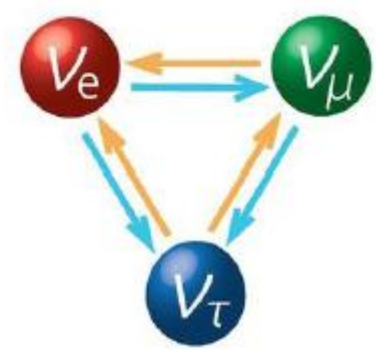
中微子理论研究进展

丁桂军
中国科学技术大学

中国物理学会高能物理分会第十三届全国粒子物理学术会议

上海交通大学，6.19-24, 2018

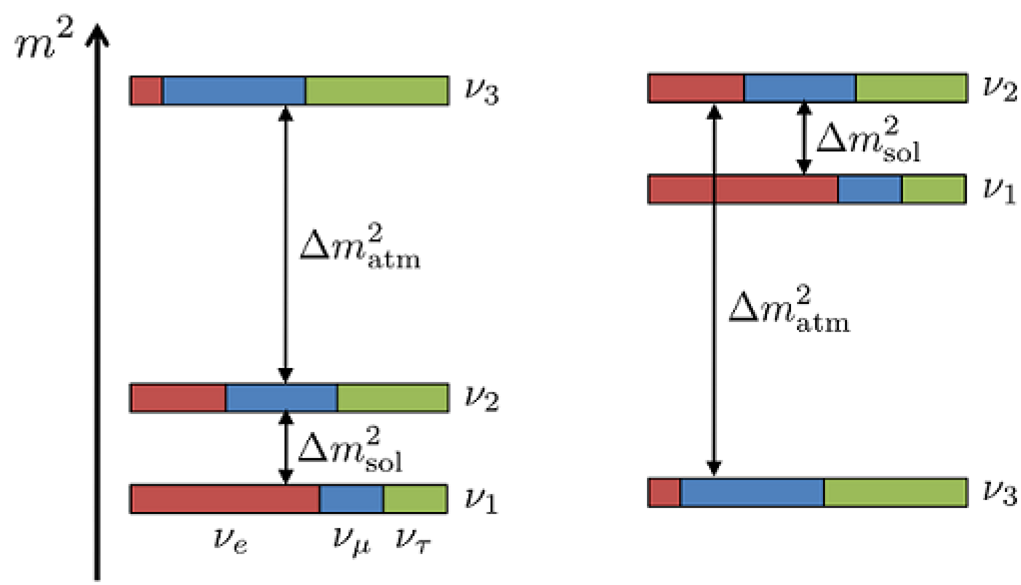


Neutrino oscillation and lepton mixing in 3-ν

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\alpha_{21}/2} & 0\\ 0 & 0 & e^{i\alpha_{31}/2}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Atmospheric mixing	Reactor mixing & Dirac CP phase	Solar mixing	Majorana CP phases
$\theta_{23} \sim 45°, \lvert\Delta m^2_{32}\rvert \sim 2.5\times10^{-3}\text{eV}^2$	$\theta_{13} \sim 9°,\ \delta_{CP} \sim ?$	$\theta_{12} \sim 34°, \lvert\Delta m^2_{21}\rvert \sim 7.5\times10^{-5}\text{eV}^2$	$\alpha_{21}, \alpha_{31} \sim ?$

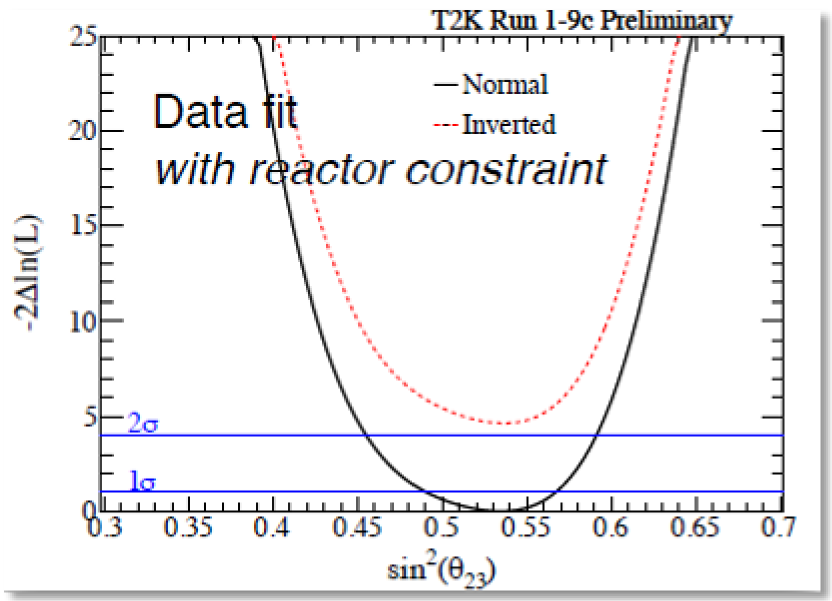
- What is the value of δ_{CP} ?
- Octant of θ_{23} : > or <45° ?
- Mass hierarchy: NO or IO?
- Absolute mass scale: $m_{lightest}$=?
- Majorana or Dirac neutrinos?
- Why m_ν so small?
- Sterile neutrino?
- Implications for BSM paradigms?
- Connections to other new physics?



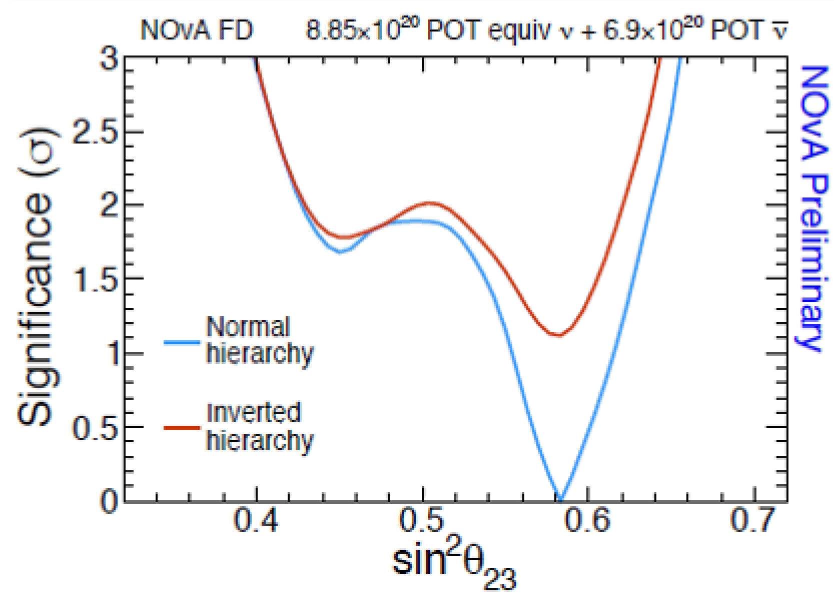
NO IO

Latest results on θ_{23} octant

T2K Wascko@ Neutrino 18



NOvA Sanchez @ Neutrino 18



- Best fit values:

	NO	IO
$\sin^2\theta_{23}$	$0.536^{+0.031}_{-0.046}$	$0.536^{+0.031}_{-0.041}$
$\lvert\Delta m^2\rvert$	2.434 ± 0.064	$2.410^{+0.062}_{-0.063}$

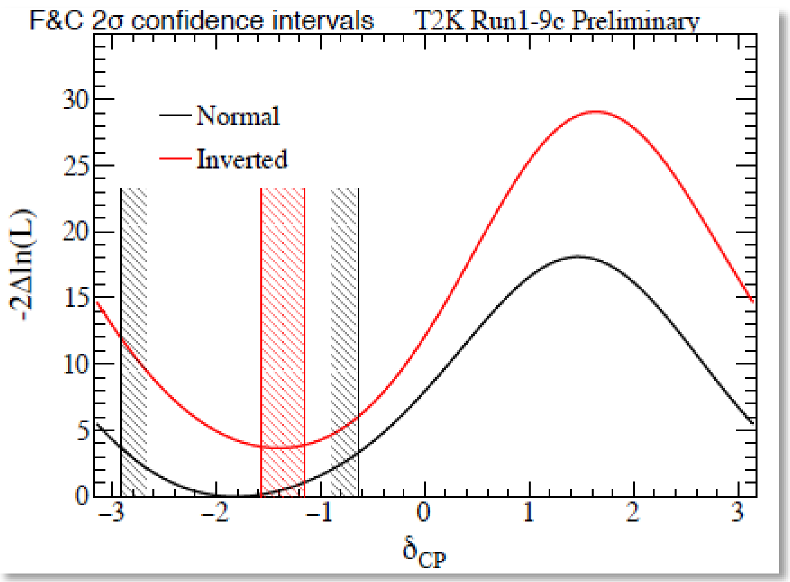
- **Best fit: $\sin^2\theta_{23}$=0.58±0.03 for NO**
- **Prefer non-maximal at 1.8σ, exclude lower octant at similar level**

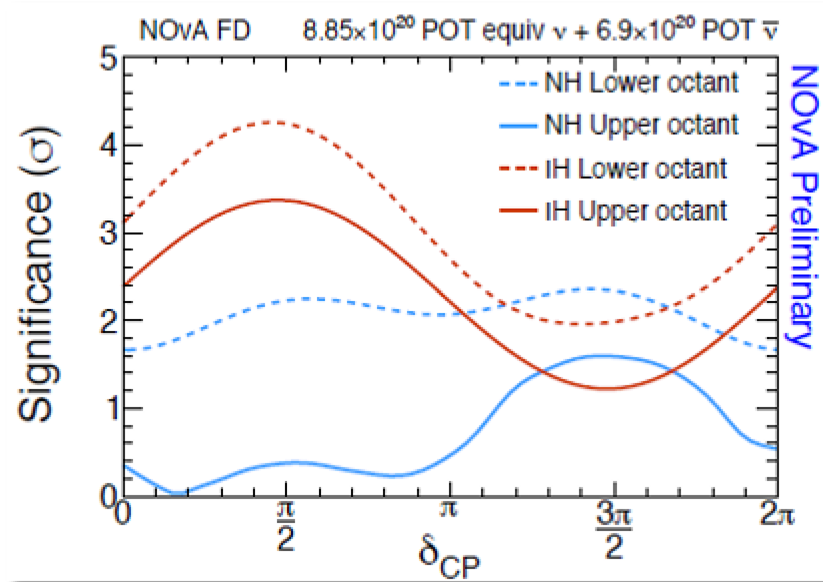
more disfavored lower octant?

Latest results on δ_{CP}

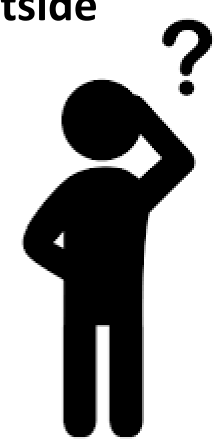
T2K Wascko@ Neutrino 18

NOνA Sanchez @ Neutrino 18





- CP conserving values δ_{CP} =0, π outside 2σ region for NO & IO
- – π< δ_{CP} <0 is favored

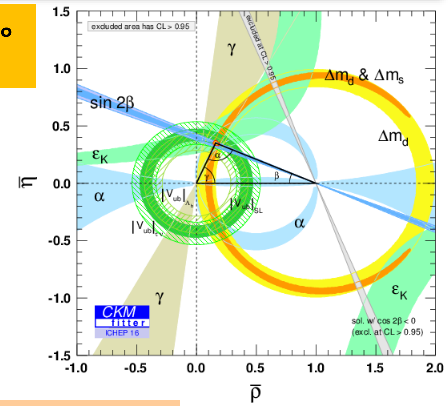


- Best fit: δ_{CP}≈0.17π for NO, δ_{CP}≈1.5π for IO
- Prefer NO by 1.8σ, exclude δ_{CP}=π/2 in the IO at > 3σ

