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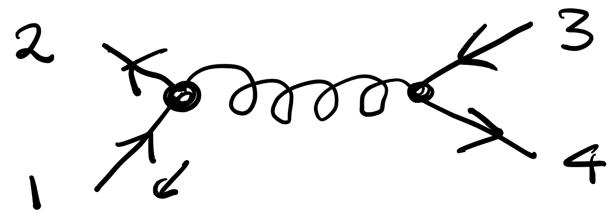
Spinor-helicity method

Example: $q\bar{q} \to q'\bar{q}'$



1 diagram

all momenta out-going

$$A(1_{\bar{q}}\, 2_q\, 3_{\bar{q}'}\, 4_{q'})$$

$$= \bar{u}(2)\gamma^\mu v(1)$$

$$\times \frac{1}{s_{12}}$$

$$\times \bar{u}(4)\gamma_\mu v(3)$$

s_{12}

$= (p_1 + p_2)^2$

$= 2 p_1 \cdot p_2$

fix helicity states of

the particles :

$$\mathcal{A}(1^-_{\bar{q}}\ 2^+_q\ 3^-_{\bar{q}'}\ 4^+_q)$$

$$\bar{u}_+(2) = [2|$$

$$v_-(1) = |1\rangle$$

$$\not{p}\, v_\pm = 0 \qquad \bar{u}_\pm \not{p} = 0$$

$$\not{p} = \begin{pmatrix} 0 & \sigma\cdot p \\ \bar{\sigma}\cdot p & 0 \end{pmatrix}$$

in reality I should write

$$v_-(1) = \begin{pmatrix} 0 \\ |1\rangle \end{pmatrix}$$

4-component dirac spinor

2-componet weyl spinor

Dirac equation :

$$\langle p | \not{p} = \langle p | (\bar{\sigma} \cdot p)$$

$$= 0$$

$$\not{p} | p] = 0$$

$$A(1^-_{\bar{q}}\ 2^+_{q}\ 3^-_{\bar{q}}\ 4^+_{q})$$

$$= [2 | \gamma^\mu | 1 \rangle \frac{1}{s_{12}}$$

$$[4 | \gamma_\mu | 3 \rangle$$

(NB drop all couplings and factors of i)

$$\langle 1\ \gamma^\mu\ 2] \langle 3\ \gamma_\mu\ 4] =$$

$$-2\langle 13\rangle[24]$$

proof using $\sigma^{\mu}_{\dot{\alpha}\alpha}\bar{\sigma}^{\mu\dot{\beta}\beta} = 2\delta_{\dot{\alpha}}^{\ \dot{\beta}}\delta_{\alpha}^{\ \beta}$

$$A\left(1_q^- 2_q^+ 3_{q'}^- 4_{q'}^+\right) =$$

$$-\frac{2\,[24]\langle 13\rangle}{s_{12}}$$

now all indices are contracted

$$s_{12} = \langle 12\rangle[21]$$

(see tomorrow's lecture)

$$A = -\frac{2\,[24]\langle 13\rangle}{}$$

$$\langle 12\rangle [21]$$

$$= -2\,\frac{\langle 13\rangle}{\langle 12\rangle}\,\frac{[24]\langle 13\rangle}{[21]\langle 13\rangle}$$

$$= -2\,\frac{\langle 13\rangle^2\,[24]}{\langle 12\rangle\,[2\,\not{p}_1\,3\rangle}$$

now use momentum conservation to show

$$\boxed{\begin{aligned} &|1]\langle 1| \\ &= \sigma\cdot p \end{aligned}}$$

$$[2\,\not{p}_1\,3\rangle = -[2\,\not{p}_4\,3\rangle$$
$$= -[24]\langle 43\rangle$$

$$A = \frac{2\langle 13\rangle^2}{\;}$$

$\langle 12 \rangle \langle 43 \rangle$

- Spinor products are Complex numbers $\left(\begin{matrix} \langle ij \rangle \\ [ij] \end{matrix} \right)$

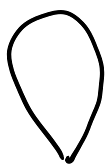
- $A(\langle ij \rangle, [ij])$

- because of momentum conservation representations of A are not unique.

