Rotational Magnetic Inhibition and Surface Magnetic Catalysis in Relativistic Fermionic Matter

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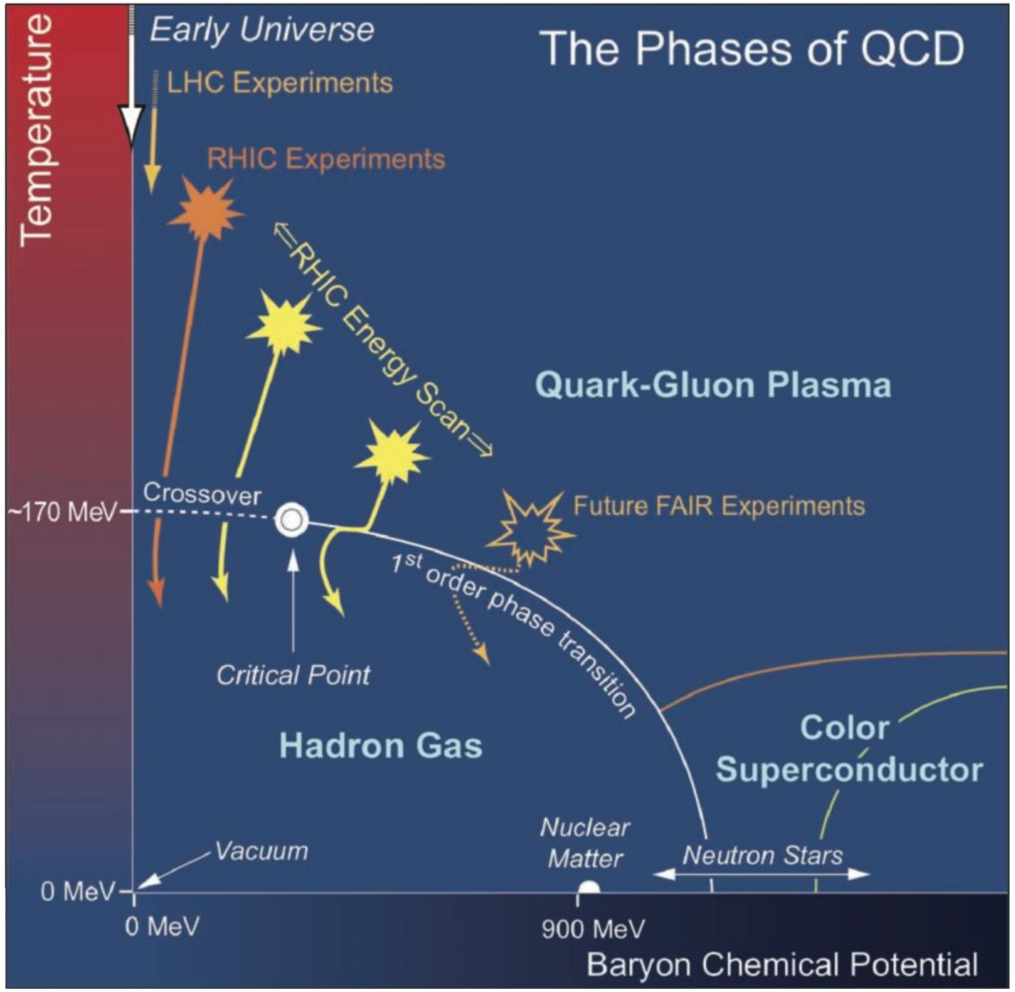
Phys. Rev. D 96, 054032 (2017), arXiv: 1707.09130

Outline

- QCD phase diagram
- (Inverse) magnetic catalysis in finite density system
- Analogy between rotation and density
- Boundary effect

$\boldsymbol{T}$–$\boldsymbol{\mu}$ phase diagram of QCD



- Other Backgrounds:

  Gravity

  Magnetic Field

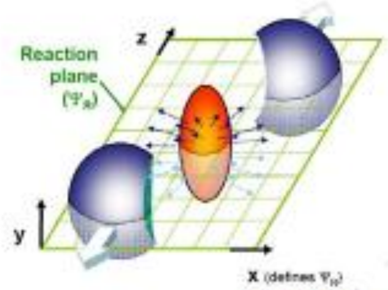
  Rotation

Chiral Symmetry Breaking

In the chiral limit $m_u = m_d = 0$

- The Lagrangian of QCD acquires chiral symmetry $SU(2)_L \times SU(2)_R$
- The ground state of QCD breaks the chiral symmetry to isospin symmetry $SU(2)_V$
- Quark condensation $\langle \bar{q}q \rangle \neq 0$
- Chiral symmetry is not broken at sufficiently high temperature or density

