2018中国物理学会高能物理分会学术年会

The study of semileptonic decay processes within LCSR in B-factory and searching for NP

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In Cooperation with

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- **Tao Zhong, Yi Zhang,**

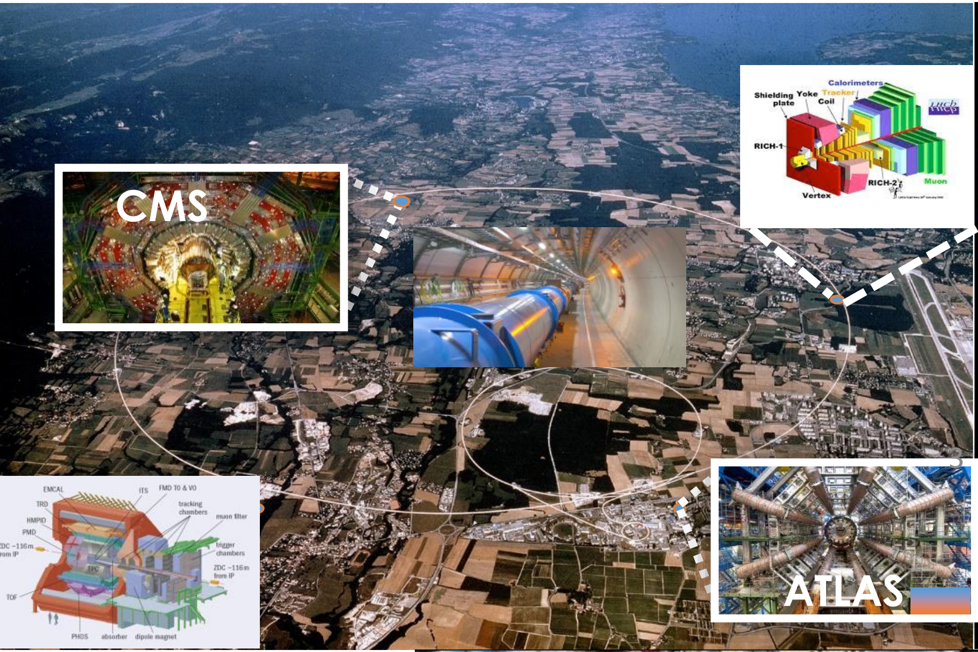
I. Introduction

II. LCSR with chiral correlator

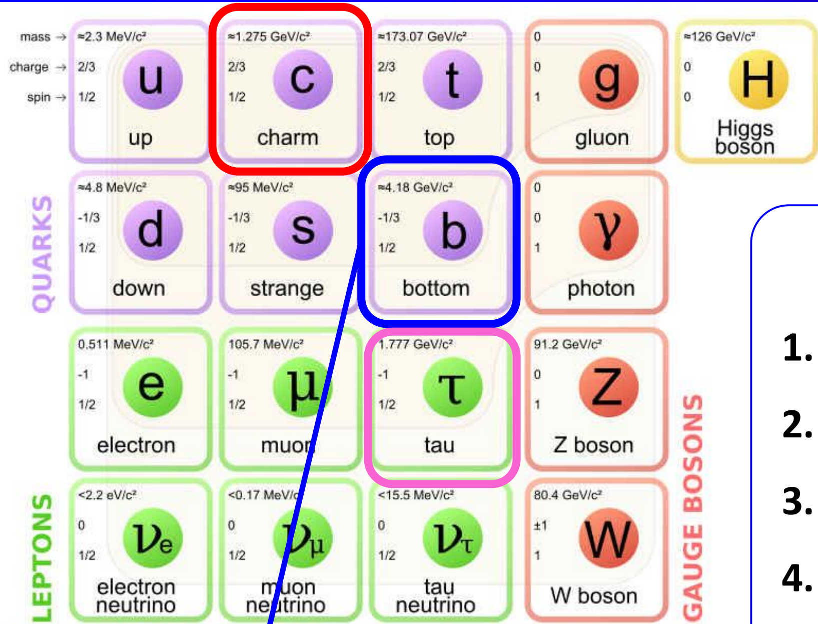
III. SVZ sum rule within BFT

IV. Summary

I. Introduction



I. Introduction



Standard_Model_of_Elementary_Particles.svg (SVG file, nominally 774 × 581 pixels, file size: 419 KB)

B/D factory
- LHCb run-II
- ATLAS
- CMS
- Belle-II
- BESIII

Flavor Physics

1. Rare FCNC and angular analyses
2. B(D) decay property and CP violation
3. Spectroscopy and exotic states
4. Hadron production and polarization

……

1. B(D) decay properties and CP violation

Pure leptonic and semileptonic B(D) decay

- Decay constant
- Transition form factors
- CKM matrix element
- Decay width and anomalous

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{tb} & V_{ts} & V_{tb} \end{pmatrix}$$

JHEP 1708 (2017) 055

Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays

The LHCb collaboration

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Abstract: A test of lepton universality, performed by measuring the ratio of the branching fractions of the $B^0 \to K^{*0}\mu^+\mu^-$ and $B^0 \to K^{*0}e^+e^-$ decays, $R_{K^{*0}}$, is presented. The K^{*0} meson is reconstructed in the final state $K^+\pi^-$, which is required to have an invariant mass within 100 MeV/c^2 of the known $K^*(892)^0$ mass. The analysis is performed using proton-proton collision data, corresponding to an integrated luminosity of about 3 fb^{-1}, collected by the LHCb experiment at centre-of-mass energies of 7 and 8 TeV. The ratio is measured in two regions of the dilepton invariant mass squared, q^2, to be

$$R_{K^{*0}} = \begin{cases} 0.66 \,^{+\,0.11}_{-\,0.07}\,(\text{stat}) \pm 0.03\,(\text{syst}) & \text{for } 0.045 < q^2 < 1.1 \ \text{GeV}^2/c^4\,, \\ 0.69 \,^{+\,0.11}_{-\,0.07}\,(\text{stat}) \pm 0.05\,(\text{syst}) & \text{for } 1.1 < q^2 < 6.0 \ \text{GeV}^2/c^4\,. \end{cases}$$

$$R_H = \frac{\int \frac{d\Gamma(B\to H\mu^+\mu^-)}{dq^2}\, dq^2}{\int \frac{d\Gamma(B\to He^+e^-)}{dq^2}\, dq^2}\,,$$

Table 1: Recent SM predictions for $R_{K^{*0}}$.

q^2 range $[\text{GeV}^2/c^4]$	$R_{K^{*0}}^{\text{SM}}$	References
[0.045, 1.1]	0.906 ± 0.028	BIP [26]
	0.922 ± 0.022	CDHMV [27–29]
	$0.919\,^{+\,0.004}_{-\,0.003}$	EOS [30, 31]
	0.925 ± 0.004	flav.io [32–34]
	$0.920\,^{+\,0.007}_{-\,0.006}$	JC [35]
[1.1, 6.0]	1.000 ± 0.010	BIP [26]
	1.000 ± 0.006	CDHMV [27–29]
	$0.9968\,^{+\,0.0005}_{-\,0.0004}$	EOS [30, 31]
	0.9964 ± 0.005	flav.io [32–34]
	0.996 ± 0.002	JC [35]

I. Introduction-Motivation

PHYSICAL REVIEW LETTERS **120,** 121801 (2018)

Measurement of the Ratio of Branching Fractions $\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$

R. Aaij *et al.**
(LHCb Collaboration)

(Received 16 November 2017; revised manuscript received 19 January 2018; published 23 March 2018; corrected 4 April 2018)

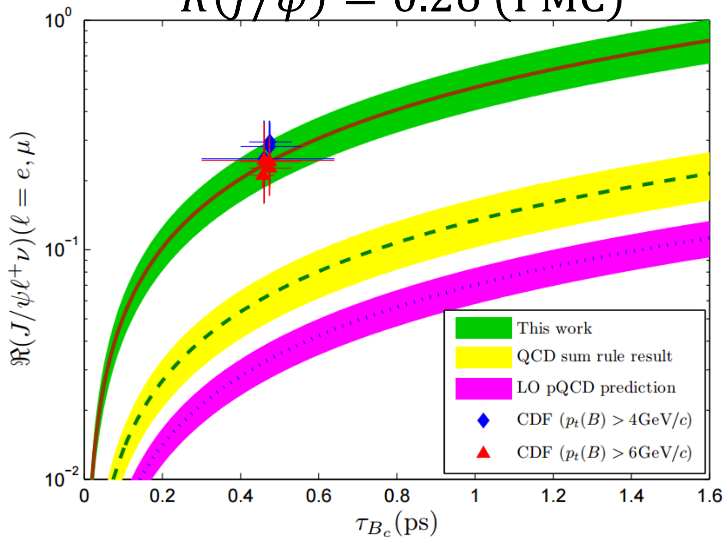
$$\mathcal{R}(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)} = 0.71 \pm 0.17(\text{stat}) \pm 0.18(\text{syst}).$$

TABLE I. Systematic uncertainties in the determination of $\mathcal{R}(J/\psi)$.

