

Mass spectra for heavy tetraquark states

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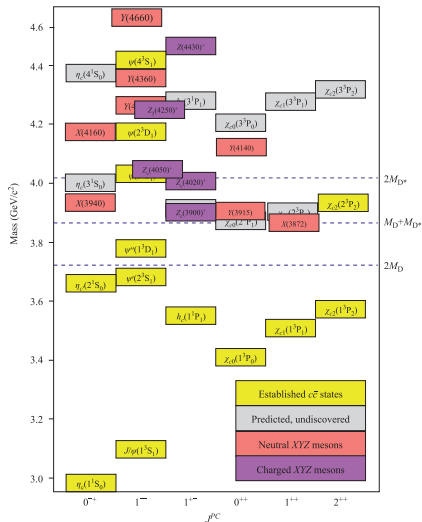
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Outline

- 1 Background of the exotic hadron states
- 2 Laplace sum rules analyses for $QQ\bar{Q}\bar{q}$ tetraquarks
- 3 Moment sum rules analyses for $QQ\bar{Q}\bar{Q}$ tetraquarks
- 4 Decay properties of the $QQ\bar{Q}\bar{Q}$ tetraquarks
- 5 Summary

Overview of XYZ States

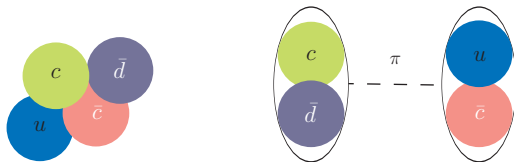


S. L. Olsen, Front. Phys. 10 (2015) 101401

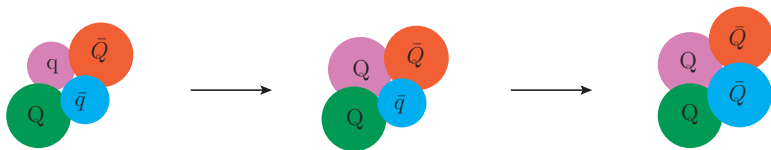
- Many charmonium-like states were discovered above the open-charm thresholds.
- Their masses and decay modes are different from the pure $c\bar{c}$ charmonium states.
- Some charged Z_c states were observed, which are evidences for four-quark states ($c\bar{c}u\bar{d}$).
- They are good candidates for exotic hadron states!

Theoretical Models

- Theoretical configurations: tetraquark, molecule, hybrid,...
- Z_c states: tetraquark, molecule



- What happens as the mass of the light quarks is raised? Binding becomes stronger?

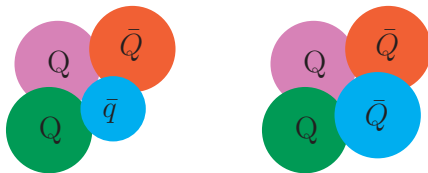


- QED analog: molecular positronium Ps_2 (bound state of $e^+e^-e^+e^-$) discovered in 2007 [Nature 449 \(09, 2007\) 195–197](#).

Triply and fully heavy tetraquarks

$QQ\bar{Q}\bar{q}$ and $QQ\bar{Q}\bar{Q}$ tetraquark states:

- They are **far away** from the mass range of the observed conventional $q\bar{q}$ hadrons and XYZ states.
- Can be clearly distinguished experimentally from the normal states.
- The light mesons ($\pi, \rho, \omega, \sigma \dots$) can not be exchanged between two charmonia/bottomonia.
- The binding force comes from the short-range gluon exchange.
- A molecule configuration is not favored and thus the $QQ\bar{Q}\bar{Q}$ is a good candidate for compact tetraquark.



Background of heavy tetraquarks

Experimental events:

- Associated production of $\Upsilon(1S)D^0$, $\Upsilon(2S)D^0$, $\Upsilon(1S)D^+$, $\Upsilon(2S)D^+$ and $\Upsilon(1S)D_s^+$: JHEP 07, 052 (2016) (LHCb).
- $J/\psi J/\psi$ pairs: Phys. Lett. B707, 52 (2012) (LHCb); JHEP 1409, 094(2014) (CMS); Phys. Rev. D90, 111101 (2014) (D0).
- $J/\psi \Upsilon(1S)$ events: Phys. Rev. Lett. 116, 082002 (2016) (D0); K. Dilsiz's talk at APS April Meeting 2016 on behalf of CMS, see <https://absuploads.aps.org/presentation.cfm?pid=11931>.
- $\Upsilon(1S)\Upsilon(1S)$ pairs: JHEP 05, 013 (2017) (CMS).

