Outlook for Perturbative QCD at Colliders

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pQCD @ West Lake, Zhejiang University

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The past (week)

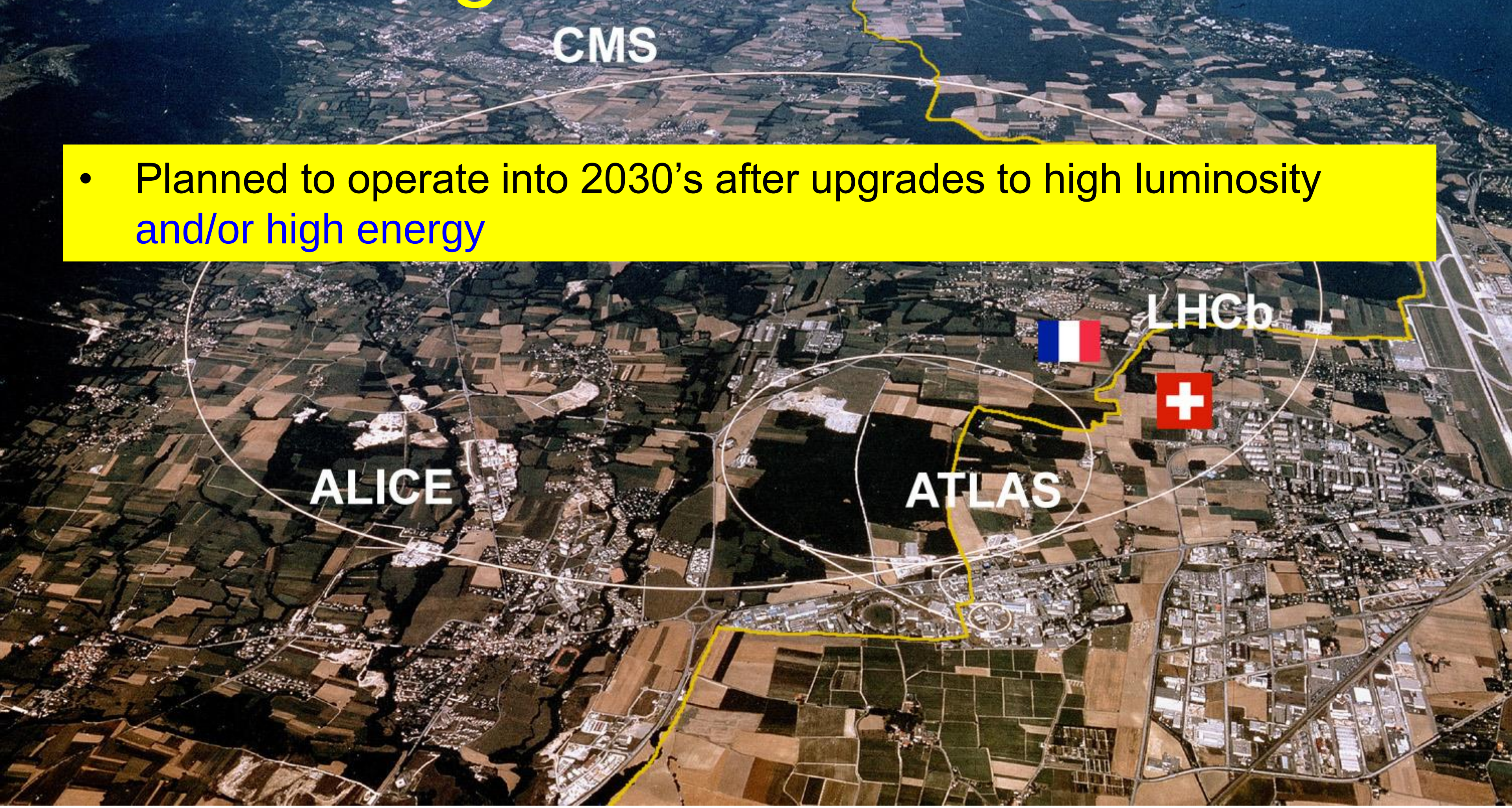
- You have learned a lot of QCD in the past week [Maybe more than you can actually absorb in one week!]
- What is the one over-riding theme?
- Factorization = “divide and conquer”:

1. In the parton model itself [Melnikov lecture]
2. For complex momenta [BCFW, Badger lecture]
3. In soft and collinear regions [Hoeche, Monni, Rontsch lectures]

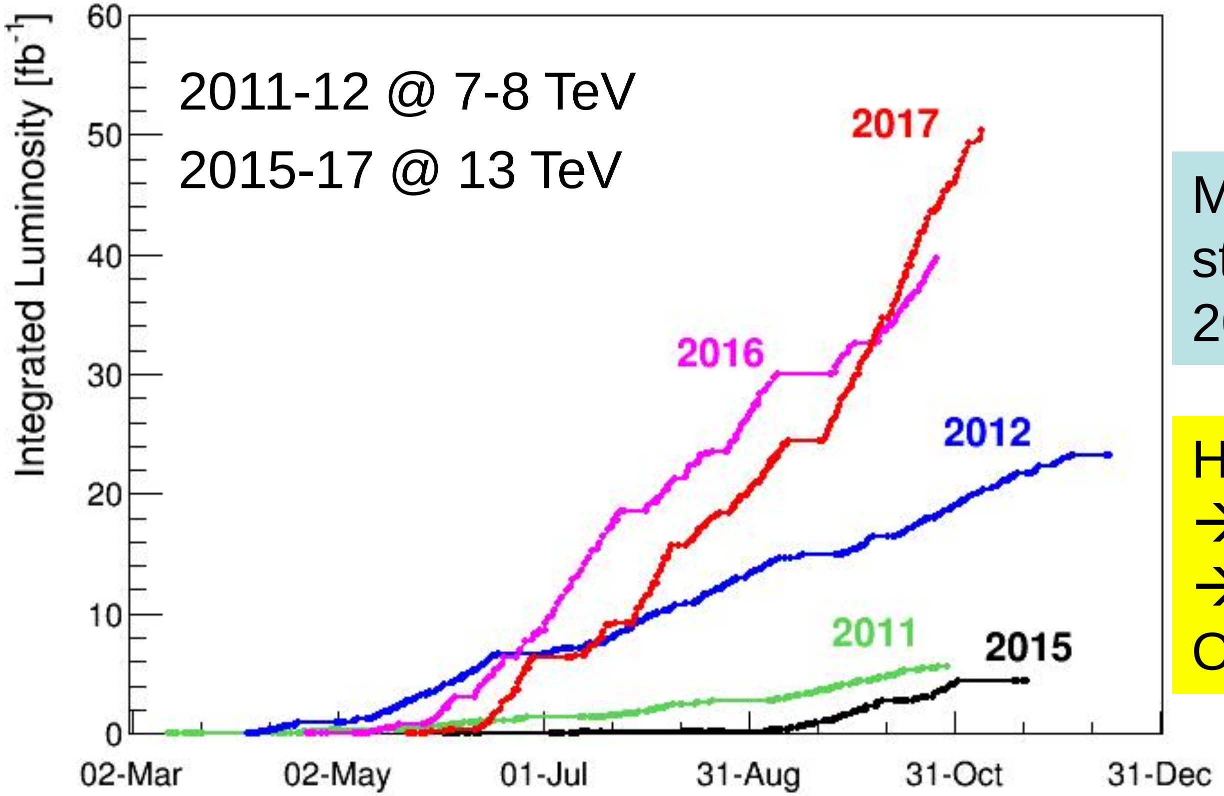
Factorization ←→ evolution

The "near" experimental future: Large Hadron Collider

- Planned to operate into 2030's after upgrades to high luminosity and/or high energy



LHC performing exceedingly well



Most analyses still based only on 2016 data, 36 fb^{-1}

High luminosity LHC
→ factor of 100 more
→ 3000 fb^{-1}
Over next decade(s)

The far(?) experimental future

- What lies beyond the LHC?
- High energy (> 220 GeV) e^+e^- collider(s):
  linear (ILC or CLIC)? or circular (FCC or CEPC)?
- Precision Higgs measurements, m_{top}

- Circular machine would likely precede a
  very large hadron collider: pp at 70 - 100 TeV

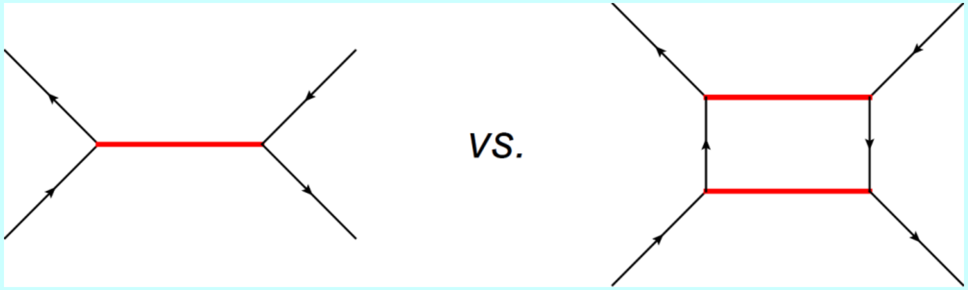
- Even longer term, plasma wakefield or laser acceleration
- Enormous acceleration gradients, GeV/m, over short distances so far. Can one iterate/stage to get to very high energy (> 10 TeV?) e^+e^-, and high enough luminosity?