Source of uncertainty	Size ($\times 10^{-2}$)
Finite simulation size	8.0
$B_c^+ \to J/\psi$ form factors	12.1
$B_c^+ \to \psi(2S)$ form factors	3.2
Fit bias correction	5.4
Z binning strategy	5.6
Mis-ID background strategy	5.6
combinatorial background cocktail	4.5
combinatorial J/ψ background scaling	0.9
$B_c^+ \to J/\psi H_c X$ contribution	3.6
$\psi(2S)$ and χ_c feed-down	0.9
Weighting of simulation samples	1.6
Efficiency ratio	0.6
$\mathcal{B}(\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau)$	0.2
Systematic uncertainty	17.7
Statistical uncertainty	17.3

1. Large discrepancy with experimental data
2. Main uncertainty is form factor
3. 3-4 times for QCDSR to the experiment

$R(J/\psi) = 0.26$ (PMC)

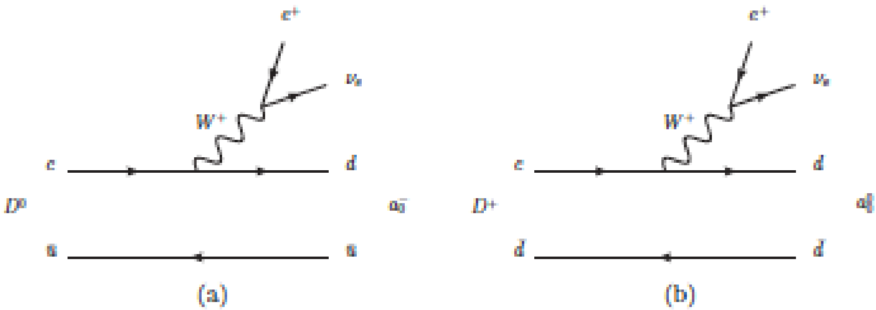


PRD90 (2014) 034025, Shen, Wu, Ma…

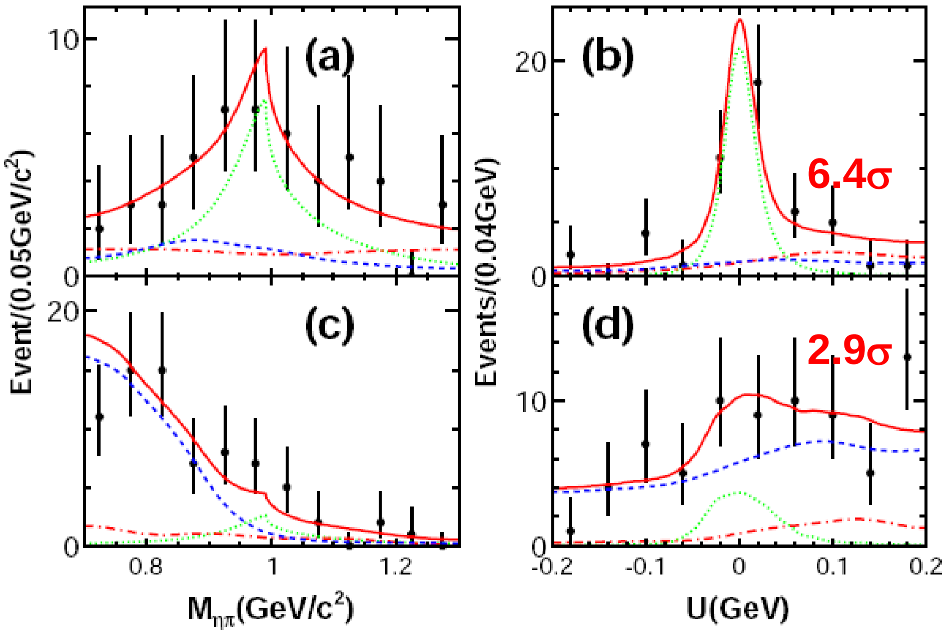
BESIII D⁺→Se⁺ν型半轻衰变的首次观测

- **Explore the nontrivial internal structure of light mesons, with clean semileptonic D/Ds decay without final state interactions.**

arXiv:1803.02166



$$R \equiv \frac{B(D^+ \to f_0 l^+ \nu) + B(D^+ \to \sigma l^+ \nu)}{B(D^+ \to a_0 l^+ \nu)}$$



$$\mathcal{B}(D^+ \to a_0(980)^0 e^+ \nu_e) \times \mathcal{B}(a_0(980)^0 \to \eta\pi^0)$$
$$= (1.66^{+0.81}_{-0.66} \pm 0.11) \times 10^{-4}, \ < \ 3.0 \times 10^{-4} \text{ at the } 90\% \text{ C.L.}$$

$$\mathcal{B}(D^0 \to a_0(980)^- e^+ \nu_e) \times \mathcal{B}(a_0(980)^- \to \eta\pi^-)$$
$$= (1.33^{+0.33}_{-0.29} \pm 0.09) \times 10^{-4}$$

$$\frac{\Gamma(D^0 \to a_0(980)^- e^+ \nu_e)}{\Gamma(D^+ \to a_0(980)^0 e^+ \nu_e)} = 2.03 \pm 0.95 \pm 0.06$$

$\delta \simeq m_V/m_b$

	Twist-2	Twist-3	Twist-4
δ^0	$\phi_{2;V}^{\perp}$	/	/
δ^1	$\phi_{2;V}^{\parallel}$	$\phi_{3;V}^{\perp}, \psi_{3;V}^{\perp}, \Phi_{3;V}^{\parallel}, \tilde{\Phi}_{3;V}^{\parallel}$	/
δ^2	/	$\phi_{3;V}^{\parallel}, \psi_{3;V}^{\parallel}, \Phi_{3;V}^{\perp}$	$\phi_{4;V}^{\perp}, \psi_{4;V}^{\perp}, \Psi_{4;V}^{\perp}, \tilde{\Psi}_{4;V}^{\perp}$
δ^3	/	/	$\phi_{4;V}^{\parallel}, \psi_{4;V}^{\parallel}$

Axial current

$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle V(p,\lambda)|\mathrm{T}\{\bar{q}_1(x)\gamma_\mu(1-\gamma_5)b(x), i\bar{b}(x)\gamma_5 q_2(x)\}|0\rangle$$

$\delta^0, \delta^1, \delta^2, \delta^3$ chiral odd and even DAs

Right-handed current

$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle V(p,\lambda)|\mathrm{T}\{\bar{q}_1(x)\gamma_\mu(1-\gamma_5)b(x), i\bar{b}(x)(1+\gamma_5) q_2(x)\}|0\rangle$$

$\phi_{2;V}^{\perp}$

δ^0, δ^2 chiral odd DAs

Left-handed current

$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle V(p,\lambda)|\mathrm{T}\{\bar{q}_1(x)\gamma_\mu(1-\gamma_5)b(x), i\bar{b}(x)(1-\gamma_5) q_2(x)\}|0\rangle$$

$\phi_{2;V}^{\parallel}$

δ^1, δ^3 chiral even DAs

Basic procedure

Decay width and Ratio

↓

***B→P/V* transition form factors (TFFs)**

↓

Light cone Sum Rule (LCSR)

↓

Light/heavy meson distribution amplitude (DAs)

1. Vector and tenser form factors

Matrix element	TFFs	Relevant decay(s)
$\langle V\|\bar{q}\gamma^\mu b\|B\rangle$	V	$B\to(\rho/\omega)\ell\nu_\ell$
$\langle V\|\bar{q}\gamma^\mu\gamma^5 b\|B\rangle$	A_0, A_1, A_2	$B\to K^*\ell^+\ell^-$
$\langle V\|\bar{q}\sigma^{\mu\nu}q_\nu b\|B\rangle$	T_1	$B\to K^*\gamma$
$\langle V\|\bar{q}\sigma^{\mu\nu}\gamma^5 q_\nu b\|B\rangle$	T_2, T_3	$B\to K^*\ell^+\ell^-$

2. Definition of TFFs

$$\langle K^*(p,\lambda)|\bar{s}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle = -ie_\mu^{*(\lambda)}(m_B+m_{K^*})A_1(q^2)+i(e^{*(\lambda)}\cdot q)\frac{(2p+q)_\mu}{m_B+m_{K^*}}A_2(q^2)$$
$$+iq_\mu(e^{*(\lambda)}\cdot q)\frac{2m_{K^*}}{q^2}[A_3(q^2)-A_0(q^2)]+\epsilon_{\mu\nu\alpha\beta}e^{*(\lambda)\nu}q^\alpha p^\beta\frac{2V(q^2)}{m_B+m_{K^*}},$$
$$\langle K^*(p,\lambda)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p+q)\rangle = 2i\epsilon_{\mu\nu\alpha\beta}e^{*(\lambda)\nu}q^\alpha p^\beta T_1(q^2)+e_\mu^{*(\lambda)}(m_B^2-m_{K^*}^2)T_2(q^2)$$
$$-(2p+q)_\mu(e^{*(\lambda)}\cdot q)\widetilde{T}_3(q^2)+q_\mu(e^{*(\lambda)}\cdot q)T_3(q^2),$$

3. Chiral current correlator

$$\Pi_\mu^{\rm I}(p,q) = i\int d^4x e^{iq\cdot x}\langle K^*(p,\lambda)|T\{\bar{s}(x)\gamma_\mu(1-\gamma_5)b(x), im_b\bar{b}(0)(1+\gamma_5)q_1(0)\}|0\rangle$$

$$\Pi_\mu^{\rm II}(p,q) = -i\int d^4x e^{iq\cdot x}\langle K^*(p,\lambda)|T\{\bar{s}(x)\sigma_{\mu\nu}q^\nu(1+\gamma_5)b(x), im_b\bar{b}(0)(1+\gamma_5)q_1(0)\}|0\rangle$$