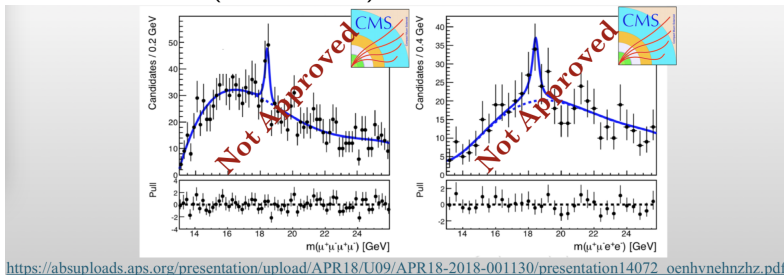
Theoretical works:

- Quark-Gluon models: Prog. Theor. Phys. 54, 492 (1975); Zeit. Phys. C7, 317 (1981).
- Potential model: Phys.Rev. D25, 2370 (1982); Phys. Lett. B123, 449 (1983).
- MIT bag model: Phys. Rev. D32, 755 (1985).
- Hyperspherical harmonic formalism: Phys. Rev. D73, 054004 (2006).
- BS or Schroedinger Eqs: Phys.Rev.D86, 034004 (2012); Phys.Lett.B718, 545 (2012).
- Recent studies: arXiv:1605.01134; 1612.00012; Phys.Lett. B773 (2017) 247-251; PRD95, 034011 (2017); EPJC77, 432 (2017); arXiv:1706.07553; 1709.09605; 1710.02540; 1710.03236.

Background of heavy tetraquarks

Search for $X_{bb\bar{b}\bar{b}}$ in CMS:

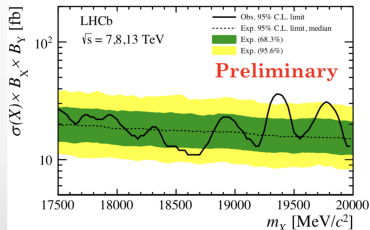
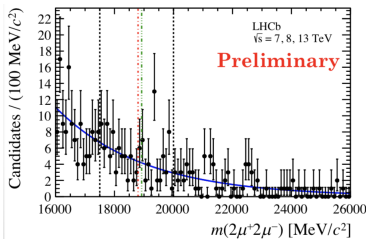
- Some recent theoretical works predict the existence of the stable $bb\bar{b}\bar{b}$ tetraquark state below $\eta_b\eta_b$ threshold.
- Preliminary observation of a peak around 18.4 GeV in four leptons channel at CMS (Thesis work).



- If it is a real physics effect, such a $X_{bb\bar{b}\bar{b}}$ state will be the first new hadron state below two-meson threshold.

Background of heavy tetraquarks

Search for X_{bbbb} in LHCb (LHCb-PAPER-2018-027):



- No significant excess was seen at LHCb at any mass hypothesis in [16, 26] GeV.

QCD Sum Rules

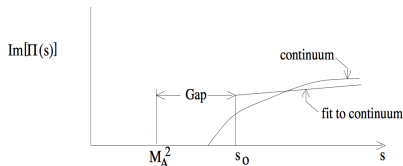
- Study **two-point correlation function** of current $J_\mu(x)$ with the same quantum numbers with hadron state:

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle \Omega | T[J_\mu(x) J_\nu^\dagger(0)] | \Omega \rangle$$

- Classify states $|X\rangle$ by coupling to current $\langle \Omega | J_\mu(x) | X \rangle \neq 0$
- **Hadron level:** described by the **dispersion relation**

$$\Pi(q^2) = \frac{(q^2)^N}{\pi} \int \frac{\text{Im}\Pi(s)}{s^N(s - q^2 - i\epsilon)} ds + \sum_{n=0}^{N-1} b_n (q^2)^n,$$

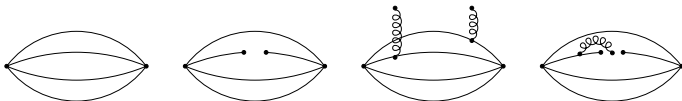
$$\rho(s) = \frac{1}{\pi} \text{Im}\Pi(s) = \sum_n \delta(s - m_n^2) \langle 0 | J | n \rangle \langle n | J^\dagger | 0 \rangle$$