QCD in strong magnetic fields

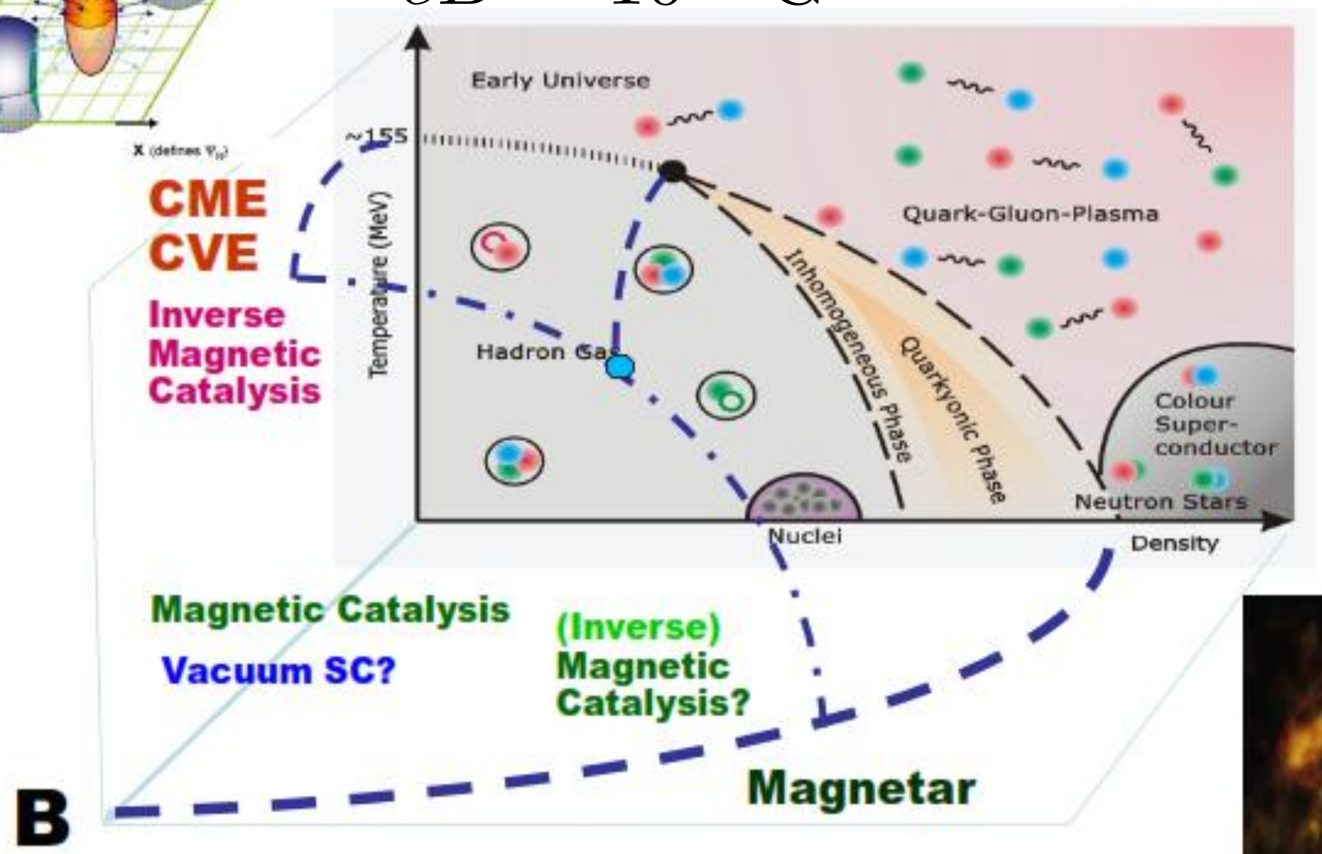


The early universe

$eB \sim 10^{18}\,\mathrm{G}$

Heavy-ion collisions

$eB \sim 10^{18}\,\mathrm{G}$

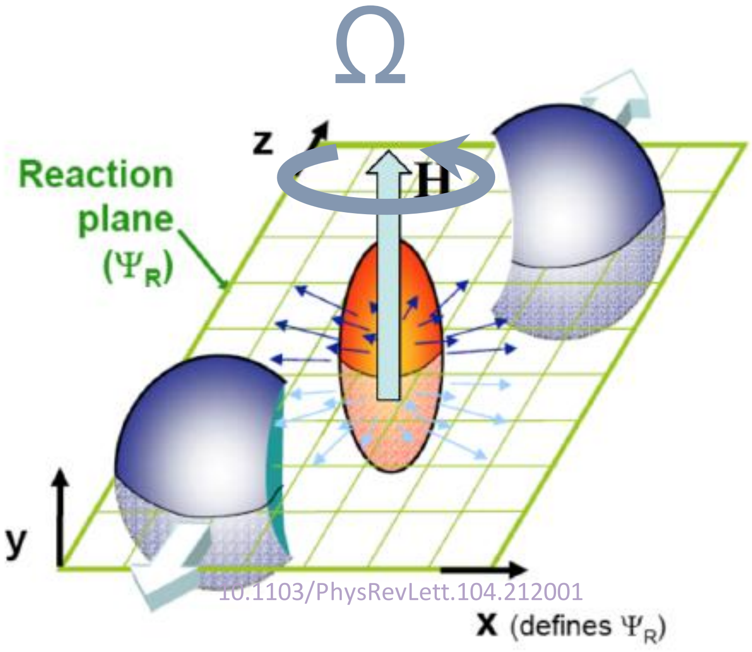


Compact stars(Magnetar)

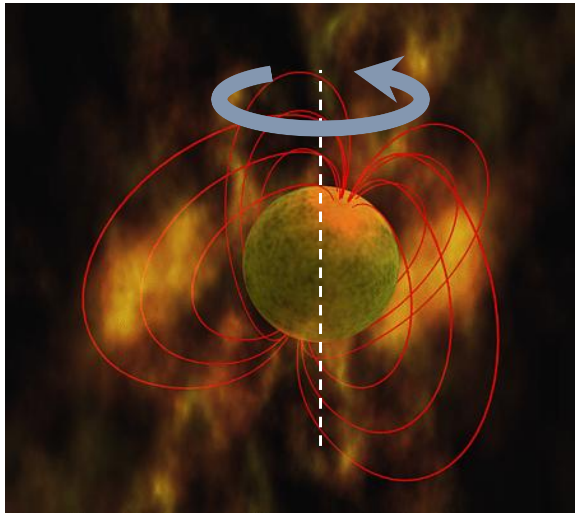
$eB \sim 10^{15}\,\mathrm{G}$

QCD in strong magnetic fields with rotation

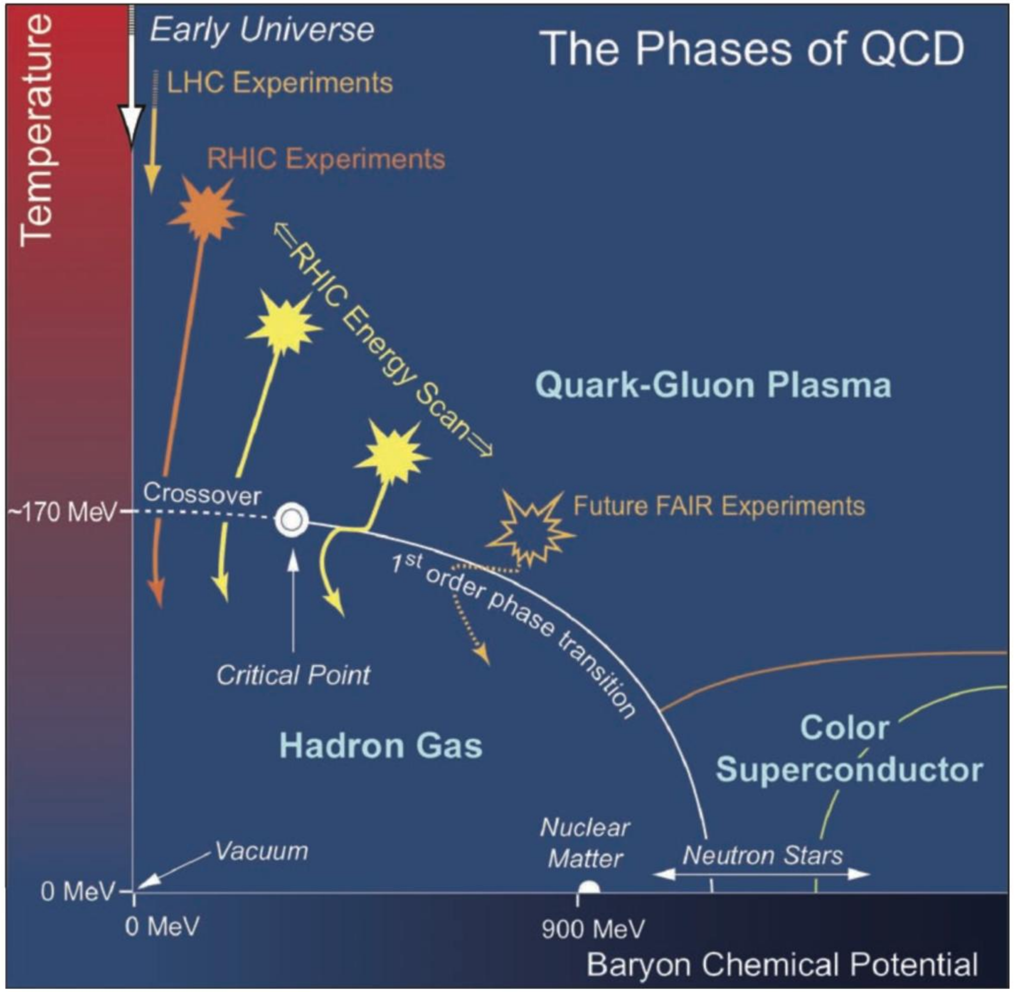
These systems also have rapid rotation



$$R\Omega \sim 10^{-1}$$



phase diagram of QCD



- Other Backgrounds:

Gravity

Magnetic Field

Rotation

The Nambu-Jona-Lasinio (NJL) model

- The effective Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\not{D} + \mu\gamma^0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau_a\psi)^2]$$

Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 124 (1961) 246.

Hatsuda T, Kunihiro T. Phys. Rep.(1994)
S.P. Klevansky. Rev.Mod.Phys. 64 (1992)

- Four fermion interaction
- Chiral symmetry breaking
- Lack of confinement
- Nonrenormalizable field theory
- The results depend on the regularization scheme and on the UV cut-off that is used

The Nambu-Jona-Lasinio (NJL) model

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$$\mathcal{L} = \bar{\psi}(i\not{D} + \mu\gamma^0)\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau_a\psi)^2]$$

- In mean field approximation (define $m = -G\langle\bar{\psi}\psi\rangle$)

$$(\bar{\psi}\psi)^2 = -\langle\bar{\psi}\psi\rangle^2 + (\bar{\psi}\psi)\langle\bar{\psi}\psi\rangle$$

$$\mathcal{L} = \bar{\psi}(i\not{D} - m + \mu\gamma^0)\psi - \frac{m^2}{2G}$$

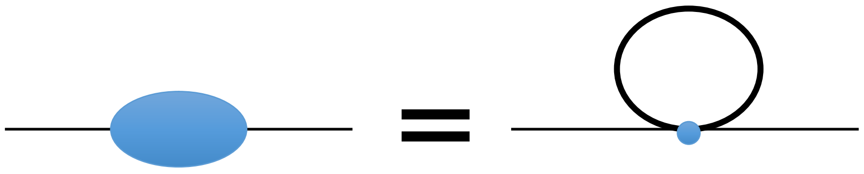
The Nambu-Jona-Lasinio (NJL) model

- Thermodynamic potential:

$$\Omega = -\frac{T}{V}\ln Z = \frac{m^2}{2G} - \frac{T}{V}\mathrm{Tr}\ln(i\not{D}_\mu - m + \mu\gamma^0)$$

