πN Drell-Yan process within TMD factorization

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Based on **Xiaoyu Wang**, Zhun Lu, Ivan Schmidt, JHEP 1708 (2017) 137
Xiaoyu Wang and Zhun Lu , Phys. Rev. D 97, 054005 (2018)

OUTLINE

Introduction

- Parton Distribution Functions(PDFs)
  - Leading twist: $f_1(x), g_1(x), h_1(x)$ describe the quark structure of hadrons
  - Only have one longitudinal freedom x, *i.e.*, quarks are perfectly collinear
- Transverse Momentum Dependent(TMD) PDFs
  - Admit a finite quark transverse momentum $k_\perp$
  - 3D internal picture of hadrons
  - Correlation between parton momentum and hadron spin

PDF

Introduction

- TMD factorization and TMD evolutions

- TMD factorization frame:

  valid in the region $q_\perp \ll Q$

  observables :convolutions of hard factor and well-defined TMD PDFs/FFs

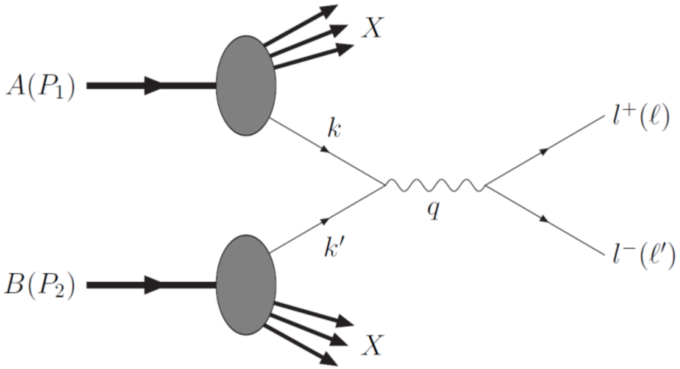
- TMD evolution :

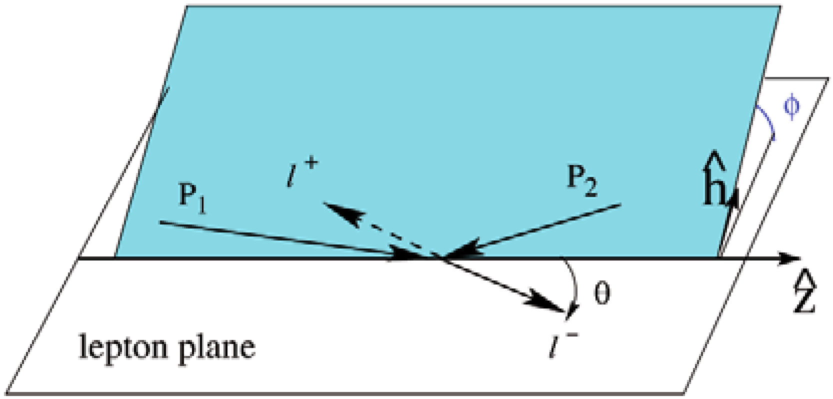
  convenient to perform in b-space (conjugate to $k_\perp$ via FT)

Introduction

◆ Drell-Yan process

$$A(P_1) + B(P_2) \to l^+(\ell) + l^-(\ell') + X\,,$$





The Collins-Soper frame

Process under study

- General form of the cross section
  (Beam: unpolarized Target: transversely polarized)

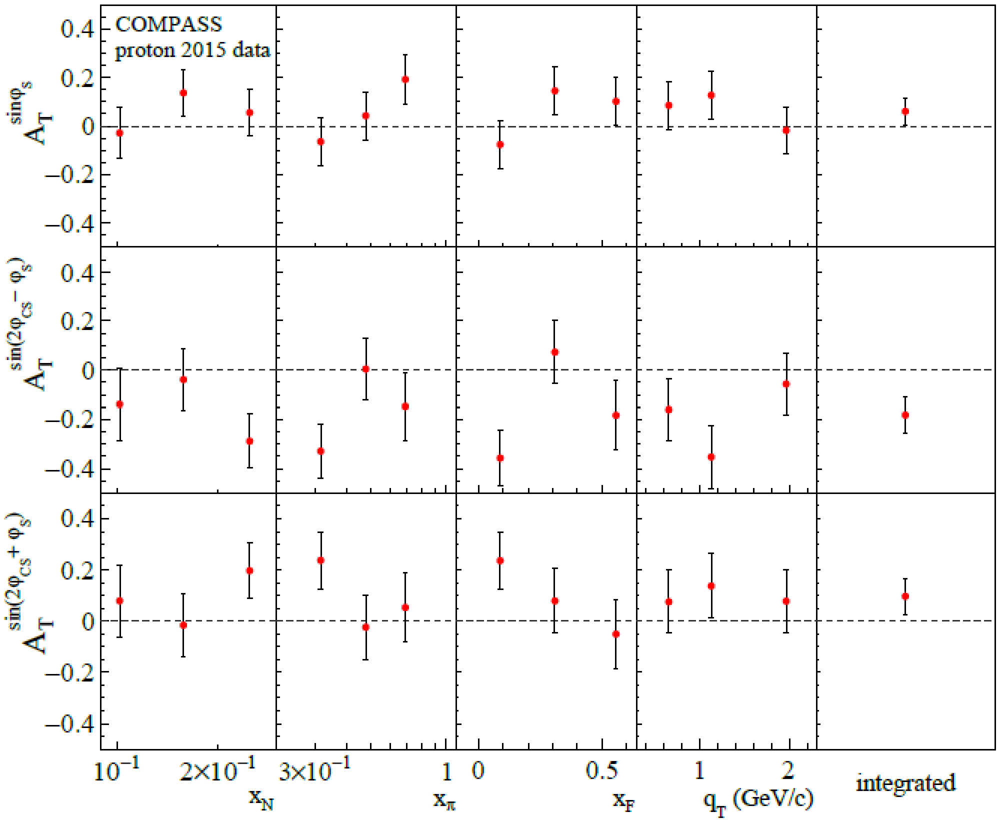
$$
\frac{d\sigma}{d^4 q d\Omega} \stackrel{\text{LO}}{=} \frac{\alpha^2}{Fq^2}\hat{\sigma_U}\Bigg\{ \Big(1+\cos^2(\theta)+\sin^2(\theta)A_{UU}^{\cos(2\phi)}\cos(2\phi)\Big)
+S_T\Big[(1+\cos^2(\theta))A_{UT}^{\sin(\phi_S)}\sin(\phi_S)
+\sin^2(\theta)\Big(A_{UT}^{\sin(2\phi+\phi_S)}\sin(2\phi+\phi_S)+A_{UT}^{\sin(2\phi-\phi_S)}\sin(2\phi-\phi_S)\Big)\Big]\Bigg\}
$$

