The QCD running coupling and the elimination of the scheme-and-scale ambiguities for fixed-order pQCD predictions

An introduction of the Principle of Maximum Conformality

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浙江大学　2018/3/27

OUTLINE

- The QCD scale-setting problem
- The principle of Maximum Conformality (PMC)
  - Extended RGE – **Conventional Coupling**
  - PMC 多能标方案- I
    PMC 多能标方案-II
    PMC 单能标方案 + **C-scheme coupling**
- Applications of the PMC
- Summary and Outlook

The QCD scale-setting problem

Basic idea of perturbative theory: A physical quantity ρ could be expanded in the following form

$$\rho = r_0 \alpha_s^{p}(\mu_R) + r_1 \alpha_s^{p+1}(\mu_R) + r_2 \alpha_s^{p+2}(\mu_R) + \ldots$$

高阶情况下需引入重整化方案、重整化能标

正规化、重整化、能标设定 → 因子化-微扰非微扰；重整化-微扰准确性

准确预言具同等重要性

Up to infinite order, there is no scheme- and scale-dependence, i.e., any choice of scheme/scale should result in the same prediction.

“Standard RGI” - 标准的重整化群不变性

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在无限阶或足够高阶的情况下，理论预言将不依赖于重整化能标选择，对重整化能标选择不敏感

$$\text{等价表述}:\frac{\partial\rho_n}{\partial\mu_R}\neq 0;\frac{\partial\rho_n}{\partial \mathrm{R}}\neq 0;\quad n-\text{微扰阶数},\mathrm{R}-\text{重整化方案}$$

当前我们的能力只在有限阶

在有限阶下，理论预言依赖于能标选择(α_s与同阶微扰系数不能很好匹配)，那么究竟那一个能标所对应的结果才是对的或最接近真实值 ？

如何获得高精度预言是可重整化理论的重要问题

途径：完成更高阶微扰阶计算、设定最优重整化能标、预言未知高阶贡献、将已知类型的项重求和至无穷阶等等

Conventional scale-setting approach－－“猜”

=>**“Choose”** the scale Q to be **typical momentum transfer**, or the one to **eliminate the large logs**

=>**“Keep it fixed”** throughout the calculation

=>**“Vary”** in a certain range, e.g. [Q/2, 2Q], [Q/3, 3Q], to discuss its uncertainty or to predict unknown high-order contributions

无招胜有招？
还是无奈的选择？

冀望于
收敛性强、完成更高阶计算；降低对重整化能标的依赖性

但在单圈、双圈等低圈情况下；如何选择能标 ？

通常方案存在致命问题

无法获得准确结果，就无法有效的限定新物理参数空间

1) 纯属“猜测”；

2) 收敛性也属运气—仅依赖于α_s-值的自然压低；
典型能标消大log-项可提高收敛性—为数学处理；
一旦出现不收敛，无法判断这是过程的本性还是因不恰当选择，
只能冀望于完成更高阶得到准确结果（或怀疑微扰论是否适用）

3)考虑更高阶，能标依赖可减小，也是因为每阶相互抵消效应
—完成再高阶也无法获得低阶准确值；
—部分可观测量正是依赖于此；

4)在自身都有很大误差前提下，预言未知高阶贡献的可靠性无法断定
—简单变化能标—无任何依据，且最多只能估算部分非共形项贡献；

高阶圈图计算的复杂性

使我们希望能在有限低阶就得到与实验测量值接近的预言

虽然，人们很早就意识到了这个问题
但为什么迄今，我们仍然坚持采用“猜”的方法呢？！

主要存在一些概念误区
这个问题本身也难解决

从重整化理论来说，
我们本应可选择任意重整化能标来完成重整化；
但，如果存在最优能标，那么我们选择能标的自由去那里了？

（最优能标之后，如何估算未知高阶误差 - 本次不涉及）

两者实际并不矛盾

我们可以首先选择任意（初始）重整化能标、任意重整化方案完成重整化步骤；当利用原微扰序列确定出最优能标之后，新的微扰序列可获得物理量的准确预言值且保证该预言值与初始重整化能标及重整化方案的选择无关。

因此，确定出最优能标，重整化能标的自由选择并没有被破坏掉，不会违背标准的重整化群不变性。

事实上，在无穷阶情况下，任意初始能标和最优能标，都能获得相同结果。

任务变成：最优重整化能标设定方案是什么？

To deal with pQCD series as a whole

Optimized perturbation theory –
minimize the higher-order contributions – **PMS**

反解有效能标

How about directly set it to satisfy the RG invariance

Any observable <=> an effective coupling constant (idea useful)
Fastest Apparent Convergence (**FAC**)

实验反定有效荷

How about directly cut off all higher-order-terms?

Local RGI
≠ RGI

PMC=> β-term

Corrected

BLM=> nf-term

To deal with
α_s-itself !

间接处理微扰序列

两种典型解决思想

直接处理微扰序列

Optimal procedure for obtaining precise QCD predictions is **to choose the (optimal) scale so that the result is scheme- and scale- independent even at fixed order !**

一个自洽的能标设定方案需要满足那些条件？

to ensure the Self-Consistence of a scale-setting method, we think it should satisfy several general properties.

PHYSICAL REVIEW D **86**, 054018 (2012)

Self-consistency requirements of the renormalization group for setting the renormalization scale

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存在性、反身性、对称性、传递性

First-belief → **Existence of such optimal scale μ**

second →

Reflexivity. Given a $\alpha_s(\mu)$ specified at a scale μ, we can express it in terms of itself but specified at another scale μ' **up to all orders**,

$$\alpha_s(\mu) = \alpha_s(\mu') + f_1(\mu,\mu')\alpha_s^2(\mu') + f_2(\mu,\mu')\alpha_s^3(\mu') + \cdots,$$

$$\frac{\partial \alpha_s(\mu)}{\partial \mu'} = 0$$

where $f_1(\mu,\mu') = -\frac{\beta_0}{4\pi}\ln(\mu^2/\mu'^2)$ and $\beta_0 = 11 - 2N_f/3$. If a scale-setting scheme is self-consistent, it should choose the **unique value** $\mu' = \mu$ on the right-hand side.

third

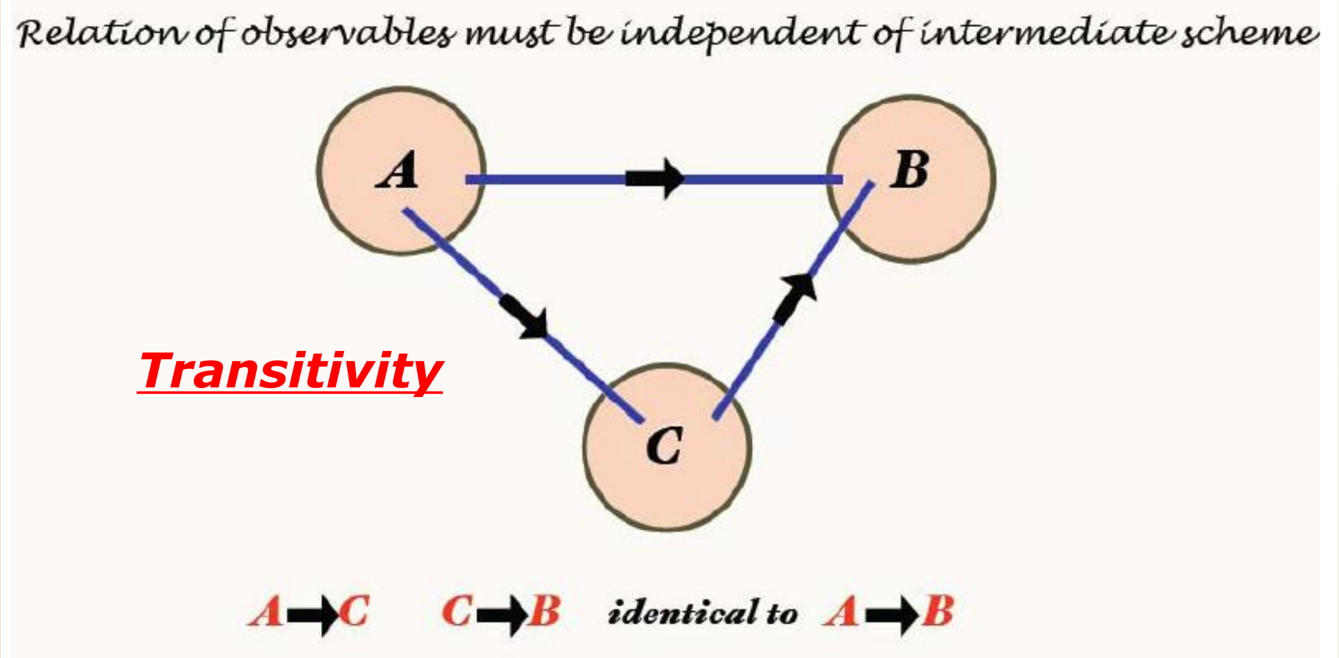
Symmetry. Given two different coupling constants $\alpha_{s1}(\mu_1)$ and $\alpha_{s2}(\mu_2)$, we can express one of them in terms of the other:

$$\alpha_{s1}(\mu_1) = \alpha_{s2}(\mu_2) + r_{12}(\mu_1, \mu_2)\alpha_{s2}^2(\mu_2) + \cdots, \quad (33)$$

$$\alpha_{s2}(\mu_2) = \alpha_{s1}(\mu_1) + r_{21}(\mu_2, \mu_1)\alpha_{s1}^2(\mu_1) + \cdots. \quad (34)$$