Beyond the Standard Model

- **Hierarchy problem:**

In SM, electroweak scale m_W looks fine-tuned as soon as ultraviolet cutoff Λ is raised well above m_W.

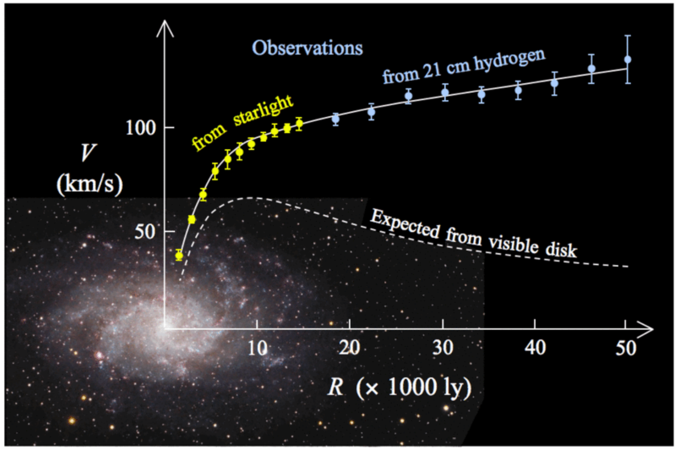
- Many theories predict a host of new massive particles with masses ~ " m_W " i.e. within reach of the LHC.
- To prevent problems from precision electroweak physics,

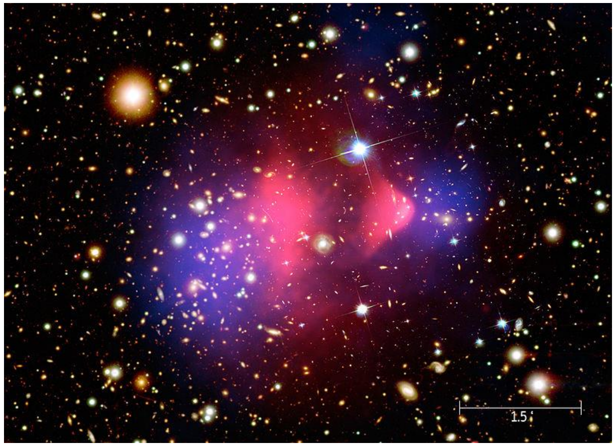


such theories often have a discrete symmetry, for which the lightest odd particle is a **dark matter candidate**

Dark matter at the LHC?

There is dark matter in the cosmos



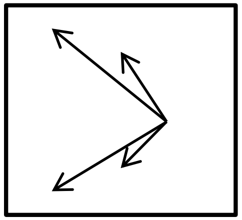


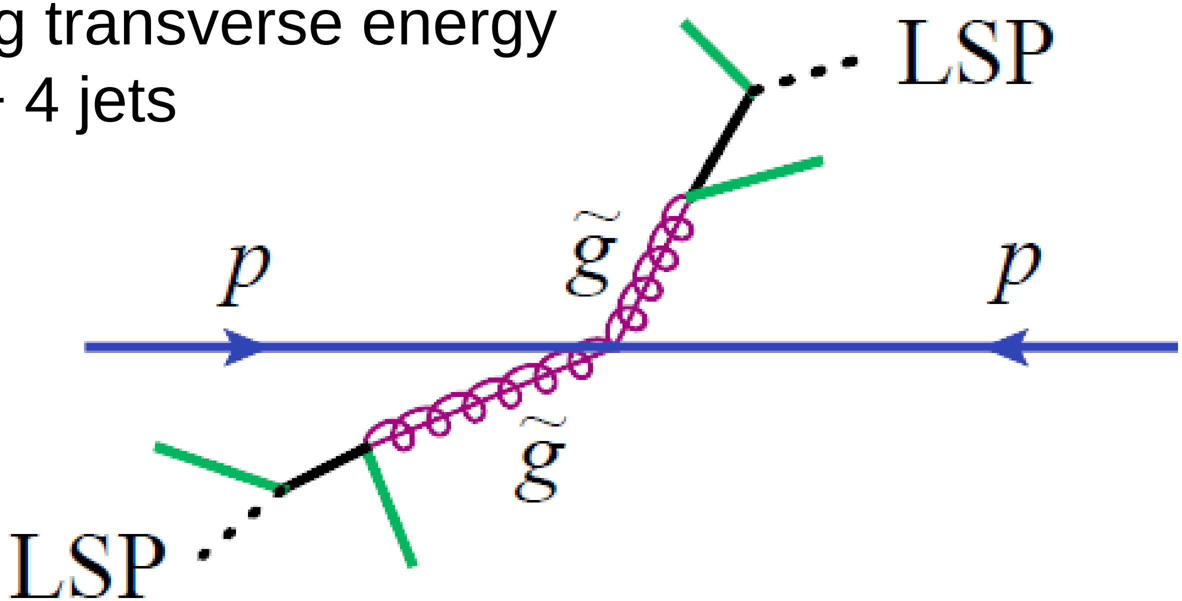
- **If it is an elementary particle with mass > GeV, the LHC could produce it, or produce other particles that decay to it.**
- **But it might be a different kind of particle (axion?) for which high-energy colliders are not very useful**

Classic SUSY dark matter signature

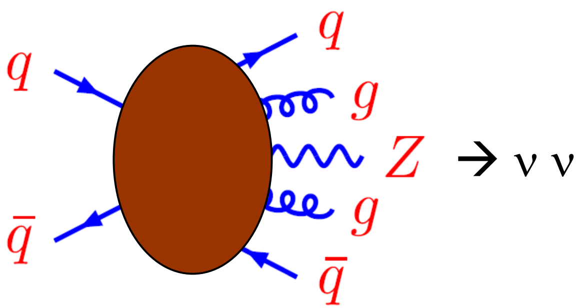
Heavy colored particles decay rapidly to stable Weakly Interacting Massive Particle (WIMP = LSP) plus jets

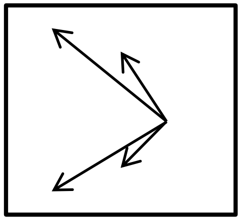
→ Missing transverse energy
MET + 4 jets





Critical to understand Standard Model backgrounds





MET + 4 jets from pp → Z + 4 jets,
Z → neutrinos

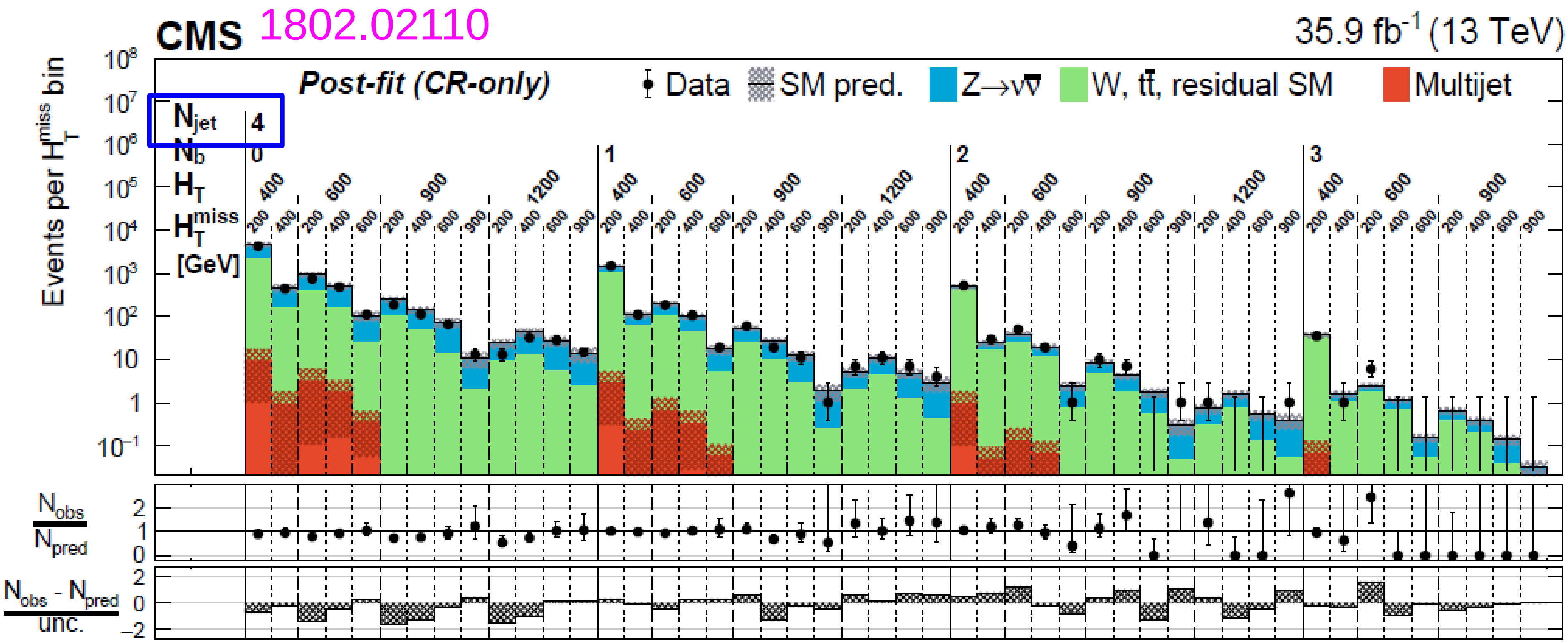
Neutrinos escape detector.