Introduction

- The asymmetries

				Beam	Target
$A_{UU}^{\cos(2\phi)}$	$\propto h_{1,\pi}^{\perp q}$	$\otimes$	$h_{1,p}^{\perp q}$	Boer-Mulders	Boer-Mulders
$A_{UT}^{\sin(\phi_S)}$	$\propto f_{1,\pi}^{q}$	$\otimes$	$f_{1T,p}^{\perp q}$	$f_{1,\pi}^{q}$	Sivers
$A_{UT}^{\sin(2\phi-\phi_S)}$	$\propto h_{1,\pi}^{\perp q}$	$\otimes$	$h_{1,p}^{q}$	Boer-Mulders	Transversity
$A_{UT}^{\sin(2\phi+\phi_S)}$	$\propto h_{1,\pi}^{\perp q}$	$\otimes$	$h_{1T,p}^{\perp q}$	Boer-Mulders	Pretzelosity

Introduction



Phys. Rev. Lett. 119, 112002 (2017)

Asymmetries

Unpolarized process

- The kinematical variables defined as

$$
\begin{aligned}
s &= (P_a + P_b)^2, && \text{the total centre-of-mass energy squared,} \\
x_{a(b)} &= q^2/(2P_{a(b)} \cdot q), && \text{the momentum fraction carried by a parton from } H_{a(b)}, \\
x_F &= x_a - x_b, && \text{the Feynman variable,} \\
M_{\mu\mu}^2 &= Q^2 = q^2 = s\ x_a\ x_b, && \text{the invariant mass squared of the dimuon.}
\end{aligned}
$$

$$\tau = Q^2/s = x_\pi x_p, \qquad y = \frac{1}{2}\ln\frac{q^+}{q^-} = \frac{1}{2}\ln\frac{x_\pi}{x_p}, \qquad x_{\pi/p} = \frac{\pm x_F + \sqrt{x_F^2 + 4\tau}}{2}, \qquad x_{\pi/p} = \sqrt{\tau}e^{\pm y}.$$

Variables definition

Unpolarized process

- The differential cross section for the unpolarized π^--proton Drell-Yan process has the form

J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B 250 (1985) 199

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{\boldsymbol{q}}_\perp \cdot \vec{\boldsymbol{b}}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q, q_\perp),$$

- $\sigma_0 = \frac{4\pi\alpha_{em}^2}{3N_C s Q^2}$ is the cross section at tree level with $N_c = 3$
- The structure function in the first term with $\widetilde{W}_{UU}(Q;b)$ is dominant at the low $q_\perp \ll$ Q value
- Y_{UU} term provides necessary correction at moderate $q_\perp \sim$ Q value, which was neglected in this work

General form of differential cross section

Unpolarized process

- The structure function $\widetilde{W}_{UU}$ can be written as

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu)\sum_{q,\bar{q}} e_q^2 \tilde{f}^{\rm sub}_{1\,\bar{q}/\pi}(x_\pi, b; \mu, \zeta_F)\tilde{f}^{\rm sub}_{1\,q/p}(x_p, b; \mu, \zeta_F),$$

- $\tilde{f}^{\rm sub}_{1\,q/H}$ is the subtracted distribution function in the b-space and universal.
- $H_{UU}(Q;\mu)$is the factor associated with hard scattering and scheme-dependent.
- The way to subtract the soft factor in the distribution function depends on the scheme to regulate the light-cone singularity in the TMD definition.

Unpolarized process

- The TMD evolution equation for the ζ_F dependence is encoded in a Collins-Soper (CS) equation through

$$\frac{\partial \ \ln \tilde{f}^{\rm sub}(x, b; \mu, \zeta_F)}{\partial \ \sqrt{\zeta_F}} = \tilde{K}(b; \mu)$$

- The TMD evolution equation for the μ dependence is encoded in a RG equation through

$$\frac{d \ \tilde{K}}{d \ \ln\mu} = -\gamma_K(\alpha_s(\mu)),$$

$$\frac{d \ \ln \tilde{f}^{\rm sub}(x, b; \mu, \zeta_F)}{d \ \ln\mu} = \gamma_F(\alpha_s(\mu); \frac{\zeta_F^2}{\mu^2}),$$

Unpolarized process

- The overall solution structure is the same as that for the Sudakov form factor.