QCD Sum Rules

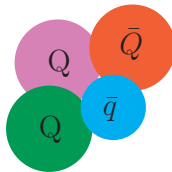
- **Quark-gluon level:** evaluated via **operator product expansion(OPE)**

$$\rho(s) = \rho^{pert}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle GG \rangle}(s) + \rho^{\langle \bar{q}Gq \rangle}(s) + \dots,$$



- Apply **Borel transform** to correlation functions
- **Quark-hadron duality:** **Laplace Sum Rules** with QCD spectral function

$$\mathcal{L}_k(s_0, M_B^2) = \int_{4m_Q^2}^{s_0} ds e^{-s/M_B^2} \rho(s) s^k = f_X^2 m_X^{2k} e^{-m_X^2/M_B^2}.$$



Interpolating currents for $QQ\bar{Q}\bar{q}$ tetraquarks

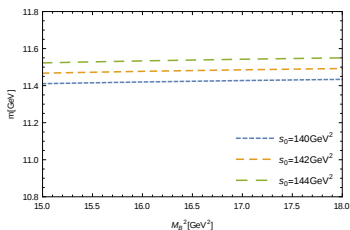
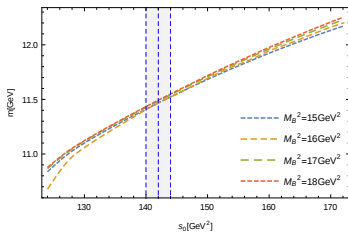
- Interpolating currents with $J^P = 0^+$:

$$\begin{aligned}J_1 &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T + \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T), \\J_2 &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma^\mu C \bar{q}_b^T + \bar{Q}_{3b} \gamma^\mu C \bar{q}_a^T), \\J_3 &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T - \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T), \\J_4 &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma^\mu C \bar{q}_b^T - \bar{Q}_{3b} \gamma^\mu C \bar{q}_a^T).\end{aligned}$$

- Interpolating currents with $J^P = 1^+$:

$$\begin{aligned}J_{1\mu} &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_\mu C \bar{q}_b^T + \bar{Q}_{3b} \gamma_\mu C \bar{q}_a^T), \\J_{2\mu} &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T + \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T), \\J_{3\mu} &= Q_{1a}^T C \gamma_5 Q_{2b} (\bar{Q}_{3a} \gamma_\mu C \bar{q}_b^T - \bar{Q}_{3b} \gamma_\mu C \bar{q}_a^T), \\J_{4\mu} &= Q_{1a}^T C \gamma_\mu Q_{2b} (\bar{Q}_{3a} \gamma_5 C \bar{q}_b^T - \bar{Q}_{3b} \gamma_5 C \bar{q}_a^T).\end{aligned}$$

For the scalar $QQ\bar{Q}\bar{q}$ tetraquarks:



System	Currents	s_0 (GeV ²)	$M_{B,min}^2, M_{B,max}^2$	m (GeV)
$bc\bar{b}\bar{q}$	j_1	133	[15, 18]	11.4 ± 0.6
	j_2	136	[15, 18]	11.3 ± 0.3
	j_3	142	[15, 18]	11.5 ± 0.3
	j_4	141	[15, 18]	11.4 ± 0.4
$cc\bar{b}\bar{q}$	j_1	70	[8, 11]	8.0 ± 0.3
	j_4	73	[8, 11]	8.2 ± 0.3
$cc\bar{c}\bar{q}$	j_1	29	[5, 8]	5.1 ± 0.2
	j_4	30	[5, 8]	5.1 ± 0.2
$bb\bar{b}\bar{q}$	j_1	189	[16, 19]	13.5 ± 0.4
	j_4	198	[16, 19]	13.7 ± 0.3

W. Chen, J. F. Jiang, S. L. Zhu, Phys.Rev. D96 (2017), 094022.

For the axial-vector $QQ\bar{Q}\bar{q}$ tetraquarks: (Phys.Rev. D96 (2017), 094022)

