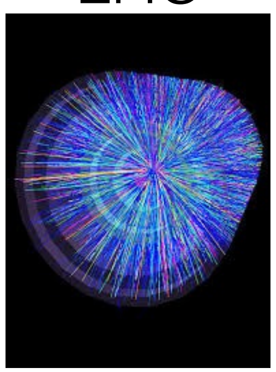
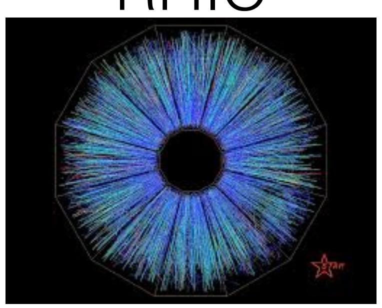
Nuclear Science
Computing Center at CCNU

QCD phase structure from Lattice QCD

LHC



RHIC



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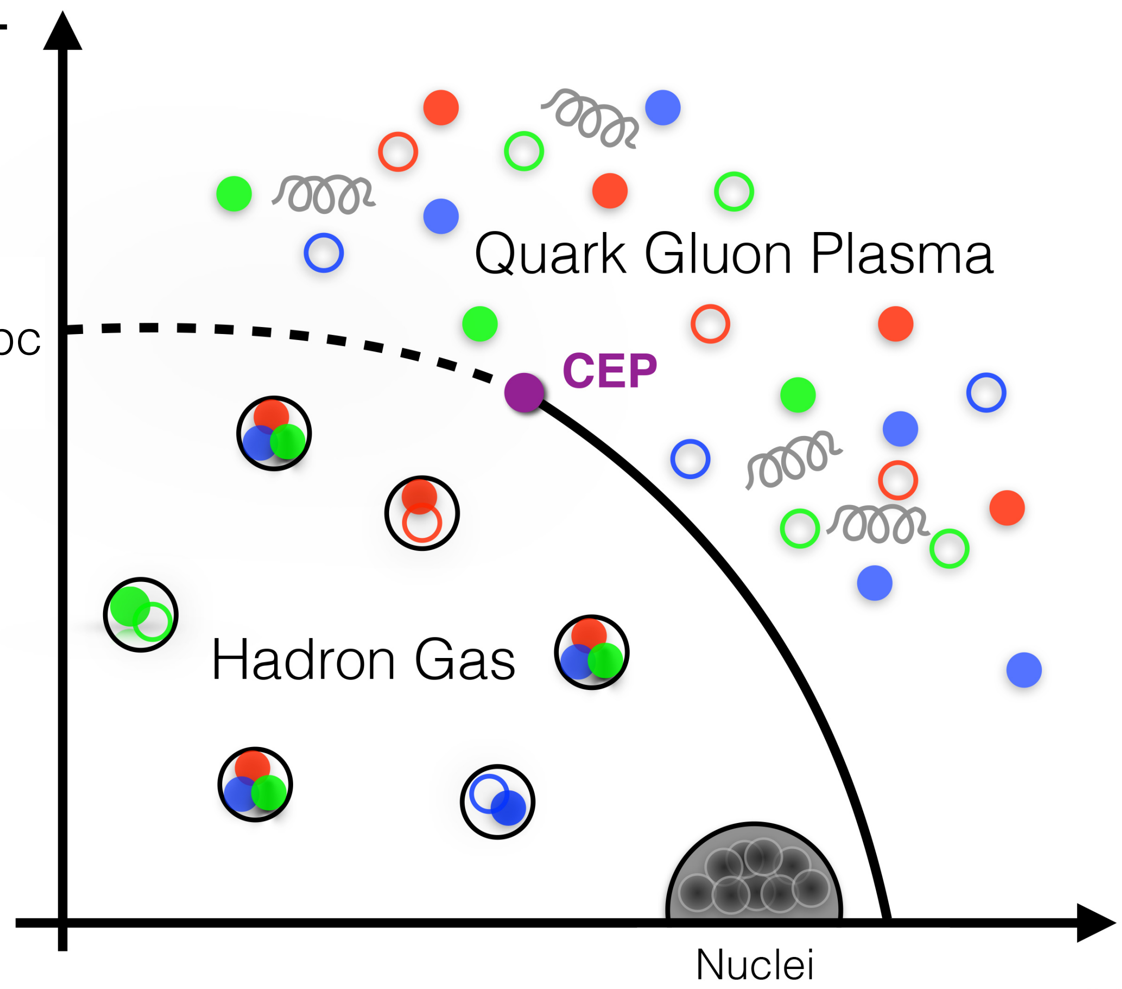
HIAF
NICA
FAIR



HTD, F. Karsch, S. Mukherjee
“Thermodynamics of
Strong-Interaction Matter from LQCD”,
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

PoS LATTICE2016 (2017) 022
PoS LATTICE2016 (2017) 372
Phys.Rev. D95 (2017) no.5, 054504
Phys.Rev. D95 (2017) no.7, 074505
Phys.Rev. D96 (2017) no.7, 074510

Heng-Tong Ding(丁亨通)
from CCNU

中国物理学会高能物理分会第十届全国会员代表大会暨学术年会

20-24 June, 2018@Shanghai

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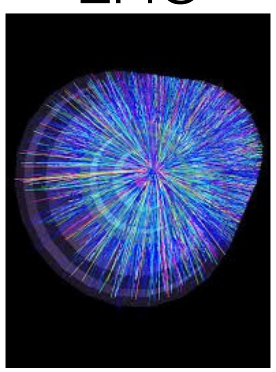

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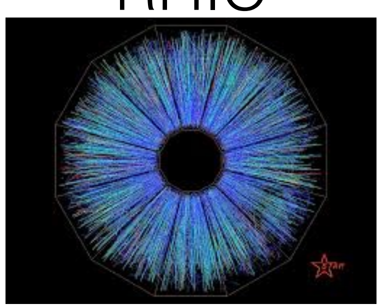

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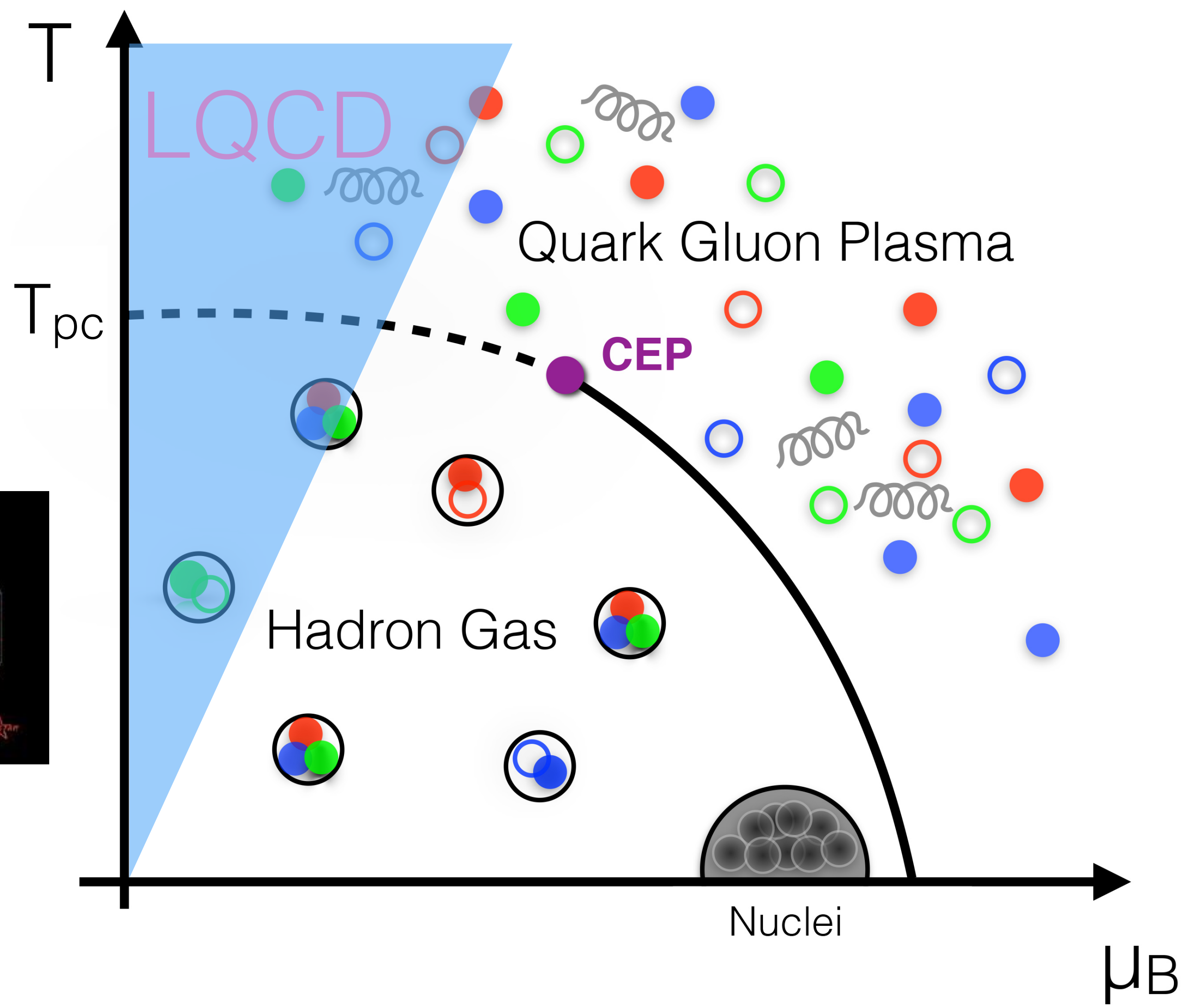

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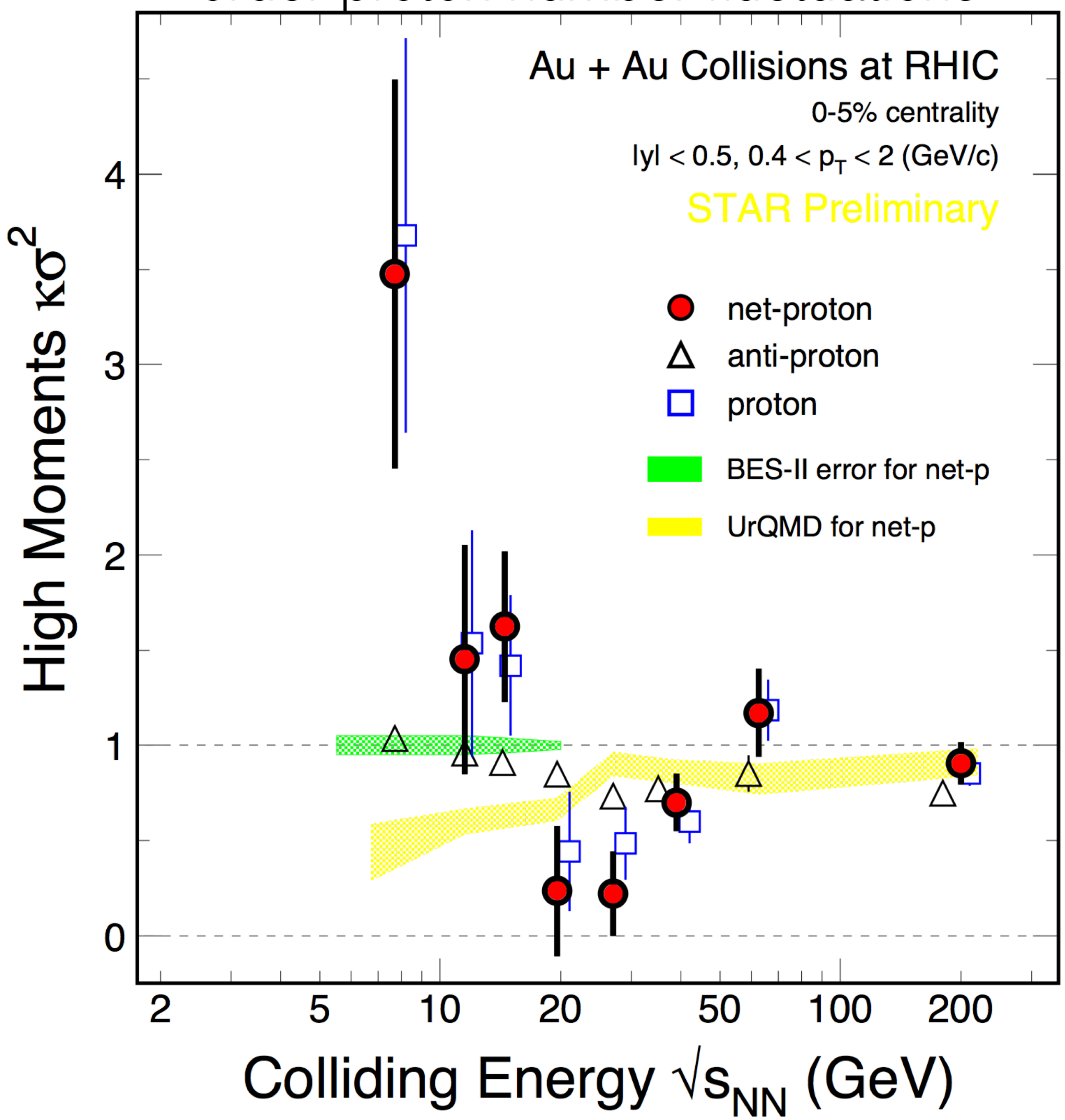
Search for the QCD critical point with fluctuations of conserved charges in HIC

Ratio of the 4th to 2nd order proton number fluctuations



HADES, Preliminary SQM 16 |y|<0.2

STAR data: X.F. Luo, 1503.02558, X.F. Luo and N. Xu, 1701.02105

Can this non-monotonic behavior be understood in terms of the QCD thermodynamics in equilibrium?

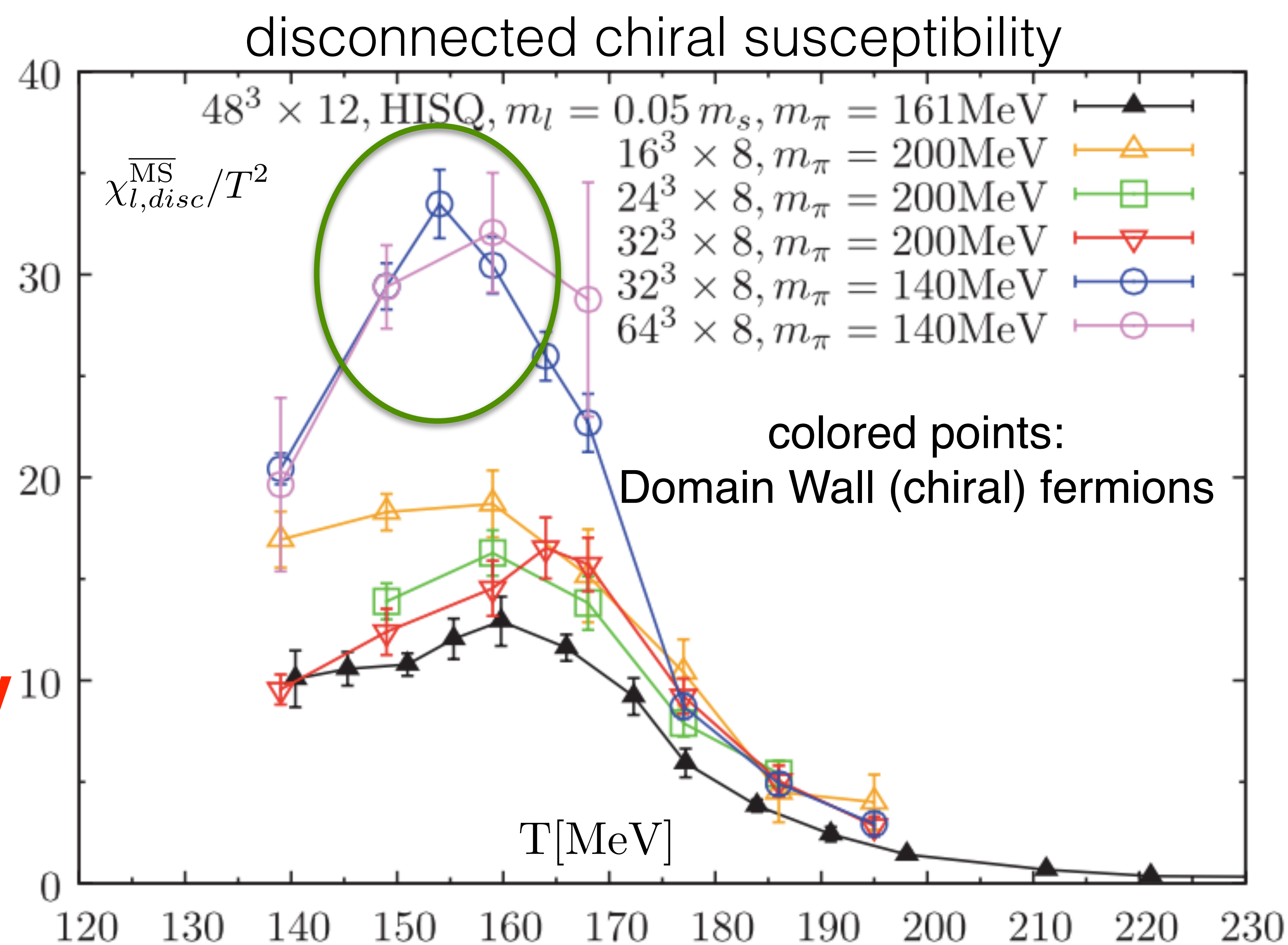
What is the relation of the observed phenomenon to the critical behavior of QCD phase transition?

QCD transition with m_π =140 MeV at μ_B=0/$\sqrt{s_{NN}}$ ≳200 GeV

"cross over" type of transition

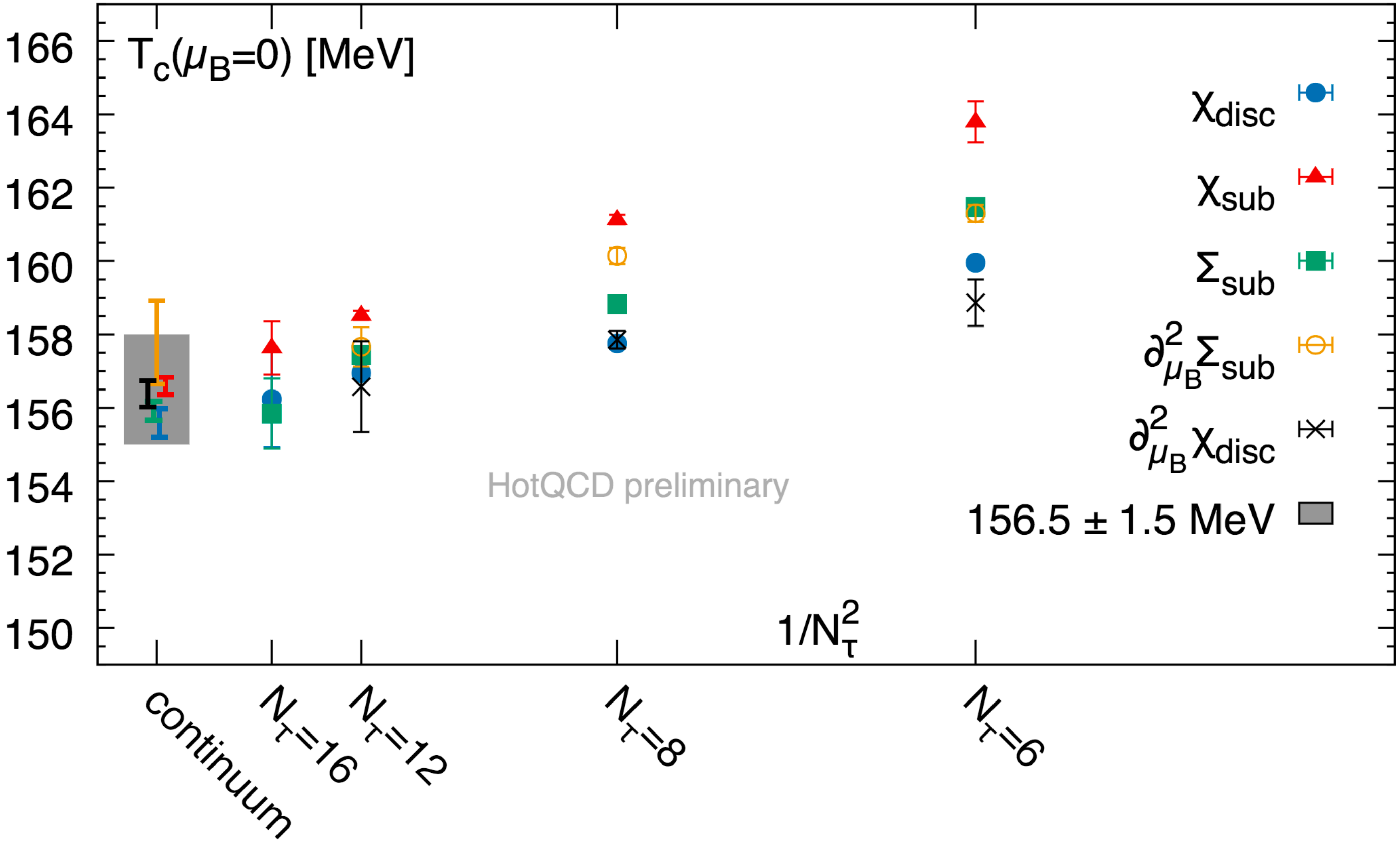
No real "PHASE transition"

T_{pc} =155(1)(8) MeV



T. Bhattacharya,…HTD,…et al. [HotQCD collaboration], Phys. Rev. Lett.,113(2014) 082001 (Editor's suggestion)

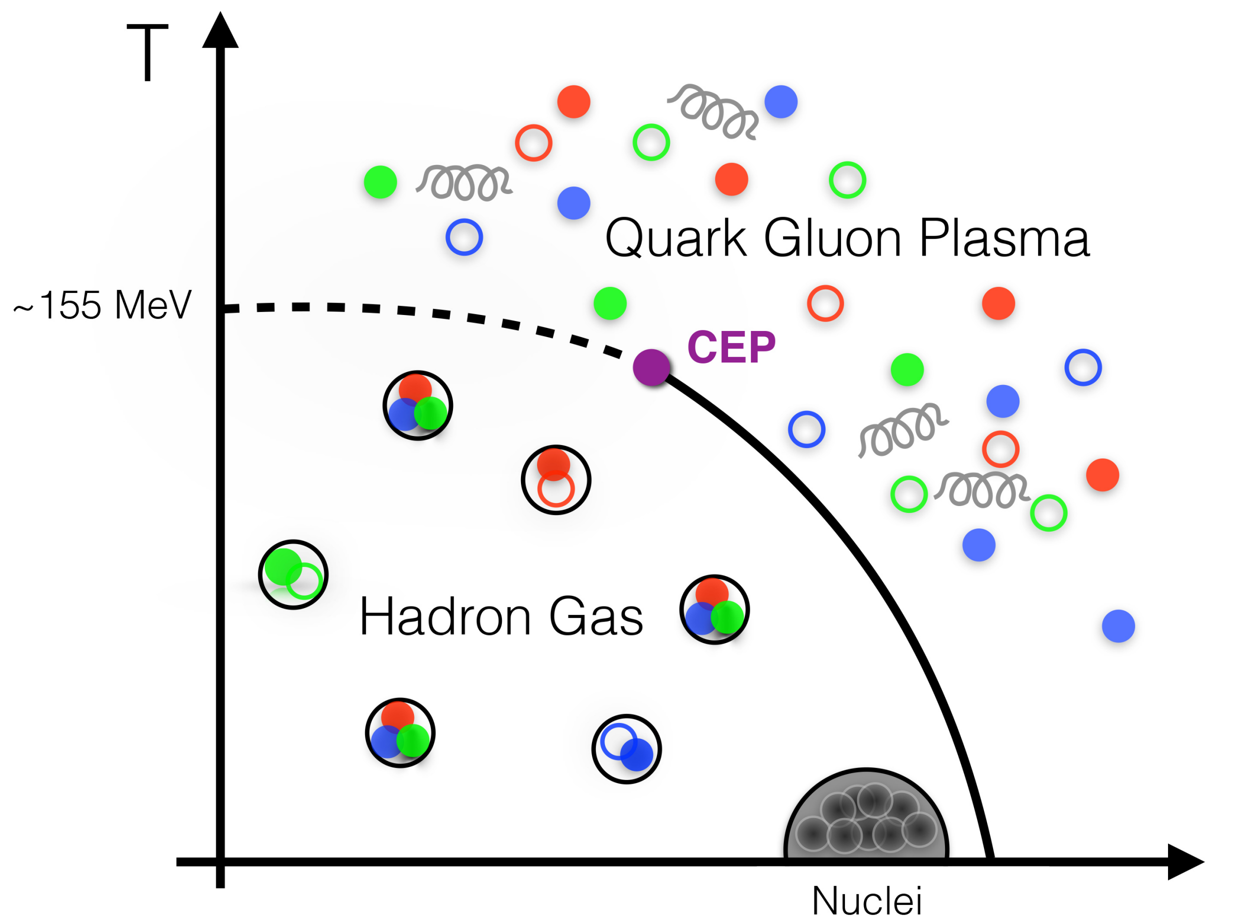
QCD transition with m_π =140 MeV at μ_B=0/$\sqrt{s_{NN}}$ ≳200 GeV



