Vacuum alignment in a composite 2HDM

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The Standard Model

What we know from the Standard Model (SM):

- Fermionic fields: quarks, leptons $\longrightarrow$ **Matter**,
- Vector fields: photon, $W^{\pm}$, Z, gluons $\rightsquigarrow$ **Force**,
- Scalar fields: Higgs boson $\dashrightarrow$ **origin of mass**.

Not explained by SM:

- Why $m_h \ll \Lambda_{GUT}$? (Hierarchy problem),
- Dark energy, **dark matters**,
- Neutrino masses and oscillation,
- Matter–antimatter asymmetry,
- Strong CP problem,...

New physics are needed!

Fundamental Composite Higgs Model

- $2N_f$ fermions ψ^i charged under some gauge Group G_{TC}.
- Global flavor symmetry $G_F = \mathrm{SU}(2N_f)$ or $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$,
- Non-abelian G_{TC}, asymptotic freedom $\longrightarrow$ ψ^i condense in the IR,

$$\langle\psi^i\psi^j\rangle \sim \Sigma^{ij} \neq 0 \quad \Rightarrow \quad G_F \to H \tag{1}$$

where H is a subgroup of G_F.
  - ψ^i: real reps. of G_{TC} $\longrightarrow$ $\mathrm{SU}(2N_f)\to\mathrm{SO}(2N_f)$,
  - ψ^i: pseudo-real reps. of G_{TC} $\longrightarrow$ $\mathrm{SU}(2N_f)\to\mathrm{Sp}(2N_f)$.
  - ψ^i: complex reps. of G_{TC} $\longrightarrow$ $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)\to\mathrm{SU}(N_f)$.
- pNGBs, coset space G_F/H,

$$U = \mathrm{e}^{i\Pi(\phi)}, \qquad \Pi(\phi) = \sum_i \phi_i X^i \tag{2}$$

- EW gauge group $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y \subset H$, Higgs doublet $\subset$ pNGBs.

Sp(2N) group

- $\mathrm{Sp}(2N) = \mathrm{Sp}(2N, C) \cap \mathrm{SU}(2N)$, $2N \times 2N$ matrices U satisfy

$$UEU^T = E, \qquad E = \begin{pmatrix} & \mathbb{1}_{N\times N} \\ -\mathbb{1}_{N\times N} & \end{pmatrix}, \tag{3}$$

$$or \quad U = e^{i\theta^a S^a}, \qquad S^a E + E(S^a)^T = 0 \tag{4}$$

- Choice of E is not unique,

$$\Sigma_0 = \begin{pmatrix} \mathcal{J} & & \\ & \pm\mathcal{J} & \\ & & \ddots \end{pmatrix}, \quad \mathcal{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \begin{matrix} \Sigma_0 = OEO^T, \quad \tilde{S}^a = OS^aO^{-1}, \\ \tilde{S}^a\Sigma_0 + \Sigma_0(\tilde{S}^a)^T = 0 \end{matrix} \tag{5}$$

- SU(4)/Sp(4): minimal model [E. Katz (2005), B. Gripaio (2009), M. Frigerio (2012), G. Cacciapaglia (2014)],
  SU(6)/Sp(6): 2HDM.

$SU(6) \to Sp(6)$ composite model

- 6 left-handed Weyl spinors ψ, fundamental reps of G_{TC} =SU(2).
- In the IR, $\langle \psi^i \psi^j \rangle \sim \Sigma^{ij}$ antisymmetric, SU(6) →Sp(6).
- NGBs: d.o.f $= 35 - 21 = 14$, decomposition:

$$14_{\mathrm{Sp}(6)} \to (2,2,1) \oplus (2,1,2) \oplus (1,2,2) \oplus (1,1,1) \oplus (1,1,1) \tag{6}$$

Case		$SU(2)_L$	$U(1)_Y$	$SU(2)_L$	Y	Higgs
A	ψ_1	**2**	0	SU(2)$_1$	$T_2^3 + \xi T_3^3$	$(2,2,1)$ [$(2,1,2)$ if $\xi = 1$]
	ψ_2	**1**	$\pm 1/2$			
	ψ_3	**1**	$\pm \xi/2$			
B	ψ_1	**2**	0	SU(2)$_1$ + SU(2)$_2$	T_3^3	$(2,1,2) + (1,2,2)$
	ψ_2	**2**	0			
	ψ_3	**1**	$\pm 1/2$			

