Soft interactions of top quarks at NNLO

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* Constructing amplitudes
* Performing loop integrals
* Dealing with IR singularities
* Resummation (analytically or numerically)

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- * Performing loop integrals
- * Dealing with IR singularities
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I'll talk about the application of some of these ideas in the context of top quark physics (with emphasis on soft gluons)

I only have 25 min, so I'll skip technical details

Why top quarks?

naturalness

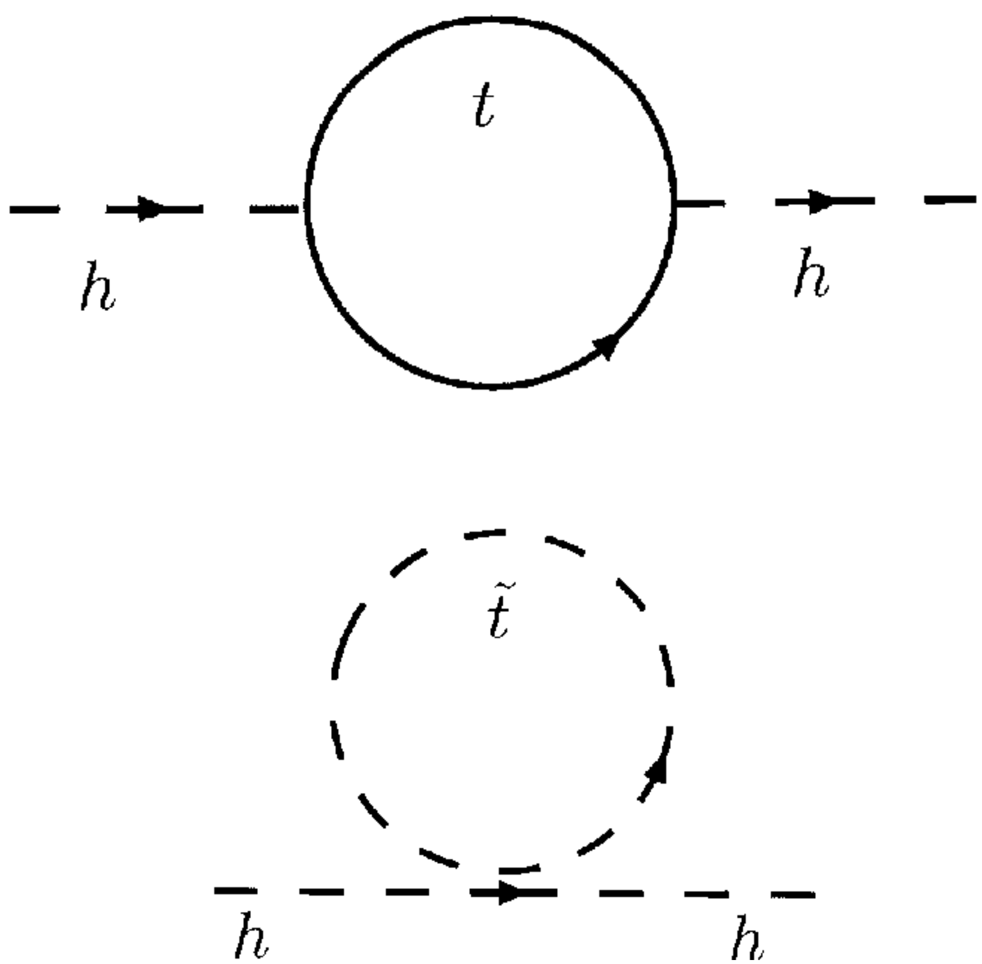
mass

vacuum

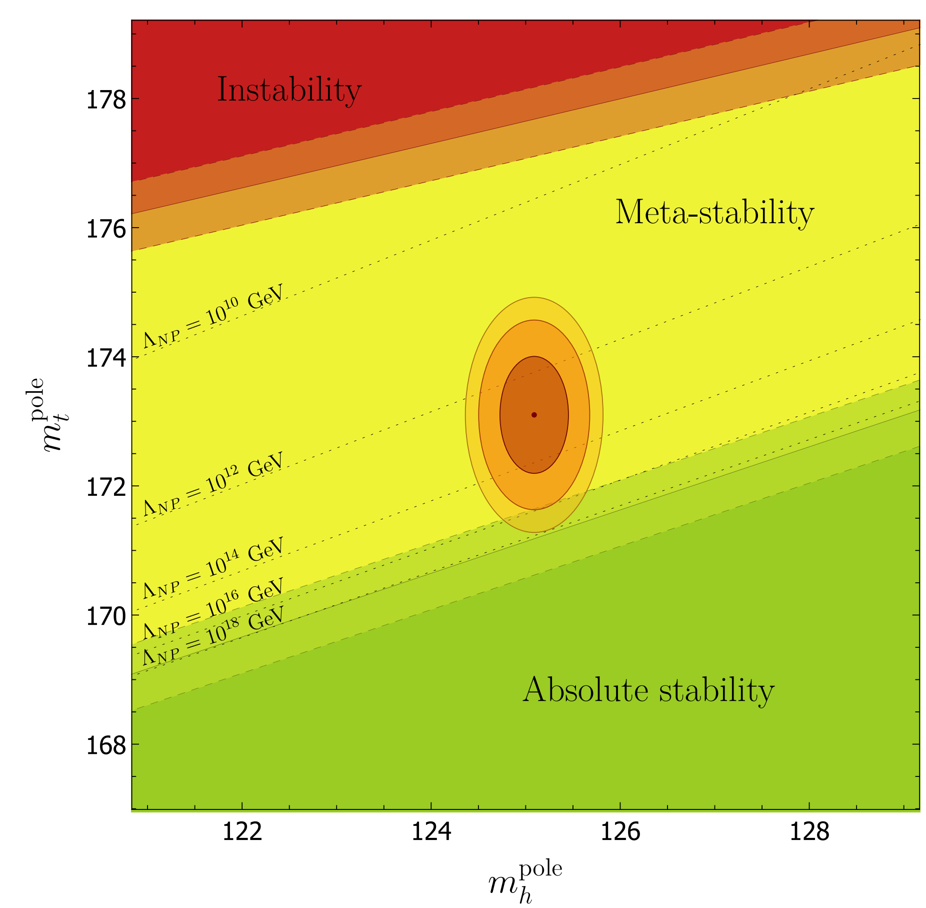
flavor

Why top quarks?

Strongly coupled to Higgs, important to gauge hierarchy problem and vacuum stability



$$\tau_{\rm SM} = 10^{139^{+102}_{-51}} \text{ years}$$

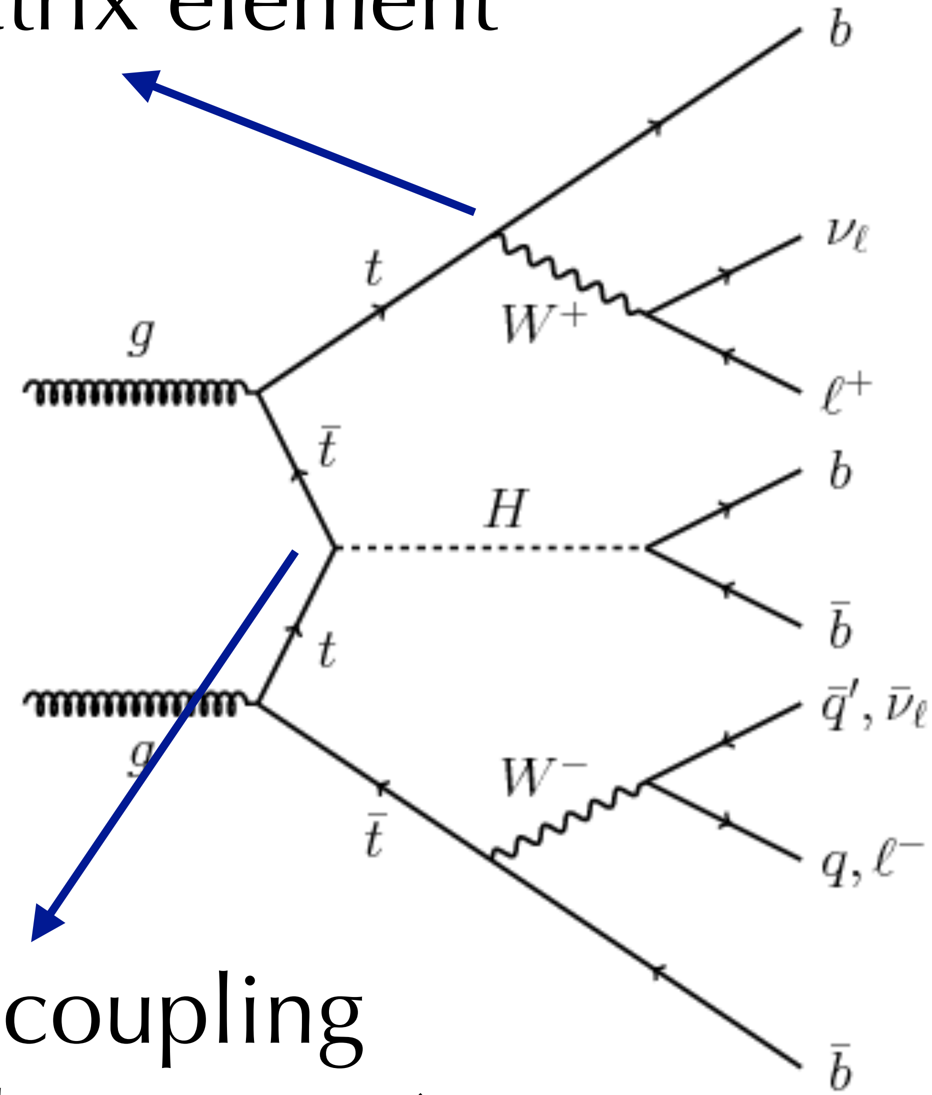


A. Andreassen, W. Frost, M. D. Schwartz: 1707.08124

Why top quarks?