- $$\langle ij \rangle = \underbrace{\sqrt{s_{ij}}}_{= (p_i + p_j)^2} \, e^{i\phi_{ij}}$$

complex phase.

TADPOLE

 $= \int D^d k \, \frac{1}{(k^2+m^2)}$

$$(d=4) \overset{?}{=} \int D^4 k \, \frac{1}{(k^2+m^2)} =$$

$$\overset{?}{=} \int d\Omega_4 \int_0^\infty dk \, \frac{k^3}{(k^2+m^2)} \to \int^\infty dk \, k \sim k^2$$

$$\int_0^\Lambda dk \, k \sim \Lambda^2$$

CUT OFF

Don't like

Dim Reg

$$\int D^d k \, \frac{1}{k^2+m^2} = T(m^2; d)$$

$m \sim k$

$$\int d\Omega_d \int_0^\infty dk \, \frac{k^{d-1}}{k^2+m^2} = \Omega(d) \int_0^\infty dk \, \frac{k^{d-1}}{k^2+m^2}$$

$$= \Omega(d)\,(m)^{d-2} \int_0^\infty d\rho \, \frac{\rho^{d-1}}{\rho^2+1}$$

$$\rho^2 = t$$

$$\rho = \sqrt{t}$$

$$d\rho = \frac{1}{2}\frac{1}{\sqrt{t}}\,dt$$

$$= \Omega(d)\, m^{d-2}\, \frac{1}{2} \int_0^\infty dt \, \frac{t^{\frac{d-1}{2}}}{t^{\frac{1}{2}}(t+1)}$$

$$= \Omega(d)\, m^{d-2}\, \frac{1}{2} \int_0^\infty dt \, \frac{t^{\frac{d}{2}-1}}{(1+t)}$$

$$B(x,y) = \int_0^\infty dt \, \frac{t^{x-1}}{(1+t)^{x+y}} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$x-1 = \frac{d}{2} - 1 \qquad x = \frac{d}{2}$$

$$x+y = 1 \qquad y = 1 - \frac{d}{2}$$

$$T(m^2, d) = \Omega(d)\, m^{d-2} \frac{1}{2} B\left(\frac{d}{2}; 1-\frac{d}{2}\right)$$

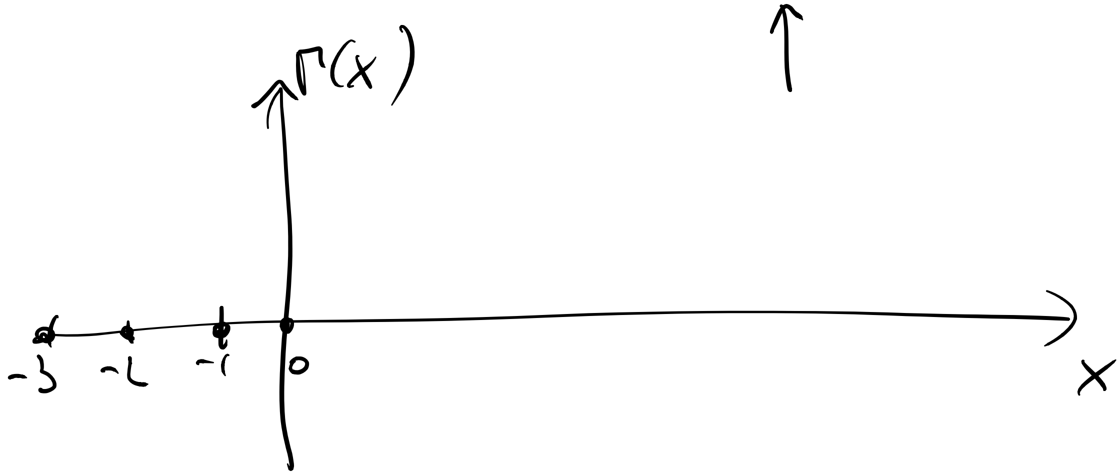
$$= \Omega(d)\, m^{d-2} \frac{1}{2} \frac{\Gamma\left(\frac{d}{2}\right)\Gamma\left(1-\frac{d}{2}\right)}{\underbrace{\Gamma(1) = 1}}$$

