NME for neutrinoless double beta decay

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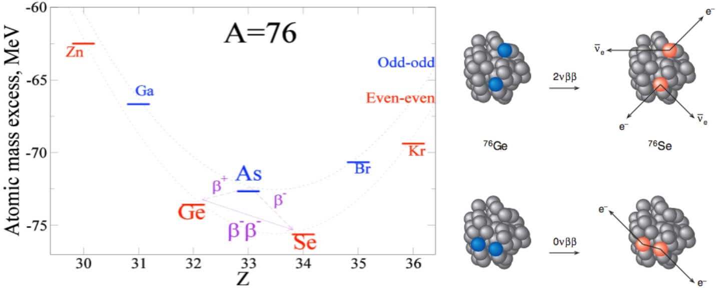
Content

Background

- Strong nuclear pairing in nuclei for neutron-neutron and proton-proton



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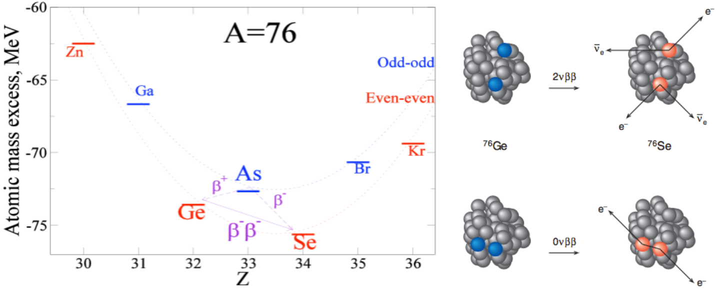
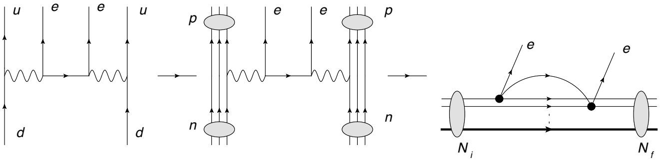


- Double beta ($\beta\beta$) decay is originating from the mass staggering
- Neutrinoless $\beta\beta$-decay is possible if $\nu = \bar{\nu}$ and $m_\nu \neq 0$

Background

- The underlying mechanism with L-R symmetry
  - left-handed and right-handed neutrino mixing
  - $SU(2)_L$ and $SU(2)_R$ gauge boson mixing



Trivial realization of neutrinoless double beta decay with L-R symmetry

M. Doi *et. al.* Prog. Theo. Phys. Suppl. 83,1(1985)

- The general decay width of this decay can be written as following the S-matrix theory:

$$d\Gamma = 2\pi \sum_{spin} |R|^2 \delta(\epsilon_1 + \epsilon_2 + E_f - M_i) \frac{d\vec{\mathbf{p}}_1}{(2\pi)^3} \frac{d\vec{\mathbf{p}}_2}{(2\pi)^3} \quad (1)$$

- Here the R-matrix can be written as follows for a general L-R symmetry model

$$\begin{aligned} R &= \frac{1}{2!}(\frac{G}{\sqrt{2}})^2 2 \sum_j \int d\vec{\mathbf{x}} \int d\vec{\mathbf{y}} \int \frac{d\vec{\mathbf{q}}}{2\omega(2\pi)^3} e^{i\vec{\mathbf{q}}\cdot(\vec{\mathbf{x}}-\vec{\mathbf{y}})} \\ &\times \sum_a [m_j (J^{\rho\sigma}_{LL} S_{L\rho\sigma} + J^{\rho\sigma}_{RR} S_{R\rho\sigma}) + (J^{\rho\sigma}_{LR} V_{L\rho\sigma} + J^{\rho\sigma}_{RL} V_{R\rho\sigma})] \quad (2) \end{aligned}$$

- The nuclear current has the form

$$J_{LL}^{\rho\sigma} = \langle f|J_{WL}^{\rho}|a\rangle\langle a|J_{WL}^{\sigma}|i\rangle \tag{3}$$