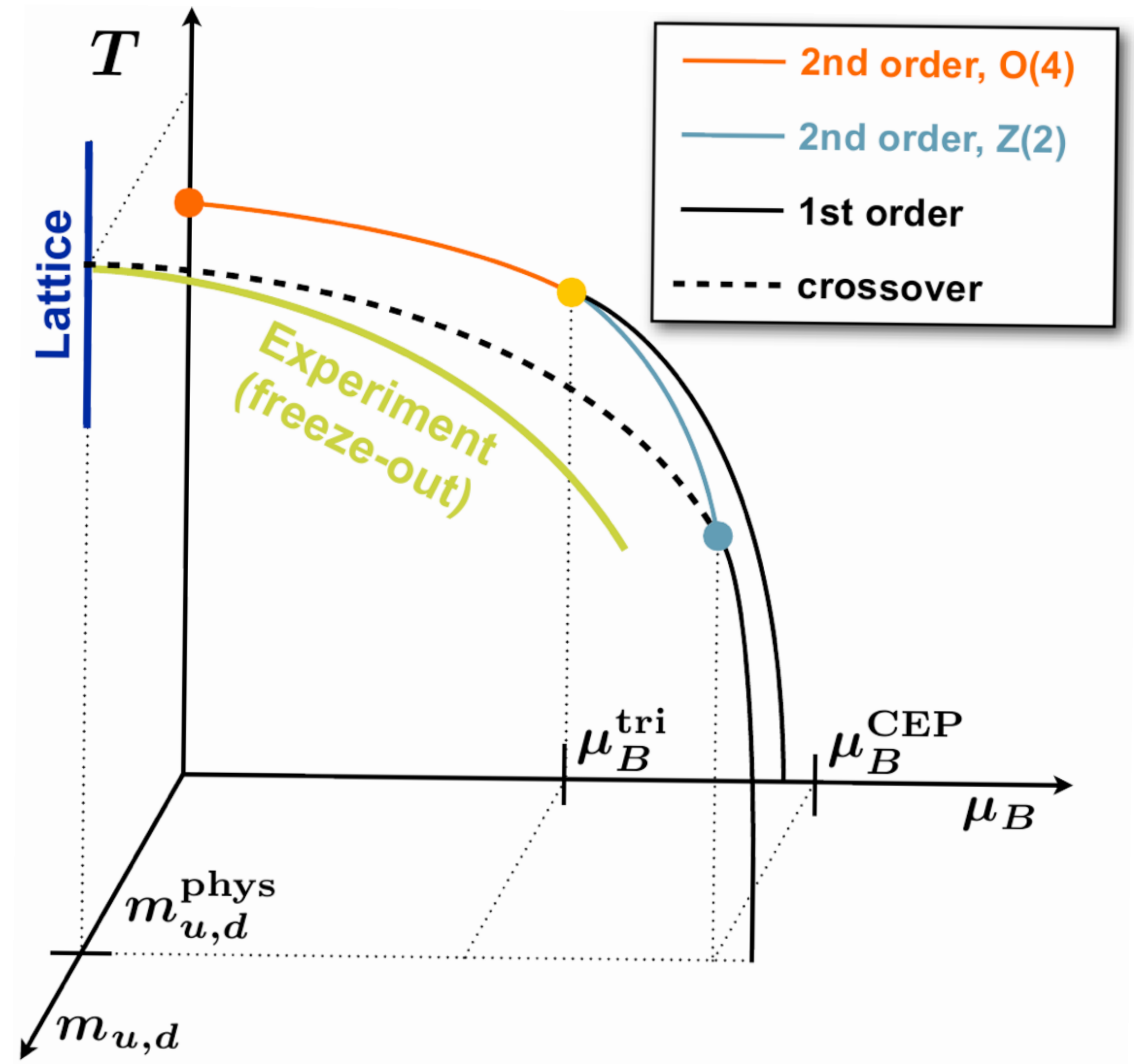
Higher precision in the continuum limit:

T_{pc} =156.5(1.5)MeV

QCD phase diagram



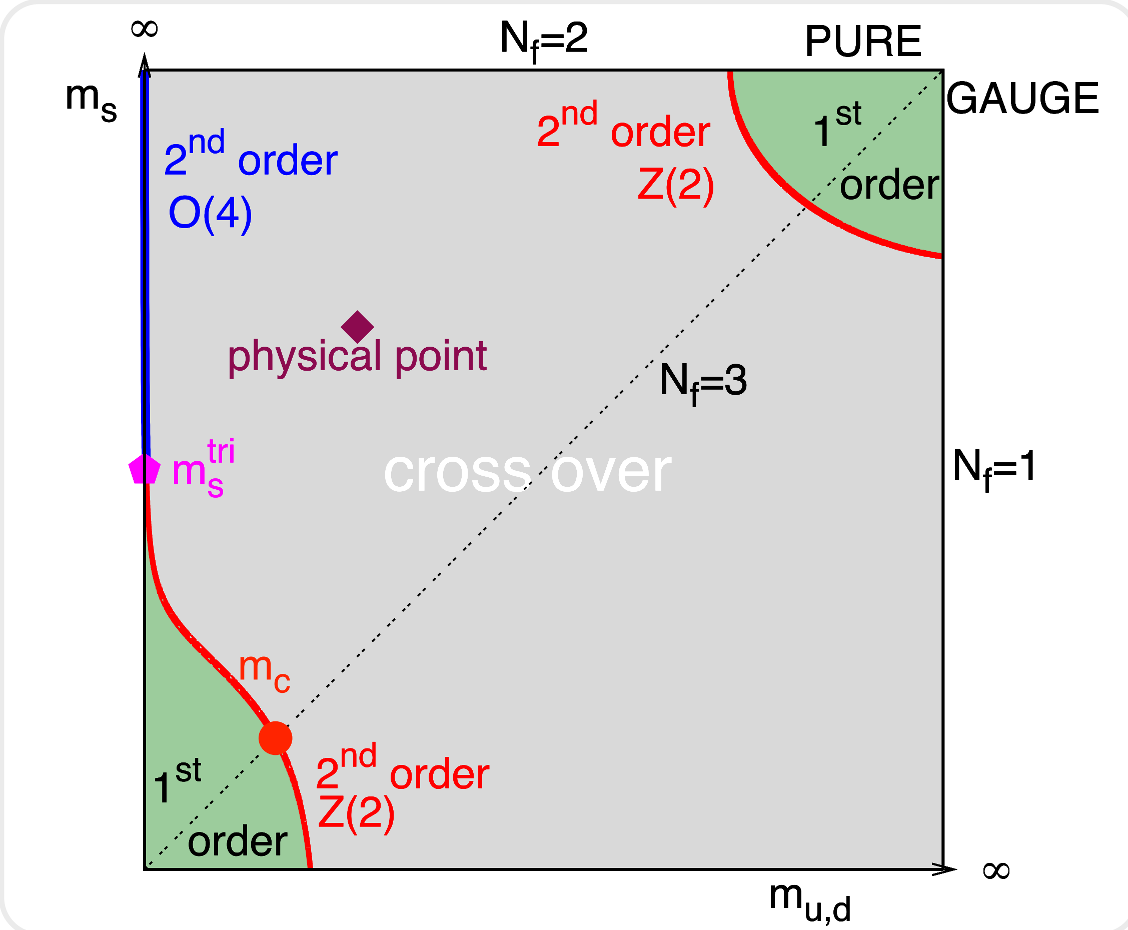
HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274
HTD, arXiv:1702.00151



- Chiral phase transitions at μ_B=0
- EoS at μ_B=/=0
- Constrains on the CEP & comparison to HIC data

QCD phase structure in the quark mass plane

columbia plot, PRL 65(1990)2491



HTD, F. Karsch, S. Mukherjee, 1504.05274

RG arguments:

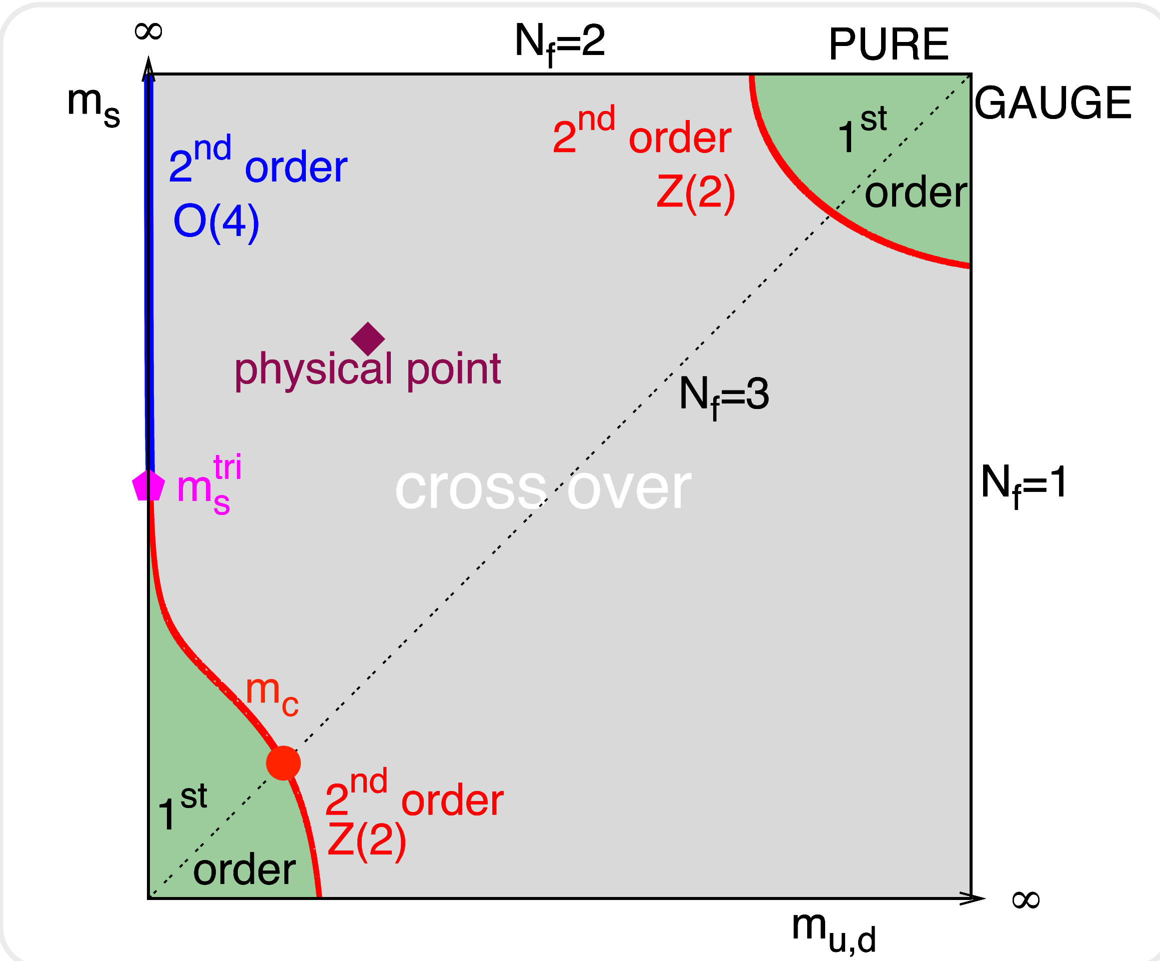
- m_q=0 or ∞ with N_f=3: a first order phase transition R. Pisarski & F. Wilczek, PRD29 (1984) 338

- Critical lines of second order transition
  N_f=2: O(4) universality class
  N_f=3: Z(2) universality class

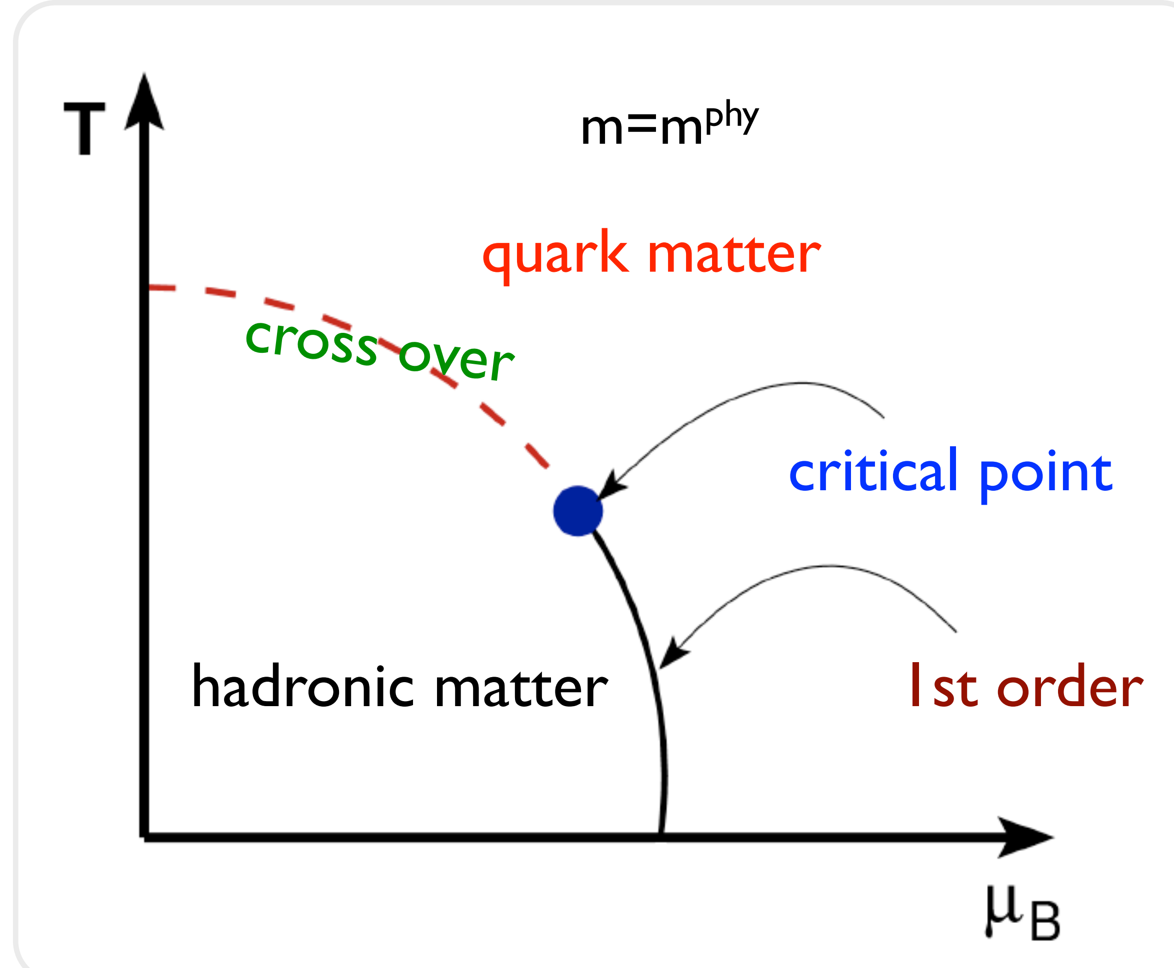
F. Wilczek, IJMPA 7(1992) 3911,6951
K. Rajagopal & F. Wilczek, NPB 399 (1993) 395
Gavin, Gocksch & Pisarski, PRD 49 (1994) 3079

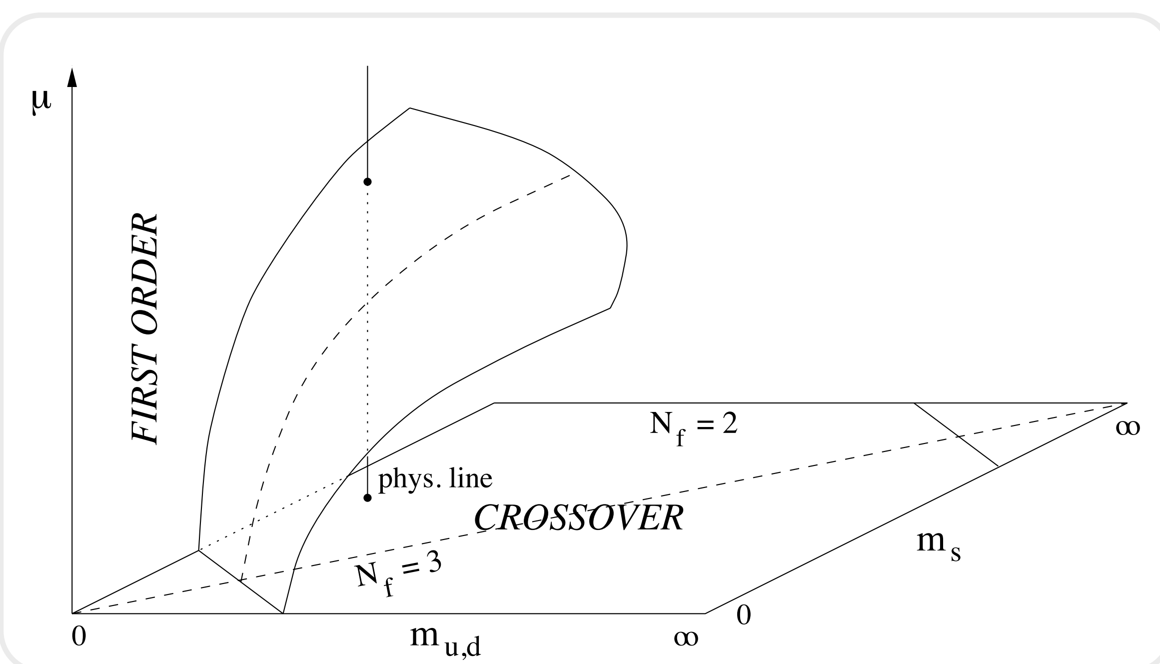
- The value of tri-critical point (m_s^{tri}) ?
- The location of 2^{nd} order Z(2) lines ?
- The influence of criticalities to the thermodynamics at the physical point ?