$$f_BA_1(q^2)e^{-m_B^2/M^2} = \frac{m_b m_{K^*}^2 f_{K^*}^\perp}{m_B^2(m_B+m_{K^*})}\bigg\{\int_0^1 \frac{du}{u}e^{-s(u)/M^2}\bigg\{\frac{\mathcal{C}}{um_{K^*}^2}\Theta(c(u,s_0))\phi_{2;K^*}^\perp(u,\mu)+\Theta(c(u,s_0))\psi_{3;K^*}^\parallel(u)-\frac{1}{4}$$
$$\times\bigg[\frac{m_b^2\mathcal{C}}{u^3M^4}\widetilde{\widetilde{\Theta}}(c(u,s_0))+\frac{\mathcal{C}-2m_b^2}{u^2M^2}\widetilde{\Theta}(c(u,s_0))-\frac{1}{u}\Theta(c(u,s_0))\bigg]\phi_{4;K^*}^\perp(u)-2\bigg[\frac{\mathcal{C}}{u^2M^2}\widetilde{\Theta}(c(u,s_0))-\frac{1}{u}\Theta(c(u,s_0))\bigg]$$
$$\times I_L(u)-\bigg[\frac{2m_b^2}{uM^2}\widetilde{\Theta}(c(u,s_0))+\Theta(c(u,s_0))\bigg]H_3(u)\bigg\}+\int\mathcal{D}\alpha_i\int_0^1 dv e^{-s(X)/M^2}\Theta(c(X,s_0))\bigg[\frac{\mathcal{C}}{2X^3M^2}-\frac{1}{2X^2}\bigg]$$
$$\times\Big[(4v-1)\Psi_{4;K^*}^\perp(\underline{\alpha})-\widetilde{\Psi}_{4;K^*}^\perp(\underline{\alpha})\Big]\bigg\} \qquad (19)$$

$$f_BA_2(q^2)e^{-m_B^2/M^2} = \frac{m_b(m_B+m_{K^*})m_{K^*}^2 f_{K^*}^\perp}{m_B^2}\bigg\{\int_0^1 \frac{du}{u}e^{-s(u)/M^2}\bigg\{\frac{1}{m_{K^*}^2}\Theta(c(u,s_0))\phi_{2;K^*}^\perp(u,\mu)-\frac{1}{M^2}\widetilde{\Theta}(c(u,s_0))\psi_{3;K^*}^\parallel(u)$$
$$-\frac{1}{4}\bigg[\frac{m_b^2}{u^2M^4}\widetilde{\widetilde{\Theta}}(c(u,s_0))+\frac{1}{uM^2}\widetilde{\Theta}(c(u,s_0))\bigg]\phi_{4;K^*}^\perp(u)+2\bigg[\frac{\mathcal{C}-2m_b^2}{u^2M^4}\widetilde{\widetilde{\Theta}}(c(u,s_0))-\frac{1}{uM^2}\widetilde{\Theta}(c(u,s_0))\bigg]I_L(u)$$
$$-\frac{1}{M^2}\widetilde{\Theta}(c(u,s_0))H_3(u)\bigg\}+\int\mathcal{D}\alpha_i\int_0^1 dv e^{-s(X)/M^2}\frac{1}{2X^2M^2}\Theta(c(X,s_0))\Big[(4v-1)\Psi_{4;K^*}^\perp(\underline{\alpha})-\widetilde{\Psi}_{4;K^*}^\perp(\underline{\alpha})$$
$$+4v\Phi_{3;K^*}^\perp(\underline{\alpha})\Big]\bigg\} \qquad (20)$$

$$f_BV(q^2)e^{-m_B^2/M^2} = \frac{m_b(m_B+m_{K^*})f_{K^*}^\perp}{m_B^2}\int_0^1 \frac{du}{u}e^{-s(u)/M^2}\bigg\{\Theta(c(u,s_0))\phi_{2;K^*}^\perp(u,\mu)-\bigg[\frac{m_b^2}{u^2M^4}\widetilde{\widetilde{\Theta}}(c(u,s_0))+\frac{1}{uM^2}$$
$$\times\widetilde{\Theta}(c(u,s_0))\bigg]\frac{m_{K^*}^2}{4}\phi_{4;K^*}^\perp(u)\bigg\} \qquad (21)$$

II. LCSR with chiral correlator —— B→K* TFFs

$$T_1(q^2)=\frac{m_b^2 m_{K^*}^2 f_{K^*}^{\perp}}{m_B^2 f_B}\left\{\int_0^1\frac{du}{u}e^{\frac{m_B^2-s(u)}{M^2}}\left\{\frac{1}{m_{K^*}^2}\Theta(c(u,s_0))\phi_{2;K^*}^{\perp}(u,\mu)-\frac{m_b^2}{4u^2M^4}\tilde{\tilde{\Theta}}(c(u,s_0))\phi_{4;K^*}^{\perp}(u)-\frac{2}{uM^2}\right.\right.$$

$$\left.\times\Theta(c(u,s_0))I_L(u)-\frac{1}{M^2}\Theta(c(u,s_0))H_3(u)+\int D\alpha_i\int_0^1 dve^{\frac{m_B^2-s(X)}{M^2}}\frac{5}{4X^2M^2}\Theta(c(X,s_0))\Psi_{4;K^*}^{\perp}(\underline{\alpha})\right\}$$

$$T_2(q^2)=\frac{m_b^2 f_{K^*}^{\perp} m_{K^*}^2}{m_B^2 f_B}\int_0^1\frac{du}{u}e^{\frac{m_B^2-s(u)}{M^2}}\left\{\frac{1-\mathcal{H}}{m_{K^*}^2}\Theta(c(u,s_0))\phi_{2;K^*}^{\perp}(u,\mu)-\frac{m_b^2}{4u^2M^4}(1-\mathcal{H})\tilde{\tilde{\Theta}}(c(u,s_0))\phi_{4;K^*}^{\perp}(u)\right.$$

$$\left.-\frac{2(1-\mathcal{H})}{uM^2}\tilde{\Theta}(c(u,s_0))I_L(u)-\frac{1}{M^2}\left[1+\left(\frac{2}{u}-1\right)\mathcal{H}\right]\tilde{\Theta}(c(u,s_0))H_3(u)\right\}$$

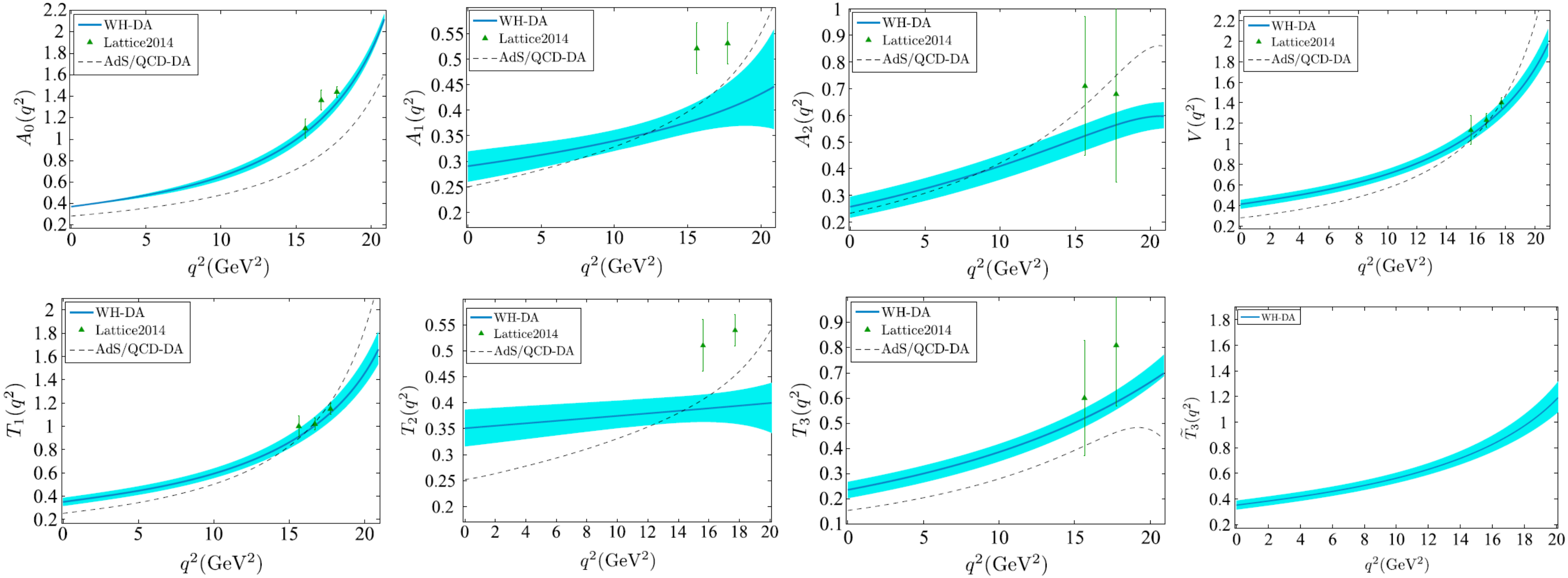
$$\tilde{T}_3(q^2)=\frac{m_b^2 f_{K^*}^{\perp} m_{K^*}^2}{m_B^2 f_B}\int_0^1\frac{du}{u}e^{\frac{m_B^2-s(u)}{M^2}}\left\{\frac{1}{m_{K^*}^2}\Theta(c(u,s_0))\phi_{2;K^*}^{\perp}(u)-\frac{m_b^2}{4u^2M^4}\tilde{\tilde{\Theta}}(c(u,s_0))\phi_{4;K^*}^{\perp}(u)-2\left[\frac{1}{uM^2}\right.\right.$$