- while the lepton current has the form

$$\begin{aligned}
S_{L\rho\sigma}(\vec{\mathbf{x}},\vec{\mathbf{y}};a) &= \frac{\bar{\psi}(\epsilon_1,\vec{\mathbf{x}})\gamma_\rho(1-\gamma_5)\gamma_\sigma\psi^C(\epsilon_2,\vec{\mathbf{y}})}{\omega+E_a+(\epsilon_2-\epsilon_1)/2} \\
&- \frac{\bar{\psi}(\epsilon_2,\vec{\mathbf{x}})\gamma_\rho(1-\gamma_5)\gamma_\sigma\psi^C(\epsilon_1,\vec{\mathbf{y}})}{\omega+E_a+(\epsilon_1-\epsilon_2)/2} \\
V_{L\rho\sigma}(\vec{\mathbf{x}},\vec{\mathbf{y}};a) &= \frac{q^\mu\bar{\psi}(\epsilon_1,\vec{\mathbf{x}})\gamma_\rho(1-\gamma_5)\gamma_\mu\gamma_\sigma\psi^C(\epsilon_2,\vec{\mathbf{y}})}{\omega+E_a+(\epsilon_2-\epsilon_1)/2} \\
&- \frac{q^\mu\bar{\psi}(\epsilon_2,\vec{\mathbf{x}})\gamma_\rho(1-\gamma_5)\gamma_\mu\gamma_\sigma\psi^C(\epsilon_1,\vec{\mathbf{y}})}{\omega+E_a+(\epsilon_1-\epsilon_2)/2}
\end{aligned}$$

- $\psi(\epsilon,\vec{\mathbf{x}})$ is Coulomb distorted electron wave function

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- For derivation of the final decay width, several assumptions are used:
- **long wavelength approximation**: s-wave electrons dominance,$\cdots$
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- With these approximations we could separate the nuclear part (Nuclear Matrix Elements) and the lepton part (Phase Space Factor)

$$\Gamma^{0\nu}(0^+ \rightarrow 0^+) = G^{01}(\langle m_\nu \rangle M_l + \langle \eta_N \rangle M_h)^2 + \langle \lambda \rangle^2 (G^{02} M^2_{\omega^-} + G^{04} M^2_{q^-}/9 - 2G^{03} M_{\omega^-} M_{q^-}) + \cdots \quad (4)$$

$\langle m_\nu \rangle = |\sum_j (U_{ej})^2 m_j|$, $\langle \eta_N \rangle = \sum_J \frac{(U_{eJ})^2 m_p}{M_J}$, $\langle \lambda \rangle = \tan\xi \sum_j U_{ej} V_{ej} (g'_V / g_V)$ are new physics parameters

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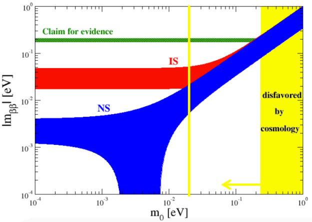

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$$\Gamma^{0\nu} = G^{01}(\langle m_\nu \rangle M_l + \langle \eta_N \rangle M_h)^2 \tag{5}$$

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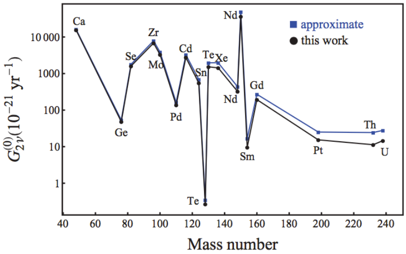
New calculations of phase space factors using numerical electron wave functions (Kotila *et al.* PRC85,034316(2012))



Nucleus	$G^{(0)}_{0\nu}$ (10^{-15} yr^{-1})	$G^{(1)}_{0\nu}$ (10^{-15} yr^{-1})	$Q_{\beta\beta}$ (MeV)
^{48}Ca	24.81	−23.09	4.27226(404)
^{76}Ge	2.363	−1.954	2.03904(16)
^{82}Se	10.16	−9.074	2.99512(201)
^{96}Zr	20.58	−18.67	3.35037(289)
^{100}Mo	15.92	−14.25	3.03440(17)
^{110}Pd	4.815	−4.017	2.01785(64)
^{116}Cd	16.70	−14.83	2.81350(13)
^{124}Sn	9.040	−7.765	2.28697(153)
^{128}Te	0.5878	−0.3910	0.86587(131)
^{130}Te	14.22	−12.45	2.52697(23)
^{136}Xe	14.58	−12.73	2.45783(37)
^{148}Nd	10.10	−8.506	1.92875(192)
^{150}Nd	63.03	−57.76	3.37138(20)
^{154}Sm	3.015	−2.295	1.21503(125)
^{160}Gd	9.559	−7.932	1.72969(126)
^{198}Pt	7.556	−5.868	1.04717(311)
^{232}Th	13.93	−10.95	0.84215(246)
^{238}U	33.61	−28.13	1.14498(125)

Formalism

Calculation of the nuclear part (NME) depends on the nuclear structure theory. Modern nuclear structure calculations face two obstacles:

- **many-body methods**
  - exact Configuration Interaction approaches
  - approximate approaches with Configuration truncations: QRPA, DFT, IBM, $\cdots$

Formalism

Calculation of the nuclear part (NME) depends on the nuclear structure theory. Modern nuclear structure calculations face two obstacles:

- **many-body methods**
  - exact Configuration Interaction approaches
  - approximate approaches with Configuration truncations: QRPA, DFT, IBM, $\cdots$
- **nuclear force**
  - *ab initio*:
    - phenomenological realistic forces
    - Chiral forces
  - Effective interactions:
    - Skyrme, Gogny, Relativistic mean fields,$\cdots$

Formalism

We use the QRPA methods based on G-matrix with CD-Bonn potential

- WS meanfield + pairing from G-matrix
- G-matrix is also used for residual interactions
- deformation of nuclei is taken into consideration

Pros:

- QRPA is capable of dealing with intermediates states
- Closure approximation is used for other approaches

Cons:

- only one phonon excitations are considered
- meanfield interactions and residual interactions are of different types

Details of the calculations, the induced weak current:

$$J^{\mu}_{WL} = \bar{\Psi}\tau^{+}[g_V(q^2)\gamma^{\mu} - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_N} - g_A(q^2)\gamma^{\mu}\gamma_5 + g_P(q^2)q^{\mu}\gamma_5]\Psi \quad (6)$$

Leads to:

$$M_{l(h)} = \langle H_{F,l(h)}(r) + H_{GT,l(h)}(r)\sigma_1 \cdot \sigma_2 + H_{T,l(h)}(r)(\sigma_1 \otimes \sigma_2)^2 : (\vec{\mathbf{r}} \otimes \vec{\mathbf{r}})^2\rangle$$

here the most important part is the "neutrino potential":

$$\begin{aligned} H_{i,l}(r) &= \frac{2}{\pi g_A^2}\frac{R}{r}\int_0^{\infty}\frac{\sin(qr)}{q+E_a}h_i(q^2)dq \\ H^{i}_{i,h}(r) &= \frac{2}{m_p \pi g_A^2}\frac{R}{r}\int_0^{\infty}\sin(qr)h_i(q^2)qdq \end{aligned}$$

Short-range correlation functions are usually multiplied

$$f(r) = c(1 - be^{-ar^2}) \quad (7)$$

DLF *et al.* Phys. Rev. C97,045503(2018)

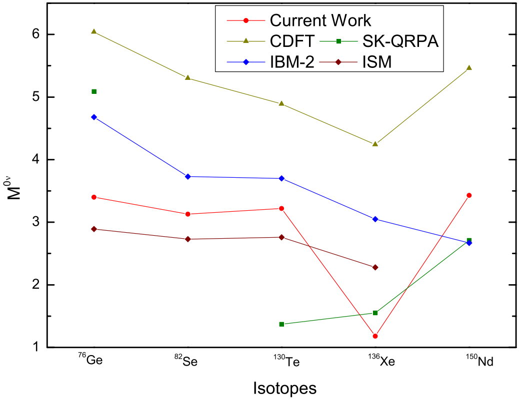
		AV18								CD Bonn							
		$g_A = g_{A0}$				$g_A = 0.75g_{A0}$				$g_A = g_{A0}$				$g_A = 0.75g_{A0}$			
		$M_F^{0\nu}$	$M_{GT}^{0\nu}$	$M_T^{0\nu}$	$M_l'^{0\nu}$	$M_{F,l}^{0\nu}$	$M_{GT,l}^{0\nu}$	$M_{T,l}^{0\nu}$	$M_l'^{0\nu}$	$M_F^{0\nu}$	$M_{GT}^{0\nu}$	$M_T^{0\nu}$	$M_l'^{0\nu}$	$M_{F,l}^{0\nu}$	$M_{GT,l}^{0\nu}$	$M_{T,l}^{0\nu}$	$M_l'^{0\nu}$
$^{76}Ge \rightarrow ^{76}Se$	a	-1.09	3.11	-0.44	3.34	-1.09	3.94	-0.46	2.63	-1.10	2.99	-0.40	3.27	-1.09	3.90	-0.42	2.64
	b	-1.06	2.92	-0.45	3.12	-1.06	3.70	-0.47	2.48	-1.15	3.09	-0.41	3.40	-1.15	4.00	-0.43	2.72
$^{82}Se \rightarrow ^{82}Kr$	a	-1.00	2.86	-0.41	3.07	-1.00	3.61	-0.43	2.41	-1.00	2.76	-0.37	3.01	-1.00	3.58	-0.42	2.41
	b	-0.98	2.68	-0.42	2.86	-0.97	3.39	-0.38	2.26	-1.05	2.85	-0.38	3.13	-1.05	3.67	-0.39	2.49
$^{130}Te \rightarrow ^{130}Xe$	a	-1.17	2.95	-0.52	3.16	-1.16	3.37	-0.55	2.31	-1.15	2.85	-0.46	3.10	-1.15	3.29	-0.49	2.29
	b	-1.13	2.73	-0.53	2.90	-1.13	3.11	-0.56	2.13	-1.21	2.95	-0.47	3.22	-1.21	3.38	-0.50	2.37
$^{136}Xe \rightarrow ^{136}Ba$	a	-0.37	1.12	-0.17	1.18	-0.37	1.39	-0.17	0.91	-0.33	1.05	-0.13	1.12	-0.33	1.29	-0.14	0.85
	b	-0.36	1.06	-0.17	1.11	-0.36	1.31	-0.17	0.86	-0.35	1.10	-0.14	1.18	-0.35	1.33	-0.14	0.89
$^{150}Nd \rightarrow ^{150}Sm$	a	-1.35	2.98	-0.53	3.28	-1.35	3.54	-0.56	2.52	-1.36	2.89	-0.45	3.28	-1.37	3.45	-0.52	2.50
	b	-1.32	2.74	-0.55	3.01	-1.31	3.26	-0.57	2.33	-1.43	3.00	-0.46	3.43	-1.43	3.55	-0.53	2.59

