DISCUSSION SESSION - FRIDAY MARCH 29th

Logarithmic counting in the branching formalism.

I will introduce a simple rule to count the logarithms arising from each correlated block $nPC^{(j)}$ defined in the lecture II.

An example:

1) $n+j \leq 1$ → LEADING LOGARITHMIC ACCURACY

· in the real matrix element this involves the diagrams (soft limit)

 + mirror conjugate

this alone is, of course, divergent in the IRC limits

· in the virtual corrections ($\mathcal{H}(Q^2)$) $n+j=1$ imply the one-loop correction

 + mirror conjugate

· the sum of the real and virtual corrections is finite, and gives rise to a term $\alpha_s L^2$ (at most) at the level of $\Sigma(\tau)$.

To see how this double logarithm arises, we look at the integrand of the real emission:

$$[dk]\,|M_{soft}(k)|^2 \sim \frac{dk_t}{k_t}\,\frac{dz^{(\ell)}}{z^{(\ell)}}$$

where we singled out only its singular structure.

it is convenient to change variables and to introduce the emission's rapidity

$$\eta^{(\ell)} =$$

$$\boxed{|\eta^{(\ell)}| < \ln \frac{Q}{k_t}}$$

and $V(k)$, i.e. the value of the observable for the emission k. In our case, this is

$$V(k) = \frac{k_t^2}{Q^2 z^{(\ell)}} = V_k$$

and we change from $\{k_t, z^{(\ell)}\} \longrightarrow \{V_k, \eta^{(\ell)}\}$

$$\frac{dk_t}{k_t}\frac{dz^{(\ell)}}{z^{(\ell)}} \longrightarrow \boxed{\frac{dV_k}{V_k}\, d\eta^{(\ell)}}$$

the advantage of using such a parametrization is that I explicitly control the 2 logarithmic integration variables:

$$|\eta^{(\ell)}| < \ln \frac{Q}{k_t} = \frac{1}{2}\ln\frac{1}{V_k}$$

and V_k itself.

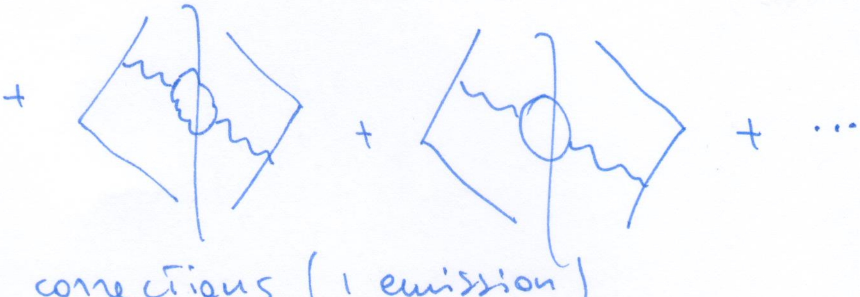
→ the number of logarithms in each block is given by the number of singular V_k, $\eta^{(\ell)}$ integrations in that block. ||

2) $\mathcal{O}(\alpha_s^2)$ ←

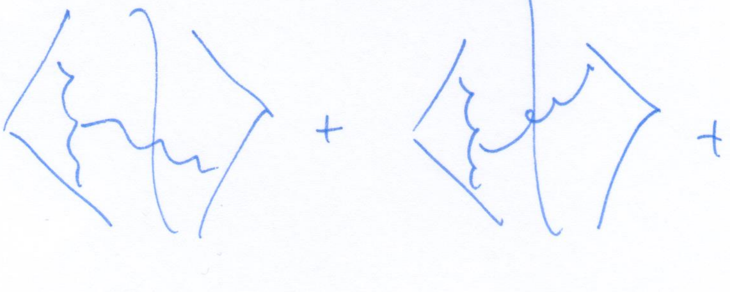
- the LL contribution is given by the iteration of the $IRC^{(0)}$ block, i.e. $|M^{(0)}(k_2)|^2 |M^{(0)}(k_1)|^2$, whose IRC divergences cancel against the iterated one-loop correction to the form factor. The combination of the two gives at most $\alpha_s^2 L^4$, according to the above counting.
- the NLL contribution is given by $\underline{n(t) \leq 2}$ in the soft limit

· virtual corrections (quark form factor) at two loops

· real corrections (2 emissions)



· real corrections (1 emission)

 + conjugate

The sum of all is finite and produces at most a term $\alpha_s^2 L^3$, subleading with respect to the iterated $IRC^{(0)}$ term ($\alpha_s^2 L^4$).

let's see why

the total number of divergent (logarithmic) degrees of freedom is given by

$$\frac{dv_1}{v_1}\frac{dv_2}{v_2}\, d\eta_1^{(e_1)}\, d\eta_2^{(e_2)}$$

so one can expect up to 4 logarithms (equivalently, a $1/\epsilon^4$ pole in the sum of all real emissions).

However, we notice that in the sum of the above diagrams, if the rapidities of the two gluons are very different, then the amplitude squared vanishes:

NB: → all emissions inside a correlated block ~~[illegible]~~ must have similar rapidities.

This implies that in the above counting of the logarithms, the two integrations

$$d\eta_1^{(e_1)}\, d\eta_2^{(e_2)}$$

should be actually ~~[illegible]~~ counted as a single one

$$d\eta_1^{(e_1)}\, d\eta_2^{(e_2)} \longmapsto d\eta_{2PC}$$

which leaves us with 3 logarithmic integrals, and hence

$$\alpha_s^2 L^3$$

3) One can repeat the exercise for any $nPC^{(j)}$.

One has

$$\frac{dv_1}{v_1}\cdots\frac{dv_n}{v_n}\,\frac{dv_{n+1}}{v_{n+1}}\cdots\frac{dv_{n+j}}{v_{n+j}}\,d\eta \longmapsto \underline{\underline{\alpha_s^{n+j}\,L^{n+j+1}}}$$

single rapidity logarithm.

rapidities cannot be very different within a correlated block

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NON-GLOBAL OBSERVABLES

the above counting also applies to non-global observables.

In this case it is important to remember that these logarithms arise from soft radiation emitted away from the collinear limit (e.g. example of the rapidity slice).

In this limit we have one single (soft) singularity per emission, hence one single logarithm.

What this means, in the above counting, is that all rapidity integrals are now non-logarithmic in the non global region, since they are away from their upper (collinear) bound.

We then find, for a purely non-global problem:

$n+j=1 \to \alpha_s^n L^n$; $n+j=2 \to \alpha_s^n L^n$; $n+j=3 \to \alpha_s^n L^n$; ...

The very same argument applies to the hard-collinear limit, in the Table we saw during lecture 2.

This method systematically allows us to identify which $nPC^{(j)}$ blocks we need to include at a given logarithmic order.