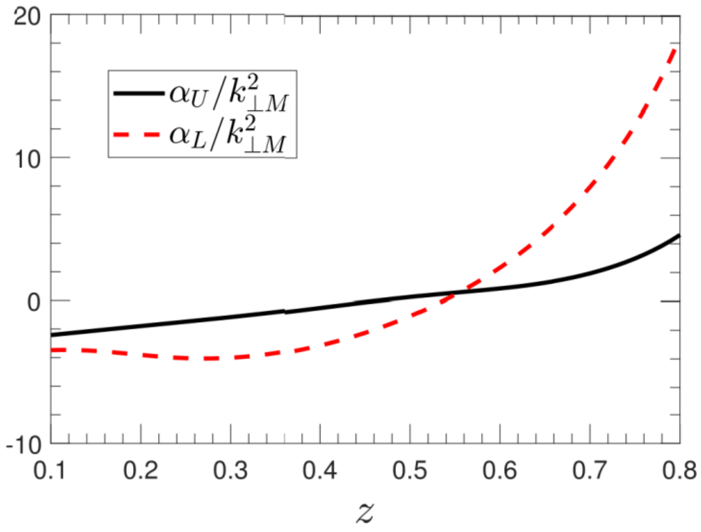
唯象学研究

4. 数值结果

$$\alpha_U \approx -k_{\perp M}^2 \left[\frac{\partial \ln T_0^q(y) D_1}{\partial \ln z} + \frac{2C(y) c_1^e c_1^q D_1}{T_0^q(y) D_1} \right]$$

$$\alpha_L \approx -k_{\perp M}^2 \left[\frac{\partial \ln T_1^q(y) G_{1L}}{\partial \ln z} - \frac{2C(y) c_1^e c_3^q G_{1L}}{T_1^q(y) G_{1L}} \right]$$

$y = 0.5, \qquad Q = M_Z.$



扭度-4修正因子相对来说还是很大的，也就是说扭度-4碎裂函数的贡献是很显著的。

总结

1. 介绍了夸克的纵向极化和夸克-反夸克横向极化关联；夸克纵向极化在$Q = M_Z$处取最大值，夸克-反夸克横向极化关联在$Q \approx M_Z$处取反向极化关联最大值。
2. 利用贡献展开技术计算了半单举e^+e^-湮灭过程中的方位角不对称，强子极化等。
3. 利用了$g = 0$ 近似估算了扭度-4碎裂函数的贡献，发现扭度-4碎裂函数的贡献是十分显著的。