The pNGBs

- pNGBs: $\Sigma(\phi) = U(\phi)\Sigma$, $U(\phi) = \exp[i\Pi(\phi)]$, $\Pi(\phi) = \sum_{i=1}^{14} \phi^i X^i$,

$$\mathcal{L}_{(p^2)} = f^2 \mathrm{Tr}\left[(D_\mu \Sigma(\phi))^\dagger \cdot D^\mu \Sigma(\phi)\right] - \mathrm{Tr}\left[\chi \cdot \Sigma^\dagger(\phi) + \chi^\dagger \cdot \Sigma(\phi)\right], \quad \chi = 2BM_\psi^\dagger \tag{7}$$

- Before EW symmetry breaking, $\langle \phi^4 \rangle = \langle \phi^8 \rangle = 0$,

$$\langle \psi^i \psi^j \rangle \sim \Sigma = \Sigma_0 = \begin{pmatrix} i\sigma_2 & 0 & 0 \\ 0 & -i\sigma_2 & 0 \\ 0 & 0 & -i\sigma_2 \end{pmatrix} \tag{8}$$

$$i\Pi(\phi') \cdot \Sigma_0 = \frac{1}{2} \begin{pmatrix} S_1 & H_1 & H_2 \\ -H_1^T & S_2 & G \\ -H_2^T & -G^T & S_3 \end{pmatrix} \tag{9}$$

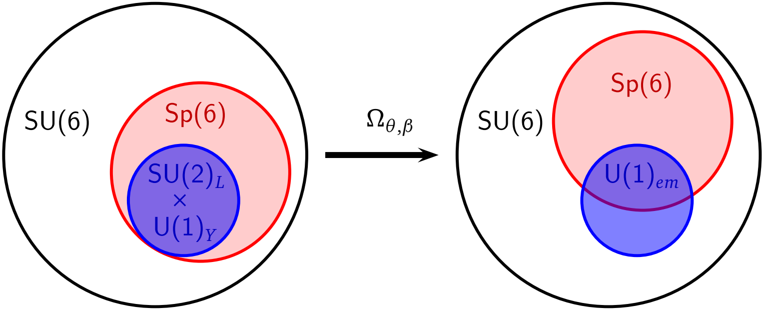
- $H_1 \sim (2,2,1)$, $H_2 \sim (2,1,2)$ —— 2HDM
- $G \sim (1,2,2)$: neutral and charged singlets
- $S_{1,2,3}$: $(1,1,1) \oplus (1,1,1)$ singlets pesudo-scalars

Vacuum misalignment and EWSB

- EW breaking, $\langle\phi^4\rangle = v_1$, $\langle\phi^8\rangle = v_2$, $\tan\beta = v_2/v_1$, $\theta = \sqrt{v_1^2+v_2^2}/2\sqrt{2}f$

$$\Sigma = \Omega_{\theta,\beta}\Sigma_0\Omega_{\theta,\beta}^T, \quad U(\phi)_{\theta,\beta} = \Omega_{\theta,\beta}U_0(\phi)\Omega_{\theta,\beta}^\dagger, \quad \Omega_{\theta,\beta} = R_\beta\Omega_\theta R_\beta^\dagger \tag{10}$$

$$R_\beta = \begin{pmatrix} \mathbb{I}_2 & 0 & 0 \\ 0 & \cos\beta\ \mathbb{I}_2 & -\sin\beta\ \mathbb{I}_2 \\ 0 & \sin\beta\ \mathbb{I}_2 & \cos\beta\ \mathbb{I}_2 \end{pmatrix}, \quad \Omega_\theta = \begin{pmatrix} \cos\frac{\theta}{2}\mathbb{I}_2 & \sin\frac{\theta}{2}\ i\sigma_2 & 0 \\ \sin\frac{\theta}{2}\ i\sigma_2 & \cos\frac{\theta}{2}\ \mathbb{I}_2 & 0 \\ 0 & 0 & \mathbb{I}_2 \end{pmatrix} \tag{11}$$