- Gap equation

$$\frac{\partial\Omega(m)}{\partial m} = 0 \quad \Rightarrow \quad \frac{m}{G} = \frac{T}{V}\mathrm{Tr}\,S(x, x')$$



- NJL model in finite density system

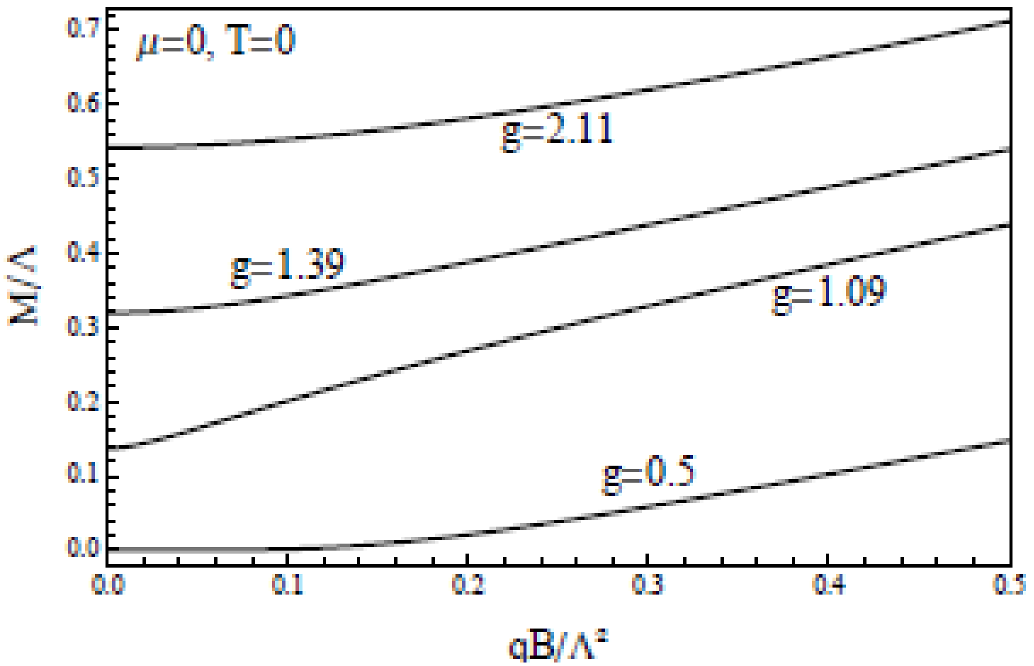
  weak coupling : Catalysis

  strong coupling : Inhibition

Ebert, Klimenko (1999)

Magnetic catalysis

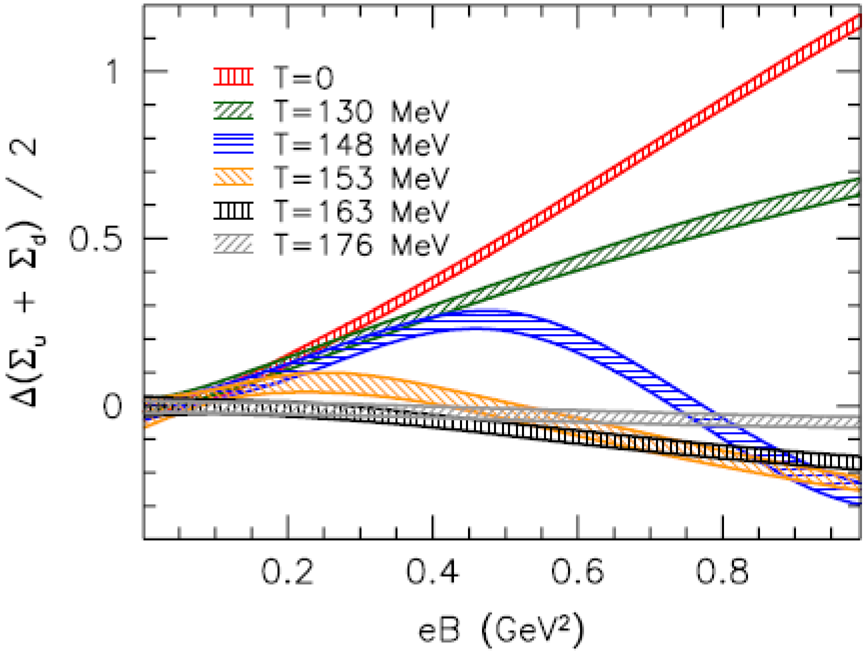
- Condensation m increases with magnetic field eB increasing



F. Preis, et al., Lect. Notes Phys. 871, 51 (2013)

Inverse magnetic catalysis

- Condensation m decreases with magnetic field eB increasing!

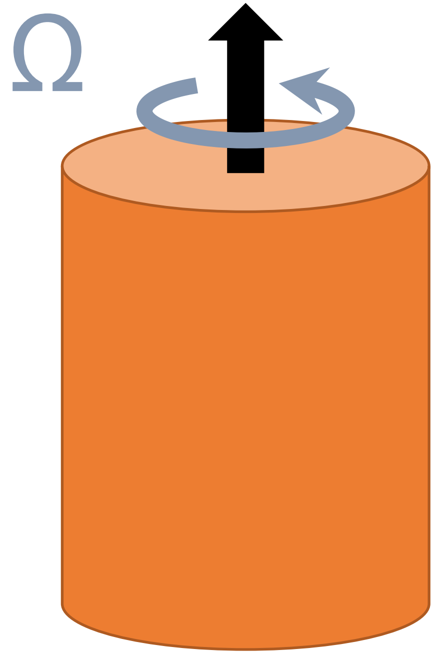


G. Bali, et al., JHEP 1202 (2012) 044

Fermions system with eB and Ω

- Dirac equation
  $$[i\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu) - m]\psi(x) = 0$$
- $e_\mu{}^i$ is vierbein, and we have
  $\gamma^\mu = e^\mu{}_i\gamma^i$ and
  $A_\mu = e_\mu{}^i A_i$ where $A_i = (0, By/2, -Bx/2, 0)$
- Γ_μ is spin connection
- The Greek letters: local orthogonal frame
  The Latin letters: tangent space
- NJL Model in Finite eB and Ω
  $$\mathrm{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu) - m]\psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2]$$