If a scale-setting method gives $\mu_2 = \lambda_{21}\mu_1$ for the first series and $\mu_1 = \lambda_{12}\mu_2$ for the second series, then this scale-setting method is said to be *symmetric* if $\lambda_{12}\lambda_{21} = 1$.

fourth

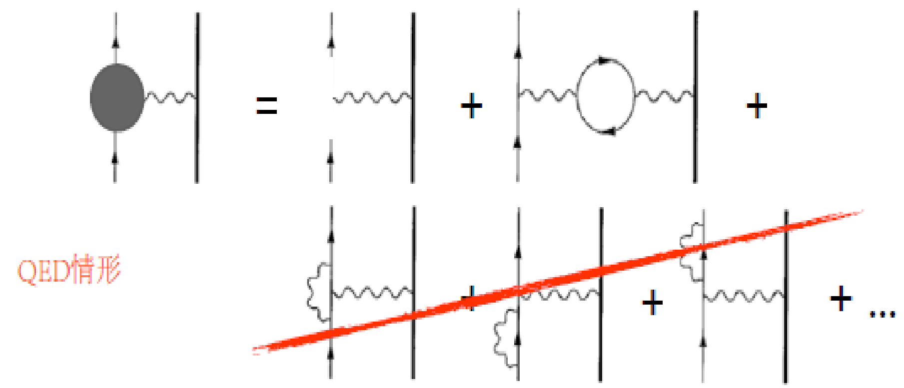


Simple review

Brodsky-Lepage-Mackenzie机制（ BLM ）

Phys. Rev.D 28, 228 (1983)

BLM-基于QED认识，以nf-项为基准来确定alphas行为，
单圈非常成功－－引用超千次



The Ward-Takahashi identity ensures exact cancellation between vertex- and wavefunction renormalizations; the renormalized coupling is the effective electric charge, which takes into account the vacuum polarization at a given momentum transfer.

等价于
QED极限下
Gell-Man – Low方案

有没有可能基于**nf**-项直接处理高阶微扰项？
问题在于如何解析延拓，**seBLM**是不成功尝试
目的仅是提高微扰收敛性，而非解决能标问题

seBLM方案

Mikhailov (Kataev), JHEP 06 (2007) 009

seBLM – 基于大β0近似，将**BLM**延拓到高阶：每阶需引入新自由度确定β – 系数
（注：其后提出，seBLM+双重展开，无需引入新自由度，但基于错误**RGE**解）

$$a_s(\mu) = \frac{a_s(\mu_0)}{1 - \beta(a_s(\mu_0))/a_s(\mu_0) \ln \frac{\mu^2}{\mu_0^2}}$$

Phys.Lett.B770(2017)494

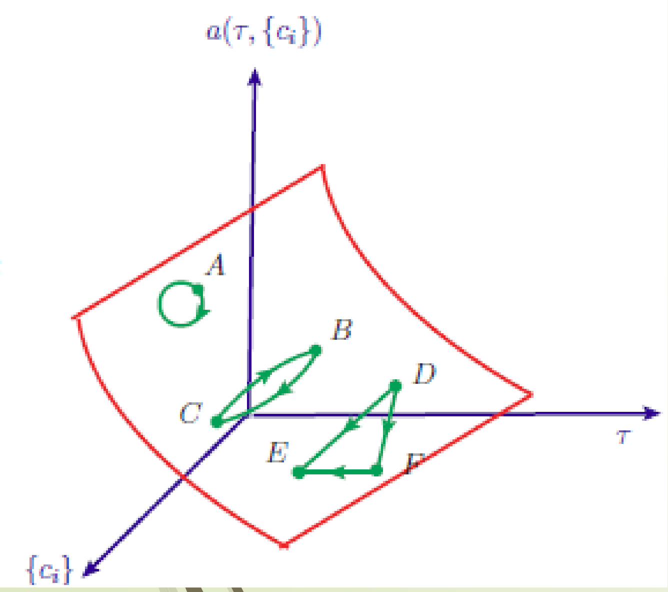
最大共形原理（PMC）提出及发展历程

1981年， Brodsky-Lepage-Mackenzie提出BLM机制
1992年，Gruberg-Kataev提出（错误）延拓方案认为BLM机制存在问题
1995年，Brodsky-Lu提出自洽能标对应关系CSR，拓展BLM机制到两圈
1997年-2011年，Brodsky提出采用β函数替换BLM中nf-项的原始理念
2011年4月，Giustino-Brodsky初步设想将BLM升级到PMC方案
（仅单圈，与BLM一致，2012年10月才正式发表）
2011年11月，Wu-Brodsky提出PMC – BLM对应原理
四圈为例，基于RGE，将BLM拓展到高圈（首篇正式论文）
2012年，Wu-Brodsky将PMC应用于Top产生取得成功（PRL, PRD）
2013年， Wu-Brodsky-Mojaza完成PPNP综述（首篇重整化能标系统综述）
2014年，Brodsky-Mojaza-Wu提出PMC第二种数学上更严格的实现方案
获赞其意义"已接近重整化理论本身"（PRL）
2015年，Bi-Wu等证明PMC两种方案的等价性
Ma-Wu等基于重整化群不变性完成PMC和PMS的深入对比（PRD）
Wu-Ma-Wang等完成RPP邀请综述、完成国内FOP综述
2016年，Shen-Wu等提出新的单能标实现方案，证明微扰等价性
2017年，Shen-Wu等将自洽能标对应关系CSR推到任意高阶
Deur-Shen-Wu等初步考虑将PMC应用于低能区
2018年，Shen-Wu等证明PMC对于任意重整化方案有方案无关性

基于《重整化群方程》讨论解决能标设定问题的可能性

Phys.Rev.D86,054018 (2012)

$\frac{\partial \rho_n}{\partial \mu_R} \neq 0; \frac{\partial \rho_n}{\partial R} \neq 0;$ 不等式—由此无法获得相应的约束方程(对比:PMS令它们为零)



只能从**重整化群方程**本身找到答案，即用它来**确定微扰展开式中的耦合常数的跑动行为，从而确定重整化能标**。

利用重整化群方程**确定每一阶所对应的beta-项**。

将**同类型的已知beta-项重求和**，确定每一阶重整化能标准确值。

从这个意义上，PMC类似于重求和方法。

PMC满足重整化群基本性质－对称性、自反性、传递性等。

总体思路

获得强耦合常数的准确行为—“基础”
拓展重整化群方程—同时处理能标及方案的跑动

连续变化

Naïve truncated series to know α_s at different scale --- not reliable !

$$\alpha_S(P)=\alpha_R(Q)+f(P,Q)\alpha_R^2(Q)$$

A way out for scale-dependence

Conventional RGE

$$\frac{d}{d\ln\mu^2}\left(\frac{\alpha_s^R(\mu)}{4\pi}\right)=-\sum_{i=0}^{\infty}\beta_i^R\left(\frac{\alpha_s^R(\mu)}{4\pi}\right)^{i+2}$$

Why it is better and useful ?

The scale is changed along the evolution trajectory with a continuous fashion, thus avoiding the presence of dissimilar scales and large expansion coefficients

universal

$\beta_0,\ \beta_1$

$$a^R=\beta_1\alpha_s^R/(4\pi\beta_0)$$

$$\tau=\frac{\beta_0^2}{\beta_1}\ln\mu^2$$

$$\frac{da^R}{d\tau}=-(a^R)^2\left[1+a^R+c_2^R(a^R)^2+c_3^R(a^R)^3+\cdots\right]$$

方案可由{β_i}-函数来标记

Each scheme leads to different c_i^R, and vice versa

Can we discuss the uncertainty of c_i^R in a consistent way as that of the scale ?

Extended RGE !

S.J. Brodsky and H.J. Lu, Phys.Rev. D**51**, 3652(1995).
G. Grunberg, Phys.Rev. D**46**, 2228(1992).

universal coupling constant $a(\tau, \{c_i\})$

Equivalent to usual RGE

$$a^R(\tau_R) = a(\tau_R, \{c_i^R\})$$

$$\beta(a, \{c_i\}) = \frac{\partial a}{\partial \tau} = -a^2\left[1 + a + c_2 a^2 + c_3 a^3 + \cdots\right]$$

Scale equation

and

$$\beta_n(a, \{c_i\}) = \frac{\partial a}{\partial c_n} = -\beta(a, \{c_i\}) \int_0^a \frac{x^{n+2} dx}{\beta^2(x, \{c_i\})}$$

Scheme equations

Useful for a reliable error analysis on higher order

PMC基本步骤

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\text{init}})$

↓

Choose μ_R^{init}; *arbitrary initial renormalization scale*

↓

Identify $\{\beta_i^R\}$ *− terms using* n_f *− terms through the PMC − BLM correspondence principle*

order-by-order

↓

Shift scale of α_s *to* μ_R^{PMC} *to eliminate* $\{\beta_i^R\}$ *− terms*

↓

Conformal Series

Result is independent of μ_R^{init} *and scheme at fixed order*

可选择任意初始(位于微扰区域)能标和方案完成微扰论计算

PMC后，结合重整化群方程得到的最优能标与初始选择无关

基于PMC序列（最大程度的）共形对称性，PMC预言与重整化方案选择无关

核心：挑出符合重整化群的nf-项；找出nf-项和β-项对应关系

$$\rho = \sum_{i=1}^{m}\left(\sum_{j=0}^{i-1} c_{i,j} n_f^j\right) a_s^{n+i-1}(\mu) + \dots,$$

出发点

$$\rho = \sum_{i=1}^{m} r_{i,0}^{M} a_s^{n+i-1}(Q_{M,i}) + \dots,$$

共形序列

如何实现，不同方案

To find out all the terms connected with the RGE, using them to determine the correct α_s-running at each order

