DYSON-SCHWINGER EQUATIONS AT LARGE N_c LIMIT AND THEIR TRUNCATION BEYOND THE RAINBOW APPROXIMATION

Hui-Feng Fu (付慧峰)

Jilin University

Collaborator: Qing Wang (王青) (THU)

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OUTLINE

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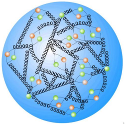
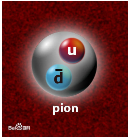
STRONG INTERACTION: QCD

- $\mathcal{L}_{QCD} = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$





- Nonperturbative phenomena:
  - Hadron spectroscopy
  - Dynamical chiral symmetry breaking
  - Confinement

- Nonperturbative methods:
  - Lattice QCD
  - Light-front dynamics
  - Dyson-Schwinger equations(DSEs)
  - ...

Strong interaction: QCD

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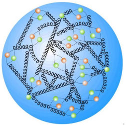

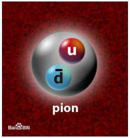

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  - Lattice QCD
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  - Dyson-Schwinger equations(DSEs)
  - . . .

APPLICATIONS OF DSEs

- DSEs: rainbow-ladder approximation

PHYSICAL REVIEW C VOLUME 56, NUMBER 6 DECEMBER 1997

π- and K-meson Bethe-Salpeter amplitudes

Pieter Maris and Craig D. Roberts
Physics Division, Building 203, Argonne National Laboratory, Argonne, Illinois 60439-4843
(Received 18 August 1997)

PHYSICAL REVIEW C, VOLUME 60, 055214

Bethe-Salpeter study of vector meson masses and decay constants

Pieter Maris and Peter C. Tandy
Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242
(Received 27 May 1999; published 21 October 1999)

- DSEs: beyond rainbow approximation

PHYSICAL REVIEW C, VOLUME 65, 065203

Bethe-Salpeter equation and a nonperturbative quark-gluon vertex

A. Bender, W. Detmold, and A. W. Thomas
Special Research Centre for the Subatomic Structure of Matter, and Department of Physics and Mathematical Physics, University of Adelaide, Adelaide SA 5005, Australia

C. D. Roberts
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843
and Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany
(Received 27 February 2002; published 12 June 2002)

PRL **103,** 122001 (2009) PHYSICAL REVIEW LETTERS week ending 18 SEPTEMBER 2009

Probing the Gluon Self-Interaction in Light Mesons

Christian S. Fischer[1,2] and Richard Williams[1]
[1]*Institute for Nuclear Physics, Darmstadt University of Technology, Schlossgartenstraße 9, 64289 Darmstadt, Germany*
[2]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1 D-64291 Darmstadt, Germany*
(Received 20 May 2009; published 15 September 2009)

APPLICATIONS OF DSEs

- Symmetry-preserving truncations

PHYSICAL REVIEW D VOLUME 52, NUMBER 8 15 OCTOBER 1995

Dynamical chiral symmetry breaking, Goldstone's theorem, and the consistency of the Schwinger-Dyson and Bethe-Salpeter equations

H. J. Munczek
Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045
(Received 21 February 1995)

$$K_{\rm EF}^{\rm GH} = -\frac{\delta\,\Sigma_{\rm EF}}{\delta\,S_{\rm GH}}$$

Pion mass and decay constant PLB420, 267

Pieter Maris [a], Craig D. Roberts [a], Peter C. Tandy [b]
[a] *Physics Division, Bldg. 203, Argonne National Laboratory, Argonne, IL 60439-4843, USA*
[b] *Centre for Nuclear Research, Department of Physics, Kent State University, Kent, OH 44242, USA*

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-)$$

LARGE N_c LIMIT

A PLANAR DIAGRAM THEORY FOR STRONG INTERACTIONS

G. 't HOOFT
CERN, Geneva

NPB72, 461 (1974)

Received 21 December 1973

Abstract: A gauge theory with colour gauge group U(N) and quarks having a colour index running from one to N is considered in the limit $N \to \infty$, g^2N fixed. It is shown that only planar diagrams with the quarks at the edges dominate; the topological structure of the perturbation series in $1/N$ is identical to that of the dual models, such that the number $1/N$ corresponds to the dual coupling constant. For hadrons N is probably equal to three. A mathematical framework is proposed to link these concepts of planar diagrams with the functional integrals of Gervais, Sakita and Mandelstam for the dual string.