Gauge bosons and fermions masses

- Gauge bosons' masses and hVV coupling are generated by

$$\mathcal{L}_{(p^2)} \supset f^2 \mathrm{Tr}\left[(D_\mu \Sigma(\phi))^\dagger \cdot D^\mu \Sigma(\phi)\right]$$
$$\Rightarrow \quad m_W^2 = 2g^2 f^2 \sin^2\theta, \quad m_Z^2 = \frac{m_W^2}{\cos^2\theta_W}, \quad v_{SM} = 2\sqrt{2} f \sin\theta \approx 246 \text{ GeV}$$
$$g_{h_1WW} = g_{h_1ZZ}\cos\theta_W^2 = \sqrt{2} g^2 f \sin\theta\cos\theta = g_{hWW}^{SM}\cos\theta \qquad (12)$$

- Top Yukawa generated by

$$\frac{y'_{t1}}{\Lambda_t^2}\left(Q_L t_R^c\right)_\alpha^\dagger \left(\psi^T P_1^\alpha \psi\right) + \frac{y'_{t2}}{\Lambda_t^2}\left(Q_L t_R^c\right)_\alpha^\dagger \left(\psi^T P_2^\alpha \psi\right) \qquad (13)$$

in the θ vacuum: $R_\beta (y_{t1} P_1^\alpha + y_{t2} P_2^\alpha) R_\beta^T = Y_{t1} P_1^\alpha + Y_{t2} P_2^\alpha$.

- Top mass and Yukawa coupling:

$$\mathcal{L}_{(p^2)} \supset f \left(Q_L t_R^c\right)_\alpha^\dagger \left(Y_{t1}\mathrm{Tr}[P_1^\alpha \cdot \Sigma(\phi')] + Y_{t2}\mathrm{Tr}[P_2^\alpha \cdot \Sigma(\phi')]\right)$$
$$\sim -f\sin\theta \; Y_{t1} \left(t_L t_R^c\right)^\dagger - \frac{Y_{t1}}{2\sqrt{2}}\left(c_\theta \; h_1 + \frac{i}{\sqrt{3}} s_\theta \; \eta_2\right)\left(t_L t_R^c\right)^\dagger + h.c.$$
$$\Rightarrow \quad m_t = Y_{t1} f \sin\theta, \qquad g_{h_1\bar{t}t} = g_{h\bar{t}t}^{SM}\cos\theta \qquad (14)$$

Three origins of the pNGBs potential

- Gauge loops

$$V_g = -C_g f^4 \left\{ \frac{g'^2}{2} + \frac{3g^2+g'^2}{2} c_\theta^2 - \frac{h_1}{2\sqrt{2}} \frac{3g^2+g'^2}{2} s_{2\theta} + \dots \right\} \tag{15}$$

- Fermions' loops

$$\begin{aligned} V_{\text{Yuk}} = -C_t f^4 \Bigg\{ & \left(|Y_{t1}|^2 + |Y_{b1}|^2\right) s_\theta^2 + \frac{h_1}{2\sqrt{2}f} \left(|Y_{t1}|^2 + |Y_{b1}|^2\right) s_{2\theta} + \\ & \frac{h_2}{\sqrt{2}f} \left(\Re\, Y_{t1} Y_{t2}^* + \Re\, Y_{b1} Y_{b2}^*\right) c_{\frac{\theta}{2}} s_\theta + \frac{\varphi_0}{\sqrt{2}f} \left(\Re\, Y_{t1} Y_{t2}^* - \Re\, Y_{b1} Y_{b2}^*\right) s_{\frac{\theta}{2}} s_\theta + \\ & \frac{A_0}{\sqrt{2}f} \left(\Im\, Y_{t1} Y_{t2}^* - \Im\, Y_{b1} Y_{b2}^*\right) c_{\frac{\theta}{2}} s_\theta + \frac{\eta_3}{\sqrt{2}f} \left(\Im\, Y_{t1} Y_{t2}^* + \Im\, Y_{b1} Y_{b2}^*\right) s_{\frac{\theta}{2}} s_\theta \Bigg\} \end{aligned} \tag{16}$$