每阶能标不一样 – RGE

PMC多能标方案I

We call it `The BLM – PMC correspondence`

PMC-I allows one to obtain the correct PMC scales using a step-by-step method without first transforming the n_f-terms into the $\{\beta_i\}$-terms. This procedure is based on the observation that one can rearrange all the Feynman diagrams of a process in form of a cascade; i.e., the "new" terms emerging at each order can be equivalently regarded as a one-loop correction to all the "old" lower-order terms. All of the n_f-terms can then be absorbed into the running coupling following the basic β-pattern in the scale-displacement formula, i.e. Eq. (3).

$$
\begin{aligned}
a_s(\mu_2) = a_s(\mu_1) &- \beta_0 \ln\left(\frac{\mu_2^2}{\mu_1^2}\right) a_s^2(\mu_1) \\
&+ \left[\beta_0^2 \ln^2\left(\frac{\mu_2^2}{\mu_1^2}\right) - \beta_1 \ln\left(\frac{\mu_2^2}{\mu_1^2}\right)\right] a_s^3(\mu_1) \\
&+ \left[\frac{5}{2}\beta_0\beta_1 \ln^2\left(\frac{\mu_2^2}{\mu_1^2}\right) - \beta_0^3 \ln^3\left(\frac{\mu_2^2}{\mu_1^2}\right) \right. \\
&\left. - \beta_2 \ln\left(\frac{\mu_2^2}{\mu_1^2}\right)\right] a_s^4(\mu_1) + \ldots.
\end{aligned}
$$

核心理念：每一阶修正项均可认为是它前一阶项的单圈修正；
按此可确定每一阶项的β-项以及消除β-项

PMC多能标方案II

R_δ-方案

核心理念：每一阶修正项均可认为是它之前所有阶强耦合常数跑动行为的线性叠加；按此可确定每一阶项的β-项

PMC-II suggests that the coefficients of the $\{\beta_i\}$-terms in the β-pattern can be fixed by requiring a "degeneracy relation" among different $\{\beta_i\}$-terms at different orders; the result resembles a skeleton-like expansion [4,5]. By resumming the $\{\beta_i\}$-series according to this expansion, one also correctly reproduces the Abelian $N_c \to 0$ limit of the observables [27]. Thus one can simultaneously determine the PMC scales $Q_{\mathrm{II},i}$ at all orders from their initial values μ; i.e. $\mu \to Q_{\mathrm{II},1}$, $\mu \to Q_{\mathrm{II},2}$, $\mu \to Q_{\mathrm{II},3}$, $\cdots$, by resumming the $\{\beta_i\}$-terms into the running couplings in the skeleton-like form.

不同δ值对应不同重整化方案

We will generalize this by defining the δ-*Renormalization scheme*, $\mathcal{R}_\delta$, where one absorbs $\ln(4\pi) - \gamma_E - \delta$, i.e.

$$\mu^2 = \mu_\delta^2 \exp(\ln 4\pi - \gamma_E - \delta), \qquad (10)$$

where δ is an arbitrary finite number, and by appropriate choice will connect all MS-type schemes. In particular,[1]

$$\mathcal{R}_0 = \overline{\mathrm{MS}}, \qquad (11a)$$

$$\mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS}, \qquad (11b)$$

$$\mathcal{R}_{-2} = \mathrm{G}, \qquad (11c)$$

$$\frac{\partial \varrho_\delta}{\partial \delta} = 0 \qquad \frac{\partial \varrho_\delta}{\partial \alpha} \neq 0$$

$$\frac{\partial \varrho_\delta}{\partial \delta} = -\beta(\alpha)\frac{\partial \varrho_\delta}{\partial \alpha}$$

$$\beta(\alpha) \to 0$$

如果β＝0, 在任意阶下，将与方案δ无关
其它任意方案间存在CSR；会有残留依赖

PMCI与PMCII的等价性

共形系数一致

$$r^{\rm I,II}_{1,0} = c_{1,0},$$

$$r^{\rm I,II}_{2,0} = c_{2,0} + \frac{11C_A}{4T_F}c_{2,1},$$

$$r^{\rm I,II}_{3,0} = \frac{1}{16T_F^2}\Big[-(84C_A^2T_F + 132C_AC_FT_F)c_{2,1} + 16T_F^2c_{3,0} + 44C_AT_Fc_{3,1} + 121C_A^2c_{3,2}\Big],$$

$$r^{\rm I,II}_{4,0} = \frac{1}{64T_F^3}[2C_AT_F^2(-287C_A^2 + 1208C_AC_F + 924C_F^2)c_{2,1} - 48C_AT_F^2(7C_A + 11C_F)c_{3,1} - 264C_A^2T_F(7C_A + 11C_F)c_{3,2} + 64T_F^3c_{4,0} + 176C_AT_F^2c_{4,1} + 484C_A^2T_Fc_{4,2} + 1331C_A^3c_{4,3}],$$

Rδ-方案下的简并关系存在于任意方案证明

$$a_A(\mu) = a_B(\mu) + (r^{AB}_{2,0} + \beta_0 r^{AB}_{2,1})a_B^2(\mu) + (r^{AB}_{3,0} + \beta_1 r^{AB}_{2,1} + 2\beta_0 r^{AB}_{3,1} + \beta_0^2 r^{AB}_{3,2})a_B^3(\mu) + (r^{AB}_{4,0} + \beta_2^A r^{AB}_{2,1} + 2\beta_1 r^{AB}_{3,1} + \frac{5}{2}\beta_0\beta_1 r^{AB}_{3,2} + 3\beta_0 r^{AB}_{4,1} + 3\beta_0^2 r^{AB}_{4,2} + \beta_0^3 r^{AB}_{4,3})a_B^4(\mu) + \cdots.$$

Phys.Lett.B748,13(2015)

微扰论意义下的等价性

差别只体现包含的高阶效应有所不同

四圈情况下的能标差别

scales by $\Delta_i = \ln Q_{{\rm I},i}/Q_{{\rm II},i}$, and the first three ones for a four-loop prediction read:

$$\Delta_1 = -\frac{3\beta_1 f}{64n^2(n+1)c_{1,0}^2c_{2,1}T_F^2}a_s(\mu)^2 + \mathcal{O}(a_s^3), \quad (12)$$

$$\Delta_2 = \frac{3\beta_0(5C_A + 3C_F)f}{16n(n+1)^2c_{1,0}c_{2,1}T_F(11c_{2,1}C_A + 4c_{2,0}T_F)}a_s(\mu) + \mathcal{O}(a_s^2), \quad (13)$$

$$\Delta_3 = 3C_A(7C_A + 11C_F)\frac{f}{g} + \mathcal{O}(a_s), \quad (14)$$

where

$$f = 6n^2(n+1)c_{1,0}^2c_{4,3} + \left(2n^3 + 8n^2 + 13n + 7\right)c_{2,1}^3 - 6n\left(n^2 + 3n + 3\right)c_{1,0}c_{3,2}c_{2,1},$$

$$g = 4n(n+1)(n+2)c_{1,0}c_{2,1}(C_A^2(84c_{2,1}T_F - 121c_{3,2}) + 44C_AT_F(3c_{2,1}C_F - c_{3,1}) - 16c_{3,0}T_F^2).$$

As indicated by Eqs. (12), (13), (14), the LO logarithmic scale difference Δ_1 starts at the order a_s^2, which changes to order a_s^1 for the NLO Δ_2, and to order a_s^0 for the NNLO Δ_3. The value of Δ_i can

PMCI与PMCII的等价性

Table 1
The logarithmic scale difference Δ_i for $R_{e^+e^-}$ ($Q = 31.6$ GeV), $\Gamma^{NS}(Z \to \text{hadrons})$, $\Gamma(\Upsilon(1S) \to e^+e^-)$, and $\Gamma(H \to b\bar{b})$ up to four-loop QCD corrections, where NS stands for the non-singlet contribution.

	Δ_1	Δ_2	Δ_3
$R_{e^+e^-}$ ($Q = 31.6$ GeV)	−0.0043	+0.0973	+1.9389
$\Gamma^{NS}(Z \to \text{hadrons})$	−0.0030	−0.0826	+2.0432
$\Gamma(\Upsilon(1S) \to e^+e^-)$	+0.0353	+0.1047	+0.0816
$\Gamma(H \to b\bar{b})$	+0.0001	−0.0014	−0.0018

found in Refs. [23,24]. In the case of $H \to b\bar{b}$, the computed scale differences are very small at any order. For example, the largest difference for the case of $H \to b\bar{b}$ is found between $Q_{I,3}$ and $Q_{II,3}$, which is less than 0.2%. The logarithmic scale differences for $R(e+e-)$ and $\Gamma^{NS}(Z \to \text{hadrons})$ are somewhat larger; their magnitudes follow the trend $|\Delta_1| \ll |\Delta_2| \ll |\Delta_3|$, indicating that the PMC-I and PMC-II scale differences quickly diminish as we include more $\{\beta_i\}$-terms to fix the PMC scales. In

能标差别趋势

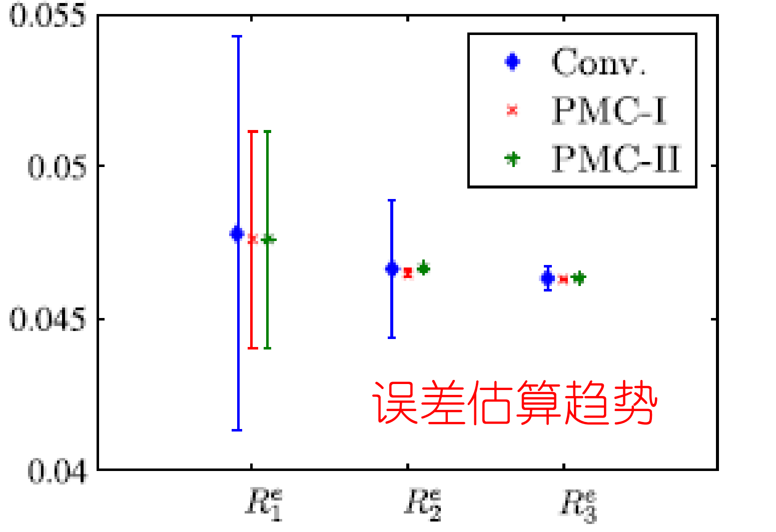
Table 2
The contributions of each of the contributions (LO, NLO, N^{2}LO and N^{3}LO) to the four-loop pQCD approximate R^e_3.