- The energy evolution of TMDs from initial energy μ_b to another energy Q is encoded in the Sudakov-like form factor S by the exponential form exp(−S)

$$f(x, b, Q) = \mathcal{F} \times e^{-S} \times f(x, b, \mu_b)$$

The Sudakov form factor

Unpolarized process

- To combine the information at small b with that at large b, a matching procedure must be introduced.

$$b_* = b/\sqrt{1 + b^2/b_{\max}^2}$$

$b_* \approx b$ at low values of b

$b_* \approx b_{\max}$ at large b values.

- In the small b region, the TMD at fixed energy μ can be expressed as the convolution of the perturbatively calculable hard coefficients and the corresponding ordinary integrated (collinear) PDFs

$$F(x, b; \mu, \zeta_F) = \sum_i C_{q \leftarrow i} \otimes f_i(x, \mu),$$

$\mu_b = c_0/b_*$ with $c_0 = 2e^{-\gamma_E}$, $\gamma_E \approx 0.577$ being the Euler Constant

Region analysis

Unpolarized process

- The Sudakov-like form factor in can be separated into a perturbatively calculable part and a nonperturbative part

$$S = S_{\text{pert}} + S_{\text{NP}}.$$

- The Sudakov form factor S for quark and antiquark can have the following relation A. Prokudin, P. Sun and F. Yuan, Phys. Lett. B 750(2015)533

$$S^q_{\text{NP}}(Q,b) + S^{\bar{q}}_{\text{NP}}(Q,b) = S_{\text{NP}}(Q,b).$$

$$S^q_{\text{pert}}(Q,b_*) = S^{\bar{q}}_{\text{pert}}(Q,b_*) = S_{\text{pert}}(Q,b_*)/2.$$

- The perturbative part of S being

$$S_{\text{pert}}(Q,b) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[A(\alpha_s(\bar{\mu}))\ln\frac{Q^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\right].$$

Sudakov form factor

Unpolarized process

- A universal non-perturbative form factor associated with the transverse momentum dependent quark distribution functions in Drell-Yan process has arXiv: 1406.3073

$$S_{\rm NP} = g_1 b^2 + g_2 \ln\frac{b}{b_*}\ln\frac{Q}{Q_0} + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda\right).$$

$$g_1 = 0.212, \quad g_2 = 0.84, \quad g_3 = 0$$

with the initial scale $Q_0^2 = 2.4\ \mathrm{GeV}^2$, limited boundary $b_{\rm max} = 1.5\ \mathrm{GeV}^{-1}$, fixed $x_0 = 0.01$ and $\lambda = 0.2$.

$$S_{\rm NP}^{f_1^{q/p}}(Q,b) = \frac{g_2}{2}\ln\frac{b}{b_*}\ln\frac{Q}{Q_0} + \frac{g_1}{2}b^2,$$

The S_{NP} for proton TMD

Unpolarized process

- With all the ingredients above, we can obtain the TMD distribution for proton

$$\tilde{f}_1^{u/p}(x,b;Q) = e^{-\frac{1}{2}S_{\text{pert}}(Q,b_*)-S_{\text{NP}}^{f_1^{q/p}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/p}(x,\mu_b)$$

$$C_{q\leftarrow q'}(x,b;\mu,\zeta_F) = \delta_{qq'}\left[\delta(1-x)+\frac{\alpha_s}{\pi}\left(\frac{C_F}{2}(1-x)\right)\right],$$

$$C_{q\leftarrow g}(x,b;\mu,\zeta_F) = \frac{\alpha_s}{\pi}T_R x(1-x),$$

- If we perform a Fourier Transformation on $\tilde{f}_{1q/p}^{\text{sub}}(x,b;Q)$

$$f_{1q/p}(x,k_\perp;Q) = \int_0^\infty \frac{dbb}{2\pi}J_0(k_\perp b)\tilde{f}_{1q/p}^{\text{sub}}(x,b;Q),$$

Unpolarized process

- Assuming the non-perturbative Sudakov form factor $S_{\rm NP}^{f_1^{q/\pi}}(Q,b)$ for quark distribution function of π meson as

X. Wang, Z. Lu, I. Schmidt, JHEP 1708 (2017) 137

$$S_{\rm NP}^{f_1^{q/\pi}} = g_1^\pi b^2 + g_2^\pi \ln\frac{b}{b_*}\ln\frac{Q}{Q_0}.$$

- With the assumption above, we can obtain the TMD distribution for pion

$$f_1^{i/\pi}(x,b;Q) = e^{-\frac{1}{2}S_{\rm pert}(Q,b_*)-S_{\rm NP}^{f_1^{q/\pi}}(Q,b)}\mathcal{F}(\alpha_s(Q))\sum_i C_{q\leftarrow i}^{f_1}\otimes f_1^{i/\pi}(x,\mu_b)$$

$$f_{1q/\pi}(x,k_\perp;Q) = \int_0^\infty \frac{dbb}{2\pi}J_0(k_\perp b)\tilde{f}_{1q/\pi}^{\rm sub}(x,b;Q).$$

The S_{NP} for pion and pion TMD

Unpolarized process

- The structure function is as follows

$$\widetilde{W}_{UU}(Q;b) = H_{UU}(Q;\mu)\sum_{q,\bar{q}} e_q^2 \tilde{f}_{q/\pi}^{\text{sub}}(x_1,b;\mu,\zeta_F)\tilde{f}_{q/p}^{\text{sub}}(x_2,b;\mu,\zeta_F),$$