- Explicitly breaking mass term, $\left(M \equiv \frac{m_L+m_R}{2},\ \delta m_R \equiv \frac{m_{R1}-m_{R2}}{2},\ \Delta \equiv \frac{m_L-m_R}{m_L+m_R}\right)$

$$\begin{aligned} V_{\text{m}} = -8B \Bigg\{ & M\ (1-\Delta+2c_\theta) - \delta m_R\ c_{2\beta}\ (1-c_\theta) \\ & - \frac{h_1}{2\sqrt{2}f} \left(2M + \delta m_R c_{2\beta}\right) s_\theta + \frac{h_2}{\sqrt{2}f} \delta m_R s_{2\beta} s_{\frac{\theta}{2}} + \dots \Bigg\} \end{aligned} \tag{17}$$

Composite inert 2HDM

- A special vacuum: all fermions couple to the same $SU(2)_R$

$$\frac{\partial V}{\partial \beta} = 0 \ + \ \text{tadpoles vanish} \Rightarrow \quad y_{f2} = Y_{f2} = 0, \quad \beta = 0, \quad Y_{f1} = y_{f1}, \tag{18}$$

$$\frac{\partial V}{\partial \theta} = 0 \quad \Rightarrow \quad \cos\theta = \frac{16BM/f^4 + 8B\delta m_R/f^4}{2C_t(|y_{t1}|^2 + |y_{b1}|^2) - C_g(3g^2 + g'^2)} \tag{19}$$

- Higgs mass and couplings:

$$m_{h_1}^2 = \frac{C_t}{4} m_t^2 - \frac{C_g}{16}(2m_W^2 + m_Z^2) \sim (125 \text{ GeV})^2 \Rightarrow C_t \sim 2 \tag{20}$$

$$g_{hXX} = g_{hXX}^{\text{SM}} c_\theta, \tag{21}$$

- $SU(2)_{R2}$ is only broken by gauging $T_{R2}^3 \sim Y$, to a remnant $U(1)_{DM}$.

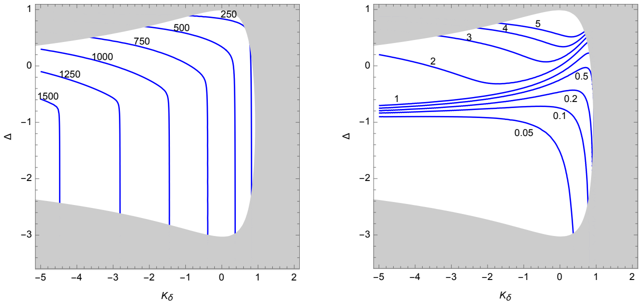
$$Q_{DM} = 1 \text{ fields}: \quad (H^+, H^0) \supset (2,1,2), \quad \eta^+, \quad \eta^0 \supset (1,2,2) \tag{22}$$

Masses spectrum

- The neutral/charged components mass matrices in basis $(H^0, \eta^0)/(H^\pm, \eta^\pm)$

$$\mathcal{M}^2_{\text{neut.}} = \frac{C_t Y_{t1}^2 f^2}{8} \begin{pmatrix} (1+c_\theta) - \frac{8K_\delta}{Y_{t1}^2} & s_\theta \\ s_\theta & (1+c_\theta - 2\Delta c_\theta) - \frac{4(1-\Delta)K_\delta}{Y_{t1}^2} - \frac{C_g(3g^2+g'^2)}{C_t Y_{t1}^2}(1-\Delta)c_\theta \end{pmatrix}$$

$$\mathcal{M}^2_{\text{charg.}} = \mathcal{M}^2_{\text{neut.}} + \frac{C_g g'^2 f^2}{4} \begin{pmatrix} (1-c_\theta) & 0 \\ 0 & (1+c_\theta) \end{pmatrix}$$