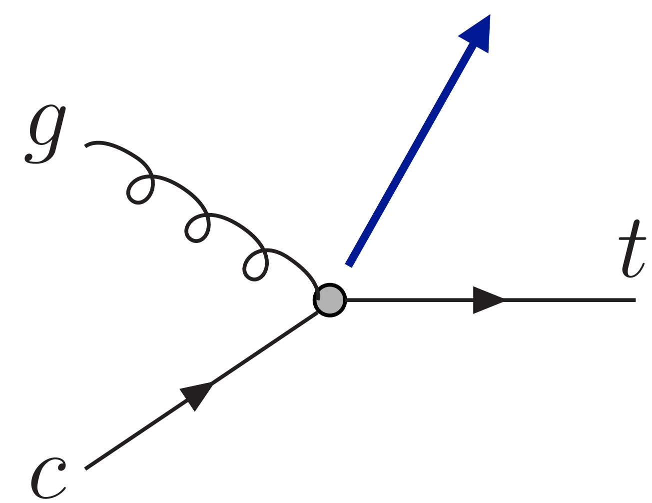
We still have many unknowns about the top quark

CKM matrix element



Yukawa coupling
(origin of large mass)

FCNC?

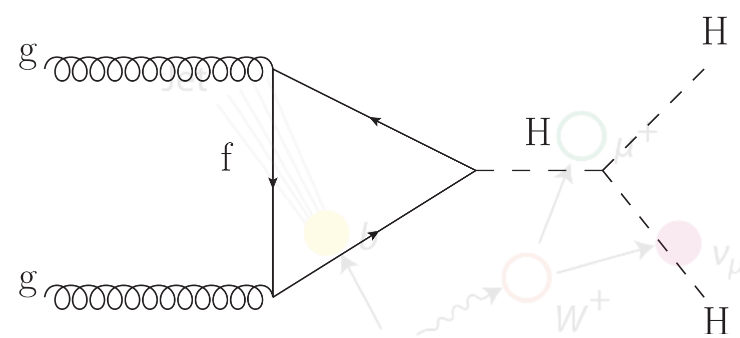


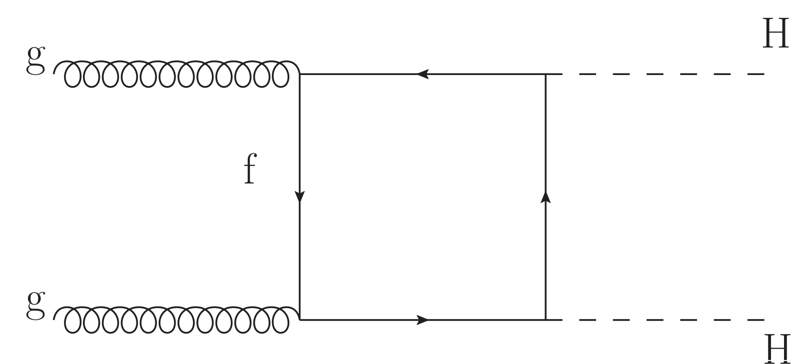
CP violation?

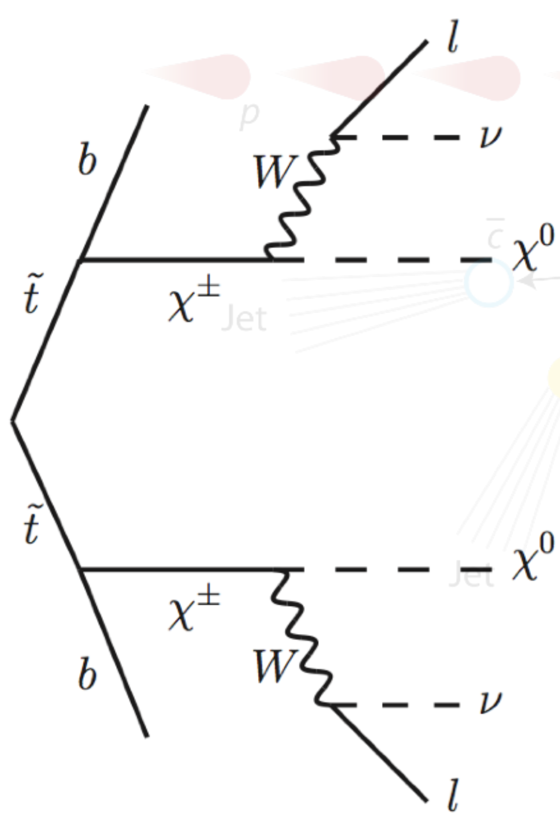
Why top quarks?

Major backgrounds to many searches

Higgs pair production
(Higgs self-coupling)

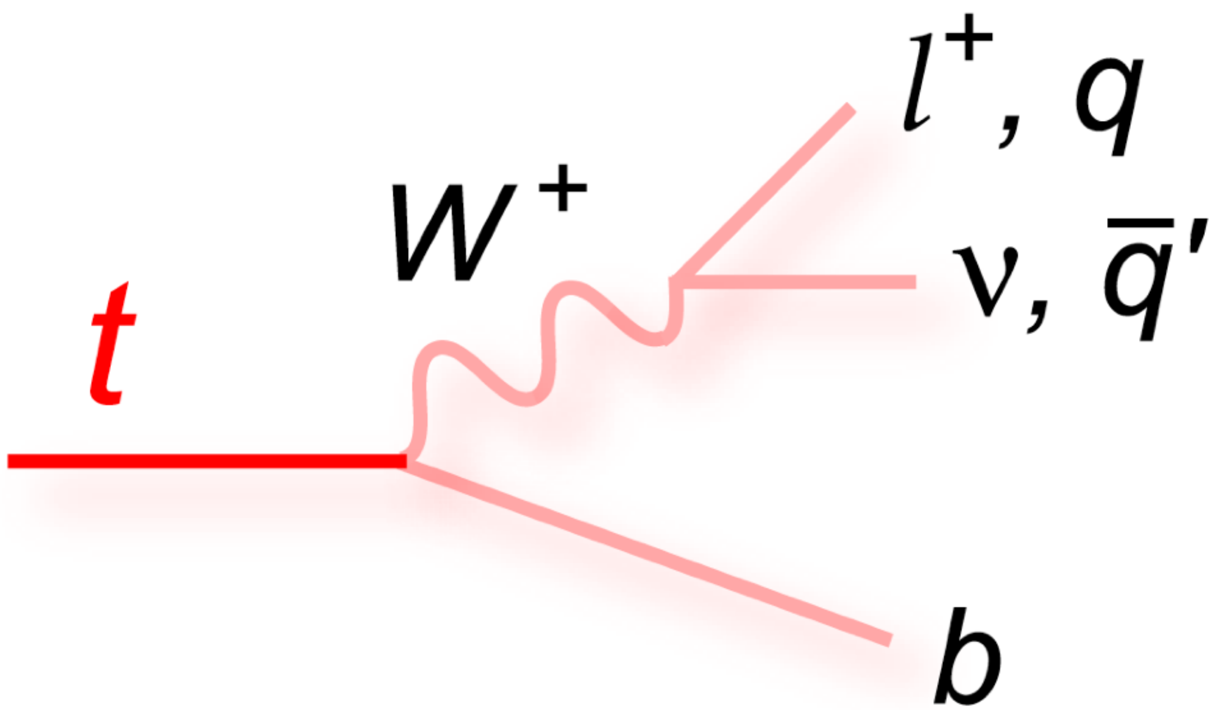






Top partner searches
(SUSY, CHM, ExD, LH)

Why top quarks?



Decay before hadronization

Perfect place to study perturbative QCD!

Why soft gluons?

JULY 15, 1937 PHYSICAL REVIEW VOLUME 52

BN theorem

Note on the Radiation Field of the Electron

F. BLOCH AND A. NORDSIECK*

OURNAL OF MATHEMATICAL PHYSICS VOLUME 3, NUMBER 4 JULY-AUGUST 1962

Mass Singularities of Feynman Amplitudes*

TOICHIRO KINOSHITA

KLN theorem

PHYSICAL REVIEW VOLUME 133, NUMBER 6B 23 MARCH 1

Degenerate Systems and Mass Singularities*

T. D. LEE† AND M. NAUENBERG‡

Intimately related to infrared (IR) divergences in scattering amplitudes (which need to be cancelled in physical observables!)

Why soft gluons?