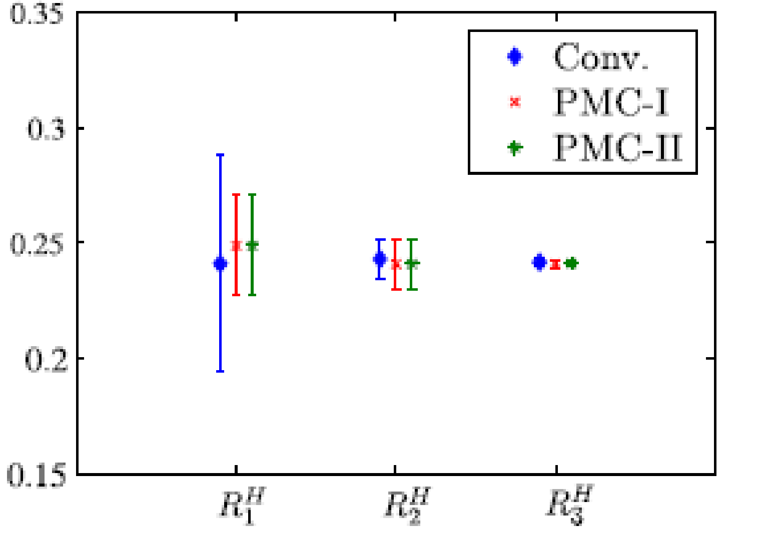
	LO	NLO	N^{2}LO	N^{3}LO	Total
PMC-I	0.04294	0.00340	−0.00002	−0.00001	0.04631
PMC-II	0.04290	0.00352	−0.00004	−0.00002	0.04636
Conv.	0.04499	0.00285	−0.00117	−0.00033	0.04634

微扰展开趋势

Table 3
The contributions of each of the contributions (LO, NLO, N^{2}LO and N^{3}LO) to the four-loop pQCD approximate R^H_3.

	LO	NLO	N^{2}LO	N^{3}LO	Total
PMC-I	0.2268	0.0249	−0.0091	−0.0012	0.2414
PMC-II	0.2268	0.0249	−0.0094	−0.0012	0.2411
Conv.	0.2037	0.0377	+0.0019	−0.0014	0.2419





PMC方案的简化

PMC的有效单能标方案

能否简化处理，一次消除所有β－项
获得有效的耦合常数、有效的有标

PMC有效单能标方案

$$\rho(Q) = r_{1,0}\alpha(\mu)^p + [r_{2,0} + p\beta_0 r_{2,1}]\,\alpha(\mu)^{p+1} + \left[r_{3,0} + p\beta_1 r_{2,1} + (p+1)\beta_0 r_{3,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2}\right]\alpha(\mu)^{p+2}$$
$$+ \left[r_{4,0} + p\beta_2 r_{2,1} + (p+1)\beta_1 r_{3,1} + \frac{p(3+2p)}{2}\beta_1\beta_0 r_{3,2} + (p+2)\beta_0 r_{4,1} + \frac{(p+1)(p+2)}{2}\beta_0^2 r_{4,2}\right.$$
$$\left.+ \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3}\right]\alpha(\mu)^{p+3} + \cdots,$$

$$\rho(Q) = \sum_{n\geq 1} r_{n,0}\alpha(\mu)^{n+p-1} + \sum_{n\geq 1}\left[(n+p-1)\alpha(\mu)^{n+p-2}\beta\right]\sum_{j\geq 1}(-1)^j \Delta_n^{(j-1)} r_{n+j,j}$$

$$r_{i,j} = \sum_{k=0}^{j} C_j^k \tilde{r}_{i-k,j-k} L^k \qquad \tilde{r}_{i,j} = r_{i,j}|_{\mu=Q} \quad L = \ln \mu^2/Q^2$$

吸收β项，确定单个有效PMC能标

$$\rho(Q) = \sum_{n\geq 1} \tilde{r}_{n,0}\alpha(Q_\star)^{n+p-1}$$

残留能标依赖性，与通常方案相比远远压低，基本可忽略不计

$$\ln\frac{Q_\star^2}{Q^2} = T_0 + T_1\alpha(Q) + T_2\alpha^2(Q) + \cdots$$

任意阶均与初始能标无关

$$T_0 = -\frac{\hat{r}_{2,1}}{\hat{r}_{1,0}},$$

$$T_1 = \frac{(p+1)(\hat{r}_{2,0}\hat{r}_{2,1} - \hat{r}_{1,0}\hat{r}_{3,1})}{p\hat{r}_{1,0}^2} + \frac{(p+1)(\hat{r}_{2,1}^2 - \hat{r}_{1,0}\hat{r}_{3,2})}{2\hat{r}_{1,0}^2}\beta_0,$$

$$T_2 = \frac{(p+1)^2(\hat{r}_{1,0}\hat{r}_{2,0}\hat{r}_{3,1} - \hat{r}_{2,0}^2\hat{r}_{2,1}) + p(p+2)(\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,0} - \hat{r}_{1,0}^2\hat{r}_{4,1})}{p^2\hat{r}_{1,0}^3} + \frac{(p+2)(\hat{r}_{2,1}^2 - \hat{r}_{1,0}\hat{r}_{3,2})}{2\hat{r}_{1,0}^2}\beta_1$$
$$- \frac{p(p+1)\hat{r}_{2,0}\hat{r}_{2,1}^2 + (p+1)^2(\hat{r}_{2,0}\hat{r}_{2,1}^2 - 2\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,1} - \hat{r}_{1,0}\hat{r}_{2,0}\hat{r}_{3,2}) + (p+1)(p+2)\hat{r}_{1,0}^2\hat{r}_{4,2}}{2p\hat{r}_{1,0}^3}\beta_0$$
$$+ \frac{(p+1)(p+2)(\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,2} - \hat{r}_{1,0}^2\hat{r}_{4,3}) + (p+1)(1+2p)(\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,2} - \hat{r}_{2,1}^3)}{6\hat{r}_{1,0}^3}\beta_0^2.$$

PMC的有效单能标方案

快速趋于稳定值

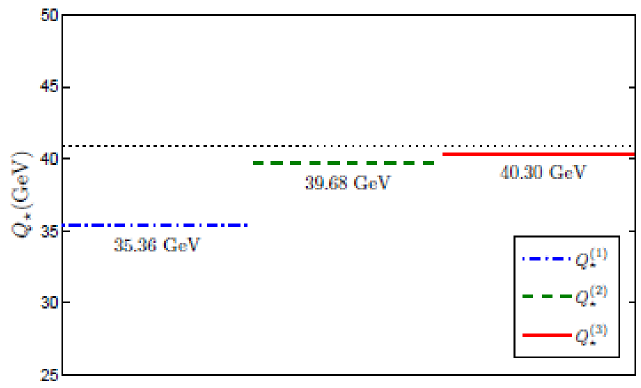


FIG. 1. The determined PMC scale $Q_\star$ for $R(Q)$ up to N^{2}LLO accuracy. $Q_\star^{(1)}$ is at the LLO accuracy, $Q_\star^{(2)}$ is at the NLLO accuracy and $Q_\star^{(3)}$ is at the N^{2}LLO accuracy. $Q = 31.6$ GeV.

$$\ln \frac{Q_\star^2}{Q^2} = 0.2249 + 1.6369\alpha_s(Q) + 1.5559\alpha_s^2(Q).$$

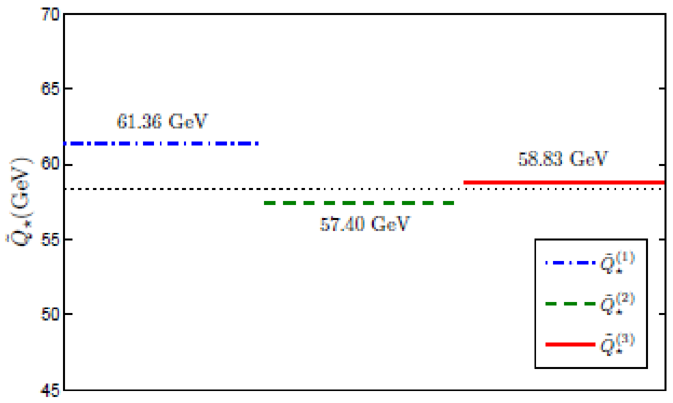


FIG. 2. The determined PMC scale $Q_\star$ for $H \to b\bar{b}$ up to N^{2}LLO accuracy. $Q_\star^{(1)}$ is at the LLO accuracy, $Q_\star^{(2)}$ is at the NLLO accuracy and $Q_\star^{(3)}$ is at the N^{2}LLO accuracy. $\mu = M_H$.

$$\ln \frac{\tilde{Q}_\star^2}{M_H^2} = -1.4389 - 1.1847\alpha_s(M_H) + 3.8753\alpha_s^2(M_H)$$

最新的考虑及尝试

残留的方案依赖性－重整化群方程本身是方案依赖的？

－引入方案无关的重整化群方程？

－C－方案耦合常数真的可行

残留的能标依赖性－越是高阶，重整化残留依赖越大？

－这是因为PMC能标的微扰收敛性

－PMC单能标方案可将其压到最低

C－方案+单能标PMC方案 ＝ 终极解决－－我们的初步想法

Colloquium: The QCD running coupling and the elimination of the scheme-and-scale ambiguities for fixed-order pQCD predictions

What's C-scheme coupling ?