Fixing
$C_g = \frac{1}{3} C_t,\ \theta = 0.2$
$(2\sqrt{2}f = 1.2\ \text{TeV})$

The most general vacuum alignment

- The vacuum $\Sigma = \Omega_{\theta,\beta,\gamma} \cdot \Sigma_0 \cdot \Omega^\dagger_{\theta,\beta,\gamma}$, $\Omega_{\theta,\beta,\gamma} = R_\beta \cdot R_\gamma \cdot \Omega_\theta \cdot R^\dagger_\gamma \cdot R^\dagger_\beta$

$$R_\beta = e^{i2\sqrt{2}\beta \ S^{21}}, \quad \Omega_\theta = e^{i\sqrt{2}\theta \ X^4}, \quad R_\gamma = e^{-i2\sqrt{2}\gamma \ S^{14}} = \begin{pmatrix} \cos\gamma \ \mathbb{I}_2 & 0 & \sin\gamma \ \sigma^1 \\ 0 & \mathbb{I}_2 & 0 \\ -\sin\gamma \ \sigma^1 & 0 & \cos\gamma \ \mathbb{I}_2 \end{pmatrix} \tag{23}$$

- EW vev: $v_{\rm SM} = 2\sqrt{2} f \sin\tau$, $\sin\frac{\tau}{2} \equiv \cos\gamma \sin\frac{\theta}{2}$,
- Higgs mixing:

$$h_1' = \frac{1}{\cos\frac{\tau}{2}}\left(\cos\gamma \ \cos\frac{\theta}{2} \ h_1 - \sin\gamma \ \varphi^0\right), \quad \varphi_0' = \frac{1}{\cos\frac{\tau}{2}}\left(\sin\gamma \ h_1 + \cos\gamma \ \cos\frac{\theta}{2} \ \varphi_0\right) \tag{24}$$

$$\frac{g_{h_1' WW}}{g^{\rm SM}_{hWW}} = \frac{g_{h_1' ZZ}}{g^{\rm SM}_{hZZ}} = \cos\tau \tag{25}$$

- Top mass and Yukawa coupling,

$$m_t = 2\cos\frac{\gamma}{2} \ \sin\frac{\theta}{2}\left(Y_{t2}\sin\frac{\gamma}{2} + Y_{t1}\cos\frac{\gamma}{2} \ \cos\frac{\theta}{2}\right) = f\sin(\tau) Y_{\rm top} \tag{26}$$

$$Y_{\rm top} = \frac{1}{\cos\frac{\tau}{2}}\left(Y_{t2}\sin\gamma \ \sin\frac{\theta}{2} + Y_{t1}\cos\frac{\theta}{2}\right) \tag{27}$$

Summary

1. SU(6)/Sp(6) model is a generalization of SU(4)/Sp(4) allowing for a second Higgs doublet.
2. In absence of CP violating phases, vacuum can be misaligned only in two exclusive ways:
   a) The misalignment is characterized by a single angle θ. Two Yukawa couplings are aligned or fermions only couple to one $\mathrm{SU}(2)_R$.
   b) The misalignment depends on 3 angles θ, β, γ. Fermions couple to both $\mathrm{SU}(2)_R$s.
3. In case a), a $\mathrm{U}(1)_{DM}$ symmetry prevent some pNGBs (DM candidate) from decaying to SM particles.
4. In case b), all pNGBs are not stable. Scalars mix with each other.

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Thanks for your attention!

Unbroken generators of Sp(6)

$$S^1=\frac{1}{2}\begin{pmatrix}\sigma_1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^2=\frac{1}{2}\begin{pmatrix}\sigma_2 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^3=\frac{1}{2}\begin{pmatrix}\sigma_3 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\tag{28}$$

$$S^4=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & -\sigma_1^T & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^5=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & -\sigma_2^T & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^6=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & -\sigma_3^T & 0\\ 0 & 0 & 0\end{pmatrix},\tag{29}$$

$$S^7=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sigma_1^T\end{pmatrix},\quad S^8=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sigma_2^T\end{pmatrix},\quad S^9=\frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -\sigma_3^T\end{pmatrix},\tag{30}$$

$$S^{10}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma_1 & 0\\ -i\sigma_1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^{11}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma_2 & 0\\ -i\sigma_2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\quad S^{12}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & i\sigma_3 & 0\\ -i\sigma_3 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\quad S^{13}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & \mathbb{I}_2 & 0\\ \mathbb{I}_2 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\tag{31}$$