Soft radiations carry universal information about the field theory (independent of particular scattering processes)

PHYSICAL REVIEW VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Infrared Photons and Gravitons*

STEVEN WEINBERG†

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Low-Energy Theorem for Graviton Scattering

DAVID J. GROSS*
Lyman Laboratory, Harvard University, Cambridge, Massachusetts
AND
ROMAN JACKIW*
Lyman Laboratory, Harvard University, Cambridge, Massachusetts

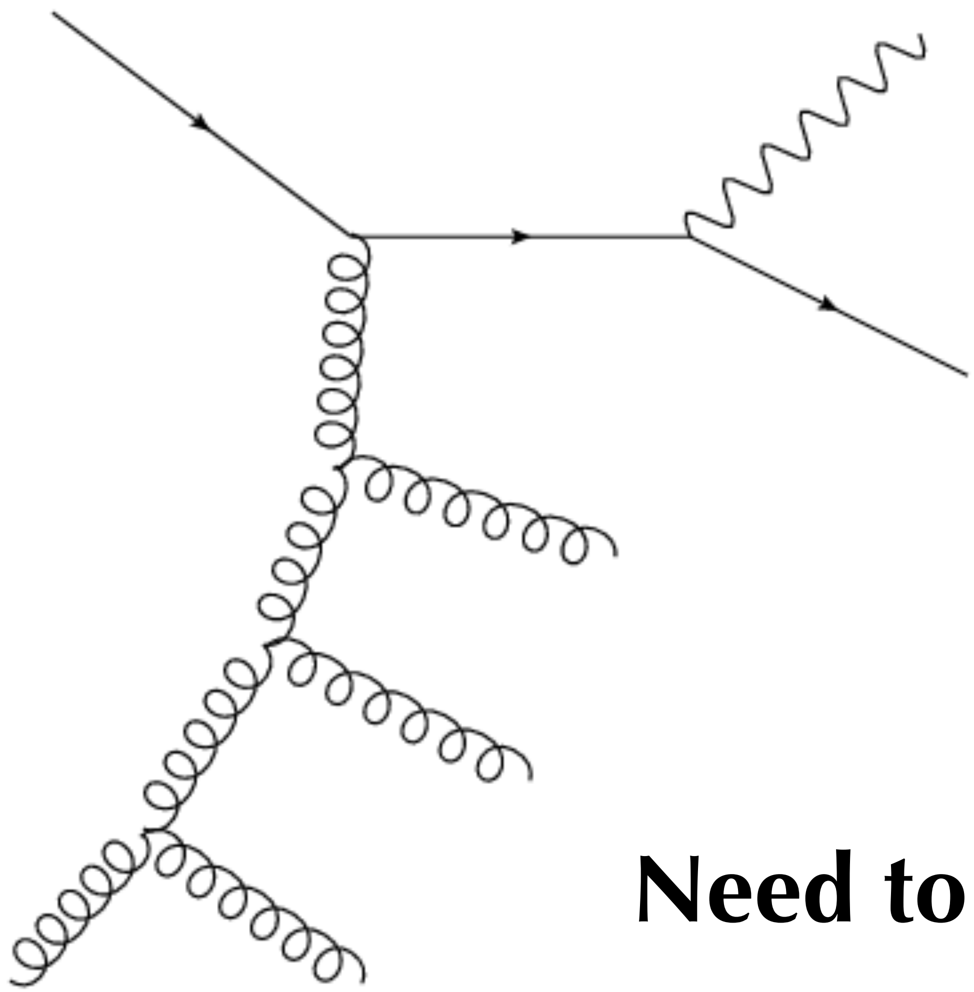
PHYSICAL REVIEW VOLUME 168, NUMBER 5 25 APRIL 1968

Low-Energy Theorems for Massless Bosons: Photons and Gravitons

R. JACKIW*

Why soft gluons?

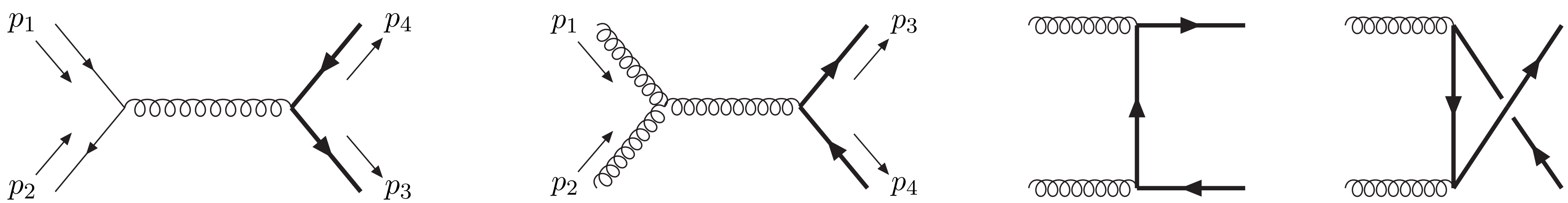
Generate large logarithmic corrections at each order in perturbation theory



Need to be resummed to all orders!

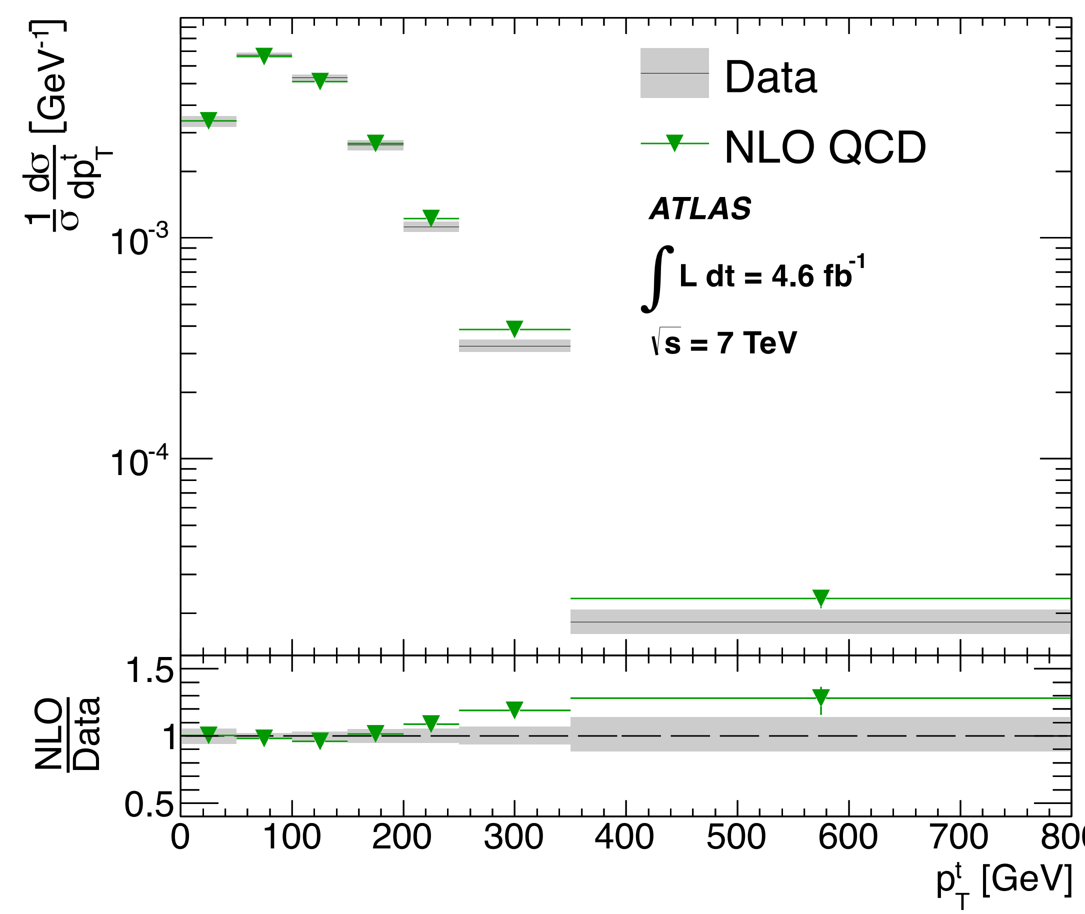
We should have heard enough of these from Dr. Monni

Top pair production



A standard candle for the LHC and future colliders

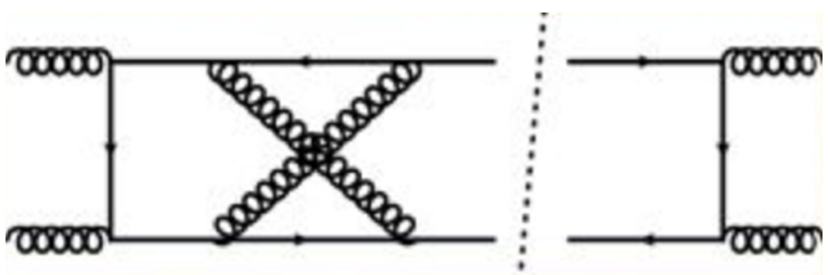
Next-to-next-to-leading order: one of the most wanted theoretical results by LHC experiments