Flavor mixing puzzle in SM

Particle Data Group 2018

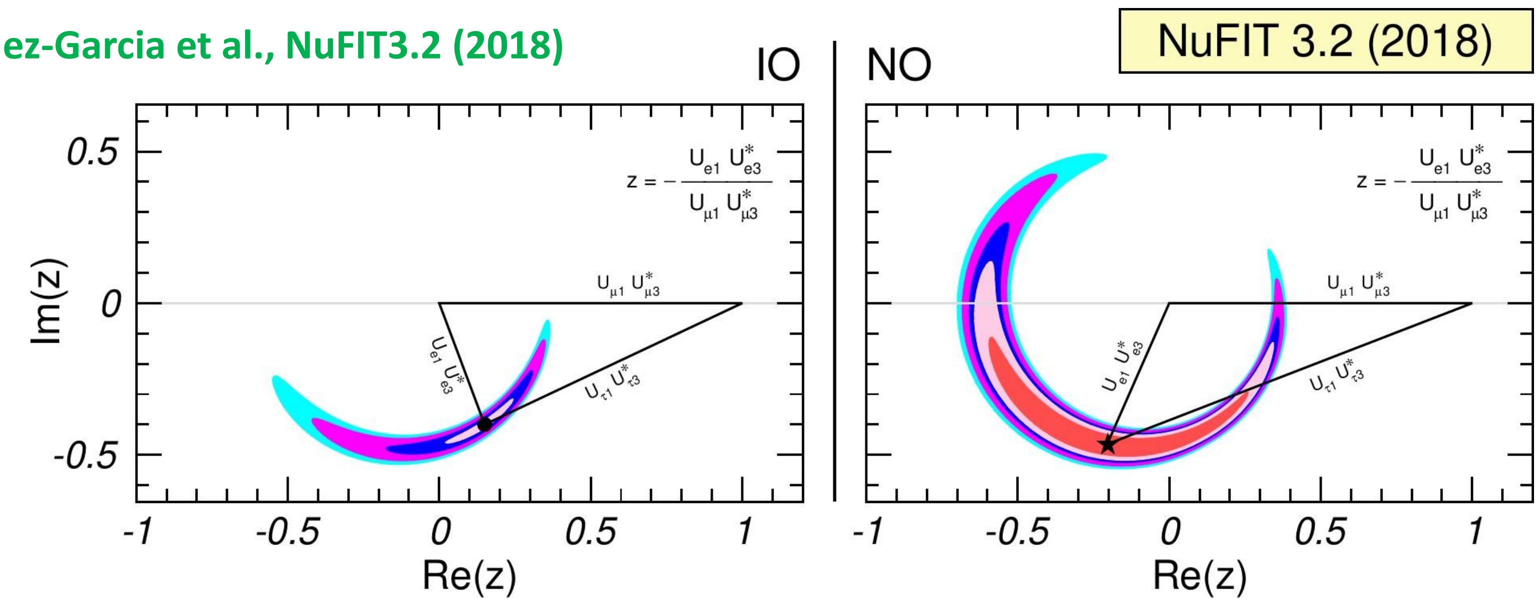
$\alpha=(88.8\pm 2.3)^\circ$



Quarks: $$\|V_{CKM}\| \approx \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 0.0403 & 0.99915 \end{pmatrix}$$

Leptons: $$\|U_{PMNS}\| = \begin{pmatrix} 0.799 \sim 0.844 & 0.516 \sim 0.582 & 0.141 \sim 0.156 \\ 0.242 \sim 0.494 & 0.467 \sim 0.678 & 0.639 \sim 0.774 \\ 0.284 \sim 0.521 & 0.490 \sim 0.695 & 0.615 \sim 0.754 \end{pmatrix}$$

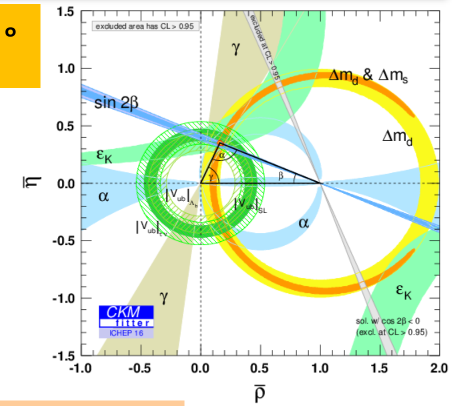
Gonzalez-Garcia et al., NuFIT3.2 (2018)



Flavor mixing puzzle in SM

Particle Data Group 2018

$\alpha=(88.8\pm 2.3)^\circ$



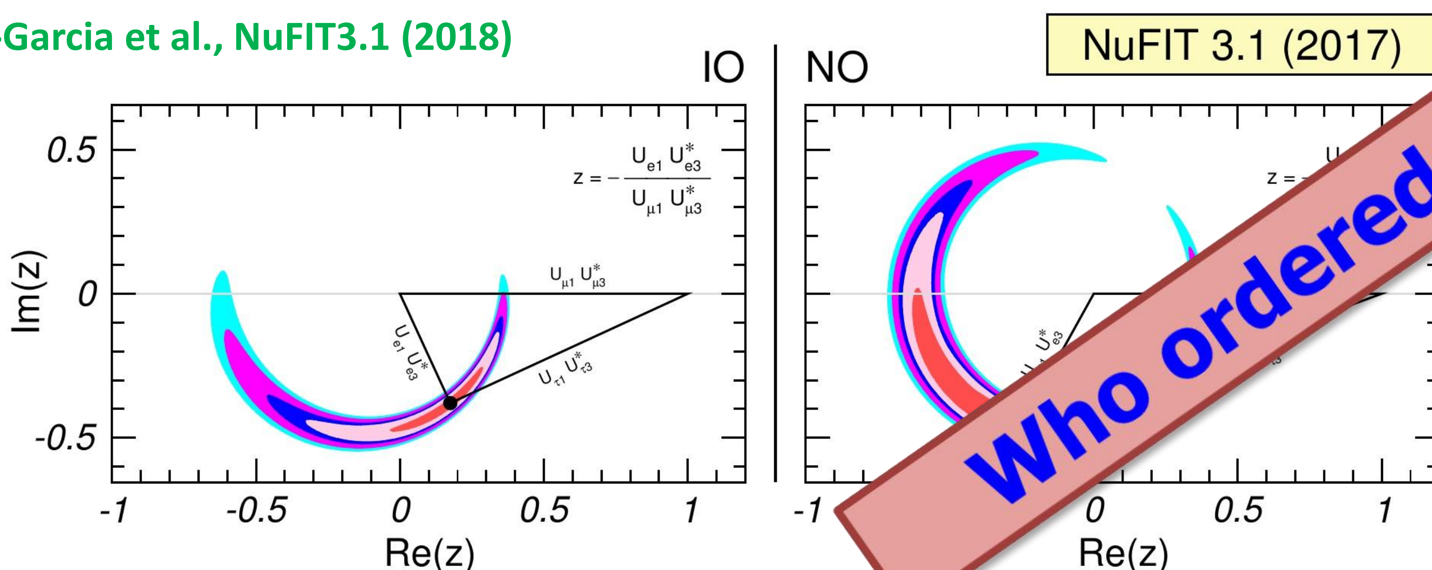
Quarks:

$$\|V_{CKM}\| \approx \begin{pmatrix} 0.97434 & 0.22506 & 0.00357 \\ 0.22492 & 0.97351 & 0.0414 \\ 0.00875 & 0.0403 & 0.99915 \end{pmatrix}$$

Leptons:

$$\|U_{PMNS}\| = \begin{pmatrix} 0.799 \sim 0.844 & 0.516 \sim 0.582 & 0.141 \sim 0.156 \\ 0.242 \sim 0.494 & 0.467 \sim 0.678 & 0.639 \sim 0.774 \\ 0.284 \sim 0.521 & 0.490 \sim 0.695 & 0.615 \sim 0.754 \end{pmatrix}$$

Gonzalez-Garcia et al., NuFIT3.1 (2018)



Who ordered that?

Pathways to to flavor mixing puzzle

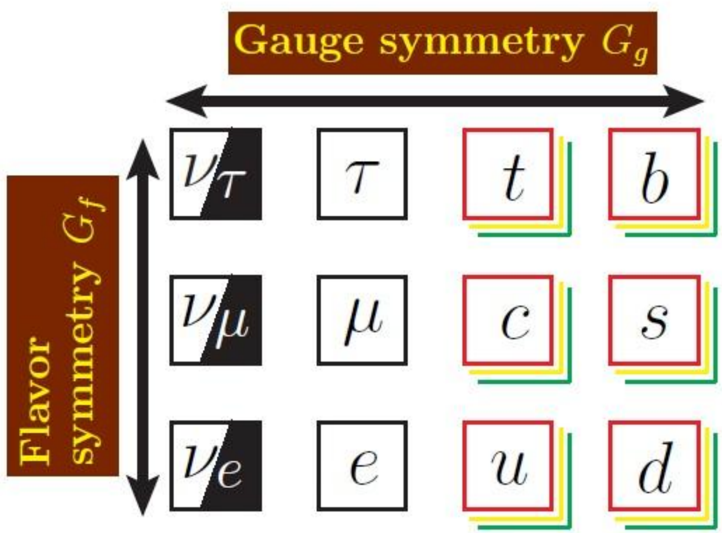
- **Anarchy** Hall, Murayama, Weiner, 99; Gouvea,Murayama,12

$$m_\nu \propto \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

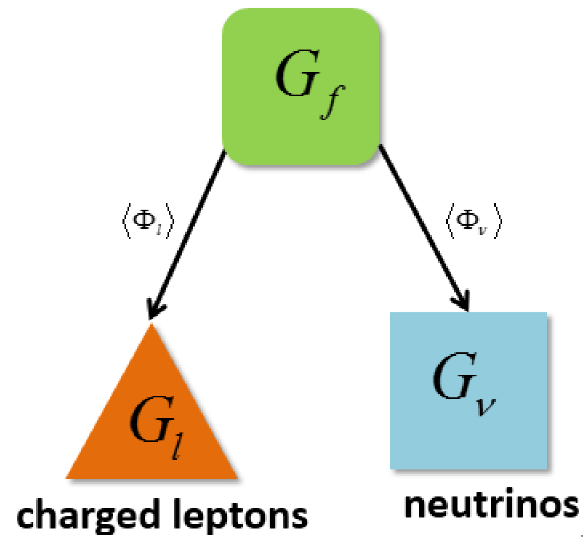
✓ Neutrino mass spectrum is approximately degenerate

✓ All mixing angles are generically large: θ_{23} is non-maximal and θ_{13} is near its upper bound

- **Flavor symmetry**

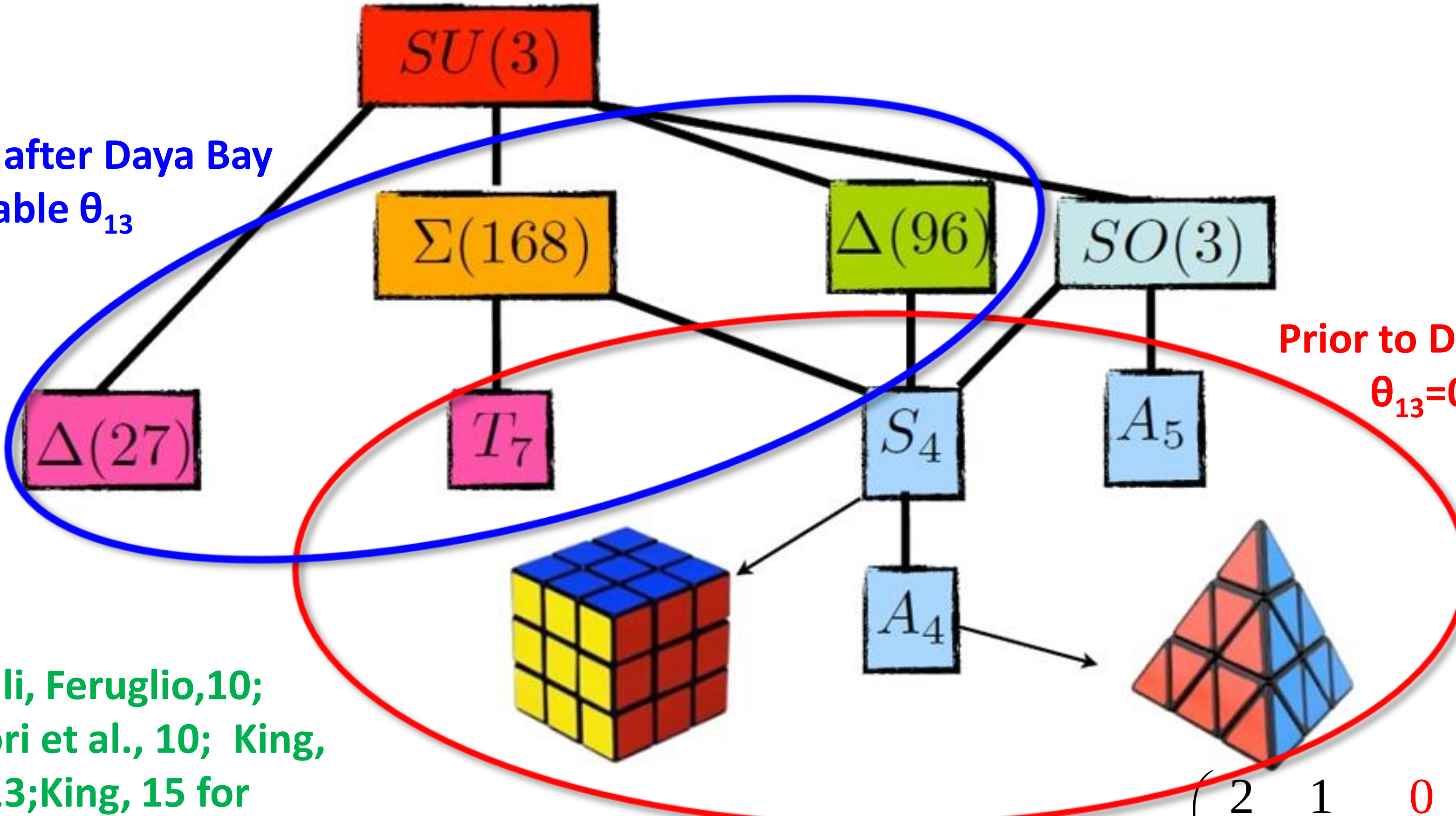


$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \end{pmatrix} \sim \mathbf{3}$$