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



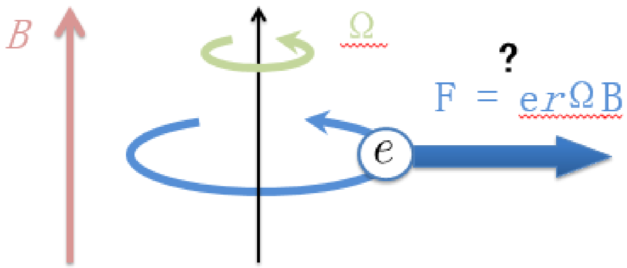
Fermions system with eB and Ω

- Dispersion relation

$$[E+\Omega(1+s_z)]^2 = p_z^2 + (2n+1-2s_z)eB + m^2$$

- No Lorentz force

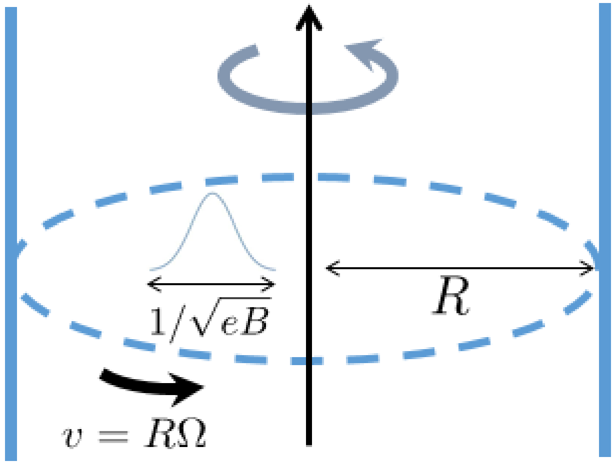
$$\gamma^\mu A_\mu = \gamma^i A_i$$



- System size

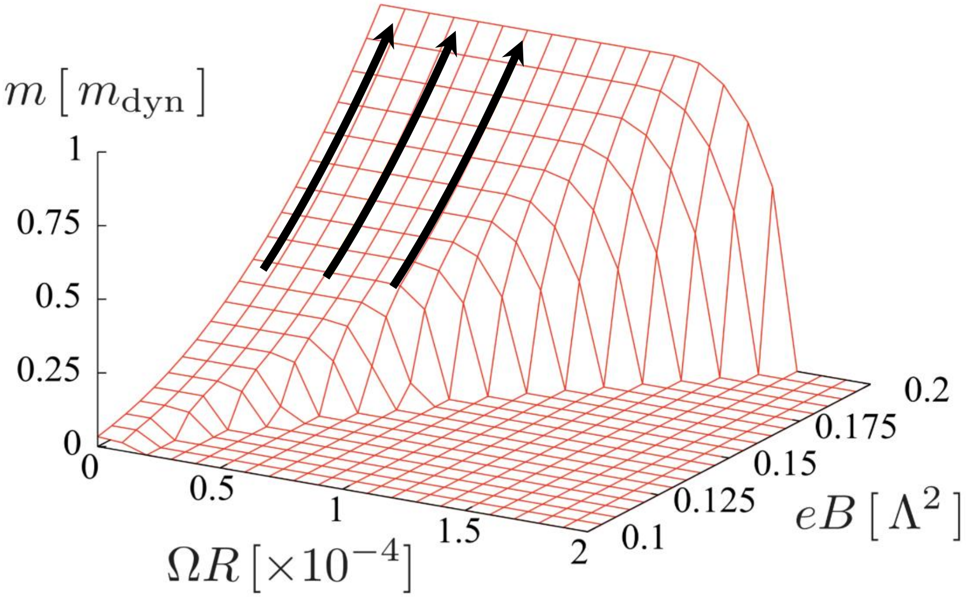
$R\ ?\ 1/\sqrt{eB}$ to ensure Landau quantization

$R \le 1/\Omega$ to ensure causality



Weak Coupling

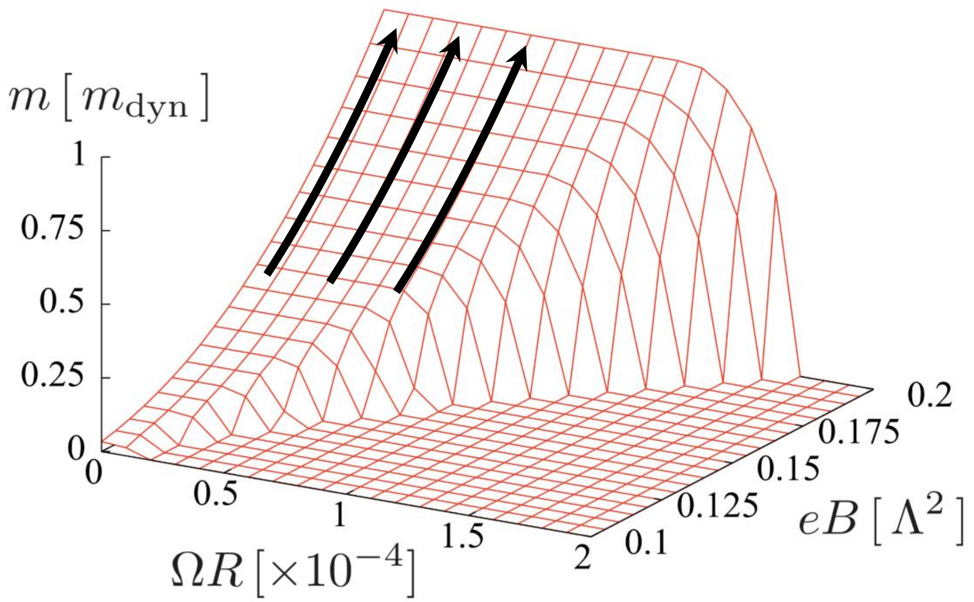
- Magnetic Catalysis



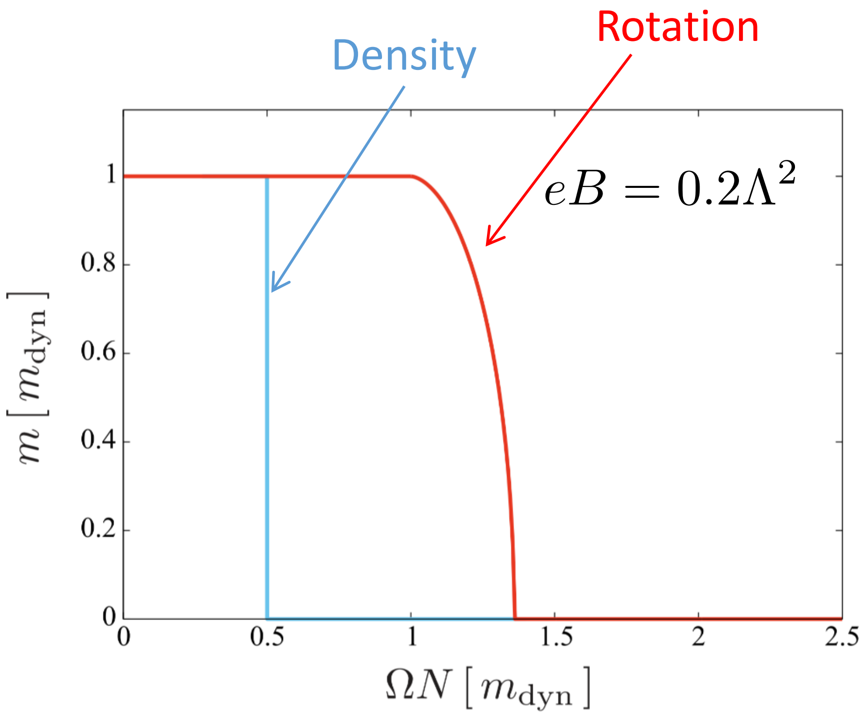
Increasing eB $\longrightarrow$ Increasing m

Weak Coupling

- Magnetic Catalysis

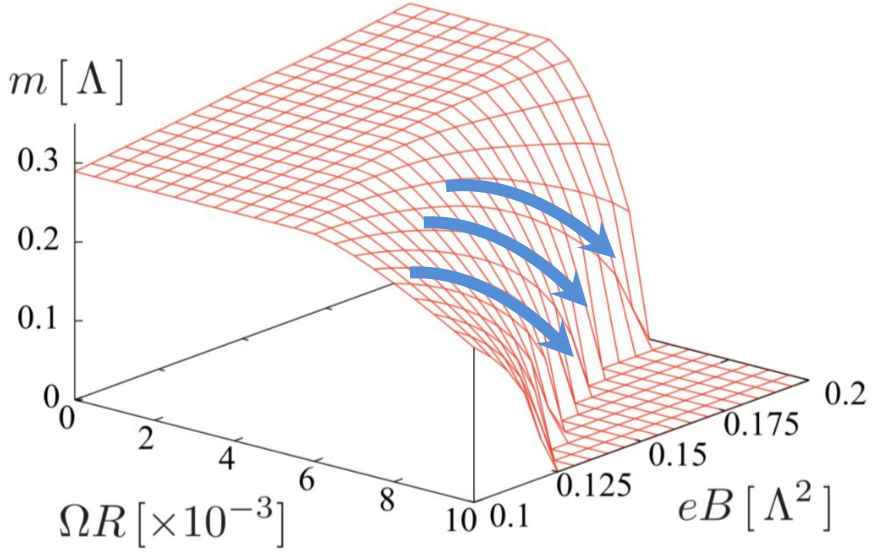


Increasing eB $\longrightarrow$ Increasing m



Strong Coupling: “Rotational Magnetic Inhibition”

- “Rotational Magnetic Inhibition”



Increasing eB $\longrightarrow$ Decreasing m

- Dropping start around $\Omega N : \sqrt{eB}$
- For finite density system the inverse magnetic catalysis start around $\mu : \sqrt{eB}$

