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Flavor SU(3) Topological Diagram and Irreducible Representation Amplitudes for Heavy Hadron Charmless Decays: Mismatch and Equivalence

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- Mismatch and equivalence between IRA and TDA approaches

- Summary

Motivation

- The flavor SU(3) symmetry is widely used in two or three body heavy meson and baryon decays. This method has an advantage of its independence on the detailed dynamics and can predict relations between various decay channels.

- In the previous literatures, the SU(3) analysis is usually formulated in two ways. One is derived from the SU(3) irreducible representaion amplitude (IRA) while another one is derived from various topologiclal diagrams (TDA).

- However, in the literatures these two methods do not match consistently. Some amplitudes of TDA are missed and some of them are not independent. In this work we will show how such mismatch occurs and the equivalence between IRA and TDA.

SU(3) analysis of $B \to PP$

IRA approach

We treat the transition operator $b \to q\bar{u}u$ as the representation of SU(3): $\bar{3} \otimes 3 \otimes \bar{3}$, it can be reduced into: $\bar{3} \oplus \bar{3} \oplus 6 \oplus \overline{15}$, which correspond to 3 tensors.

$\Delta S = 0(b \to d)$

$$(H_{\mathbf{3}})^2 = 1, \quad (H_6)^{12}_1 = -(H_6)^{21}_1 = (H_6)^{23}_3 = -(H_6)^{32}_3 = 1,$$
$$2(H_{\overline{15}})^{12}_1 = 2(H_{\overline{15}})^{21}_1 = -3(H_{\overline{15}})^{22}_2 = -6(H_{\overline{15}})^{23}_3 = -6(H_{\overline{15}})^{32}_3 = 6.$$

$\Delta S = -1(b \to s)$ Just change the index $2 \leftrightarrow 3$ corresponding to the d $\leftrightarrow$ s exchange.

The B meson and pseudoscalar mesons are constructed by suitable SU(3) representations:

$$B_i = (B^-, \bar{B}_0, \bar{B}_s)$$

$$(M^i_j) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta_8}{\sqrt{6}}, \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1, \end{pmatrix}$$

IRA method

Irreducible Representation tree Amplitudes:

$$\mathcal{A}_t^{IRA} = A_3^T B_i (H_{\bar{3}})^i (M)_k^j (M)_j^k + C_3^T B_i (M)_j^i (M)_k^j (H_{\bar{3}})^k + B_3^T B_i (H_3)^i (M)_k^k (M)_j^j + D_3^T B_i (M)_j^i (H_{\bar{3}})^j (M)_k^k$$
$$+ A_6^T B_i (H_6)_k^{ij} (M)_j^l (M)_l^k + C_6^T B_i (M)_j^i (H_6)_k^{jl} (M)_l^k + B_6^T B_i (H_6)_k^{ij} (M)_j^k (M)_l^l$$
$$+ A_{15}^T B_i (H_{\overline{15}})_k^{ij} (M)_j^l (M)_l^k + C_{15}^T B_i (M)_j^i (H_{\overline{15}})_l^{jk} (M)_k^l + B_{15}^T B_i (H_{\overline{15}})_k^{ij} (M)_j^k (M)_l^l.$$

For penguin diagrams, the QCD penguin operators behave as the $\overline{3}$ while the electroweak penguin operators behave the same as the tree operator.

$$\mathcal{A}_p^{IRA} = \mathcal{A}_t^{IRA}(A_i^T \to A_i^P,\ B_i^T \to B_i^P,\ C_i^T \to C_i^P,\ D_i^T \to D_i^P)$$

$$\mathcal{A}^{IRA} = V_{ub} V_{uq}^* \mathcal{A}_t^{IRA} + V_{tb} V_{tq}^* \mathcal{A}_p^{IRA}$$