Uncertainties of the calculations

- nuclear force, quenching of g_A in nuclei, SRC

Results

DLF *et al.* Phys. Rev. C97,045503(2018)



Deviations between different calculations are still large

DLF *et al.* Phys. Rev. C97,045503(2018)

- In case of normal hierarchy, we may have other mechanism dominant over light neutrino mass mechanism, *i.e.* heavy neutrino mass

		Argonne								CD Bonn							
		$g_A = g_{A0}$				$g_A = 0.75g_{A0}$				$g_A = g_{A0}$				$g_A = 0.75g_{A0}$			
		$M^{0\nu}_{F,l}$	$M^{0\nu}_{GT,l}$	$M^{0\nu}_{T,l}$	$M'^{0\nu}_{h}$	$M^{0\nu}_{F,h}$	$M^{0\nu}_{GT,h}$	$M^{0\nu}_{T,h}$	$M'^{0\nu}_{h}$	$M^{0\nu}_{F,l}$	$M^{0\nu}_{GT,l}$	$M^{0\nu}_{T,l}$	$M'^{0\nu}_{h}$	$M^{0\nu}_{F,h}$	$M^{0\nu}_{GT,h}$	$M^{0\nu}_{T,h}$	$M'^{0\nu}_{h}$
^{76}Ge→^{76}Se	a	-109.7	369.7	-59.0	378.7	-109.5	423.1	-63.5	270.2	-111.0	370.8	-54.2	385.4	-110.8	426.6	-58.2	275.9
	b	-83.2	198.0	-62.2	187.3	-83.1	206.1	-67.0	129.7	-102.1	287.8	-57.4	293.7	-101.9	317.8	-61.8	207.2
^{82}Se→^{82}Kr	a	-102.3	345.9	-54.1	355.3	-102.2	397.0	-58.0	254.1	-102.5	344.2	-48.9	358.7	-102.4	397.1	-52.4	257.4
	b	-77.4	184.9	-57.0	175.9	-77.3	193.2	-61.2	122.1	-94.2	267.0	-51.8	273.6	-94.1	295.8	-55.6	193.4
^{130}Te→^{130}Xe	a	-116.1	393.0	-68.8	396.3	-116.0	440.6	-74.5	277.9	-115.9	391.5	-62.3	401.1	-115.9	439.7	-67.5	281.2
	b	-87.7	209.5	-72.5	191.4	-87.6	213.6	-78.7	130.2	-106.4	303.4	-65.9	303.5	-106.4	326.9	-62.5	209.5
^{136}Xe→^{136}Ba	a	-37.8	133.6	-21.8	135.2	-37.8	153.2	-23.2	96.5	-32.5	113.0	-16.1	117.1	-32.4	128.5	-17.1	82.7
	b	-28.8	72.0	-23.0	66.9	-28.8	75.0	-24.5	46.3	-30.2	88.7	-16.9	90.5	-30.2	96.5	-18.1	62.8
^{150}Nd→^{150}Sm	a	-127.4	414.7	-70.2	423.5	-127.4	466.8	-75.3	299.2	-130.8	420.0	-59.0	442.0	-127.8	466.1	-69.2	302.5
	b	-96.3	220.4	-74.0	206.1	-96.2	226.5	-79.5	142.3	-120.1	325.0	-62.4	337.0	-117.3	346.6	-73.4	226.4

- SRC becomes more important
- Pion's contributions may be important

Conclusion

- Neutrinoless double beta decay is very good probe for new physics beyond Standard Model
- Calculations of NME is important for the determinations of new physics parameters
- NMEs from various nuclear many-body approaches don't converge at present
- We need to understand the underlying mechanisms of this rare process

END

Thank You