谢谢！

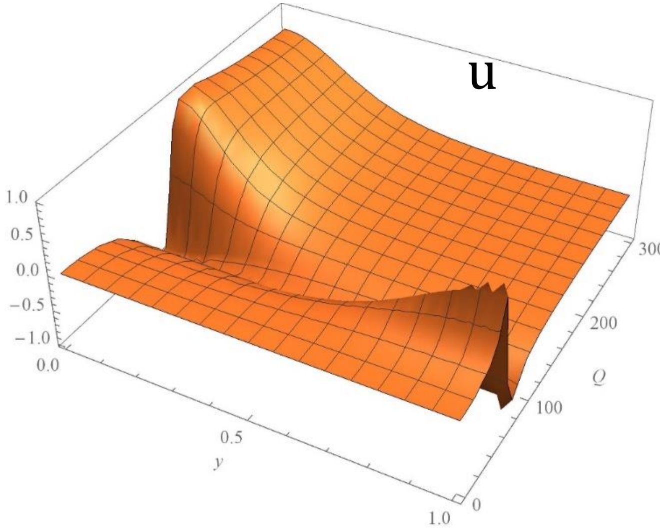
备份——$e^+e^- \to q\,\bar{q}$

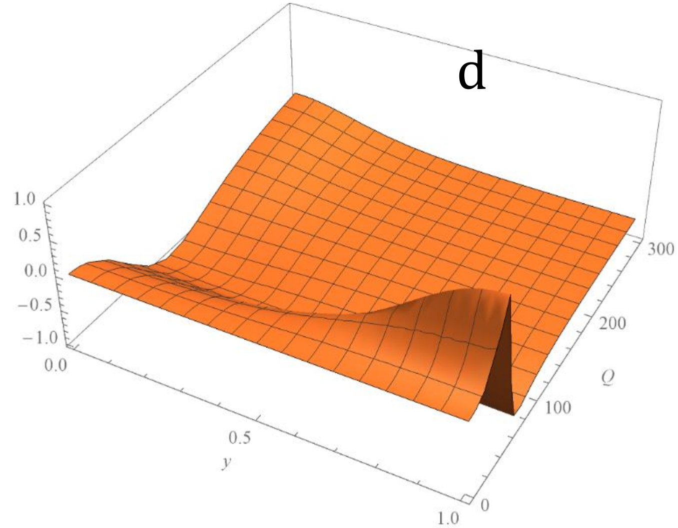
1. 夸克的纵向极化

$$P_q(y,Q) = \frac{\chi\, T_1^q(y) + \chi_{int}^q T_A^q(y)}{\chi\, T_0^q(y) + \chi_{int}^q T_V^q(y) + e_q^2 A(y)},$$

$$T_1^q(y) = -c_1^e c_3^q A(y) + c_3^e c_1^q B(y),$$

$$T_A^q(y) = -c_V^e c_A^q A(y) + c_A^e c_V^q B(y).$$



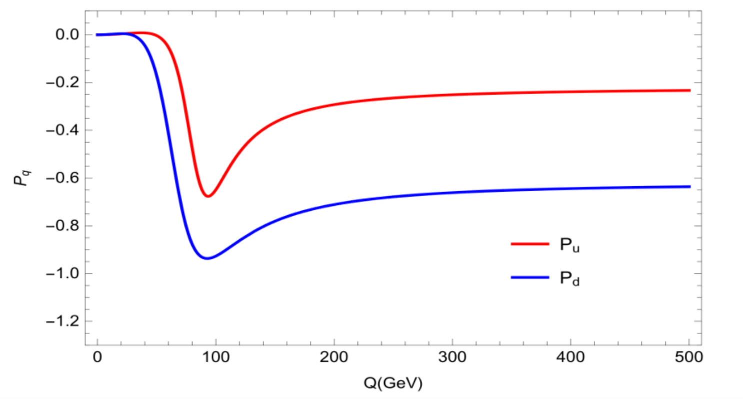


备份——$e^+e^- \to q\,\bar{q}$

1. 夸克的纵向极化

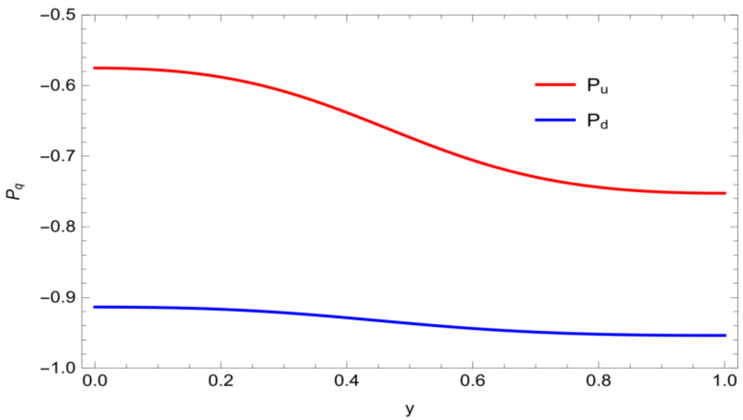
- 纵向极化的能量依赖

$$P_q(Q) = -\frac{\chi c_1^e c_3^q + \chi_{int}^q c_V^e c_A^q}{\chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q + e_q^2},$$