***b → d* transition:**

channel	IRA
$B^- \to \pi^0 \pi^-$	$4\sqrt{2} C_{15}^T$
$B^- \to \pi^- \eta_8$	$\sqrt{\frac{2}{3}} (A_6^T + 3A_{15}^T + C_3^T - C_6^T + 3C_{15}^T)$
$B^- \to \pi^- \eta_1$	$\frac{1}{\sqrt{3}}(2A_6^T + 6A_{15}^T + 3B_6^T + 9B_{15}^T + 2C_3^T + C_6^T + 3C_{15}^T + 3D_3^T)$
$B^- \to K^0 K^-$	$A_6^T + 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$
$\overline{B}^0 \to \pi^+ \pi^-$	$2A_3^T - A_6^T + A_{15}^T + C_3^T + C_6^T + 3C_{15}^T$
$\overline{B}^0 \to \pi^0 \pi^0$	$2A_3^T - A_6^T + A_{15}^T + C_3^T + C_6^T - 5C_{15}^T$
$\overline{B}^0 \to \pi^0 \eta_8$	$\frac{1}{\sqrt{3}}(-A_6^T + 5A_{15}^T - C_3^T + C_6^T + C_{15}^T)$
$\overline{B}^0 \to \pi^0 \eta_1$	$-\frac{1}{\sqrt{6}}(2A_6^T - 10A_{15}^T + 3B_6^T - 15B_{15}^T + 2C_3^T + C_6^T - 5C_{15}^T + 3D_3^T)$
⋮	⋮

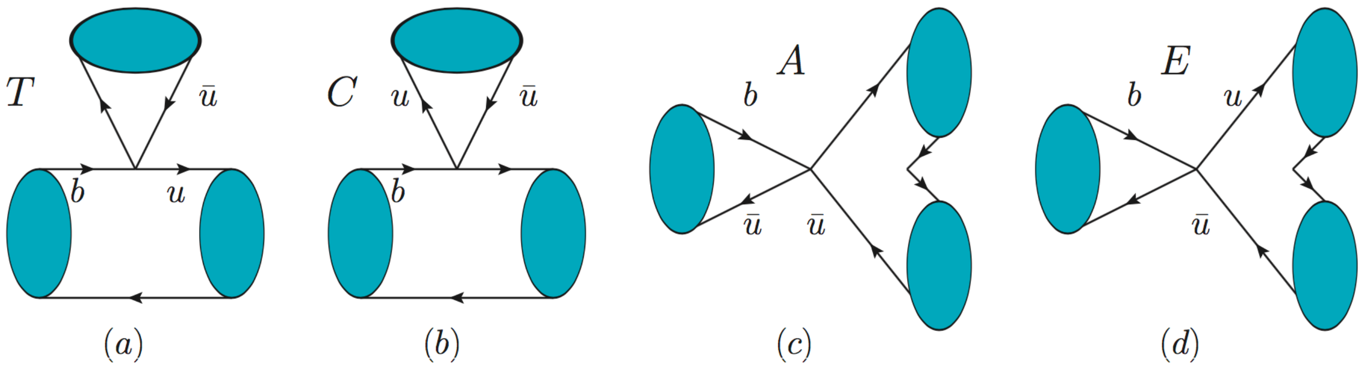
It should be noticed that these 10 coefficients are not all independent. We can do a redefinition to remove a redundant degree of freedom:

$$C_6^{T\prime} = C_6^T - A_6^T$$

$$B_6^{T\prime} = B_6^T + A_6^T$$

TDA method

Find out all the possible topology diagrams of $B \to PP$ decay. In the literature they are:



Here the transition operator $b \to q\bar{u}u$ is treated as only one operator $\bar{H}_k^{ij}$, where the 3 indexes correspond to real quark flavors.

$$\bar{H}_1^{12} = 1\ (\Delta s = 0),\quad \bar{H}_1^{13} = 1\ (\Delta s = -1)$$

The TDA Halmiltonian is

$$\mathcal{A}_t^{TDA} = T \times B_i (M)_j^i \bar{H}_k^{jl} (M)_l^k + C \times B_i (M)_j^i \bar{H}_k^{lj} (M)_l^k + A \times B_i \bar{H}_j^{il} (M)_k^j (M)_l^k + E \times B_i \bar{H}_j^{li} (M)_k^j (M)_l^k$$

Each term corresponds to a diagram above. But the question is whether they are complete and independent?