存在残留方案依赖性原因，实际在于RGE本身是依赖于方案的；
为最大程度的降低方案依赖性，有没有可能RGE是不依束于方案的？

重新定义强耦合常数有可能实现目标

$$\frac{1}{a_\mu} + \frac{\beta_1}{\beta_0}\ln a_\mu = \beta_0\left(\ln\frac{\mu^2}{\Lambda^2} - \int_0^{a_\mu}\frac{da}{\tilde{\beta}(a)}\right).$$

As suggested by Ref.(Boito *et al.*, 2016), one can define a new coupling $\hat{a}_\mu = \hat{\alpha}_s(\mu)/\pi$ in the following way:

$$\frac{1}{\hat{a}_\mu} + \frac{\beta_1}{\beta_0}\ln\hat{a}_\mu = \beta_0\left(\ln\frac{\mu^2}{\Lambda^2} + C\right),$$

$$\hat{\beta}(\hat{a}_\mu) = \mu^2\frac{\partial\hat{a}_\mu}{\partial\mu^2} = -\frac{\beta_0\hat{a}_\mu^2}{1-\frac{\beta_1}{\beta_0}\hat{a}_\mu} = -\beta_0\hat{a}_\mu^2\sum_{i=0}^{\infty}(\beta_1/\beta_0)^i\,\hat{a}_\mu^i.$$

$$\frac{\partial\hat{a}_\mu}{\partial C} = \hat{\beta}(\hat{a}_\mu).$$

随能标及方案跑动**RGE**

$$\begin{aligned}\hat{a}_\mu &= a_\mu - C\beta_0 a_\mu^2 + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} + \beta_0^2C^2 - \beta_1 C\right)a_\mu^3 \\ &\quad + \left[\frac{\beta_1^3}{2\beta_0^3} - \frac{\beta_3}{2\beta_0} + \left(2\beta_2 - \frac{3\beta_1^2}{\beta_0}\right)C + \frac{5}{2}\beta_0\beta_1C^2 - \beta_0^3C^3\right]a_\mu^4 + \mathcal{O}(a_\mu^5).\end{aligned}$$

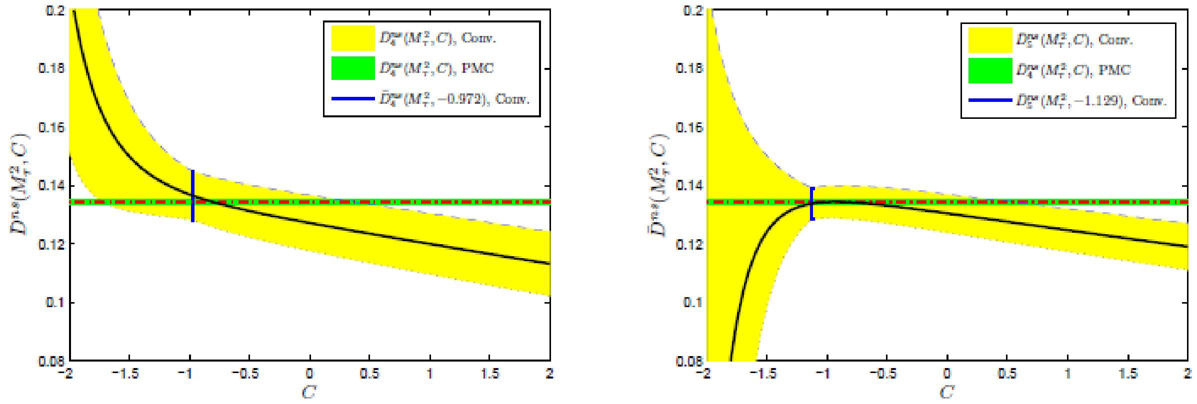


FIG. 4 (color online) $\bar{D}^{ns}(M_\tau^2,C)$ for the Adler function as a function of the parameter C. The solid line is the prediction using conventional scale setting, the lighter-shaded band is the uncertainty for a four-loop prediction $\Delta = \pm|\hat{c}_4(\mu/M_\tau)\hat{a}_\mu^4|_{\rm MAX}$ (Left) and for an approximate five-loop prediction $\Delta = \pm|\hat{c}_5(\mu/M_\tau)\hat{a}_\mu^5|_{\rm MAX}$ (Right), where MAX is the maximum value for $\mu \in [M_\tau, 4M_\tau]$. When $C = -0.972$ (Left) and $C = -1.129$ (Right), the error bar as shown by a vertical solid line is the minimum. The dash-dot line represents the four-loop PMC prediction, and the darker shaded band is for $\Delta = \pm|\hat{r}_{4,0}\hat{a}_{Q_*}^4|$. The independence of the PMC prediction on the parameter C demonstrates its scheme-independence.

$$\frac{\partial\hat{\rho}_n}{\partial C} = -\frac{\partial\hat{a}_\mu}{\partial C}\frac{\partial\hat{\rho}_n}{\partial\hat{a}_\mu} = -\mu^2\frac{\partial\hat{a}_\mu}{\partial\mu^2}\frac{\partial\hat{\rho}_n}{\partial\hat{a}_\mu} = -\hat{\beta}(\hat{a}_\mu)\frac{\partial\hat{\rho}_n}{\partial\hat{a}_\mu},$$

消除β-项，即消除方案C依赖

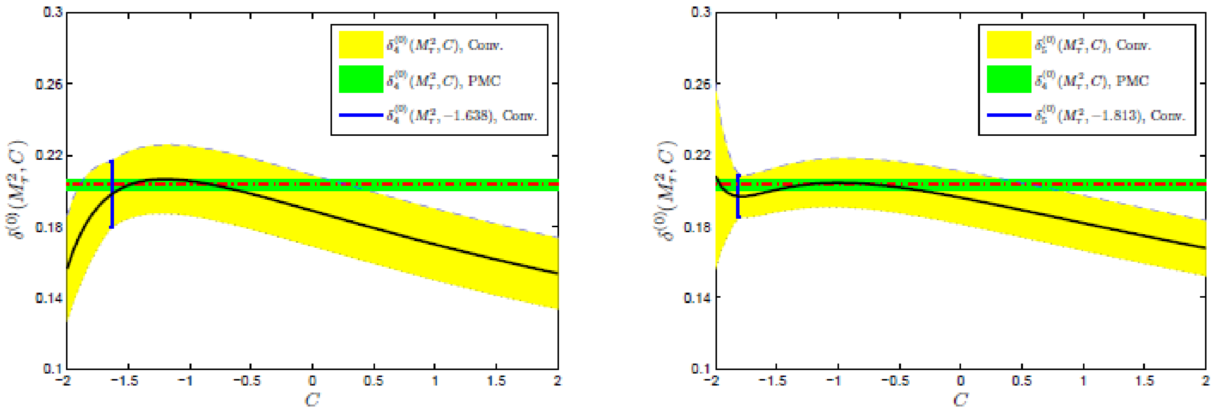


FIG. 8 (color online) $\delta^{(0)}(M_\tau^2,C)$ for τ decay as a function of the parameter C. The solid line is the prediction using conventional scale setting, the lighter-shaded band is the uncertainty for a four-loop prediction $\Delta = \pm|\hat{c}_4(\mu/M_\tau)\hat{a}_\mu^4|_{\rm MAX}$ (Left) and for an approximate five-loop prediction $\Delta = \pm|\hat{c}_5(\mu/M_\tau)\hat{a}_\mu^5|_{\rm MAX}$ (Right), where MAX is the maximum value for $\mu \in [M_\tau, 4M_\tau]$. When $C = -1.638$ (Left) and $C = -1.183$ (Right), the error bar as shown by a vertical solid line is the minimum. The dash-dot line represents the four-loop PMC prediction, and the darker shaded band is for $\Delta = \pm|\hat{r}_{4,0}\hat{a}_{Q_*}^4|$. The independence of the PMC prediction on the parameter C demonstrates its scheme-independence.

最大共形原理的一些实例

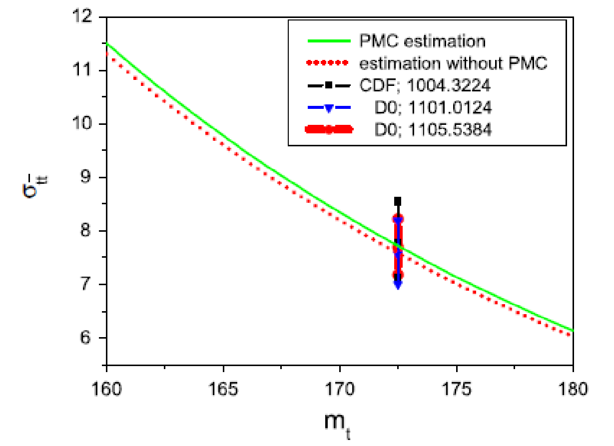
顶夸克对的强产生

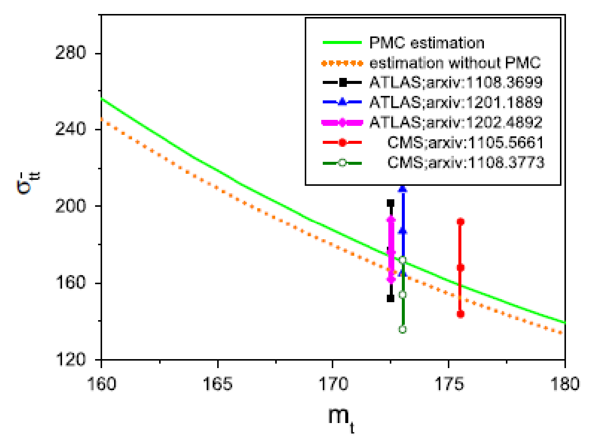
重整化能标不确定性的消除

TABLE I. Dependence of the $t\bar{t}$ production cross sections (in unit: pb) at the Tevatron and LHC on the initial renormalization scale $\mu_R^{init} = Q$. Here, $m_t = 172.9$ GeV. The number in parenthesis shows the Monte Carlo uncertainty in the last digit.