- If we absorb the hard factors H_{UU} and $\mathcal{F}(\alpha_s(Q))$ into the definition of C-coefficients, the C-coefficients become process-dependent

S. Catani, D. de Florian and M. Grazzini, Nucl. Phys. B 596 (2001) 299

$$C_{q\leftarrow q'}(x,b;\mu_b) = \delta_{qq'}\left[\delta(1-x) + \frac{\alpha_s}{\pi}\left(\frac{C_F}{2}(1-x) + \frac{C_F}{4}(\pi^2-8)\delta(1-x)\right)\right],$$

$$C_{q\leftarrow g}(x,b;\mu_b) = \frac{\alpha_s}{\pi}T_R x(1-x).$$

Unpolarized process

- The structure function W_{UU} in b-space can be written as

$$\widetilde{W}_{UU}(Q;b) = e^{-S(Q^2,b)} \times \sum_{q,\bar{q}} e_q^2 C_{q\leftarrow i} \otimes f_{i/\pi^-}(x_1,\mu_b) C_{\bar{q}\leftarrow j} \otimes f_{j/p}(x_2,\mu_b)$$

- The differential cross section is

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int_0^\infty \frac{dbb}{2\pi} J_0(q_\perp b) \times \widetilde{W}_{UU}(Q;b),$$

Differential cross section

Unpolarized process

- In E615 experiment, 252 GeV pions colliding on tungsten, and kinematics

$$0.2 < x_\pi < 1, \quad 0.04 < x_N < 1, \quad 0 < x_F < 1, \quad 4.05 \text{ GeV} < Q < 8.55 \text{ GeV}.$$

- The experimental observables measured at E615 are

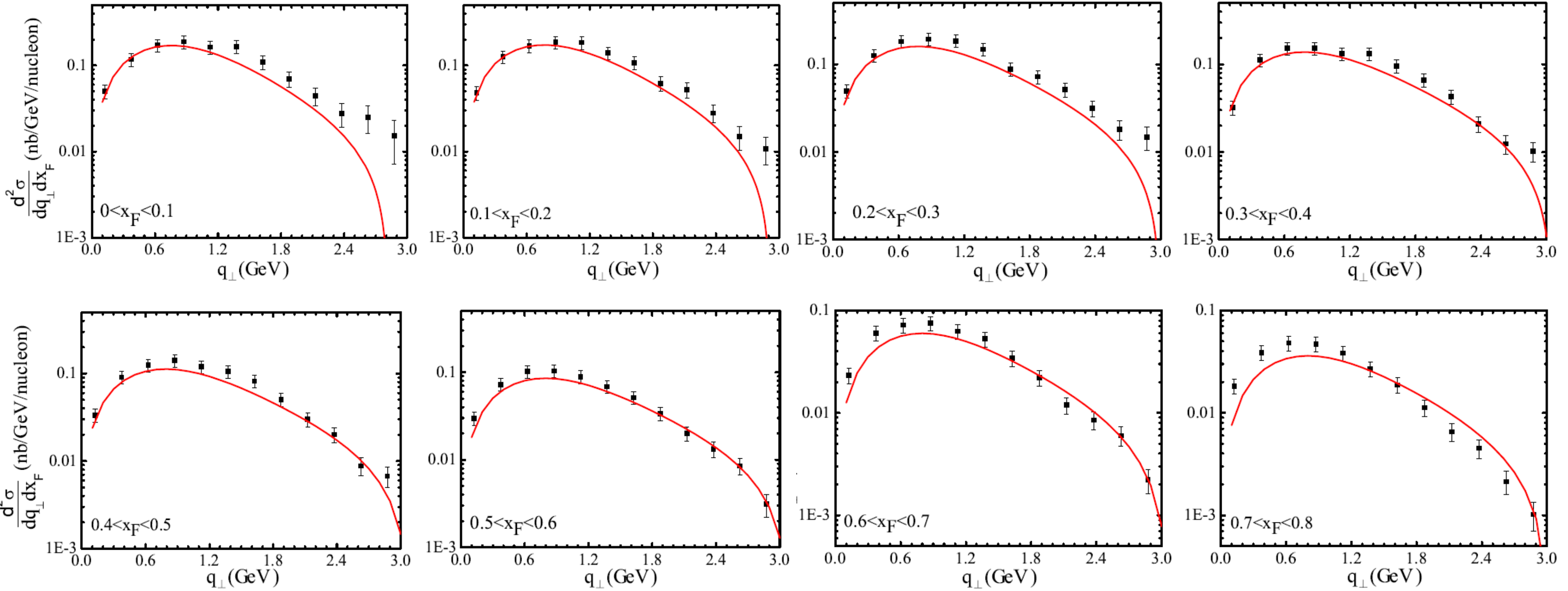
$$\frac{d^2\sigma}{dx_F dq_\perp} = \sigma_0 \frac{1}{\sqrt{x_F^2 + 4\frac{Q^2}{s}}} 2\pi q_\perp \int_{4.05^2}^{8.55^2} dQ^2 \int_0^\infty \frac{dbb}{2\pi} J_0(q_\perp b) \times \widetilde{W}_{UU}(Q;b).$$