QCD transitions at the physical point

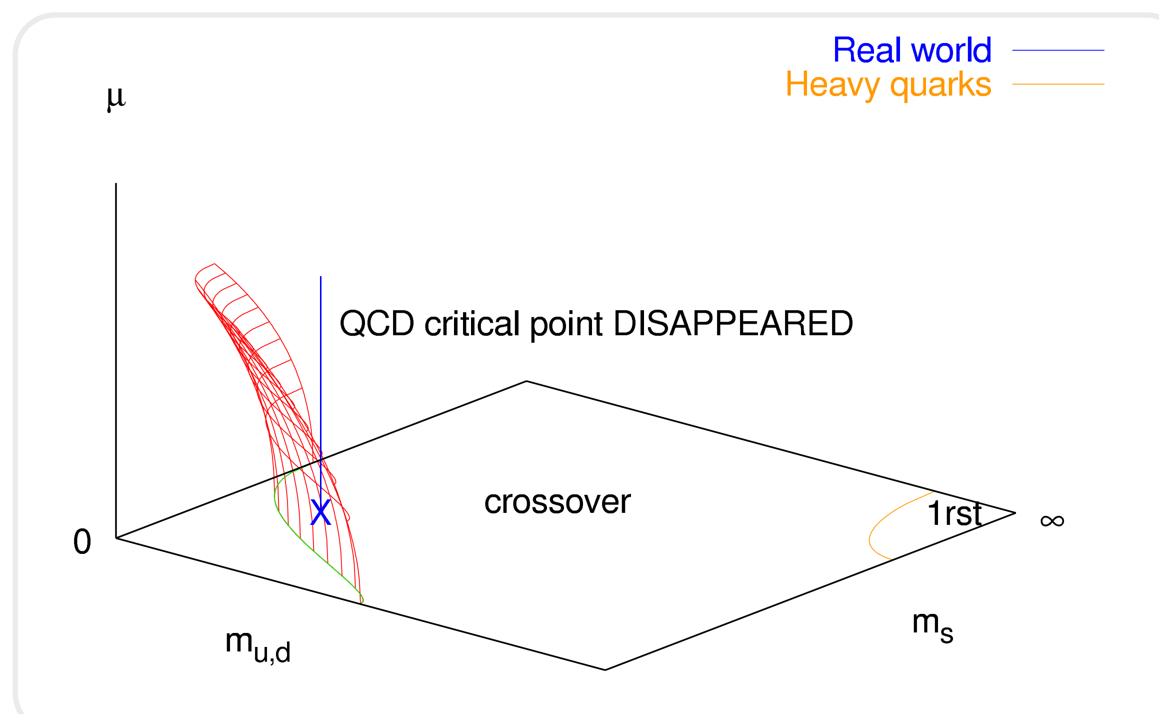


HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274



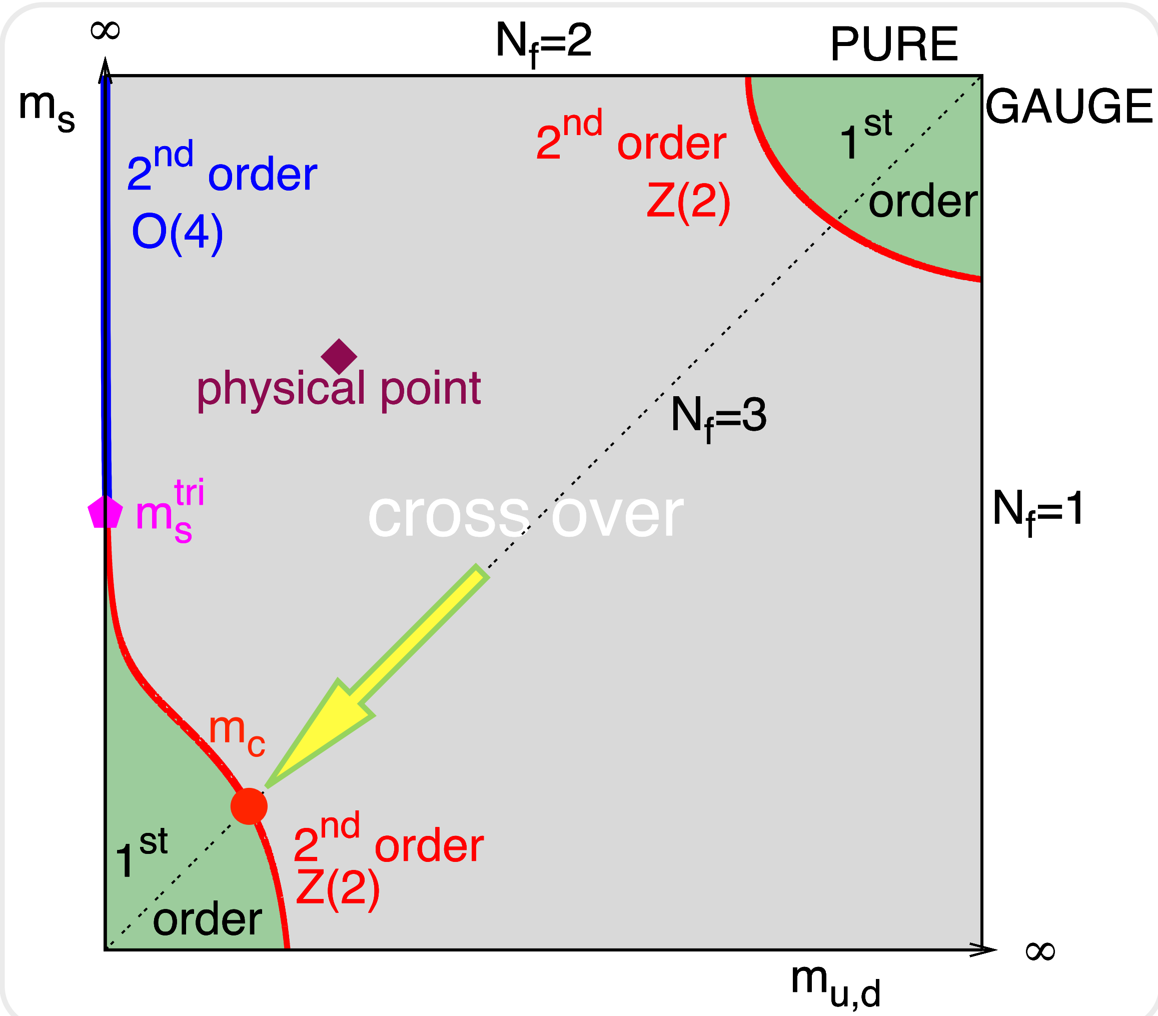


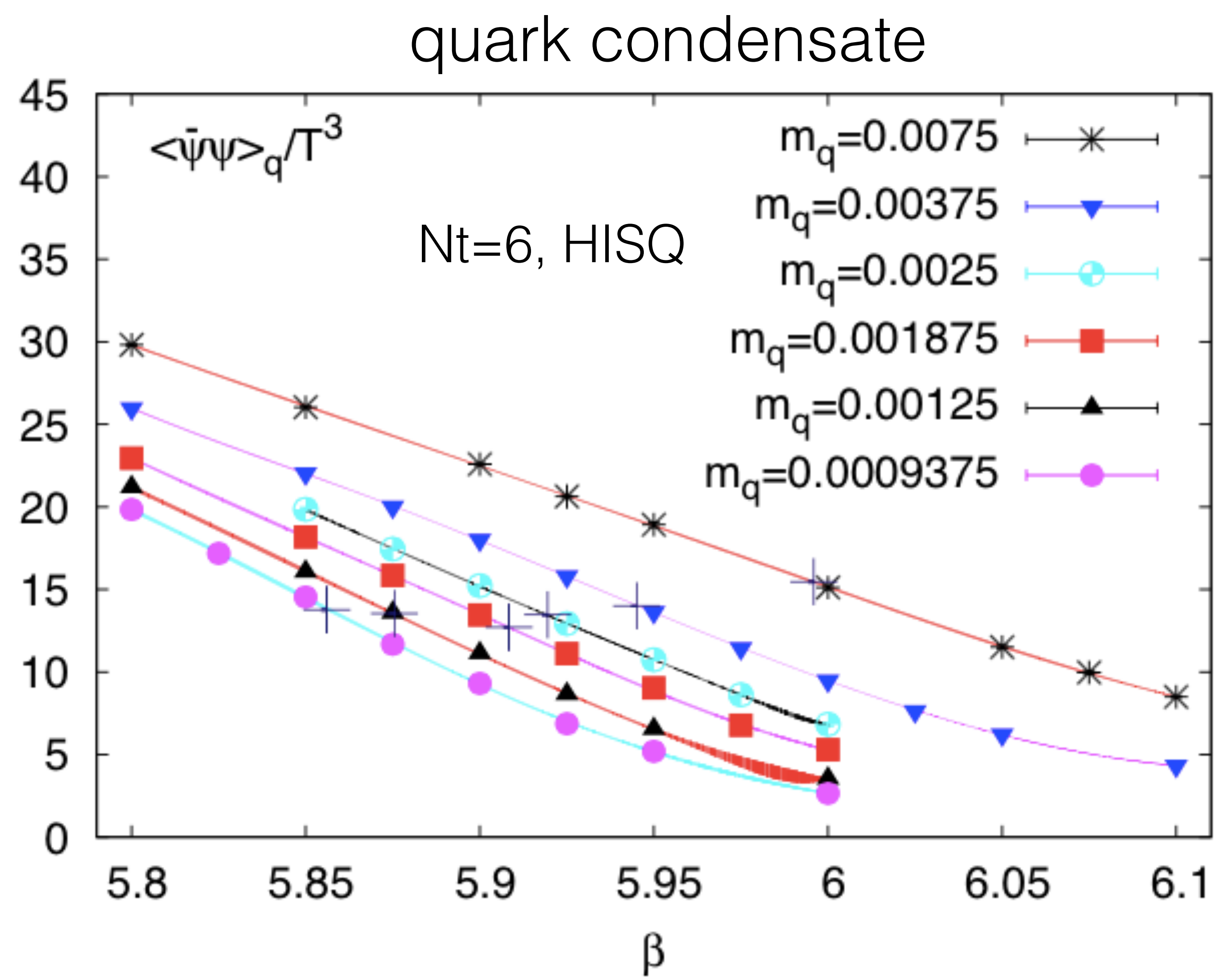
Karsch et al., '03, X.-Y. Jin et al., '15



de Forcrand & Philipsen, '07

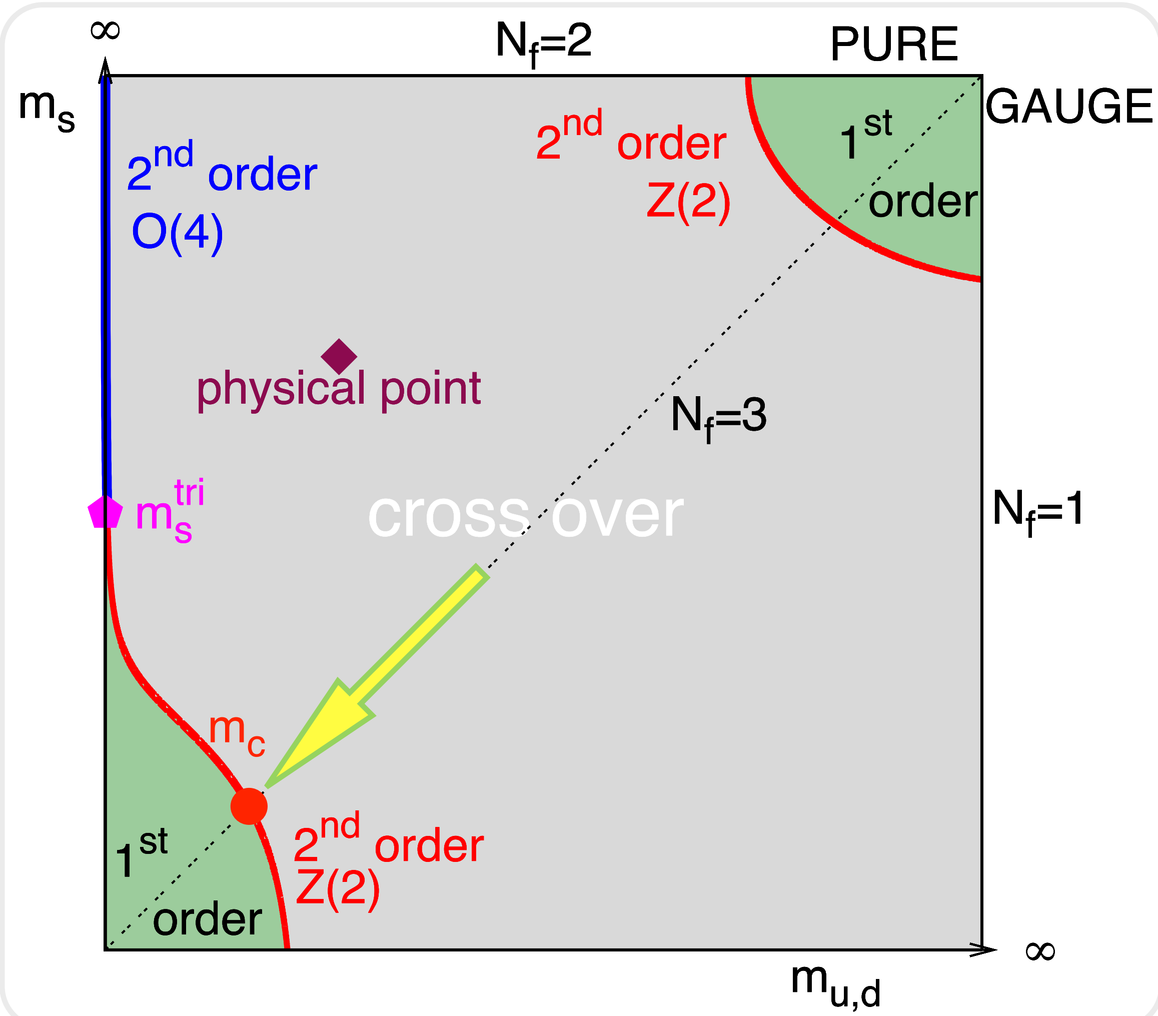
chiral phase transition in Nf=3 QCD at μB=0



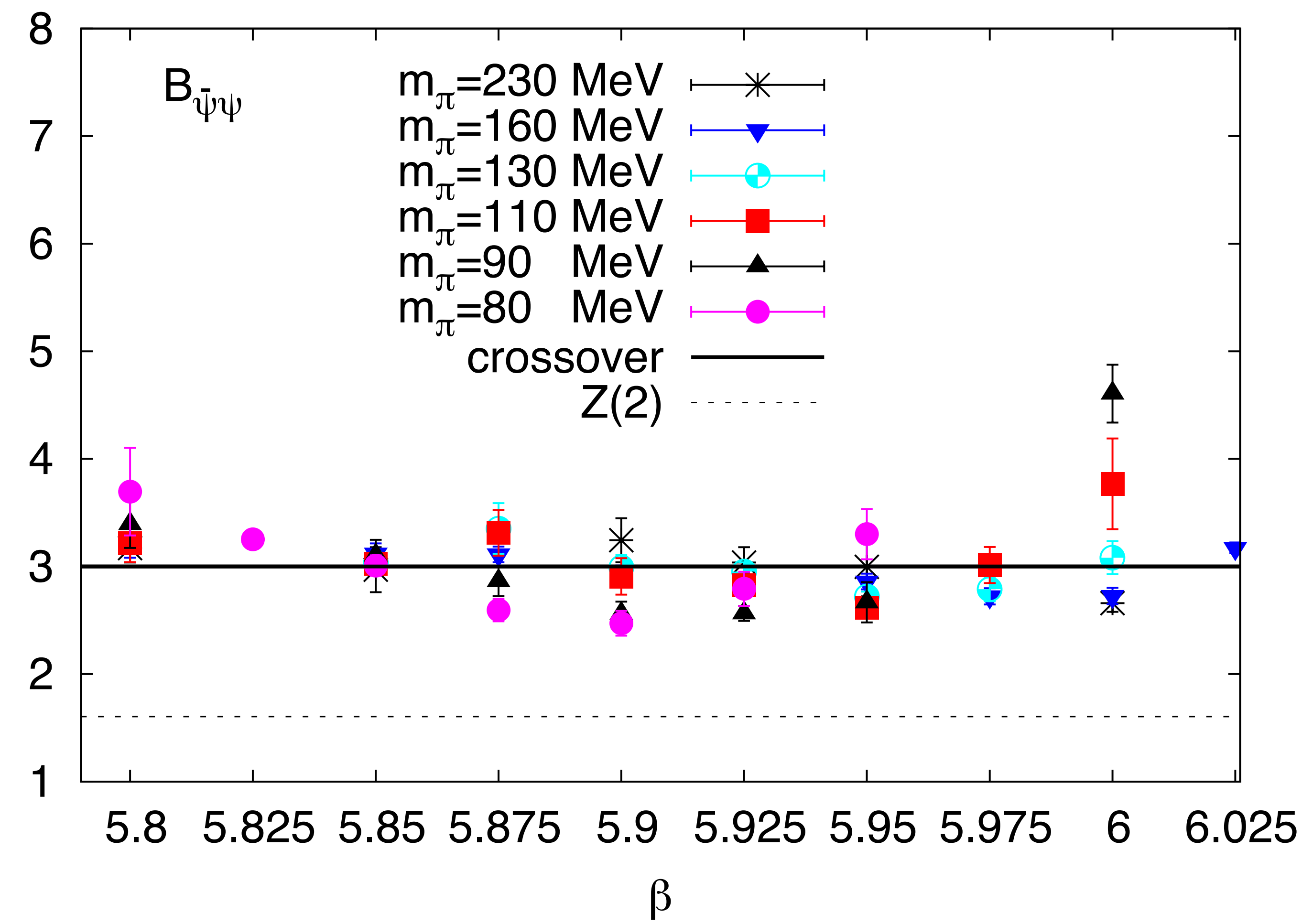


mass region: $200 \text{ MeV} \gtrsim m_\pi \gtrsim 80 \text{ MeV}$

chiral phase transition in Nf=3 QCD at μB=0



Binder cumulant of chiral condensate



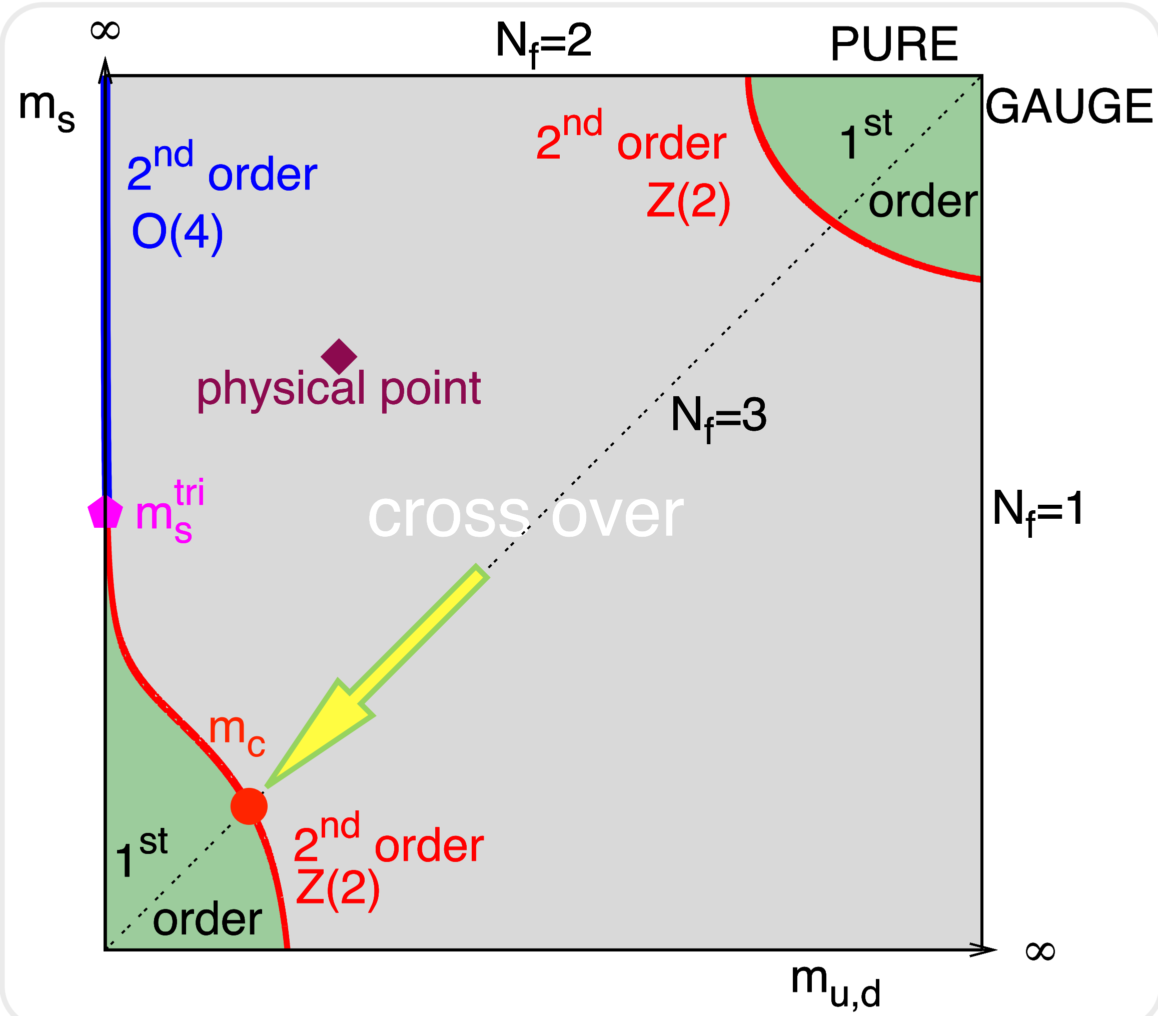
$$B_{\bar{\psi}\psi} = \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2}$$

B=3: crossover

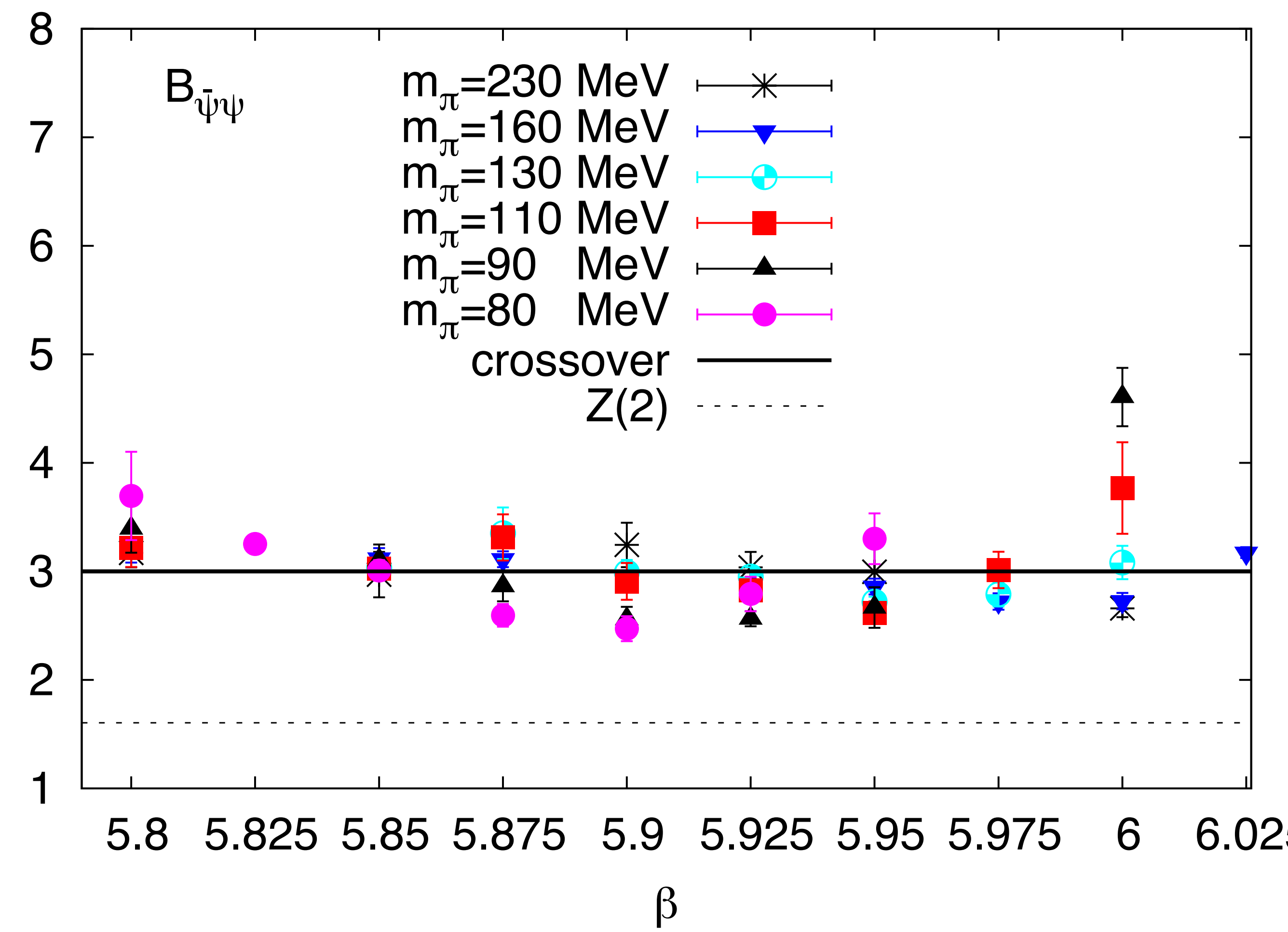
B=1.604: 2nd order phase transition with Z(2) universality class