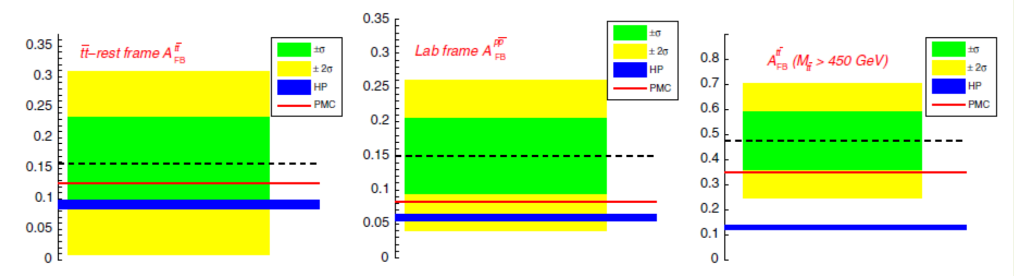
	PMC scale setting					Conventional scale setting		
	$Q = m_t/4$	$Q = m_t$	$Q = 10m_t$	$Q = 20m_t$	$Q = \sqrt{s}$	$\mu_R \equiv m_t/2$	$\mu_R \equiv m_t$	$\mu_R \equiv 2m_t$
Tevatron (1.96 TeV)	7.620(5)	7.626(3)	7.625(5)	7.624(6)	7.628(5)	7.742(5)	7.489(3)	7.199(5)
LHC (7 TeV)	171.6(1)	171.8(1)	171.7(1)	171.7(1)	171.7(1)	168.8(1)	164.6(1)	157.5(1)
LHC (14 TeV)	941.8(8)	941.3(5)	942.0(8)	941.4(8)	942.2(8)	923.8(7)	907.4(4)	870.9(6)

总截面与实验值更好的符合





解释前后不对称性



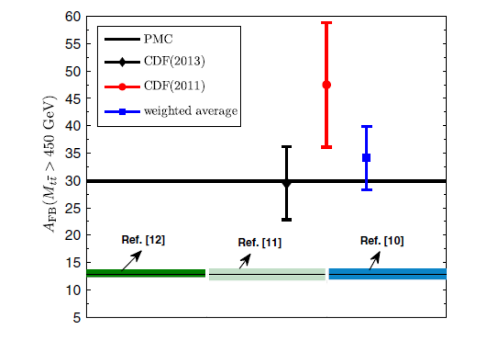


FIG. 1. Comparison of the PMC prediction for the top-pair asymmetry $A_{FB}(M_{t\bar{t}} > 450$ GeV) with the CDF measurement [5,6]. The NLO results predicted by Ref. [12], Ref. [11], and Ref. [10] under conventional scale setting are presented as a comparison, and are shown by shaded bands.

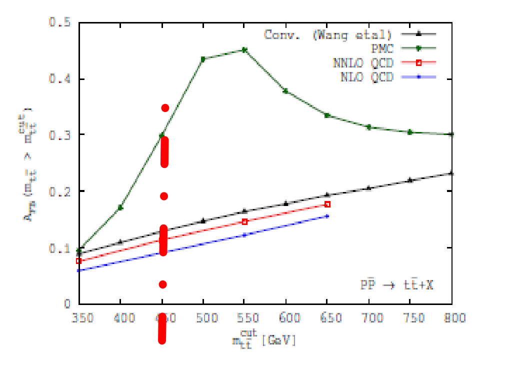


Figure 11. Predictions for the $m_{t\bar{t}}$ cumulative asymmetry: pure QCD at NLO and NNLO (as derived in this work), NLO prediction of Ref. [11] including EW corrections, as well as the PMC scale-setting prediction of Ref. [11].

Bc介子半轻衰变

消除重整化能标不确定性

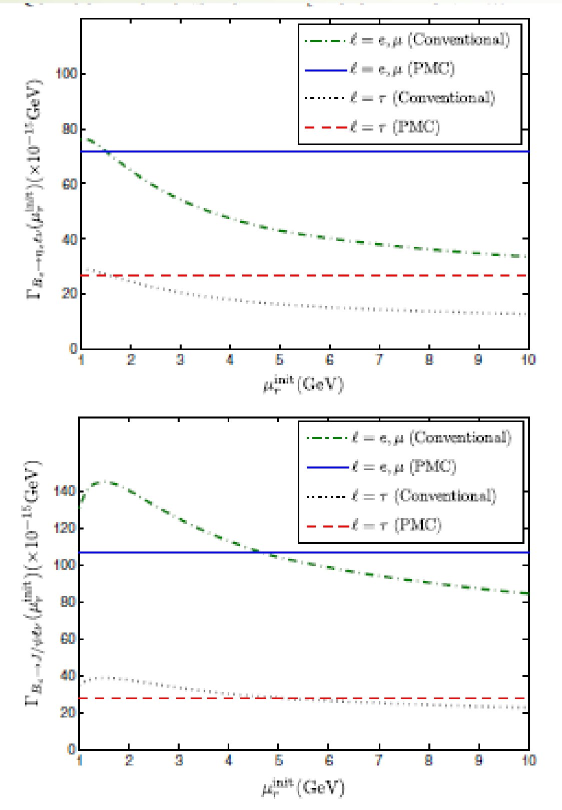


FIG. 3 (color online). The decay widths for $B_c \to \eta_c(J/\psi)\ell\nu$ up to NLO level versus the initial scale $\mu_r^{\rm init}$ under the conventional and the PMC scale settings, respectively.

与实验一致的R值

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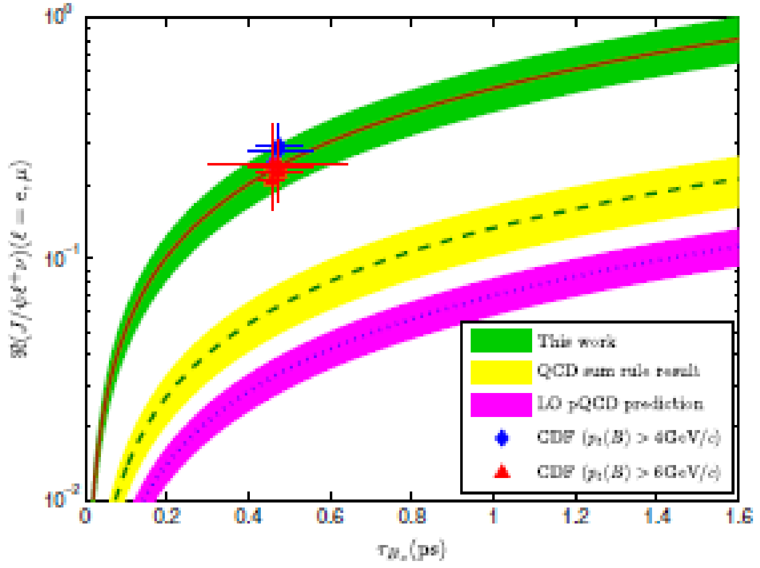

FIG. 4 (color online). The value of $\Re(J/\psi\ell^+\nu)$ after the PMC scale setting, which is shown by the upper shaded band. The CDF predictions [7–10], the QCD sum rule (SR) prediction [28], and the LO pQCD prediction [60] are presented as a comparison. The middle shaded band represents the QCD sum rule prediction and the lower shaded band is the LO pQCD prediction under the conventional scale setting.

$$\Re(J/\psi\ell^+\nu) = \frac{\sigma(B_c^+){\rm BR}(B_c^+ \to J/\psi\ell^+\nu)}{\sigma(B^+){\rm BR}(B^+ \to J/\psi K^+)}.$$

电弱参数ρ

$\rho = 1 + \Delta\rho$

PMC后更快趋于稳定值

消除重整化能标不确定性

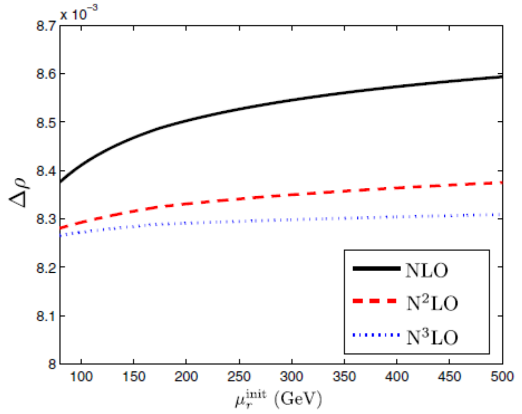

FIG. 1 (color online). The $\Delta\rho$ parameter versus the initial renormalization scale μ_r^{init} under the conventional scale setting. The solid, dashed and doted lines stand for QCD corrections up to NLO/two-loop, N^{2}LO/three-loop and N^{3}LO/four-loop, respectively.