$$S^{14}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & i\sigma_1\\ 0 & 0 & 0\\ -i\sigma_1 & 0 & 0\end{pmatrix},\quad S^{15}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & i\sigma_2\\ 0 & 0 & 0\\ -i\sigma_2 & 0 & 0\end{pmatrix},\quad S^{16}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & i\sigma_3\\ 0 & 0 & 0\\ -i\sigma_3 & 0 & 0\end{pmatrix},\quad S^{17}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & \mathbb{I}_2\\ 0 & 0 & 0\\ \mathbb{I}_2 & 0 & 0\end{pmatrix},\tag{32}$$

$$S^{18}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \sigma_1\\ 0 & \sigma_1 & 0\end{pmatrix},\quad S^{19}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \sigma_2\\ 0 & \sigma_2 & 0\end{pmatrix},\quad S^{20}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & \sigma_3\\ 0 & \sigma_3 & 0\end{pmatrix},\quad S^{21}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & i\mathbb{I}_2\\ 0 & -i\mathbb{I}_2 & 0\end{pmatrix},\tag{33}$$

Broken generators

$$X^1 = \frac{1}{2\sqrt{2}}\begin{pmatrix} \mathbb{I}_2 & 0 & 0 \\ 0 & -\mathbb{I}_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^2 = \frac{1}{2\sqrt{6}}\begin{pmatrix} \mathbb{I}_2 & 0 & 0 \\ 0 & \mathbb{I}_2 & 0 \\ 0 & 0 & -2\mathbb{I}_2 \end{pmatrix}, \tag{34}$$

$$X^3 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_1 & 0 \\ \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^4 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_2 & 0 \\ \sigma_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^5 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & \sigma_3 & 0 \\ \sigma_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad X^6 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & i\mathbb{I}_2 & 0 \\ -i\mathbb{I}_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{35}$$

$$X^7 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & \sigma_1 \\ 0 & 0 & 0 \\ \sigma_1 & 0 & 0 \end{pmatrix}, \quad X^8 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & \sigma_2 \\ 0 & 0 & 0 \\ \sigma_2 & 0 & 0 \end{pmatrix}, \quad X^9 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & \sigma_3 \\ 0 & 0 & 0 \\ \sigma_3 & 0 & 0 \end{pmatrix}, \quad X^{10} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 0 & i\mathbb{I}_2 \\ 0 & 0 & 0 \\ -i\mathbb{I}_2 & 0 & 0 \end{pmatrix}, \tag{36}$$

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WZW terms

The Wess-Zumino-Witten topological term [Wess (1971), Witten (1983)] reads:

$$\mathcal{L}_{WZW} = \frac{d_{\rm FCD}\, g_{V_1V_2}}{16\sqrt{2}\pi^2 f}\left(c_\theta\ \eta_1 + \frac{1}{\sqrt{3}c_\theta}\ \eta_2\right)\ \epsilon_{\mu\nu\rho\sigma}V_1^{\mu\nu}V_2^{\rho\sigma}\,, \tag{38}$$

where $d_{\rm FCD}$ is the dimension of the FCD representation of the underlying fermions ($d_{\rm FCD} = 2$ in the minimal $SU(2)_{\rm TC}$ model), and

$$g_{WW} = g^2\,, \quad g_{ZZ} = (g^2 - g'^2), \quad g_{Z\gamma} = g\,g'\,. \tag{39}$$

The couplings above also shows that η_1 and η_2 are pseudo-scalars under CP.

Projectors for the top mass

$$P_1^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad P_1^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{40}$$

$$P_2^1 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad P_2^2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{41}$$

Similar for the bottom Yukawas.

Masses spectrum of the pseudo-scalars

- The pseudo-scalars' mass matrix

$$\mathcal{M}_\eta^2 = \frac{m_h^2}{s_\theta^2}\begin{pmatrix} 1 & \frac{1}{\sqrt{3}}\Delta c_\theta \\ \frac{1}{\sqrt{3}}\Delta c_\theta & \frac{1}{3}(2(1-\Delta)+c_\theta)c_\theta \end{pmatrix} - C_t f^2 K_\delta \begin{pmatrix} 0 & \frac{1+\Delta}{2\sqrt{3}} \\ \frac{1+\Delta}{2\sqrt{3}} & \frac{3-\Delta}{3} \end{pmatrix} \quad (42)$$