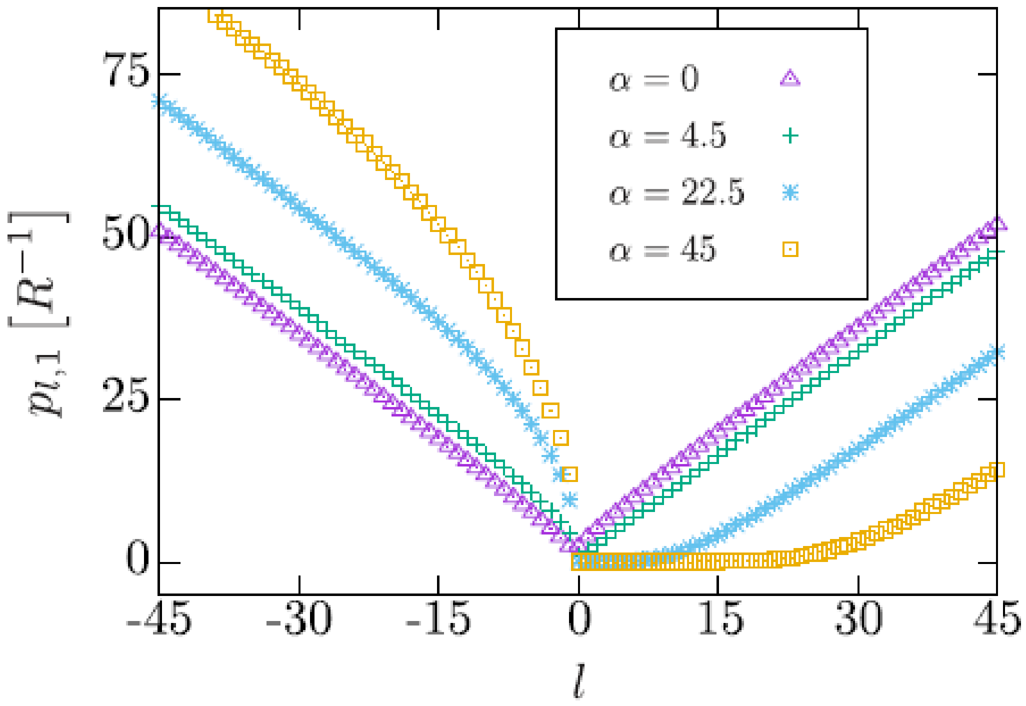
Preis, Rebhan, Schmitt (2012)

Boundary effect (without rotation)

- Boundary condition

$$R \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\theta \, \bar{\psi} \gamma^r \psi \bigg|_{r=R} = 0$$

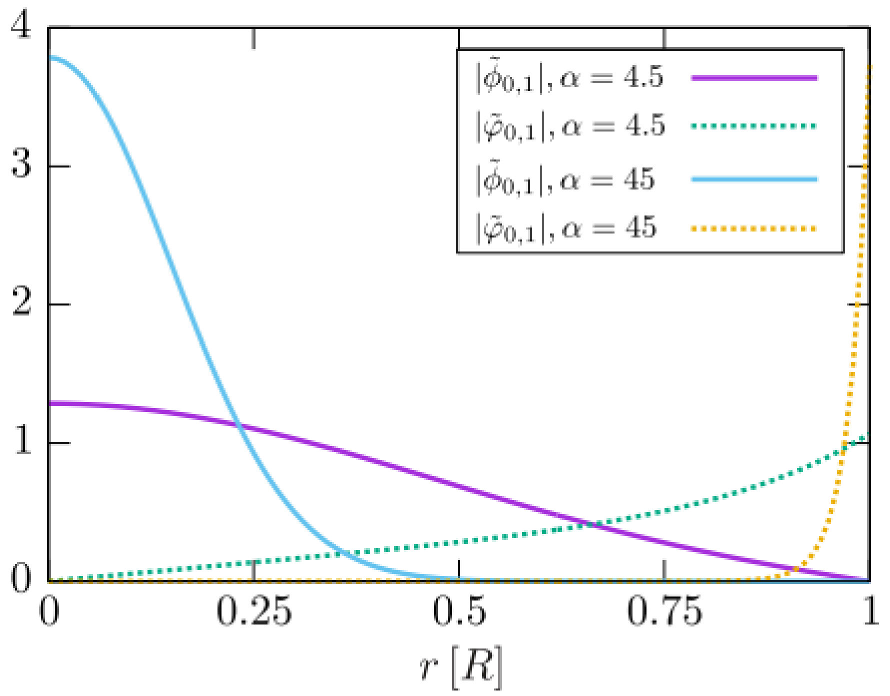
- Dispersion relation

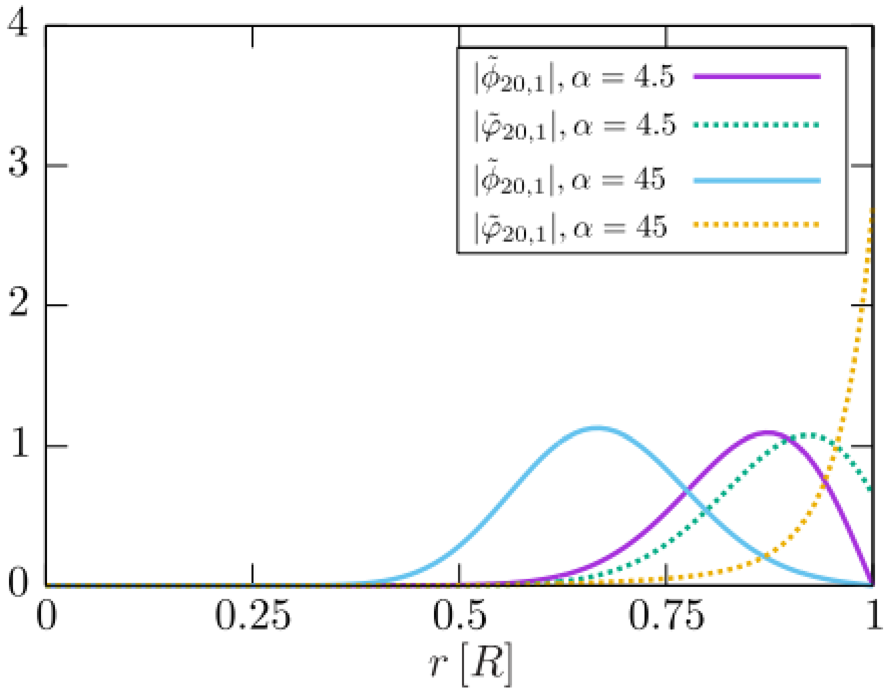


$$\alpha \equiv \frac{1}{2} eBR^2$$

Boundary effect (without rotation)

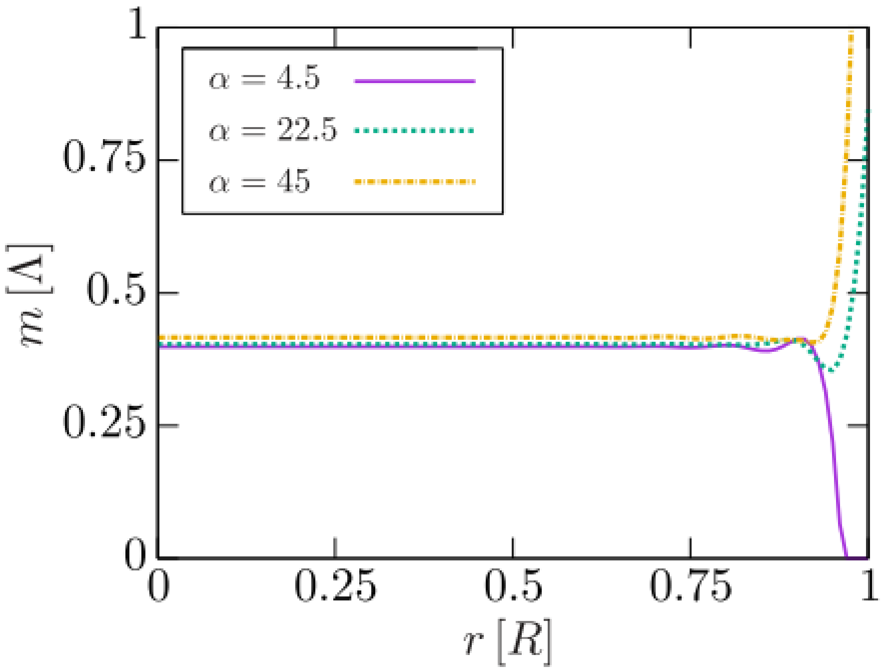
- Profile





Boundary effect (without rotation)