BARYONS IN THE 1/N EXPANSION

NPB60, 57 (1979)

Edward Witten

Lyman Laboratory of Physics
Harvard University
Cambridge, Massachusetts 02138

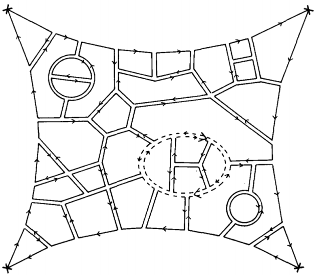

Fig. 3. One of the leading diagrams for the four-point function.

OUTLINE

- Introduction
- The Symmetry-Preserving Truncation Scheme

THE GENERATING FUNCTIONAL

- QCD generating functional:

$$
\begin{aligned}
Z[J,\mathcal{I},\bar{I},I] &= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \exp i\int d^4x\{\mathcal{L}(\psi,\bar{\psi},A_\mu)+\bar{\psi}J\psi+\mathcal{I}_i^\mu A_\mu^i+\bar{I}\psi+\bar{\psi}I\} \\
&= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\exp\left\{i\int d^4x\{\bar{\psi}(i\partial\!\!\!/+J)\psi+\bar{I}\psi+\bar{\psi}I\}\right\} \\
&\quad \times \int \mathcal{D}A_\mu \Delta_F(A_\mu)\exp\left\{i\int d^4x\left[\mathcal{L}_G(A)-\frac{1}{2\xi}[F^i(A_\mu)]^2+\mathcal{I}_i'^\mu A_\mu^i\right]\right\},
\end{aligned}
$$

- Integrating out gluon fields formally

$$
\begin{aligned}
&\int \mathcal{D}A_\mu \Delta_F(A_\mu)\exp\left\{i\int d^4x\left[\mathcal{L}_G(A)-\frac{1}{2\xi}[F^i(A_\mu)]^2+\mathcal{I}_i'^\mu A_\mu^i\right]\right\} \\
&= \exp\ i\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n \frac{i^n}{n!}G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n)\mathcal{I}'^{\mu_1}_{i_1}(x_1)\cdots\mathcal{I}'^{\mu_n}_{i_n}(x_n),
\end{aligned}
$$

- Fierz reordering $\to$ Factor out N_c

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&= \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left\{i\int d^4x\{\bar{\psi}(i\partial\!\!\!/ + J)\psi + \bar{I}\psi + \bar{\psi}I\}\right\} \\
&\quad \times \int \mathcal{D}A_\mu \Delta_F(A_\mu) \exp\left\{i\int d^4x\left[\mathcal{L}_G(A) - \frac{1}{2\xi}[F^i(A_\mu)]^2 + \mathcal{I}_i'^\mu A_\mu^i\right]\right\},
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$$= \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left\{i\int d^4x\{\bar{\psi}(i\partial\!\!\!/ + J)\psi + \bar{I}\psi + \bar{\psi}I\}\right\}$$

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$$= \exp\ i\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n \frac{i^n}{n!} G^{i_1\cdots i_n}_{\mu_1\cdots\mu_n}(x_1,\cdots,x_n)\mathcal{I}_{i_1}'^{\mu_1}(x_1)\cdots\mathcal{I}_{i_n}'^{\mu_n}(x_n),$$