B=1: 1st order phase transition

chiral phase transition in Nf=3 QCD at μ_B=0



Binder cumulant of chiral condensate

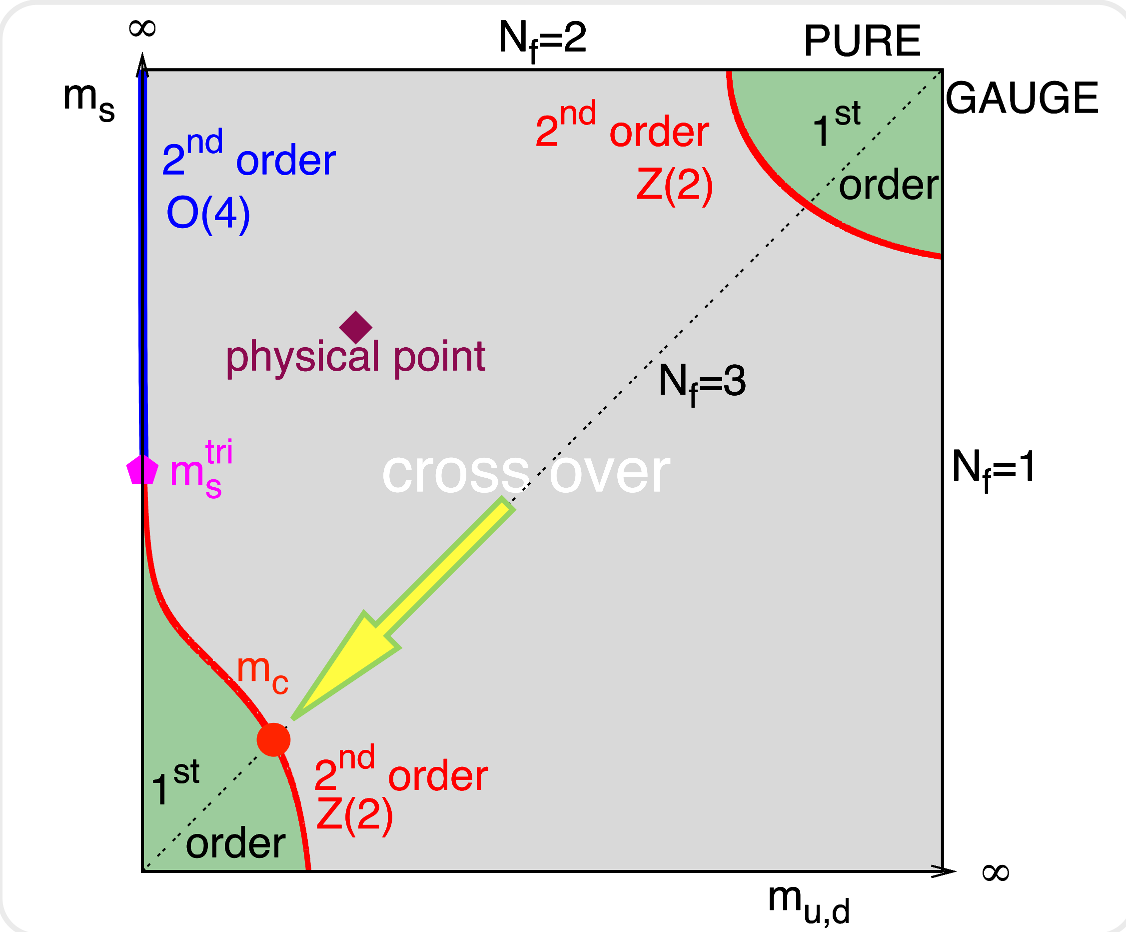


$$B_{\bar\psi\psi} = \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2}$$

No evidence of a first order phase transition

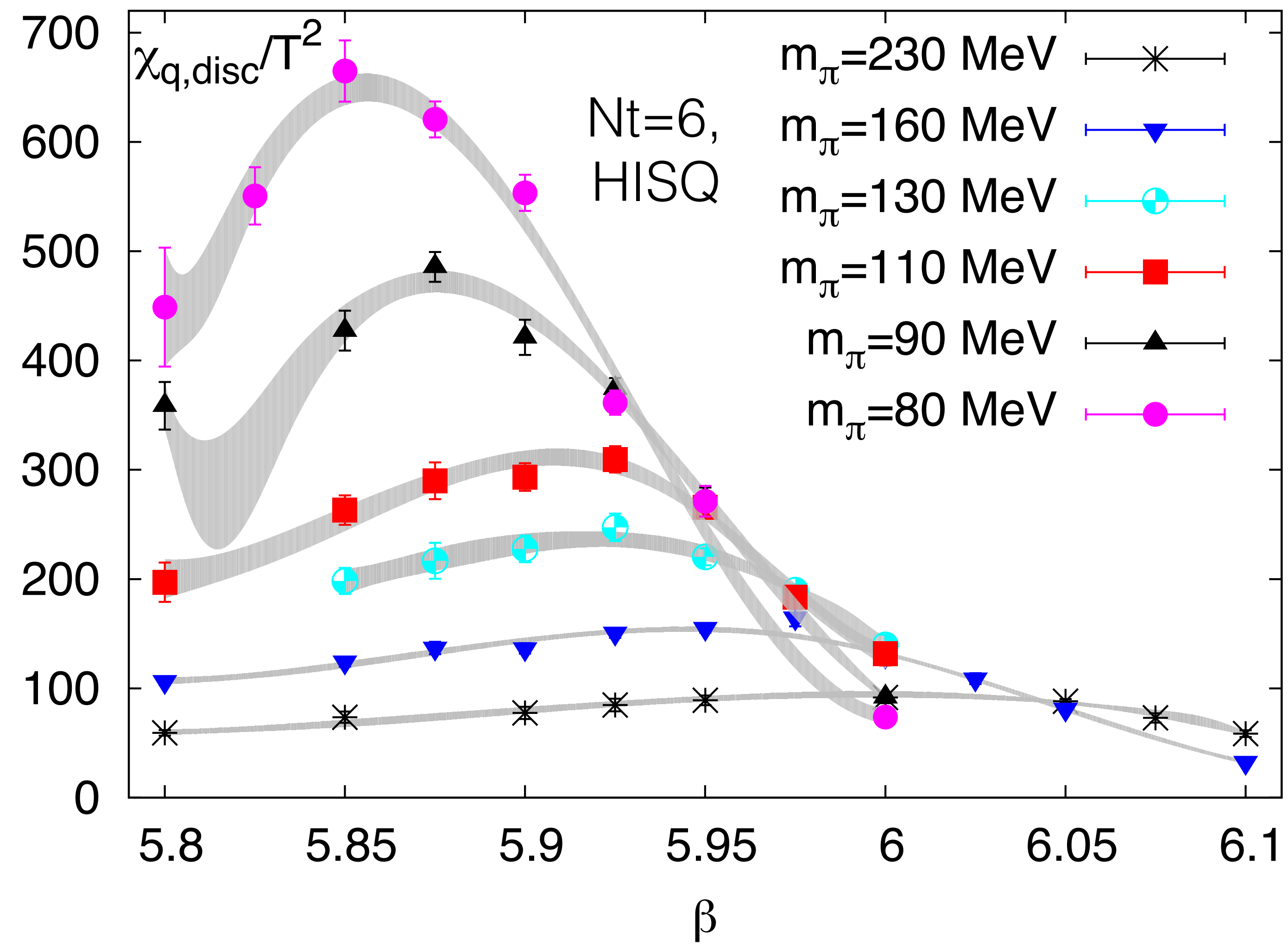
Bielefeld-BNL-CCNU,
Phys.Rev. D 95 (2017) no.7, 074505

Chiral phase transition in Nf=3 QCD at $\mu_B=0$



physical point:
(ml, ms):(0.00375,0.10125)

disconnected chiral susceptibility

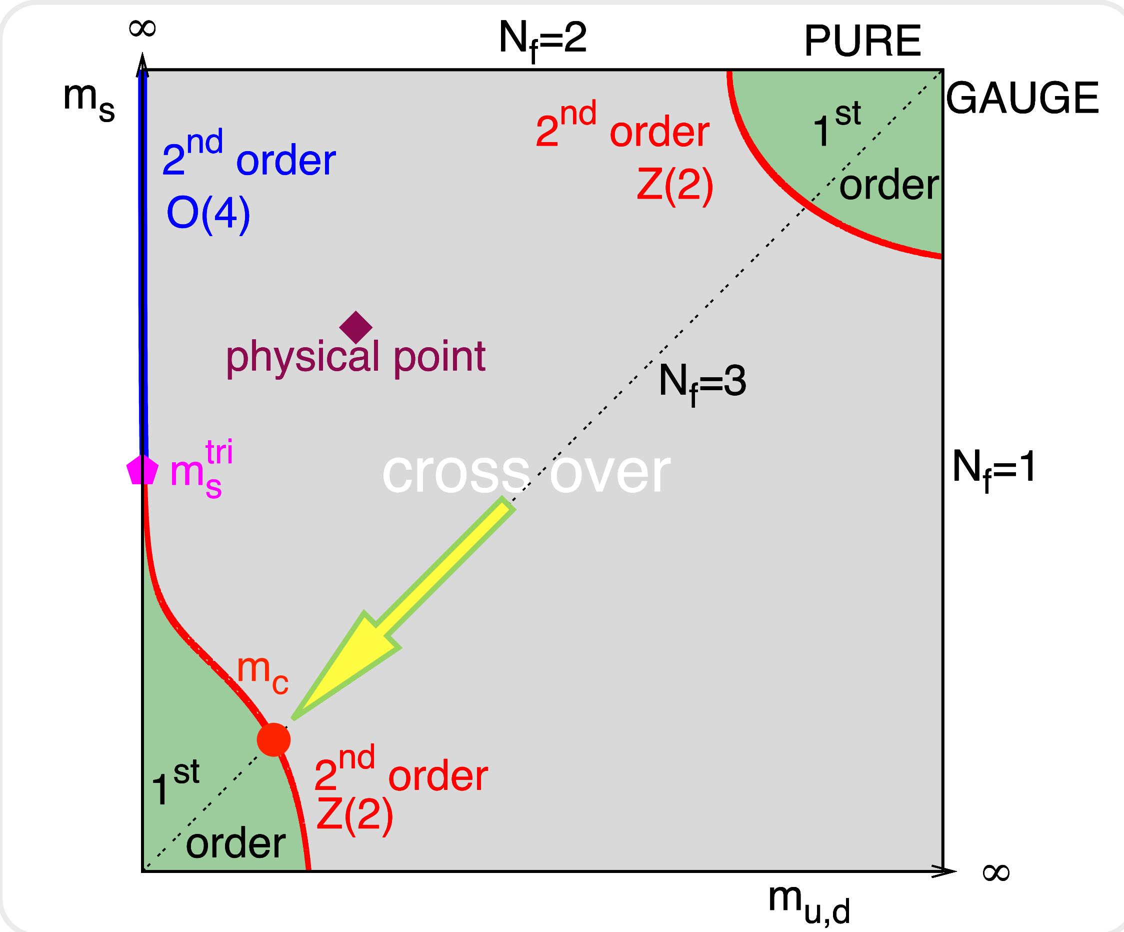


Bielefeld-BNL-CCNU
Phys.Rev. D 95 (2017) no.7, 074505

Close to Z(2) phase transition line:

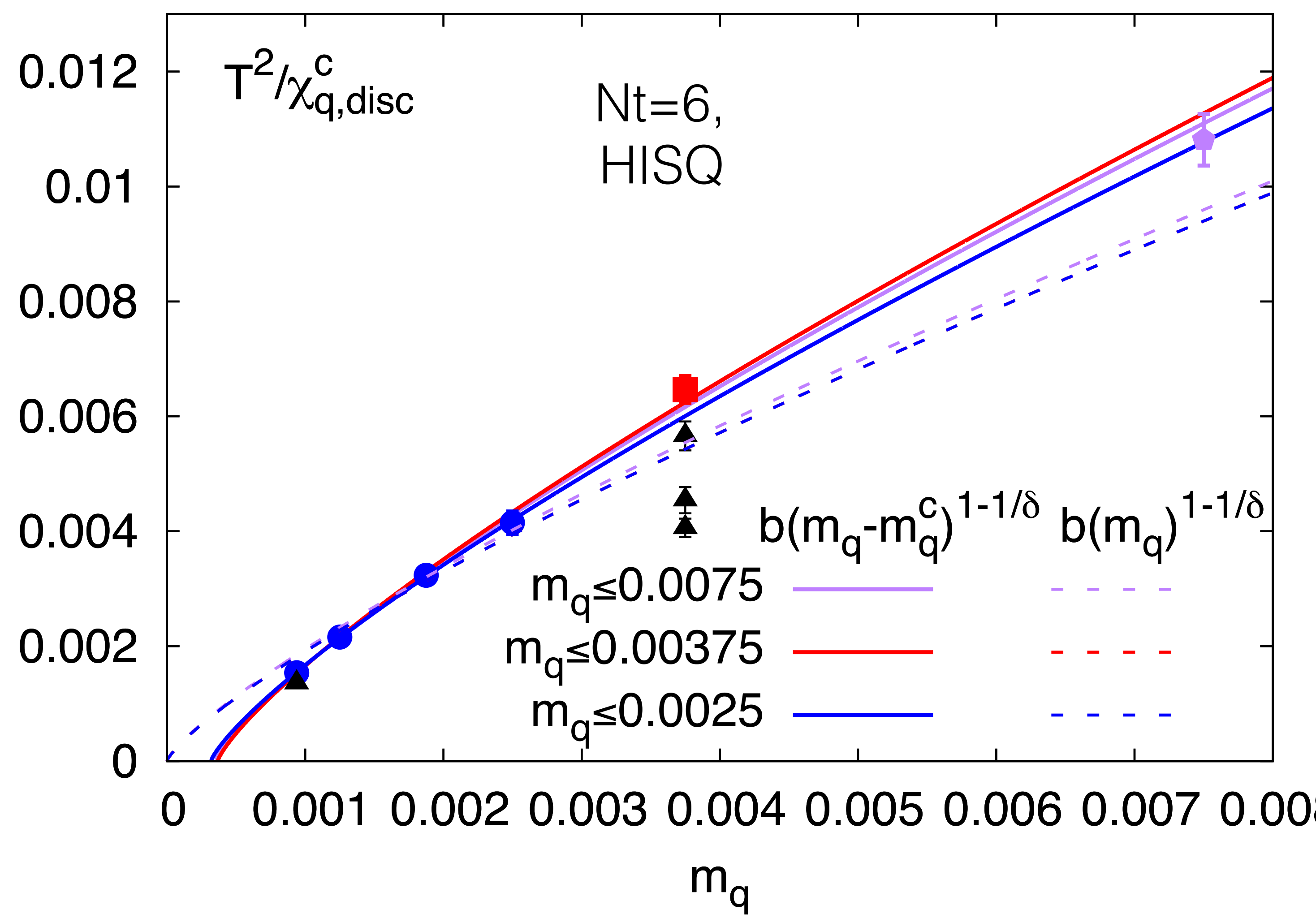
$$\chi_{q,disc}^{max} \sim (m - m_c)^{1/\delta - 1}$$

Chiral phase transition in Nf=3 QCD at μB=0



physical point:
(ml, ms):(0.00375,0.10125)

inverse peak height of
disconnected chiral susceptibility



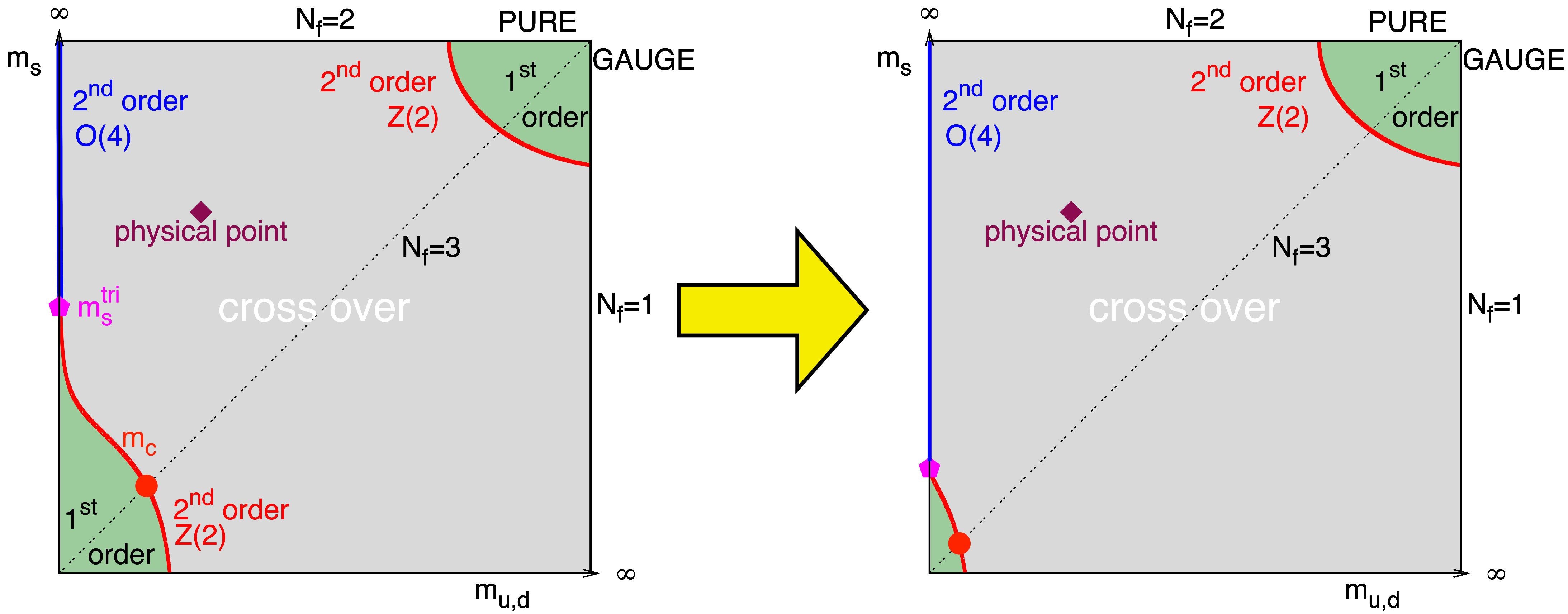
Bielefeld-BNL-CCNU
Phys.Rev. D 95 (2017) no.7, 074505

Close to Z(2) phase transition line:

$$\chi_{q,disc}^{max} \sim (m - m_c)^{1/\delta - 1}$$

critical quark mass m_c ~ 0.0004 $\Rightarrow$ $m_\pi^c \lesssim 50\mathrm{MeV}$

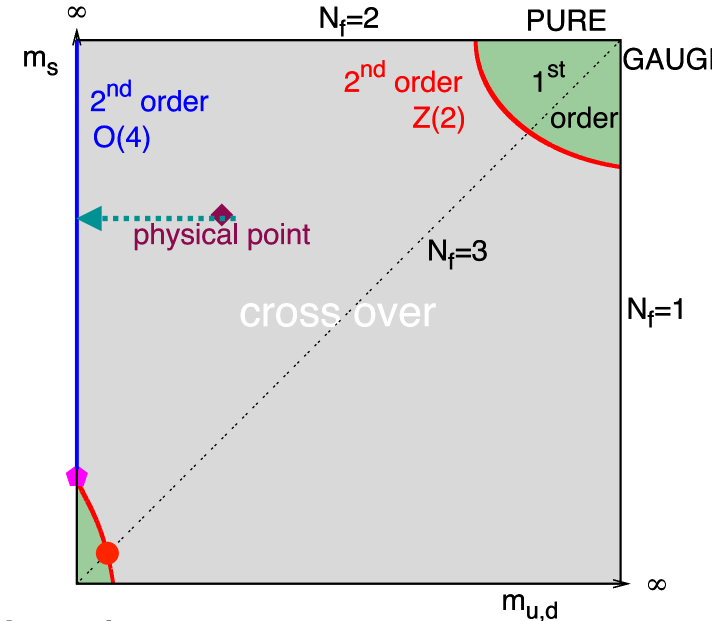
Chiral phase transition region in Nf=3 QCD at μ_B=0



1st order chiral phase transition seem to be not much relevant for thermodynamics at the physical point

How about the 2nd order O(4) transition line?

Universal behavior of chiral phase transition in N_f=2+1 QCD at μ_B=0



Behavior of the free energy close to critical lines

$$f(m,T)=h^{1+1/\delta}\, f_s(z) + f_{reg}(m,T), \qquad z=t/h^{1/\beta\delta}$$

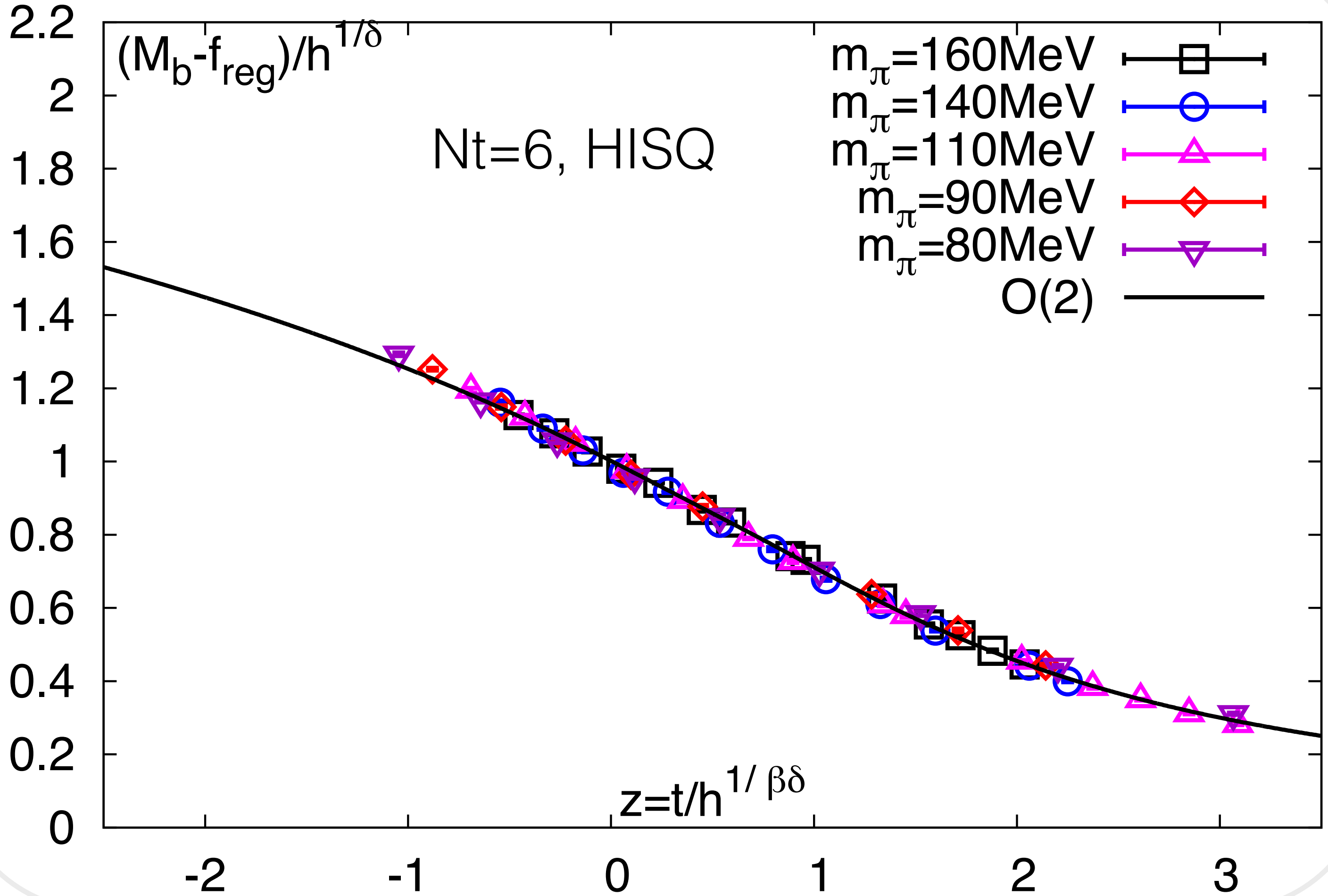
h: external field, t: reduced temperature, β,δ: universal critical exponents

$$M = -\partial f(t,h)/\partial h = h^{1/\delta} f_G(z) + f_{reg}(t,h)$$

$h \sim m$; $t \sim T-T_C$

$f_G(z)$: O(2) scaling functions

QCD: SU(2)xSU(2)≃O(4)