- Surface magnetic catalysis



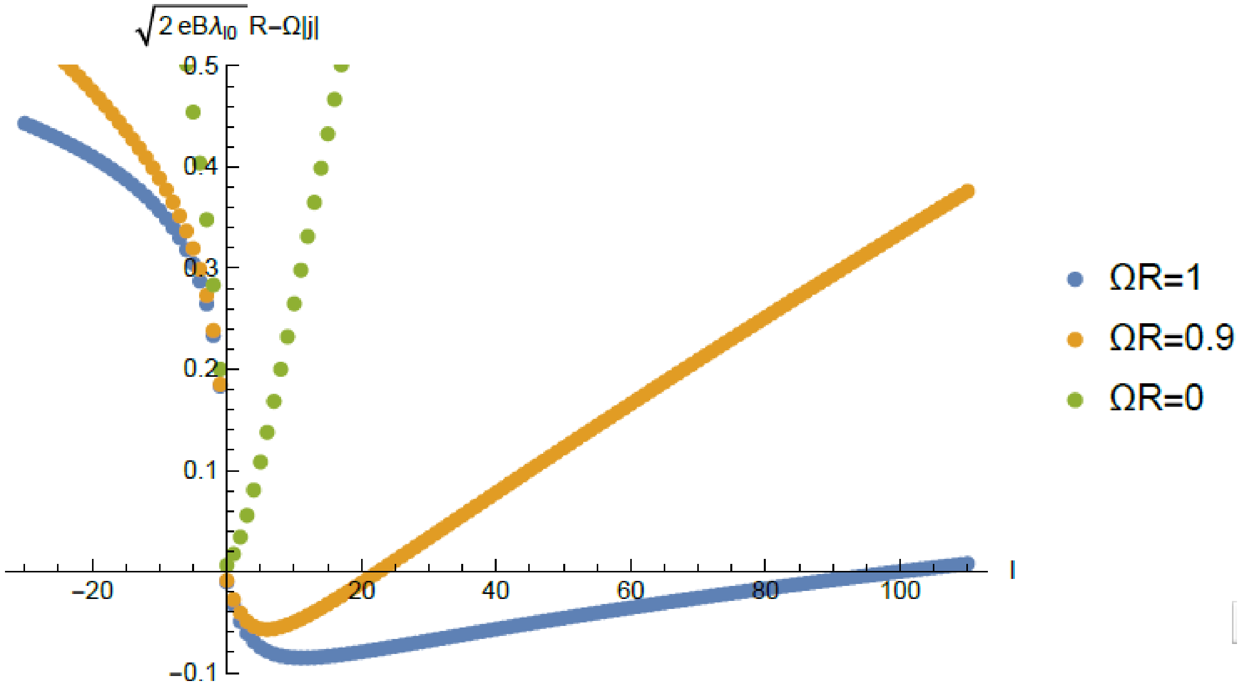
Boundary effect (with rotation)

Non-rotation part

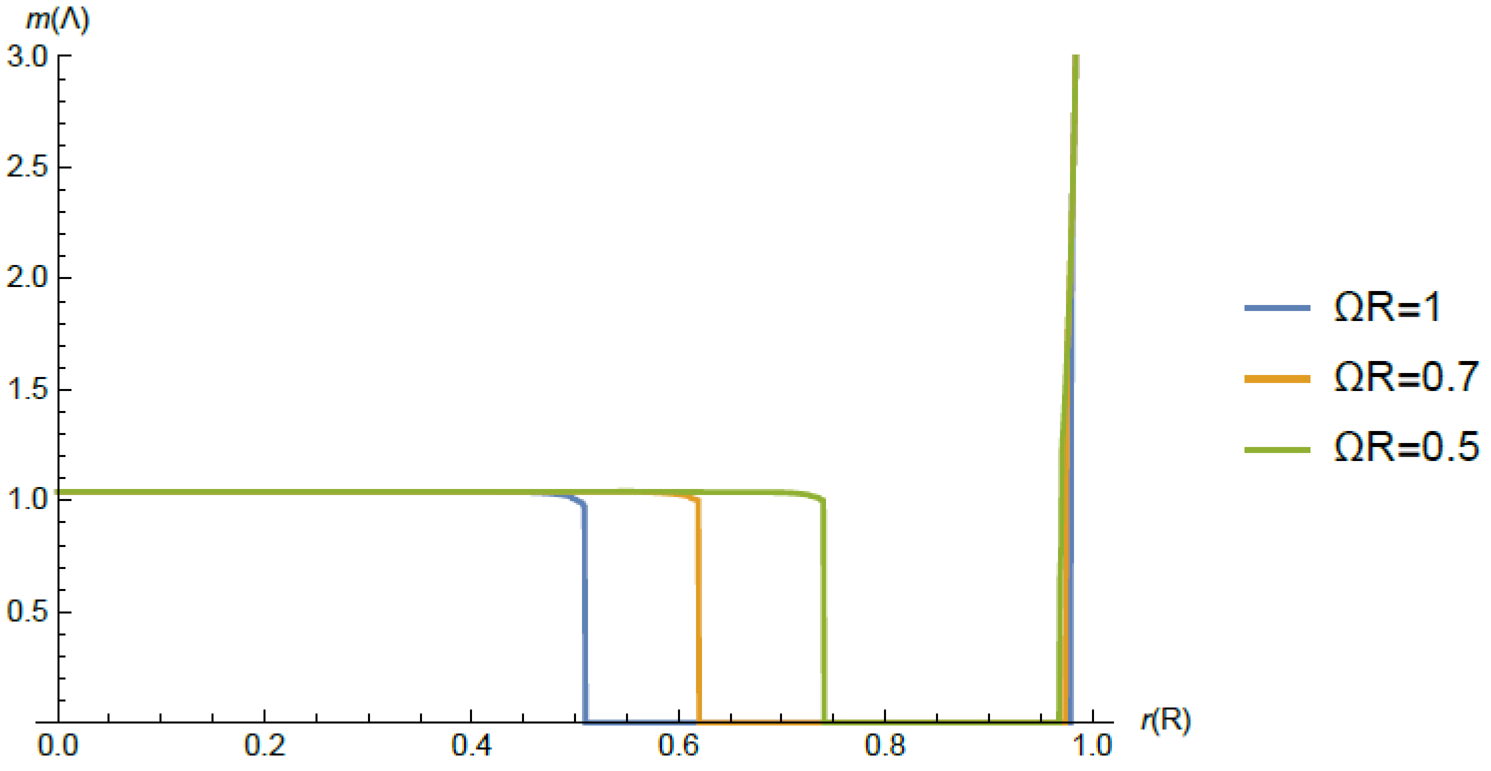
- If $eB = 0$, it can be proved: $E = E_0 - \Omega J_z > 0$

Rotation does not change condenste Ebihara S, Fukushima K, Mameda K. Phys.Lett. B764 (2017)

- If $eB \neq 0$,



Rotational Magnetic Inhibition



Conclusions and Outlook

- Rotation provide similar effect as chemical potential
  $E \to E - \Omega(L_z + S_z)$ $\qquad$ $E \to E - \mu$
- "rotational magnetic inhibition“
- Condensed matter physics system(e.g. graphene or 3D Dirac semimetal)
- Spatially inhomogeneous condensates and boundary effect
- Pion condensate

Thank you very much!