Irreducible background.

Plus there are many reducible backgrounds from W + jets, tt + jets, ...

Precision theory (typically NLO) can help with this, usually when embedded in parton shower Monte Carlos [Hoeche]

SUSY (and other) searches now very advanced

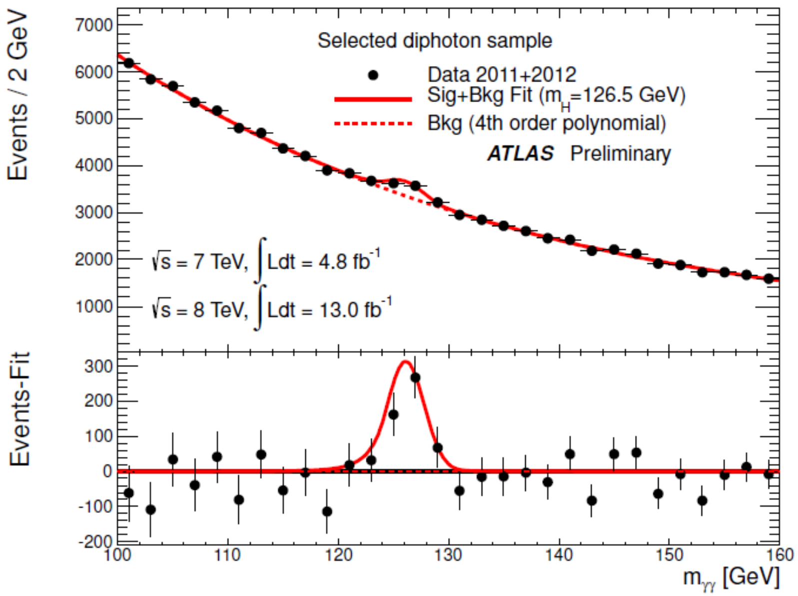


- **Also N_{jet} = 1,2,3,5,6**
- **No significant excesses seen, so set lower limits on masses of superparticles.**
- **Can better simulations, (N)NLO + parton showers, help further?**

From searches to measurements

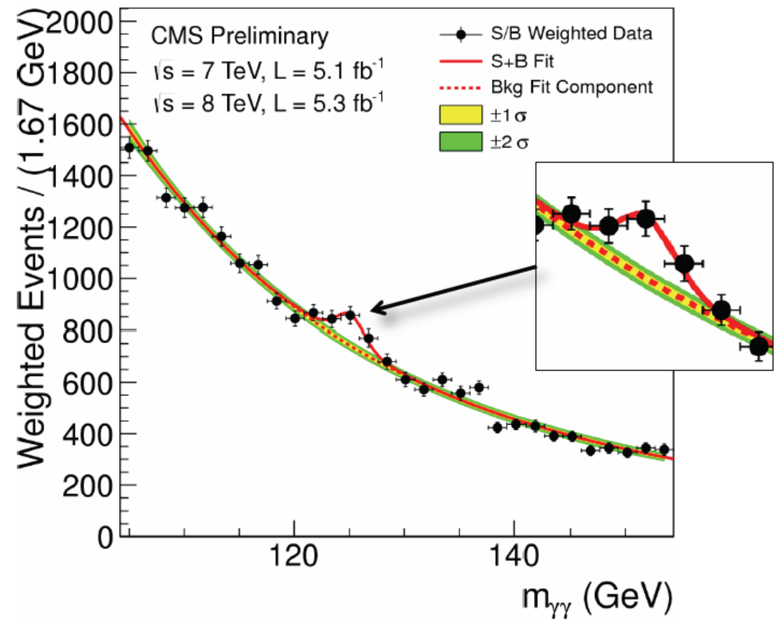
- No convincing evidence for SUSY, or any other direct production of new particles.
- Also look for deviations in rates for Standard Model processes, especially involving the brand-new Higgs boson.
- Measurements are hard, take a while to perform.
- Even more precise (QCD) theory typically needed. Also NLO electroweak will become important at mass scales ~ TeV