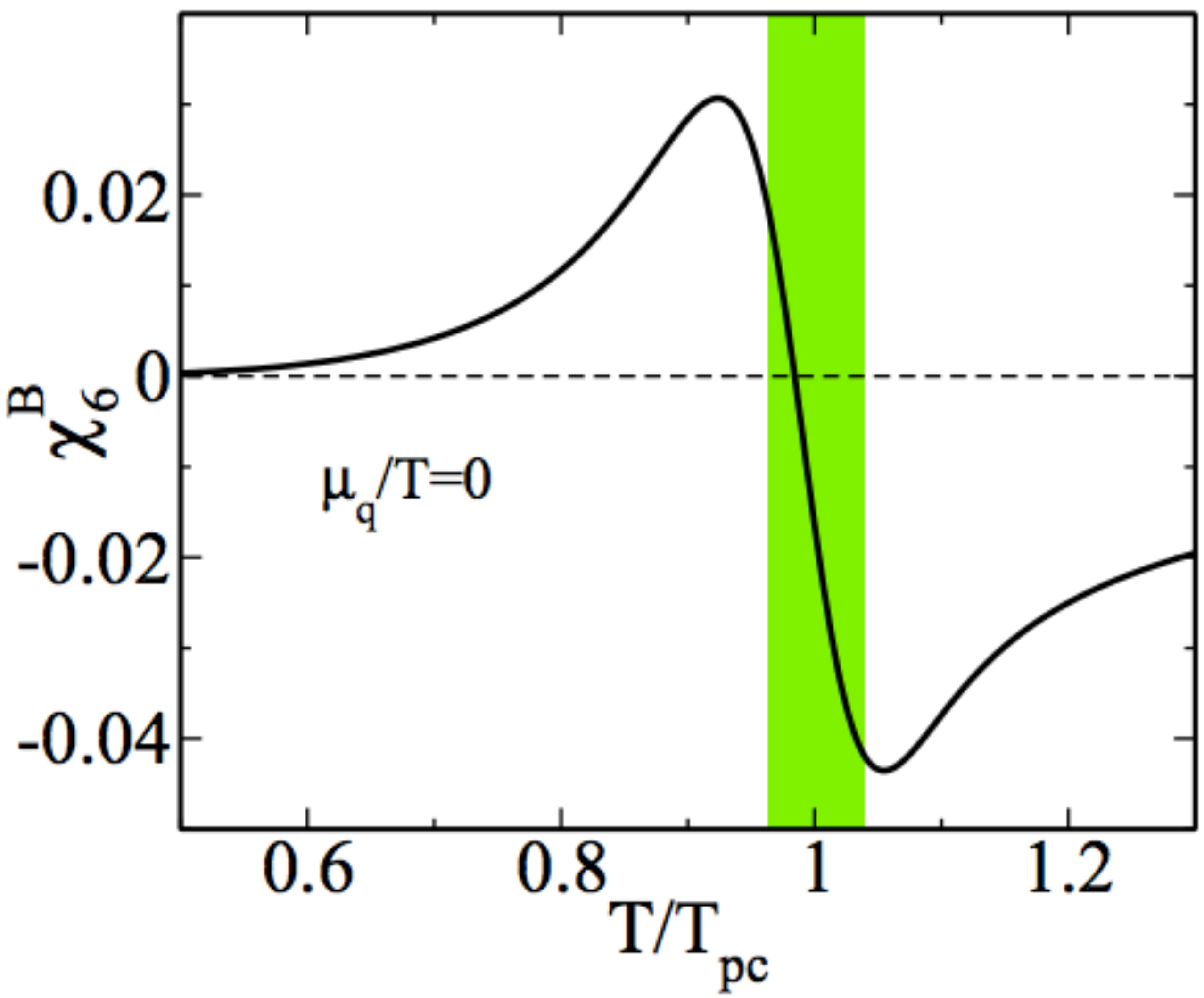


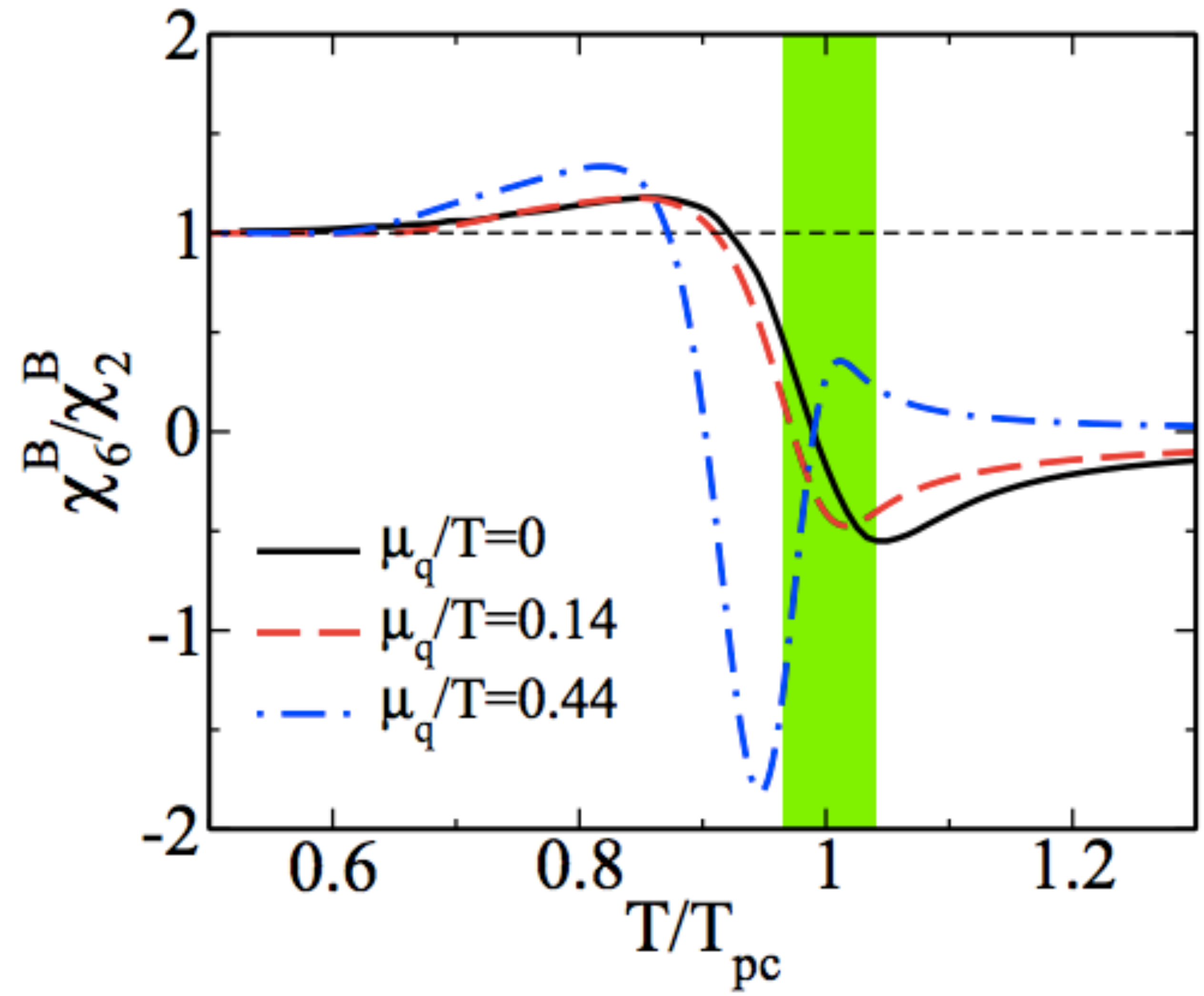
Some evidence of O(N) scaling for chiral phase transition

Ongoing analyses with Nt=8,12 towards continuum limit

S.-T. Li(李胜泰), Lattice 2016, Bielefeld-BNL-CCNU, PoS LATTICE2016 (2017) 372

Baryon number fluctuations according to 3-d O(4) universality class





$$\chi_n^B \sim \begin{cases} -(2\kappa_q)^{n/2}|t|^{2-\alpha-n/2} f_\pm^{(n/2)} & , \text{ for } \mu_q/T = 0, \text{ and } n \text{ even} \\ -(2\kappa_q)^n \left(\frac{\mu_q}{T}\right)^n |t|^{2-\alpha-n} f_\pm^{(n)} & , \text{ for } \mu_q/T > 0 , \end{cases}$$

$$t = \frac{1}{t_0}\left(\frac{T - T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2\right)$$

3-d O(4) : $\boldsymbol{\alpha}$=-0.21

B. Friman et al., Eur.Phys.J. C71 (2011) 1694

Lattice simulations at nonzero μ_B

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

fluctuations of conserved charges:

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial P(T, \hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\bigg|_{\hat{\mu}=0}$$

- Thermodynamic quantities can be obtained using relations, e.g.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial P/T^4}{\partial T} = \sum_{i,j,k=0}^{\infty} \frac{T \, \mathrm{d}\chi_{ijk}^{BQS}/\mathrm{d}T}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

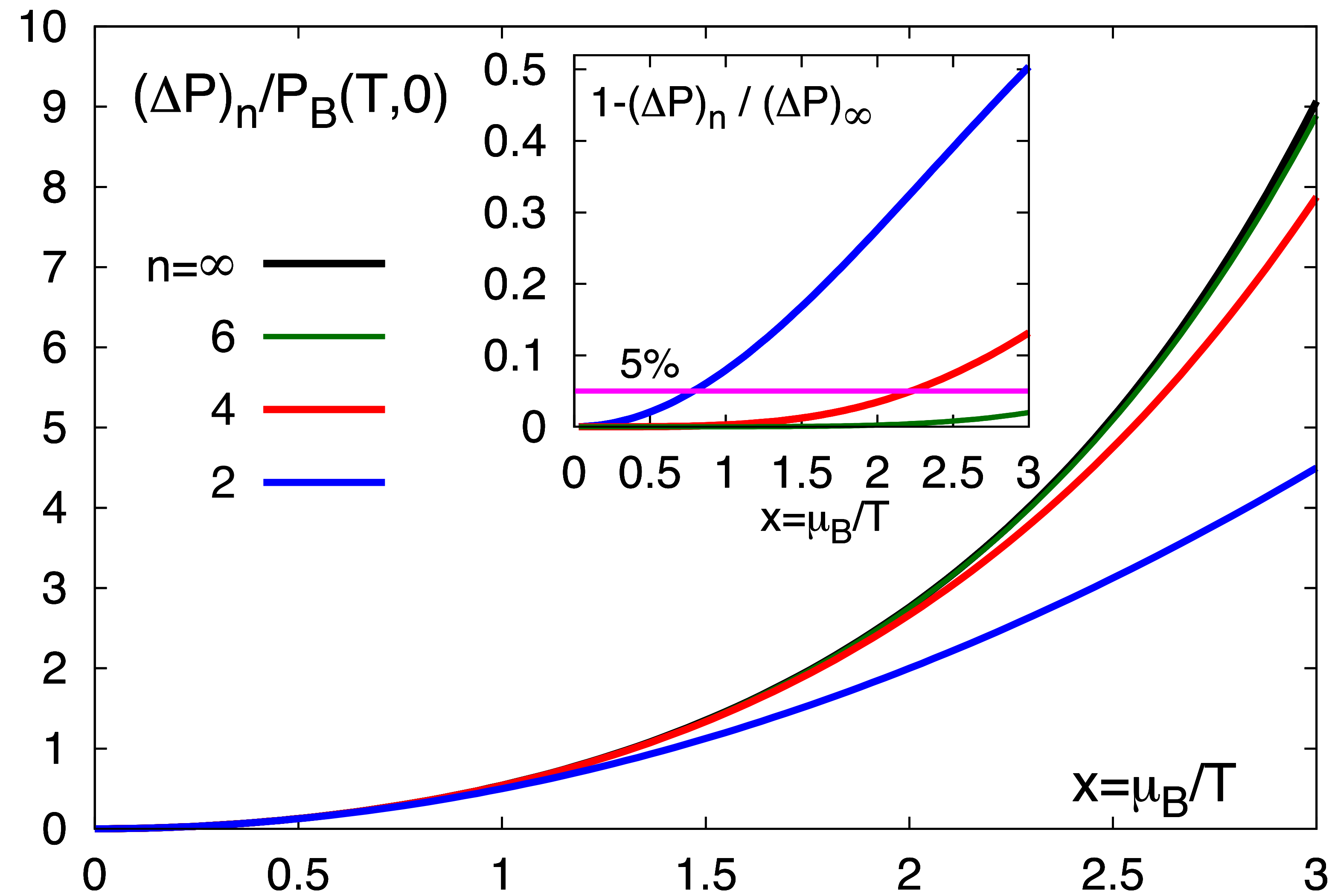
Truncation effects of pressure in HRG

Pressure of hadron resonance gas (HRG)

$$P(T,\mu_B) = P_M(T) + P_B(T,\hat{\mu}_B)$$
$$= P_M(T) + P_B(T,0) + P_B(T,0)(\cosh(\hat{\mu}_B) - 1)$$

Truncate the Taylor expansion at (2n)-th order:

$$(\Delta P)_n = \Big(P_B(T,\mu_B) - P_B(T,0)\Big)_n$$
$$= \sum_{k=1}^{n} \frac{\chi_{2k}^{B,HRG}(T)}{(2k)!}\hat{\mu}_B^{2k}$$
$$\simeq P_B(T,0)\sum_{i=1}^{n} \frac{1}{(2k)!}\hat{\mu}_B^{2k}$$



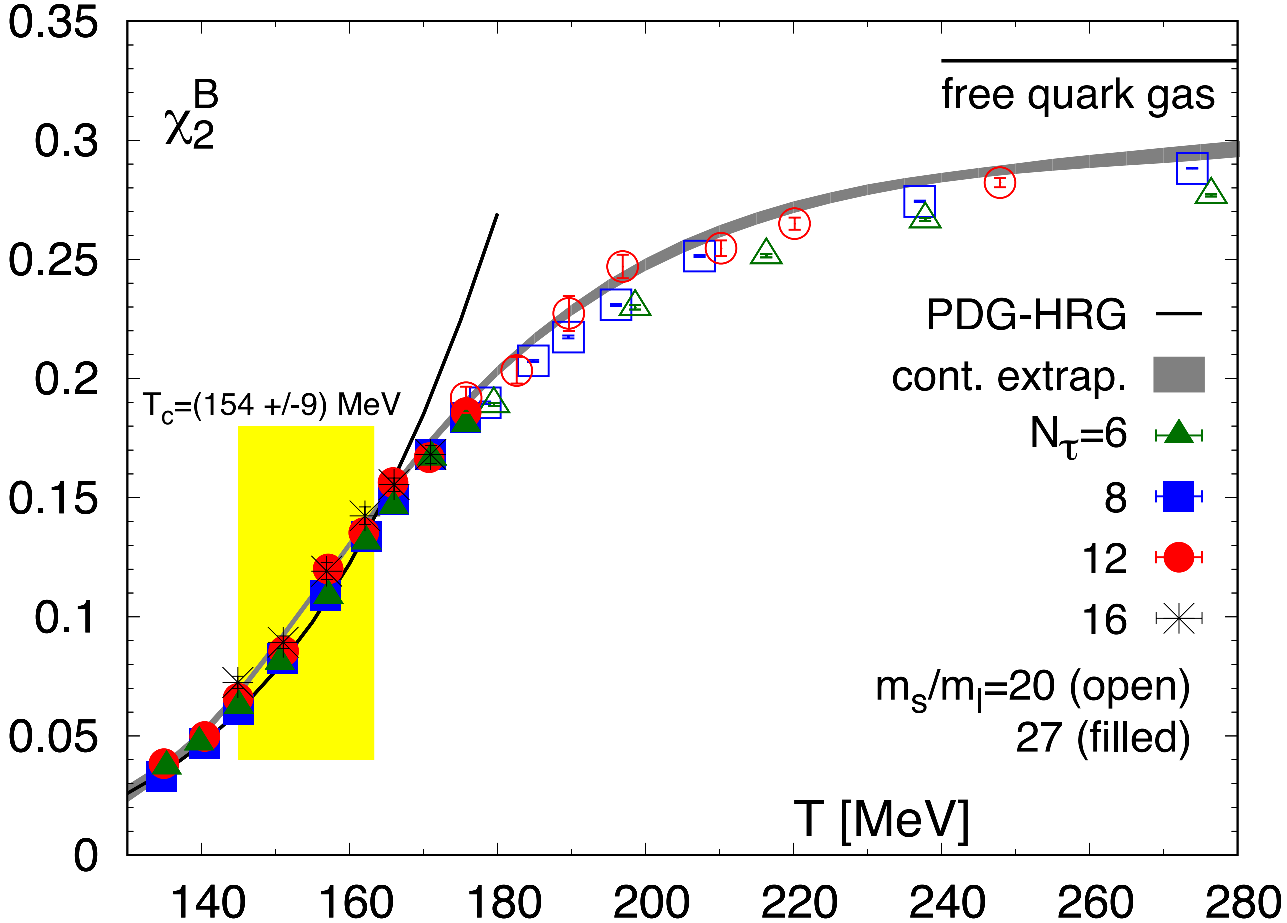
Radius of convergence from HRG is infinity

Pressure of QCD at μ_B =/=0

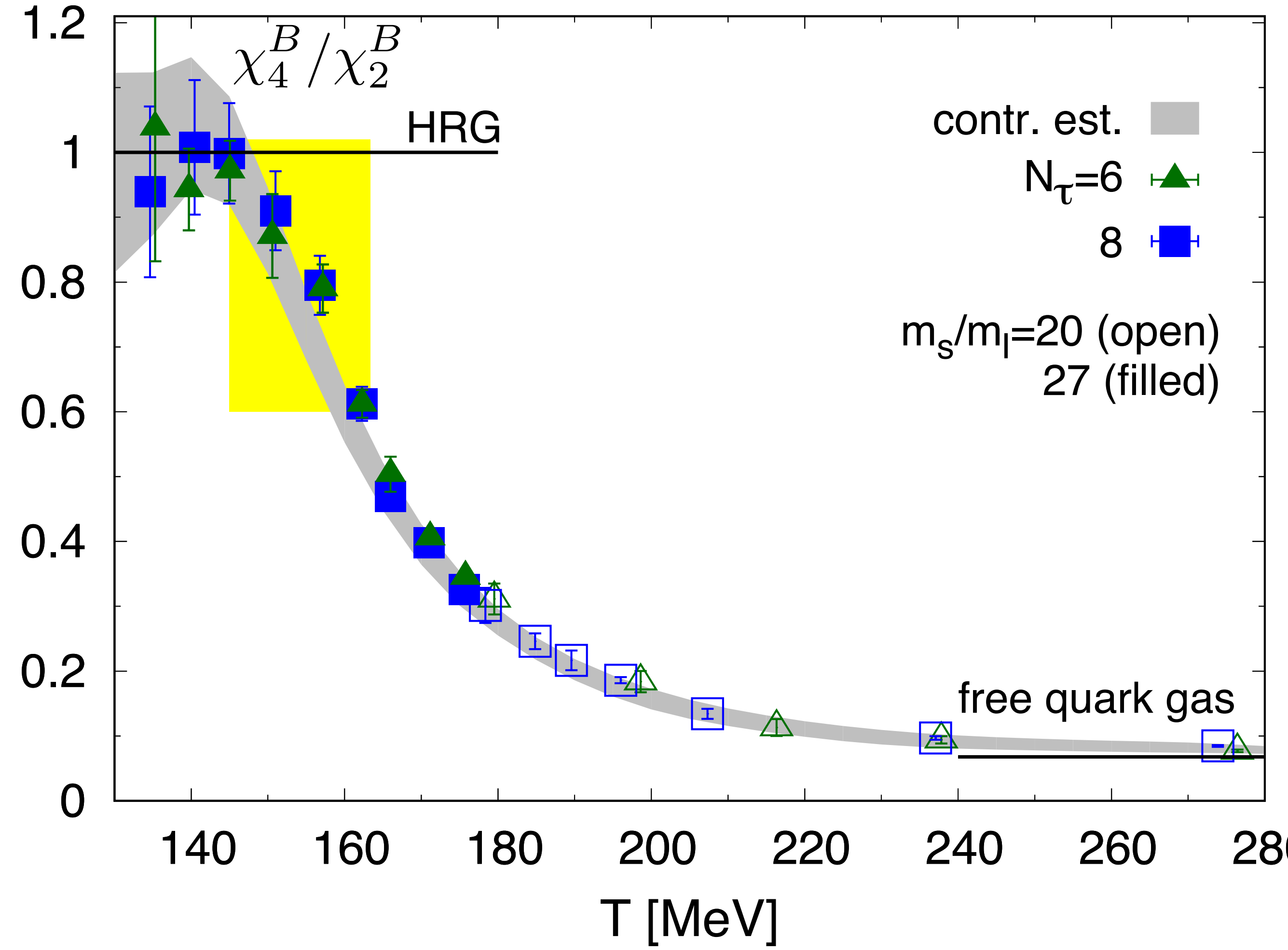
$\mu_Q=\mu_S=0$:

$$\Delta(P/T^4) = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\cdots\right)$$

LO expansion coefficient
variance of net-baryon number distri.