System	Currents	s_0 (GeV ²)	$M_{B,min}^2, M_{B,max}^2$	m (GeV)
$bcb\bar{q}$	$j_{1\mu}$	142	[15, 18]	11.5 ± 0.5
	$j_{2\mu}$	136	[15, 18]	11.3 ± 0.4
	$j_{3\mu}$	144	[15, 18]	11.6 ± 0.4
	$j_{4\mu}$	140	[15, 18]	11.4 ± 0.5
$cc\bar{b}\bar{q}$	$j_{1\mu}$	71	[8, 11]	8.1 ± 0.3
	$j_{4\mu}$	73	[8, 11]	8.2 ± 0.3
$cc\bar{c}\bar{q}$	$j_{1\mu}$	30	[5, 8]	5.1 ± 0.2
	$j_{4\mu}$	30	[5, 8]	5.1 ± 0.2
$bb\bar{b}\bar{q}$	$j_{1\mu}$	189	[16, 19]	13.5 ± 0.4
	$j_{4\mu}$	189	[16, 19]	13.5 ± 0.4

- Both the $J^P = 0^+$ and 1^+ triply bottomed $bb\bar{b}\bar{q}$ tetraquarks are lower than the thresholds $T_{\eta_b B} = 14.68\text{GeV}$ and $T_{\eta_b B^*} = 14.72\text{GeV}$.
- They cannot decay into a bottomonia plus a $B^{(*)}$ meson.
- The process $bb\bar{b}\bar{q} \rightarrow (b\bar{q}) + (q\bar{q})$ may contribute significantly.
- The electromagnetic and weak decays are important, such as $bb\bar{b}\bar{q} \rightarrow \bar{B}^{(*)}\gamma, J/\psi\Upsilon K$ processes.

Moment Sum Rules

- Define **moments** in Euclidean region $Q^2 = -q^2 > 0$:

$$\begin{aligned} M_n(Q_0^2) &= \frac{1}{n!} \left(-\frac{d}{dQ^2} \right)^n \Pi(Q^2) |_{Q^2=Q_0^2} \\ &= \int_{m_H^2}^{\infty} \frac{\rho(s)}{(s + Q_0^2)^{n+1}} ds = \frac{f_X^2}{(m_X^2 + Q_0^2)^{n+1}} [1 + \delta_n(Q_0^2)], \end{aligned}$$

where $\delta_n(Q_0^2)$ contains the higher states and continuum.

- Ratio of the moments

$$r(n, Q_0^2) \equiv \frac{M_n(Q_0^2)}{M_{n+1}(Q_0^2)} = (m_X^2 + Q_0^2) \frac{1 + \delta_n(Q_0^2)}{1 + \delta_{n+1}(Q_0^2)}.$$



Limitations for (n, ξ) parameter space:

$$\xi = Q_0^2/16m_c^2, \text{ for } cc\bar{c}\bar{c} \text{ system};$$

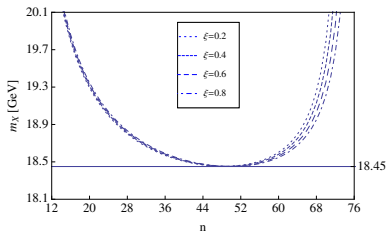
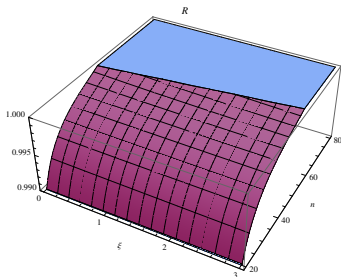
$$\xi = Q_0^2/m_b^2, \text{ for } bb\bar{b}\bar{b} \text{ system}.$$

- **Small ξ** : higher dimensional condensates give large contributions to $M_n(Q_0^2)$, leading to bad OPE convergence.
- **Large ξ** : slower convergence of $\delta_n(Q_0^2)$. This can be compensated by taking higher derivative n for the lowest lying resonance to dominate.
- **Large n** : moving further away from the asymptotically free region. The OPE convergence would also become bad.
- Requiring $\Pi^{\langle GG \rangle}(s) \leq \Pi^{pert}(s)$ to obtain an upper limit n_{max} , which will increase with respect to ξ .
- **Good (n, ξ) region**: the lowest lying resonance dominates the moments while the OPE series has good convergence.