Discrete flavor symmetry

Popular after Daya Bay
sizeable θ_{13}



Prior to Daya Bay
θ_{13}=0

Altarelli, Feruglio,10; Ishimori et al., 10; King, Luhn,13;King, 15 for reviews

$$\frac{1}{2}\begin{pmatrix} -\sqrt{2} & \sqrt{2} & 0 \\ 1 & 1 & \sqrt{2} \\ 1 & 1 & -\sqrt{2} \end{pmatrix}$$

Vissiani, 97; Barger et al.,98

$$\frac{1}{\sqrt{6}}\begin{pmatrix} 2 & 1 & 0 \\ -1 & 1 & \sqrt{3} \\ -1 & 1 & -\sqrt{3} \end{pmatrix}$$

Harrison, Pekins, Scott, 02; Xing, 02; He, Zee, 03

Predictions of flavor symmetry

- **If the lepton mixing matrix is fully determined by the flavor symmetry G_f and its breaking into G_l, G_ν**

$$U_{PMNS}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\cos\vartheta & 1 & -\sqrt{2}\sin\vartheta\\ -\sqrt{2}\cos(\vartheta-\pi/3) & 1 & \sqrt{2}\sin(\vartheta-\pi/3)\\ -\sqrt{2}\cos(\vartheta+\pi/3) & 1 & \sqrt{2}\sin(\vartheta+\pi/3)\end{pmatrix}$$

Lindner et al.,12; Fonseca, Grimus, 14; Yao, Ding,15

✓ mixing angles:

$$\sin^2\theta_{12}=\sec^2\theta_{13}/3\simeq 0.341,\quad \sin^2\theta_{23}=\frac{1}{2}\pm\frac{1}{2}\tan\theta_{13}\sqrt{2-\tan^2\theta_{13}}=0.395\ \text{ or }\ 0.605$$

✓ Dirac CP phase is conserved : $\sin\delta_{CP}=0$

✓ Extension to quark sector

$$V_{CKM}=\begin{pmatrix}\cos\theta_C & \sin\theta_C & 0\\ -\sin\theta_C & \cos\theta_C & 0\\ 0 & 0 & 1\end{pmatrix},\qquad \theta_C=\frac{\pi}{14}$$

Lam,07; Lindner et al.,07; Yao, Ding,15

- **If the lepton mixing matrix is partially determined by the flavor symmetry G_f, G_l and G_ν, e.g. $G_\nu=Z_2$**

Ge, Dicus and Repko,11; Hernandez and Smirnov,12

For example, two deformations of TBM

TM_1

$$U = U_{TBM}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{-i\delta} \\ 0 & -\sin\theta e^{i\delta} & \cos\theta \end{pmatrix},$$

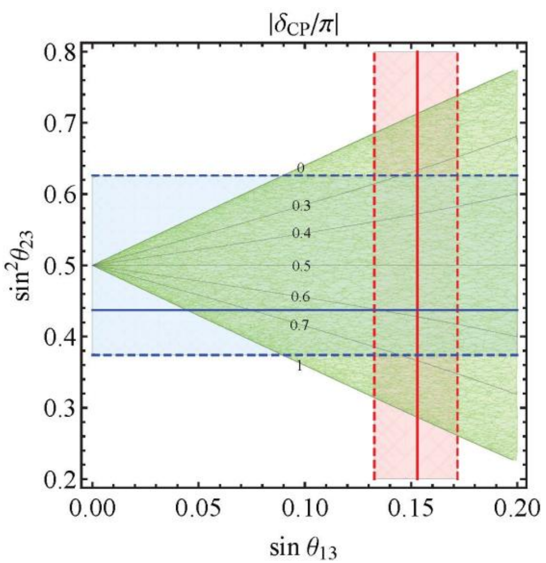
TM_2

$$U = U_{TBM}\begin{pmatrix} \cos\theta & 0 & \sin\theta e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta e^{i\delta} & 0 & \cos\theta \end{pmatrix}$$

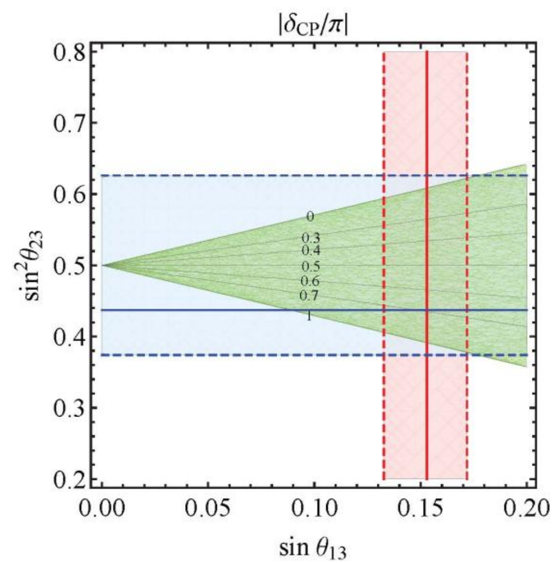
Two predictions in terms of sum rules

$$3\cos^2\theta_{12}\cos^2\theta_{13} = 2$$
$$\sin^2\theta_{23} \simeq \frac{1}{2} - \sqrt{2}\sin\theta_{13}\cos\delta_{CP}$$

$$3\sin^2\theta_{12}\cos^2\theta_{13} = 1$$
$$\sin^2\theta_{23} \simeq \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\theta_{13}\cos\delta_{CP}$$



He, Zee, 07 and 11; Grimus, Lavoura, 08; Albright, Rodejohann, 09; King, Luhn 11; Xing, Zhou, 14 ……



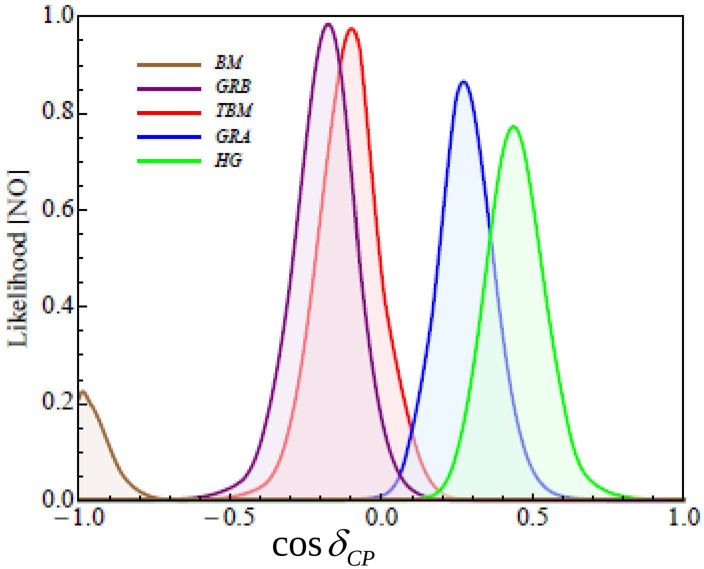
- **LO flavor symmetry + NLO charged lepton corrections**
  - ✓ assume a certain charged lepton correction scheme:

$$U_{PMNS} = U_e^{\dagger} \Psi U_{\nu}, \qquad U_e = R_{23}(\theta_{23}^e) R_{12}(\theta_{12}^e)$$

Ψ is a phase matrix, and U_ν is tribimaximal (TBM), bimaximal (BM), golden ratio (GR) mixings or hexagonal mixing (HG) .

  - ✓ sum rule for δ_{CP}

$$\cos\delta_{CP} = \frac{\tan\theta_{23}}{\sin 2\theta_{12} \sin\theta_{13}} \left[\cos 2\theta_{12}^{\nu} + (\sin^2\theta_{12} - \cos^2\theta_{12}^{\nu})(1 - \cot^2\theta_{23}\sin^2\theta_{13}) \right]$$



Girardi,Petcov and Titov,14

Many contributions from Chinese physicists

Q.H.Cao,P.Chen,S.L.Chen, G.J.Ding, S.F.Ge, P.H.Gu, H.J.He, X.G.He, B.Hu,C.C.Li, G.N.Li, X.Q.Li,Y.F.Li,Y.Liao, W.Liao, C.Liu, B.Q.Ma, S.J.Rong, Y.-L.Wu, X.J.Xu, Z.Z.Xing, C.Y.Yao, F.R.Yin, H.Zhang, J.Zhang, X.Y.Zhang, Z.H. Zhao, S. Zhou, Y.L. Zhou, J.Y. Zhu ……

Lepton mixing from Flavor+CP symmetries

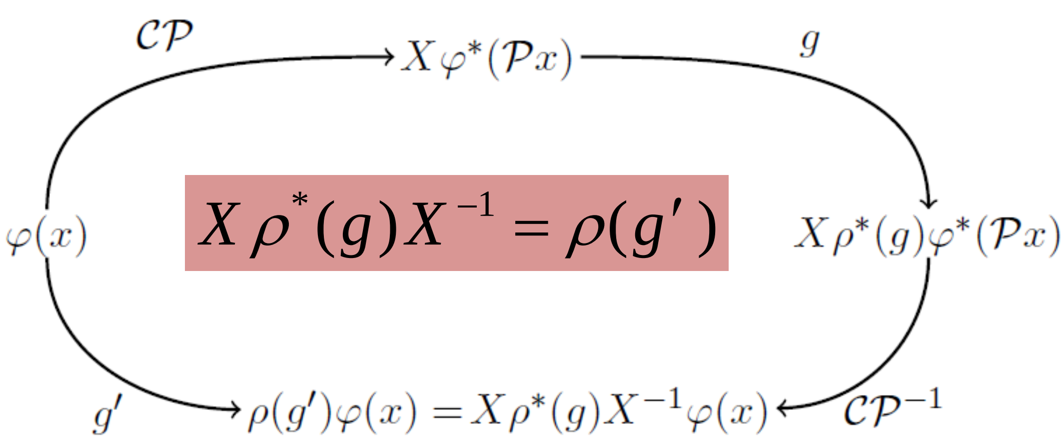
- Simplest example: μτ reflection =μτ exchange+canonical CP

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \to \begin{pmatrix}\nu_e^c\\ \nu_\tau^c\\ \nu_\mu^c\end{pmatrix} = \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}\nu_e^c\\ \nu_\mu^c\\ \nu_\tau^c\end{pmatrix} \Rightarrow \theta_{23}=\frac{\pi}{4},\quad \delta_{CP}=\pm\frac{\pi}{2}$$

Harrison, Scott, 02; Grimus, Lavoura,03; Xing, Zhao,15

- Consistency condition

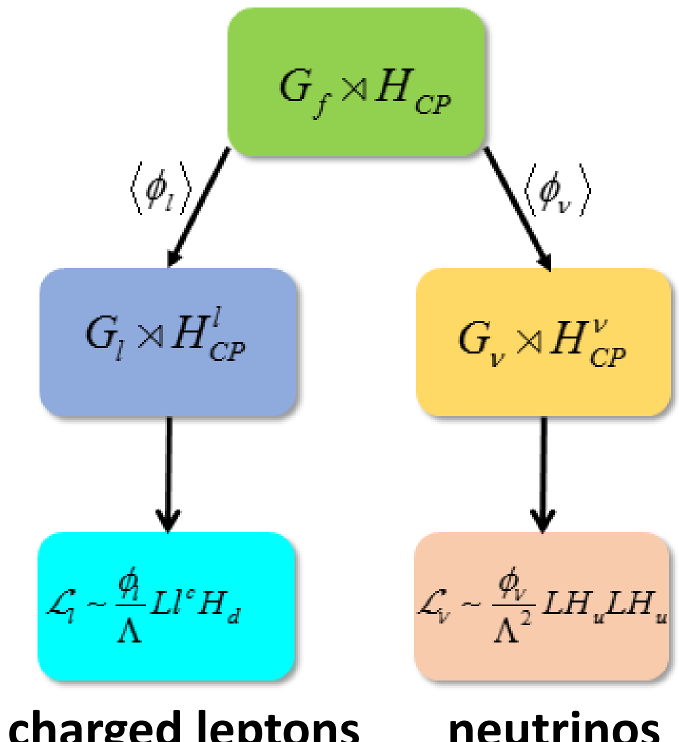
generalized CP: $\varphi_i(x) \to X_{ij}\varphi_j^*(\mathcal{P}x)$



$$X\rho^*(g)X^{-1}=\rho(g')$$

Feruglio et al., 12; Lindner et al., 12; Ding et al.,13

- Symmetry breaking→flavor mixing



Semi-direct approach to lepton mixing

Feruglio et al., 12; Ding et al.,13

$$G_f \rtimes H_{CP}$$

charged lepton | neutrino

$$Z_n^{g_l}, n \geq 3$$

$$Z_2^{g_\nu} \times X_\nu$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T,$$
$$\Sigma_\nu^\dagger \rho_3(g_\nu)\Sigma_\nu = \pm\text{diag}(1,-1,-1)$$

$$U_l^\dagger \rho_3(g_l) U_l = \rho_3(g_l)_{diag}$$

$$U_l$$

$$U_\nu = \Sigma_\nu R_{23}(\theta) Q_\nu$$

$$U_{PMNS} = U_l^\dagger \Sigma_\nu R_{23}(\theta) Q_\nu$$

- The mixing angles and CP violating phases are predicted in terms of a single real parameter $0 \leq \theta < \pi$. One column is fixed.