In terms of penguin diagrams:

1. The QCD penguin operators just behave as the $\overline{3}$ representation $\overline{H}^i$.
2. For the electroweak penguin operators, we can use the following trick:

$$\bar{q}b\sum_{q'} e_q \bar{q}' q' = \bar{q}b\bar{u}u - \frac{1}{3}\bar{q}b\sum_{q'}\bar{q}'q'$$

Similar to tree operators and is represented by $\overline{H}^{ij}_k$

behaves as the $\overline{3}$ representaion $\overline{H}^i$

Where $\overline{H}^2 = 1(\Delta s = 0)$ *and* $\overline{H}^3 = 1(\Delta s = -1)$

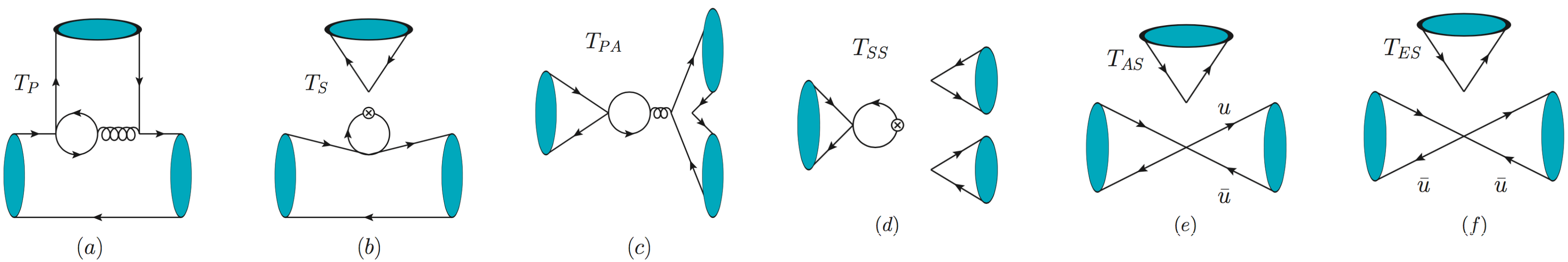
$$\mathcal{A}_p^{TDA} = \underline{P} \times B_i(M)^i_j(M)^j_k\bar{H}^k + \underline{S} \times B_i(M)^i_j\bar{H}^j(M)^k_k + \underline{P_A} \times B_i\bar{H}^i(M)^j_k(M)^k_j$$
$$+\underline{P_T} \times B_i(M)^i_j\bar{H}^{jl}_k(M)^k_l + \underline{P_C} \times B_i(M)^i_j\bar{H}^{lj}_k(M)^k_l.$$

Complete and Independent ?

Mismatch between IRA and TDA

Actually some topological diagrams are missed in such TDA analysis.

The IRA has 10-1 independent Hamiltonians while the TDA shows only 4 Hamiltonians. To solve this mismatch, some missed topological diagrams must be added (for tree amplitudes):



$$
\begin{aligned}
\mathcal{A}_t^{\prime TDA} &= T_S B_i (M)^i_j \bar{H}^{lj}_l (M)^k_k + T_P B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l + T_{PA} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j + T_{SS} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k \\
&\quad + T_{AS} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k + T_{ES} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k, \\
\mathcal{A}_p^{\prime TDA} &= P_{SS} B_i \bar{H}^i (M)^j_j (M)^k_k + P_{TA} B_i \bar{H}^{il}_j (M)^j_k (M)^k_l + P_{TE} B_i \bar{H}^{ji}_k (M)^k_l (M)^l_j \\
&\quad + P_{AS} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k + P_{ES} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k.
\end{aligned}
$$

Now the number of degree of freedom are matched between IRA and TDA !

Equivalence between IRA and TDA

The TDA analysis must be equivalent with the IRA analysis.

To see this we need to obtain the relationship between the two sets of amplitudes. Actually the TDA coefficients can be expressed as a composition of IRA coefficients :

$$A_3^T = -\frac{A}{8} + \frac{3E}{8} + T_{PA}, \quad B_3^T = T_{SS} + \frac{3T_{AS} - T_{ES}}{8}, \quad C_3^T = \frac{1}{8}(3A - C - E + 3T) + T_P$$

$$D_3^T = T_S + \frac{1}{8}(3C - T_{AS} + 3T_{ES} - T), \quad B_6'^T = \frac{1}{4}(A - E + T_{ES} - T_{AS}), \quad C_6'^T = \frac{1}{4}(-A - C + E + T),$$

$$A_{15}^T = \frac{A + E}{8}, \quad B_{15}^T = \frac{T_{ES} + T_{AS}}{8}, \quad C_{15}^T = \frac{C + T}{8}.$$

It should be noticed that among the 10 IRA amplitudes, there's one redundant degree of freedom. This redundancy must also happen in the case of TDA amplitudes.

