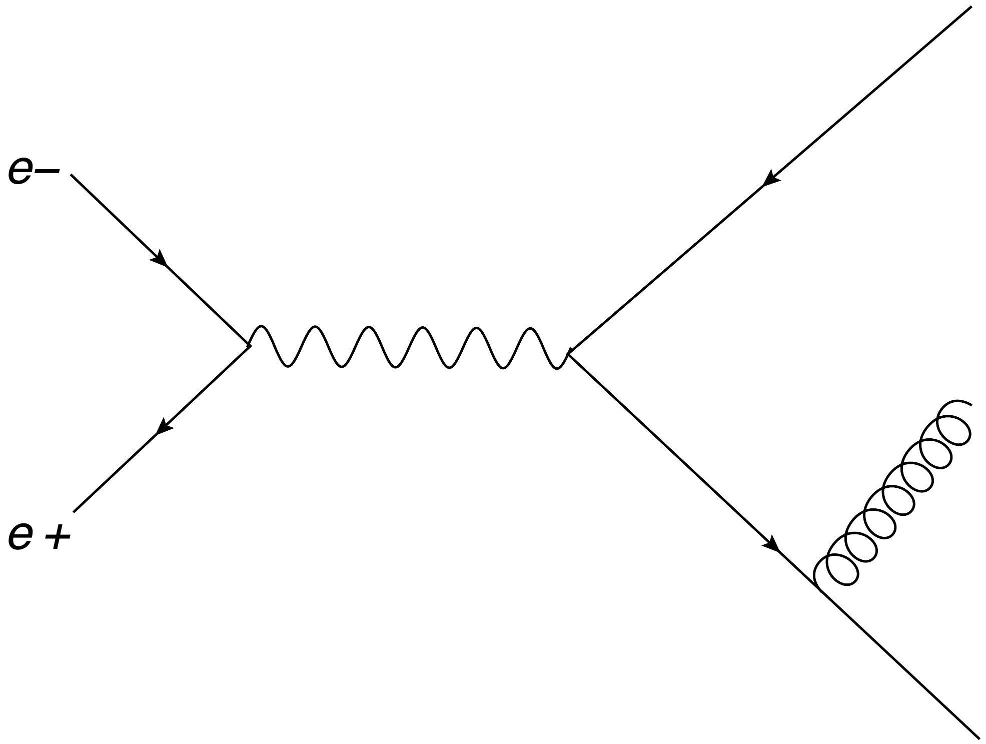
An Introduction to Resummation

Pier F. Monni
CERN

QCD@West Lake School and Workshop
March 2018, Hangzhou

Structure of a QCD event

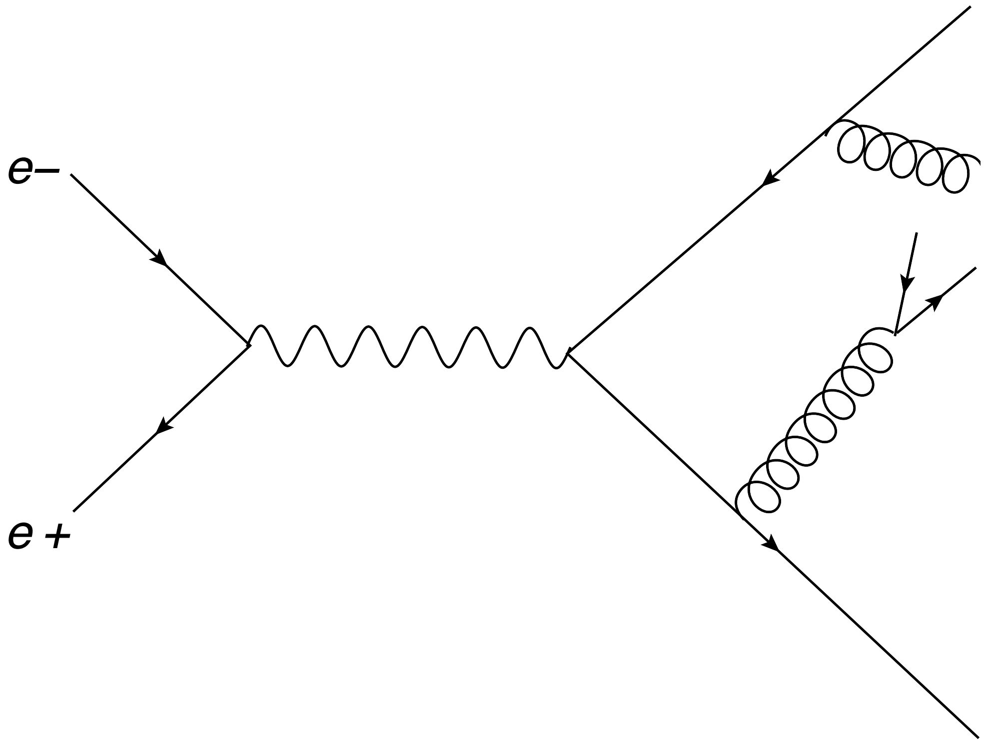


$$\sigma \sim 1 + \alpha_s$$

- Hard scattering with large momentum transfer into the hadronic system:
  - Theory description relies on perturbative expansion of observables in powers of the strong coupling constant
  - Coupling associated with extra radiation evaluated at typical "hardness" scales of the emissions (e.g. transverse momentum)