- 纵向极化的角度依赖

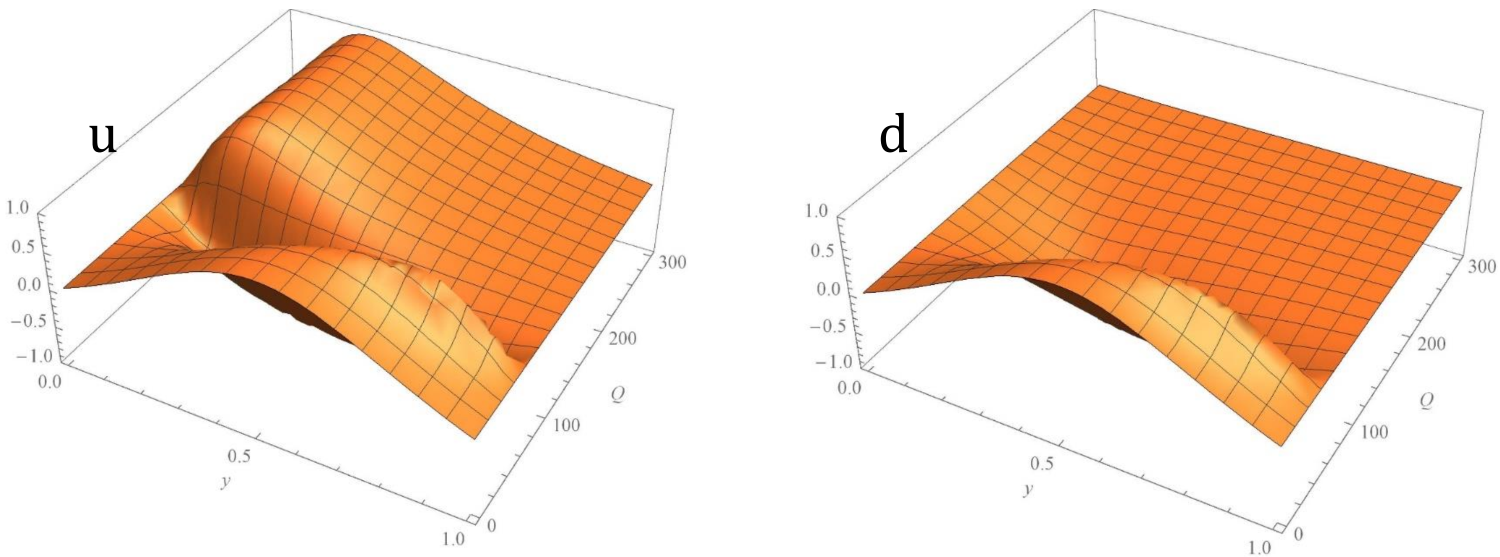
$$P_q(y) = \frac{T_1^q(y)}{T_0^q(y)} = -\frac{c_1^e c_3^q A(y) - c_3^e c_1^q B(y)}{c_1^e c_1^q A(y) - c_3^e c_3^q B(y)}.$$



备份——$e^+e^- \to q\,\bar{q}$

2. 夸克反夸克横向极化关联

$$c_{nn}^q \equiv \frac{|\hat{m}_{n++}|^2+|\hat{m}_{n--}|^2-|\hat{m}_{n-+}|^2-|\hat{m}_{n+-}|^2}{|\hat{m}_{n++}|^2+|\hat{m}_{n--}|^2+|\hat{m}_{n-+}|^2+|\hat{m}_{n+-}|^2}, \qquad c_{nn}^q(y,Q) = \frac{C(y)\left[\chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q + e_q^2\right]}{2\left[\chi\, T_0^q(y) + \chi_{int}^q T_V^q(y) + e_q^2 A(y)\right]}.$$