So we have the inverse relations:

$$T + E = 4A_{15}^T + 2C_6'^T + 4C_{15}^T, \quad C - E = -4A_{15}^T - 2C_6'^T + 4C_{15}^T,$$

$$A + E = 8A_{15}^T, \quad T_P - E = -5A_{15}^T + C_3^T - C_6'^T - C_{15}^T,$$

$$T_{PA} + \frac{E}{2} = A_3^T + A_{15}^T, \quad T_{AS} + E = 4A_{15}^T - 2B_6'^T + 4B_{15}^T,$$

$$T_{ES} - E = -4A_{15}^T + 2B_6'^T + 4B_{15}^T, \quad T_{SS} - \frac{E}{2} = -2A_{15}^T + B_3^T + B_6'^T - B_{15}^T,$$

$$T_S + E = 4A_{15}^T - B_6'^T - B_{15}^T + C_6'^T - C_{15}^T + D_3^T.$$

Where we have absorbed The coefficient E.

Implication from the fit of $B \to PP$

In the previous work Y. K. Hsiao, C. F. Chang and X. G. He, Phys. Rev. D 93, no. 11, 114002 (2016) , a global fit was performed for $B \to PP$ decays in IRA scheme.
For example, the result of two coefficients are:

$$|C_3^T| = -0.211 \pm 0.027, \ \ \delta_3^T = (-140 \pm 6)^\circ, \ \ |B_{\overline{15}}^T| = -0.038 \pm 0.016, \ \ \delta_{B_{\overline{15}}^T} = (78 \pm 48)^\circ$$

It seems that $|B_{\overline{15}}^T|/|C_3^T| \sim 20\%$, both contributions are significant.

However, $B_{\overline{15}}^T$ is constructed by two originally missed TDA terms:

$$B_{15}^T = \frac{T_{ES} + T_{AS}}{8}$$

So it's obvious that they have non-negligible contributions and cannot be missed.

Furthermore, the one redundant degree of freedom among the 10 IRA amplitudes means only 9 coefficients need to be fitted.

Implication from CP violation

	IRA	TDA
$\overline{B}^0 \to K^+K^-$	$2\left(A_3^T + A_{15}^T\right)$	$E + 2T_{PA}$
$\overline{B}^0 \to K^0\overline{K}^0$	$2A_3^T + A_6^T - 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$	$T_P + 2T_{PA}$
$\overline{B}^0 \to \eta_8\eta_8$	$2A_3^T + A_6^T - A_{15}^T + \frac{C_3^T}{3} - C_6^T + C_{15}^T$	$\frac{1}{3}\left(C + E + T_P + 6T_{PA}\right)$
$\overline{B}_s^0 \to K^+K^-$	$2A_3^T - A_6^T + A_{15}^T + C_3^T + C_6^T + 3C_{15}^T$	$E + T_P + 2T_{PA} + T$
$\overline{B}_s^0 \to K^0\overline{K}^0$	$2A_3^T + A_6^T - 3A_{15}^T + C_3^T - C_6^T - C_{15}^T$	$T_P + 2T_{PA}$
$\overline{B}_s^0 \to \eta_8\eta_8$	$2A_3^T - 2A_{15}^T + \frac{4C_3^T}{3} - 4C_{15}^T$	$\frac{1}{3}\left(-2C + E + 4T_P + 6T_{PA}\right)$

If these TDA terms are missed, the amplitudes of Channel $\overline{B}^0 \to K^0\overline{K}^0$, $\overline{B}_s^0 \to K^0\overline{K}^0$ are only proportional to $V_{tq}V_{tb}^*$ which implies no CP violation.

However, from above corrected TDA analysis, they are contributed by $T_P + 2T_{PA}$, which is multiplied by $V_{uq}V_{ub}^*$ and have non-zero CP violation.

$D \to PP$ decays

The effective Hamiltonian is:

Cabibbo-Allowed　　Cabibbo-Suppressed

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\{V_{cs}V_{ud}^*[C_1O_1^{sd} + C_2O_2^{sd}] + V_{cd}V_{ud}^*[C_1O_1^{dd} + C_2O_2^{dd}]$$
$$+V_{cs}V_{us}^*[C_1O_1^{ss} + C_2O_2^{ss}] + V_{cd}V_{us}^*[C_1O_1^{ds} + C_2O_2^{ds}]\},$$

Doubly Cabibbo-Suppressed

$$O_1^{sd} = [\bar{s}^i\gamma_\mu(1-\gamma_5)c^j][\bar{u}^i\gamma^\mu(1-\gamma_5)d^j], \quad O_2^{sd} = [\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{u}\gamma^\mu(1-\gamma_5)d]$$