- Fierz reordering → Factor out N_c

THE GENERATING FUNCTIONAL

- Introducing auxiliary fields:

$$\int \mathcal{D}\Phi\, \delta\Big(N_c \Phi^{(a\eta)(b\zeta)}(x,x') - \bar{\psi}^{a\eta}_{\alpha}(x)\psi^{b\zeta}_{\alpha}(x')\Big).$$

$$\delta\Big(N_c\Phi(x,x') - \bar{\psi}(x)\psi(x')\Big) \sim \int \mathcal{D}\Pi e^{i\int d^4x d^4x' \Pi(x,x')\cdot\big(N_c\Phi(x,x')-\bar{\psi}(x)\psi(x')\big)}.$$

- Integrating out quark fields:

$$Z[J,\bar{I},I] \overset{\text{large } N_c \text{ limit}}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=} \text{const} \times \exp i\Big\{ -iN_c \mathrm{Tr}\ln[i\partial\!\!\!/ + J - \Pi_c] - \bar{I}[i\partial\!\!\!/ + J - \Pi_c]^{-1} I$$
$$+ \int d^4x d^4x' N_c \Phi_c^{\sigma\rho}(x,x')\Pi_c^{\sigma\rho}(x,x') + N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x_1' \cdots d^4x_n' \frac{(-i)^n (N_c g^2)^{n-1}}{n!}$$
$$\times \bar{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x_1',\cdots,x_n,x_n')\Phi_c^{\sigma_1\rho_1}(x_1,x_1')\cdots\Phi_c^{\sigma_n\rho_n}(x_n,x_n')\Big\},$$

where $\Phi_c(\Pi_c) \equiv \frac{\int \mathcal{D}\Phi\mathcal{D}\Pi\Phi(\Pi)e^{iS}}{\int \mathcal{D}\Phi\mathcal{D}\Pi e^{iS}}$

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$$\times \bar{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x'_1,\cdots,x_n,x'_n)\Phi_c^{\sigma_1\rho_1}(x_1,x'_1)\cdots\Phi_c^{\sigma_n\rho_n}(x_n,x'_n)\Big\},$$

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BENEFITS OF THIS FORM

- Explicit expression of interaction kernel
- Allows truncations at generating functional level
  $\rightarrow$ symmetry-preserving guaranteed

◇ Stationary equations:

$$\frac{-i\delta \ln Z[J,\mathcal{I},\bar{I},I]}{\delta \Pi_c^{\sigma\rho}(x,y)} = 0$$

$$\frac{-i\delta \ln Z[J,\mathcal{I},\bar{I},I]}{\delta \Phi_c^{\sigma\rho}(x,y)} = 0$$

$\Rightarrow$

$$\begin{aligned}
\Phi_c^{\sigma\rho}(x,y) &= -i[i\partial\!\!\!/ - M - \Pi_c]^{-1,\rho,\sigma}(y,x), \\
\Pi_c^{\sigma\rho}(x,y) &= -\sum_{n=2}^{\infty}\int d^4x_2\cdots d^4x_n d^4x_2'\cdots d^4x_n' \frac{(-i)^n(N_cg^2)^{n-1}}{(n-1)!}\bar{G}^{\sigma\sigma_2\cdots\sigma_n}_{\rho\rho_2\cdots\rho_n}(x,y,x_2,x_2',\cdots,x_n,x_n') \\
&\quad \times\Phi_c^{\sigma_2\rho_2}(x_2,x_2')\cdots\Phi_c^{\sigma_n\rho_n}(x_n,x_n').
\end{aligned}$$

THE GAP EQUATION

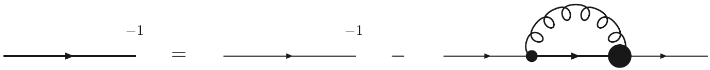


Figure 1: The DSE for the quark propagator.