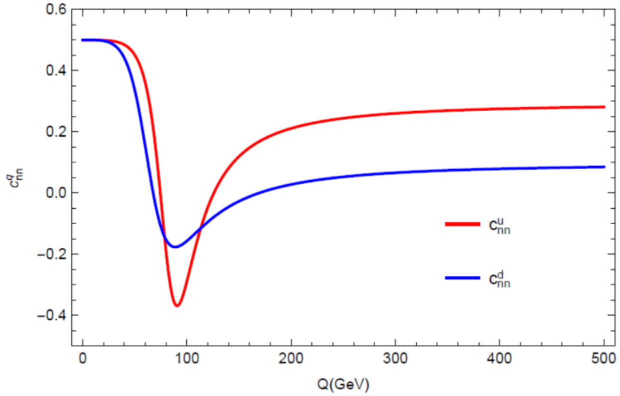


备份——$e^+e^- \to q\,\bar{q}$

2. 夸克反夸克横向极化关联

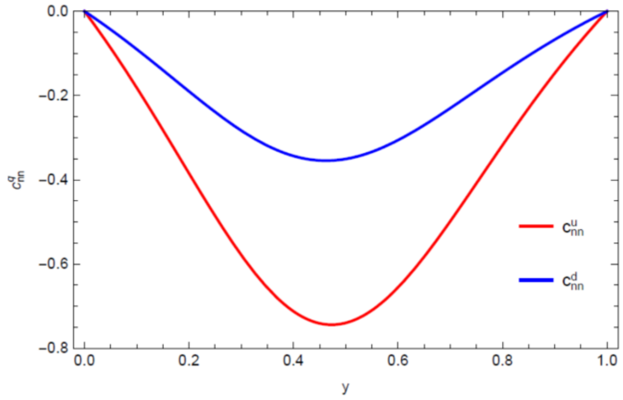
- 横向极化关联的能量依赖

$$c_{nn}^q(Q) = \frac{\chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q + e_q^2}{2\left[\chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q + e_q^2\right]},$$



- 横向极化关联的角度依赖

$$c_{nn}^q(y) = \frac{c_1^e c_2^q C(y)}{2\left[c_1^e c_1^q A(y) - c_3^e c_3^q B(y)\right]}.$$



备份——$e^+e^- \to q\,\bar{q}$

3. 夸克反夸克横向极化关联

$$c_{nn}^{q}(y,Q)=\frac{c_1^e c_2^q C(y)}{2\,T_0^q(y)}\xrightarrow{D_1(z_1)\bar{D}_1(z_2)}c_{nn}^{h_1h_2}=|s_{1T}||s_{2T}|\cdot\frac{c_1^e c_2^q C(y)}{2T_0^q(y)}\cdot\frac{D_1(z_1)\bar{D}_1(z_2)}{D_1(z_1)\bar{D}_1(z_2)}$$

s_{1T}, s_{2T} 分别是夸克反夸克自旋.

利用公式：$s_T D_1(z)=S_T H_{1T}(z)$，有

$$c_{nn}^{h_1h_2}=|S_{1T}||S_{2T}|\cdot\frac{c_1^e c_2^q C(y)}{2T_0^q(y)}\cdot\frac{H_{1T}(z_1)\bar{H}_{1T}(z_2)}{D_1(z_1)\bar{D}_1(z_2)}.$$

如果不限制极化方向，那么就有

$$c^{h_1h_2}=|S_{1T}||S_{2T}|\cdot\frac{c_1^e c_2^q C(y)\cos(2\phi-\phi_{s_1}-\phi_{s_2})}{2T_0^q(y)}\cdot\frac{H_{1T}(z_1)\bar{H}_{1T}(z_2)}{D_1(z_1)\bar{D}_1(z_2)}=\mathrm{A}_{\mathrm{TT}}^{\mathrm{Z}}.$$

夸克的横向极化关联在强子层次对应的是强子的横向极化关联，也就是强子的双自旋不对称.