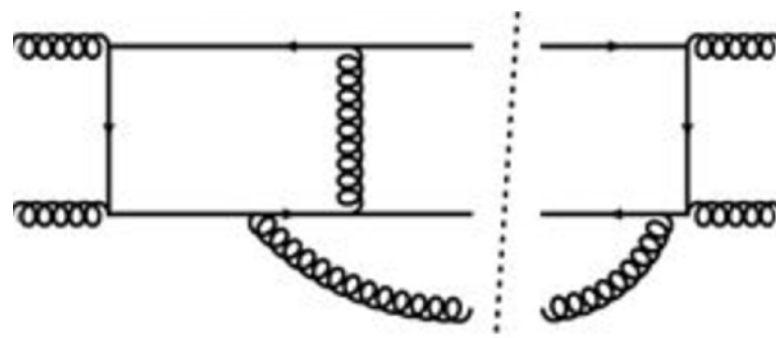
ATLAS Collaboration: 1407.0371

Top pair at NNLO

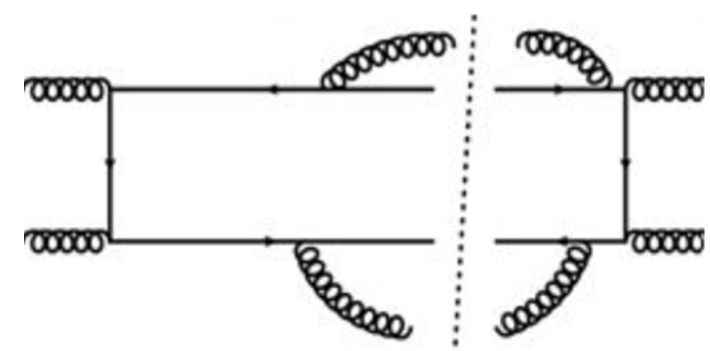
3 ingredients



two-loop virtual



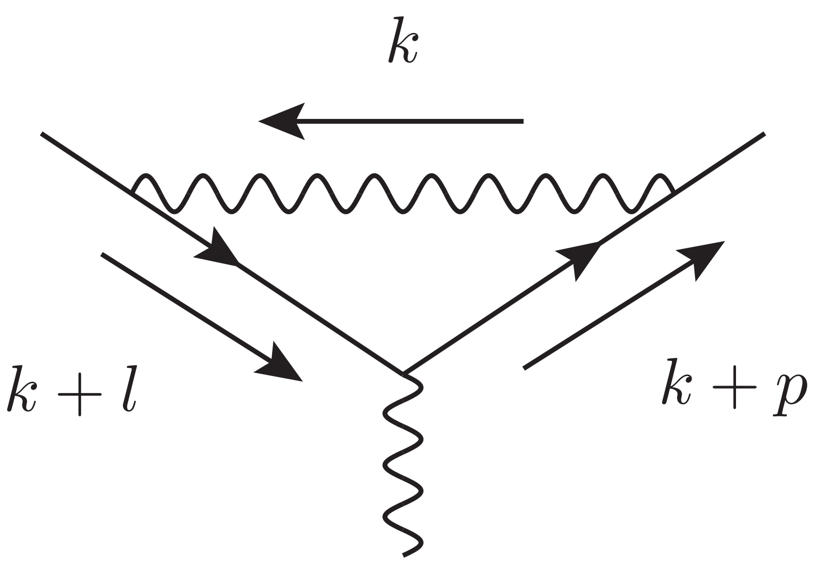
virtual-real



double real

An important issue: understanding IR divergences and their cancellation among the 3 contributions

IR divergences in scattering amplitudes



Collinear divergence $k^\mu \parallel l^\mu$ or $k^\mu \parallel p^\mu$

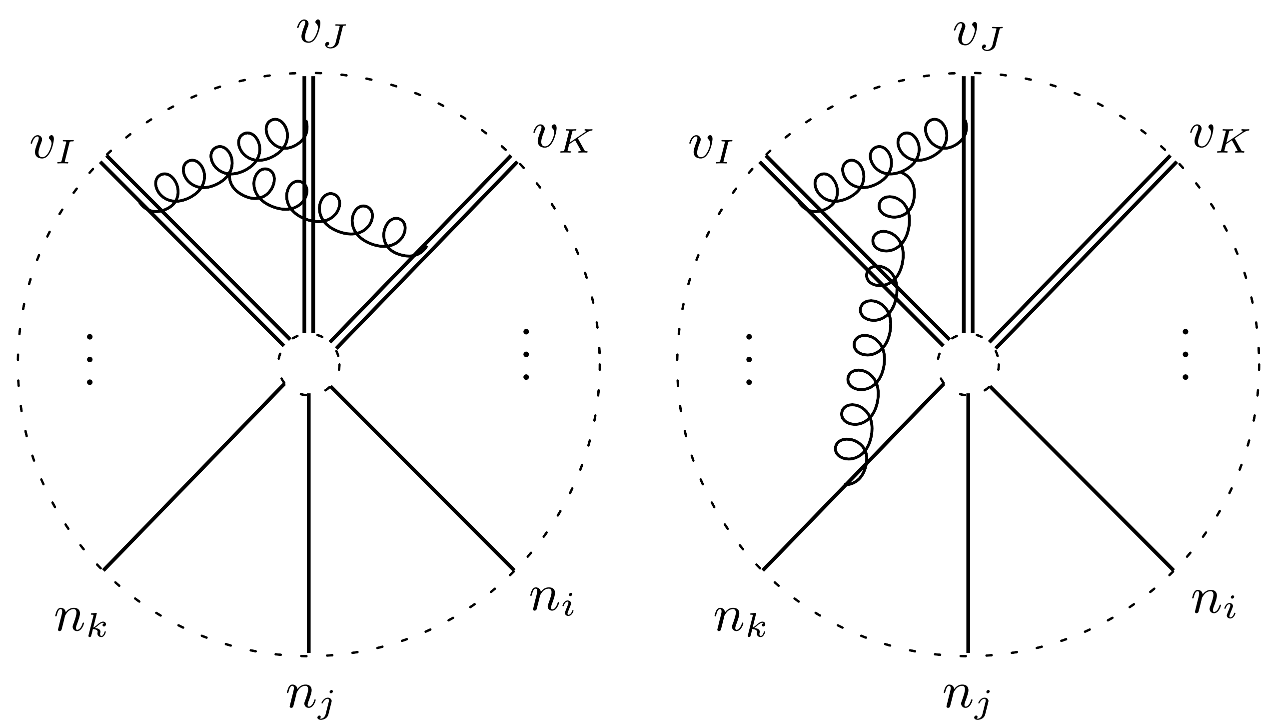
Easy: only knows about one leg

Soft divergence $k^\mu \to 0$

Hard: probes all legs

Wilson lines

$$\boldsymbol{S}_i(x) = \mathcal{P} \exp\left(i g_s \int_{-\infty}^{0} ds\, v_i \cdot A^a(x + s v_i)\, \boldsymbol{T}_i^a \right)$$

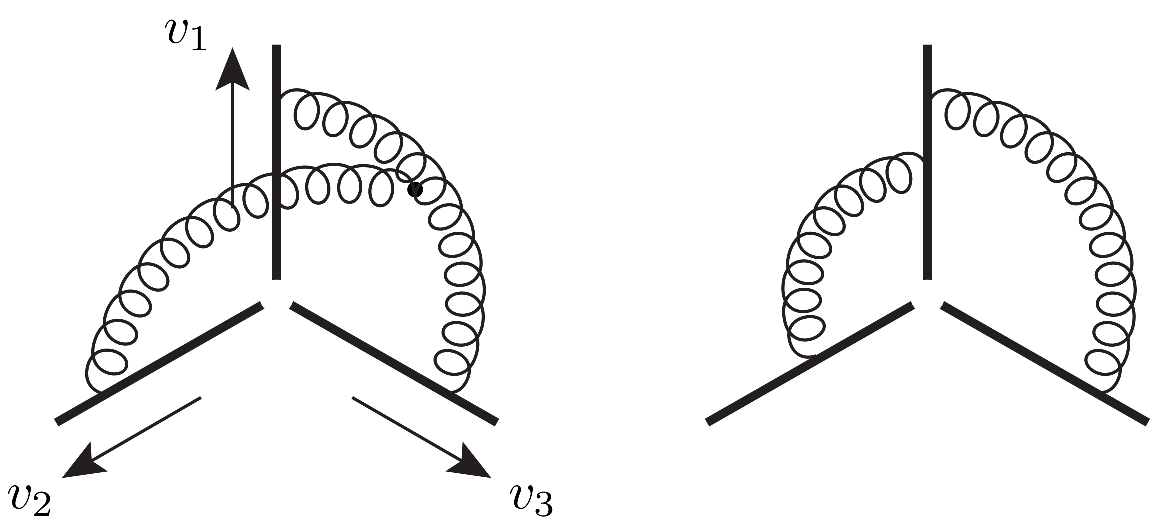


Soft interactions are described by Wilson lines

Two-loop IR massless vs. massive

Purely massless case simple (due to the fact that 3-parton correlations vanish)

Aybat, Dixon, Sterman: hep-ph/0606254



Massive case (top quark) notably more complicated!