$$g_1^\pi = 0.082 \pm 0.022, \quad g_2^\pi = 0.394 \pm 0.103,$$

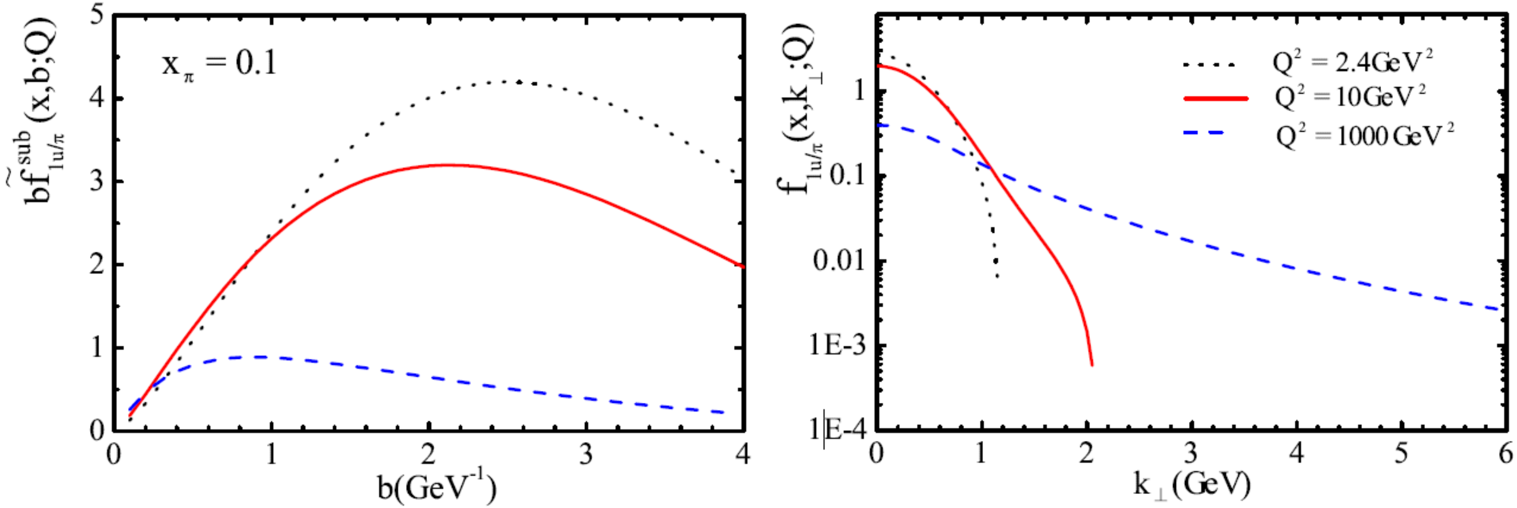
E615 differential cross section

Unpolarized process

- Fit the theoretical estimate with the experimental data from E615, we can obtain the parameters in $S_{\rm NP}^{f_1^{q/\pi}}(Q,b)$ X. Wang, Z. Lu, I. Schmidt, JHEP 1708 (2017) 137



Unpolarized process



Subtracted unpolarized TMD distribution of the pion meson for valence quarks in b-space (left panel) and $k_\perp$-space (right panel), at energies: $Q^2 = 2.4\ \text{GeV}^2$ (dotted lines), $Q^2 = 10\ \text{GeV}^2$ (solid lines) and $Q^2 = 1000\ \text{GeV}^2$ (dashed lines).

X. Wang, Z. Lu, I. Schmidt, JHEP 1708 (2017) 137

Unpolarized process

- In COMPASS experiment, 190 GeV pions colliding on NH_3, and kinematics

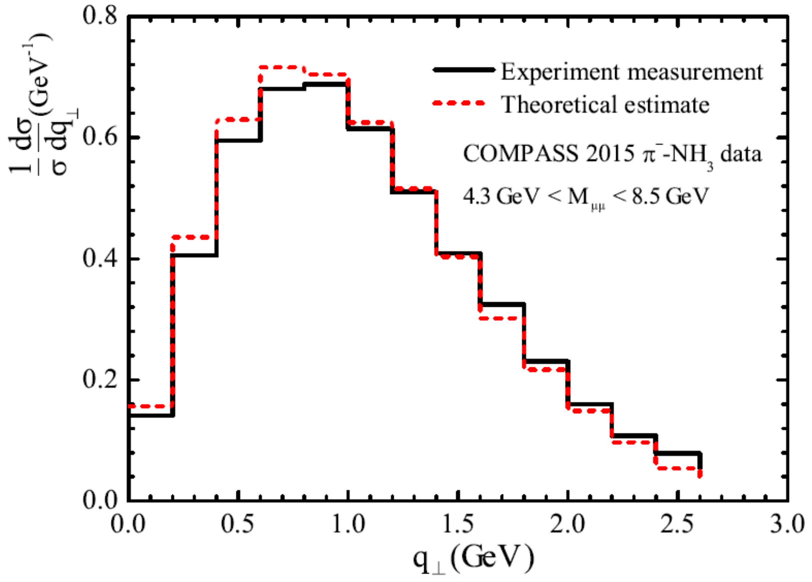
$$0.05 < x_N < 0.4, \quad 0.05 < x_\pi < 0.9, \quad 4.3\ \mathrm{GeV} < Q < 8.5\ \mathrm{GeV}.$$

- The experimental observables: normalized differential cross section

$$\frac{1}{\sigma}\frac{d\sigma}{dq_\perp} = \frac{1}{\Sigma_i N_i}\frac{N_i}{\Delta q_\perp}.$$

COMPASS

Unpolarized process



The transverse spectrum of lepton pair production in the unpolarized pion-nucleon Drell-Yan process, with an NH_3 target at COMPASS. The dashed line is our theoretical calculation using the extracted Sudakov form factor for the pion TMD PDF. The solid line shows the experimental measurement at COMPASS.