NLO expansion coefficient
kurtosis * variance



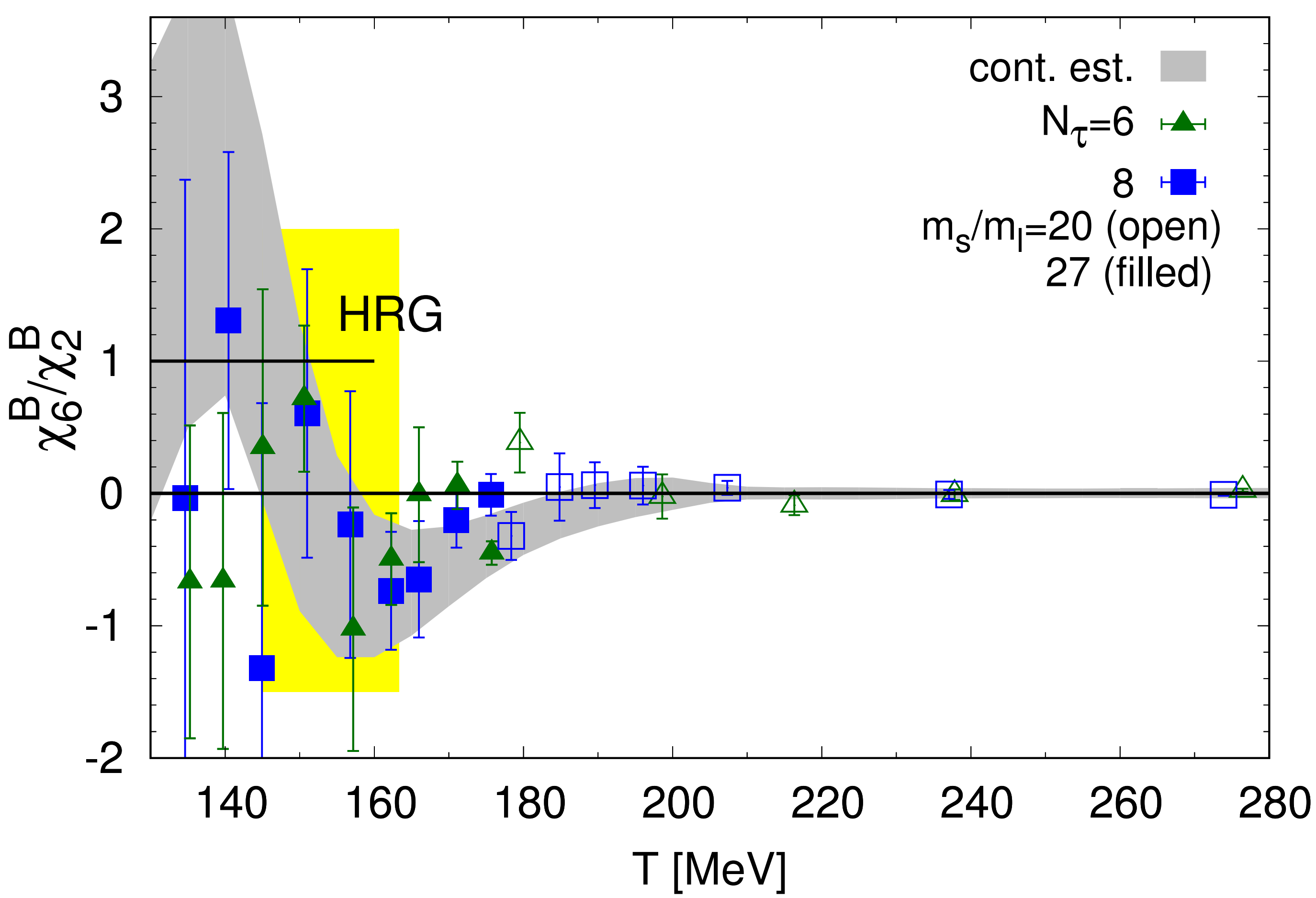
Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

Pressure of QCD at μ_B =/=0

$\mu_Q=\mu_S=0$:

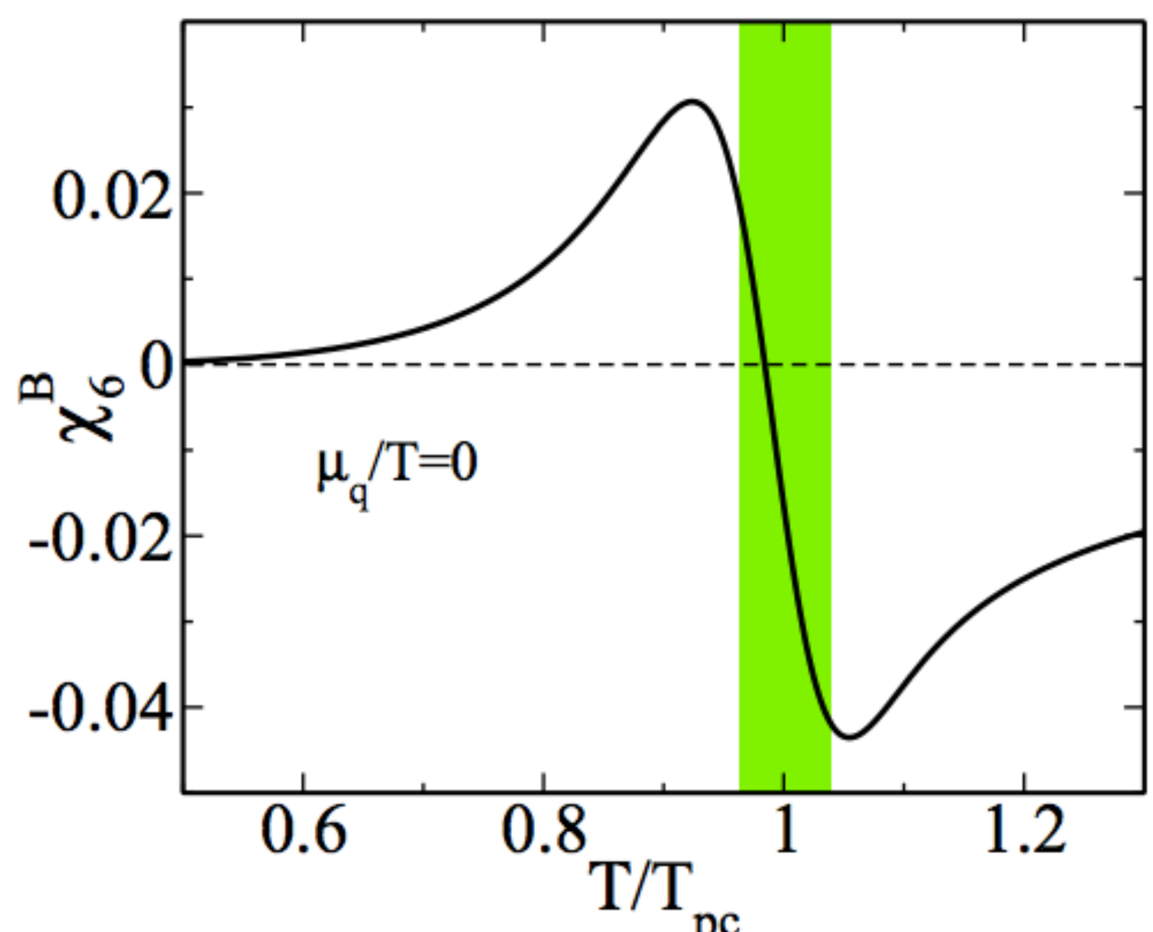
$$\Delta(P/T^4) = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\cdots\right)$$

NNLO expansion coefficient



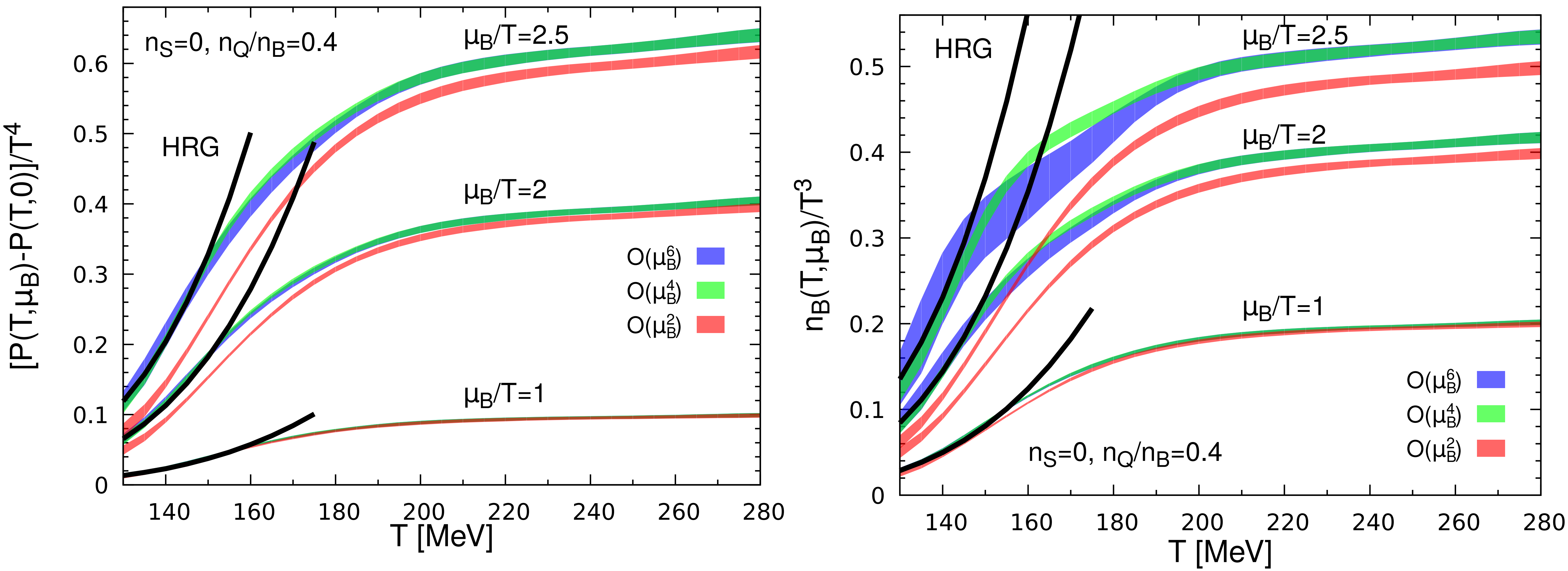
Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

PQM with O(4) symmetry



B. Friman et al.,EPJC71 (2011) 1694

Pressure and baryon number density in the strangeness neutral case



Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

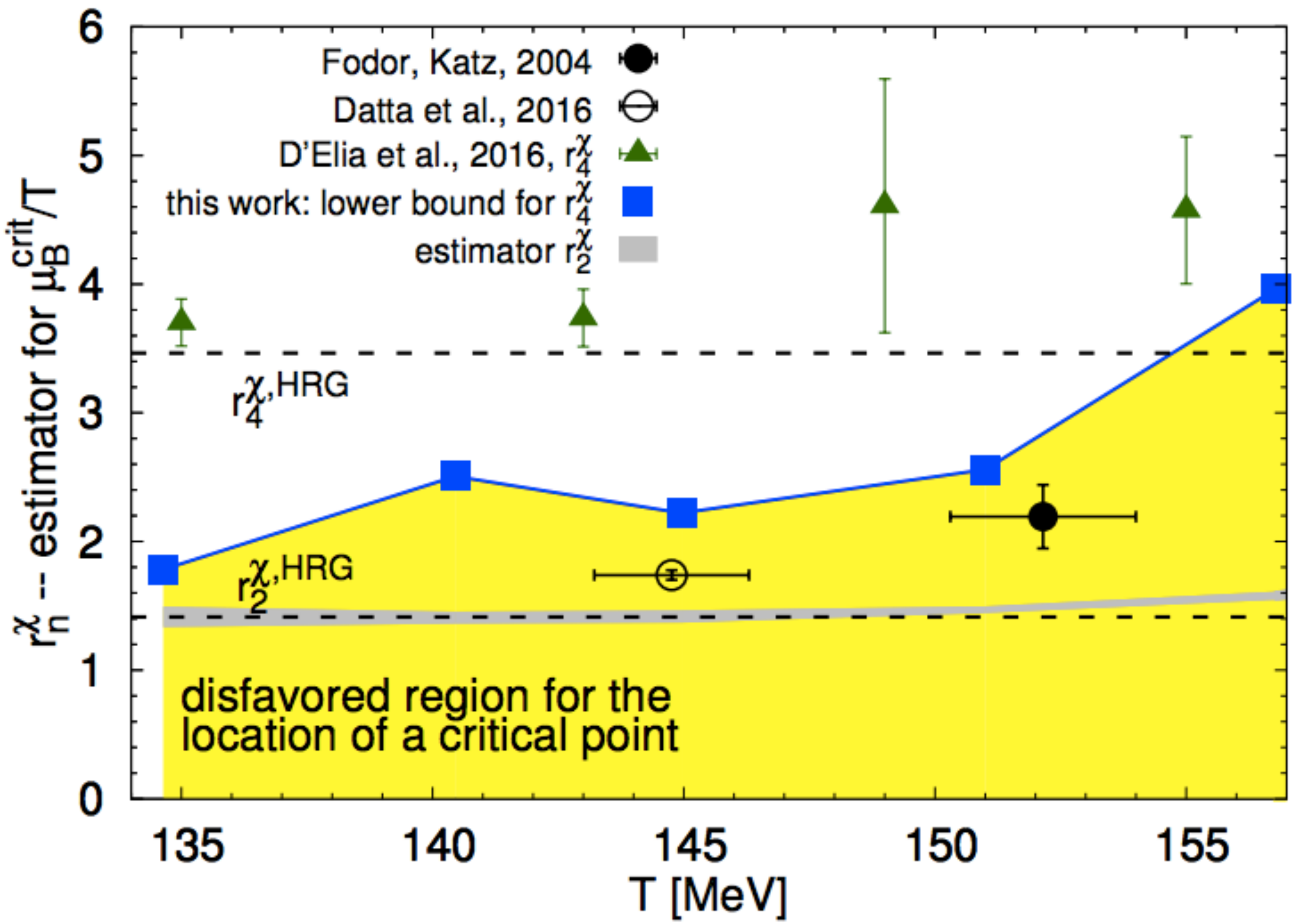
The EoS is well under control at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV

Consistent results obtained using analytic continuations from the imaginary mu

Wuppertal-Budapest-Houston: EPJ Web Conf. 137(2017) 07008

Estimates of the radius of convergence

radius of convergence = $\lim_{n\to\infty} r^{\chi}_{2n} = \lim_{n\to\infty} \left| \frac{2n(2n-1)\chi^B_{2n}}{\chi^B_{2n+2}} \right|^{1/2}$



HISQ + Taylor Exp. (this work):
Nf=2+1, Nt=8
Bielefeld-BNL-CCNU,
PRD 95 (2017) no.5, 054504

stout + Img. mu:
Nf=2+1,Nt=8
D'Elia et al., PRD 95 (2017) 094503

unimproved staggered + Taylor Exp.:
Nf=2, Nt=4,6,8
Datta et al., PRD 95 (2017) 054512

unimproved staggered + Reweighting:
Nf=2+1, Nt=4
Fodor and Katz, JHEP 0404 (2004) 050

A QCD critical point is disfavored at $\mu_B/T \lesssim 2$ at $T \gtrsim 135$ MeV

A. Bazavov, HTD et al., [Bielefeld-BNL-CCNU], Phys.Rev. D95 (2017) no.5, 054504

Explore the QCD phase diagram through fluctuations of conserved charges

Comparison of experimentally measured higher order cumulants of conserved charges to those from LQCD, e.g.:

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = R_{12}^Q(T,\mu_B)$$

$$\frac{S_Q(\sqrt{s})\,\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)} = R_{31}^Q(T,\mu_B)$$

HIC

mean: M_Q
variance: σ_Q^2
skewness: S_Q
kurtosis: κ_Q

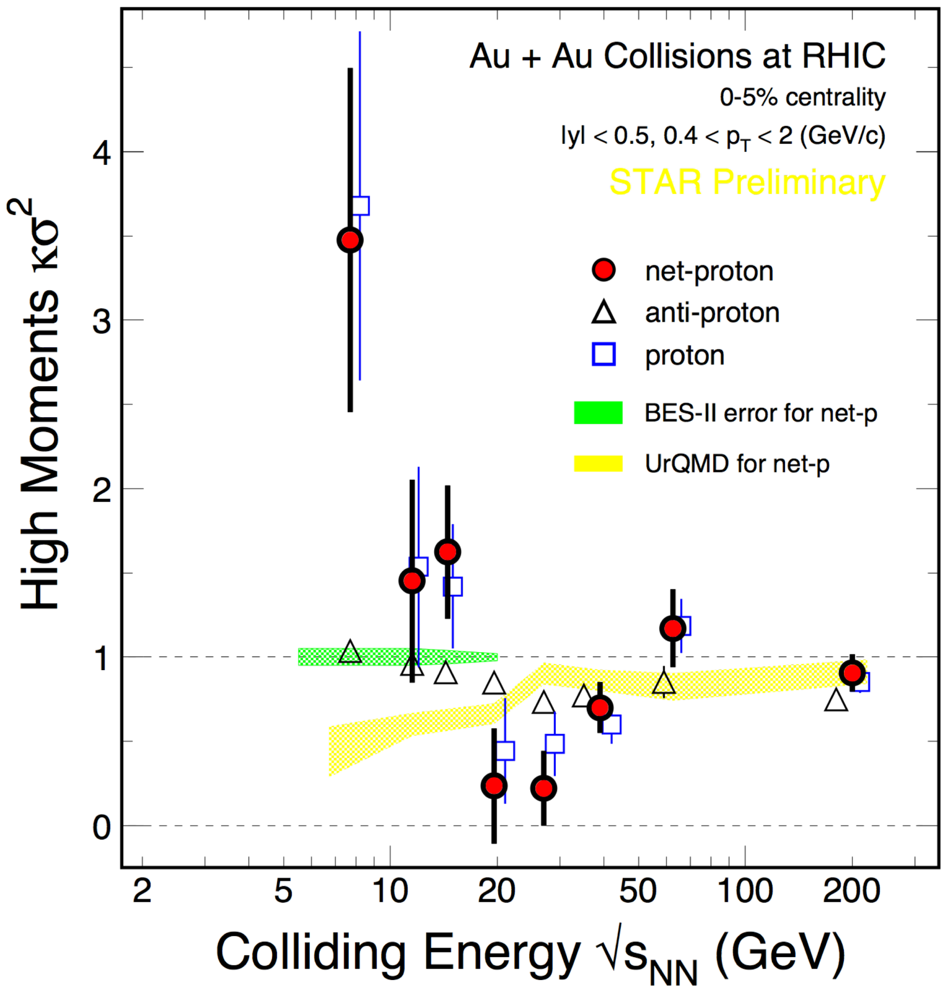
LQCD

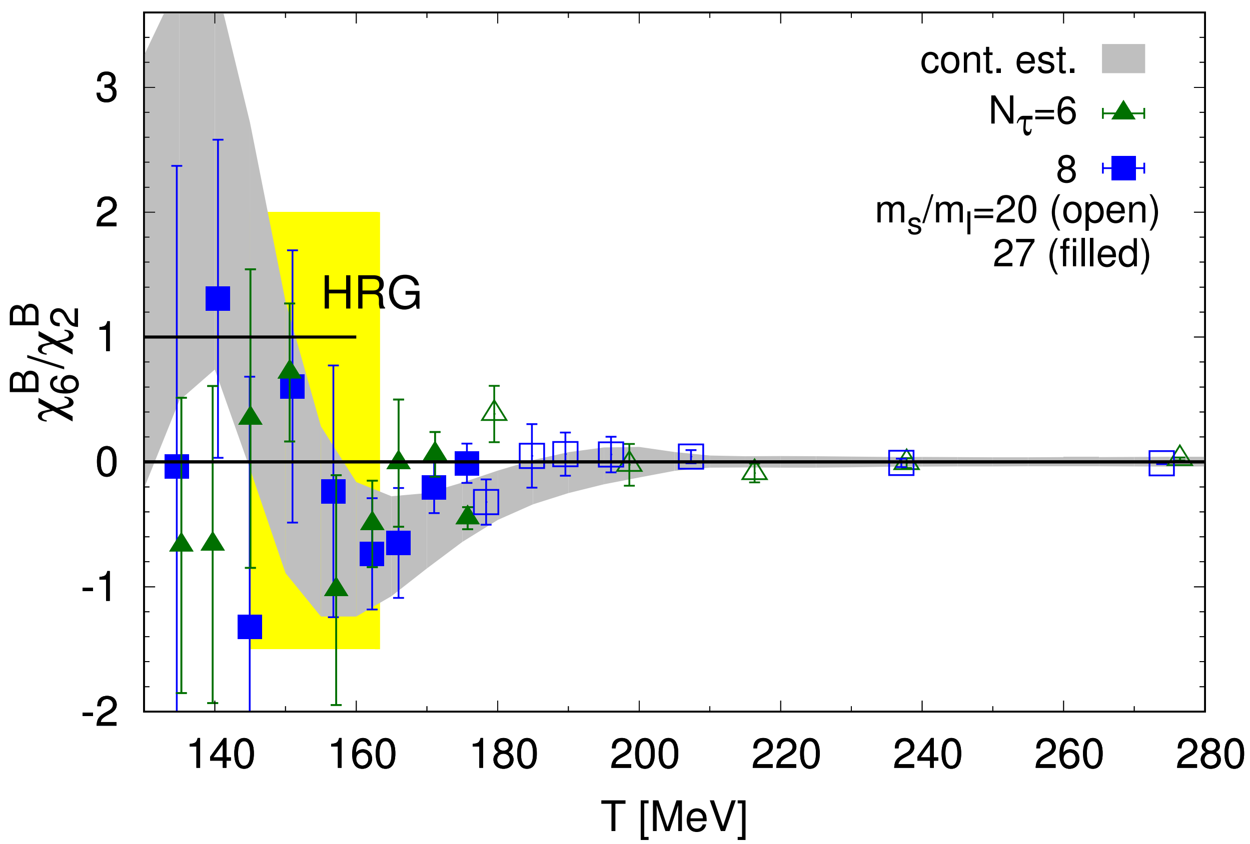
generalized susceptibilities

$$\chi_n^Q(T,\vec{\mu}) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_Q/T)^n}$$

BNL-Bielefeld, Phys. Rev. Lett. 109 (2012) 192302

Cumulant ratios of proton (baryon) number fluctuations: HIC data v.s. Lattice results





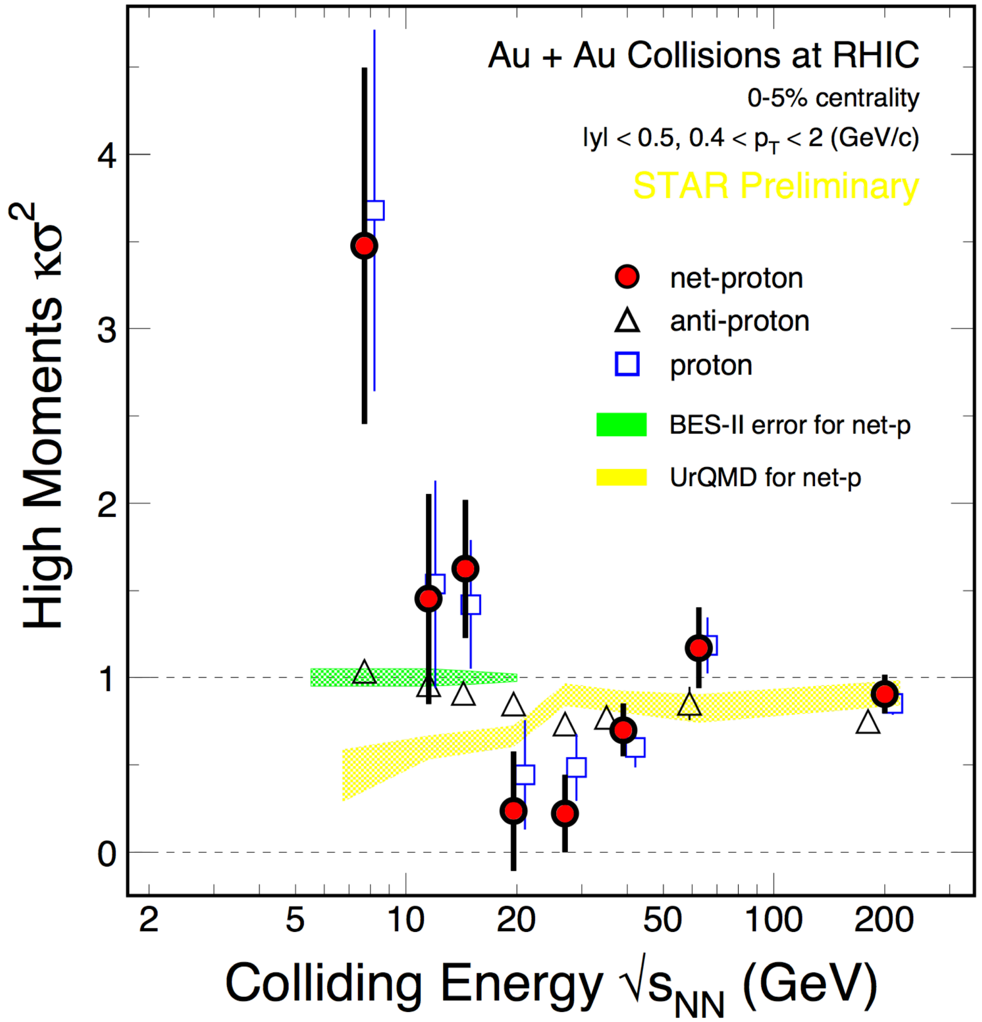
Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