$$= \Omega(d)\, m^{d-2} \frac{\cancel{\Gamma\left(\frac{d}{2}\right)}\,\Gamma\left(\frac{2-d}{2}\right)}{\cancel{2}}$$

$$\Omega(d) = \frac{\cancel{2}\pi^{\frac{d}{2}}}{\cancel{\Gamma\left(\frac{d}{2}\right)}}$$

$$= \pi^{\frac{d}{2}} m^{d-2} \Gamma\left(\frac{2-d}{2}\right)$$

$$d=4 \quad \sim \pi^2 m^2 \Gamma(-1) \quad ?$$



$$T(m^2, d) = \pi^{\frac{d}{2}} m^{d-2} \Gamma\left(\frac{2-d}{2}\right)$$

$$\Gamma(1+x) = x\,\Gamma(x)$$

$$\Gamma\left(\frac{2-d}{2}\right) = \Gamma\left(\frac{4-d}{2} - 1\right) = \frac{2}{2-d}\Gamma\left(\frac{4-d}{2}\right)$$

$$= \frac{4}{(2-d)(4-d)} \Gamma\left(\frac{6-d}{2}\right)$$

$$T(m^2, d) = \pi^{\frac{d}{2}} m^{d-2} \frac{4}{(2-d)(4-d)} \Gamma\left(\frac{6-d}{2}\right)$$

$d = 2$

$d = 4$

$$T(m^2, d) = \int D^d k \frac{1}{k^2+m^2} \qquad \underline{d=3}$$

$$= \int D^3 k \frac{1}{k^2+m^2} \sim \int^\infty dk \frac{k^2}{k^2+m^2} \sim k$$

$$T(m^2, 3) = \pi^{3/2} m^1 \frac{4}{(-1)} \Gamma\left(\frac{6-3}{2}\right)$$

$$= -\pi^{3/2}\, m\, 4\, \Gamma\left(\frac{3}{2}\right)$$ ← negative result

$$I(m^2, 3) = \int d^3k \, \frac{1}{k^2+m^2}$$

POSITIVE FUNCTION

$$\left(\frac{1}{d-3}\right)$$

$$\int \frac{d^dk}{k^2} = \nabla$$

$$= \Omega(d) \int_0^\infty dk\, \frac{k^{d-1}}{k^2} = \Omega(d) \int_0^\infty dk\, k^{d-3}$$

$$= 0$$

$d = ?$

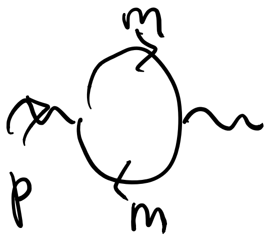
$$\int d^d k \, \frac{1}{k^2(k^2+m^2)} \sim \int_0^\infty dt \, \frac{t^{\alpha}}{(1+t)^\beta}$$

$$= B(\alpha, \beta) \ ?$$

$$\sim \left(\underbrace{\int d^d k \, \frac{1}{k^2}}_{=0} - \int d^d k \, \frac{1}{k^2+m^2} \right) \frac{1}{m^2}$$