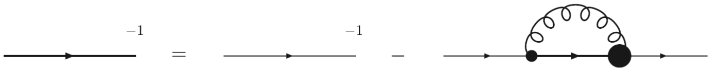
- In momentum space:

$$S(p,\mu)^{-1} = Z_2(\mu,\Lambda)\left(i\not p + m_b(\Lambda)\right) + Z_{1F}(\mu,\Lambda) C_F \int^{\Lambda} \frac{d^4q}{(2\pi)^4} \times g^2(\mu)\gamma_\mu S(q,\mu) G_{\mu\nu}(p-q,\mu)\Gamma_\nu(p,q,\mu)$$

- The rainbow approximation (RA) $\Leftrightarrow$ Keeping only gluon propagator

$$\Gamma_\nu(p,q,\mu) \rightarrow \gamma_\nu$$

THE GAP EQUATION



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- The rainbow approximation (RA) $\Leftrightarrow$ Keeping only gluon propagator

$$\Gamma_\nu(p,q,\mu) \to \gamma_\nu$$

THE BETHE-SALPETER EQUATION

- The Bethe-Salpeter equation:

$$\chi_{P,s}^{\rho\sigma} \quad = \quad -i[i\partial\!\!\!/ - M - \Pi_c]^{-1,\rho,\sigma'} K_{\sigma_2\rho_2}^{\sigma'\rho'} [i\partial\!\!\!/ - M - \Pi_c]^{-1,\rho',\sigma} \chi_{P,s}^{\rho_2\sigma_2}$$

- The BS kernel $K_{\sigma_2\rho_2}^{\sigma'\rho'}$ is defined as

$$K_{\sigma_2\rho_2}^{\sigma'\rho'} \equiv -\sum_{n=2}^{\infty} \frac{(-i)^n (N_c g^2)^{n-1}}{(n-2)!} \bar{G}_{\rho'\rho_2\rho_3'\cdots\rho_n'}^{\sigma'\sigma_2\sigma_3'\cdots\sigma_n'} \Phi_c^{\sigma_3'\rho_3'} \ldots \Phi_c^{\sigma_n'\rho_n'} = \frac{\delta\Pi_c^{\sigma'\rho'}}{\delta\Phi_c^{\sigma_2\rho_2}}$$

DIAGRAMMATIC EXPRESSIONS

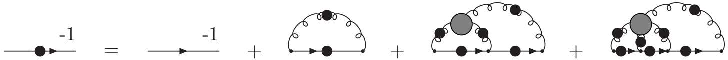


Figure 2: The gap equation up to the NNL-order truncation.

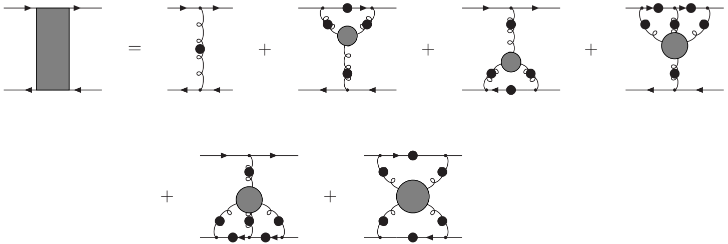


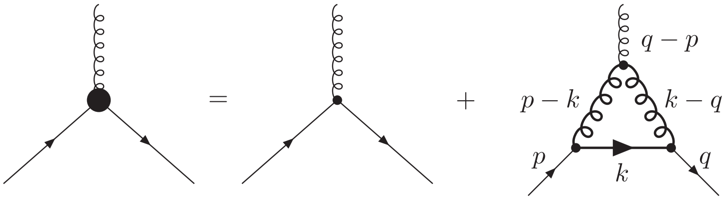
Figure 3: The meson BS kernel up to the NNL-order truncation.