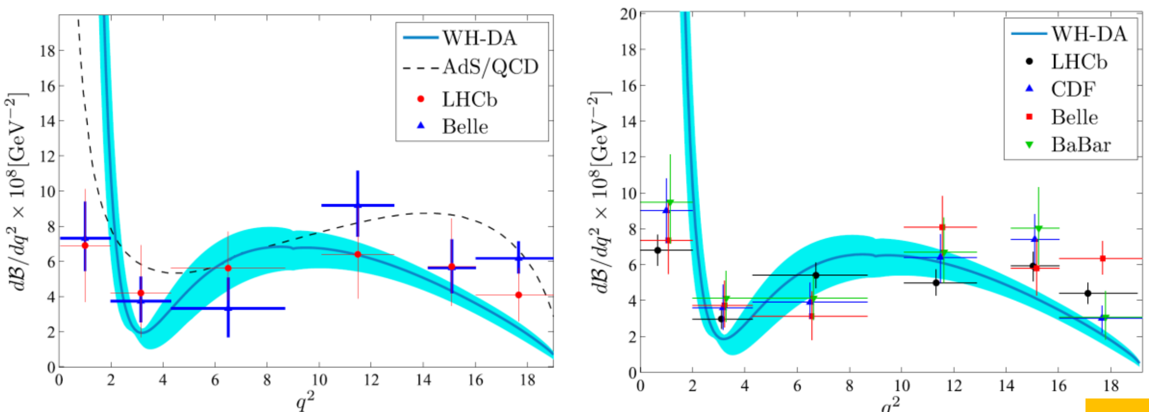
$$\left.\left.\times\tilde{\Theta}(c(u,s_0))+\frac{2q^2}{u^2M^4}\tilde{\tilde{\Theta}}(c(u,s_0))\right]I_L(u)-\frac{1}{M^2}\tilde{\Theta}(c(u,s_0))H_3(u)\right\}$$

$$T_3(q^2)=\frac{m_b^2 f_{K^*}^{\perp} m_{K^*}^2}{m_B^2 f_B}\int_0^1\frac{du}{u}e^{\frac{m_B^2-s(u)}{M^2}}\left\{\frac{1}{m_{K^*}^2}\Theta(c(u,s_0))\phi_{2;K^*}^{\perp}(u,\mu)-\frac{m_b^2}{4u^2M^4}\tilde{\tilde{\Theta}}(c(u,s_0))\phi_{4;K^*}^{\perp}(u)-\left[\frac{2}{uM^2}\right.\right.$$

$$\left.\left.\times\tilde{\Theta}(c(u,s_0))+\frac{4}{u^2M^4}\tilde{\tilde{\Theta}}(c(u,s_0))(m_B^2-m_{K^*}^2)\right]I_L(u)+\left[\frac{2}{uM^2}-\frac{1}{M^2}\right]\tilde{\Theta}(c(u,s_0))H_3(u)\right\}$$

II. LCSR with chiral correlator —— B→K* TFFs





$$\mathcal{B} = \left(1.088^{+0.261}_{-0.205}\right) \times 10^{-6},$$

$$\mathcal{B}(B \to K^*\mu^+\mu^-) = (1.19 \pm 0.39) \times 10^{-6} \quad \text{pQCD}$$

Ali A, Lunghi E, Greub C and Hiller G 2002 Improved model independent analysis of semileptonic and radiative rare *B* decays *Phys. Rev.* D **66** 034002

JPG 43 (2016) 015002

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II. LCSR with chiral correlator —— $B \to K^* \mu^+ \mu^-$ Asymmetry

- **Forward-Backward asymmetry**

$$\frac{dA_{\rm FB}}{dq^2} \equiv \frac{1}{d\Gamma/dq^2}\left(\int_0^1 d(\cos\theta)\frac{d^2\Gamma[B\to K^*\ell^+\ell^-]}{dq^2 d\cos\theta} - \int_{-1}^0 d(\cos\theta)\frac{d^2\Gamma[B\to K^*\ell^+\ell^-]}{dq^2 d\cos\theta}\right),$$

- **Isospin asymmetry**

$$\frac{dA_I}{dq^2} \equiv \frac{d\Gamma[B^0\to K^{*0}\ell^+\ell^-]/dq^2 - d\Gamma[B^\pm\to K^{*\pm}\ell^+\ell^-]/dq^2}{d\Gamma[B^0\to K^{*0}\ell^+\ell^-]/dq^2 + d\Gamma[B^\pm\to K^{*\pm}\ell^+\ell^-]/dq^2}.$$

→ **Decay Width**

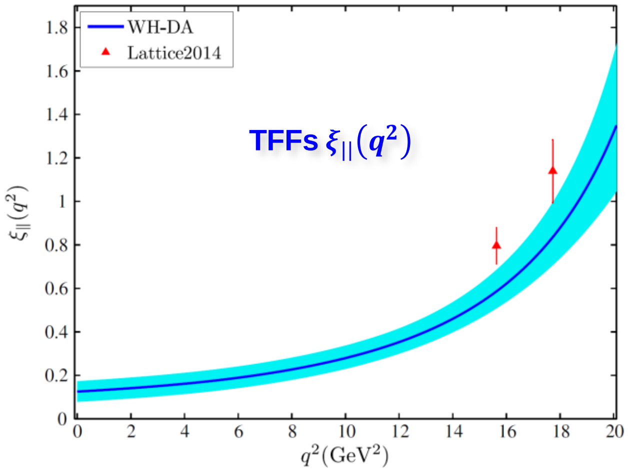
→ **TFFs $\xi_{\perp,||}(q^2)$**

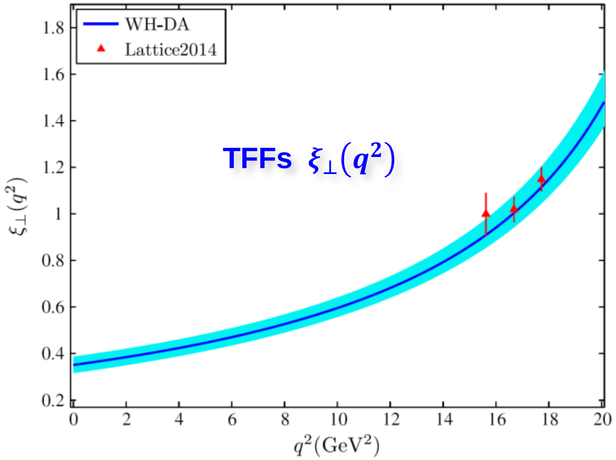
→ **LCSR with Chiral correlator**

→ **WH model for K* meson twist-2 LCDA**

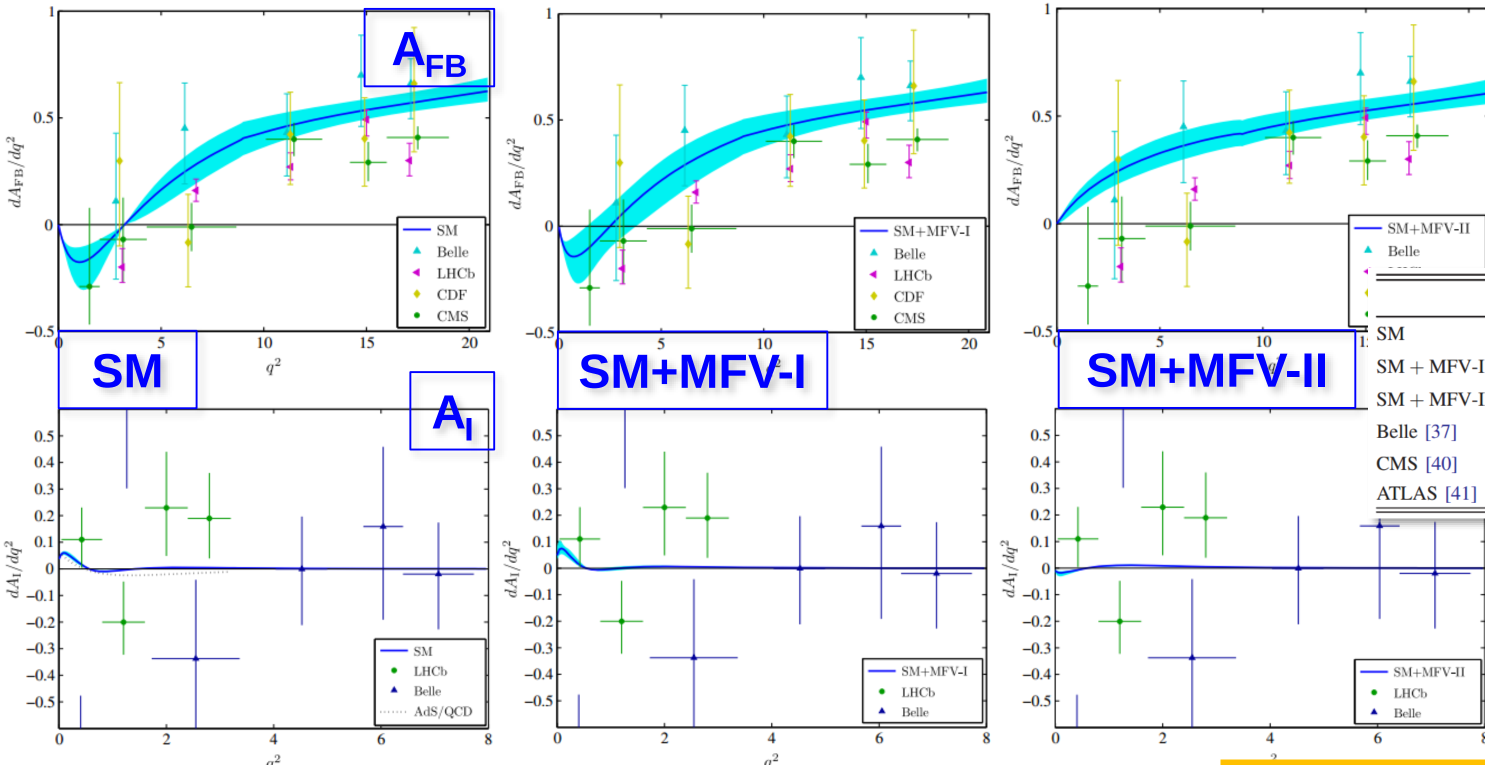
	$\xi_{\parallel}(0)$	$\xi_{\perp}(0)$
Our prediction	$0.129^{+0.006}_{-0.009}$	$0.351^{+0.036}_{-0.035}$
LCSR1 [9]	0.126(11)	0.333(28)
LCSR2 [32]	0.118(8)	0.266(32)
AdS/QCD [20]	0.076	0.245
Empirical estimate [5]	0.16(3)	0.26(6)

TABLE I: The $B \to K^*$ SFFs at the large recoil region ξ_λ, where the errors are squared average of all mentioned error sources. As a comparison, the results derived by light-cone sum rule [9, 32], AdS/QCD [20] predictions and empirical estimate [5] are also presented.