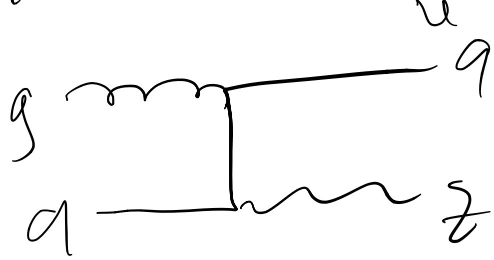
$$\int D^d k \, f(k) \quad \left\{ \begin{array}{l} k \to k + \lambda \\ k \to \lambda k \end{array} \right]$$

$$\Rightarrow \int D^d k \, k^\alpha = 0 \ !$$



~ Appel Function

$qg \to e^- e^+ + q$

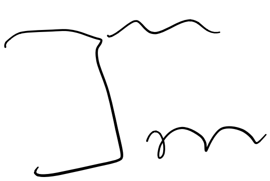
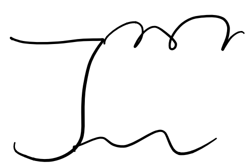


$E_3 \to 0$

$|\mathcal{M}|^2 \sim \frac{1}{p_2 \cdot p_3}$

$\sim \frac{1}{E_3}$

$q\bar{q} \to e^- e^+ + g$

 +  $\sim \frac{1}{(p_1 \cdot p_3)(p_2 \cdot p_3)}$

$\int_0^{E_{max}} dE_3\, E_3 \frac{1}{E_3^2} = \int_0^{E_{max}} \frac{dE}{E} \to \infty \quad \sim \frac{1}{E_3^2}$

$\int^{E_{max}} dE \quad E \quad \frac{1}{\ } \qquad \int^{E_{max}} dE$

$$\int_0 \frac{dk_3\, k_3}{E_3} = \int_0 dk_3$$

converges

$$\sum \bar{u}u \sim \not{p} \sim E$$

$$\sum \epsilon_\mu \epsilon^*_\nu = -g^{\mu\nu} \sim E^0$$

Real : $\sim \frac{1}{\epsilon^2}, \frac{1}{\epsilon}$

Virtual : $\frac{1}{\epsilon^2}, \frac{1}{\epsilon}$

R + V : $\frac{1}{\epsilon^2}$ cancel

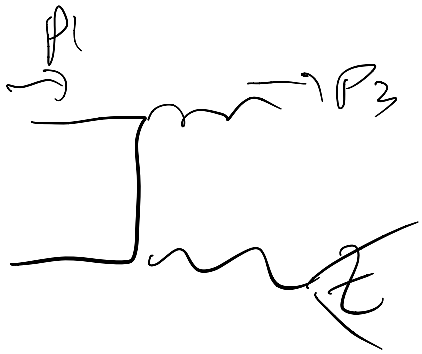
<u>some</u> $\frac{1}{\epsilon}$ cancel

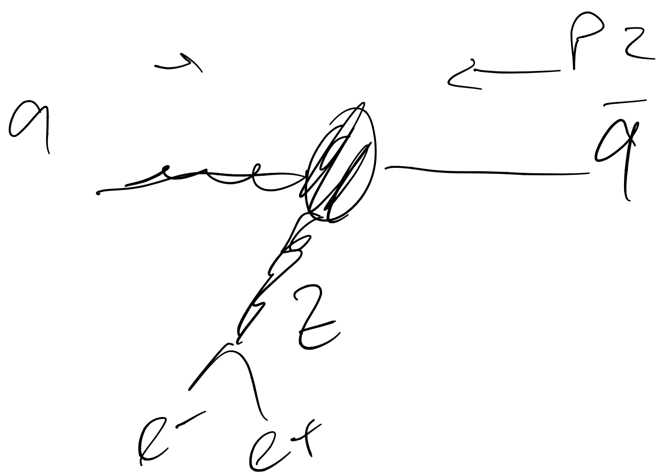
$q\bar{q} \to V$ @ LO

$q\bar{q} \to V + g$

$$\lim_{\theta_{13}\to 0} |M|^2 \sim \frac{1}{\theta_{13}^2}$$

$$\int_0^\pi \frac{d\theta_{13}}{\theta_{13}} \to \infty$$


p1
p3
Z


q
p2
q̄
Z
e-
e+