OUTLINE

- Introduction
- The Symmetry-Preserving Truncation Scheme
- The Gap Equation Beyond the Rainbow Approximation

The gap equation at NL order truncation

- Study the impacts of going beyond the RA on the quark propagator and DCSB.
- Quantitative studies $\rightarrow$ construct a model interaction respecting the UV behavior of QCD.
- Vertex to one loop level $\Leftrightarrow$ keeping three gluon Green's function



INTERACTION MODEL

- At **large** space-like momentum, at leading log order

$$G^{\text{tr}}_{\mu\nu}(q) \sim \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{1}{q^2}\left[\frac{1}{2}\ln(q^2)\right]^{-d_G}$$
$$d_G = (39 - 9\xi - 4N_f)/[2(33 - 2N_f)]$$

- Inspired by **the MT model**, which is extensively used in the rainbow approximation studies, we take

$$G_{\mu\nu}(k^2) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{D(k^2)}{k^2}$$
$$g^2(\mu)\frac{D(k^2)}{k^2} = \frac{4\pi^2 D k^2}{\omega^6}e^{-k^2/\omega^2} + \frac{4\pi^2\gamma_m \ln(\mu^2/\Lambda^2_{\text{QCD}})^{d_G-1}F(k^2)}{\left(\frac{1}{2}\ln\left\{e^2 - 1 + (1 + \frac{k^2}{\Lambda^2_{\text{QCD}}})^2\right\}\right)^{d_G}}$$
$$F(k^2) = \{1 - \exp(-k^2/(4m_t^2))\}/k^2$$

- $d_G = 1 \Rightarrow$ the MT model

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F(k^2) &= \{1 - \exp(-k^2/(4m_t^2))\}/k^2
\end{aligned}
$$

- $d_G = 1 \Rightarrow$ the MT model

RENORMALIZATION

- **UV divergences** indeed appear in our model

With definitions

$$\Gamma_\nu(p,q) = Z_{1F}\gamma_\nu + \Lambda_\nu(p,q)$$

$$Z_{1F} = 1 + C_{1F}$$

We found

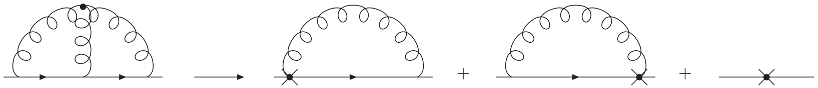
$$\Lambda_\nu(p,q) \quad = \underset{\substack{\uparrow \\ \text{Divergent term}}}{f_1(p,q)\gamma_\nu} \quad + \underset{\substack{\uparrow \\ \text{Convergent terms}}}{\tilde{\Lambda}_\nu(p,q)}$$

OVERLAPPING DIVERGENCE

- Overlapping term

$$
\begin{aligned}
S(p)^{-1} \quad = \quad & Z_2\left(i\not{p}+m_b\right)+C_F\int\frac{d^4q}{(2\pi)^4}g^2\gamma_\mu S(q)G_{\mu\nu}(p-q)\gamma_\nu \\
& +2C_{1F}C_F\int\frac{d^4q}{(2\pi)^4}g^2\gamma_\mu S(q)G_{\mu\nu}(p-q)\gamma_\nu \\
& \underline{+C_F\int\frac{d^4q}{(2\pi)^4}g^2\gamma_\mu S(q)G_{\mu\nu}(p-q)\Lambda_\nu(p,q)}
\end{aligned}
$$

- Overlapping diagram



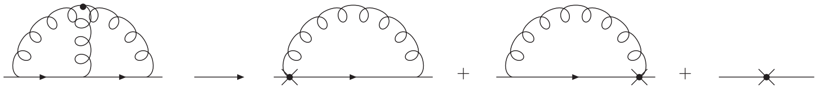
$+$ Finite terms

OVERLAPPING DIVERGENCE

- Overlapping term

$$\begin{aligned} S(p)^{-1} \quad = \quad & Z_2\left(i\not{p} + m_b\right) + C_F \int \frac{d^4q}{(2\pi)^4} g^2 \gamma_\mu S(q) G_{\mu\nu}(p-q)\gamma_\nu \\ & \underline{+2C_{1F}C_F \int \frac{d^4q}{(2\pi)^4} g^2 \gamma_\mu S(q) G_{\mu\nu}(p-q)\gamma_\nu} \\ & +C_F \int \frac{d^4q}{(2\pi)^4} g^2 \gamma_\mu S(q) G_{\mu\nu}(p-q)\Lambda_\nu(p,q) \end{aligned}$$