Back up

Phase Space

Magnetic Field

$$E^2 = eB(2n+1-2s_z) + p_z^2 + m$$

$$\sum_{s_z=\pm} \int \frac{dp_x dp_y}{(2\pi)^2} \Longrightarrow \frac{eB}{2\pi} \sum_{n=0}^{\infty} \alpha_n \qquad \alpha_n = \begin{cases} 1 & \text{for } n = 0 \\ 2 & \text{for } n \geq 1 \end{cases}$$

Magnetic Field + Rotation

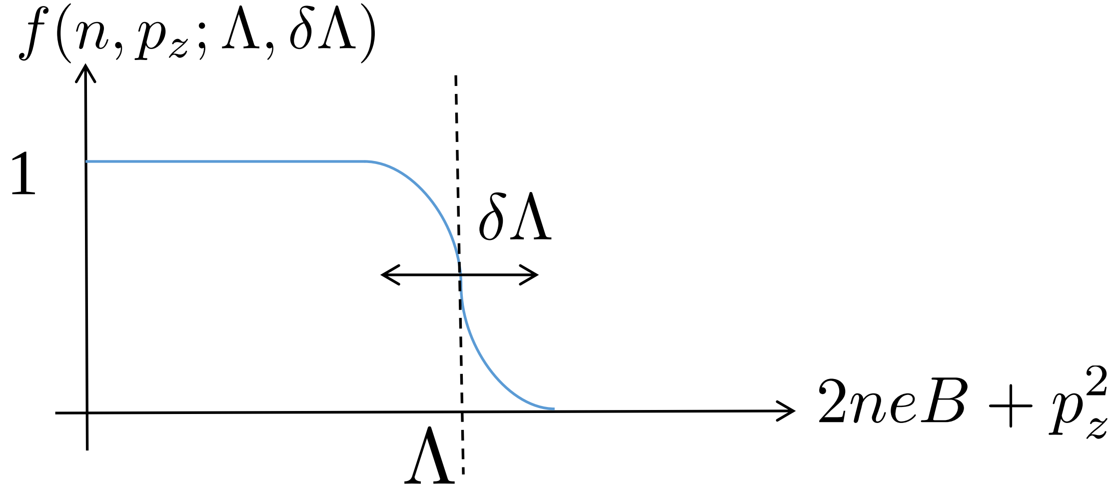
$$(E + \underline{\Omega \ell} + \Omega s_z)^2 = eB(2n+1-2s_z) + p_z^2 + m$$

$$\sum_{s_z=\pm} \int \frac{dp_x dp_y}{(2\pi)^2} \Longrightarrow \frac{1}{S} \sum_{n=0}^{\infty} \alpha_n \sum_{\ell=-n}^{N-n}$$

Landau degeneracy factor $\quad N = \dfrac{eBS}{2\pi}$

Regularization Scheme

- Naïve cutoff for n & p_z



- Not gauge invariant but **Qualitatively correct** (as long as the regulator is smooth) Gorbar, Miransky, Shovkovy (2011)

Fermions in Curved Spacetime

$$S = \int dx \sqrt{-\det g}\bar{\psi}\Big[i\gamma^\mu(D_\mu + \Gamma_\mu) - m\Big]\psi$$

Affine connection $\Gamma_\mu = -\frac{i}{4}\sigma^{ij} g_{\alpha\beta} e_i^\alpha \left(\partial_\mu e_j^\beta + \Gamma^\beta_{\nu\mu} e_j^\mu\right)$

Spin tensor $\sigma^{ij} = \frac{i}{4}[\gamma^i, \gamma^j]$

vierbein

$$e_0^t = e_1^x = e_2^y = e_3^z = 1 \;,$$
$$e_0^x = y\Omega \;, \quad e_0^y = -x\Omega$$

$$V^\mu = e_i^\mu V^i$$

Greek: coordinate space

Latin: tangent space

Parameters

$$R = 10^3/\Lambda \qquad eB = (0.1 - 0.2)\Lambda^2$$

Condition for the Landau quantization $1/\sqrt{eB} \ll R$

Critical coupling in the vacuum $G_c = 19.65/\Lambda^2$

Weak coupling $G = 0.622 G_c$

Strong coupling $G = 1.11 G_c$

NJL in finite density Ebert, Klimenko (1999)

Catalysis: $G < G_c$ Inhibition: $G > G_c$

Gap Equation (T = 0)

$$\frac{m}{G} = \frac{m}{\pi}\left(F_0 - F_\Omega\right)$$

Regulator

Pure-magnetic
$$F_0 \equiv \frac{eB}{2\pi}\sum_{n=0}^{\infty}\alpha_n \int_0^{\infty} \frac{dp_z\, f(p_z, n; \Lambda)}{\sqrt{p_z^2 + m^2 + 2neB}}$$

Rotational
$$F_\Omega \equiv \frac{1}{S}\sum_{n=0}^{\infty}\alpha_n \sum_{\ell=-n}^{N-n} \theta(\Omega|j| - m^2 - 2neB)\int_0^{k_{nj}} \frac{dp_z\, f(p_z, n; \Lambda)}{\sqrt{p_z^2 + m^2 + 2neB}}$$

$$k_{nj}^2 = (\Omega j)^2 - m^2 - 2neB$$

If N is very large

$$F_\Omega \; ; \; F_\mu(\mu = \mu_N) - \frac{eB}{2\pi}\sum_{n=0}^{\infty}\alpha_n \sqrt{1 - \frac{m_n^2}{\mu_N^2}}\,\theta(\mu_N - m_n)$$

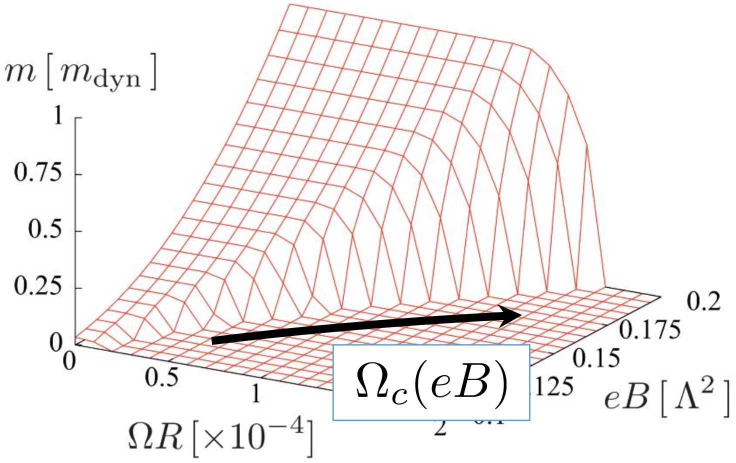
Condensed matter physics system

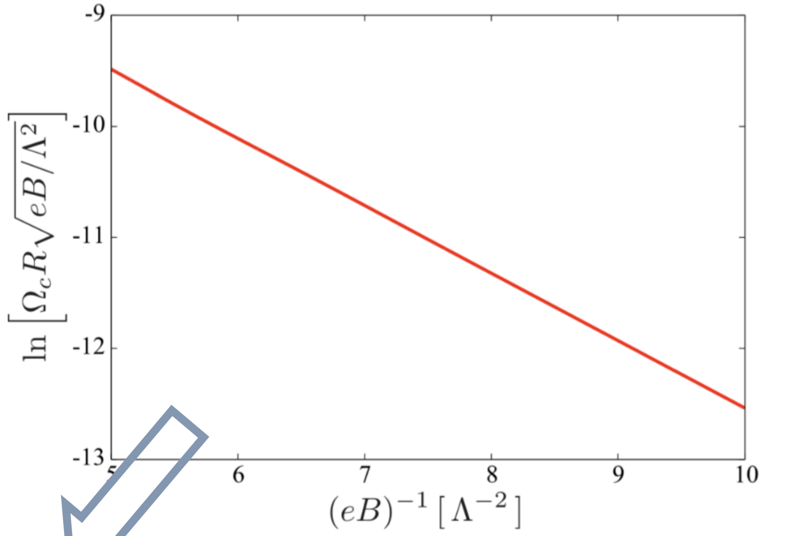
- For a material with R = 1 cm under the magnetic field B = 1.7*10^6 G, the rotational magnetic inhibition takes place around

$$\Omega \sim \sqrt{eB} v_F \,/\, N \; ; \;\; 2.5 \times 10^2 \, \mathrm{s}^{-1}$$

