Introduction to Parton Showers

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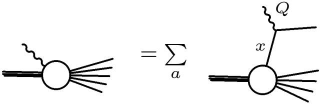
The DGLAP equation

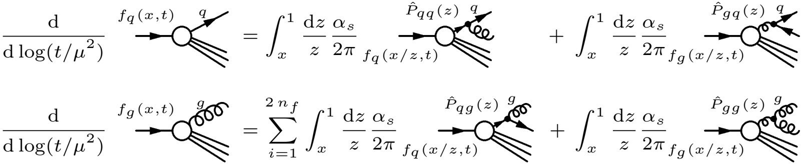
[Altarelli,Parisi] NPB126(1977)298

- Hadronic cross section factorizes into perturbative & non-perturbative piece

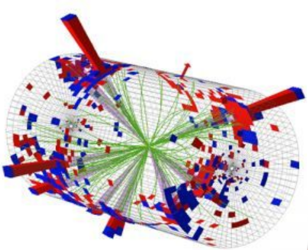
$$\sigma = \sum_{a=q,g} \int \mathrm{d}x\, f_a(x,\mu_F^2)\hat\sigma_a(\mu_F^2)$$

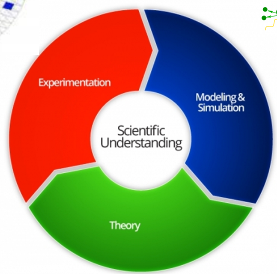


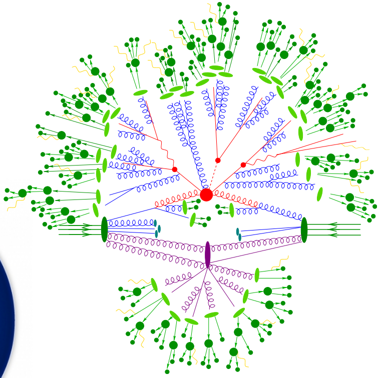
- Evolution from previous slide turns into evolution equation for $f_a(x,\mu_F^2)$
- $f_a(x,\mu_F^2)$ cannot be predicted as a function of x, but dependence on μ_F^2 can be computed order by order in pQCD due to invariance of σ under change of μ_F
- DGLAP equation $\leftrightarrow$ renormalization group equation



How event generators fit in





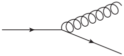


$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\Psi} \not{D} \psi + h.c.$$

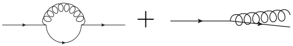
Radiative corrections as a branching process

[Marchesini,Webber] NPB238(1984)1, [Sjöstrand] PLB157(1985)321

- Make two well motivated assumptions
  - Parton branching can occur in two ways



- observed



- unobserved

  - Evolution conserves probability
- The consequence is Poisson statistics
  - Let the decay probability be λ
  - Assume indistinguishable particles $\to$ naive probability for n emissions

$$P_{\text{naive}}(n, \lambda) = \frac{\lambda^n}{n!}$$

  - Probability conservation (i.e. unitarity) implies a no-emission probability

$$P(n, \lambda) = \frac{\lambda^n}{n!} \exp\{-\lambda\} \qquad \longrightarrow \qquad \sum_{n=0}^{\infty} P(n, \lambda) = 1$$

- In the context of parton showers $\Delta = \exp\{-\lambda\}$ is called Sudakov factor

Radiative corrections as a branching process

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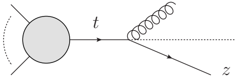
- Decay probability for parton state in collinear limit

$$\lambda \to \frac{1}{\sigma_n} \int_t^{Q^2} \mathrm{d}\bar{t}\, \frac{\mathrm{d}\sigma_{n+1}}{\mathrm{d}\bar{t}} \approx \sum_{\text{jets}} \int_t^{Q^2} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int \mathrm{d}z\, \frac{\alpha_s}{2\pi} P(z)$$



Parameter t identified with evolution "time"