II. LCSR with chiral correlator —— $B \to K^*\mu^+\mu^-$ Asymmetry



	$A_{FB}(q^2 \in [1,6]\ \mathrm{GeV}^2)$
SM	$0.121^{+0.212}_{-0.357}$
SM + MFV-I	$0.425^{+0.225+0.395}_{-0.369-0.424}$
SM + MFV-II	$1.297^{+0.203+0.227}_{-0.189-0.213}$
Belle [37]	$0.26^{+0.27}_{-0.30} \pm 0.07$
CMS [40]	-0.05 ± 0.03
ATLAS [41]	$0.07 \pm 0.20 \pm 0.07$

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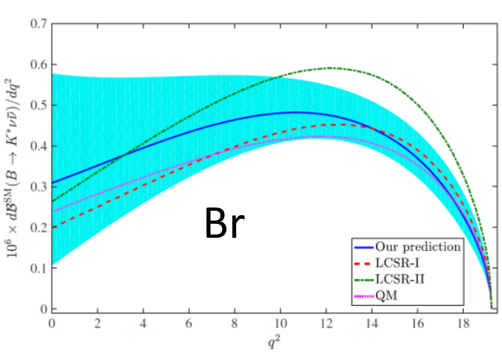
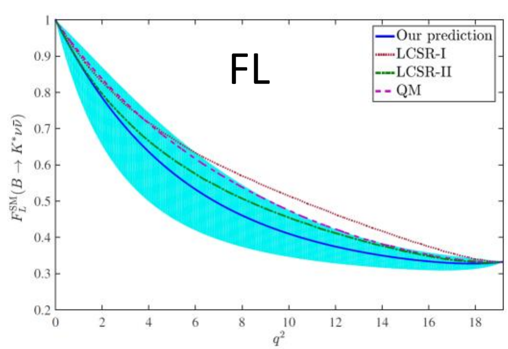
$B \to K^*\nu\bar{\nu}$ Branching Ratio and F_L





	$\mathcal{B} \times 10^6$	$\langle F_L \rangle$
SM	$7.60^{+2.16}_{-1.70}$	$0.49^{+0.09}_{-0.10}$
SM + GFV-I	$5.92^{+1.68}_{-1.33}$	...
SM + GFV-II	$9.72^{+2.76}_{-2.18}$	...
Belle [49]	<18	...
ABSW [47] (SM)	$6.8^{+1.0}_{-1.1}$	0.54(1)
ABSW [47] (GFV-I)	5.3	...
ABSW [47] (GFV-II)	8.7	...
NWA(SM) [50]	9.49(101)	0.49(4)

By Suppressing the value of threshold S_0, the contribution from the added scalar state can be highly reduced or eliminated.

	$A_1(0)$	$A_2(0)$	$V(0)$	$T_1(0)[T_2(0), \widetilde{T}_3(0)]$	$T_3(0)$
LCSR-$\mathcal{R}$	$0.310^{+0.030}_{-0.037}$	$0.260^{+0.055}_{-0.058}$	$0.332^{+0.051}_{-0.051}$	$0.254^{+0.046}_{-0.049}$	$0.152^{+0.039}_{-0.043}$
LCSR-$\mathcal{U}$	$0.308^{+0.032}_{-0.028}$	$0.257^{+0.028}_{-0.026}$	$0.307^{+0.024}_{-0.023}$	$0.251^{+0.028}_{-0.024}$	$0.145^{+0.020}_{-0.020}$
LCSR [38]	$0.25^{+0.16}_{-0.10}$	$0.23^{+0.19}_{-0.10}$	$0.36^{+0.23}_{-0.12}$	$0.31^{+0.18}_{-0.10}$	$0.22^{+0.17}_{-0.10}$
BZ [12]	0.292 ± 0.028	0.259 ± 0.027	0.411 ± 0.033	0.333 ± 0.028	0.202 ± 0.018
AdS [36]	0.249	0.235	0.277	0.255	0.155

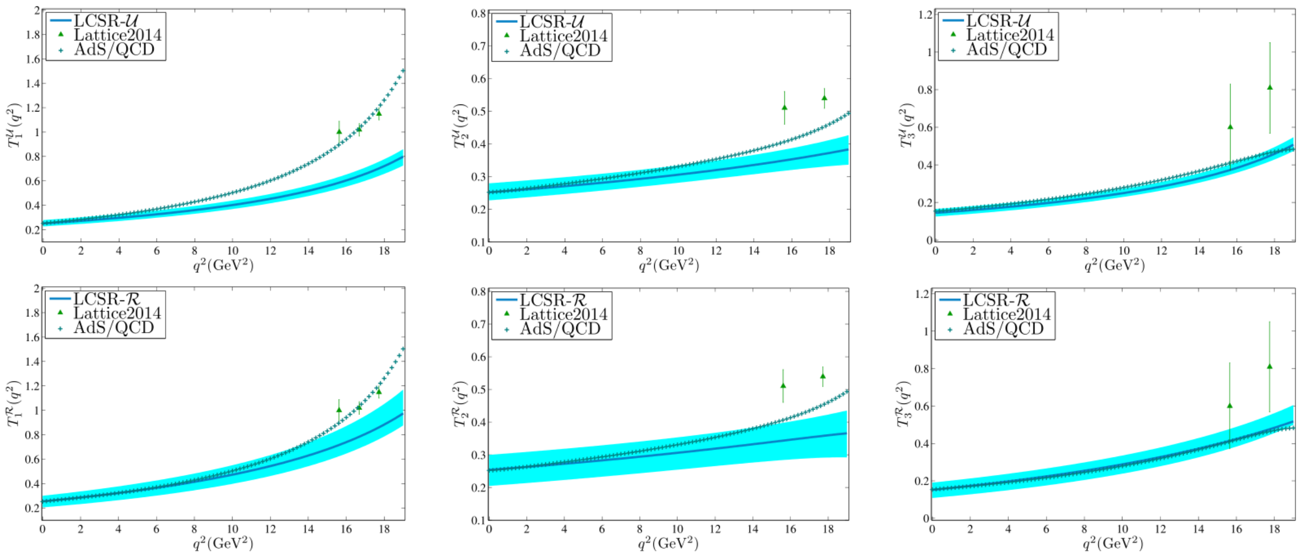
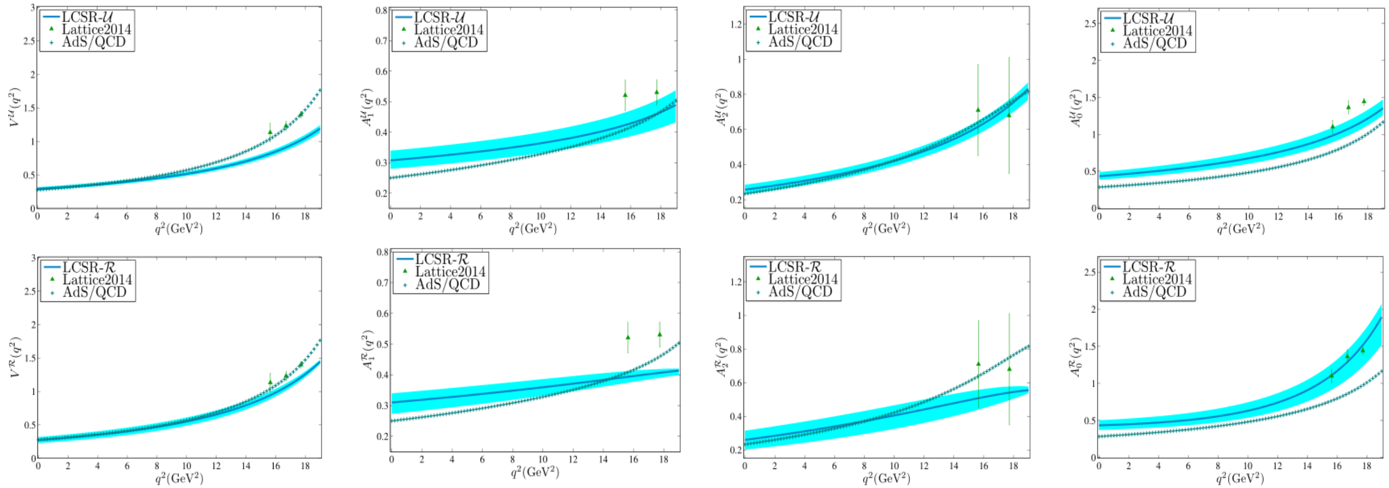
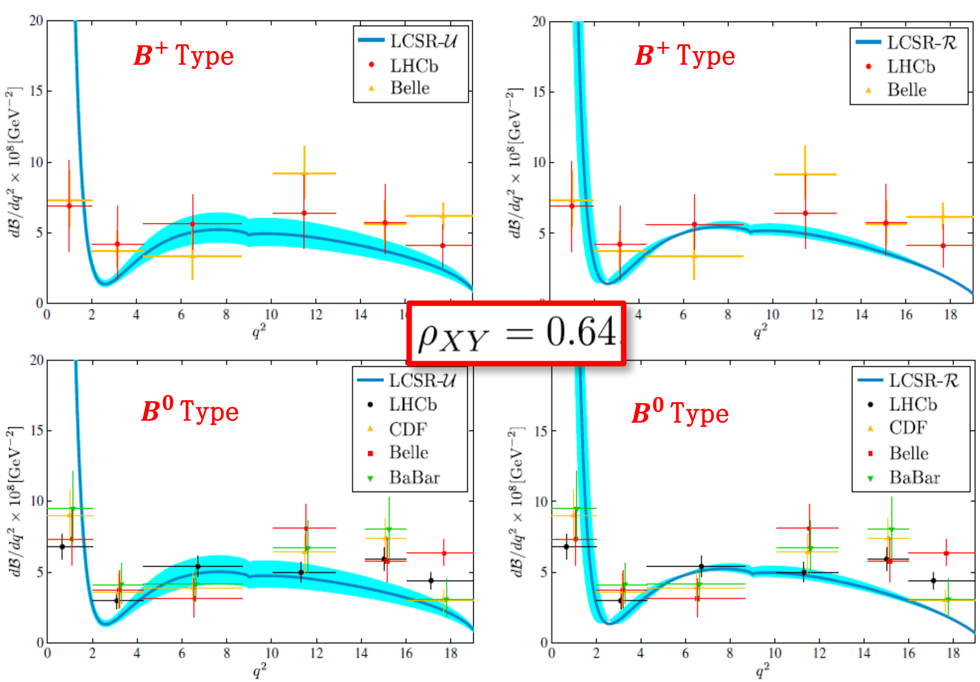