Early attempt: Mitov, Sterman, Sung: 0903.3241

Universal two-loop IR

Ferroglia, Neubert, Pecjak, **LLY**: PRL 103, 201601 (2009)

The first derivation of the universal two-loop IR structure

$$\begin{aligned}\boldsymbol{\Gamma} = & \sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2} \gamma_{\rm cusp}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\ & - \sum_{(I,J)} \frac{\boldsymbol{T}_I \cdot \boldsymbol{T}_J}{2} \gamma_{\rm cusp}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) \\ & + \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j \, \gamma_{\rm cusp}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}} \qquad (2 \\ & + \sum_{(I,J,K)} i f^{abc} \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_K^c \, F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI}) \\ & + \sum_{(I,J)} \sum_k i f^{abc} \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_k^c \, f_2\Big(\beta_{IJ}, \ln \frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\Big)\end{aligned}$$

Compact analytic expression (important for precise cancellation)

$$F_1(\beta_{12}, \beta_{23}, \beta_{31}) = \frac{\alpha_s^2}{12\pi^2} \sum_{i,j,k} \epsilon_{ijk} \, g(\beta_{ij}) \, r(\beta_{ki})$$

$$\begin{aligned} r(\beta) &= \beta \coth \beta \,, \\ g(\beta) &= \coth \beta \left[\beta^2 + 2\beta \ln(1 - e^{-2\beta}) - \mathrm{Li}_2(e^{-2\beta}) + \frac{\pi^2}{6} \right] \\ & \quad - \beta^2 - \frac{\pi^2}{6} \,. \qquad (5) \end{aligned}$$

Generalizing Catani's formula to massive cases

Universal two-loop IR

Ferroglia, Neubert, Pecjak, **LLY**:
PRL 103, 201601 (2009)

Any gauge theory
(any gauge group,
SUSY or not)

Any external particles
(massless or massive,
any representation,
boson or fermion)

First application: top quark production in QCD

Two-loop IR for top pair

Ferroglia, Neubert, Pecjak, **LLY**: 0908.3676

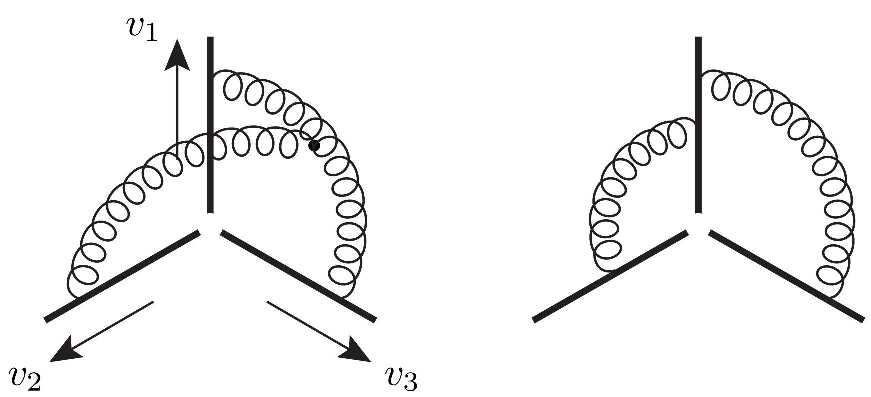
$$2\,\mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle_{gg} = (N^2-1)\Bigg(N^3A^g + N\,B^g + \frac{1}{N}\,C^g + \frac{1}{N^3}\,D^g + N^2n_l\,E_l^g + N^2n_h\,E_h^g + n_l\,F_l^g + n_h\,F_h^g + \frac{n_l}{N^2}\,G_l^g + \frac{n_h}{N^2}\,G_h^g + Nn_l^2H_l^g + Nn_ln_h\,H_{lh}^g + Nn_h^2H_h^g + \frac{n_l^2}{N}\,I_l^g + \frac{n_ln_h}{N}\,I_{lh}^g + \frac{n_h^2}{N}\,I_h^g\Bigg)$$

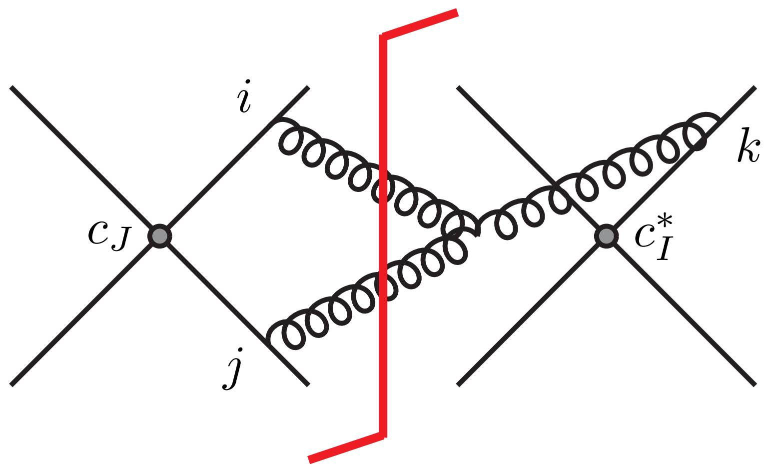
	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}
A^g	10.749	18.694	−156.82	262.15
B^g	−21.286	−55.990	−235.04	1459.8
C^g		−6.1991	−68.703	−268.11
D^g			94.087	−130.96
E_l^g		−12.541	18.207	27.957
E_h^g			0.012908	11.793
F_l^g		24.834	−26.609	−50.754
F_h^g			0.0	−23.329
G_l^g			3.0995	67.043
G_h^g				0.0
H_l^g			2.3888	−5.4520
H_{lh}^g				−0.0043025
H_h^g				
I_l^g			−4.7302	10.810
I_{lh}^g				0.0
I_h^g				

Table 1: Numerical results for the IR poles in the color coefficients (65) for top-quark pair production in the $gg \to t\bar{t}$ channel, evaluated at the point $t_1 = -0.45s$, $s = 5m_t^2$, and $\mu = m_t$. The blank entries are not present in general, while the entries with value 0.0 vanish only for the particular choice $\mu = m_t$.

Fully analytic results given: important ingredient for the NNLO calculation!

Soft gluons in cross sections

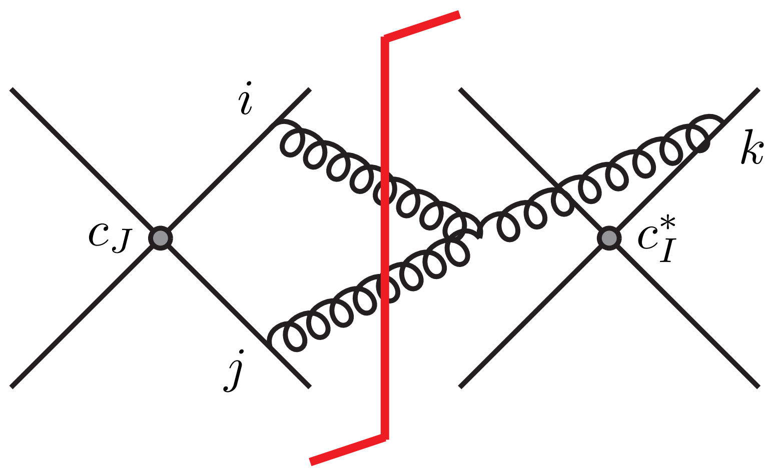




Soft gluons in amplitudes → IR divergences

Soft gluons in cross sections → Soft functions

NNLO soft function



Only calculated in the massless limit

Ferroglia, Pecjak, **LLY**: 1207.4798

Not really heavy quarks...

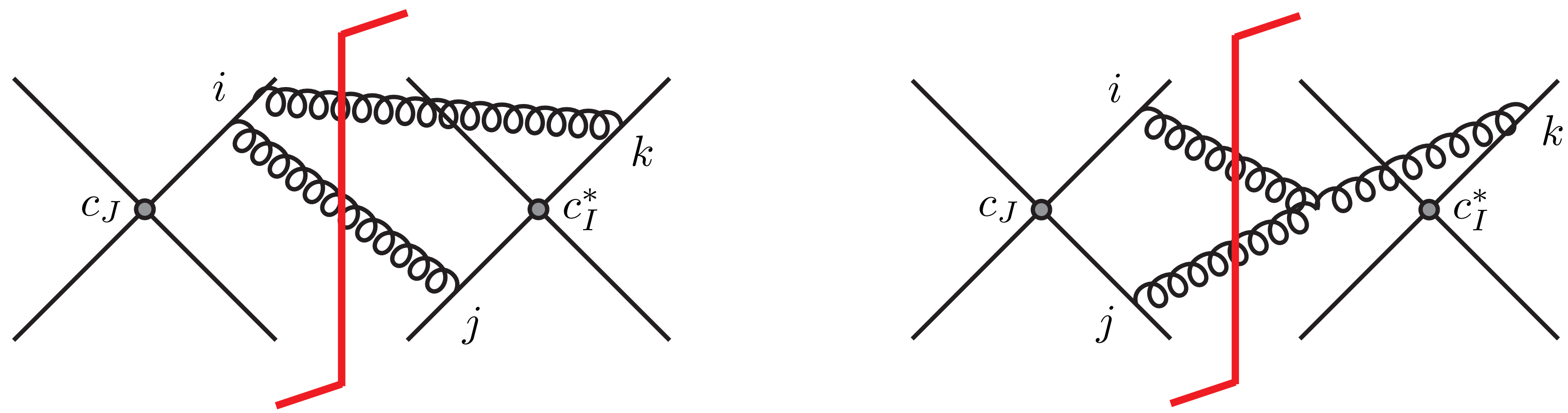
Why do we care?