- Splitting function $P(z)$ spin & color dependent

$$P_{qq}(z) = C_F \left[\frac{2}{1-z} - (1+z) \right] \qquad P_{gq}(z) = T_R \left[z^2 + (1-z)^2 \right]$$

$$P_{gg}(z) = C_A \left[\frac{2}{1-z} - 2 + z(1-z) \right] + (z \leftrightarrow 1-z)$$

- Matching to soft limit will requires some care, because full soft emission probability present in all collinear sectors

$$\frac{1}{t} \frac{2}{1-z} \xrightarrow{z \to 1} \frac{p_i p_k}{(p_i q)(q p_k)}$$

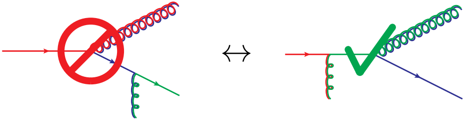
Soft double counting problem [Marchesini, Webber] NPB310(1988)461

- Let us first see how to compute the Poissonian in practice

Color coherence and the dipole picture

[Marchesini,Webber] NPB310(1988)461

- Individual color charges inside a color dipole cannot be resolved if gluon wavelength larger than dipole size $\rightarrow$ emission off "mother"

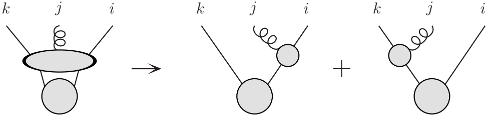


- Net effect is destructive interference outside cone with opening angle set by emitting color dipole $\rightarrow$ phase space for soft radiation halved

[Gustafsson,Pettersson] NPB306(1988)746

- Alternative description of effect in terms of dipole evolution
- Modern approach is to partial fraction soft eikonal and match to collinear sectors [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \rightarrow \frac{1}{p_i p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j} + \frac{1}{p_k p_j}\,\frac{p_i p_k}{(p_i + p_k) p_j}$$



Color coherence and the dipole picture

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- Splitting kernels become dependent on anti-collinear direction usually defined by color spectator in large-N_c limit
- Singularity confined to soft-collinear region only captures all coherence effects at leading color, NLL

$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- Complete set of leading-order splitting functions now given by

$$P_{qq}(z,\kappa^2) = C_F\left[\frac{2\,(1-z)}{(1-z)^2+\kappa^2} - (1+z)\right]$$

$$P_{qg}(z,\kappa^2) = C_F\left[\frac{1+(1-z)^2}{z}\right], \qquad P_{gq}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\frac{1-z}{(1-z)^2+\kappa^2} + \frac{1}{z} - 2 + z(1-z)\right]$$

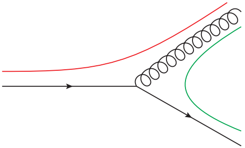
Color flow

- Parton showers replace gluon propagators by means of the identity

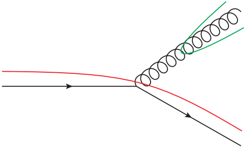
$$\underbrace{\delta^{ab}}_{\text{standard}} = 2\,\mathrm{Tr}(T^aT^b) = 2\,T^a_{ij}T^b_{ji} = T^a_{ij}\,\underbrace{2\,\delta_{ik}\delta_{jl}}_{\text{parton shower}}\,T^b_{lk}$$

- Quark-gluon vertex

$$T^a_{ij}T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$

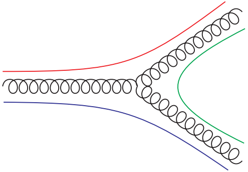


$$-\frac{1}{N_c}$$



- Gluon-gluon vertex

$$f^{abc}T^a_{ij}T^b_{kl}T^c_{mn} = \delta_{il}\delta_{kn}\delta_{mj} - \delta_{in}\delta_{ml}\delta_{kj}$$



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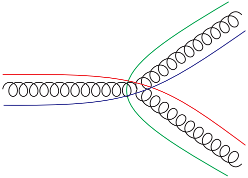
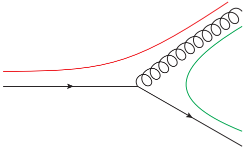