$$n_{max} = 75, 76, 77, 78 \text{ for } \xi = 0.2, 0.4, 0.6, 0.8$$

- Mass for scalar $bb\bar{b}\bar{b}$ tetraquark: mass curves have plateaus at $(n, \xi) = (48, 0.2), (49, 0.4), (49, 0.6), (50, 0.8)$.
- Hölder's inequality:

$$R = \frac{M_n(Q_0^2)^2}{M_r(Q_0^2)M_{2n-r}(Q_0^2)} \leq 1$$



$$m_{X_{bb\bar{b}\bar{b}}} = (18.45 \pm 0.15) \text{ GeV}.$$

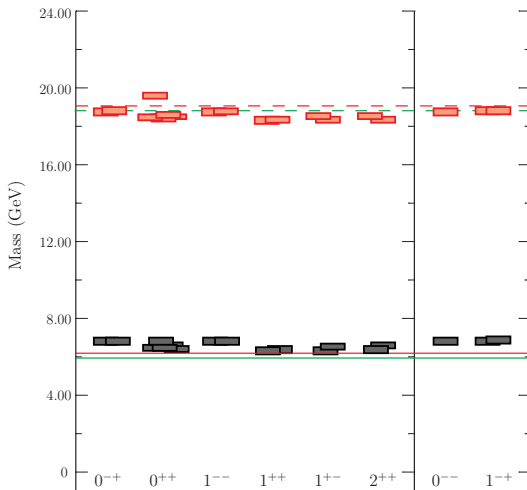
W. Chen, et. al, Phys.Lett. B773 (2017) 247-251.

Mass spectra for the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ tetraquarks:

J^{PC}	Currents	$m_{X_c}(\text{GeV})$	$m_{X_b}(\text{GeV})$
0^{++}	J_1	6.44 ± 0.15	18.45 ± 0.15
	J_2	6.59 ± 0.17	18.59 ± 0.17
	J_3	6.47 ± 0.16	18.49 ± 0.16
	J_4	6.46 ± 0.16	18.46 ± 0.14
	J_5	6.82 ± 0.18	19.64 ± 0.14
1^{++}	$J_{1\mu}^+$	6.40 ± 0.19	18.33 ± 0.17
	$J_{2\mu}^+$	6.34 ± 0.19	18.32 ± 0.18
1^{+-}	$J_{1\mu}^-$	6.37 ± 0.18	18.32 ± 0.17
	$J_{2\mu}^+$	6.51 ± 0.15	18.54 ± 0.15
2^{++}	$J_{1\mu\nu}$	6.51 ± 0.15	18.53 ± 0.15
	$J_{2\mu\nu}$	6.37 ± 0.19	18.32 ± 0.17
0^{-+}	J_1^+	6.84 ± 0.18	18.77 ± 0.18
	J_2^+	6.85 ± 0.18	18.79 ± 0.18
0^{--}	J_1^-	6.84 ± 0.18	18.77 ± 0.18
1^{-+}	$J_{1\mu}^+$	6.84 ± 0.18	18.80 ± 0.18
	$J_{2\mu}^+$	6.88 ± 0.18	18.83 ± 0.18
1^{--}	$J_{1\mu}^-$	6.84 ± 0.18	18.77 ± 0.18
	$J_{2\mu}^-$	6.83 ± 0.18	18.77 ± 0.16

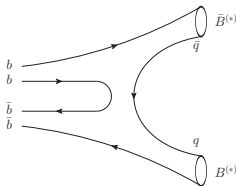
W. Chen, et. al, Phys.Lett. B773 (2017) 247-251.

Spontaneous dissociation thresholds:



Decay behavior: $bb\bar{b}\bar{b}$ tetraquarks

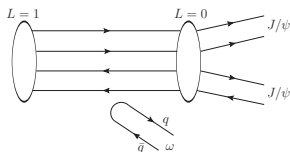
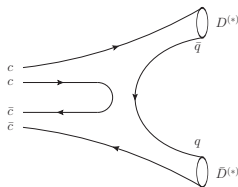
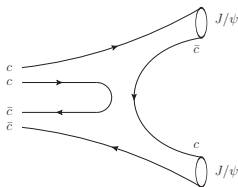
- $X_{bb\bar{b}\bar{b}} \rightarrow (b\bar{b}) + (b\bar{b})$: **kinematically forbidden**.
- $X_{bb\bar{b}\bar{b}} \rightarrow (bbq) + (\bar{b}\bar{b}\bar{q})$: **kinematically forbidden**.
- $X_{bb\bar{b}\bar{b}} \rightarrow (bqq) + (\bar{b}\bar{q}\bar{q})$: **suppressed** by two light quark pair creation.
- $X_{bb\bar{b}\bar{b}} \rightarrow (q\bar{b}) + (b\bar{q})$: **possible** in $B^{(*)}\bar{B}^{(*)}$ final states, with large phase space.
- $X_{bb\bar{b}\bar{b}} \rightarrow (b\bar{b}) + \gamma$: electromagnetic decay via $b\gamma_\mu\bar{b} \rightarrow \gamma$.
- $X_{bb\bar{b}\bar{b}} \rightarrow \Upsilon(1S)X \rightarrow l^+l^-l^+l^-$: multi-lepton final states could provide clean signals although the branching fraction may be small.



- **These $bb\bar{b}\bar{b}$ states are expected to be very narrow.** They are good candidates for compact tetraquarks, if they do exist.

Decay behavior: $cc\bar{c}\bar{c}$ tetraquarks

- $cc\bar{c}\bar{c} \rightarrow (ccq) + (\bar{c}\bar{c}\bar{q})$: kinematically forbidden.
- $cc\bar{c}\bar{c} \rightarrow (cq\bar{q}) + (\bar{c}\bar{q}\bar{q})$: suppressed by two light quark pair creation.
- $cc\bar{c}\bar{c} \rightarrow (c\bar{c}) + (c\bar{c})$: charm quark pair rearrangement or annihilation (suppressed). Phase space is small.
- $cc\bar{c}\bar{c} \rightarrow (q\bar{c}) + (c\bar{q})$: possible via a heavy quark pair annihilation and a light quark pair creation, with large phase space.
- $cc\bar{c}\bar{c}(L=1) \rightarrow cc\bar{c}\bar{c}(L=0) + (q\bar{q})_{L=0}$: OZI forbidden.



Spontaneous dissociations

J^{PC}	S-wave	P-wave
0^{++}	$\eta_c(1S)\eta_c(1S), J/\psi J/\psi$	$\eta_c(1S)\chi_{c1}(1P), J/\psi h_c(1P)$
0^{-+}	$\eta_c(1S)\chi_{c0}(1P), J/\psi h_c(1P)$	$J/\psi J/\psi$
0^{--}	$J/\psi \chi_{c1}(1P)$	$J/\psi \eta_c(1S)$
1^{++}	$J/\psi J/\psi$	$J/\psi h_c(1P), \eta_c(1S)\chi_{c1}(1P),$ $\eta_c(1S)\chi_{c0}(1P)$
1^{+-}	$J/\psi \eta_c(1S)$	$J/\psi \chi_{c0}(1P), J/\psi \chi_{c1}(1P),$ $\eta_c(1S)h_c(1P)$
1^{-+}	$J/\psi h_c(1P), \eta_c(1S)\chi_{c1}(1P)$	—
1^{--}	$J/\psi \chi_{c0}(1P), J/\psi \chi_{c1}(1P),$ $\eta_c(1S)h_c(1P)$	$J/\psi \eta_c(1S)$

Summary

- We have calculated the mass spectra for the triply heavy $QQ\bar{Q}\bar{q}$ and fully heavy $QQ\bar{Q}\bar{Q}$ tetraquark states.
- The $X_{bb\bar{b}\bar{q}}$ and $X_{bb\bar{b}\bar{b}}$ states are below the $\eta_b B^{(*)}$ and $\eta_b \eta_b$ thresholds respectively, and thus are expected to be narrow. They are good candidate compact tetraquarks.
- The $X_{bb\bar{b}\bar{q}}$ states could be searched for in final states $B^{(*)} + \text{light meson}$, $B^{(*)}\gamma$, $J/\psi \Upsilon K$.
- The possible CMS observation of $X_{bb\bar{b}\bar{b}}$ may be the first new hadron state below two-meson threshold, if it is a real physics effect.
- The $X_{bb\bar{b}\bar{b}}$ states could be searched for in final states $B^{(*)}\bar{B}^{(*)}$, bottomonia $+\gamma$, $l^+ l^- l^+ l^-$.

Thank you for your attention!