Possible mixing patterns from finite flavor and CP symmetries

Only eight kinds of mixing matrices consistent with experimental data can be obtained up to row and column permutations.

$$U^{I}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\sin\varphi_1 & e^{i\varphi_2} & \sqrt{2}\cos\varphi_1\\ \sqrt{2}\cos\left(\varphi_1-\frac{\pi}{6}\right) & -e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1-\frac{\pi}{6}\right)\\ \sqrt{2}\cos\left(\varphi_1+\frac{\pi}{6}\right) & e^{i\varphi_2} & -\sqrt{2}\sin\left(\varphi_1+\frac{\pi}{6}\right)\end{pmatrix}R_{23}(\theta)Q_\nu$$

Yao, Ding, 16

$$U^{II}=\frac{1}{\sqrt{3}}\begin{pmatrix}e^{i\varphi_1} & 1 & e^{i\varphi_2}\\ \omega e^{i\varphi_1} & 1 & \omega^2 e^{i\varphi_2}\\ \omega^2 e^{i\varphi_1} & 1 & \omega e^{i\varphi_2}\end{pmatrix}R_{13}(\theta)Q_\nu$$

$R_{ij}(\theta)$ is the rotation matrix in the ij plane

$$U^{III}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}e^{i\varphi_1}\sin\varphi_2 & 1 & \sqrt{2}e^{i\varphi_1}\cos\varphi_2\\ \sqrt{2}e^{i\varphi_1}\cos\left(\varphi_2+\frac{\pi}{6}\right) & 1 & -\sqrt{2}e^{i\varphi_1}\sin\left(\varphi_2+\frac{\pi}{6}\right)\\ -\sqrt{2}e^{i\varphi_1}\cos\left(\varphi_2-\frac{\pi}{6}\right) & 1 & \sqrt{2}e^{i\varphi_1}\sin\left(\varphi_2-\frac{\pi}{6}\right)\end{pmatrix}R_{13}(\theta)Q_\nu$$

$$U^{IV(a)}=\begin{pmatrix} -\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ \sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ \sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)Q_\nu, \qquad U^{IV(b)}=\begin{pmatrix} -i\sqrt{\frac{\phi_g}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi_g}} & 0 \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & -\frac{1}{\sqrt{2}} \\ i\sqrt{\frac{1}{2\sqrt{5}\phi_g}} & \sqrt{\frac{\phi_g}{2\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix} R_{13}(\theta)Q_\nu$$

$$U^{V}=\frac{1}{2}\begin{pmatrix} \phi_g & 1 & \phi_g-1 \\ \phi_g-1 & -\phi_g & 1 \\ 1 & 1-\phi_g & -\phi_g \end{pmatrix} R_{23}(\theta)Q_\nu, \quad U^{VI}=\frac{1}{2\sqrt{3}}\begin{pmatrix} (\sqrt{3}-1)e^{i\varphi} & 2 & -(\sqrt{3}+1)e^{i\left(\varphi+\frac{3\pi}{4}\right)} \\ -(\sqrt{3}+1)e^{i\varphi} & 2 & (\sqrt{3}-1)e^{i\left(\varphi+\frac{3\pi}{4}\right)} \\ 2e^{i\varphi} & 2 & 2e^{i\left(\varphi+\frac{3\pi}{4}\right)} \end{pmatrix} R_{13}(\theta)Q_\nu$$

$$U^{VII}=\frac{1}{2\sqrt{6}}\begin{pmatrix} -\frac{\sqrt{3}}{s_3} & 2\sqrt{2} & \frac{s_2-s_1}{s_1 s_2} \\ \frac{\sqrt{3}}{s_2} & 2\sqrt{2} & -\frac{s_1+s_3}{s_1 s_3} \\ \frac{\sqrt{3}}{s_1} & 2\sqrt{2} & \frac{s_2+s_3}{s_2 s_3} \end{pmatrix} R_{23}(\theta)Q_\nu, \qquad U^{VIII}=\frac{1}{2}R_{13}^{T}(\theta)\begin{pmatrix} \sqrt{2}e^{i\varphi_1} & -\sqrt{2}e^{i\varphi_1} & 0 \\ 1 & 1 & -\sqrt{2}e^{i\varphi_2} \\ 1 & 1 & \sqrt{2}e^{i\varphi_2} \end{pmatrix} Q_\nu$$

Results collected on the website

I(a)	I(b)	II	III	IV	V	VI	VII	VIII

$$U_{\rm PMNS}^{I(b)} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2}\cos\varphi_1 & e^{i\varphi_2} & \sqrt{2}\sin\varphi_1 \\ -\sqrt{2}\sin\left(\varphi_1 - \frac{\pi}{6}\right) & -e^{i\varphi_2} & \sqrt{2}\cos\left(\varphi_1 - \frac{\pi}{6}\right) \\ -\sqrt{2}\sin\left(\varphi_1 + \frac{\pi}{6}\right) & e^{i\varphi_2} & \sqrt{2}\cos\left(\varphi_1 + \frac{\pi}{6}\right) \end{pmatrix} S_{12}(\theta)$$

Group ID	(φ_1, φ_2)
[648,259]	$(\frac{\pi}{18}, -\frac{\pi}{6}), (\frac{\pi}{18}, 0), (\frac{\pi}{18}, \frac{\pi}{3}), (\frac{\pi}{18}, \frac{\pi}{2}), (\frac{17\pi}{18}, -\frac{\pi}{6}), (\frac{17\pi}{18}, 0), (\frac{17\pi}{18}, \frac{\pi}{3}), (\frac{17\pi}{18}, \frac{\pi}{2})$
[726,5]	$(\frac{2\pi}{33}, -\frac{2\pi}{11}), (\frac{2\pi}{33}, 0), (\frac{2\pi}{33}, \frac{\pi}{11}), (\frac{2\pi}{33}, \frac{3\pi}{11}), (\frac{2\pi}{33}, \frac{4\pi}{11}), (\frac{2\pi}{33}, \frac{5\pi}{11}), (\frac{31\pi}{33}, -\frac{2\pi}{11}), (\frac{31\pi}{33}, 0), (\frac{31\pi}{33}, \frac{\pi}{11}), (\frac{31\pi}{33}, \frac{3\pi}{11}), (\frac{31\pi}{33}, \frac{4\pi}{11}), (\frac{31\pi}{33}, \frac{5\pi}{11})$
[1734,5]	$(\frac{\pi}{17}, -\frac{8\pi}{17}), (\frac{\pi}{17}, -\frac{6\pi}{17}), (\frac{\pi}{17}, 0), (\frac{\pi}{17}, \frac{\pi}{17}), (\frac{\pi}{17}, \frac{2\pi}{17}), (\frac{\pi}{17}, \frac{3\pi}{17}), (\frac{\pi}{17}, \frac{4\pi}{17}), (\frac{\pi}{17}, \frac{5\pi}{17}), (\frac{\pi}{17}, \frac{7\pi}{17}), (\frac{16\pi}{17}, -\frac{8\pi}{17}), (\frac{16\pi}{17}, -\frac{6\pi}{17}), (\frac{16\pi}{17}, 0), (\frac{16\pi}{17}, \frac{\pi}{17}), (\frac{16\pi}{17}, \frac{2\pi}{17}), (\frac{16\pi}{17}, \frac{3\pi}{17}), (\frac{16\pi}{17}, \frac{4\pi}{17}), (\frac{16\pi}{17}, \frac{5\pi}{17}), (\frac{16\pi}{17}, \frac{7\pi}{17})$

http://staff.ustc.edu.cn/~dinggj/cp_scan.html

Results collected on the website

Group ID

[24,12]

All of the groups are available here!

Group ID	[24,12]
Structure	S_4
3-Dimensional Representation	$\rho(g_1)=\begin{pmatrix}-1&0&0\\0&0&-1\\0&-1&0\end{pmatrix}, \quad \rho(g_2)=\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix},$ $\rho(g_3)=\begin{pmatrix}-1&0&0\\0&1&0\\0&0&-1\end{pmatrix}, \rho(g_4)=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&1\end{pmatrix}$
Class-inverting Automorphism	$u: g_1 \mapsto g_1,\ g_2 \mapsto g_2,\ g_3 \mapsto g_3,\ g_4 \mapsto g_4$
Generalized CP	$X_0=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

http://staff.ustc.edu.cn/~dinggj/cp_scan.html

Results collected on the website

PMNS matrix predicted in semidirect approach (G_l =Abelian subgroup, $G_\nu = Z_2$)

#	Res Sym.	Σ	f_c	$\theta_{\rm bf}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	δ_{CP}/π	α_{21}/π (mod 1)	α'_{31}/π (mod 1)	$\chi^2_{\rm min}$	NO\IO	45°	Check
1	$(1_a, 1_1)$	$\begin{pmatrix} 0.289-0.500i & 0.577 & -0.289-0.500i \\ 0.289+0.500i & 0.577 & -0.289+0.500i \\ -0.577 & 0.577 & 0.577 \end{pmatrix}$	2	0.750	0.167	0.400	0.200	0.000	0.000	0.000	21121.88	NO	<	×
			2	0.500	0.333	0.500	0.500	0.500	0.333	0.667	97281.18	NO	>	×
			2	0.750	0.167	0.400	0.200	0.000	0.000	0.000	17479.61	IO	<	×
			2	0.500	0.333	0.500	0.500	0.500	0.333	0.667	80389.49	IO	>	×
		$\begin{pmatrix} 0.289-0.500i & 0.577 & -0.289-0.500i \\ -0.577 & 0.577 & 0.577 \\ 0.289+0.500i & 0.577 & -0.289+0.500i \end{pmatrix}$	2	0.500	0.333	0.500	0.500	1.500	0.333	0.667	97281.18	NO	<	×
			2	0.750	0.167	0.400	0.800	1.000	0.000	0.000	21085.67	NO	>	×
			2	0.500	0.333	0.500	0.500	1.500	0.333	0.667	80389.49	IO	<	×
			2	0.750	0.167	0.400	0.800	1.000	0.000	0.000	17452.83	IO	>	×
		$\begin{pmatrix} -0.577 & 0.577 & 0.577 \\ 0.289-0.500i & 0.577 & -0.289-0.500i \\ 0.289+0.500i & 0.577 & -0.289+0.500i \end{pmatrix}$	2	0.192	0.022	0.341	0.500	1.500	0.000	0.000	8.84	NO	=	√
			2	0.192	0.022	0.341	0.500	1.500	0.000	0.000	12.56	IO	=	√
2	$(1_a, 1_2)$	$\begin{pmatrix} 0.289-0.500i & 0.577 & -0.500+0.289i \\ 0.289+0.500i & 0.577 & 0.500+0.289i \\ -0.577 & 0.577 & -0.577i \end{pmatrix}$	2	0.000	0.333	0.500	0.500	1.500	0.667	0.333	97281.18	NO	<	×
			2	0.250	0.045	0.349	0.651	1.000	0.500	0.000	549.09	NO	>	×
			2	0.000	0.333	0.500	0.500	1.500	0.667	0.333	80389.49	IO	<	×
			2	0.250	0.045	0.349	0.651	1.000	0.500	0.000	448.30	IO	>	×
		$\begin{pmatrix} 0.289-0.500i & 0.577 & -0.500+0.289i \\ -0.577 & 0.577 & -0.577i \\ 0.289+0.500i & 0.577 & 0.500+0.289i \end{pmatrix}$	2	0.250	0.045	0.349	0.349	0.000	0.500	0.000	547.99	NO	<	×
			2	0.000	0.333	0.500	0.500	0.500	0.667	0.333	97281.18	NO	>	×
			2	0.250	0.045	0.349	0.349	0.000	0.500	0.000	478.65	IO	<	×
			2	0.000	0.333	0.500	0.500	0.500	0.667	0.333	80389.49	IO	>	×
		$\begin{pmatrix} -0.577 & 0.577 & -0.577i \\ 0.289-0.500i & 0.577 & -0.500+0.289i \\ 0.289+0.500i & 0.577 & 0.500+0.289i \end{pmatrix}$	2	0.482	0.333	0.500	0.452	0.482	0.965	0.000	97280.33	NO	<	×
			2	0.000	0.333	0.500	0.500	1.500	0.000	0.000	97281.18	NO	>	×
			2	0.500	0.333	0.500	0.500	0.500	0.000	0.000	80389.49	IO	<	×
			2	0.529	0.333	0.500	0.579	0.529	0.058	0.000	80384.93	IO	>	×

http://staff.ustc.edu.cn/~dinggj/cp_scan.html

Test these mixing patterns at JUNO, DUNE, Hyper-K…

Benchmark examples

For popular flavor symmetry **A_4, S_4, A_5**:

$\delta_{CP} = \pm\pi/2,\ \theta_{23} = \pi/4$ or $\delta_{CP} = 0, \pi,\ \theta_{23} \neq \pi/4$

- U^{II} with $\varphi_4 = \pi/4, \varphi_5 = 0$

Ding et al., 14; Hagedorn et al., 14

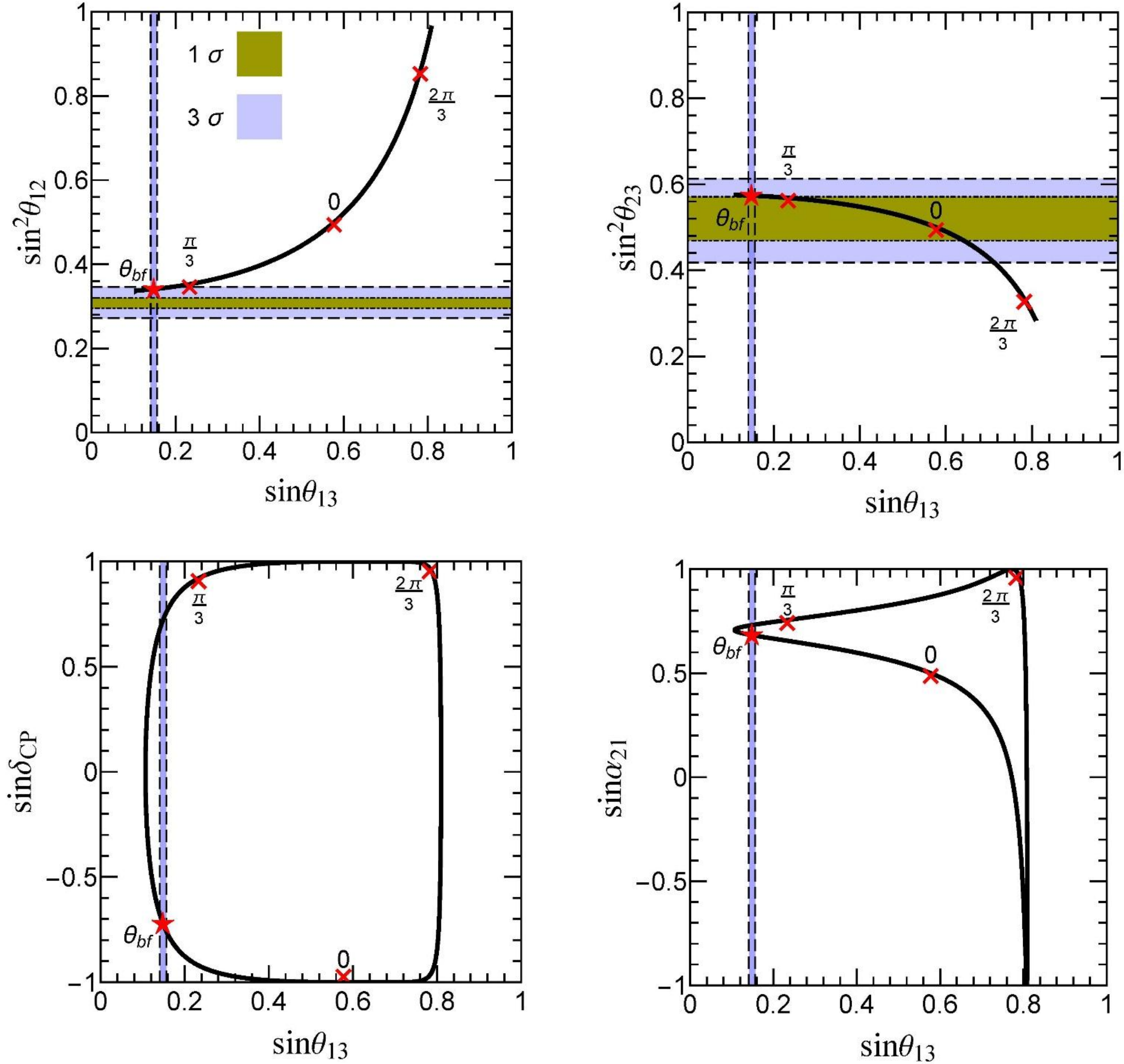
$$U^{II} = \frac{1}{\sqrt{3}} \begin{pmatrix} \omega^2 e^{i\pi/4}\cos\theta - \omega\sin\theta & 1 & \omega^2 e^{i\pi/4}\sin\theta + \omega\cos\theta \\ e^{i\pi/4}\cos\theta - \sin\theta & 1 & e^{i\pi/4}\sin\theta + \cos\theta \\ \omega e^{i\pi/4}\cos\theta - \omega^2\sin\theta & 1 & \omega e^{i\pi/4}\sin\theta + \omega^2\cos\theta \end{pmatrix}, \qquad \omega \equiv e^{2\pi i/3}$$

$$\Rightarrow \begin{cases} \sin^2\theta_{13} = \frac{1}{3} - \frac{\sqrt{3}+1}{6\sqrt{2}}\sin 2\theta, \quad \sin^2\theta_{12} = \frac{2\sqrt{2}}{4\sqrt{2} + \left(\sqrt{3}+1\right)\sin 2\theta}, \quad J_{CP} = -\frac{\cos 2\theta}{6\sqrt{3}}, \\ \sin^2\theta_{23} = \frac{1}{2} + \frac{\left(3-\sqrt{3}\right)\sin 2\theta}{8\sqrt{2} + 2\left(\sqrt{3}+1\right)\sin 2\theta}, \quad I_1 = \frac{1}{18}\left(1 + \left(\sqrt{3}-1\right)\sin^2\theta + \sqrt{2}\sin 2\theta\right), \quad I_2 = \frac{\cos 2\theta}{18} \end{cases}$$

Best fit values:

$$\theta_{bf} = 0.209\pi, \quad \chi^2_{min} = 7.960, \quad \sin^2\theta_{12} = 0.341, \quad \sin^2\theta_{13} = 0.0220,$$

$$\sin^2\theta_{23} = 0.574, \quad \sin\delta_{CP} = -0.722, \quad \sin\alpha_{21} = 0.683, \quad \sin\alpha_{31} = -0.091.$$



1 σ
3 σ
θ_{bf}
$\frac{\pi}{3}$
0
$\frac{2\pi}{3}$
$\sin^2\theta_{12}$
$\sin^2\theta_{23}$
$\sin\delta_{CP}$
$\sin\alpha_{21}$
$\sin\theta_{13}$

Another scheme to predict lepton mixing from flavor and CP

Lu, Ding, 16,18; Li, Lu, Ding, 17

$$G_f \rtimes H_{CP}$$

charged lepton

neutrino

$$Z_2^{g_l} \times X_l \qquad Z_2^{g_\nu} \times X_\nu$$

$$X_l = \Sigma_l \Sigma_l^T, \quad \Sigma_l^\dagger \rho_3(g_l) \Sigma_l = \pm \mathrm{diag}(1,-1,-1)$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad \Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) \qquad U_\nu = \Sigma_\nu R_{23}(\theta_\nu) Q_\nu$$

$$U_{PMNS}(\theta_l, \theta_\nu) = R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) Q_\nu$$

- All mixing angles and CP phases are expressed in terms of two free parameters $\theta_{l,\nu} \in [0,\pi)$
- This scheme can be extended to quark sector, and the CKM mixing matrix is of similar form

Drastically different quark and lepton mixing angles can be explained simultaneously in the $\Delta(6n^2)$ flavor group, and $\Delta(6\cdot 7^2)$ with n=7 is the smallest group.

- Quark sector : $\begin{cases} g_u = bc^x d^x, \quad X_u = c^\gamma d^{-2x-\gamma} \\ g_d = bc^{x-3} d^{x-3}, \quad X_d = c^{\gamma+2} d^{-2x-\gamma+4} \end{cases}$

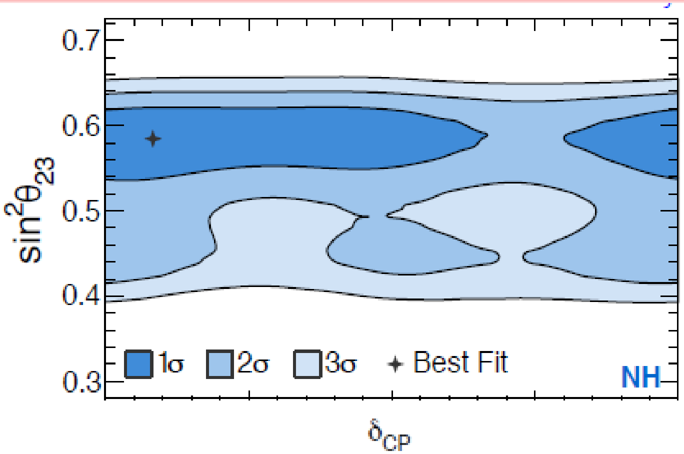
UTfit

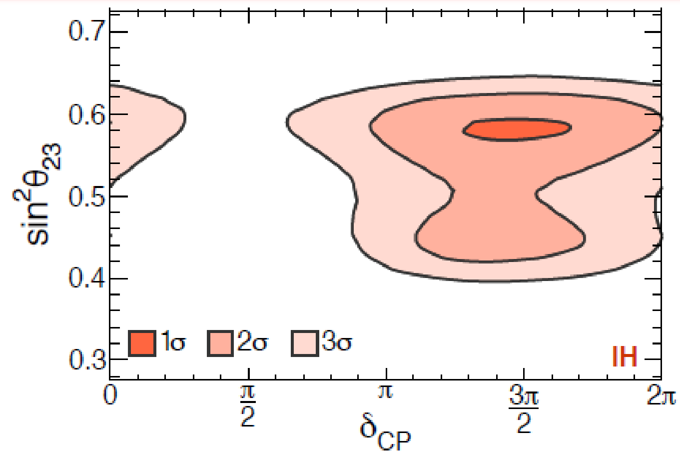
	$\theta_u^{\rm bf}/\pi$	$\theta_d^{\rm bf}/\pi$	$\sin\theta_{12}^q$	$\sin\theta_{23}^q$	$\sin\theta_{13}^q$	J_{CP}^q
Our	0.4867	0.4988	0.22252	0.04204	0.00359	3.202×10^{-5}
Data	—	—	0.22497 ± 0.00069	0.04229 ± 0.00057	0.00368 ± 0.00010	$(3.115 \pm 0.093)\times 10^{-5}$

- Lepton sector : $\begin{cases} g_l = bc^x d^x, \quad X_l = c^\gamma d^{-2x-\gamma} \\ g_\nu = abc^{-x+(1-3\gamma)/2}, \quad X_\nu = c^\gamma d^{1-2x-\gamma} \end{cases}$

Li, Lu, Ding, 17

	$\theta_l^{\rm bf}/\pi$	$\theta_\nu^{\rm bf}/\pi$	$\chi^2_{\rm min}$	$\sin^2\theta_{13}$	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin\delta_{CP}$	$\lvert\sin\alpha_{21}\rvert$	$\lvert\sin\alpha_{31}\rvert$
Our	0.911	0.0347	2.416	0.0222	0.319	0.579	-0.802	0.391	0.596
Data	—	—	—	$0.0198 \to 0.0244$	$0.272 \to 0.346$	$0.418 \to 0.613$	$-1 \to 0.588$	$0 \to 1$	$0 \to 1$





NOvA,Sanchez @ Neutrino 18

Origin of neutrino masses

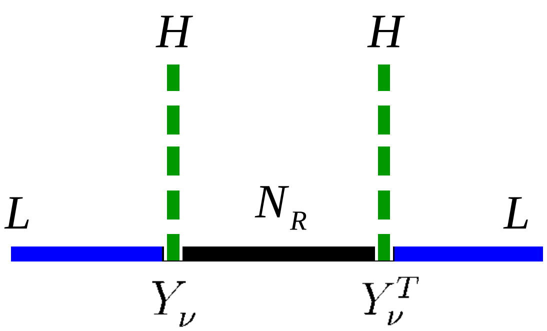
If neutrinos are **Majorana** particles, the effective mass operators are

$$\mathscr{L}^{M}_{5+2n} = -\frac{1}{2}\frac{g_{\alpha\beta}}{\Lambda}\left(\overline{\ell^{C}_{L\alpha}}\tilde{H}^{*}\right)\left(\tilde{H}^{\dagger}\ell_{L\beta}\right)\left(\frac{H^{\dagger}H}{\Lambda^{2}}\right)^{n} + \text{H.c.}$$