The Higgs boson



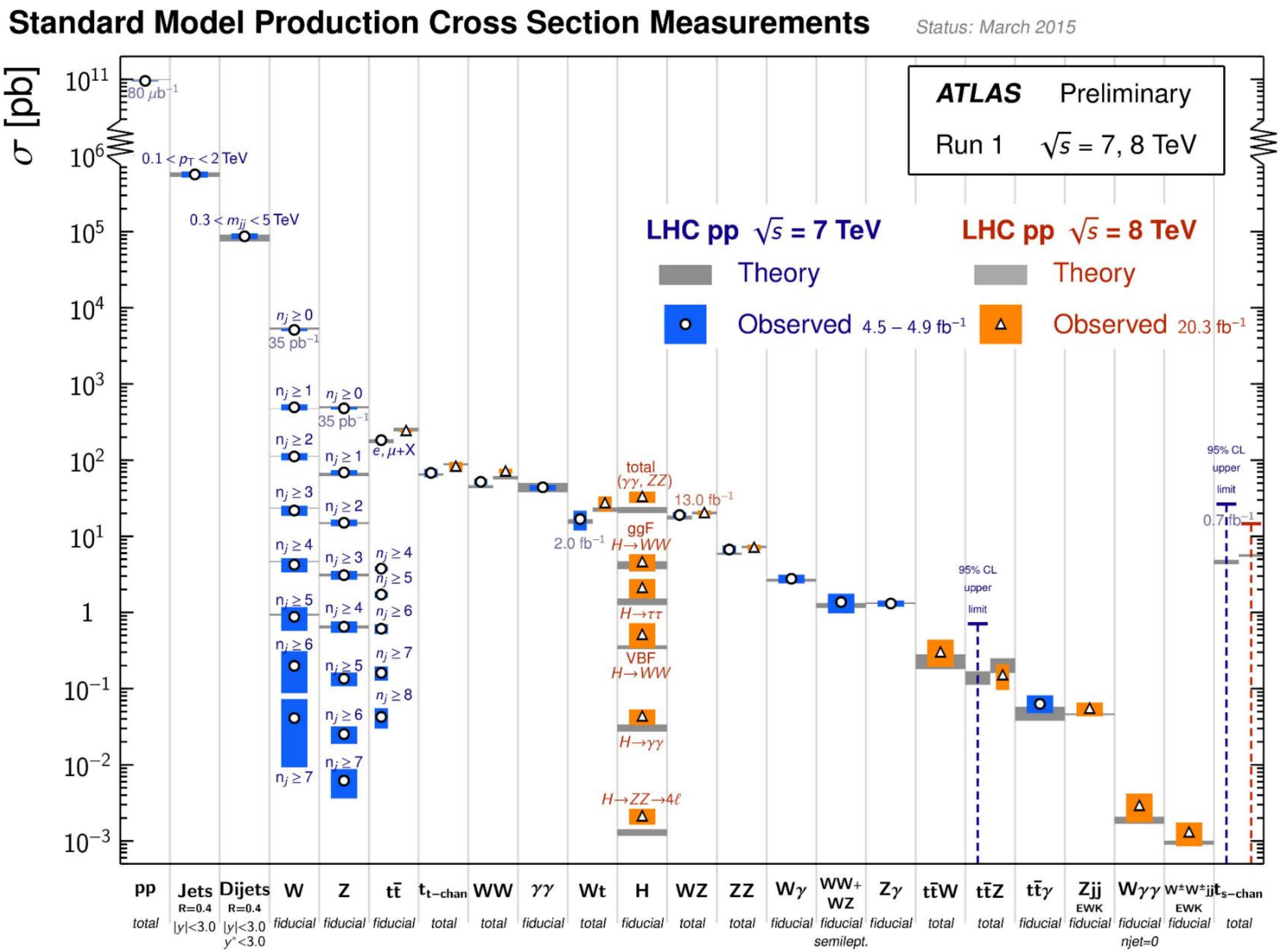
July 4
2012

H → γγ



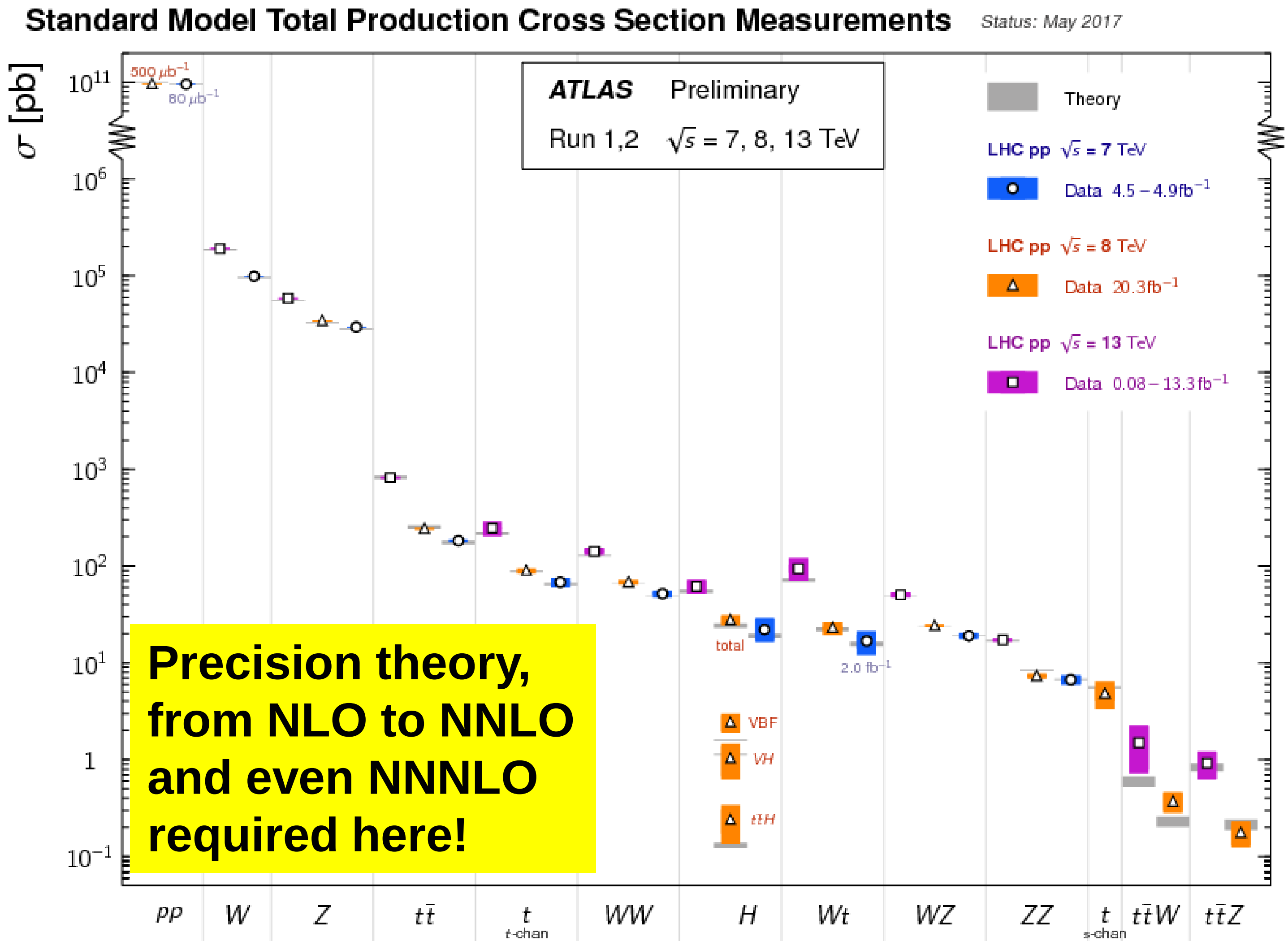
- Newest and most direct window into electroweak symmetry breaking.
- Clearly need to study its properties in as much detail as possible at LHC and at any future e^+e^- colliders [e.g. Jun Gao talk]
- Complicated production at LHC, lots of soft gluons, additional jets, …

2015



Standard Model Production Cross Section Measurements
Status: March 2015
σ [pb]
ATLAS Preliminary
Run 1 √s = 7, 8 TeV
LHC pp √s = 7 TeV
Theory
Observed 4.5 – 4.9 fb⁻¹
LHC pp √s = 8 TeV
Theory
Observed 20.3 fb⁻¹
80 μb⁻¹
0.1 < pT < 2 TeV
0.3 < mjj < 5 TeV
35 pb⁻¹
e, μ+X
total (γγ, ZZ)
13.0 fb⁻¹
2.0 fb⁻¹
ggF H→WW
H→ττ
VBF H→WW
H→γγ
H→ZZ→4ℓ
95% CL upper limit
0.7 fb⁻¹
pp total
Jets R=0.4 |y|<3.0
Dijets R=0.4 |y|<3.0 y*<3.0
W fiducial
Z fiducial
tt̄ fiducial
tt-chan total
WW total
γγ fiducial
Wt total
H fiducial
WZ total
ZZ total
Wγ fiducial
WW+WZ fiducial semilept.
Zγ fiducial
tt̄W total
tt̄Z total
tt̄γ fiducial
Zjj EWK fiducial
Wγγ fiducial njet=0
W±W±jj EWK fiducial
ts-chan total

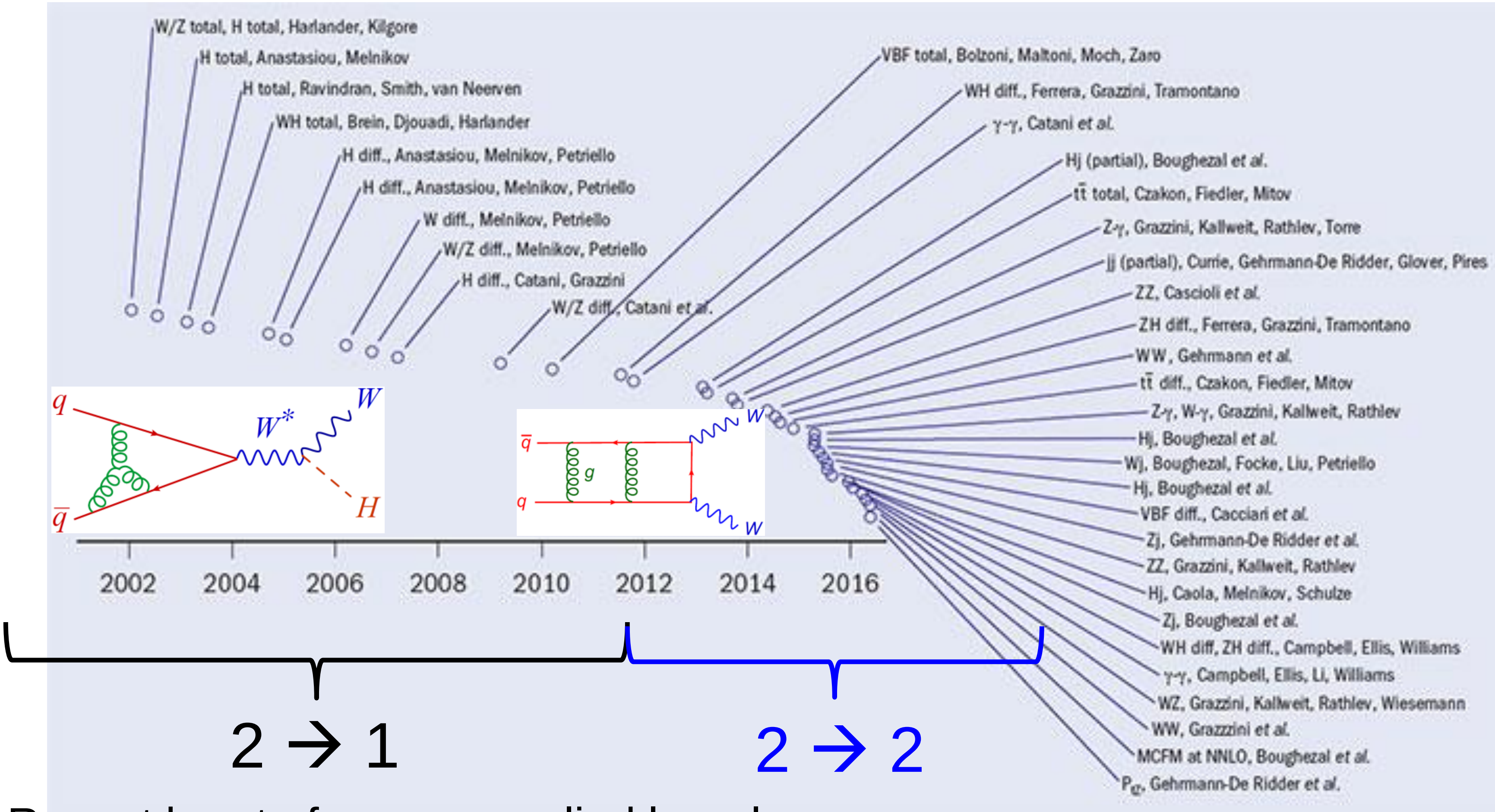
2017



Precision theory, from NLO to NNLO and even NNNLO required here!