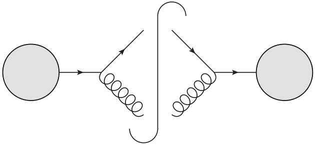
Color flow

- Typically, parton showers also make the leading-color approximation

$$T^a_{ij}T^a_{kl} \to \frac{1}{2}\delta_{il}\delta_{jk} \qquad \leftrightarrow$$



- If used naively, this would overestimate the color charge of the quark: Consider process $q \to qg$ attached to some larger diagram



$$\propto \qquad T^a_{ij}T^a_{jk} = C_F\delta_{ik}$$

but now we have $\dfrac{1}{2}\,\delta_{il}\delta_{jm}\delta_{mj}\delta_{lk} = \dfrac{C_A}{2}\,\delta_{ik}$

- While color assignments in the parton shower are made at leading color the color charge of quarks is actually kept at C_F

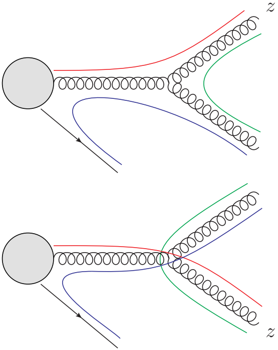
Color flow

- Having matched the eikonal to two collinear sectors implies that in $g \to gg$ splittings color and kinematics are entangled

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j} \frac{p_i p_k}{(p_i + p_k) p_j} + \ldots \to \frac{1}{p_i p_j} \frac{1 - z}{(1-z)^2 + \kappa^2} \ldots$$

- There is only one possible color assignment for each leading-color dipole

$$\frac{1 - z}{(1-z)^2 + \kappa^2} \qquad \leftrightarrow$$



Parton-shower kinematics: Final state radiation

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- Want to construct three (massless) on-shell momenta from two, corresponding to branching process $\widetilde{ij} \to i, j$ in presence of $\tilde{k} \to k$
- Calculate p_{ij}^2 and $\tilde{z} = (p_i\tilde{p}_k)/(\tilde{p}_{ij}\tilde{p}_k)$ from PS variables t and z
- First generate the propagator mass by rescaling

$$p_{ij}^\mu = \tilde{p}_{ij}^\mu + \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\,\tilde{p}_k^\mu\,, \qquad p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

- Then branch off-shell momentum into two on-shell momenta

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\,\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$

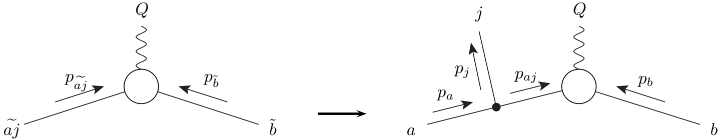
- On-shell conditions require that

$$\vec{k}_T^2 = p_{ij}^2\,\tilde{z}(1-\tilde{z}) \qquad \leftrightarrow \qquad \tilde{z}_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{4\vec{k}_T^2}{p_{ij}^2}}\right)$$

$\to$ for any finite $\vec{k}_T$ we have $0 < \tilde{z} < 1$

Parton-shower kinematics: Initial-state radiation

- Initial-state kinematics slightly more involved
  as recoil should not be taken by opposite-side beam



- Compute new beam momentum by rescaling to new partonic cms energy

$$p_a^\mu = \frac{2\, p_a p_b}{2\, \tilde{p}_{aj} \tilde{p}_b}\, \tilde{p}_{aj}^\mu$$

- Compute final-state momentum and internal momentum as

$$p_{aj}^\mu = \tilde{z}\, p_a^\mu + \frac{p_{aj}^2}{2 p_b p_a}\, p_b^\mu + k_\perp^\mu$$

$$p_j^\mu = (1 - \tilde{z})\, p_a^\mu - \frac{p_{aj}^2}{2 p_b p_a}\, p_b^\mu - k_\perp^\mu$$