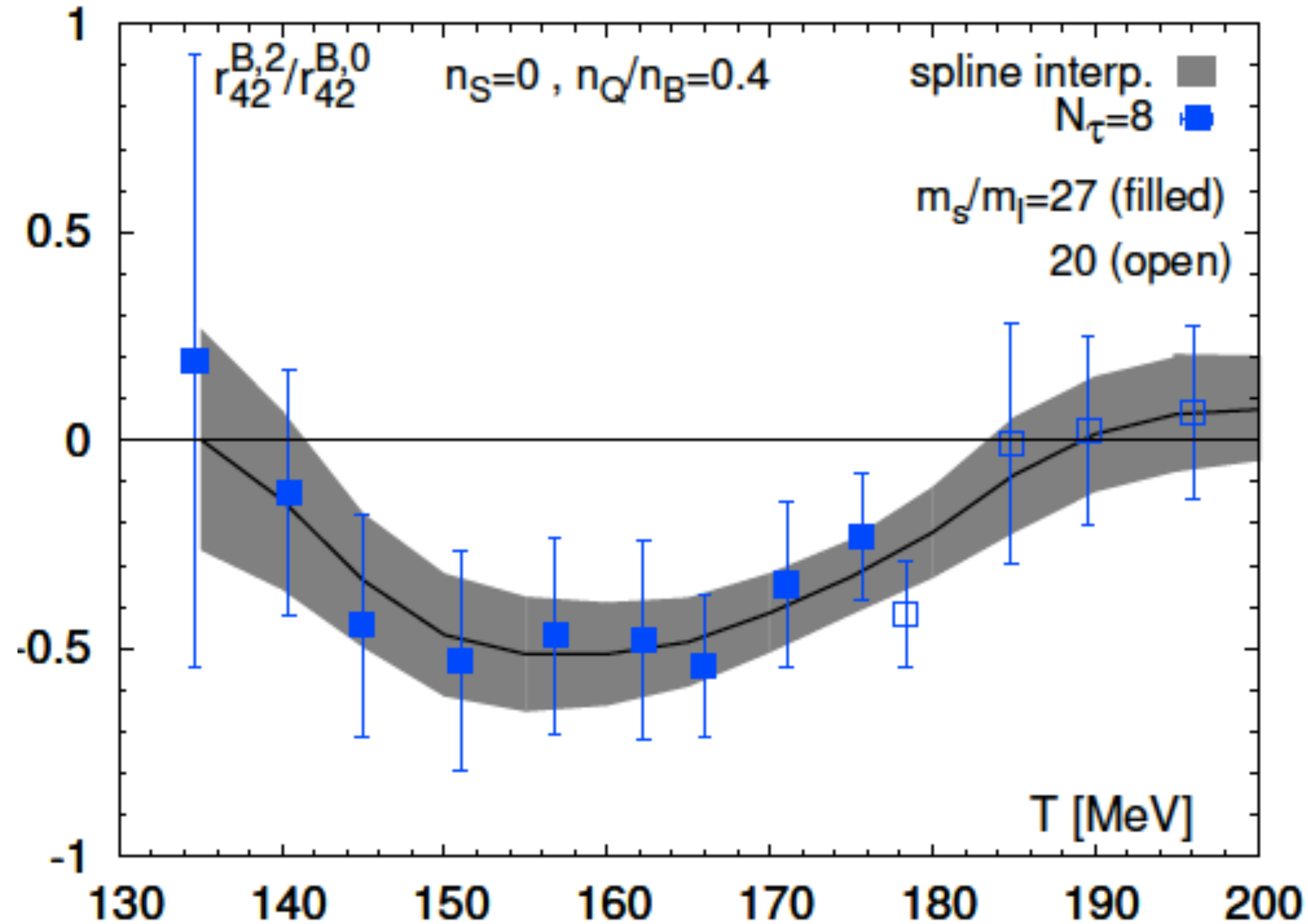
$\mu_Q = \mu_S = 0$:

$$(\kappa\sigma^2)_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}} = \frac{\chi^B_4}{\chi^B_2}\left[1 + \left(\frac{\chi^B_6}{\chi^B_4} - \frac{\chi^B_4}{\chi^B_2}\right)\left(\frac{\mu_B}{T}\right)^2 + \cdots\right]$$

HRG: $\chi^B_6/\chi^B_4 = \chi^B_4/\chi^B_2 = 1$, O(4) & LQCD: $\chi^B_6/\chi^B_2 < 0$ at $T \sim T_c$

Cumulant ratios of proton (baryon) number fluctuations: HIC data v.s. Lattice results





Bazavov et al., [HotQCD], Phys.Rev. D96 (2017) no.7, 074510

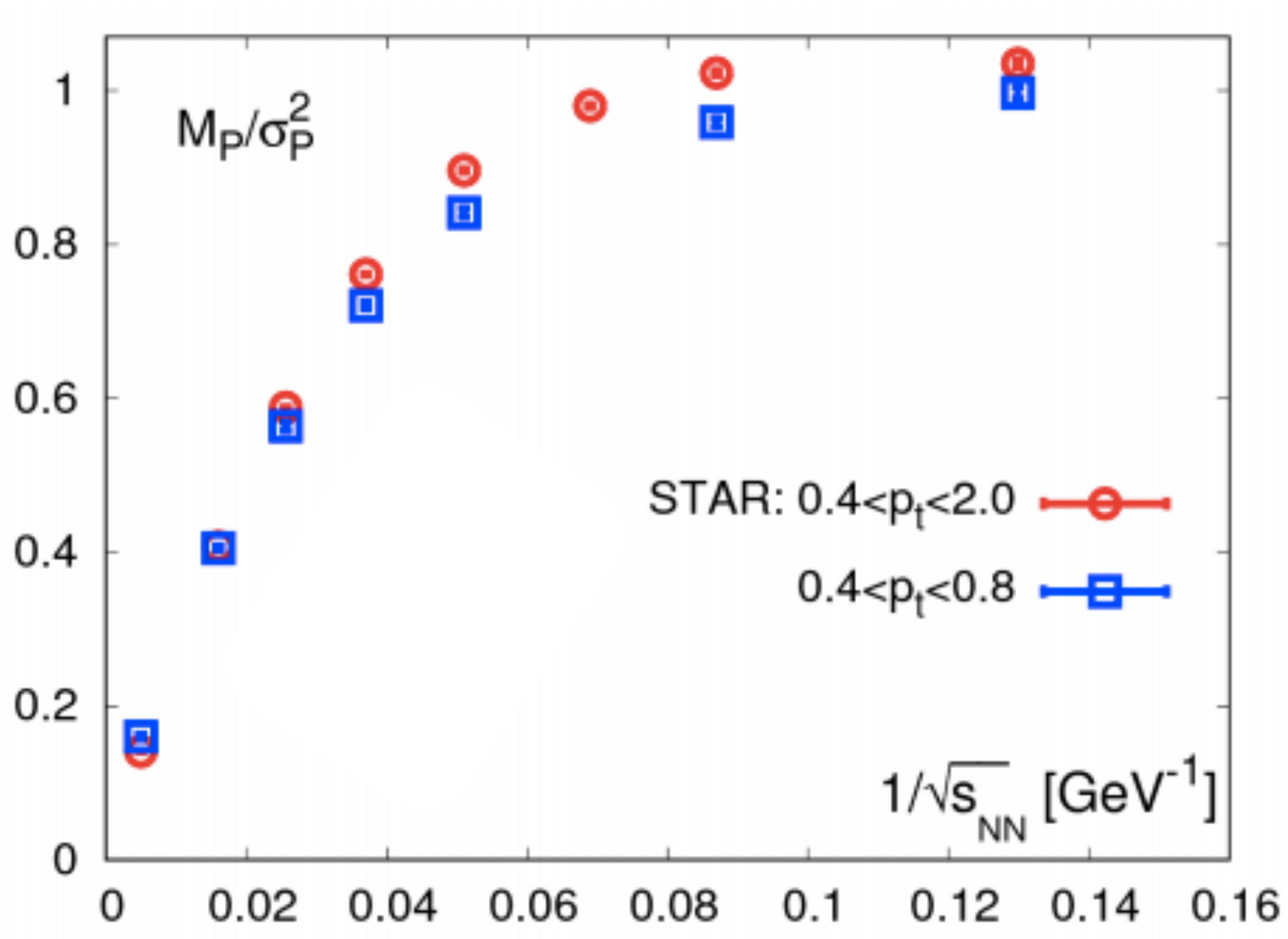
strangeness neutral case:

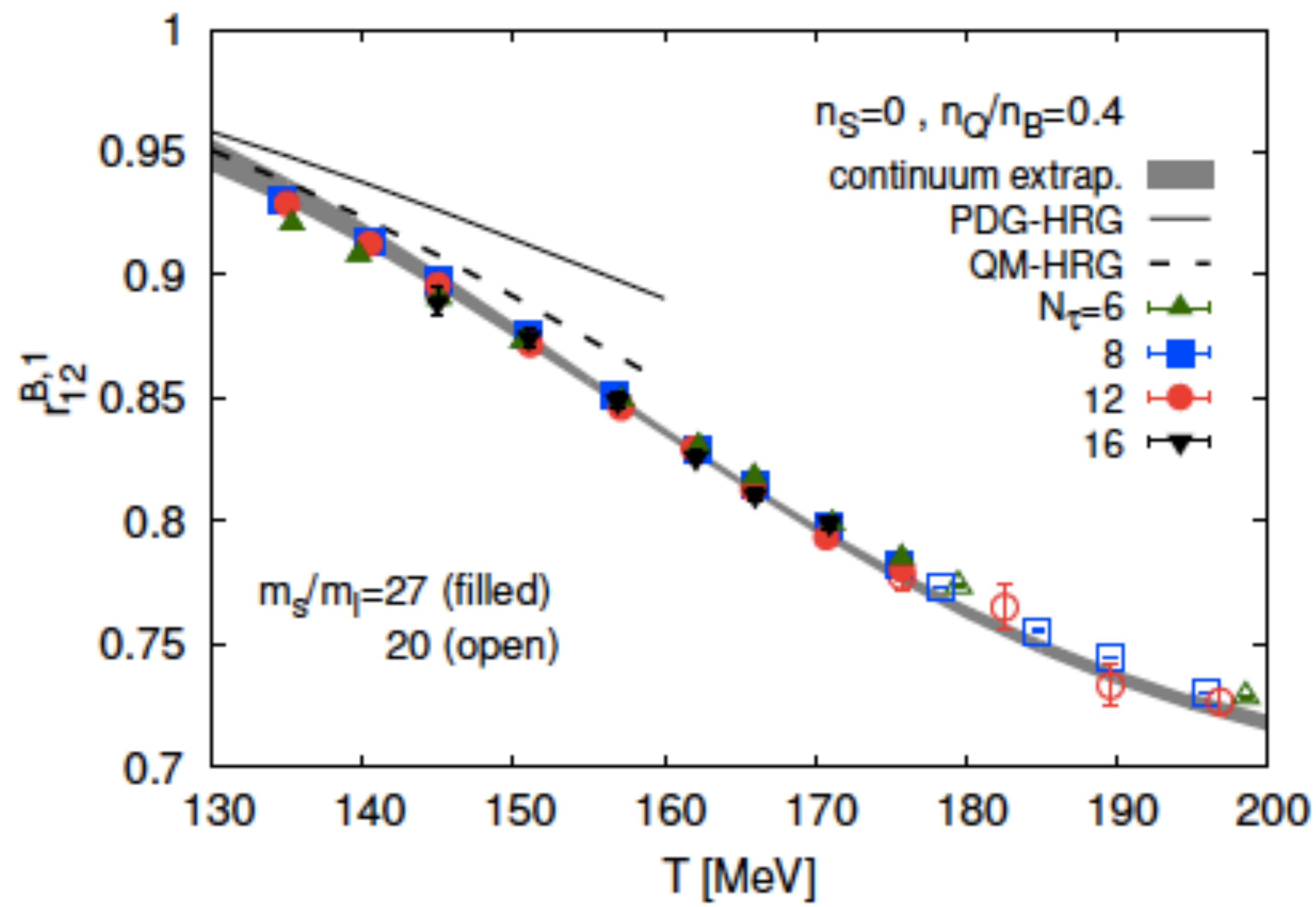
$$(\kappa\sigma^2)_B = \frac{\chi_{4,\mu}^{B,\mathrm{SN}}}{\chi_{2,\mu}^{B,\mathrm{SN}}} = r_{42}^{B,0}\left[1 + \frac{r_{42}^{B,2}}{r_{42}^{B,0}}\left(\frac{\mu_B}{T}\right)^2 + \cdots\right]$$

$\sqrt{s_{NN}} \gtrsim 20$ GeV:

$\kappa\sigma^2$ is consistent with QCD in equilibrium

Cumulant ratios of proton (baryon) number fluctuations: HIC data v.s. Lattice results





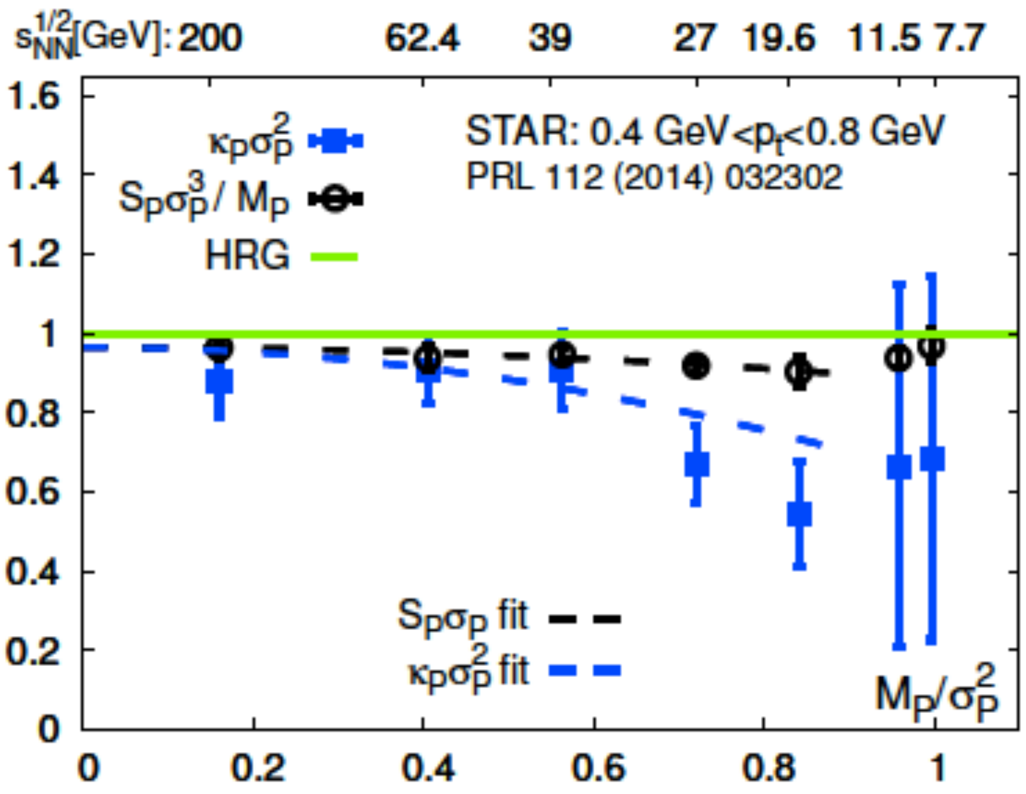
BNL-Bielefeld-CCNU, PRD 93 (2016)014512

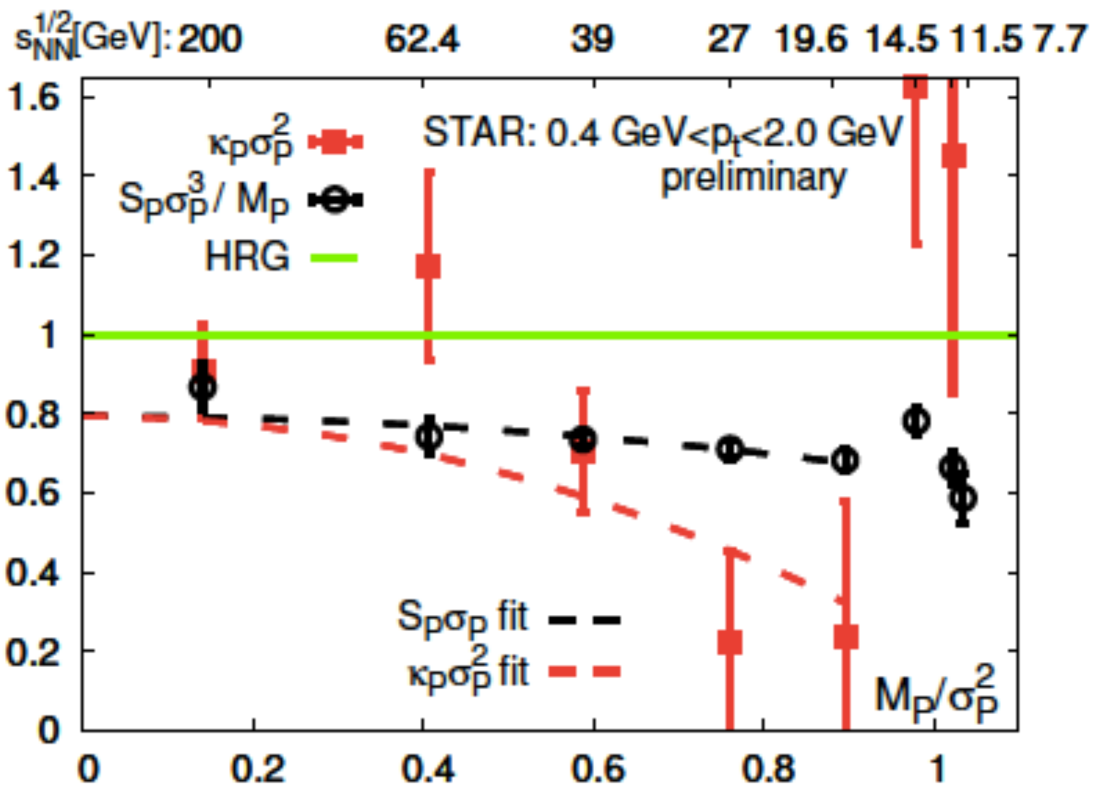
HIC: a function of M_p/σ_p^2 rather than $\sqrt{s_{NN}}$

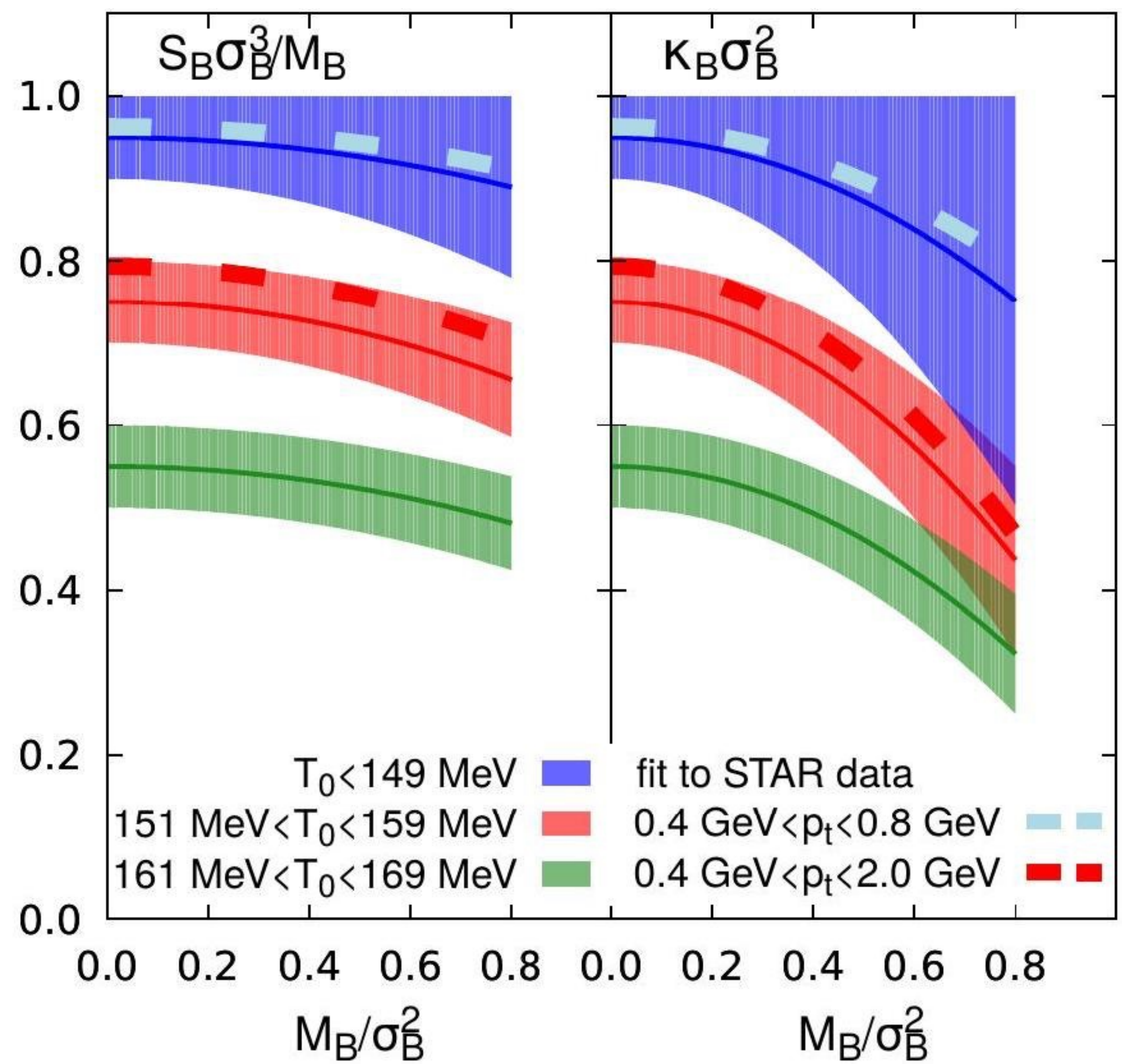
Lattice: $\mu_B \Leftrightarrow M_B/\sigma_B^2/R_{12}^{B,1}$, $R_{12}^B(T_f,\mu_B) \equiv \frac{M_B}{\sigma_B^2}(T_f,\mu_B) = \left.\frac{\partial R_{12}^B}{\partial \hat{\mu}_B}\right|_{\hat{\mu}_B=0} \hat{\mu}_B + \mathcal{O}(\hat{\mu}_B^3)$

$$\frac{\partial R_{12}^B}{\partial \hat{\mu}_B}\Big|_{\hat{\mu}_B=0} \equiv r_{12}^{B,1}$$

Cumulant ratios of proton (baryon) fluctuations: HIC data v.s. Lattice results







Bazavov et al., [HotQCD]
Phys.Rev. D96 (2017) no.7, 074510

Conclusions

- ☑ The 2nd O(4) chiral phase transition seems more relevant to the thermodynamics at the physical point at vanishing baryon density

- ☑ EoS from Taylor expansions of QCD partition functions are now reliable in the region $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV

- ☑ Properties of cumulants measured in BES-I for $\sqrt{s_{NN}} \gtrsim 20$ GeV clearly differs from HRG thermodynamics but are consistent to QCD thermodynamics close to the transition region

- ☑ A QCD critical point is disfavored at $\mu_B/T \lesssim 2$ at $T \gtrsim 135$ MeV

NSC³

Nuclear Science Computing Center at CCNU

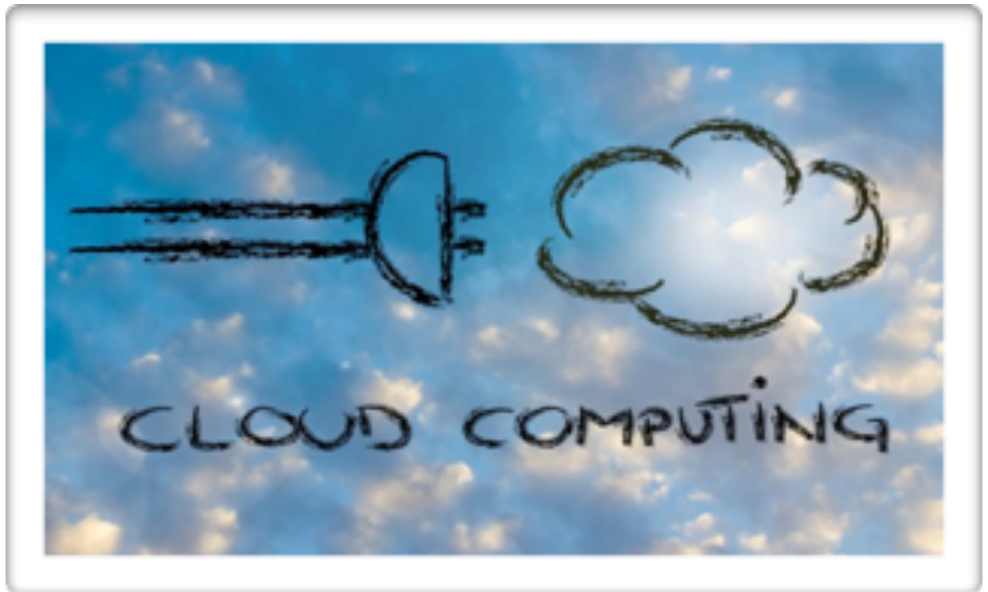
华中师范大学
核科学计算中心

计算决定未来!