NNLO timeline

G. Zanderighi
CERN Courier (2017)

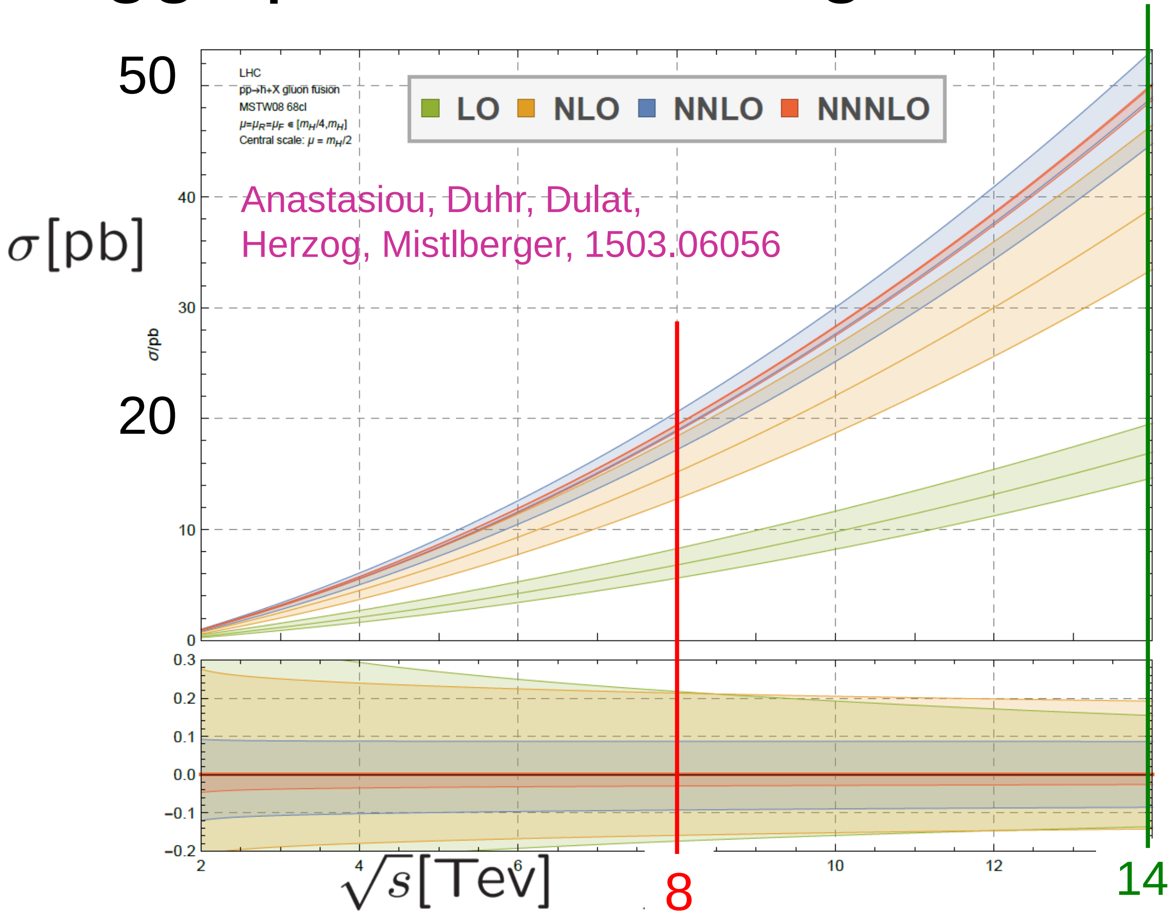


Recent burst of progress relied largely on:

a) 2 loop 2 → 2 amplitudes, loop integration methods from Tancredi lectures

b) NNLO subtraction (or slicing) methods, a la Rontsch lectures

The one high p_T LHC cross section known at NNNLO
Higgs production via gluon fusion

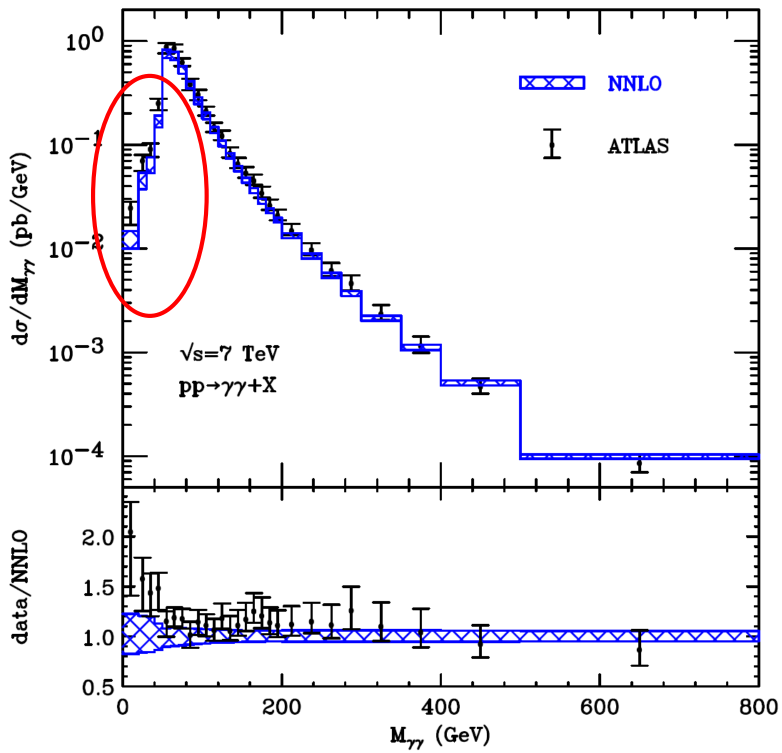


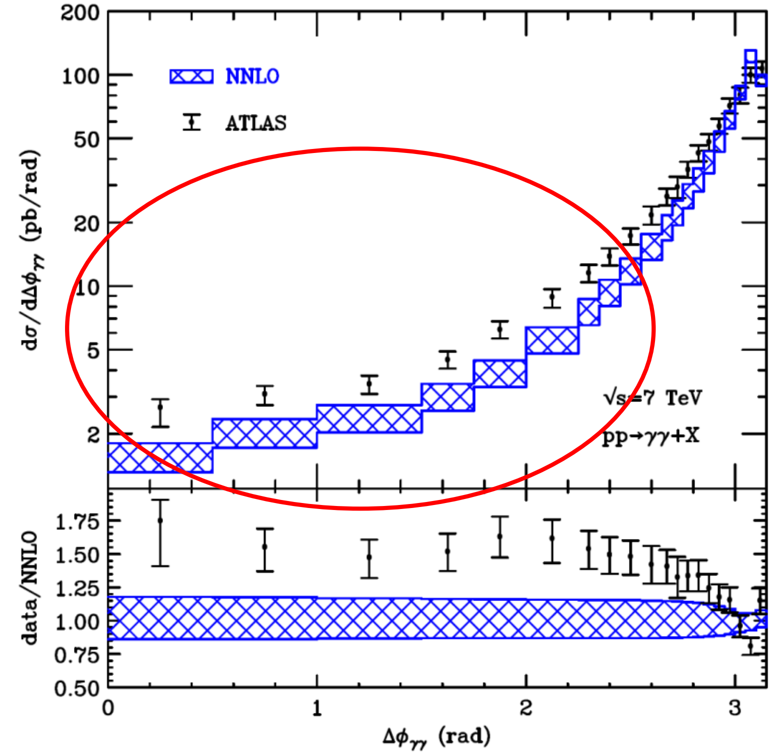
LO → NNNLO gives factor of 2.7 increase!