- Recoil is taken by complete final state via Lorentz transformation

$$p_i^\mu = p_{\tilde{\imath}}^\mu - \frac{2\, p_{\tilde{\imath}}(K + \tilde{K})}{(K + \tilde{K})^2}\,(K + \tilde{K})^\mu + \frac{2\, p_{\tilde{\imath}} \tilde{K}}{\tilde{K}^2}\, K^\mu\ ,$$

where $K^\mu = p_a^\mu - p_j^\mu + p_b^\mu$ and $\tilde{K}^\mu = p_{\widetilde{aj}}^\mu + p_b^\mu$

Connection to the DGLAP formalism

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- DGLAP equation for fragmentation functions

$$\frac{\mathrm{d}\,xD_a(x,t)}{\mathrm{d}\ln t} = \sum_{b=q,g}\int_0^1 \mathrm{d}\tau\int_0^1 \mathrm{d}z\,\frac{\alpha_s}{2\pi}\left[zP_{ab}(z)\right]_+\,\tau D_b(\tau,t)\,\delta(x-\tau z)$$

- Refine plus prescription $\left[zP_{ab}(z)\right]_+ = \lim_{\varepsilon\to 0} zP_{ab}(z,\varepsilon)$

$$P_{ab}(z,\varepsilon) = P_{ab}(z)\,\Theta(1-\varepsilon-z) - \delta_{ab}\sum_{c\in\{q,g\}}\frac{\Theta(z-1+\varepsilon)}{\varepsilon}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,P_{ac}(\zeta)$$

- Rewrite for finite ε

$$\frac{\mathrm{d}\ln D_a(x,t)}{\mathrm{d}\ln t} = -\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta) + \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}$$

- First term is derivative of Sudakov factor $\Delta = \exp\{-\lambda\}$

$$\Delta_a(t,Q^2) = \exp\left\{-\int_t^{Q^2}\frac{\mathrm{d}\bar{t}}{\bar{t}}\sum_{c=q,g}\int_0^{1-\varepsilon}\mathrm{d}\zeta\,\zeta\,\frac{\alpha_s}{2\pi}P_{ac}(\zeta)\right\}$$

Connection to the DGLAP formalism

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- Use generating function $\Pi_a(x,t,Q^2) = D_a(x,t)\Delta_a(t,Q^2)$ to write

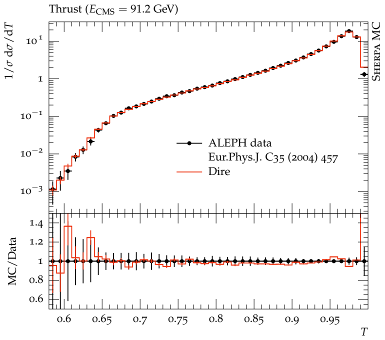
$$\frac{\mathrm{d}\ln\Pi_a(x,t,Q^2)}{\mathrm{d}\ln t/Q^2} = \sum_{b=q,g}\int_x^{1-\varepsilon}\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)\,\frac{D_b(x/z,t)}{D_a(x,t)}\;.$$

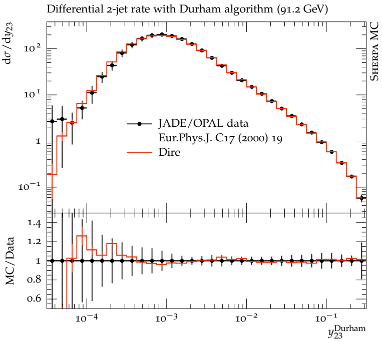
- If hadron not resolved, obtain

$$\frac{\mathrm{d}}{\mathrm{d}\ln t/Q^2}\ln\left(\frac{\Pi_a(x,t,Q^2)}{D_a(x,t)}\right) = \frac{\mathrm{d}\Delta_a(t,Q^2)}{\mathrm{d}\ln t/Q^2} = \sum_{b=q,g}\int_0^{1-\varepsilon}\mathrm{d}z\,z\,\frac{\alpha_s}{2\pi}\,P_{ab}(z)$$