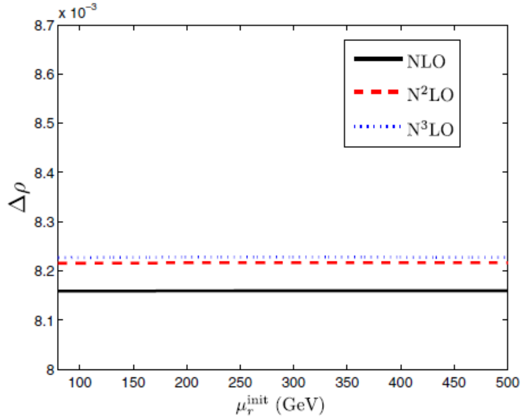

FIG. 2 (color online). The $\Delta\rho$ parameter versus the initial renormalization scale μ_r^{init} under the PMC scale setting. The solid, dashed and doted lines stand for QCD corrections up to NLO/two-loop, N^{2}LO/three-loop and N^{3}LO/four-loop, respectively.

TABLE II. The parameter $\Delta\rho$, the shift $\delta\rho$, and the K factor before and after the PMC scale setting. $\Delta\rho|_i$ with i = LO, NLO, N^{2}LO and N^{3}LO denote the QCD corrections up to one-loop, two-loop, three-loop, and four-loop levels, respectively. The $\delta\rho|_i$ and K_i stand for the shift of $\Delta\rho|_i$ and K factor for the two-loop, three-loop or four-loop level, respectively. $\mu_r^{init} = M_t$.

	Conventional scale setting				PMC scale setting			
	LO	NLO	N^{2}LO	N^{3}LO	LO	NLO	N^{2}LO	N^{3}LO
$\Delta\rho\|_i(\times 10^{-3})$	9.411	8.483	8.305	8.257	9.411	8.175	8.217	8.228
$\delta\rho\|_i(\times 10^{-3})$	⋯	−0.928	−0.178	−0.048	⋯	−1.236	0.042	0.011
K_i	⋯	9.8%	2.1%	0.6%	⋯	13%	0.5%	0.1%

TABLE III. The shifts δM_W and $\delta \sin^2\theta_{\rm eff}^{\rm left}$ due to the QCD improved ρ parameter before and after the PMC scale setting, where the symbols NLO, N^{2}LO, and N^{3}LO shift due to the QCD corrections up to two-loop, three-loop, and four-loop levels, respectively. $\mu_r^{init} = M_t$.

	Conventional			PMC		
	NLO	N^{2}LO	N^{3}LO	NLO	N^{2}LO	N^{3}LO
$\delta M_W\|_i$ (MeV)	−52.3	−10.0	−2.7	−69.7	+2.4	+0.7
$\delta\sin^2\theta_{\rm eff}^{\rm left}\|_i(\times 10^{-5})$	+29.0	+5.6	+1.5	+38.6	−1.3	−0.4

Z玻色子的强衰变

消除重整化能标不确定性

PHYSICAL REVIEW D 90, 037503 (2014)

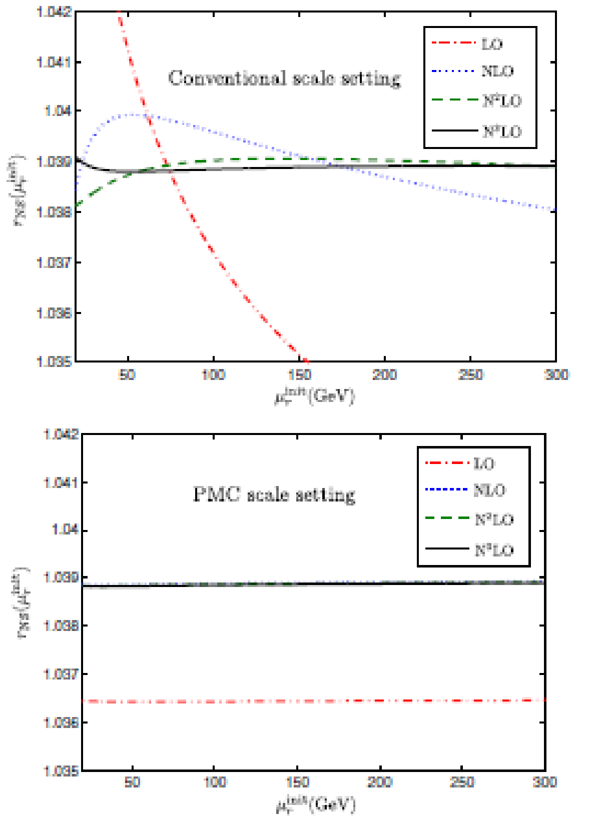

FIG. 1 (color online). The nonsinglet contribution r_{NS} up to four-loop QCD corrections versus μ_r^{init} before and after PMC scale setting. After PMC scale setting, the curves for NLO, N²LO, and N³LO almost coincide with each other.

TABLE I. Perturbative contributions for the nonsinglet r_{NS} under the conventional (Conv.) and the PMC scale settings. Here, $R_i^{NS} = C_i^{NS} a_s^i$ with $i = (1, \ldots, 4)$ stand for the one-, two-, three-, and four-loop terms, respectively. $\mu_r^{init} = M_Z$.

	R_1^{NS}	R_2^{NS}	R_3^{NS}	R_4^{NS}	$\sum_{i=1}^4 R_i^{NS}$
Conv.	0.03769	0.00200	−0.00069	−0.00016	0.03884
PMC	0.03636	0.00252	−0.00003	−0.00001	0.03885

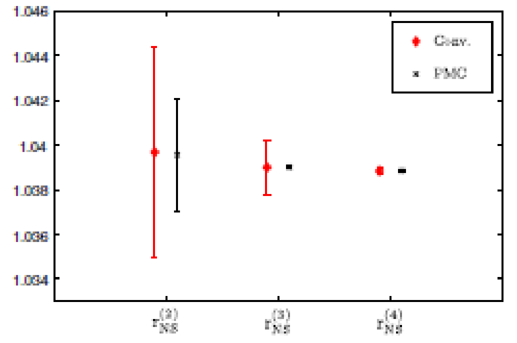

FIG. 4 (color online). The values of $r_{NS}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{NS} a_s^i$ and their errors $\pm|C_n^{NS} a_s^n|_{MAX}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{init} = M_Z$.

快速收敛

Higgs玻色子的强产生

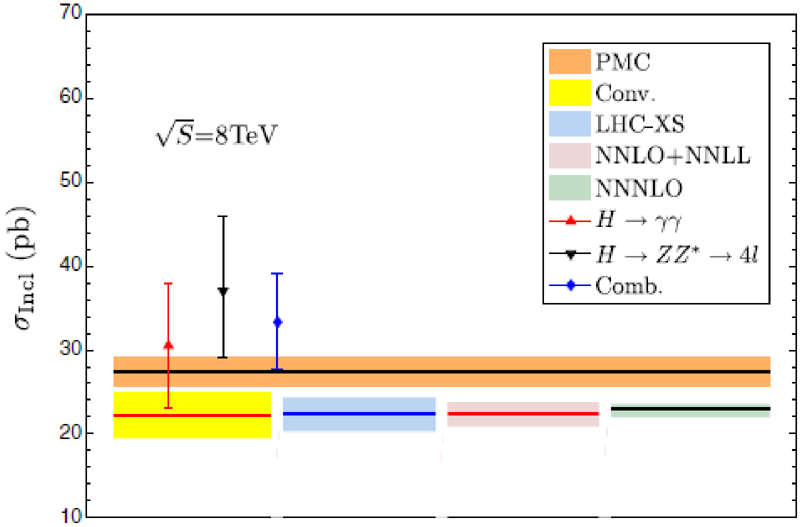

FIG. 6. Comparison of the NNLO conventional vs PMC predictions for the total inclusive cross section $\sigma_{\rm Incl}$ with the latest ATLAS measurements at 8 TeV [4]. The LHC-XS predictions [3], the NNLO + NNLL prediction [9], and the NNNLO prediction [10] are presented as a comparison. The solid lines are central values.

TABLE V. The fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ (in units of fb) at the LHC for c.m. collision energies $\sqrt{S} = 7, 8$ and 13 TeV, respectively.

$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [51]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.3^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

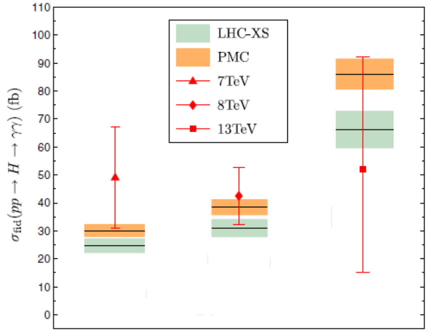

FIG. 7. Comparison of the PMC predictions for the fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ with ATLAS measurements at various collision energies [51]. The LHC-XS predictions [3] are presented as a comparison.

TABLE V. The fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ (in units of fb) at the LHC for c.m. collision energies $\sqrt{S} = 7, 8$ and 13 TeV, respectively.

$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [51]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.3^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

A solution to the $\gamma\gamma^* \to \eta_c$ puzzle using the Principle of Maximum Conformality

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(Dated: March 22, 2018)

Preliminary

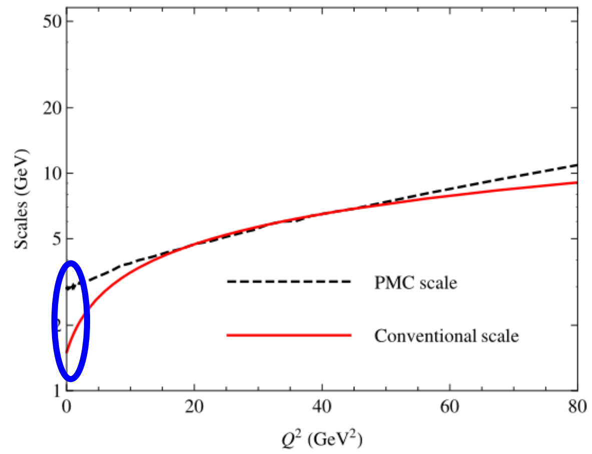


FIG. 1: The PMC scale of the NNLO form factor $F(Q^2)$ versus the momentum transfer squared Q^2. The usual choice of $\mu_r = \sqrt{Q^2 + m_c^2}$ is presented as a comparison.