Where "NNLO" is still way off

- Diphoton production at LHC

Catani, Cieri, de Florian Ferrera, Grazzini, 1802.02095





NNLO is really NLO in worst regions

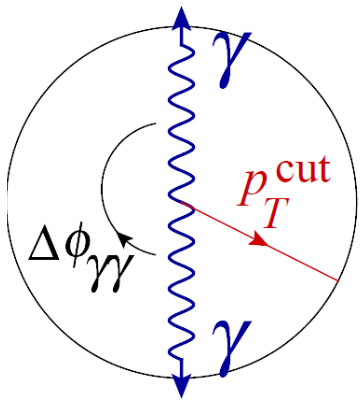
LO kinematics, with

$p_T(\gamma) > p_T^{\text{cut}} = 25$ GeV

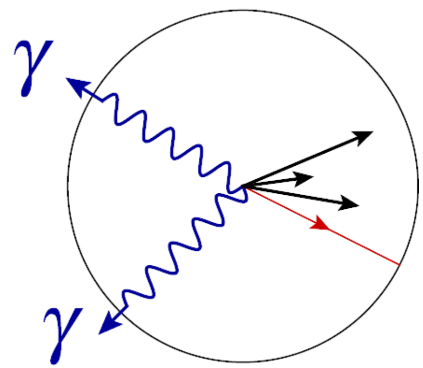
$\Rightarrow$

$\Delta\phi_{\gamma\gamma} = \pi$

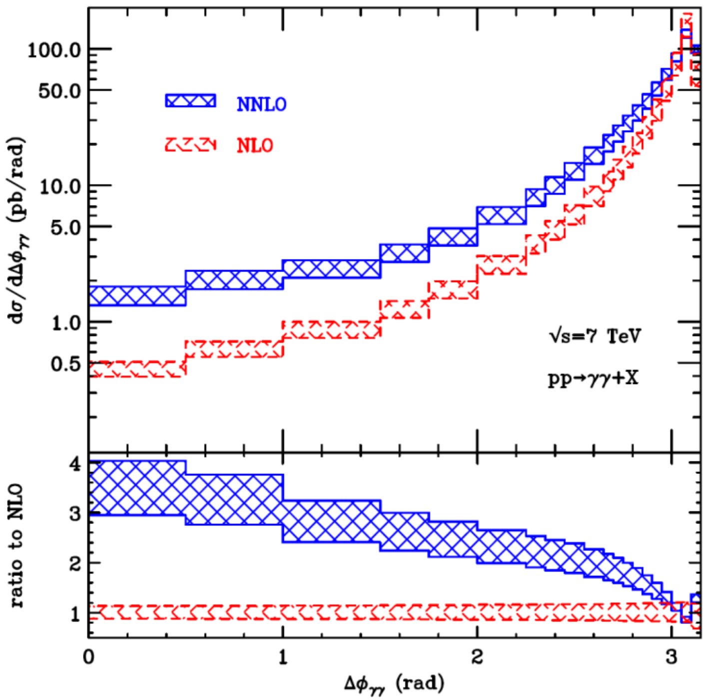
$M_{\gamma\gamma} > 2\, p_T^{\text{cut}}$



LO

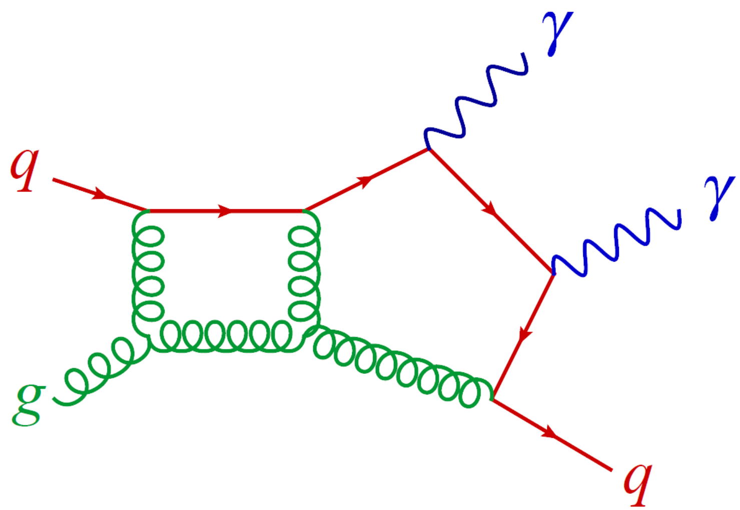


beyond LO



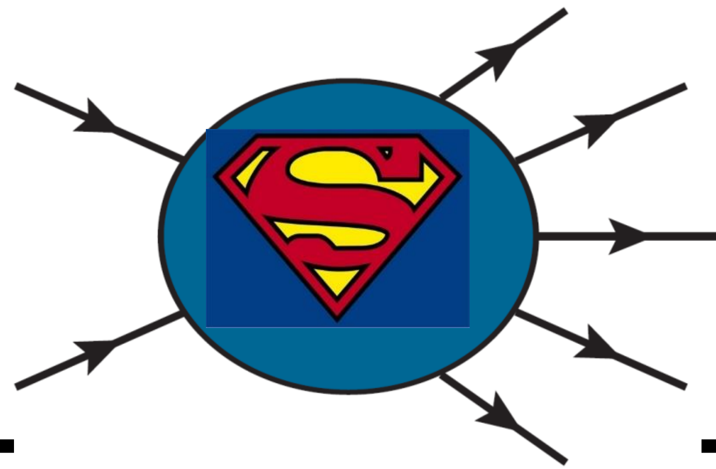
→ "only" need NNLO "$\gamma\gamma$ + jet" to fix

- Good application for ~ first(?) 2 → 3 two-loop amplitudes?



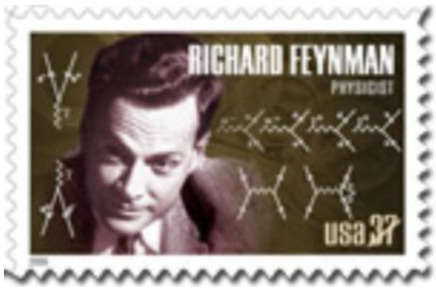
- IR subtractions no harder than processes that have already been done.

Short-distance cross sections built out of scattering amplitudes, ***S*-matrix elements**

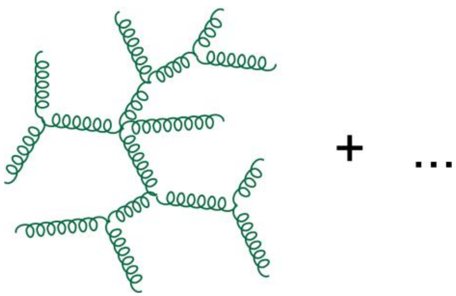
$$\sigma = \int \Bigg[$$  $$\Bigg]$$

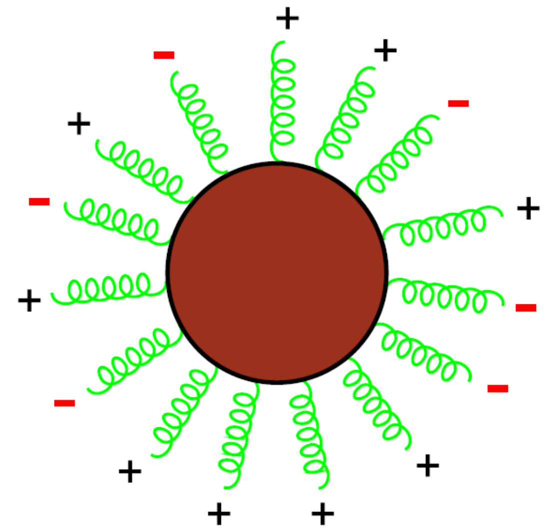
Rontsch lectures [Badger + Tancredi lectures]

Granularity vs. Fluidity



Badger lectures

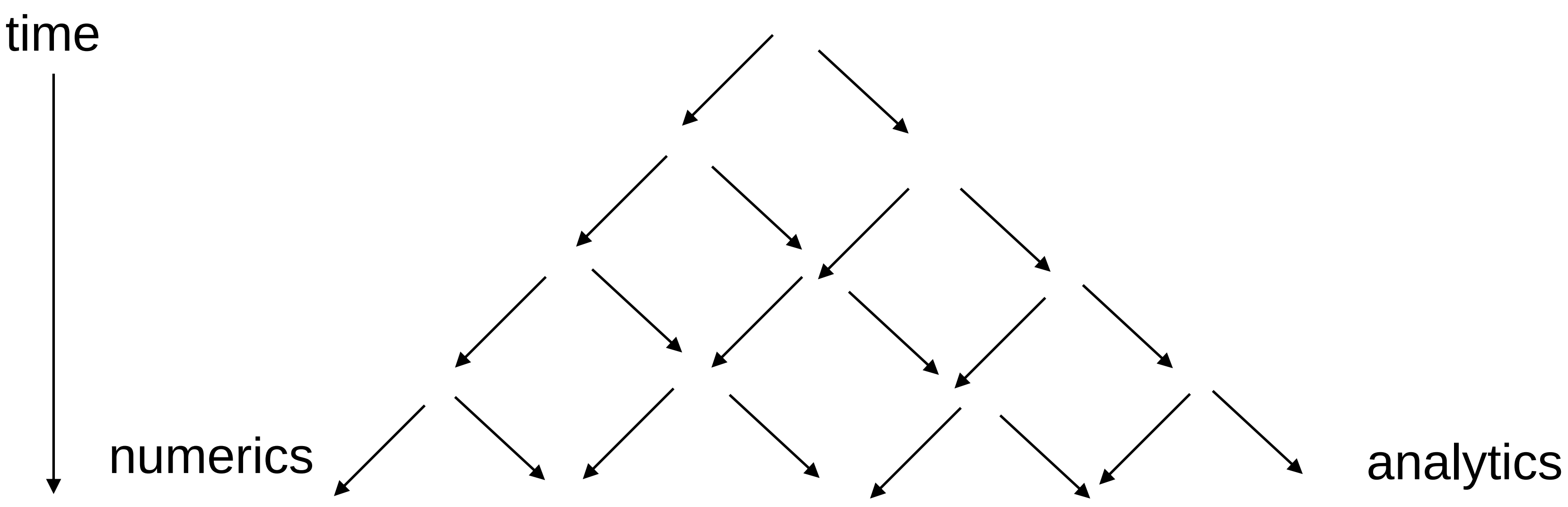




Future of QCD@collider theory?