Weinberg,79; Babu, Leung, 01; Liao, 10

➢tree level UV completion--- seesaw mechanism

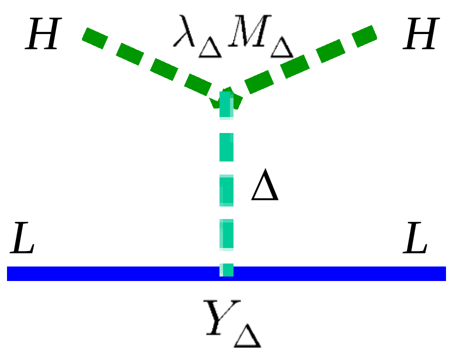
Type-I:SM+singlet fermion N_R



$$m_\nu \approx -v^2 Y_\nu \frac{1}{M_N} Y_\nu^T$$

Minkowski,77; Yanagida,1979; Glashow ,79; Gell-Mann, Ramond, Slansky,79; Mohapatra, Senjanovic,80

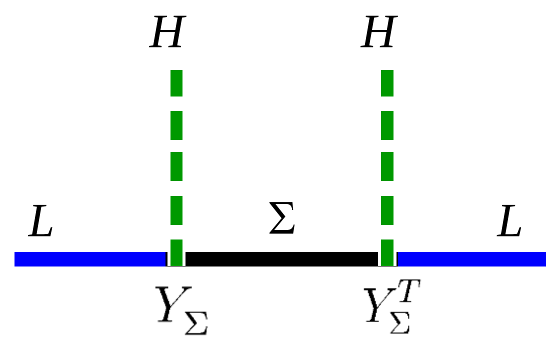
Type-II:SM+triplet scalar Δ



$$m_\nu \approx \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta}$$

Magg, Wetterich,80; Schechter, Valle ,80; Mohapatra, Senjanovic,80

Type-III:SM+triplet fermion Σ



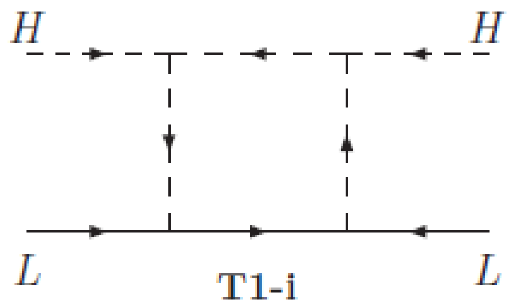
$$m_\nu \approx -v^2 Y_\Sigma \frac{1}{M_\Sigma} Y_\Sigma^T$$

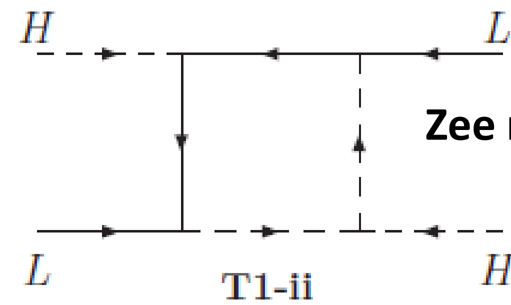
Foot, Lew, He, Joshi, 89

- One-Loop realizations: only 4 independent topologies

Bonnet,Hirsch,Ota, Winter,12

Dark doublet model, Ma, 06

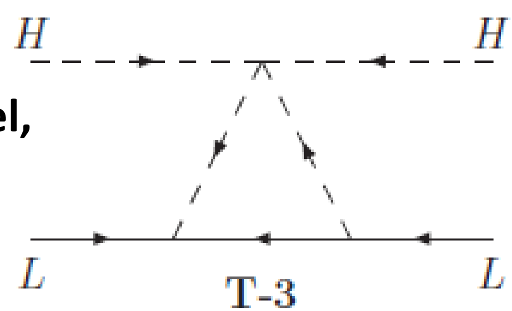


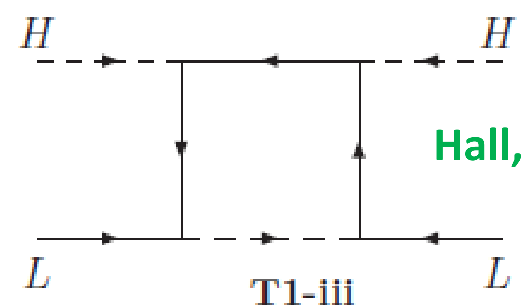


Zee model, Zee,80

many, many more references ...

Scotogenic model, Ma,98





Hall,Suzuki,84; Ma, 98

✓at least 2 new multiplets required as intermediate states

✓The intermediate states could be light and probed at existing facilities (the fermion singlets N_R in seesaw are at the GUT scale)

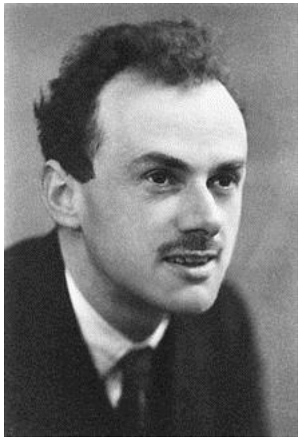
✓new states in the loop can be DM candidates

✓Disadvantages: uniqueness of tree-level seesaw lost

For recent review: Cai, Volkas et al., Front.in Phys.5 (2017) 63

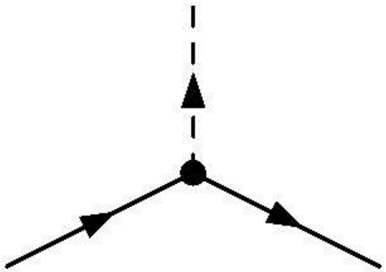
If neutrinos are **Dirac** particles, the effective mass operators are

$$\mathscr{L}^{D}_{4+2n} = -y_{\alpha\beta}\overline{\ell_{L\alpha}}\widetilde{H}\nu_{R\beta}\left(\frac{H^{\dagger}H}{\Lambda^{2}}\right)^{n} + \text{H.c.}$$

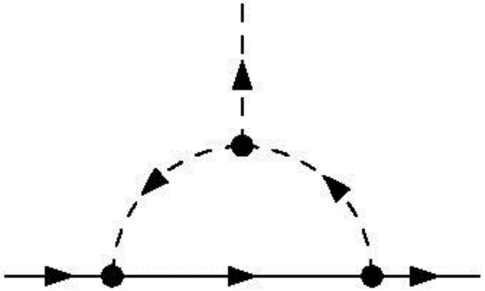


- **Lowest order contribution → d=4**

tree level

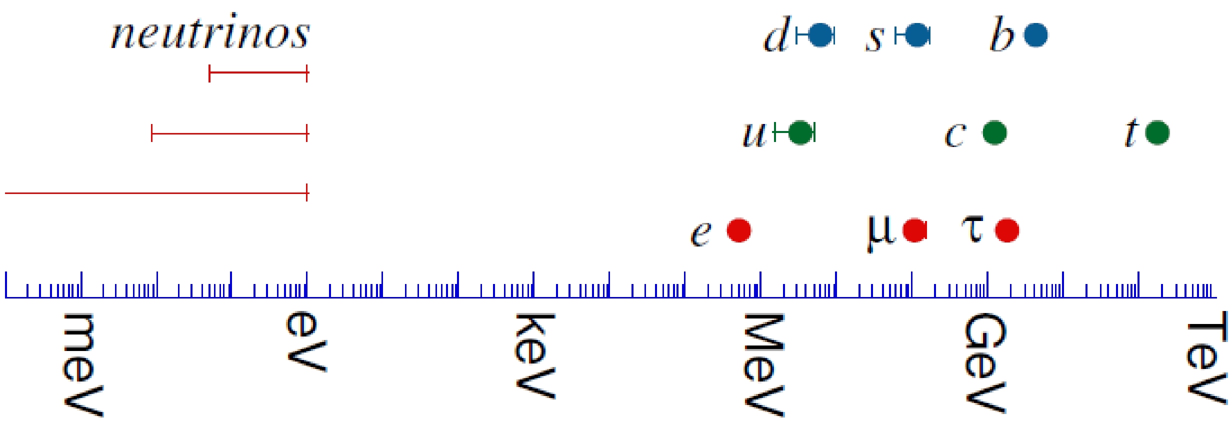


one-loop level



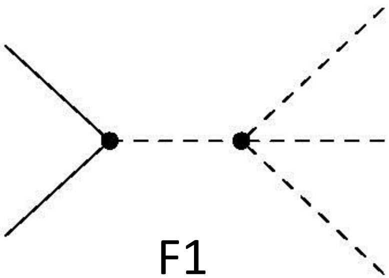
Ma,Popov,16; Wang, Han,16; Yao, Ding, 17

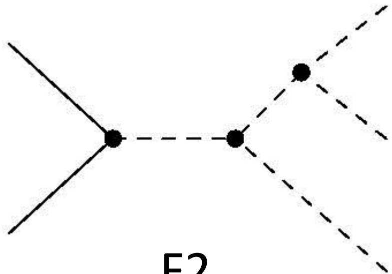
The fermion mass hierarchy problem is worsened.(**i.e., $m_i/m_t < 10^{-12}$**)

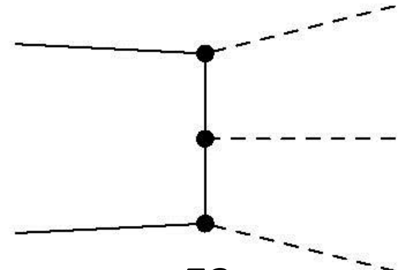


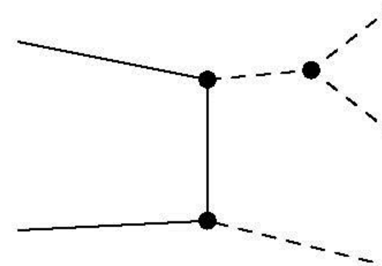
➢ Next to lowest order contribution → d=6

✓tree level realization : 4 independent topologies

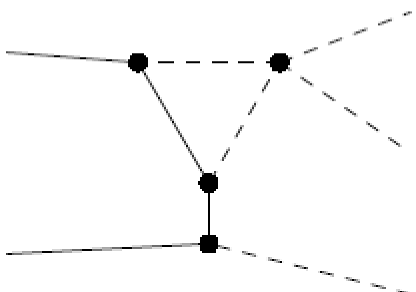

F1

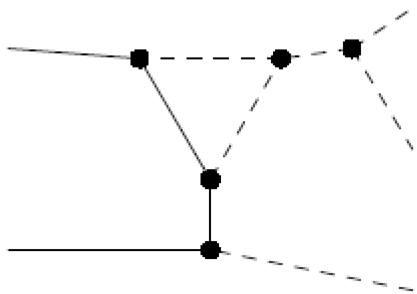

F2

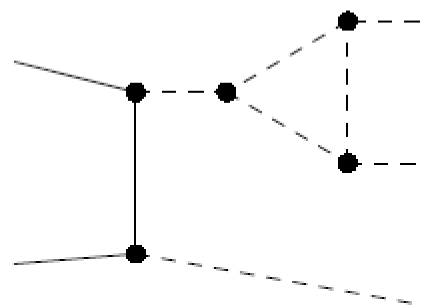

F3


F4

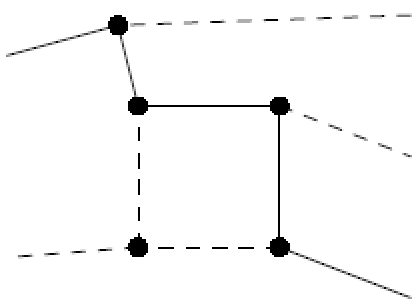
✓One-loop realization : 6 independent topologies

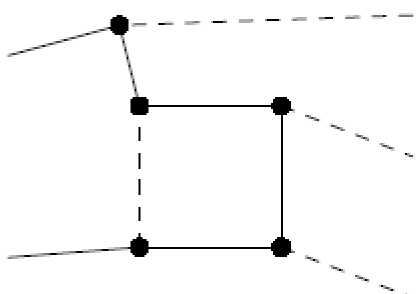

T2-2

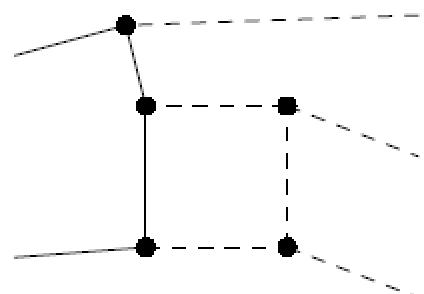

T3-6


T6-4

Yao, Ding, 17


T8-3


T8-4


T8-5

Dirac neutrino masses at dimension five

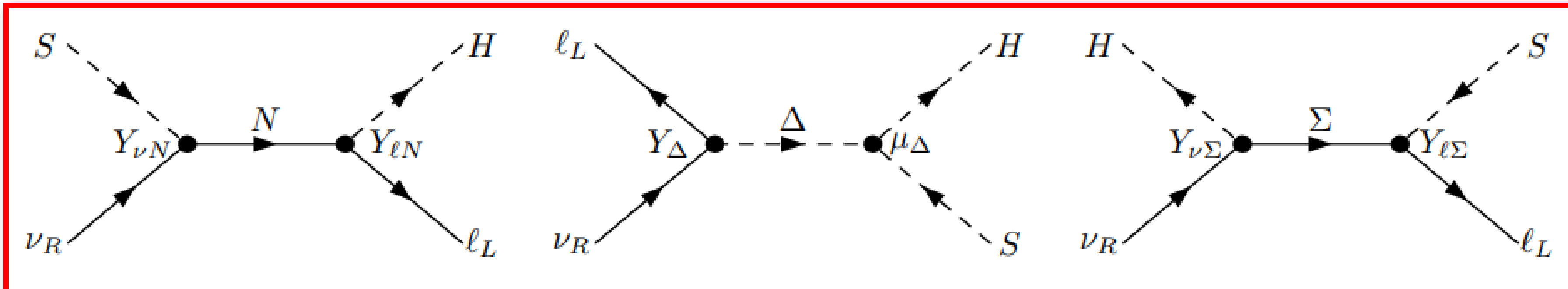
The effective mass operator: **Yao, Ding, 18**

$$\mathscr{L}_5^D = -\frac{g_{\alpha\beta}}{\Lambda}\overline{\ell_{L\alpha}}\widetilde{H}\nu_{R\beta}S + \text{H.c.}$$