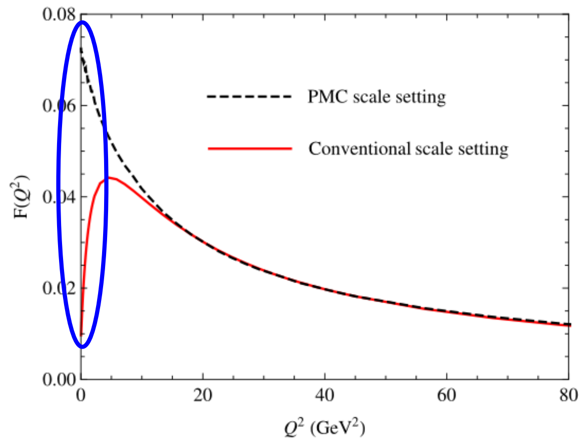
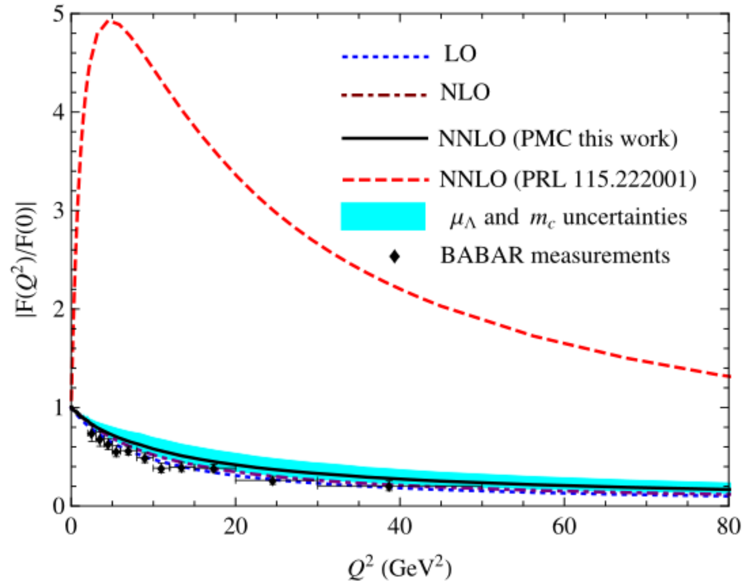


FIG. 2: The form factor $F(Q^2)$ versus the momentum transfer squared Q^2 under conventional and PMC scale setting.



Summary and Outlook

Up to infinite order, the predictions are scheme and scale independent, there is no scale ambiguity

At fixed-order, guessing/using typical momentum flow as the scale, one cannot get precise value for all-orders, and also for each order, becoming an important systematic error

希望即使是在有限阶，也可以找到一个普适方案确定出最优能标：**得到已知阶下的准确理论预言；同时通过提高收敛性得到更快更接近于真实值的理论预言；为新物理的寻找提供更好的依据**

PMC不是简单的选择“特殊/有效-能标”
而是基于-重整化群方程以及基本重整化群不变性--提供具普适性的可系统设定高能物理过程“最优”重整化能标的方案

有限阶下，PMC最优能标的思想
并不违背微扰论展开到无穷阶下与能标选择无关的基本理念

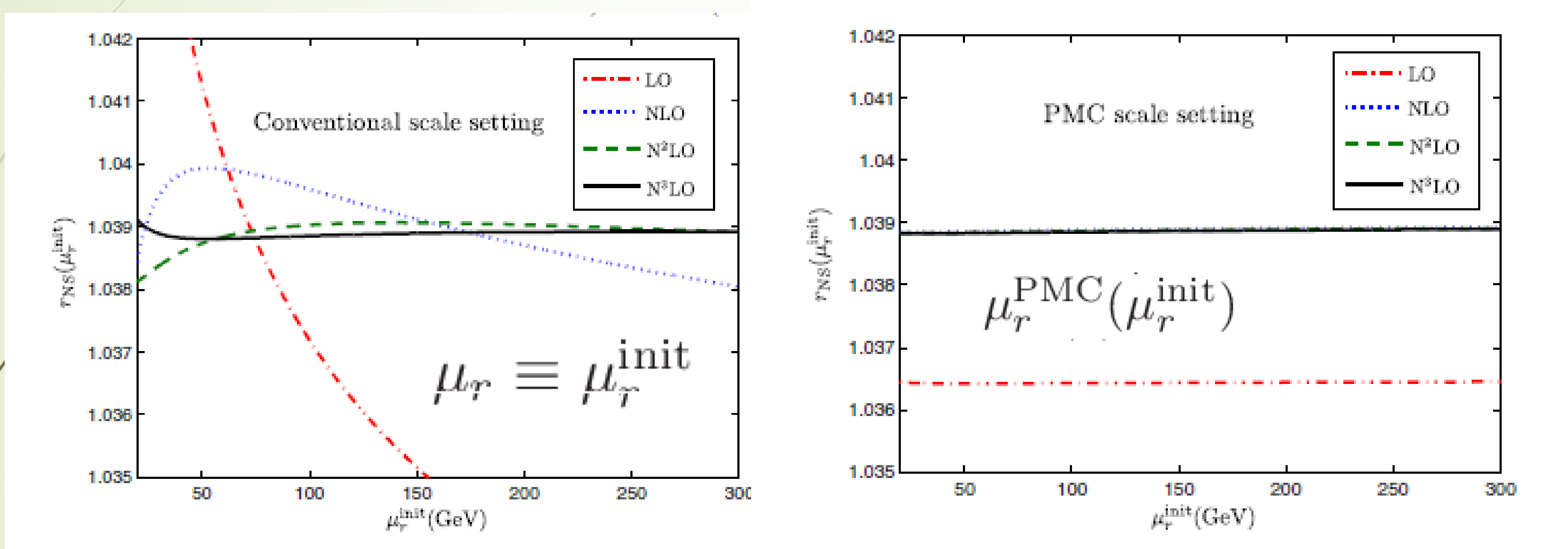
采用PMC最优能标
1）快速收敛
2）快速稳定，获得物理量真实值
3）低阶下就与初始能标、重整化方案选择无关，获得每一阶的准确值

传统方案
1）收敛慢－收敛性纯属猜测和运气
2）计算到任意高阶也无法获得每一阶真实值
3）非常高阶时才能获得与能标选择无关的预言值

希望大家在完成复杂圈图计算之后，更进一步，讨论相对简单的重整化能标设定

注：我们已表明PMC能标与初始能标选择无关，那么真正在做时，可以直接采用通常的典型能标做为初始能标，然后再利用标准PMC步骤处理获得预言 －－ 所以并不难做

Before and after applying the PMC, the issues are always like this



PMC predictions:
Quickly approaches its "true" value
More accurate predictions for low orders without initial scale dependence
Residual scale uncertainty is small

I) PMC solves the problem on when there are several momentum flows, which one is preferable for setting the scale. — example for hadronic Z decay, the PMC scale is close to mZ, but not mt. — Phys.Rev.D90,037503(2014)

II) How to apply PMC to deal with such kind of multiscale problem is in progress; There is no conflict on setting the scale and resuming large logs

III) How to use PMC predictions in connection with the low-energy data to achieve a better prediction of alphas in whole region is in progress

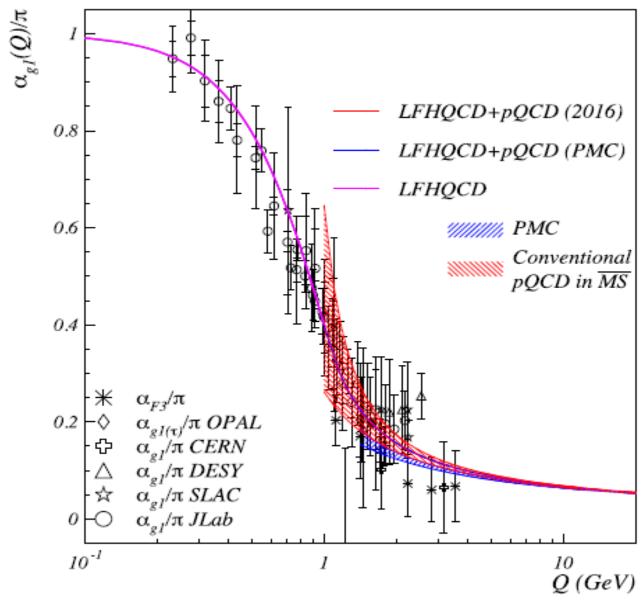


Fig. 2. (Color online.) Matching procedure applied to PMC calculation (blue line). It is matched to the LFHQCD results (magenta line) by requiring the continuity of both α_{g_1} and its β-function. The blue band is the PMC prediction evaluated down to $Q = 1.5$ GeV, and the red band shows the conventional $\overline{MS}$ prediction from Ref. [31]. We use $\kappa = 0.523$ GeV for LFHQCD.

Phys.Lett.B773,98(2017)

Our initial results indicates PMC may work, but we should do some proper alterations.

e.g. **how to get reliable PMC prediction in the region close to critical point**

Thanks