Cabibbo-Allowed: $(H_6)_2^{31} = -(H_6)_2^{13} = 1, \quad (H_{\overline{15}})_2^{31} = (H_{\overline{15}})_2^{13} = 1 \quad \bar{H}_2^{31} = 1$

Cabibbo-Suppressed: $(H_6)_3^{31} = -(H_6)_3^{13} = (H_6)_2^{12} = -(H_6)_2^{21} = \sin(\theta_C), \ \bar{H}_2^{21} = -sin\theta_c$

$(H_{\overline{15}})_3^{31} = (H_{\overline{15}})_3^{13} = -(H_{\overline{15}})_2^{12} = -(H_{\overline{15}})_2^{21} = \sin(\theta_C). \ \bar{H}_3^{31} = sin\theta_c$

Since $V_{cd}V_{ud}^* = -V_{cs}V_{us}^* - V_{cb}V_{ub}^* \approx -V_{cs}V_{us}^*$, the $\bar{3}$ representation vanishes.

Doubly Cabibbo-Suppressed:

$(H_6)_3^{21} = -(H_6)_3^{12} = -\sin^2\theta_C, \quad (H_{\overline{15}})_3^{21} = (H_{\overline{15}})_3^{12} = -\sin^2\theta_C \quad \bar{H}_3^{21} = -sin^2\theta_c$

$D \to PP$ decays

For D decays both the IRA and TDA Hamiltonians are almost the same as that of B decays. The only difference is that there's no $H_{\overline{3}}$ term for IRA and no $\overline{H}_l^{li}$ term for TDA.

The relations between the two sets of amplitudes are:

$$\begin{aligned} A_{15}^T &= \frac{1}{2}(A+E), \\ B_6^{\prime T} &= \frac{1}{2}(T_{ES}-T_{AS}+A-E), \;\; B_{15}^T = \frac{1}{2}(T_{ES}+T_{AS}), \\ C_6^{\prime T} &= \frac{1}{2}(T-C-A+E), \;\; C_{15}^T = \frac{1}{2}(T+C), \end{aligned}$$

The inverse relations are:

$$\begin{aligned} T &= A_{15}^T + C_6^{\prime T} + C_{15}^T - E, \;\; C = -A_{15}^T - C_6^{\prime T} + C_{15}^T + E, \;\; A = 2A_{15}^T - E, \\ T_{ES} &= -A_{15}^T + B_6^{\prime T} + B_{15}^T + E, \;\; T_{AS} = A_{15}^T - B_6^{\prime T} + B_{15}^T - E. \end{aligned}$$

$B \to VP$ decays

For B decays into a vector and a pseudoscalar, the IRA and TDA Hamiltonians are almost a double of that of $B \to PP$ due to the interchanging of the final two mesons.