- Survival probabilities for one parton between scales t_1 and t_2:
  - $\dfrac{\Pi_a(x,t_2,Q^2)}{\Pi_a(x,t_1,Q^2)}$ Resolved hadron $\leftrightarrow$ constrained (backward) evolution
  - $\dfrac{\Delta_a(t_2,Q^2)}{\Delta_a(t_1,Q^2)}$ No resolved hadron $\leftrightarrow$ unconstrained (forward) evolution
- Parton-showers draw t_2-points starting from t_1 based on these probabilities

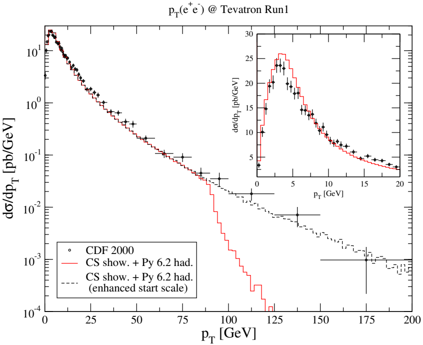
Effects of the parton shower





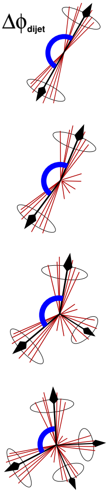
- Thrust and Durham $2 \to 3$-jet rate in $e^+e^- \to$hadrons
- Hadronization region to the right (left) in left (right) plot

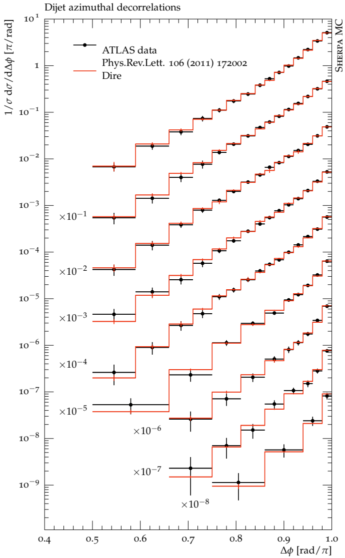
Effects of the parton shower

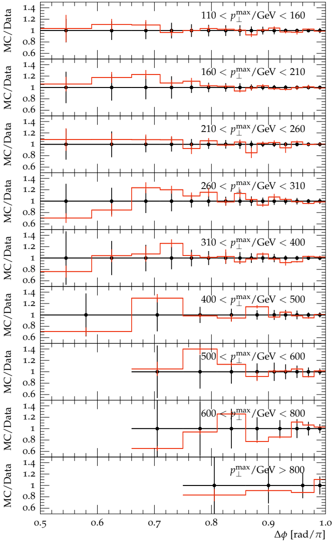


- Drell-Yan lepton pair production at Tevatron
- If hard cross section computed at leading order, then parton shower is only source of transverse momentum

Effects of the parton shower







Hands on tutorials

- Great resource for learning parton showers:
  "Hackathons" at CTEQ/MCnet schools
  `http://www.slac.stanford.edu/~shoeche/cteq17`
  `svn co svn://svn.slac.stanford.edu/mc/ps`

Tutorial on MC event generators

Held by the MCnet collaboration at CTEQ 2017.

Instructions

PS coding tutorial
MC running tutorial

TASI Lectures

arXiv:1411.4085

Tutorial on Parton Showers and Matching

1 Introduction

In this tutorial we will discuss the construction of a parton shower, the implementation of on-the-fly uncertainty estimates, and of matrix-element corrections, and matching at next-to-leading order. At the end, you will be able to run your own parton shower for $e^+e^- \to$hadrons at LEP energies and compare its predictions to results from the event generator Sherpa (using a simplified setup). You will also have constructed your first MC@NLO and POWHEG generator.

2 Getting started

In order to run this tutorial you should install PyPy and Rivet on your PC. The following command will