Z_2 sym: ν_R, S are odd

- Tree-level UV completion--- Dirac seesaw mechanism

Type-I: singlet fermion *N* **Type-II: doublet scalar Δ** **Type-III: doublet fermion Σ**



$$(m_\nu)_{\alpha\beta} = -\frac{(Y_{\ell N})_{\alpha i}(Y_{\nu N})_{i\beta} v_H v_S}{M_N^{(i)}}$$

Roncadelli, D. Wyler, 83; Ma, Srivastava, 15; Chuli, Srivastava, Valle,17

$$(m_\nu)_{\alpha\beta} = -\frac{\mu_\Delta (Y_\Delta)_{\alpha\beta} v_H v_S}{M_\Delta}$$

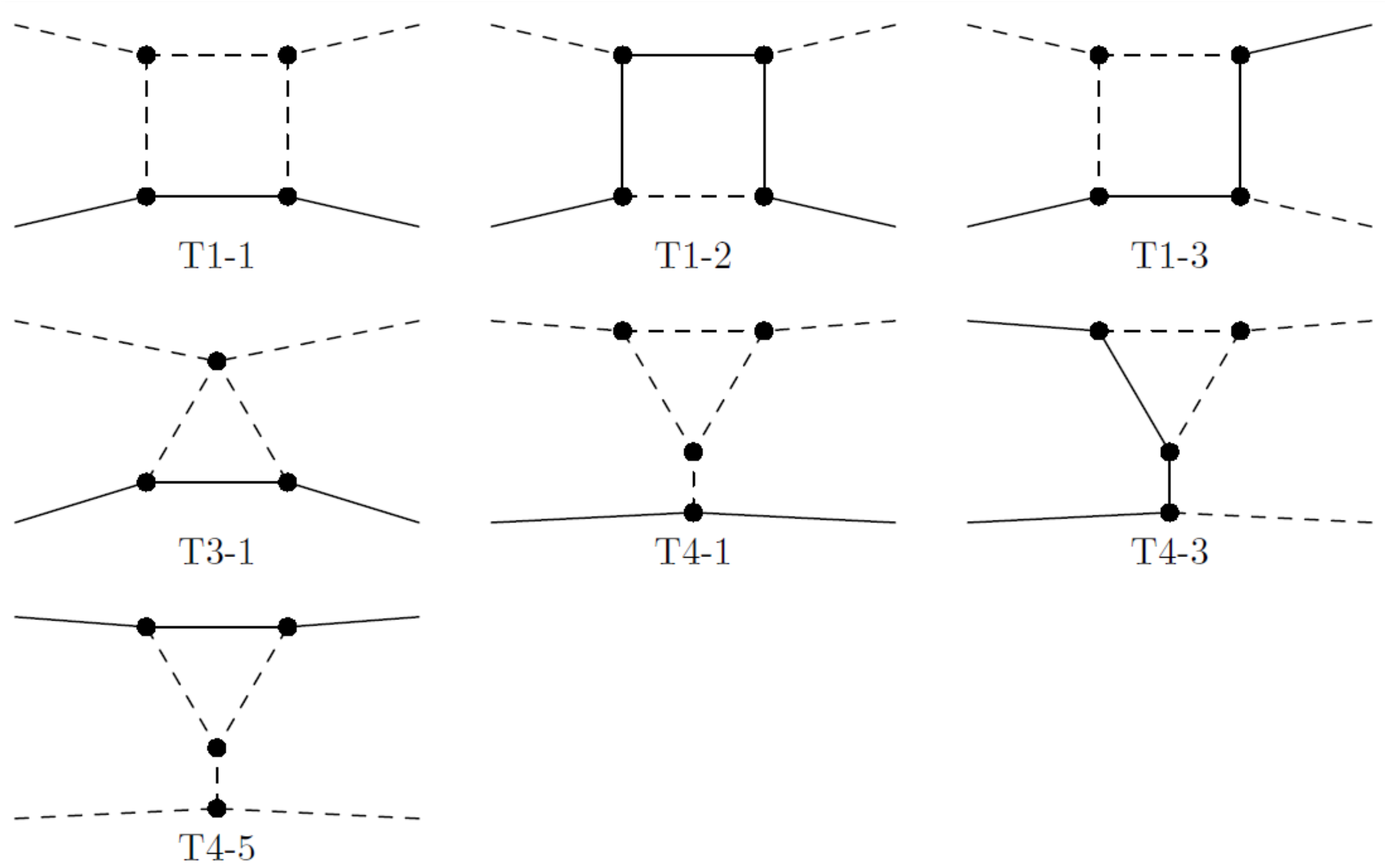
Gu, He,06; Bonilla, Lamprea, Peinado,Valle,17

$$(m_\nu)_{\alpha\beta} = -\frac{(Y_{\ell\Sigma})_{\alpha i}(Y_{\nu\Sigma})_{i\beta} v_H v_S}{M_\Sigma^{(i)}}$$

Yao, Ding,18 ; Chuli, Srivastava, Valle,18

➢ One-Loop UV completion

Yao, Ding, 18



✓Messengers in the loop can be dark matter candidates

✓ Very rich phenomena are expected as the Majorana case

Conclusions

- Flavor and CP symmetry is a powerful framework to understand the neutrino mixing angles and predict leptonic CP violation phases.
- The drastically different quark and lepton mixing patterns can be explained from the same flavor symmetry combined with CP.
- Understanding the origin of neutrino masses requires lots of theoretical and experimental efforts:
  - ✓ precise measurement of θ_{23} and δ_{CP}: guiding us in the search of first principles of flavor mixing
  - ✓ search for lepton number violation: $0\nu\beta\beta$
  - ✓ search for lepton flavor violation: $\mu \to e\gamma$ and μ-e conversion etc
  - ✓ collider experiments: directly revealing the new physics behind small neutrino masses
  - ✓ DM experiments: the connection between neutrino and DM?

  ……

Thank you!

Backup

$\Delta(6n^2)$ flavor group and CP symmetry

- $\Delta(6n^2)$ is a non-abelian finite subgroup of SU(3), it is isomorphic to $(Z_n \times Z_n) \rtimes S_3$. Its four generators satisfy:

$$a^3 = b^2 = (ab)^2 = 1,$$

$$c^n = d^n = 1, \qquad cd = dc,$$

$$aca^{-1} = c^{-1}d^{-1}, \quad ada^{-1} = c, \quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}$$

- Familiar examples: $\Delta(6\times1^2) \cong S_3, \quad \Delta(6\times2^2) \cong S_4$
- Irreducible representations : 1-dim, 2-dim, 3-dim, 6-dim $\qquad \eta \equiv e^{2\pi i/n}$

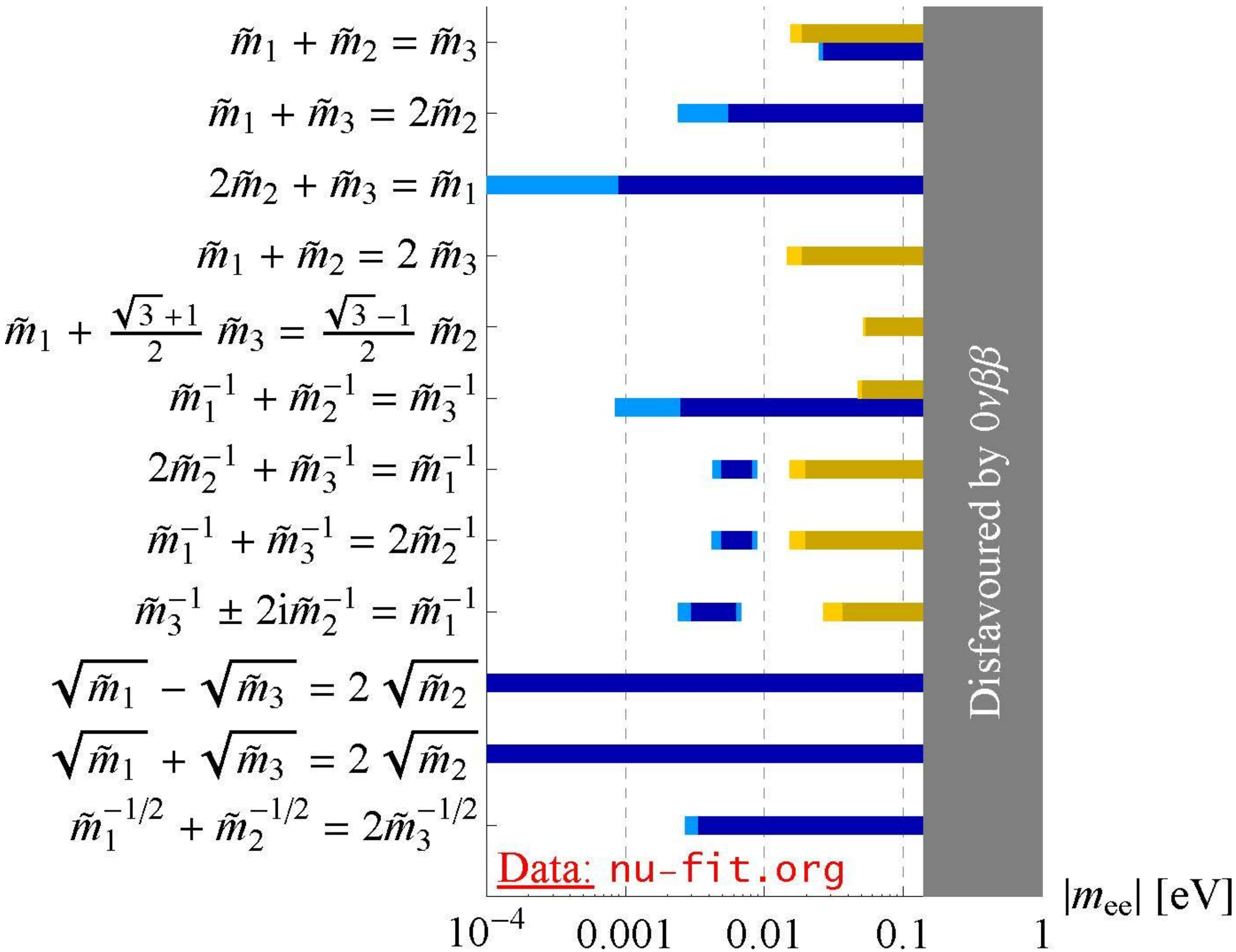
$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad b = -\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}$$

- Physical CP transformations are of the same form as the flavor symmetry transformations in the chosen basis.