→ Structure of soft real emissions (in particular, 3-parton correlations)

→ Higher accuracy in resummation

NNLO soft function

Wang, Xu, **LLY**, Zhu: 1804.xxxxx



Rule of thumb: finite piece significantly more difficult than divergent piece

Requires more systematic methods!
→ Integration-by-parts identities
→ Differential equations

We should have heard enough of these from Dr. Tancredi

Integration-by-parts

$$\int \ldots \int \mathrm{d}^d k_1 \mathrm{d}^d k_2 \ldots \frac{\partial}{\partial k_i}\left(p_j \frac{1}{E_1^{a_1} \ldots E_n^{a_n}}\right) = 0$$

⬇

Leads to relations among different Feynman integrals

⬇

Significantly reduces number of integrals to compute
("master integrals")

Chetyrkin, Tkachov: NPB 192, 159 (1981)

Differential equations

The "master integrals" form a basis in the space of all integrals

They satisfy a system of linear differential equations

Kotikov: PLB 254, 158 (1991); Remiddi: hep-th/9711188

New development: "canonical form" greatly simplifying the solution (when applicable)

Henn: 1304.1806

Differential equations

Henn: 1412.2296

$$G_{a_1,a_2,a_3,a_4} = \int \frac{d^D k}{i\pi^{D/2}} \frac{1}{[-k^2]^{a_1}[-(k+p_1)^2]^{a_2}[-(k+p_1+p_2)^2]^{a_3}[-(k+p_1+p_2+p_3)^2]^{a_4}}$$

$$p_i^2 = 0 \qquad s = (p_1+p_2)^2 \text{ and } t = (p_2+p_3)^2$$

$$\begin{aligned} g_1 =& c(-s)^\epsilon t G_{0,1,0,2}\,, \\ g_2 =& c(-s)^\epsilon s G_{1,0,2,0}\,, \\ g_3 =& c\epsilon(-s)^\epsilon st G_{1,1,1,1}\,, \end{aligned}$$

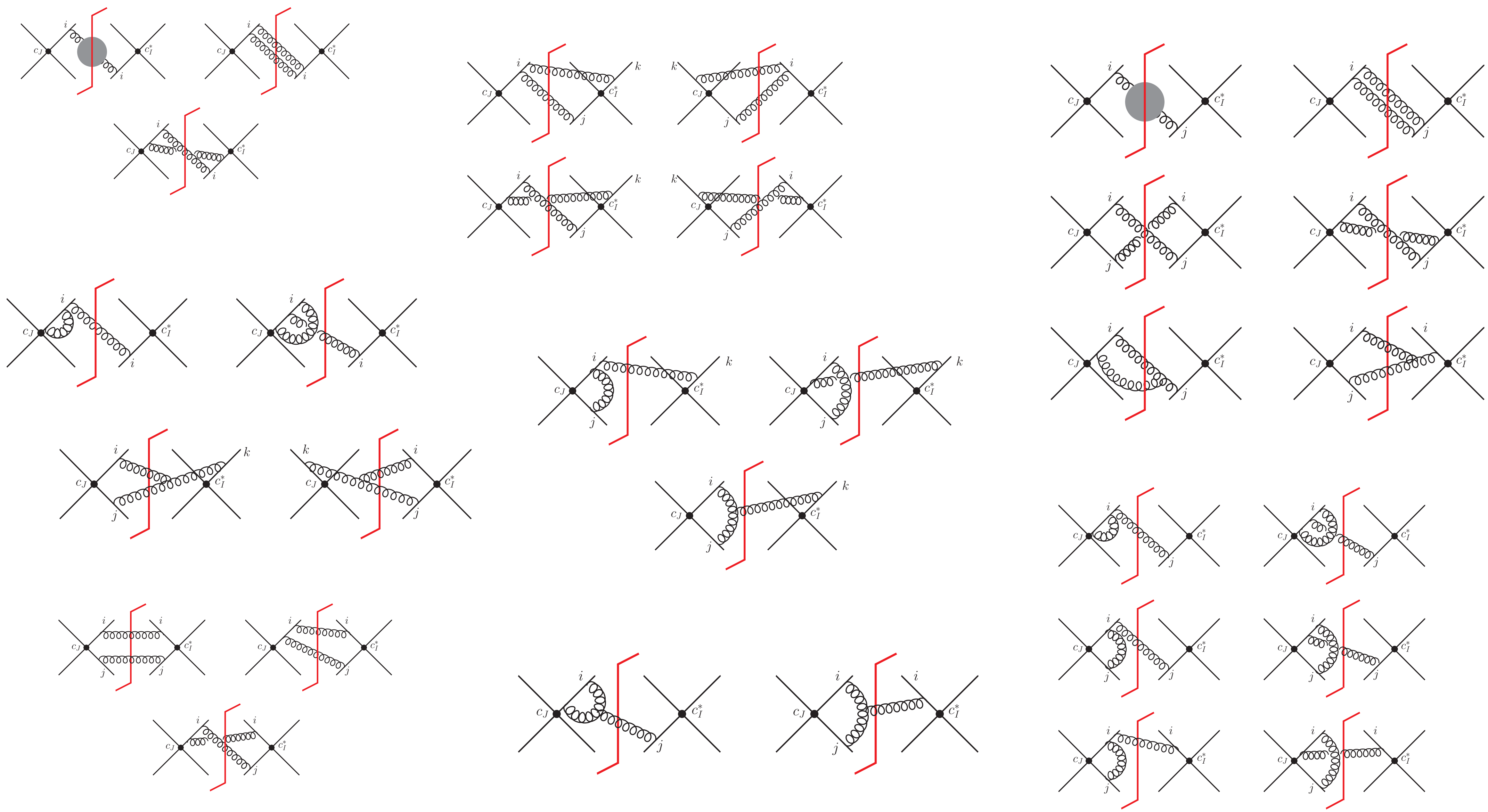
$$\partial_x \vec{g}(x;\epsilon) = \epsilon \left[\frac{a}{x} + \frac{b}{1+x}\right] \vec{g}(x,\epsilon)$$

$$a = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & -1 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 2 & 1 \end{pmatrix}$$

Solution in terms of iterated integrals order by order in ε

NNLO diagrams

Wang, Xu, **LLY**, Zhu: 1804.xxxxx



Solving integrals

Wang, Xu, **LLY**, Zhu: 1804.xxxxx

~60 master integrals

Differential equations

matrices

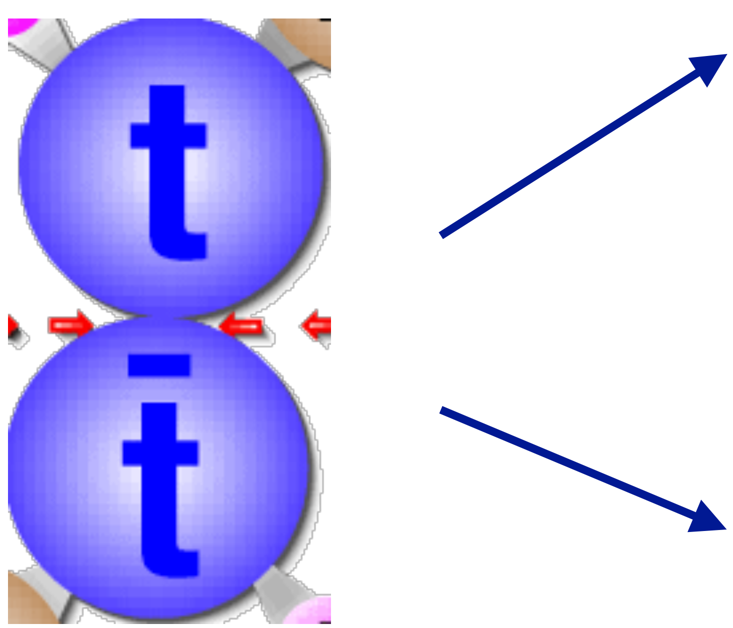
$$\partial_\beta \vec{f}(\epsilon, \beta, \cos\theta) = \epsilon \left(\frac{A}{\beta - 1} + \frac{B}{\beta} + \frac{C}{\beta + 1} + \frac{D}{\beta - 1/\cos\theta} + \frac{E}{\beta + 1/\cos\theta} \right) \vec{f}(\epsilon, \beta, \cos\theta)$$

Solution in terms of generalized polylogarithms

Difficult part: boundary conditions

Validation: threshold limit

It is interesting to check the threshold limit where the top quarks are produced at rest



Color singlet: same as Drell-Yan and Higgs production

Belitsky: hep-ph/9808389

Color octet Czakon, Fiedler: 1311.2541

Validation: boosted limit

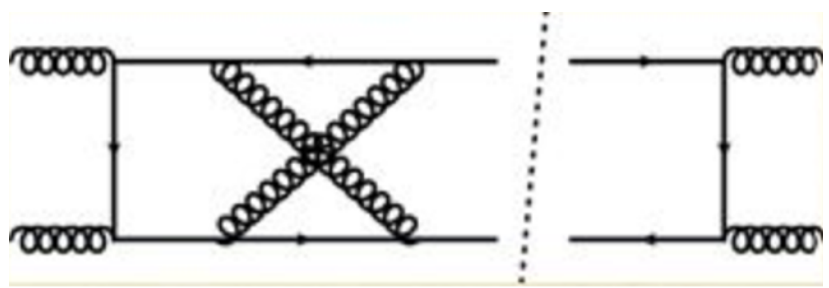
In the limit where the top quarks are highly boosted