At Weak Coupling

$m \sim \sqrt{eB} e^{-\#/eB}$ (Magnetic Catalysis)





$$\Omega_c \sim \frac{1}{\sqrt{eB}} e^{-\#/eB}$$

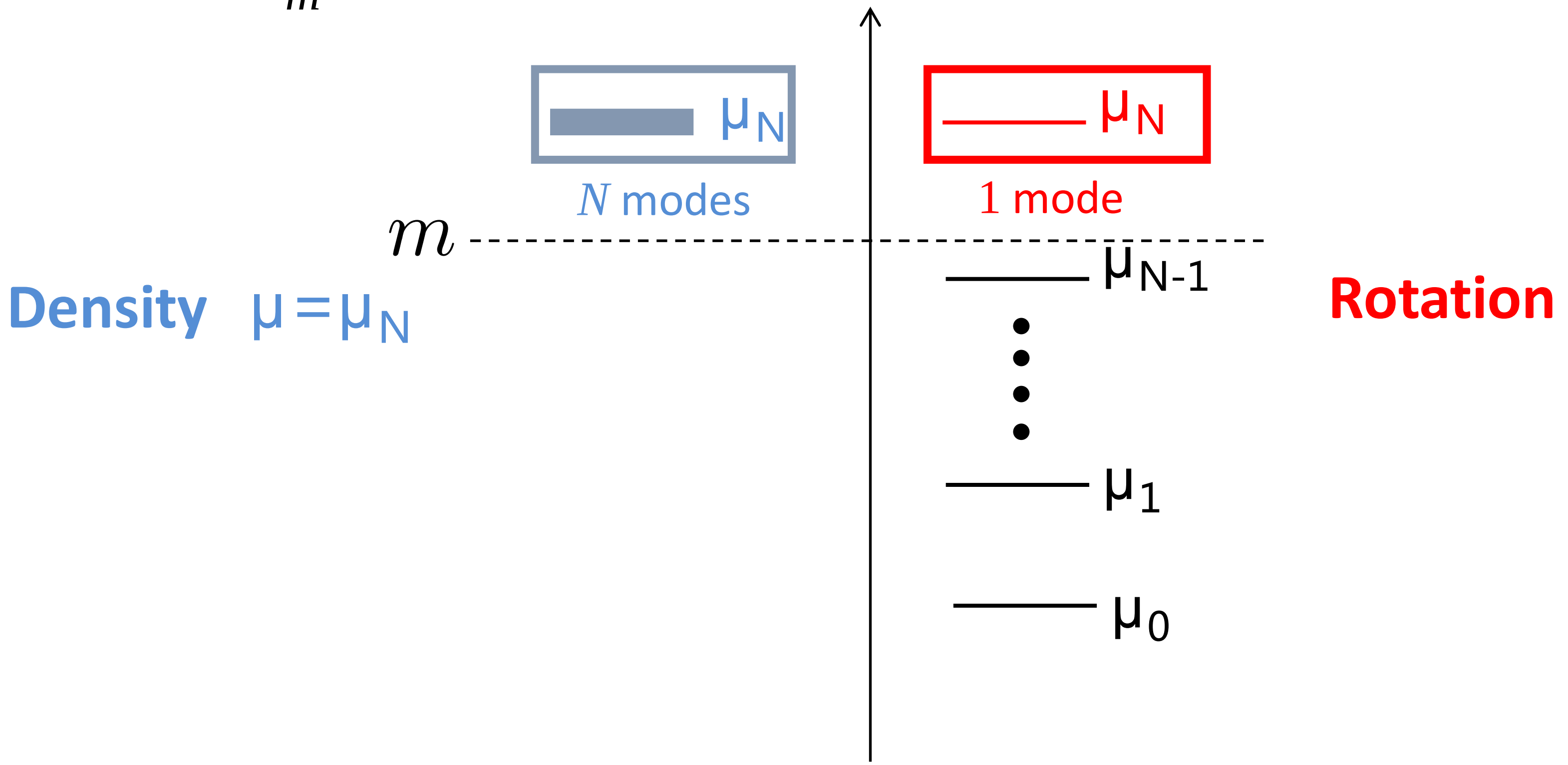
Critical angular velocity $\Omega_c \sim m/eB \sim m/N$

Weak Coupling

- Lowest Landau Level

$$F_{\mu} = \frac{eB}{2\pi} f(0,n;\Lambda)\ln\left(\frac{|\mu|+\sqrt{\mu^2-m^2}}{m}\right)\theta(|\mu|-m)$$

$$F_{\Omega} \simeq \frac{1}{S}\sum_{n=0}^{\infty}\alpha_n\sum_{l=-n}^{N-n}\theta(\Omega|j|-m)\, f(0,n;\Lambda)\ln\left(\frac{\Omega|j|+\sqrt{(\Omega j)^2-m^2}}{m}\right)$$



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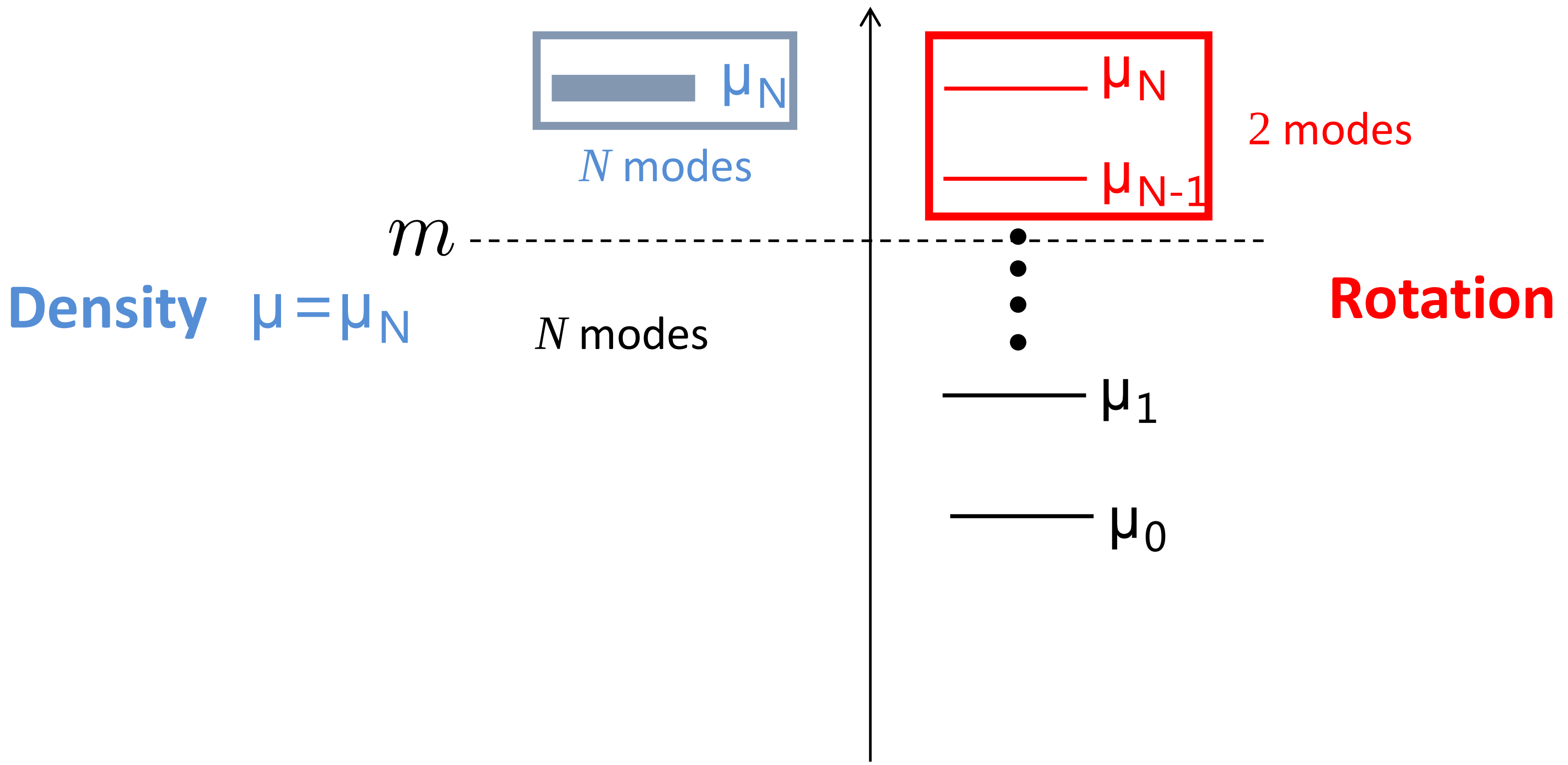

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