备注——扭度

OPE

$$\bar{q}\gamma^{\mu}q(x)\,\bar{q}\gamma^{\nu}q(0) = \bar{q}(x)\gamma^{\mu}\overbrace{q(x)\bar{q}(0)}\gamma^{\nu}q(0) + \overbrace{\bar{q}(x)\gamma^{\mu}q(x)\,\bar{q}(0)\gamma^{\nu}q(0)} + \cdots \quad (1)$$

利用

$$\frac{1}{(i\partial+q)^2} = \frac{-1}{Q^2-2iq\cdot\partial+\partial^2} = -\frac{1}{Q^2}\sum_{n=0}^{\infty}\left(\frac{2iq\cdot\partial-\partial^2}{Q^2}\right)^n \quad (2)$$

$$\frac{1}{2}(\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu}+\gamma^{\nu}\gamma^{\alpha}\gamma^{\mu}) = g^{\mu\alpha}\gamma^{\nu}+\gamma^{\mu}g^{\alpha\nu}-g^{\mu\nu}\gamma^{\alpha} \quad (3)$$

有

$$\int d^4x\, e^{iqx}\,\bar{q}(x)\gamma^{\mu}\overbrace{q(x)\,\bar{q}(0)}\gamma^{\nu}q(0) = \bar{q}\gamma^{\mu}\frac{i(i\partial\!\!\!/+q\!\!\!/)}{(i\partial+q)^2}\gamma^{\nu}q(0)$$

$$= -i\bar{q}(2\gamma^{\mu}(i\partial^{\nu})-g^{\mu\nu}q\!\!\!/)\frac{1}{Q^2}\sum_{n=0}^{\infty}\left(\frac{2iq\cdot\partial}{Q^2}\right)^n q \quad (4)$$

$$= -i\bar{q}\,2\gamma^{\mu}(i\partial^{\nu})(i\partial)^n\cdot q\sum_{n=0}^{\infty}\frac{(2q)^n}{Q^{2n+2}} \quad (\text{两段和.})$$

$\Downarrow$

$$i\int d^4x\, e^{iq\cdot x}J^{\mu}(x)J^{\nu}(0) = \sum_f Q_f^2\Big[4\sum_{n=2}^{\infty}\frac{(2q^{\mu_1})\cdots(2q^{\mu_{n-2}})}{(Q^2)^{n-1}}O_f^{(n)\mu\nu\mu_1\cdots\mu_{n-2}}$$

$$-g^{\mu\nu}\sum_{n=2}^{\infty}\frac{(2q^{\mu_1})\cdots(2q^{\mu_n})}{(Q^2)^n}O_f^{(n)\mu_1\cdots\mu_n}\Big]+\cdots \quad (5)$$

$$\langle J^{\mu}(x)J^{\nu}(0)\rangle = \sum_i C_i\, O_i \quad (6)$$

$$\langle J^{\mu}(x)J^{\nu}(0)\rangle \longrightarrow 6;\quad O_i \rightarrow d;\quad C_i \rightarrow 6-d \quad (7)$$

$$\int d^4x\, e^{iqx}\langle J^{\mu}(x)J^{\nu}(0)\rangle = \sum_i\int d^4x\, e^{iqx}C_i\,O_i$$

$$= \sum_i\tilde{C}_i\,O_i \quad (8)$$

$$\tilde{C}_i \sim 2-d \sim \left(\frac{1}{Q}\right)^{d-2}.$$

考虑自旋. $\sim \left(\frac{1}{Q}\right)^{d-2-s} = \left(\frac{1}{Q}\right)^{t-2}$ (9)

t: twist.