Factorization   Ferroglia, Pecjak, **LLY**: 1205.3662

$$S_{\text{massive}}(s,t,m_t) \to S_{\text{massless}}(s,t)\, S_D^2(m_t)$$

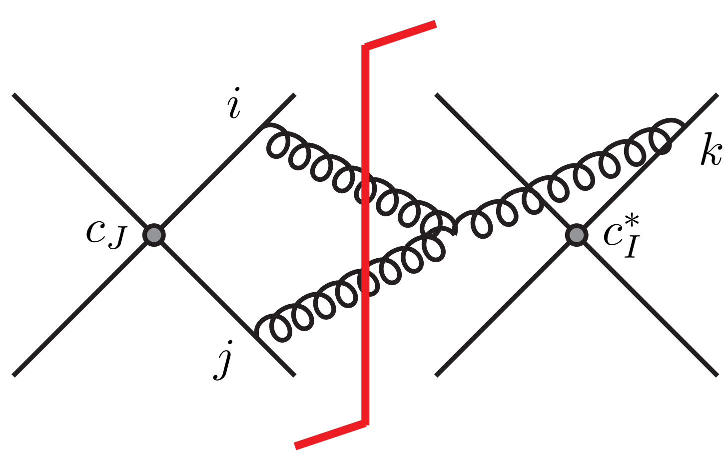
soft fragmentation function

Soft gluon resummation

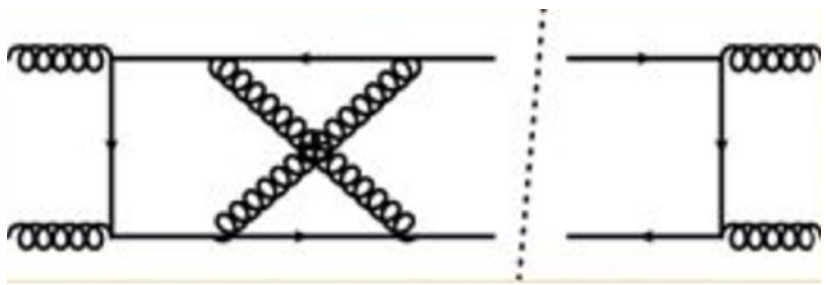


Resummation is achieved by evolving from the scale of hard scatterings

to the scale of soft (and/or collinear) interactions



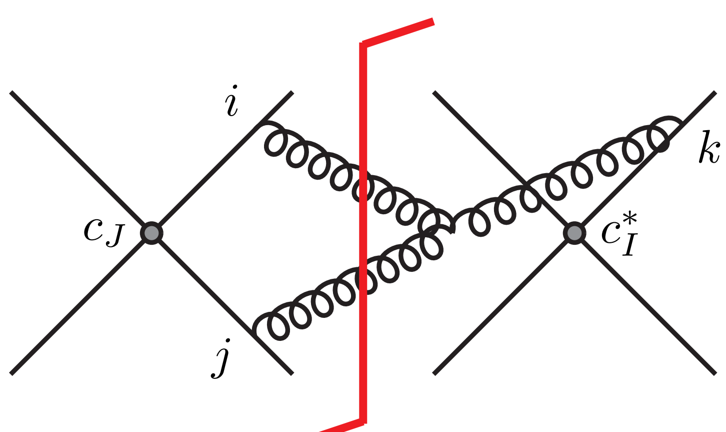
Soft gluon resummation



Resummation is achieved by evolving from the scale of hard scatterings

Governed by IR structure

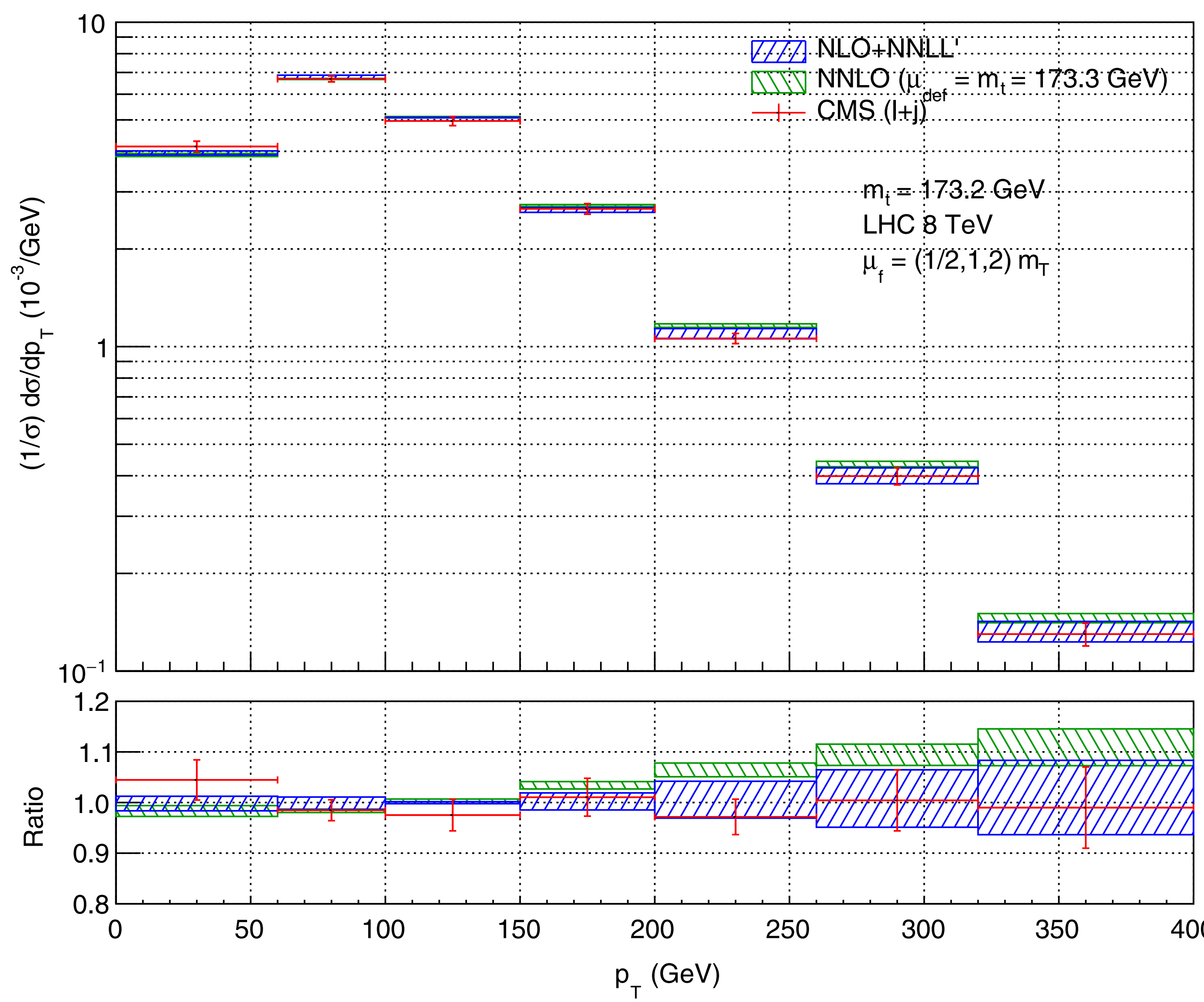
to the scale of soft (and/or collinear) interactions



NLO+NNLL′ (boosted) top

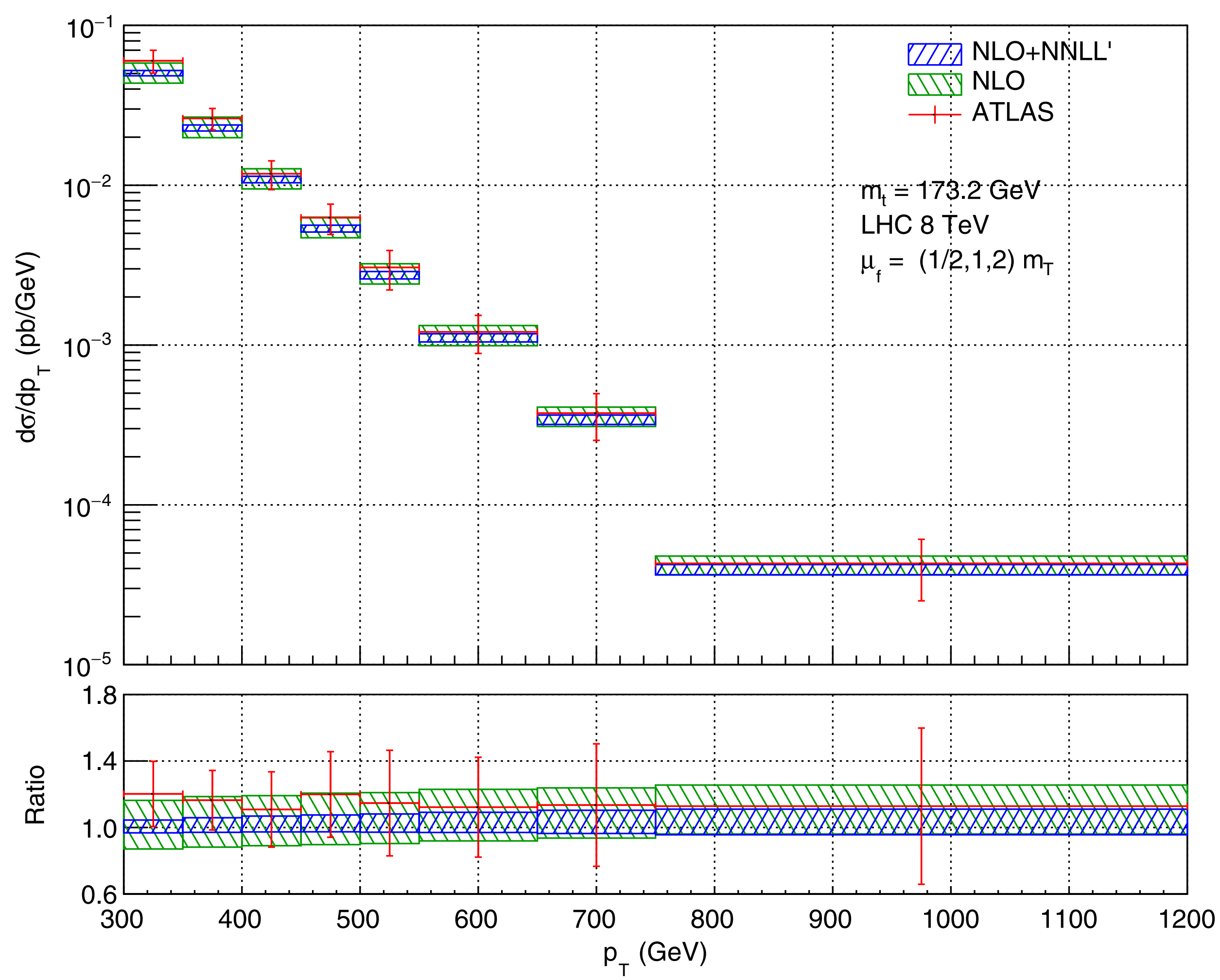
Pecjak, Scott, Wang, **LLY**: PRL 116, 202001 (2016)