- Overlapping diagram



$+$ Finite terms

RENORMALIZATION SCALE DEPENDENCE

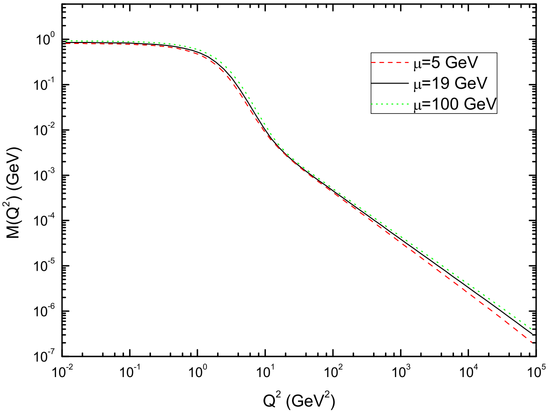


Figure 4: The running quark mass at different renormalization points μ.

Definition for $M(p^2)$ and $Z(p^2)$ $\quad S(p,\mu) = \frac{Z(p^2,\mu^2)}{i\not{p}+M(p^2)}$

RENORMALIZATION SCALE DEPENDENCE

Table 1: Pion mass m_π, decay constant f_π, quark condensate $-\langle\bar{q}q\rangle^0$ and the renormalization independent current quark mass $\hat{m}_{u/d}$ at different renormalization points μ.

	f_π(MeV)	m_π(MeV)	$-\langle\bar{q}q\rangle^0$(MeV)^3	$\hat{m}_{u/d}$ (MeV)
$\mu = 5$ GeV	105	128	$(250)^3$	3.9
$\mu = 19$ GeV	106	181	$(289)^3$	7.2
$\mu = 100$ GeV	111	189	$(311)^3$	7.5

$$-\langle\bar{q}q\rangle^0_\mu \equiv N_c Z_4 \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[S(p,\mu)\right].$$

$$f_\pi^2 = \frac{3}{4\pi} \int dp^2 \frac{p^2 Z(p^2) M(p^2)}{(p^2 - M^2(p^2))^2} \left(M(p^2) - \frac{p^2}{2}\frac{dM}{dP^2} \right),$$

$$m_\pi^2 = \frac{-2m(\mu)\langle\bar{q}q\rangle^0_\mu}{f_\pi^2}$$

OUTLINE

- Introduction
- The Symmetry-Preserving Truncation Scheme
- The Gap Equation Beyond the Rainbow Approximation
- Numerical Results

BEYOND RA VERSUS RA

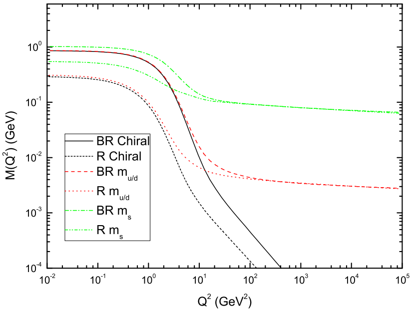
- Parameters

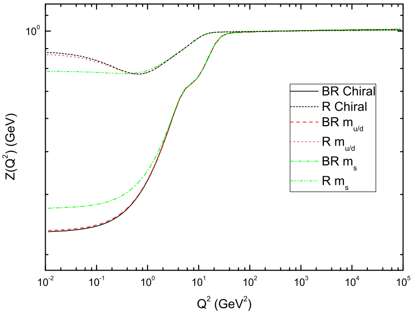
$$D = 0.74\ \mathrm{GeV}^2 \qquad m_{u/d}(\mu)|_{\mu=19\ \mathrm{GeV}} = 3.7\ \mathrm{MeV},$$
$$\omega = 0.5\ \mathrm{GeV} \qquad m_s(\mu)|_{\mu=19\ \mathrm{GeV}} = 85\ \mathrm{MeV}.$$