X. Wang, Z. Lu, I. Schmidt, JHEP 1708 (2017) 137

Prediction

Unpolarized process

- The theoretical is compatible with the COMPASS measurement at small $q_\perp$ region with $q_\perp \ll Q$, indicating that our approach can be used as a first step to study the Drell-Yan process at COMPASS.
- Our study may provide a better understanding on the pion TMD distribution as well as its role in Drell-Yan process.
- The framework applied in this work can also be extended to the study of the azimuthal asymmetries in the $\pi^- N$ Drell-Yan process.

Transversely polarized process

- The transverse single spin asymmetry can be defined as

M. G. Echevarria, A. Idilbi, Z. B. Kang, and I. Vitev, Phys. Rev. D 89, 074013(2014)

$$A_{UT} = \frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} \Big/ \frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp},$$

Spin-dependent

Spin-independent(Unpolarized)

$$\frac{d^4\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UU}(Q;b) + Y_{UU}(Q,q_\perp).$$

$$\frac{d^4\Delta\sigma}{dQ^2 dy d^2\boldsymbol{q}_\perp} = \sigma_0 \epsilon_\perp^{\alpha\beta} S_\perp^\alpha \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}} \widetilde{W}_{UT}^\beta(Q;b) + Y_{UT}^\beta(Q,q_\perp).$$

Sivers Asymmetry

Transversely polarized process

- Spin-dependent structure function

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = H_{UT}(Q;\mu) \sum_{q,\bar{q}} e_q^2 \tilde{f}_{1\,\bar{q}/\pi}(x_\pi, b; \mu, \zeta_F)\, \tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x_p, b; \mu, \zeta_F).$$

$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x, b; \mu, \zeta_F) = \int d^2\boldsymbol{k}_\perp e^{-i\vec{k}_\perp\cdot\vec{b}} \frac{k_\perp^\alpha}{M_p} f_{1T,q/p}^{\perp(\mathrm{DY})}(x, \boldsymbol{k}_\perp; \mu),$$

Follow the same evolution equations and the solution structure can be written in the same form

Transversely polarized process

- Perturbative Sudakov form factor has the same form as unpolarized PDF
- Nonperturbative Sudakov form factor has the parameterization as

M. G. Echevarria, A. Idilbi, Z. B. Kang, and I. Vitev, Phys. Rev. D 89, 074013(2014)

$$S_{\rm NP}^{\rm Siv} = \left(g_1^{\rm Siv} + g_2^{\rm Siv} \ln\frac{Q}{Q_0}\right) b^2,$$

$$g_1^{\rm Siv} = \langle k_{s\perp}^2\rangle_{Q_0}/4 = 0.071{\rm GeV}^2 \qquad g_2^{\rm Siv} = \frac{1}{2}g_2 = 0.08{\rm GeV}^2$$

Sudakov form factor

Transversely polarized process

- In the small b region, the Sivers function can be also expressed as the convolution of perturbatively calculable hard coefficients and the corresponding collinear correlation functions as

$$\tilde{f}_{1T\,q/p}^{\perp\alpha(\mathrm{DY})}(x,b;\mu) = (\frac{-ib^{\alpha}}{2}) \sum_i \Delta C^{T}_{q\leftarrow i} \otimes f^{(3)}_{i/p}(x',x'';\mu).$$

Qiu-Sterman matrix element $T_{q,F}(x,x)$ is the most relevant one

$$T_{q,F}(x,x) = \int d^2k_\perp \frac{|k_\perp^2|}{M_p} f_{1T\,q/p}^{\perp\mathrm{DY}}(x,k_\perp) = 2M_p\, f_{1T\,q/p}^{\perp(1)\mathrm{DY}}(x),$$

Low b region

Transversely polarized process

- Sivers function in the b space

$$\tilde{f}^{\perp}_{1T,q/p}(x,b;Q)=\frac{b^2}{2\pi}\sum_i \Delta C^T_{q\leftarrow i}\otimes T_{i,F}(x,x;\mu_b)e^{-S^{\text{siv}}_{\text{NP}}-\frac{1}{2}S_{\text{P}}},$$

- Sivers function in the transverse momentum space

$$\frac{k_\perp}{M_p}f^{\perp}_{1T,q/p}(x,k_\perp;Q)=\int_0^\infty db\frac{b^2}{2\pi}J_1(k_\perp b)\sum_i \Delta C^T_{q\leftarrow i}\otimes f^{\perp(1)}_{1T,i/p}(x,\mu_b)e^{-S^{\text{siv}}_{\text{NP}}-\frac{1}{2}S_{\text{P}}}.$$

Transversely polarized process

- The spin-dependent differential cross section

$$\frac{d^4\Delta\sigma}{dQ^2dyd^2\boldsymbol{q}_\perp} = \sigma_0\epsilon^{\alpha\beta}S_\perp^\alpha \int \frac{d^2b}{(2\pi)^2}e^{i\vec{q}_\perp\cdot\vec{b}}\widetilde{W}_{UT}^\beta(Q;b)$$

$$= \frac{\sigma_0}{4\pi}\int_0^\infty dbb^2 J_1(q_\perp b)\sum_{q,i,j} e_q^2 \Delta C_{q\leftarrow i}^T T_{i,F}(x_p, x_p;\mu_b)$$

$$\times\, C_{\bar{q}\leftarrow j}\otimes f_{1,j/\pi}(x_\pi,\mu_b)e^{-\left(S_{\rm NP}^{\rm Siv}+S_{\rm NP}^{f_{1q/\pi}}+S_{\rm P}\right)}.$$

$$\Delta C_{q\leftarrow q'}^T(x,b;\mu_b) = \delta_{qq'}\left[\delta(1-x)+\frac{\alpha_s}{\pi}\left(-\frac{1}{4N_c}(1-x)+\frac{C_F}{4}(\pi^2-8)\delta(1-x)\right)\right].$$