- **N**: Nuclear   **National**
- **S**: Science
- **C³**: Color 3 -> QCD

“道生一，一生二，二生三，三生万物” —- 《道德经》老子 600 BC

High Performance, Low Power Consumption
~100% utilized



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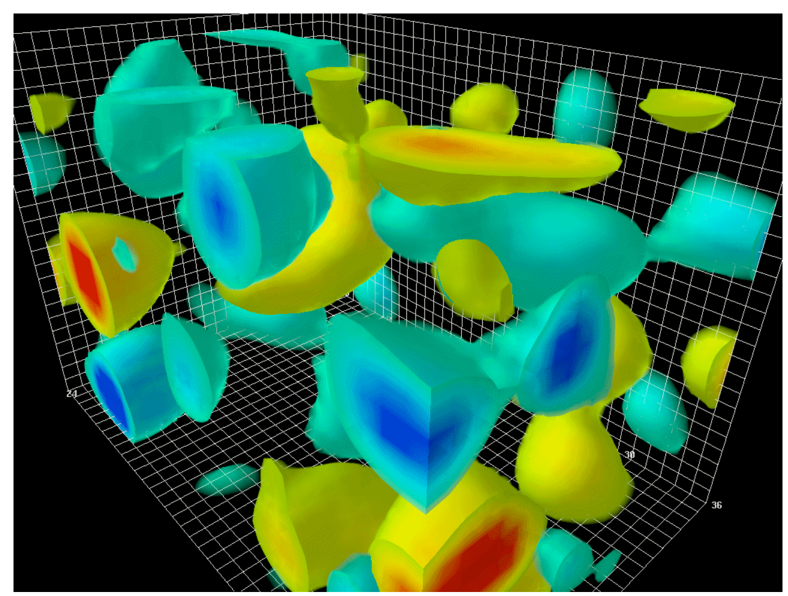
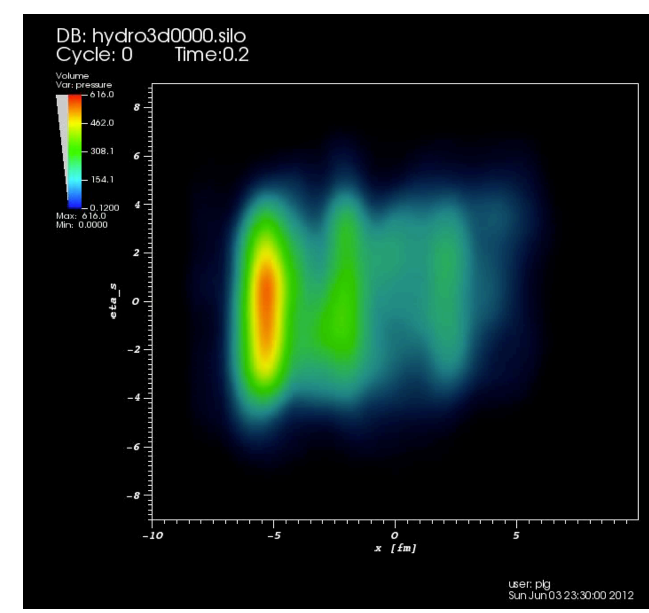
To be online in Sep. 2018
Theoretical Peak Performance: 1PFlops/s
(10^{15} floating operations per second)
Storage: 500 + 500 TB

- Lattice QCD (hep-lat)
- Data analyses & simulation in HIC (nucl-exp)
- Phenomenology (hep-ph)
- …





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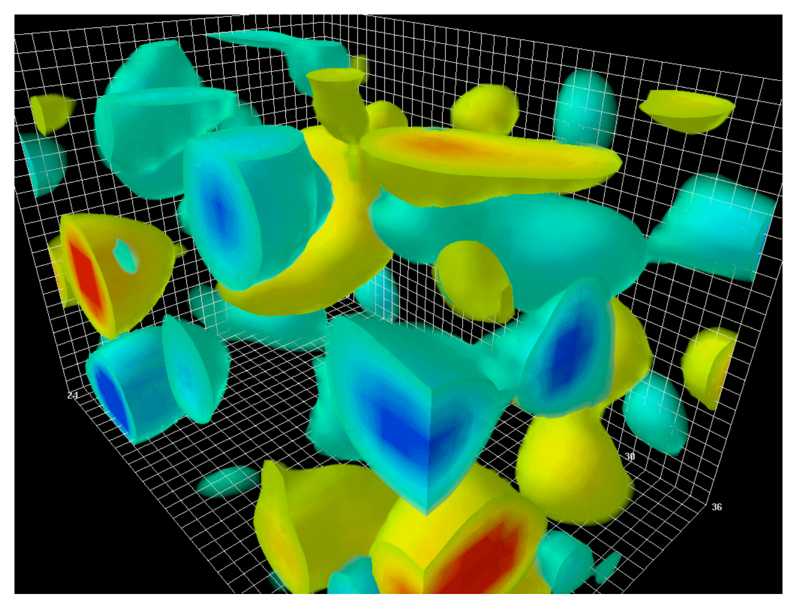
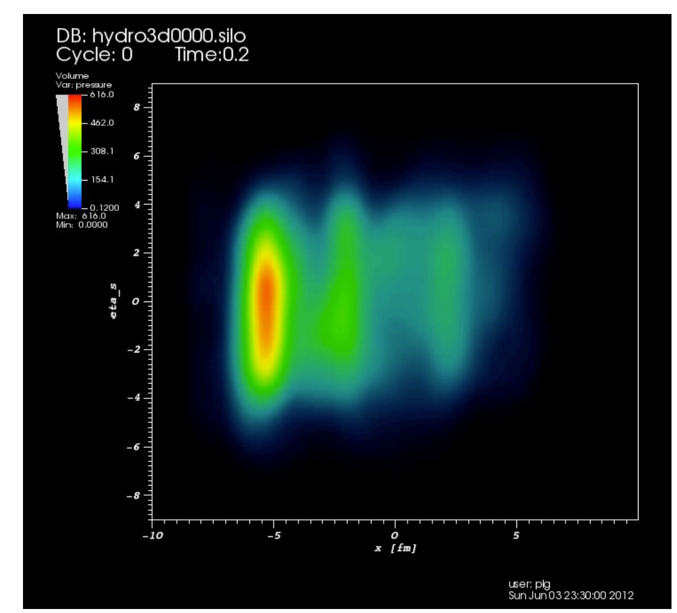
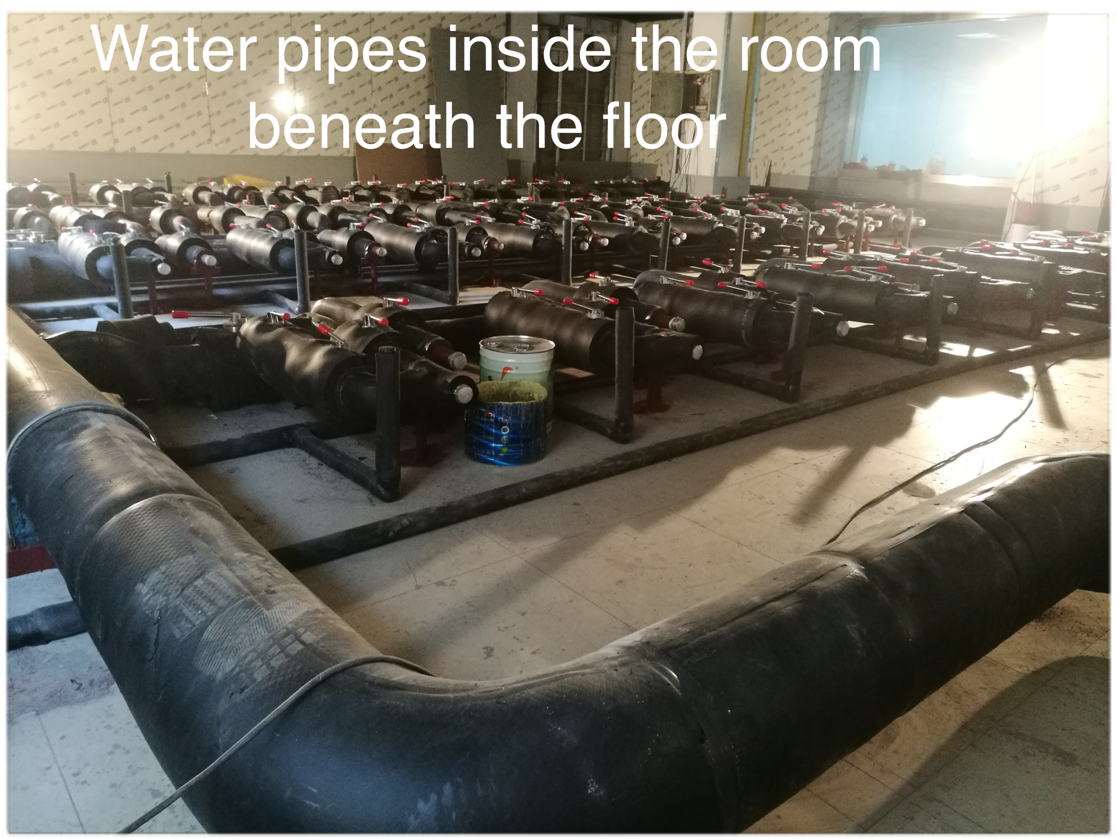





Racks


GPU nodes


8 V100-GPUs in one node


Water pipes inside the room
beneath the floor


Water pipes & chillers
outside the room

谢谢！

Thanks for your attention!

Outlook: Mapping out the QCD phase diagram

2019-2020: RHIC Beam Energy Scan, Phase II (BES-II)

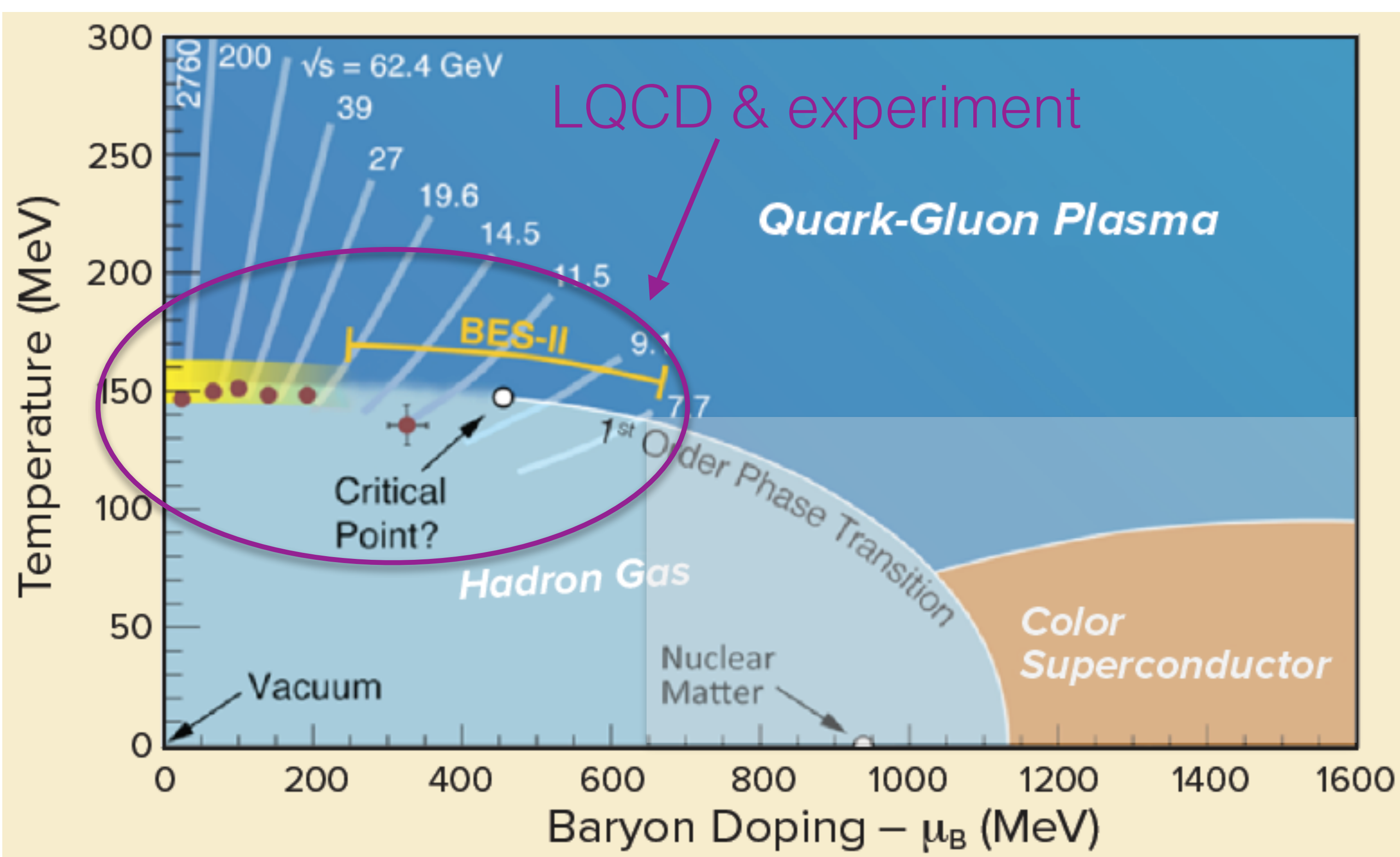
at least 10 times more statistics for each small $\sqrt{s_{NN}}$

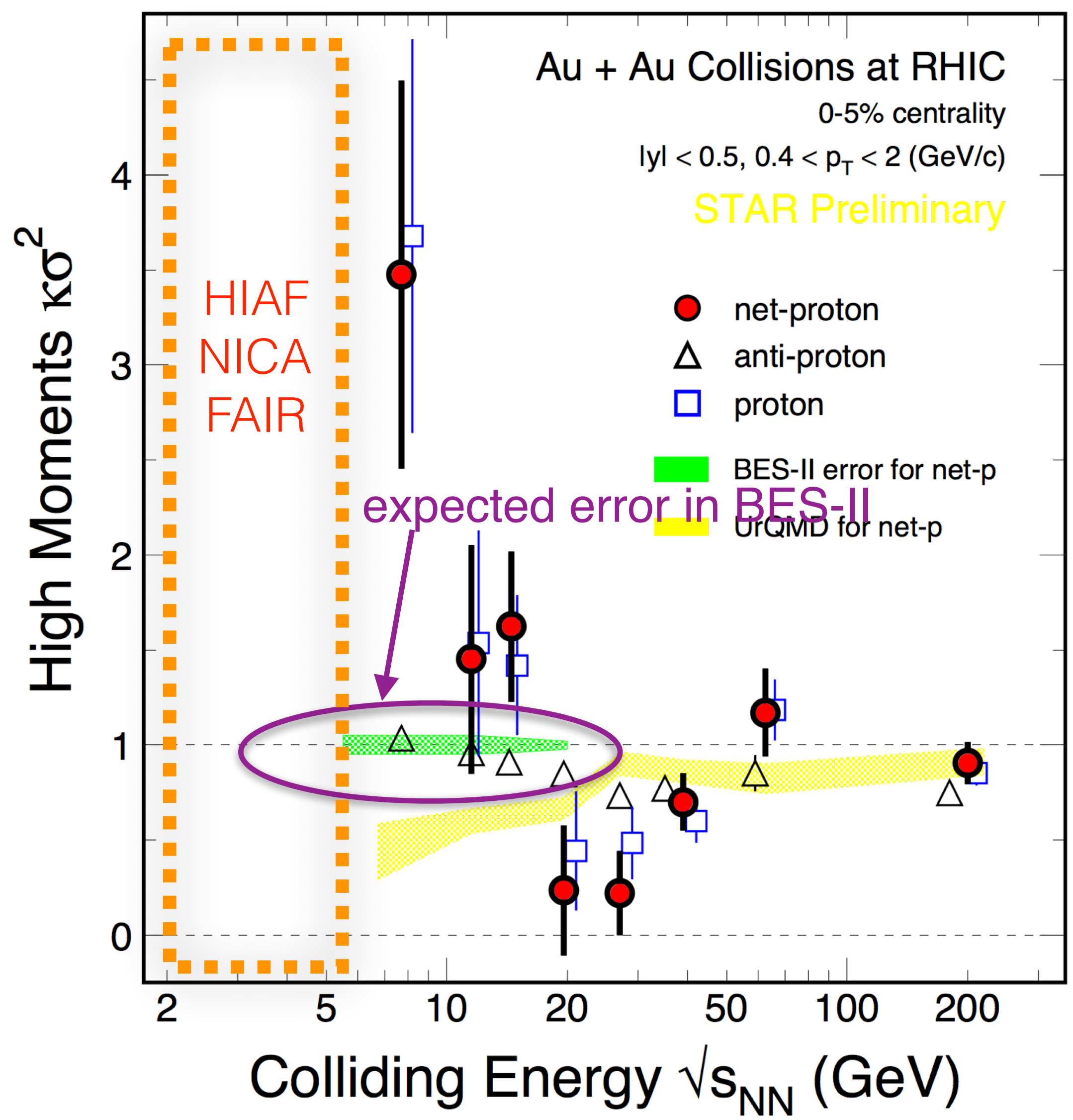
≳2021: NICA, ≳2024: FAIR, HIAF

LQCD:

higher accuracy for the 6th & 8th or even higher order Taylor exp. coeff.

Imaginary chemical potential approach …





Beam Energy Scan at RHIC

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	BES II / BES I	Weeks	μ_B (MeV)	T_{CH} (MeV)
200	350	2010		25	166
62.4	67	2010		73	165
39	39	2010		112	164
27	70	2011		156	162
19.6	**400** / 36	**2019-20** / 2011	**3**	206	160
14.5	**300** / 20	**2019-20** / 2014	**2.5**	264	156
11.5	**230** / 12	**2019-20** / 2010	**5**	315	152
9.2	**160** / 0.3	**2019-20** / 2008	**9.5**	355	140
7.7	**100** / 4	**2019-20** / 2010	**14**	420	140

Courtesy of N. Xu

Theoretically it is crucial to know:

QCD Equation of State for $\mu_B/T \lesssim 3$

Location of the transition and freeze out lines

The possible location of CP or window of criticality

Conditions meet in heavy ion collisions

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$\mu_Q=\mu_S=0$:
$$\frac{p}{T^4} = \sum_{n=0} \frac{\chi_{2n}^{B}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

strangeness neutral case:
$$\frac{p}{T^4} = \sum_{n=0} \frac{\chi_{2n,\mathrm{SN}}^{B}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

- Expand μ_Q and μ_S in terms of μ_B

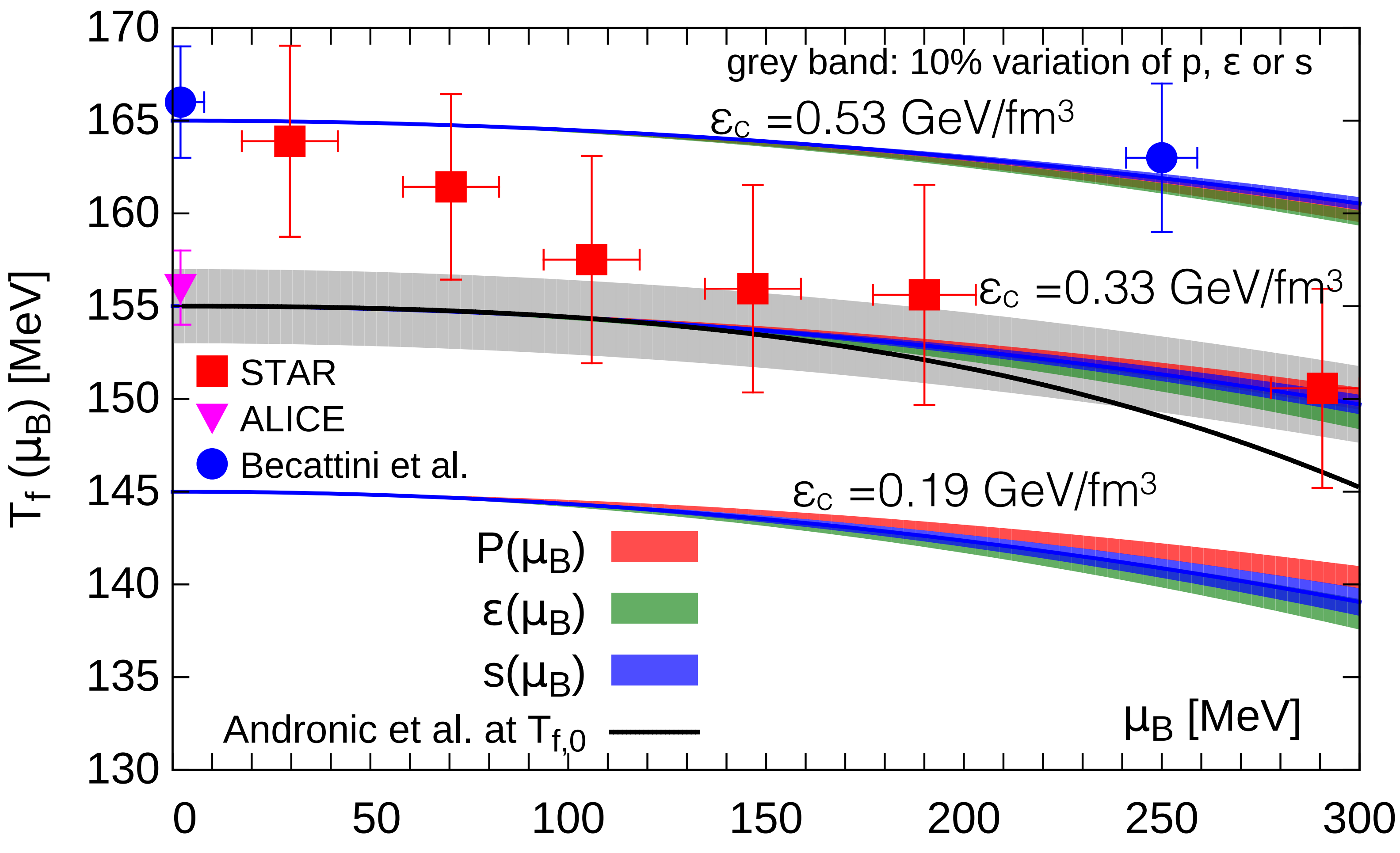
$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots \, , \quad \frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots$$

- With constrains from isospin symmetry etc., one can derive q_i and s_i order by order and then the pressure etc.

A. Bazavov, HTD et al., Phys. Rev. Lett. 109 (2012)192302

Line of constant physics to O($\hat{\mu}_B^4$) and freeze-out

Parameterization: $T(\mu_B) = T(0)(1 - \kappa_2 \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4))$



Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

curvature at constant b:

$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

curvature of transition line:

$$\kappa_2^t \approx 0.006 - 0.013$$

Cea et al., PRD 93 (2016) no. 1, 014507
Bellwied et al., PLB 751 (2015) 559
Bonati, PRD 92 (2015) no. 5, 054503
Kaczmarek et al., PRD 83 (2011) 014504,
Endrodi et al,, JHEP 1104 (2011) 001

curvature of freeze-out line:

$$\kappa_2^f \lesssim 0.011$$

Bielefeld-BNL-CCNU, PRD93 (2016) no.1, 014512