$$X\rho^*(g)X^{-1} = \rho(g')$$

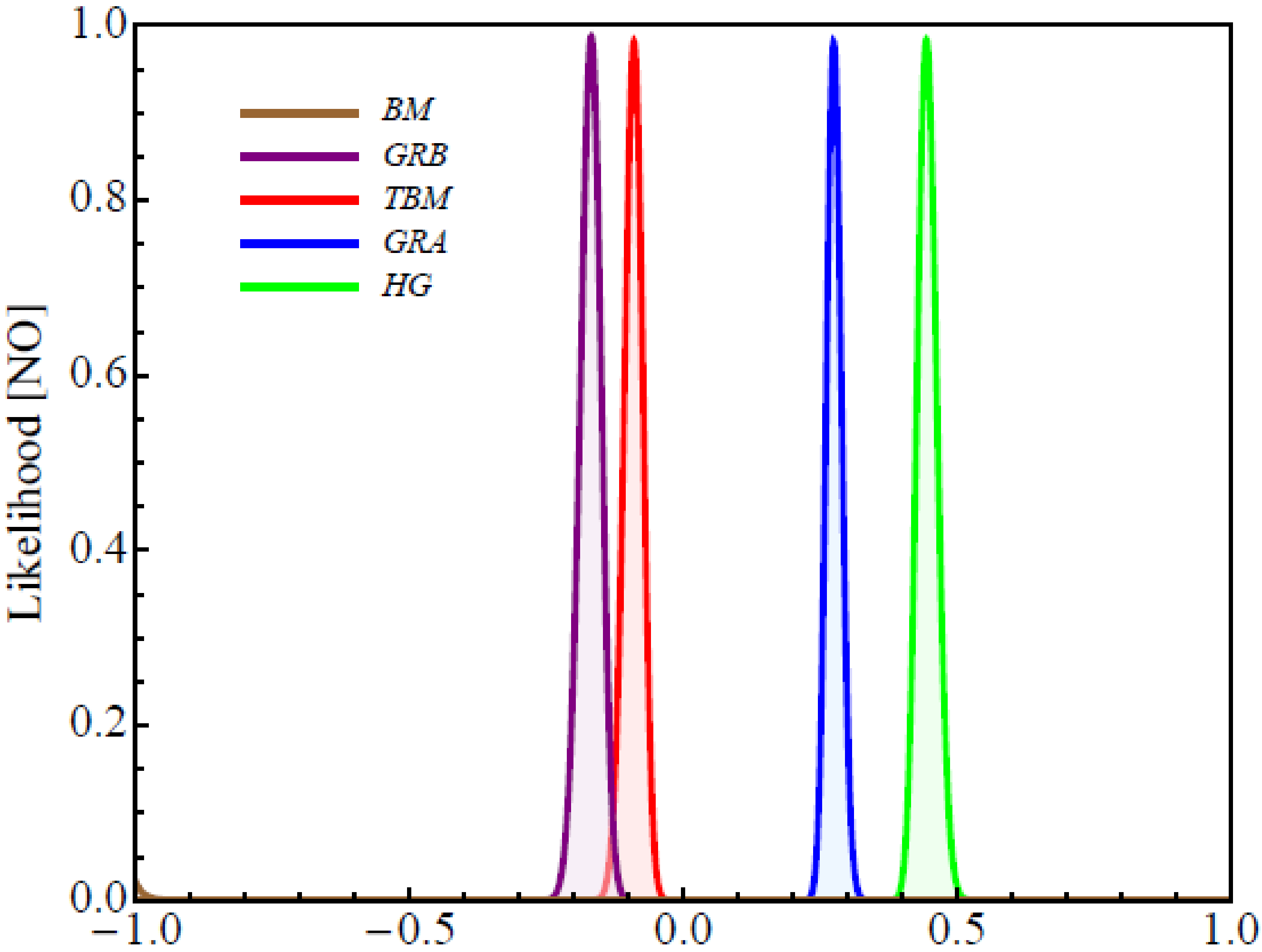
[Ding,King and Neder, JHEP 1412, 007 (2014) ;
Hagedorn, Meroni and Molinaro, Nucl. Phys. B 891, 499 (2015)]

Mass sum rules:



King,Merle,Stuart,13

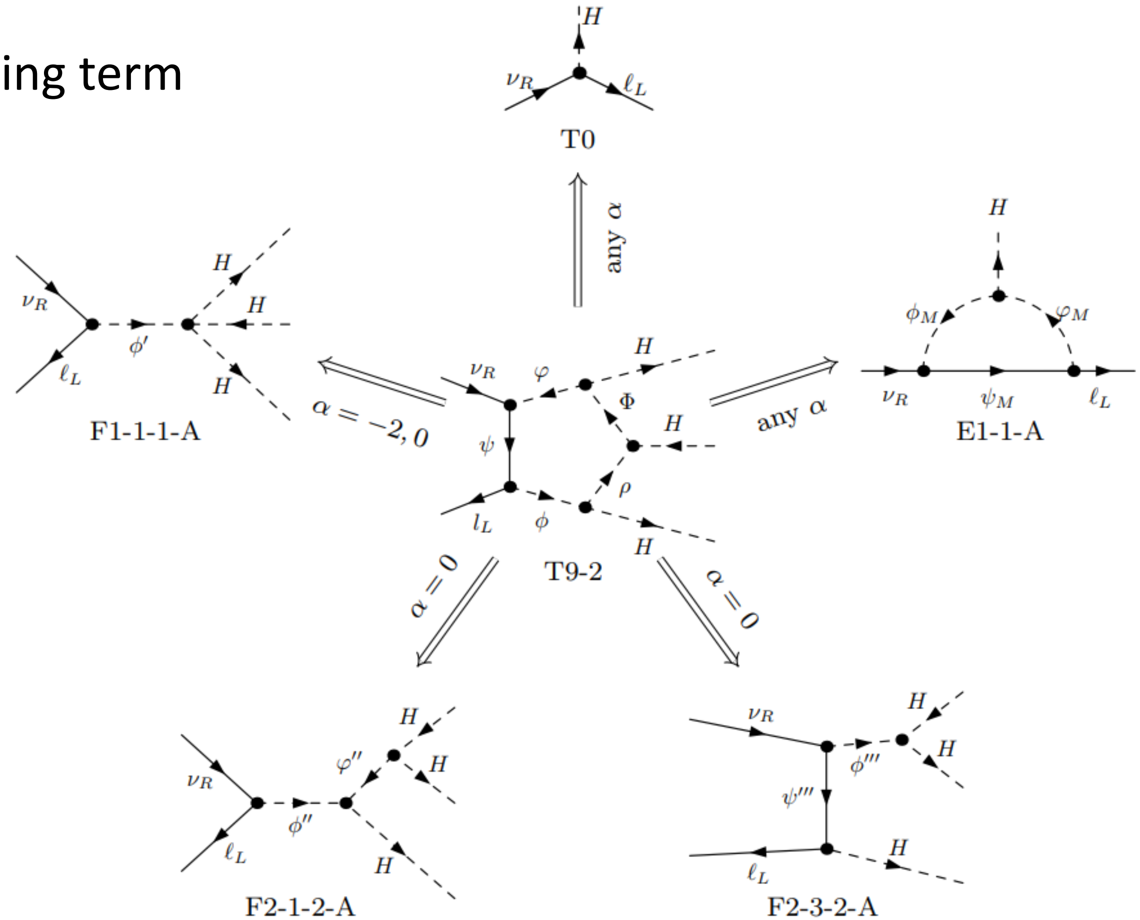
Mixing sum rules:



Petcov, Girardi, Titov, 15

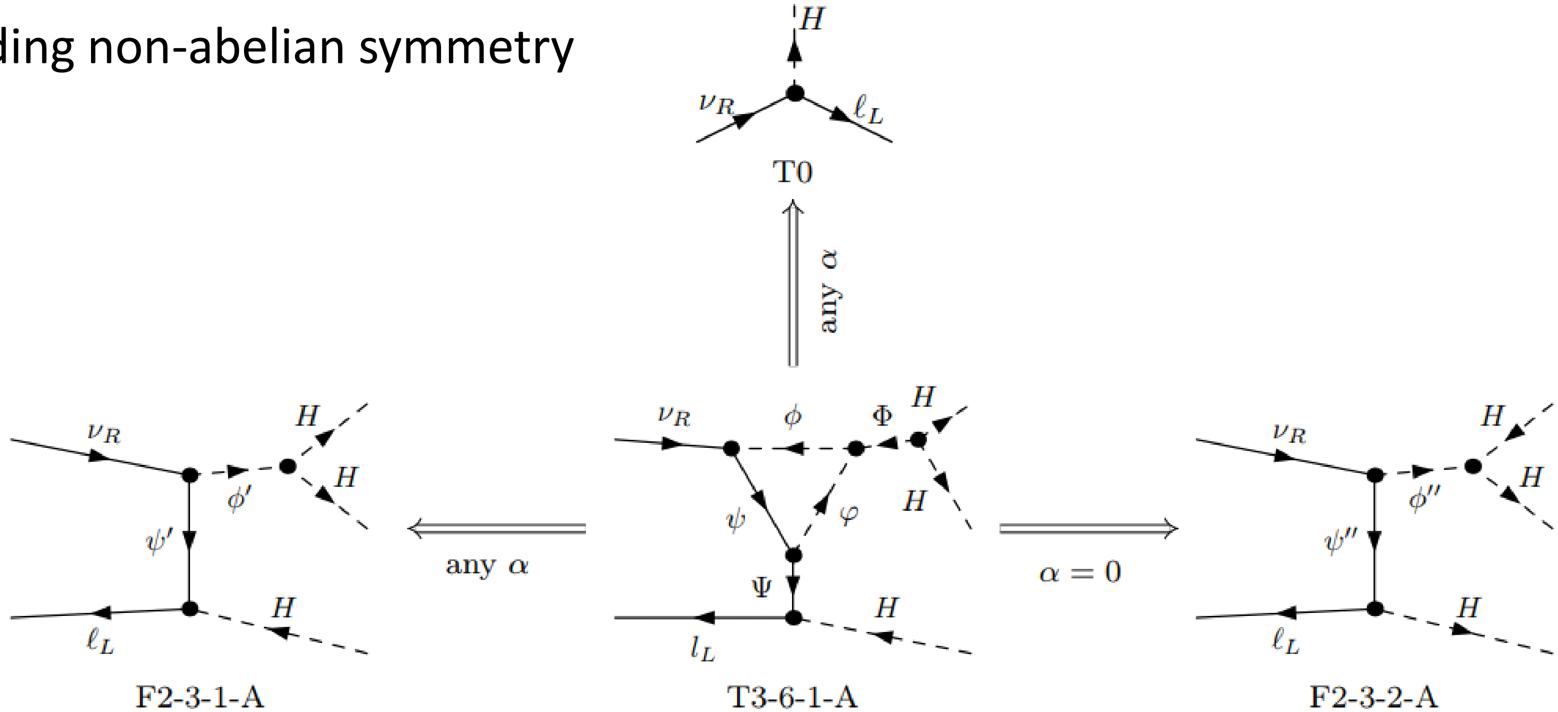
➢ Forbid lower level contribution

✓ By adding soft-breaking term



	T0			T9-2					E1-1-A			Viable
Fields	ℓ_L	ν_R	H	ψ	ϕ	φ	Φ	ρ	ψ_M	ϕ_M	φ_M	
Gauge Sym.	$\mathbf{2}^F_{-1}$	$\mathbf{1}^F_0$	$\mathbf{2}^S_1$	$\mathbf{1}^F_\alpha$	$\mathbf{2}^S_{\alpha+1}$	$\mathbf{1}^S_\alpha$	$\mathbf{2}^S_{\alpha+1}$	$\mathbf{1}^S_\alpha$	$\mathbf{1}^F_\alpha$	$\mathbf{1}^S_\alpha$	$\mathbf{2}^S_{\alpha+1}$	
Z_3 Sym.	ω	ω	ω	$1_\blacktriangle$	$1_\blacktriangle$	ω^2	1	ω^2	$1_\blacktriangle$	ω^2	$1_\blacktriangle$	✗
				1	$\omega^2_\blacktriangle$	ω^2	1	$\omega^2_\blacktriangle$	1	$\omega^2_\blacktriangle$	$\omega^2_\blacktriangle$	✗
				1	ω^2	ω^2	$1_\blacktriangle$	$\omega_\blacktriangle$	1	$\omega_\blacktriangle$	$1_\blacktriangle$	✓
				1	ω^2	$\omega^2_\blacktriangle$	$\omega^2_\blacktriangle$	ω	1	$\omega^2_\blacktriangle$	$\omega^2_\blacktriangle$	✗
				$1_\blacktriangle$	ω^2	$\omega_\blacktriangle$	ω^2	ω	$1_\blacktriangle$	$\omega_\blacktriangle$	ω^2	✗

✓ By adding non-abelian symmetry



S_4:

$$\mathbf{1}\otimes R = R\otimes \mathbf{1} = R,\quad \mathbf{1}'\otimes\mathbf{1}' = \mathbf{1},\quad \mathbf{1}'\otimes\mathbf{2} = \mathbf{2},\quad \mathbf{1}'\otimes\mathbf{3} = \mathbf{3}',\quad \mathbf{1}'\otimes\mathbf{3}' = \mathbf{3},$$
$$\mathbf{2}\otimes\mathbf{2} = \mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{2},\quad \mathbf{2}\otimes\mathbf{3} = \mathbf{2}\otimes\mathbf{3}' = \mathbf{3}\oplus\mathbf{3}',$$
$$\mathbf{3}\otimes\mathbf{3} = \mathbf{3}'\otimes\mathbf{3}' = \mathbf{1}\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}',\quad \mathbf{3}\otimes\mathbf{3}' = \mathbf{1}'\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}',$$

	T0			T3-6-1-A					F2-3-1-A		F2-3-2-A	
Fields	ℓ_L	ν_R	H	ψ	Ψ	ϕ	φ	Φ	ψ'	ϕ'	ψ''	ϕ''
Gauge Sym.	$\mathbf{2}^F_{-1}$	$\mathbf{1}^F_0$	$\mathbf{2}^S_1$	$\mathbf{1}^F_\alpha$	$\mathbf{3}^F_{-2}$	$\mathbf{1}^S_\alpha$	$\mathbf{3}^S_{\alpha+2}$	$\mathbf{3}^S_{-2}$	$\mathbf{3}^F_{-2}$	$\mathbf{3}^S_2$	$\mathbf{1}^F_0$	$\mathbf{1}^S_0$
S_4 Sym.	$\mathbf{3}$	$\mathbf{3}'$	$\mathbf{1}$	$\mathbf{3}'$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$ (or $\mathbf{3}$)	$\mathbf{3}'$	$\mathbf{3}$