FIG. 4. The extrapolated $B \to K^*$ tensor TFFs $T_{1,2,3}(q^2)$. The left and right figures stand for LCSR with the usual and right current, respectively. The solid lines are central values and the shaded bands are their errors. As a comparison, the AdS/QCD [36] and the lattice QCD [41] and predictions are presented.

$$\rho_{XY} = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}.$$



	A_1	A_0	T_2	A_2	V	T_3	T_1
ρ_{XY}	0.89	0.79	0.71	0.59	0.54	0.49	0.37



$\rho_{XY} = 0.64$

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III. SVZ sum rule within BFT

- Background Field Theory (BFT)

Gluon field $\mathcal{A}_\mu^A(x) \rightarrow \mathcal{A}_\mu^A(x) + \phi_\mu^A(x),$

Quark field $\psi(x) \rightarrow \psi(x) + \eta(x).$

- Equation of the motion

$$(i\not{D} - m)\psi(x) = 0,$$
$$\widetilde{D}_\mu^{AB} G^{B\nu\mu}(x) = g_s \bar{\psi}(x)\gamma^\nu T^A \psi(x),$$

- Fundamental and adjoint representations of the gauge covariant derivatives

$$D_\mu = \partial_\mu - i g_s T^A \mathcal{A}_\mu^A(x)$$

$$\widetilde{D}_\mu^{AB} = \delta^{AB} - g_s f^{ABC} \mathcal{A}_\mu^C(x)$$

- Quark propagator (up to 6-dimension)

$$S_F(x,0) = S_F^0(x,0) + S_F^2(x,0) + S_F^3(x,0) + \sum_{i=1}^{2} S_F^{4(i)}(x,0) + \sum_{i=1}^{3} S_F^{5(i)}(x,0) + \sum_{i=1}^{5} S_F^{6(i)}(x,0),$$

- Vertex operator

$$\begin{aligned} z\cdot B &= -2iz\cdot\mathcal{A} \\ &= -ix^\mu z^\nu G_{\mu\nu} - \frac{2i}{3}x^\mu x^\rho z^\nu G_{\mu\nu;\rho} \\ &\quad -\frac{i}{4}x^\mu x^\rho x^\sigma z^\nu G_{\mu\nu;\rho\sigma} - \frac{i}{15}x^\mu x^\rho x^\sigma x^\lambda z^\nu G_{\mu\nu;\rho\sigma\lambda} \\ &\quad -\frac{i}{72}x^\mu x^\rho x^\sigma x^\lambda x^\tau z^\nu G_{\mu\nu;\rho\sigma\lambda\tau} + \cdots . \end{aligned}$$

III. SVZ sum rule within BFT

$$S_F^{d\leqslant 3}(x,0) = \mathrm{i}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot x}\left\{-\frac{m+\not p}{m^2-p^2}+\frac{\gamma^\nu(\not p-m)\gamma^\mu}{(m^2-p^2)^2}b_{0\nu\mu}-\mathrm{i}\left[2\frac{\gamma^\nu(\not p-m)p^\rho}{(m^2-p^2)^3}+\frac{g^{\nu\rho}}{(m^2-p^2)^2}\right]\gamma^\mu b_{1\nu\mu|\rho}\right\}$$

$$S_F^{4(1)}(x,0) = \mathrm{i}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot x}\left\{\left[\frac{1}{4}\left(\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^3}-2\frac{p^\mu}{(m^2-p^2)^3}\right)\gamma^\nu\gamma^\rho\gamma^\sigma+\frac{1}{2}\left(\frac{(m+\not p)\gamma^\mu}{(m^2-p^2)^3}g^{\nu\sigma}+4\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}p^\nu p^\sigma\right)\gamma^\rho\right]G_{\mu\nu}G_{\rho\sigma}\right\}$$

$$S_F^{4(2)}(x,0) = \mathrm{i}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot x}\left\{\frac{\mathrm{i}}{4}\left[\frac{g^{\{\mu\rho}\gamma^{\sigma\}}(m-\not p)}{(m^2-p^2)^3}-2\frac{g^{\{\mu\rho}p^{\sigma\}}}{(m^2-p^2)^3}+4\frac{\gamma^{\{\mu}p^\rho p^{\sigma\}}(m-\not p)}{(m^2-p^2)^4}\right]\gamma^\nu G_{\mu\nu;\rho\sigma}\right\}$$

$$S_F^{5(1)}(x,0) = \mathrm{i}\int \frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot x}\left\{-\frac{\mathrm{i}}{3}\left[\left(3\frac{\gamma^\mu(m-\not p)\gamma^\nu}{(m^2-p^2)^4}(p^\lambda\gamma^\rho+p^\rho\gamma^\lambda)-\frac{\gamma^\nu(g^{\mu\lambda}\gamma^\rho+g^{\mu\rho}\gamma^\lambda)}{(m^2-p^2)^3}\right)\gamma^\sigma\right.\right.$$
$$\left.\left.+4\left(\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}g^{\{\nu\sigma}p^{\lambda\}}+2\frac{p^\mu g^{\{\nu\sigma}p^{\lambda\}}}{(m^2-p^2)^4}+6\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^5}p^\nu p^\sigma p^\lambda\right)\gamma^\rho\right]\times G_{\mu\nu}G_{\rho\sigma;\lambda}\right\}$$

$$S_F^{5(2)}(x,0) = i\int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\left\{\frac{2i}{3}\left[\left(\frac{g^{\mu\lambda}}{(m^2-p^2)^3}+\frac{6p^\mu p^\lambda}{(m^2-p^2)^4}\right)\gamma^\nu\gamma^\rho\gamma^\sigma\right.\right.$$
$$\left.\left.-2\left(\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}g^{\{\nu\sigma}p^{\lambda\}}+2\frac{p^\mu g^{\{\nu\sigma}p^{\lambda\}}}{(m^2-p^2)^4}+6\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^5}p^\nu p^\sigma p^\lambda\right)\gamma^\rho\right]G_{\mu\nu;\lambda}G_{\rho\sigma}\right\},$$

$$S_F^{5(3)}(x,0) = i\int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\left\{\frac{4}{15}\left[\frac{g^{\{\rho\sigma}p^\lambda\gamma^{\mu\}}(m-\not p)}{(m^2-p^2)^4}-2\frac{g^{\{\rho\sigma}p^\lambda p^{\mu\}}}{(m^2-p^2)^4}+6\frac{\gamma^{\{\mu}p^\rho p^\sigma p^{\lambda\}}(m-\not p)}{(m^2-p^2)^5}\right.\right.$$
$$\left.\left.-\frac{g^{(\mu\rho\sigma\lambda)}}{(m^2-p^2)^3}\right]\gamma^\nu G_{\mu\nu;\rho\sigma\lambda}\right\},$$

$$S_F^{6(1)}(x,0) = i\int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\frac{i}{8}\left\{\left[\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}-4\frac{p^\mu}{(m^2-p^2)^4}\right]\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\lambda\gamma^\tau\right.$$
$$\left.+2\left[3\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}g^{\sigma\tau}+16\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^5}p^\sigma p^\tau-4\frac{g^{\mu\sigma}p^\tau+g^{\mu\tau}p^\sigma}{(m^2-p^2)^4}\right]\gamma^\nu\gamma^\rho\gamma^\lambda\right\}G_{\mu\nu}G_{\rho\sigma}G_{\lambda\tau},$$

$$S_F^{6(2)}(x,0) = i\int \frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}\left(-\frac{1}{8}\right)\left\{\left[3\frac{\gamma^\mu(m-\not p)\gamma^\nu}{(m^2-p^2)^4}g^{\{\lambda\tau}\gamma^{\rho\}}+16\frac{\gamma^\mu(m-\not p)\gamma^\nu}{(m^2-p^2)^5}\gamma^{\{\rho}p^\lambda p^{\tau\}}\right.\right.$$
$$-4\frac{\gamma^\nu}{(m^2-p^2)^4}g^{\mu\{\lambda}p^\tau\gamma^{\rho\}}\Bigg]\gamma^\sigma+4\left[\frac{m+\not p}{(m^2-p^2)^4}g^{(\nu\sigma\tau\lambda)}+6\frac{m+\not p}{(m^2-p^2)^5}g^{\{\nu\sigma}p^\tau p^{\lambda\}}\right.$$
$$\left.\left.+48\frac{m+\not p}{(m^2-p^2)^6}p^\nu p^\sigma p^\tau p^\lambda\right]\gamma^\mu\gamma^\rho\right\}G_{\mu\nu}G_{\rho\sigma;\lambda\tau},$$