Table 2: Comparing the pion mass, pion decay constant, the quark condensate at the renormalization point and the renormalization point independent quark condensate between the beyond-the-rainbow scheme (denoted as “BR”) and the rainbow approximation scheme (denoted as “R”).

	f_π(MeV)	m_π(MeV)	$-\langle\bar{q}q\rangle^0_{\mu=19\mathrm{GeV}}$(MeV)^3	$-\langle\bar{q}q\rangle^0$(MeV)^3
BR	106	181	$(369)^3$	$(289)^3$
R	74.5	148	$(254)^3$	$(229)^3$

BEYOND RA VERSUS RA

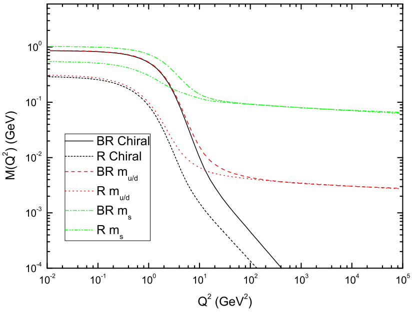


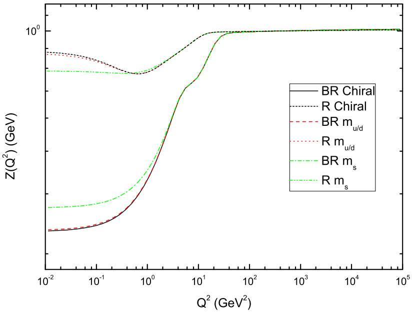


- Significant impacts.
- At large energy region, all functions coincide except $M(Q)$ **at chiral limit.**
- **Non-perturbative effects**, difference between quark condensate.

$$M(p^2) \xrightarrow{\text{large } p^2} \frac{2\pi^2\gamma_m(-\langle\bar{q}q\rangle^0)}{3p^2}$$

BEYOND RA VERSUS RA





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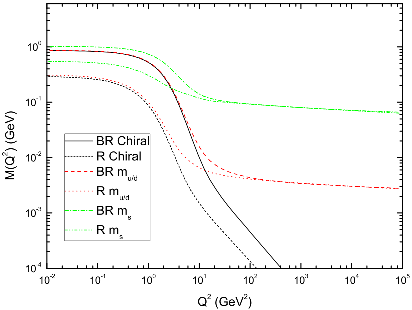

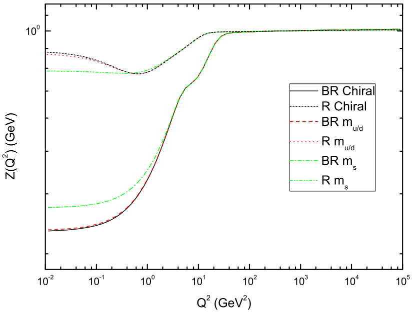

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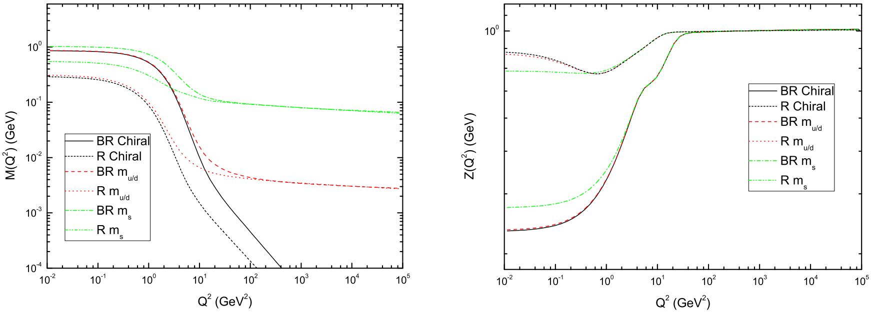