部分子分布函数

夸克极化 \ 强子极化	U	L	T
U	f_1 数密度		$f_{1T}^{\perp}$ Sivers
L		g_{1L}	g_{1T}
T	$h_1^{\perp}$ Boer-Mulders	$h_{1L}^{\perp}$	h_{1T} Transversity $h_{1T}^{\perp}$

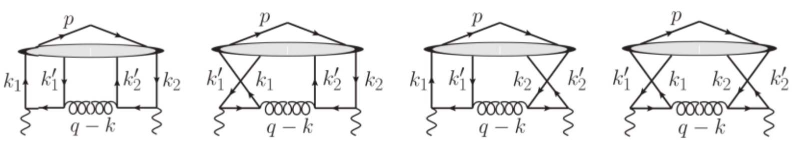
部分子碎裂函数

强子极化 / 夸克极化	U	L	T
U	D_1 数密度		$D_{1T}^{\perp}$ −
L		G_{1L} − 纵向极化	G_{1T} −
T	$H_1^{\perp}$ − Collins	$H_{1L}^{\perp}$ −	H_{1T} − 横向极化 $H_{1T}^{\perp}$ −

4-夸克关联函数贡献

5. 4−夸克关联函数贡献

不仅考虑夸克-j-胶子-夸克关联函数的贡献，
还要考虑4−夸克关联函数的贡献。



$$W^{(g/q)}_{4q\mu\nu} = \frac{1}{p\cdot q}\int dz dz_1 dz_2 h^{g/q}_{4q}\Big[(c_1^q g_{\perp\mu\nu} + i c_3^q \varepsilon_{\perp\mu\nu})C_s + (c_3^q g_{\perp\mu\nu} + i c_1^q \varepsilon_{\perp\mu\nu})C_{ps}\Big].$$

$$h^{qL}_{4q} = \frac{z z_B^3 \delta(z_1 - z_B)}{(z - z_B - i\epsilon)(z_2 - z_B - i\epsilon)} - \Big(\frac{1}{z_2} \to \frac{1}{z} - \frac{1}{z_2}\Big) - \frac{z z_B^3 \delta(z_1 + z_B - \frac{z_1 z_B}{z})}{(z - z_B - i\epsilon)(z_2 - z_B - i\epsilon)} + \Big(\frac{1}{z_2} \to \frac{1}{z} - \frac{1}{z_2}\Big),$$

$$h^{g}_{4q} = \frac{z z_B^3 \delta(z - z_B)}{(z_1 - z_B + i\epsilon)(z_2 - z_B - i\epsilon)} + \frac{z_B^2/z_1 z_2 \delta(z - z_B)}{(1/z_1 + i\epsilon)(1/z_2 - i\epsilon)} - \frac{z_B^3/z_2 \delta(z - z_B)}{(z_1 - z_B + i\epsilon)(1/z_2 - i\epsilon)} - (1 \leftrightarrow 2)^*.$$

$h^{qR}_{4q}(z_1, z, z_2) = h^{qL*}_{4q}(z_2, z, z_1)$， $h_{4q} = h^{qL}_{4q} + h^{qR}_{4q} + h^{g}_{4q}$.

$$C_j = \int d^4k_1 d^4k d^4k_2 \delta\Big(z - \frac{p^+}{k^+}\Big)\delta(k_1^+ z_1 - p^+)\delta(k_2^+ z_2 - p^+)(2\pi)^2 \delta^2(\vec{k}_\perp + \vec{k}'_\perp)\Xi^{(0)}_{(4q)j}(k_1, k, k_2; p, S)\,.$$

备注——4-夸克关联函数贡献

5. 4-夸克关联函数贡献

将关联函数分解就可以得到：

$$z\int D[z]C_s = M^2\Big(\mathcal{D}_{4q} - \frac{\varepsilon_\perp^{kS}}{M}D_{4qT}^\perp + \frac{k_\perp\cdot S_{LT}}{M}D_{4qLT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{4qTT}^\perp\Big),$$

$$z\int D[z]C_{ps} = M^2\Big(\lambda_h G_{4qL} - \frac{k_\perp\cdot S_T}{M}G_{4qT}^\perp + \frac{\varepsilon_\perp^{kS_{LT}}}{M}G_{4qLT}^\perp + \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{4qTT}^\perp\Big).$$

其中： $\int D[z] \equiv \int dzdz_1dz_2h_{4q}$， $\mathcal{D}_{4q} \equiv D_{4q} + S_{LL}D_{4qLL}$ 。