… …

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T.Zhong, X.G.Wu, Z.G.Wang,

T.Huang, H.B.Fu, H.Y.Han

$$\phi^{\|}_{2;\rho}(x,\mu) = 6x(1-x)\left(1+\sum_n C_n^{3/2}(\xi)\times a^{\|}_{n;\rho}(\mu)\right),$$

$$a^{\|}_{2;\rho} = \frac{7}{12}(5\langle\xi^{\|}_{2;\rho}\rangle - 1),$$

$$a^{\|}_{4;\rho} = -\frac{11}{24}(14\langle\xi^{\|}_{2;\rho}\rangle - 21\langle\xi^{\|}_{4;\rho}\rangle - 1),$$

$$a^{\|}_{6;\rho} = \frac{5}{64}(135\langle\xi^{\|}_{2;\rho}\rangle - 495\langle\xi^{\|}_{4;\rho}\rangle + 429\langle\xi^{\|}_{6;\rho}\rangle - 5).$$

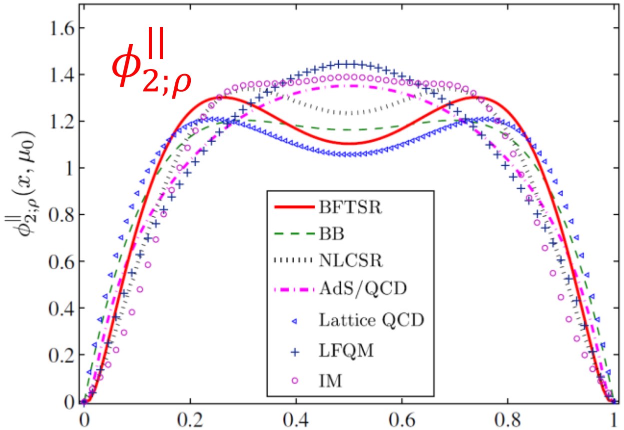
- Correlation function

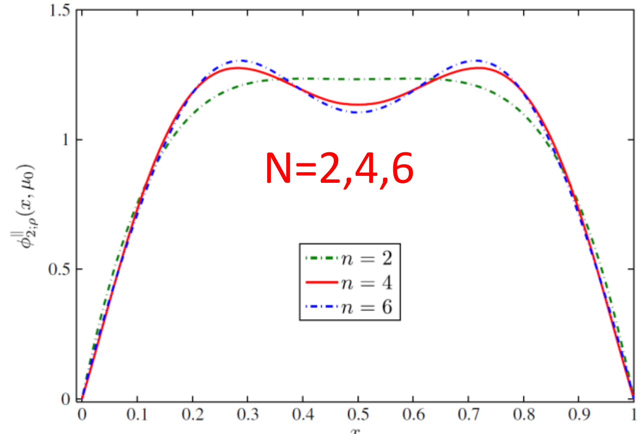
$$\Pi^{\|(n,0)}_{\rho}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_n(x)J_0^{\dagger}(0)\}|0\rangle = (z\cdot q)^{n+2} I^{\|(n,0)}(q^2)$$

$$J_n(x) = \bar{d}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n u(x) \quad J_0^{\dagger}(0) = \bar{u}(0)\not{z}d(0)$$

$$\langle\xi^{\|}_{n;\rho}\rangle = \frac{M^2}{f_\rho^2}e^{m_\rho^2/M^2}\left\{\frac{3}{4\pi^2(n+1)(n+3)}\left(1+\frac{\alpha_s}{\pi}A'_n\right)(1-e^{-s_\rho/M^2}) + \sum_{q=u,d}\left(\frac{m_q\langle\bar{q}q\rangle}{M^4} - \frac{8n+1}{18}\frac{m_q\langle g_s\bar{q}\sigma TGq\rangle}{M^6} + \frac{4n+2}{81}\frac{\langle g_s\bar{q}q\rangle^2}{M^6}\right) + \frac{1+n\theta(n-2)}{12\pi(n+1)}\frac{\langle\alpha_s G^2\rangle}{M^4} + \frac{1}{16\pi}\frac{\langle g_s^3 fG^3\rangle}{M^6}\left\{\frac{8\delta^{n0}+405n+192}{36}\ln\frac{M^2}{\mu^2} - \frac{16\delta^{n0}+810n+363}{72} \times\gamma_E + \frac{7}{24}\psi(n+1) + \frac{8\delta^{n0}+405n+826}{72} + \theta(n-2)\left[\frac{16-22n}{72}\ln\frac{M^2}{\mu^2} - \frac{788n+421}{72}\psi(n+1) - \frac{766n+437}{72}\gamma_E - \frac{68n^2-37n-11}{144n} + \sum_{k=0}^{n-2}(-1)^k\frac{1}{144}\left(\frac{3(135k+128)}{n-k} + \frac{383k+399}{k-n+1} - \frac{106kn-410k+617n-415}{(k+1)(k+2)} + 106\right)\right]\right\}\right\}, \tag{15}$$

III. SVZ sum rule within BFT— rho meson longitudinal twist-2 DA





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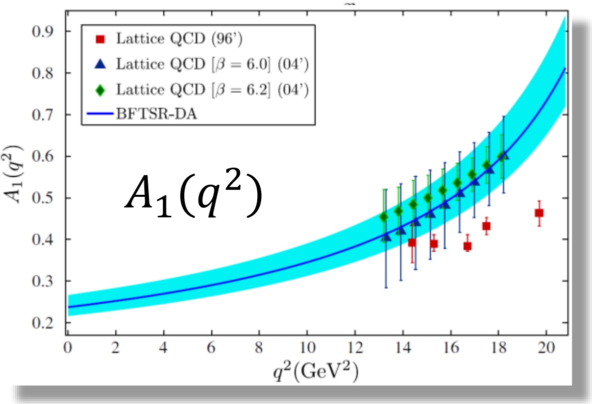
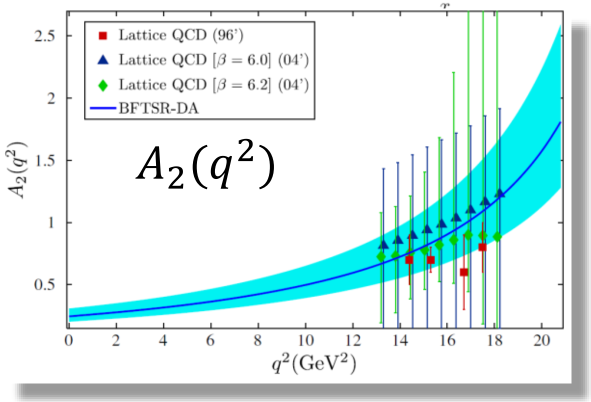
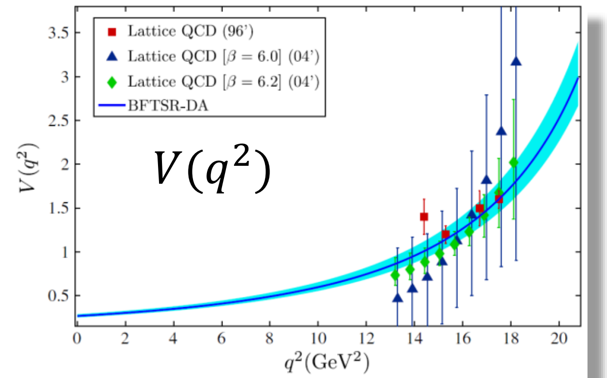
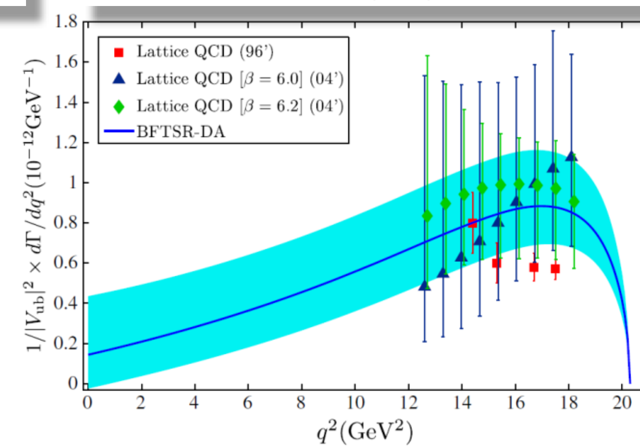






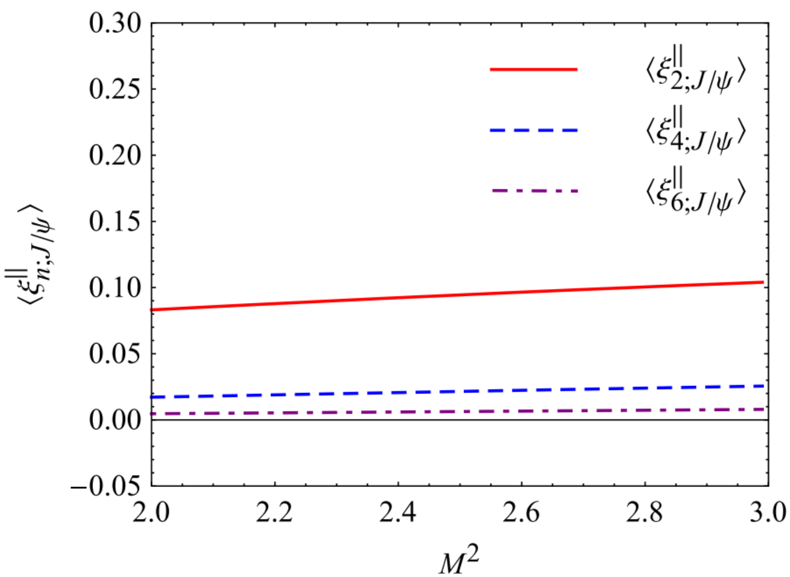
V_{ub}

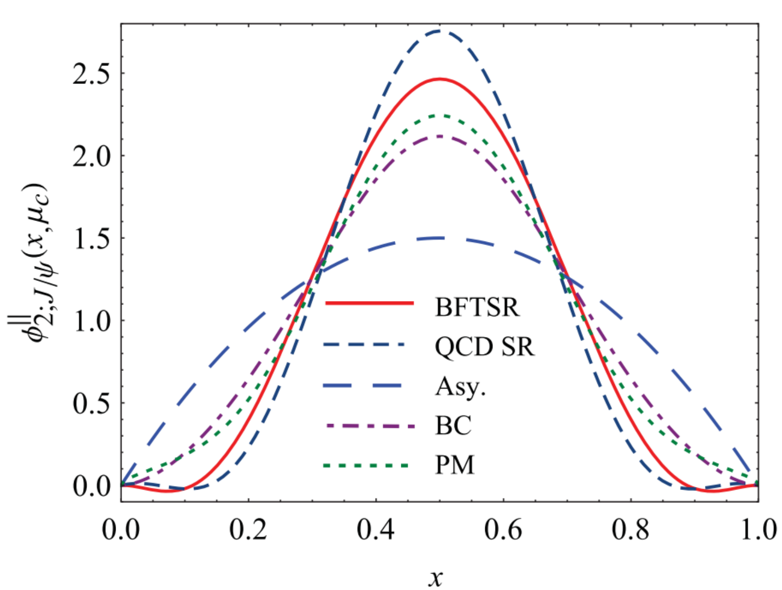
Our prediction		$3.19^{+0.65}_{-0.62}$
Omnès parametrization [67]		2.80(20)
BABAR [34]	LCSR [5]	2.75(24)
	ISGW [68]	2.83(24)
BABAR [35]	LCSR [5]	2.85(40)
	ISGW [68]	2.91(40)