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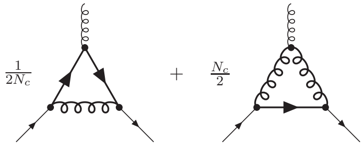
Non-Abelian versus Abelian

- $1/N_c^2$ order correction at one loop level:



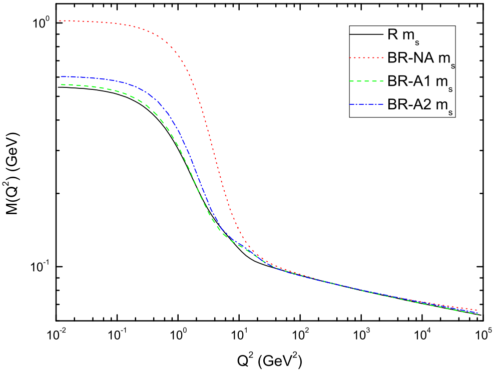


Figure 5: Comparisons of non-Abelian contribution and Abelian contribution to $M(Q^2)$. "BR-NA" represents using Λ_ν^{NA} only; "BR-A1" represents using Λ_ν^{A} only; "BR-A2" represents using re-scaled $N_c^2\Lambda_\nu^{A}$.

UV BEHAVIOR

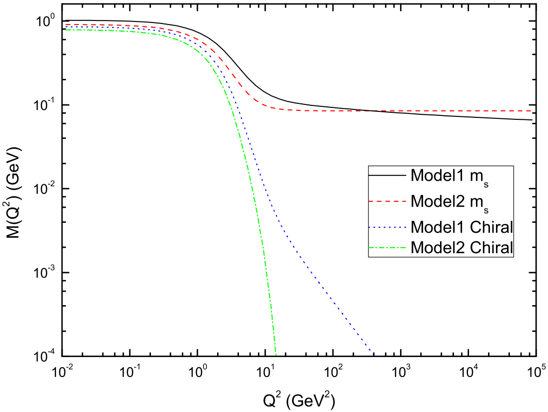


Figure 6: Comparing $M(Q^2)$ resulted from Our model Eq. (1) and the model from PRL103,122001.

SUPPRESSION OF CRITICAL STRENGTH

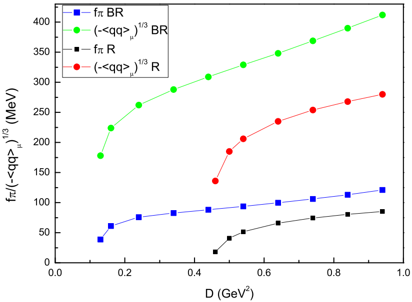


Figure 7: Quark condensate and the pion decay constant vs D, i.e. the strength of the interaction at low momentum.

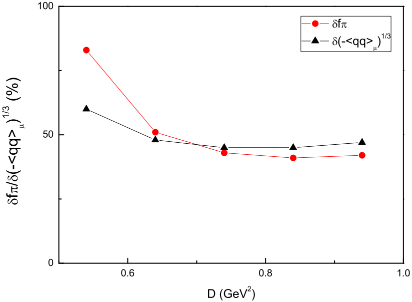


Figure 8: Relative deviations (i.e. $\delta X \equiv \frac{X_{BR}-X_R}{X_R}$ where $X_{R(BR)}$ represents quantity under the RA(BR) scheme) of pion decay constant and the quark condensate vs D.

OUTLINE

SUMMARY

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- We have studied DCSB and quark propagators with DSEs beyond RA with an interaction model respecting the asymptotic behavior of QCD.
- The impacts of beyond RA are shown. A pure non-perturbative effect makes the beyond-rainbow term contribute even at large momentum in the chiral limit.
- Critical strength for DCSB is suppressed.

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Thank you for your attention!