相应的强子张量为：

$$W_{4q\mu\nu} = \frac{M^2}{z(p\cdot q)}\Big\{w_{\mu\nu}^{13}\mathrm{Re}\Big(\mathcal{D}_{4q} - \frac{\varepsilon_\perp^{kS}}{M}D_{4qT}^\perp + \frac{k_\perp\cdot S_{LT}}{M}D_{4qLT}^\perp + \frac{S_{TT}^{kk}}{M^2}D_{4qTT}^\perp\Big)$$
$$+ w_{\mu\nu}^{31}\mathrm{Re}\Big(\lambda_h D_{4qL} - \frac{k_\perp\cdot S_T}{M}D_{4qT}^{\perp 3} + \frac{\varepsilon_\perp^{kS_{LT}}}{M}D_{4qLT}^{\perp 3} + \frac{S_{TT}^{\tilde{k}k}}{M^2}D_{4qTT}^{\perp 3}\Big)\Big\}.$$

其中 $w_{\mu\nu}^{13} = c_1^q g_{\perp\mu\nu} + ic_3^q\varepsilon_{\perp\mu\nu}$， $w_{\mu\nu}^{31} = c_3^q g_{\perp\mu\nu} + ic_1^q\varepsilon_{\perp\mu\nu}$ 。

备注——4-夸克关联函数贡献

5. 4-夸克关联函数

$$\hat{\Xi}^{(0)}_{(4q)s} = \frac{g^2}{8}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4} e^{-ik_1y+i(k_1-k)y_1-i(k_2-k)y_2}$$

$$\sum_X \Big\{\langle 0|\bar{\psi}(y_2)\not{n}\psi(0)|hX\rangle\langle hX|\bar{\psi}(y)\not{n}\psi(y_1)|0\rangle$$

$$+\langle 0|\bar{\psi}(y_2)\gamma^5\not{n}\psi(0)|hX\rangle\langle hX|\bar{\psi}(y)\gamma^5\not{n}\psi(y_1)|0\rangle\Big\},$$

$$\hat{\Xi}^{(0)}_{(4q)ps} = \frac{g^2}{8}\int \frac{d^4y}{(2\pi)^4}\frac{d^4y_1}{(2\pi)^4}\frac{d^4y_2}{(2\pi)^4} e^{-ik_1y+i(k_1-k)y_1-i(k_2-k)y_2}$$

$$\sum_X \Big\{\langle 0|\bar{\psi}(y_2)\gamma^5\not{n}\psi(0)|hX\rangle\langle hX|\bar{\psi}(y)\not{n}\psi(y_1)|0\rangle$$

$$+\langle 0|\bar{\psi}(y_2)\not{n}\psi(0)|hX\rangle\langle hX|\bar{\psi}(y)\gamma^5\not{n}\psi(y_1)|0\rangle\Big\}.$$

备注——4-夸克关联函数贡献

5. 4−夸克关联函数贡献

- 4-夸克关联函数贡献

$$z^2W_{4qU1} = -\kappa_M^2 c_1^e c_1^q D_{4q},\qquad z^2W_{4qU3} = -2\kappa_M^2 c_3^e c_3^q D_{4q},$$

$$z^2\tilde{W}_{4qL1} = -\kappa_M^2 c_1^e c_3^q G_{4qL},\qquad z^2\tilde{W}_{4qL3} = -2\kappa_M^2 c_3^e c_1^q G_{4qL},$$

$$z^2W_{4qT1}^{\sin(\varphi-\varphi_S)} = -k_{\perp M}\kappa_M^2 c_1^e c_1^q D_{4qT}^{\perp},\qquad z^2W_{4qT3}^{\sin(\varphi-\varphi_S)} = -2k_{\perp M}\kappa_M^2 c_3^e c_3^q D_{4qT}^{\perp},$$