Propagator and vertex operator contain m_c which are more complex than ρ-meson !!

$$\langle\xi^{\|}_{n;J/\psi}\rangle = \frac{e^{m^2_{J/\Psi}/M^2}}{f^{\|2}_{J/\Psi}}\left\{\frac{3}{8\pi^2(n+1)(n+3)}\left(1+\frac{\alpha_s}{\pi}A'_n\right)\int_{t_{\min}}^{s_{J/\psi}} ds e^{-s/M^2}\left[v^{n+1}\frac{2(n+1)m_c^2+s}{s}-(v\to -v)\right]+\frac{\langle\alpha_s G^2\rangle}{6\pi M^2}\right.$$
$$\times\int_0^1 dx\, e^{-\frac{m_c^2}{x\bar{x}M^2}}\frac{\xi^{n-2}}{x^2\bar{x}^2}\left[n(n-1)x^3\bar{x}^3+\frac{\xi^2}{2}\left(1-\frac{m_c^2(x^3+\bar{x}^3)}{x^3\bar{x}^3M^2}\right)\right]+\frac{\langle g_s^3fG^3\rangle}{16\pi^2M^4}\int_0^1 dx\, e^{-\frac{m_c^2}{x\bar{x}M^2}}\frac{\xi^{n-2}}{2}$$
$$\times\left\{\left[-\xi^2\left(\frac{69+2n(11+64x\bar{x})}{72x\bar{x}}+\frac{45(1-3x\bar{x})}{8x^2\bar{x}^2}\right)-\frac{n(n-1)}{9}[16+(n-31)x\bar{x}]\right]+\frac{1}{3M^2}\left[\xi^2\left(\frac{m_c^2(1+2x\bar{x})}{12x^2\bar{x}^2}\right.\right.\right.$$
$$\left.\left.\left.\left.-\frac{8nm_c^2}{3x\bar{x}}-\frac{3m_c^2(x^4+\bar{x}^4)}{4x^3\bar{x}^3}\right)+\xi\frac{11nm_c^2(x^3-\bar{x}^3)}{6x^2\bar{x}^2}-\frac{n(n-1)m_c^2}{3}\right]+\xi^2\frac{m_c^2(x^5+\bar{x}^5)}{30M^4x^4\bar{x}^4}\right\}\right\}. \quad (14)$$





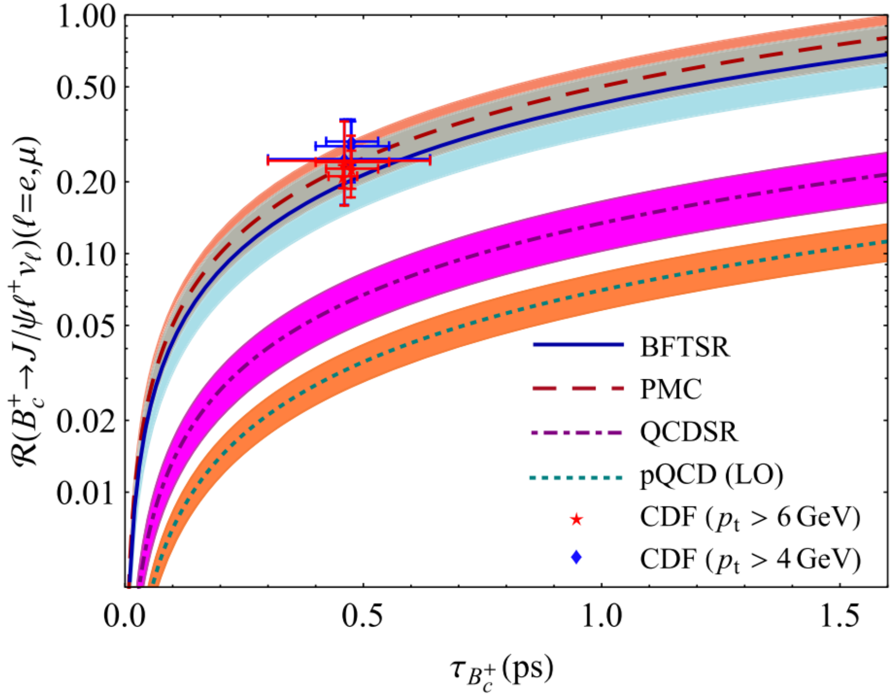
Borel window for SVZSR

III. SVZ sum rule within BFT— J/ψ longitudinal twist-2 DA

$\langle\xi^{\parallel}_{n;J/\psi}\rangle$	$n=2$	$n=4$	$n=6$
Our prediction	0.083(12)	0.015(5)	0.003(2)
QCD SR [47]	0.070(7)	0.012(2)	0.0031(8)
BT model [48]	0.086	0.020	0.0066
Cornell model [49]	0.084	0.019	0.0066
NRQCD [50]	0.075(11)	0.010(3)	0.0017(7)

	$A_1(0)$	$A_2(0)$	$V(0)$
This work	$1.13^{+0.13}_{-0.11}$	$1.20^{+0.14}_{-0.12}$	$1.50^{+0.17}_{-0.15}$
PMC [17]	1.07(52)	1.15(55)	1.47(72)
QCD SR [9]	0.75	1.69	1.69
3PSR [8]	0.63	0.69	1.03
QM [52]	0.68	0.66	0.96

References	$\Re(J/\psi\ell^+\nu_\ell)$
This work	$0.217^{+0.069}_{-0.057}$
CDF2016 [2]	$0.211\pm0.012(\mathrm{st})^{+0.021}_{-0.020}(\mathrm{sy})$
PMC [17]	$0.257^{+0.045}_{-0.034}$
NLO pQCD [12]	$0.235^{+0.088}_{-0.049}$
QCDSR-LCSR [9]	0.068(12)
QCDSR-3PSR [8]	0.084
LO pQCD [7]	$0.036^{+0.005}_{-0.004}$
PM [6]	0.073
BS equation [5]	0.083
CQM-I [4]	$0.053^{+0.003}$
CQM-II [3]	0.068



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IV. Summary and Outlook

1. LCSR with chiral correlator

   B-K* TFFs, Asymmetry, Equivalence of two LCSR

2. SVZSR with BFT

   ρ, J/ψ and semileptonic decay

- $B, D \to \pi, K, S, V, T$ ... ... at BESIII、 LHCb、Belle-II
- NLO correction for semileptonic decay processes
- Heavy baryon decay processes

IV. Summary and Outlook

1	PRD 97 (2018) 074025	H.B.Fu, L.Zeng, W.Cheng, T.Zhong, X.G.Wu
2	PRD 97 (2018) 055037	H.B.Fu, X.G.Wu, W.Cheng, T.Zhong, Z.Sun
3	EPJC 78 (2018) 76	Y.Zhang, T.Zhong, X.G.Wu, K.Li, H.B.Fu, T.Huang
4	PRD 95 (2017) 094023	W.Cheng, X.G.Wu, H.B.Fu
5	PRD 94 (2016) 074004	H.B.Fu, X.G.Wu, W.Cheng, T.Zhong
6	EPJC 76 (2016) 509	T.Zhong, X.G.Wu, K.Li, T.Huang, H.B.Fu
7	JPG 43 (2016) 015002	H.B.Fu, X.G.Wu, Y.Ma, W.Cheng, T.Zhong
8	JPG 42 (2015) 055002	H.B.Fu, X.G.Wu, H.Y.Han, Y.Ma
9	ROPP 78 (2015) 126201	X.G.Wu, Y.Ma, S.Q.Wang, H.B.Fu, H.H.Ma, S.J.Brodsky,
10	PLB 738 (2014) 228	H.B.Fu, X.G.Wu, H.Y.Han, Y.Ma, H.Y.Bi
11	NPB 884 (2014) 172	H.B.Fu, X.G.Wu, H.Y.Han, Y.Ma, T.Zhong
12	PRD 90 (2014) 034004	G.Chen, X.G.Wu, H.B.Fu, H.Y.Han, Z.Sun
13	JHEP 1412 (2014) 018	G.Chen, X.G.Wu, Z.Sun, Y.Ma, H.B.Fu
14	PRD 90 (2014) 016004	T.Zhong, X.G.Wu, Z.G.Wang, T.Huang, H.B.Fu, H.Y.Han
15	PRD 89 (2014) 074020	G.Chen, X.G.Wu, J.W.Zhang, H.Y.Han, H.B.Fu
	…….	…….

Thanks for your attention!