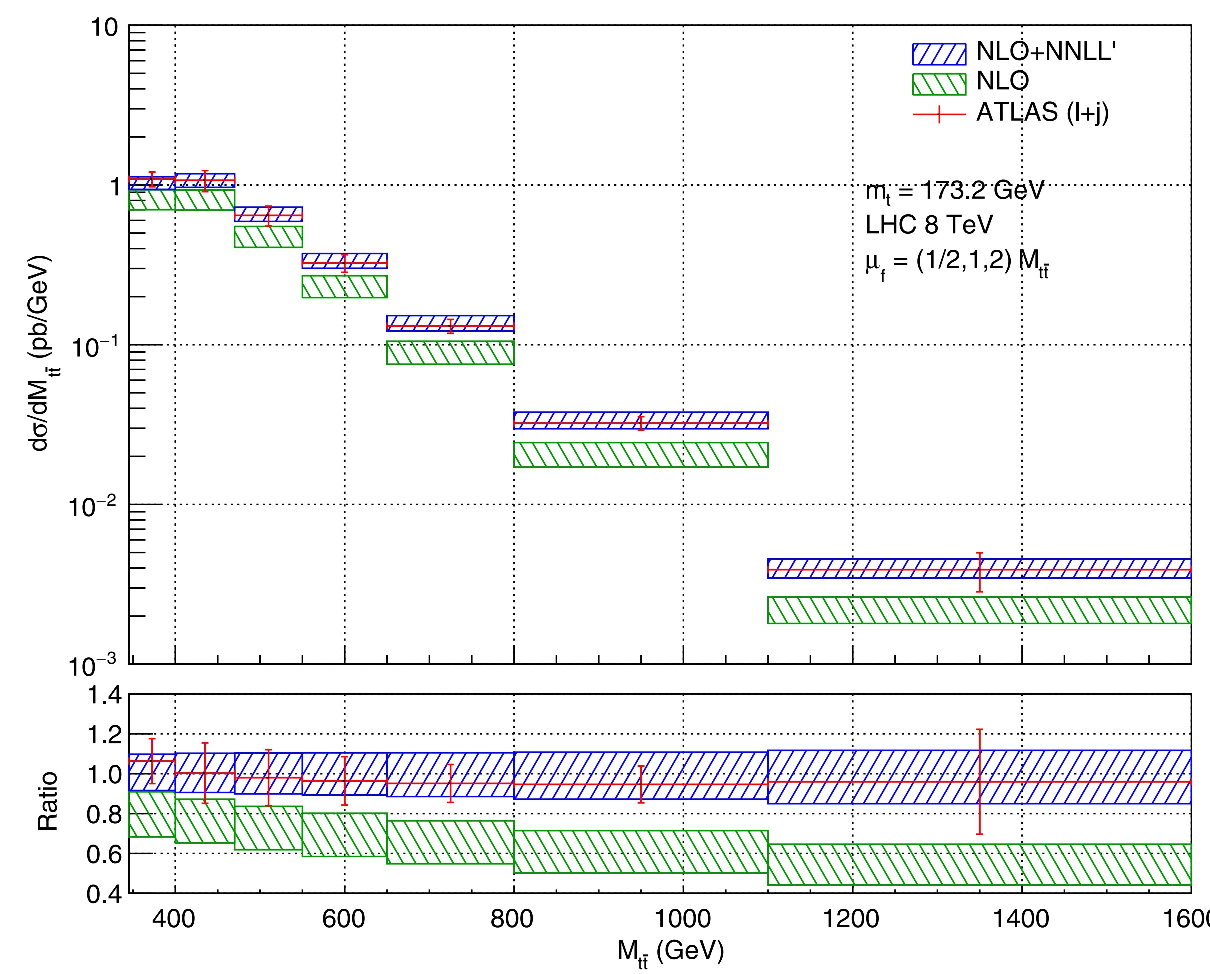
Resum soft logarithms and small-mass logarithms beyond NNLO



NLO+NNLL′ (boosted) top

Pecjak, Scott, Wang, **LLY**: PRL 116, 202001 (2016)





Matching to NNLO

A joint effort of the NNLO group and the resummation group

Czakon, Heymes, Mitov
Ferroglia, Pecjak, Scott, Wang, **LLY**
arXiv:1803.07623

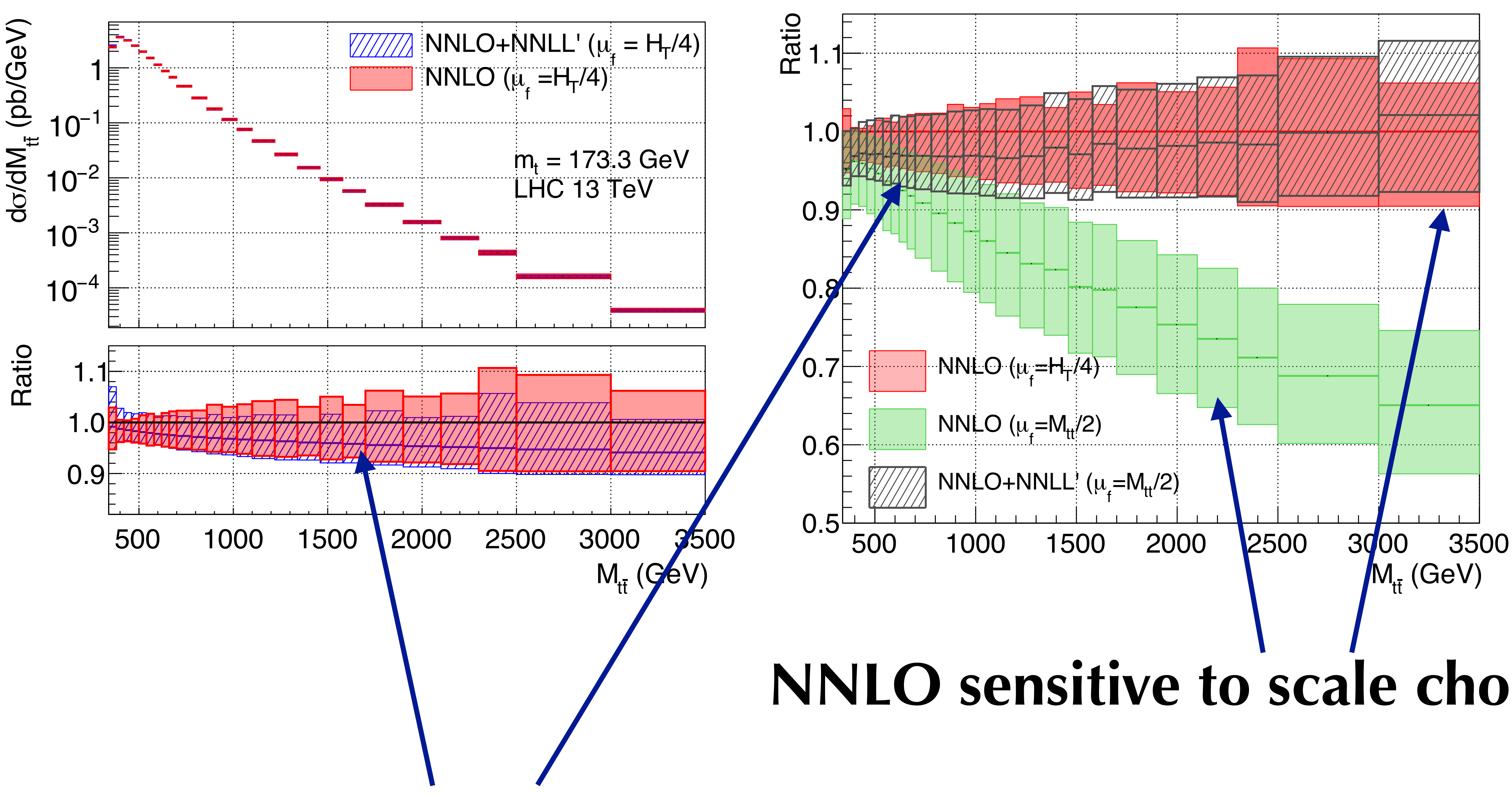
$$d\sigma^{\text{NNLO+NNLL}'} = d\sigma^{\text{NNLL}'_b} + \left(d\sigma^{\text{NNLL}_m} - d\sigma^{\text{NNLL}_b}\Big|_{\substack{\mu_{ds}=\mu_s \\ \mu_{dh}=\mu_h}} \right) + \left(d\sigma^{\text{NNLO}} - d\sigma^{\text{“top line”}}\Big|_{\substack{\text{NNLO} \\ \text{Expansion}}} \right)$$

boosted resummation

match to un-boosted resummation

match to NNLO

Matching to NNLO



NNLO sensitive to scale choice

Resummed result much less sensitive!

Summary and outlook

* Soft gluons and top quarks are important and interesting
* We have thoroughly studied their interactions at NNLO
    * Universal two-loop IR structure
    * NNLO soft real emissions
    * Resummation of soft logarithms

Thank you!