Prediction:

- Numerics will get “more numerical”
- Analytics will get “more analytical”



Possible future of numerical approaches

- Computation is getting much cheaper
- Still, integrals needed at state-of-art, for either fixed-order, or high-precision resummation, or matched Monte Carlos, are getting more difficult.
- Find better ways to integrate approximate integrands?

Possible future of numerical approaches (cont.)

- With all the analytic and numerical knowledge we have accumulated, can machine learning be used in pQCD?
- For example, to predict approximate amplitudes?
- Already being used in jet substructure analyses to optimize separation of new physics from QCD.

Possible future of analytical approaches

- More sophisticated mathematics for complicated loop integrals [Tancredi lectures, L. B. Chen talk,…]
- Polylogarithms → elliptic integrals → hyperelliptic integrals → …
- Lots of interesting math including algebraic geometry and number theory.

Multiple Zeta Values (MZVs)

- Classical polylogs

evaluate to Riemann zeta values

$$\mathrm{Li}_n(u) = \int_0^u \frac{dt}{t} \mathrm{Li}_{n-1}(t) = \sum_{k=1}^{\infty} \frac{u^k}{k^n}$$

$$\mathrm{Li}_n(1) = \sum_{k=1}^{\infty} \frac{1}{k^n} = \zeta(n) \equiv \zeta_n$$

- HPL's evaluate to nested sums called multiple zeta values (MZVs):

$$\zeta_{n_1,n_2,\ldots,n_m} = \sum_{k_1>k_2>\cdots>k_m>0}^{\infty} \frac{1}{k_1^{n_1} k_2^{n_2} \cdots k_m^{n_m}}$$

Weight $n = n_1 + n_1 + \ldots + n_m$

- MZV's obey many identities, e.g. stuffle

$$\zeta_{n_1}\zeta_{n_2} = \zeta_{n_1,n_2} + \zeta_{n_2,n_1} + \zeta_{n_1+n_2}$$

- All reducible to Riemann zeta values until weight 8. Irreducible MZVs: $\zeta_{5,3}, \zeta_{7,3}, \zeta_{5,3,3}, \zeta_{9,3}, \zeta_{6,4,1,1}, \ldots$

Possible future of analytical approaches (cont.)

- Use N=4 super-Yang Mills theory as a guide to the "most complicated" (highest weight, or leading transcendentality) part of the problem.
- This is more useful when the problem is soft-gluon dominated.
- E.g. constants entering resummation formulae [Monni lectures].

Why N=4 super-Yang-Mills theory?

- Most supersymmetric theory possible without gravity
- Uniquely specified by local internal symmetry group
  – *e.g.*, number of colors N_c for $SU(N_c)$
- Exactly scale-invariant (conformal) field theory:
  for any coupling g, $\beta(g) = 0$
- Similar IR behavior to pQCD
- Uniform weight for iterated integrals [Tancredi lectures]

weight = $2L$ at L loops (for ε^0 terms)

[Also connected to gravity and/or string theory by AdS/CFT correspondence, weak/strong duality]

N=4 SYM particle content

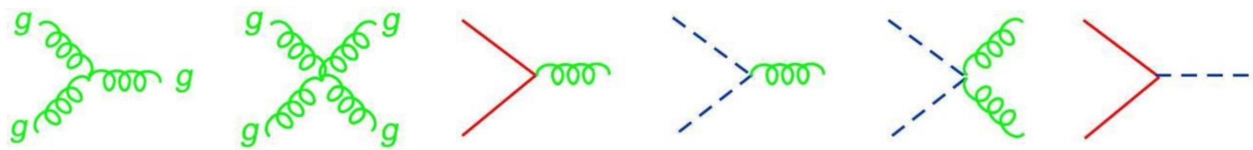
massless spin 1 gluon
4 massless spin 1/2 gluinos
6 massless spin 0 scalars

SUSY
Q_a, a=1,2,3,4
shifts helicity
by 1/2 $\longleftrightarrow$

$\mathcal{N}=4$	$1 \longleftrightarrow$	$4 \longleftrightarrow$	$6 \longleftrightarrow$	$4 \longleftrightarrow$	1
	g^-	$\lambda^-_{\bar{i}}$	$\bar{\phi}_{\bar{i}\bar{j}}, \phi_{ij}$	λ^+_i	g^+
helicity	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1

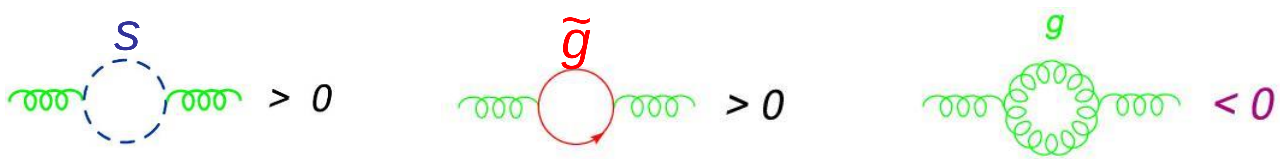
all in adjoint representation

N=4 SYM interactions



All proportional to same *dimensionless* coupling constant, *g*

- SUSY cancellations: scale invariance *preserved* quantum mechanically



$$\frac{d\alpha}{d\ln\mu^2} = \beta(\alpha) = \left[6\times\frac{1}{6} + 4\times\frac{2}{3} - \frac{11}{3}\right]\frac{N_c\,\alpha^2}{4\pi} = 0 \quad \text{(true to all orders in } \alpha\text{)}$$

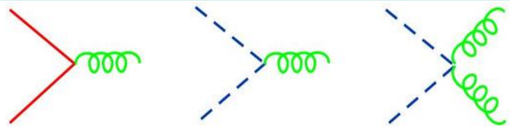
Just the beginning of N=4 “miracles”

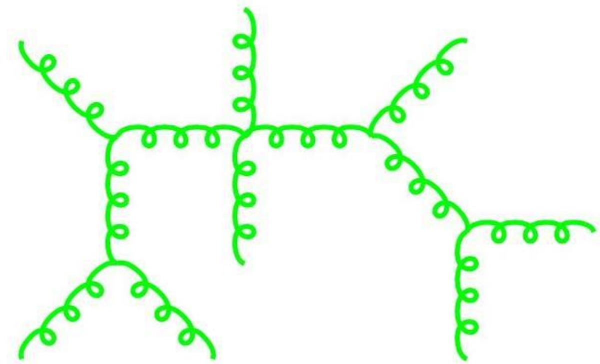
How are QCD and N=4 SYM related?

At tree level they are essentially identical

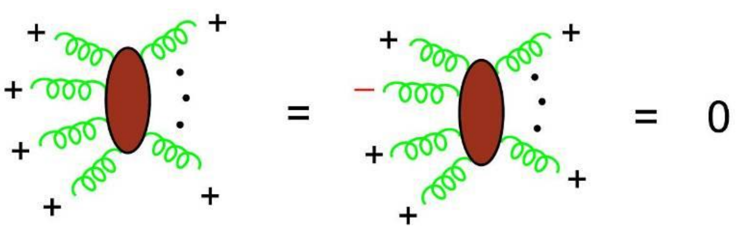
Consider a tree amplitude for *n* gluons.

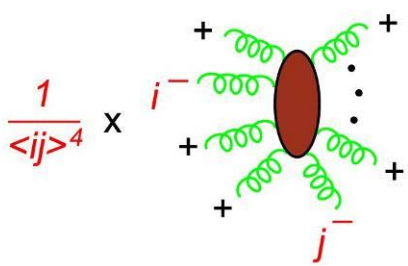
Fermions and scalars cannot appear because they are produced in pairs





Hence the amplitude is the **same** in QCD and N=4 SYM.
So the QCD tree amplitude "secretly" obeys all identities of N=4 supersymmetry:





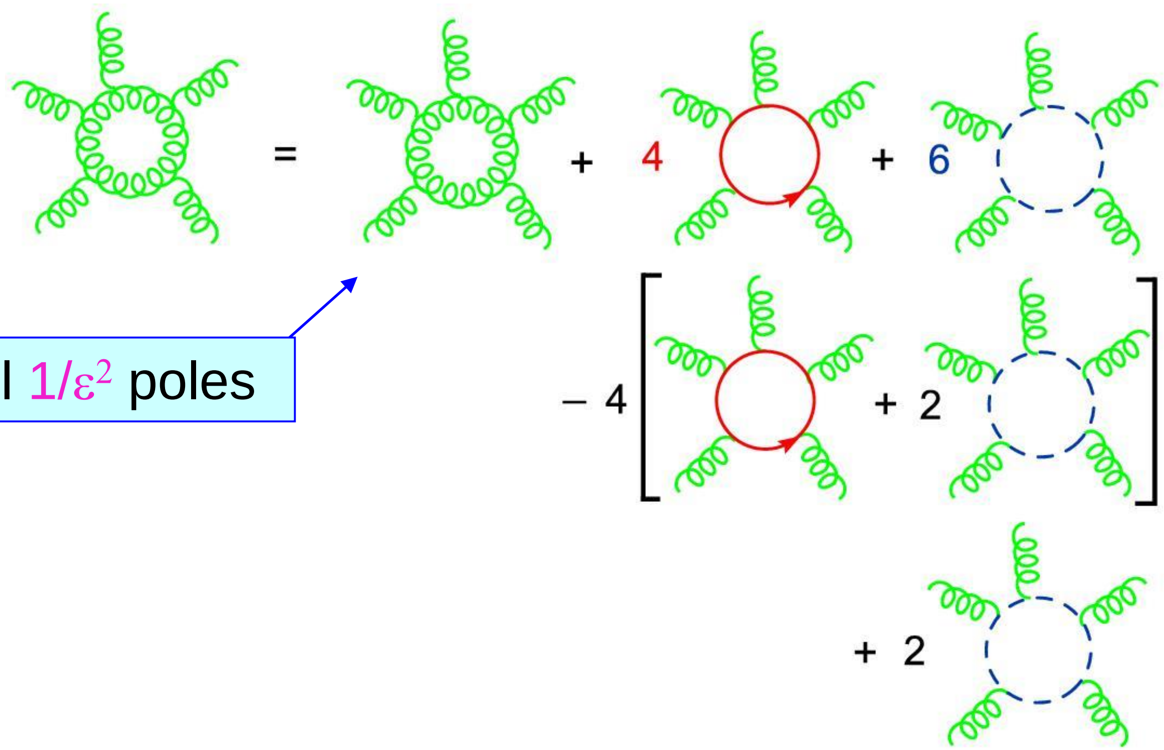
$$\frac{\langle i\, j\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\cdots\langle n\,1\rangle}$$

independent of *i,j*

At loop level, QCD and N=4 SYM differ

However, it is profitable to rearrange the QCD computation to exploit supersymmetry

gluon loop



N=4 SYM

contains all $1/\varepsilon^2$ poles

N=1 multiplet

scalar loop
--- no SUSY,
but also no
spin tangles

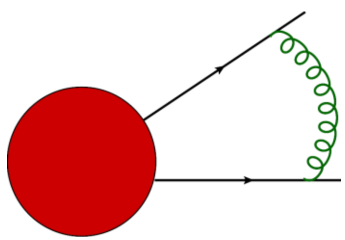
Cusp anomalous dimension γ_K

- Leading behavior of DGLAP splitting kernels as $x \to 1$,

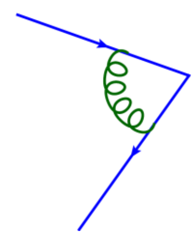
$$P_{ii}(x) \;=\; \frac{\gamma_K}{2(1-x)_+} + B_i\,\delta(1-x) + \ldots$$

$$\gamma_K = C_i \sum_{L=1}^{\infty} \gamma_K^{(L)} \left(\frac{\alpha_s}{4\pi}\right)^L$$

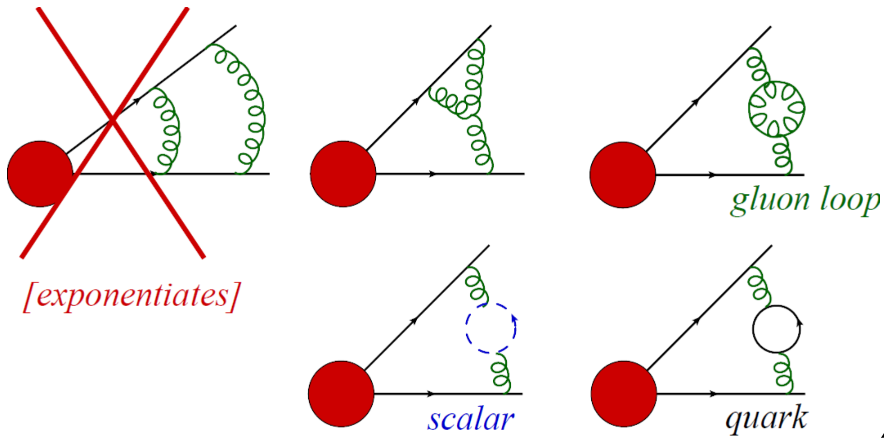
- Appears in all soft-gluon resummation at higher orders in 1/*L* expansion [Monni lectures]
- Soft radiation ←→ Wilson line expectation value
- One loop



$$\gamma_K^{(1)} = 8$$



Two loop cusp anomalous dimension



Needed for 2-loop CMW scheme [Hoeche lectures]

$$\gamma_K^{(2)} = 16\left[C_A\left(\frac{67}{18} - \zeta_2\right) - \frac{5}{9}n_f - \frac{2}{9}n_s\right]$$

$n_f = 4$, $n_s = 6$, and $\overline{MS} \Rightarrow \overline{DR}$ $[16C_A(-1/6)]$

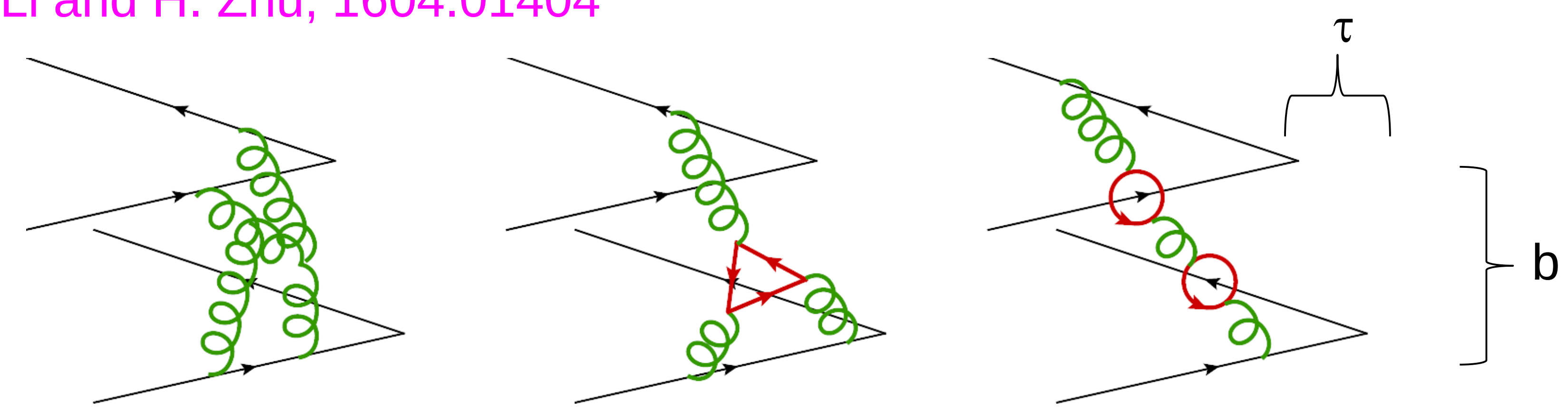
$$\Rightarrow \gamma_K^{(2),\mathcal{N}=4\ \text{SYM}} = 16C_A(-\zeta_2)$$

"leading transcendental part" of QCD result [Kotikov, Lipatov, Onishchenko, Velizhanin]

Similar results for many other soft/collinear quantities

- E.g. rapidity anomalous dimension and soft functions at 3 loops for Higgs boson p_T resummation

Y. Li and H. Zhu, 1604.01404



N=4 SYM ←→ leading transcendental part of QCD result

$$S^{\mathrm{F.D.}}_{3,\mathcal{N}=4}\Big|_{\mu=\nu} = c^s_{3,\mathcal{N}=4} + N_c^3\Big(16\zeta_2 H_4 + 48\zeta_2 H_{2,2} + 64\zeta_2 H_{3,1} + 96\zeta_2 H_{2,1,1} + 120\zeta_4 H_2 + 48 H_6 + 24 H_{2,4} + 40 H_{3,3}$$
$$+ 72 H_{4,2} + 128 H_{5,1} + 16 H_{2,1,3} + 56 H_{2,2,2} + 80 H_{2,3,1} + 80 H_{3,1,2} + 144 H_{3,2,1} + 224 H_{4,1,1}$$
$$+ 64 H_{2,1,1,2} + 96 H_{2,1,2,1} + 160 H_{2,2,1,1} + 256 H_{3,1,1,1} + 192 H_{2,1,1,1,1}\Big)$$

Conclusions

- You have seen this week that pQCD for colliders is a very rich field of study
- It is also incredibly important for the near term understanding of physics at the LHC
- Wide range of skills are needed, to help improve:
  - parton shower accuracy
  - matching parton showers to fixed order
  - fixed order real radiation methods, especially NNLO
  - multi-loop amplitudes numerically
  - multi-loop amplitudes analytically
- Go forth, factorize and evolve!

Thank you!

Thanks to Hua Xing, Kirill, and everyone else who helped to put on such a great school and workshop at West Lake!