$$\begin{aligned}
\mathcal{A}^{IRA}_{B\to VM(t)} &= A_3^T B_i(H_{\bar{3}})^i M_k^j V_j^k + C_3^{T1} B_i(H_{\bar{3}})^k M_j^i V_k^j + C_3^{T2} B_i(H_{\bar{3}})^k V_j^i M_k^j + B_3^T B_i(H_{\bar{3}})^i M_k^k V_j^j \\
&+ D_3^{T1} B_i(H_{\bar{3}})^j M_j^i V_k^k + D_3^{T2} B_i(H_{\bar{3}})^j V_j^i M_k^k + A_6^{T1} B_i(H_6)_k^{ij} M_j^l V_l^k + A_6^{T2} B_i(H_6)_k^{ij} V_j^l M_l^k \\
&+ C_6^{T1} B_i(H_6)_k^{jl} M_j^i V_l^k + C_6^{T2} B_i(H_6)_k^{jl} V_j^i M_l^k + B_6^{T1} B_i(H_6)_k^{ij} M_j^k V_l^l + B_6^{T2} B_i(H_6)_k^{ij} V_j^k M_l^l \\
&+ A_{15}^{T1} B_i(H_{\overline{15}})_k^{ij} M_j^l V_l^k + A_{15}^{T2} B_i(H_{\overline{15}})_k^{ij} V_j^l M_l^k + C_{15}^{T1} B_i(H_{\overline{15}})_l^{jk} M_j^i V_k^l + C_{15}^{T2} B_i(H_{\overline{15}})_l^{jk} V_j^i M_k^l \\
&+ B_{15}^{T1} B_i(H_{\overline{15}})_k^{ij} M_j^k V_l^l + B_{15}^{T2} B_i(H_{\overline{15}})_k^{ij} V_j^k M_l^l
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}^{TDA}_{B\to VM(t)} &= T_1 B_i \bar{H}_k^{jl} M_j^i V_l^k + T_2 B_i \bar{H}_k^{jl} V_j^i M_l^k + C_1 B_i \bar{H}_k^{lj} M_j^i V_l^k + C_2 B_i \bar{H}_k^{lj} V_j^i M_l^k \\
&+ A_1 B_i \bar{H}_j^{il} M_k^j V_l^k + A_2 B_i \bar{H}_j^{il} V_k^j M_l^k + E_1 B_i \bar{H}_j^{li} M_k^j V_l^k + E_2 B_i \bar{H}_j^{li} V_k^j M_l^k \\
&+ T_{S1} B_i \bar{H}_l^{lj} M_j^i V_k^k + T_{S2} B_i \bar{H}_l^{lj} V_j^i M_k^k + T_{P1} B_i \bar{H}_l^{lk} M_j^i V_k^j + T_{P2} B_i \bar{H}_l^{lk} V_j^i M_k^j \\
&+ T_{PA} B_i \bar{H}_l^{li} M_k^j V_j^k + T_{SS} B_i \bar{H}_l^{li} M_j^j V_k^k + T_{AS1} B_i \bar{H}_l^{ji} M_j^l V_k^k + T_{AS2} B_i \bar{H}_l^{ji} V_j^l M_k^k \\
&+ T_{ES1} B_i \bar{H}_l^{ij} M_j^l V_k^k + T_{ES2} B_i \bar{H}_l^{ij} V_j^l M_k^k
\end{aligned}$$

Note that There's two Hamiltonians cannot be doubled because of the symmetric contraction between M and V.

$B \to VP$ decays

The relation between IRA and TDA:

$$
\begin{aligned}
A_3^T &= -\frac{1}{8}(A_1 + A_2 - 3E_1 - 3E_2) + T_{PA} \\
C_3^{T1} &= \frac{1}{8}(3T_1 - C_1 + 3A_1 - E_1) + T_{P1} \\
C_3^{T2} &= \frac{1}{8}(3T_2 - C_2 + 3A_2 - E_2) + T_{P2} \\
B_3^T &= T_{SS} + \frac{1}{8}(3T_{AS1} + 3T_{AS2} - T_{ES1} - T_{ES2}) \\
D_3^{T1} &= \frac{1}{8}(3C_1 - T_1 - T_{AS1} + 3T_{ES1}) + T_{S1} \\
D_3^{T2} &= \frac{1}{8}(3C_2 - T_2 - T_{AS2} + 3T_{ES2}) + T_{S2} \\
A_6^{T1} &= \frac{1}{4}(A_2 - E_2) \\
A_6^{T2} &= \frac{1}{4}(A_1 - E_1) \\
C_6^{T1} &= \frac{1}{4}(T_1 - C_1) \\
C_6^{T2} &= \frac{1}{4}(T_2 - C_2) \\
B_6^{T1} &= \frac{1}{4}(T_{ES1} - T_{AS1})
\end{aligned}
$$

$$
\begin{aligned}
B_6^{T2} &= \frac{1}{4}(T_{ES2} - T_{AS2}) \\
A_{15}^{T1} &= \frac{1}{8}(A_2 + E_2) \\
A_{15}^{T2} &= \frac{1}{8}(A_1 + E_1) \\
C_{15}^{T1} &= \frac{1}{8}(T_1 + C_1) \\
C_{15}^{T2} &= \frac{1}{8}(T_2 + C_2) \\
B_{15}^{T1} &= \frac{1}{8}(T_{AS1} + T_{ES1}) \\
B_{15}^{T2} &= \frac{1}{8}(T_{AS2} + T_{ES2})
\end{aligned}
$$

Conclusion

- Through $\boldsymbol{B} \to \boldsymbol{PP}$ decays, we find that in previous literatures where the TDA method was used some topology diagrams are missed. Furthermore, there's a redundant degree of freedom among them.

- In this work we correct such disadvantage of TDA analysis and proved that the TDA analysis is actually equal to IRA analysis.

- We also show that, according to the previous fitting results, these originally missed TDA amplitudes have significant contribution and have non-negligible CP violation effects.

Thank you for your attention !