$$z^2\tilde{W}_{4qT1}^{\cos(\varphi-\varphi_S)} = -k_{\perp M}\kappa_M^2 c_1^e c_3^q G_{4qT}^{\perp},\qquad z^2\tilde{W}_{4qT3}^{\cos(\varphi-\varphi_S)} = -2k_{\perp M}\kappa_M^2 c_3^e c_1^q G_{4qT}^{\perp},$$

$$z^2W_{4qLL1} = -\kappa_M^2 c_1^e c_1^q D_{4qLL},\qquad z^2W_{4qLL3} = -2\kappa_M^2 c_3^e c_3^q D_{4qLL},$$

$$z^2\tilde{W}_{4qLT1}^{\sin(\varphi-\varphi_{LT})} = k_{\perp M}\kappa_M^2 c_1^e c_3^q G_{4qLT}^{\perp},\qquad z^2\tilde{W}_{4qLT3}^{\sin(\varphi-\varphi_{LT})} = 2k_{\perp M}\kappa_M^2 c_3^e c_1^q G_{4qLT}^{\perp},$$

$$z^2W_{4qLT1}^{\cos(\varphi-\varphi_{LT})} = k_{\perp M}\kappa_M^2 c_1^e c_1^q D_{4qLT}^{\perp},\qquad z^2W_{4qLT3}^{\cos(\varphi-\varphi_{LT})} = 2k_{\perp M}\kappa_M^2 c_3^e c_3^q D_{4qLT}^{\perp},$$

$$z^2\tilde{W}_{4qTT1}^{\sin(2\varphi-2\varphi_{TT})} = -k_{\perp M}^2\kappa_M^2 c_1^e c_3^q G_{4qTT}^{\perp},\qquad z^2\tilde{W}_{4qTT3}^{\sin(2\varphi-2\varphi_{TT})} = -2k_{\perp M}^2\kappa_M^2 c_3^e c_1^q G_{4qTT}^{\perp},$$

$$z^2W_{4qTT1}^{\cos(2\varphi-2\varphi_{TT})} = -k_{\perp M}^2\kappa_M^2 c_1^e c_1^q D_{4qTT}^{\perp},\qquad z^2W_{4qTT3}^{\cos(2\varphi-2\varphi_{TT})} = -2k_{\perp M}^2\kappa_M^2 c_3^e c_3^q D_{4qTT}^{\perp}.$$

备注——4-夸克关联函数贡献

5. 4-夸克关联函数贡献

- 强子极化，4-夸克关联函数的贡献

$$\alpha_{4qU} = \frac{T_0^q(y)D_{4q}}{zT_0^q(y)D_1}, \quad \alpha_{4qL} = \frac{T_1^q(y)G_{4qL}}{zT_1^q(y)G_{1L}}, \quad \alpha_{4qLL} = \frac{T_0^q(y)D_{4qLL}}{zT_0^q(y)D_{1LL}},$$

$$\alpha_{4qT}^t = \frac{T_1^q(y)G_{4qT}^\perp}{zT_1^q(y)G_{1T}^\perp}, \quad \alpha_{4qT}^n = \frac{T_0^q(y)D_{4qT}^\perp}{zT_0^q(y)D_{1T}^\perp}, \quad \alpha_{4qLT}^n = -\frac{T_1^q(y)G_{4qLT}^\perp}{zT_1^q(y)G_{1LT}^\perp},$$

$$\alpha_{4qLT}^t = \frac{T_0^q(y)D_{4qLT}^\perp}{zT_0^q(y)D_{1LT}^\perp}, \quad \alpha_{4qTT}^{nt} = \frac{T_1^q(y)G_{4qTT}^\perp}{zT_1^q(y)G_{1LT}^\perp}, \quad \alpha_{4qTT}^{nn} = \frac{T_0^q(y)D_{4qTT}^\perp}{zT_0^q(y)D_{1TT}^\perp}.$$

4-夸克关联函数的贡献和领头扭度的贡献形式上完全一致，是领头扭度的直接修正。