Spin-dependent

Transversely polarized process

- Qiu-Sterman function parameterization

M. G. Echevarria, A. Idilbi, Z. B. Kang, and I. Vitev, Phys. Rev. D 89, 074013(2014)

$$T_{q,F}(x,x;\mu) = N_q \frac{(\alpha_q+\beta_q)^{(\alpha_q^{\alpha_q}+\beta_q^{\beta_q})}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}} x^{\alpha_q}(1-x)^{\beta_q} f_{q/p}(x,\mu),$$

Energy dependence

Set 1:propotional to unpolarized PDF

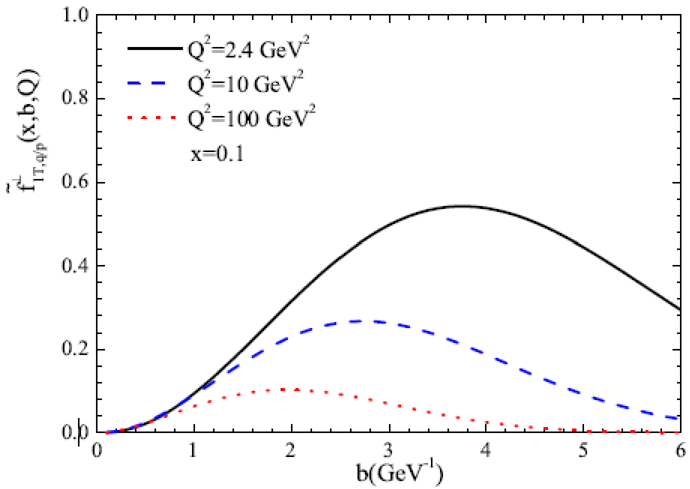
Set 2:adopt approximate evolution kernel

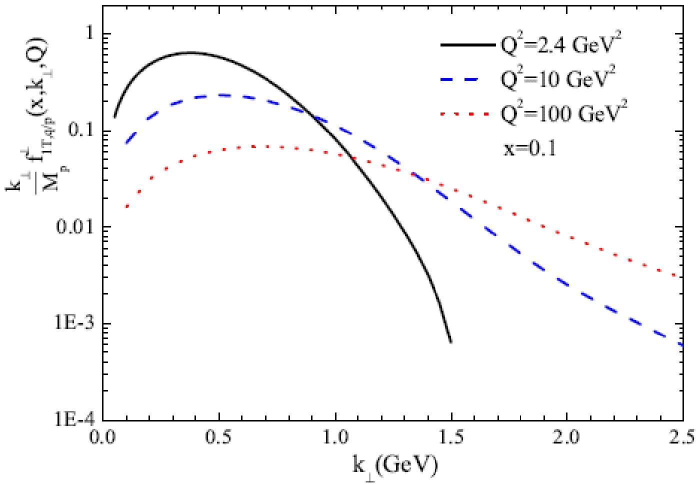
$$P_{qq}^{\mathrm{QS}} \approx P_{qq}^{f_1} - \frac{N_c}{2}\frac{1+z^2}{1-z} - N_c\delta(1-z), \qquad P_{qq}^{f_1} = \frac{4}{3}\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right).$$

Qiu-Sterman

Transversely polarized process

◆ Set 1

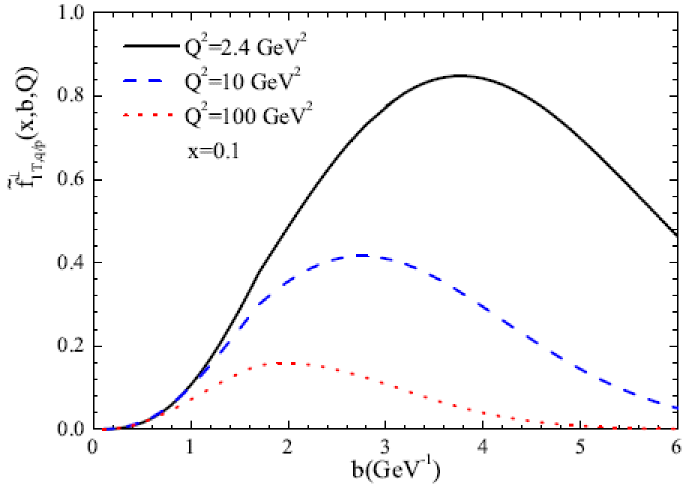


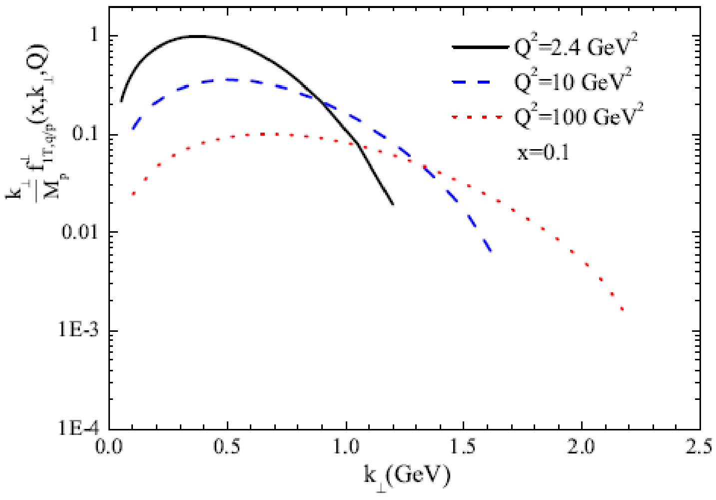


X. Wang and Z. Lu, Phys. Rev. D 97, 054005 (2018)

Transversely polarized process

- Set 2



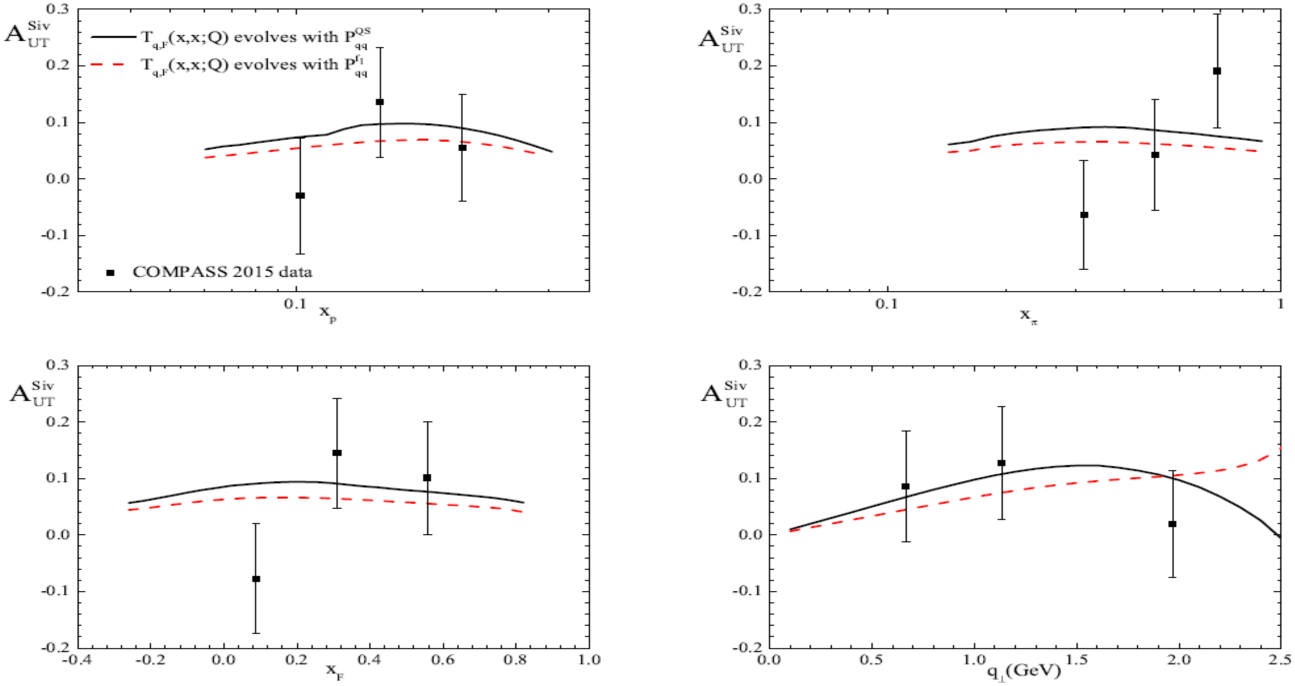


X. Wang and Z. Lu, Phys. Rev. D 97, 054005 (2018)

Transversely polarized process

- Sivers asymmetry with the COMPASS measurement

X. Wang and Z. Lu, Phys. Rev. D 97, 054005 (2018)



Conclusion

- The Sivers asymmetry calculated from the TMD evolution formalism is consistent with the COMPASS measurement.

- The scale dependence of the Qiu-Sterman function will play a role in the interpretation of the experimental data, and it should also be considered in the phenomenological studies.

THANK YOU!

BACKUPS

Quark, Gluon / Nucleon	U	L	T
U	number density $f_1^{q,g}(x,k_T^2)$		Boer-Mulders $h_1^{\perp q,g}(x,k_T^2)$
L		Helicity $g_{1L}^{q,g}(x,k_T^2)$	worm-gear L $h_{1L}^{\perp q,g}(x,k_T^2)$
T	Sivers $f_{1T}^{\perp q,g}(x,k_T^2)$	Kotzinian-Mulders worm-gear T $g_{1T}^{\perp q,g}(x,k_T^2)$	Transversity $h_1^{q,g}(x,k_T^2)$ Pretzelosity $h_{1T}^{\perp q,g}(x,k_T^2)$

Leading twist transverse momentum dependent parton distribution functions(TMDs)

spin of the nucleon

spin of the parton

k_T of the parton

Unpolarized process

- The fit coincides with the experimental data well when $0 < x_F < 0.8$.
- The fit breaks down when x_F is above 0.8 since at that region the TMD factorization is invalid and the higher twist effects dominant.
- For the pion-induced Drell-Yan process in fixed-target scattering, the NLL threshold resummation effects are also important in the kinematic higher x_F.