$$\alpha_s(k_t) \sim \alpha_s(Q) \ll 1$$

Structure of a QCD event

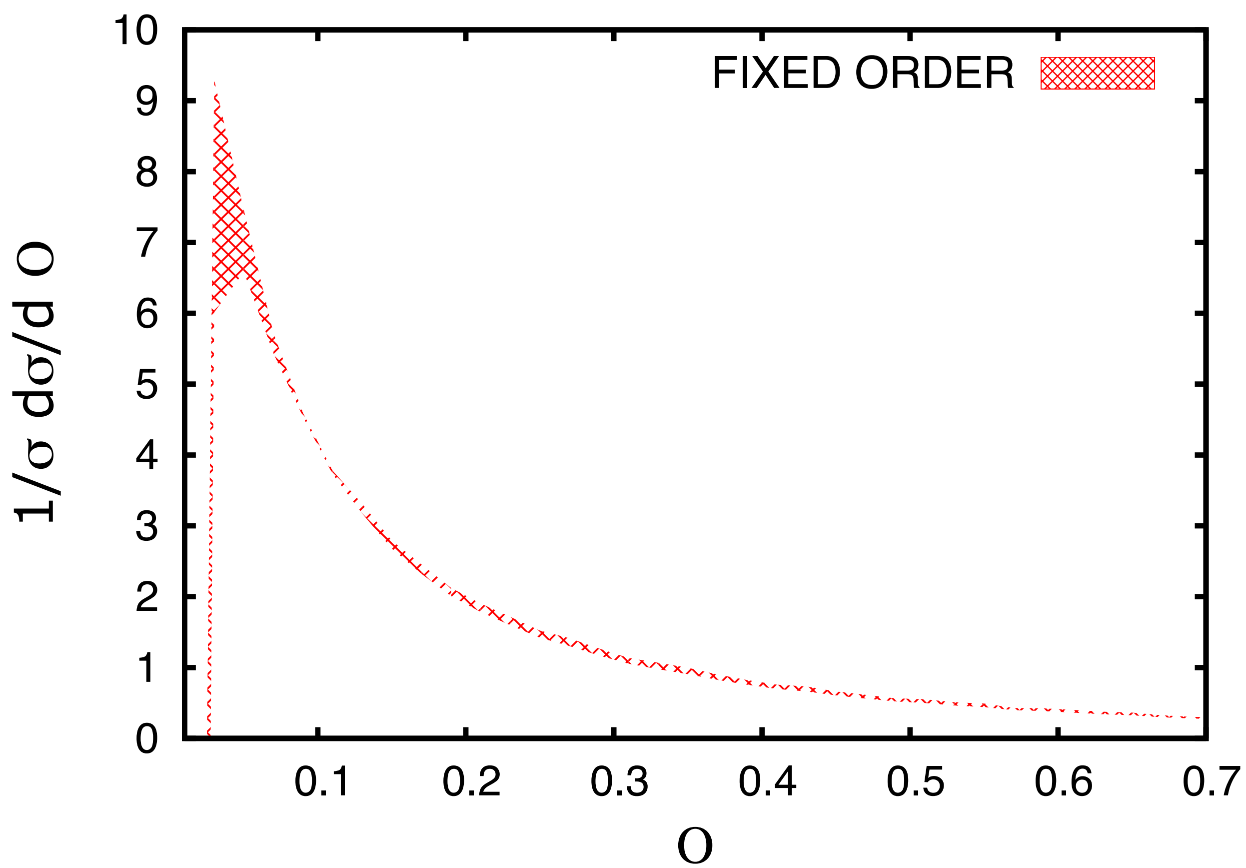


$$\sigma \sim 1 + \alpha_s + \alpha_s^2 + \alpha_s^3$$

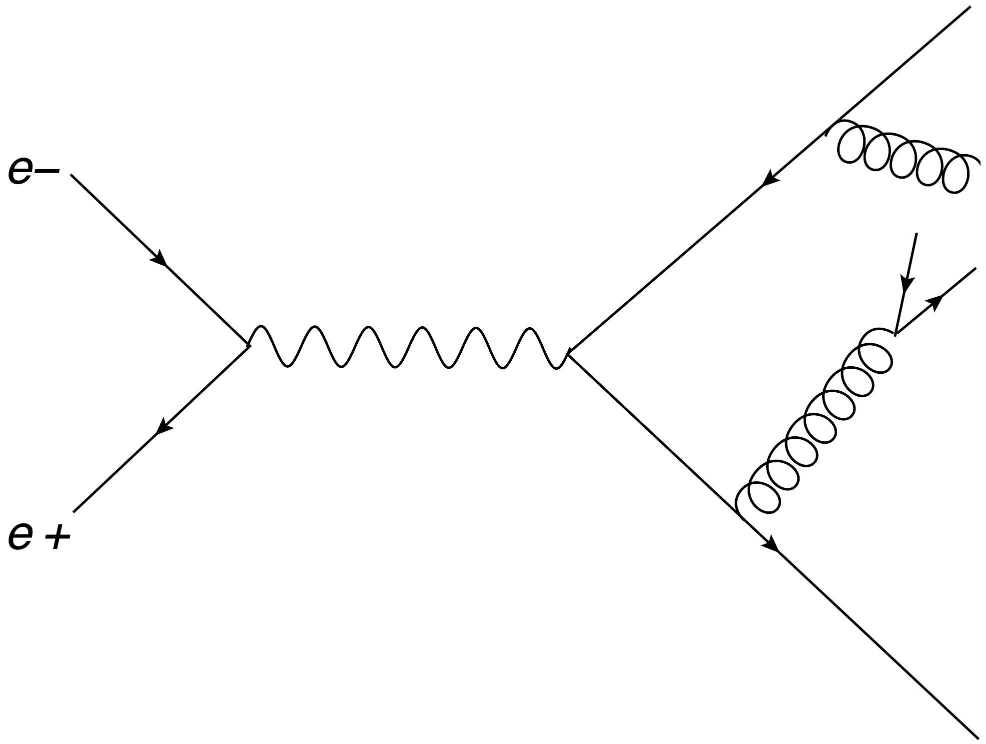
- Hard scattering with large momentum transfer into the hadronic system:
  - Theory description relies on perturbative expansion of observables in powers of the strong coupling constant
  - Coupling associated with extra radiation evaluated at typical "hardness" scales of the emissions (e.g. transverse momentum)

$$\alpha_s(k_t) \sim \alpha_s(Q) \ll 1$$

- First *few* terms of the series sufficient



Structure of a QCD event

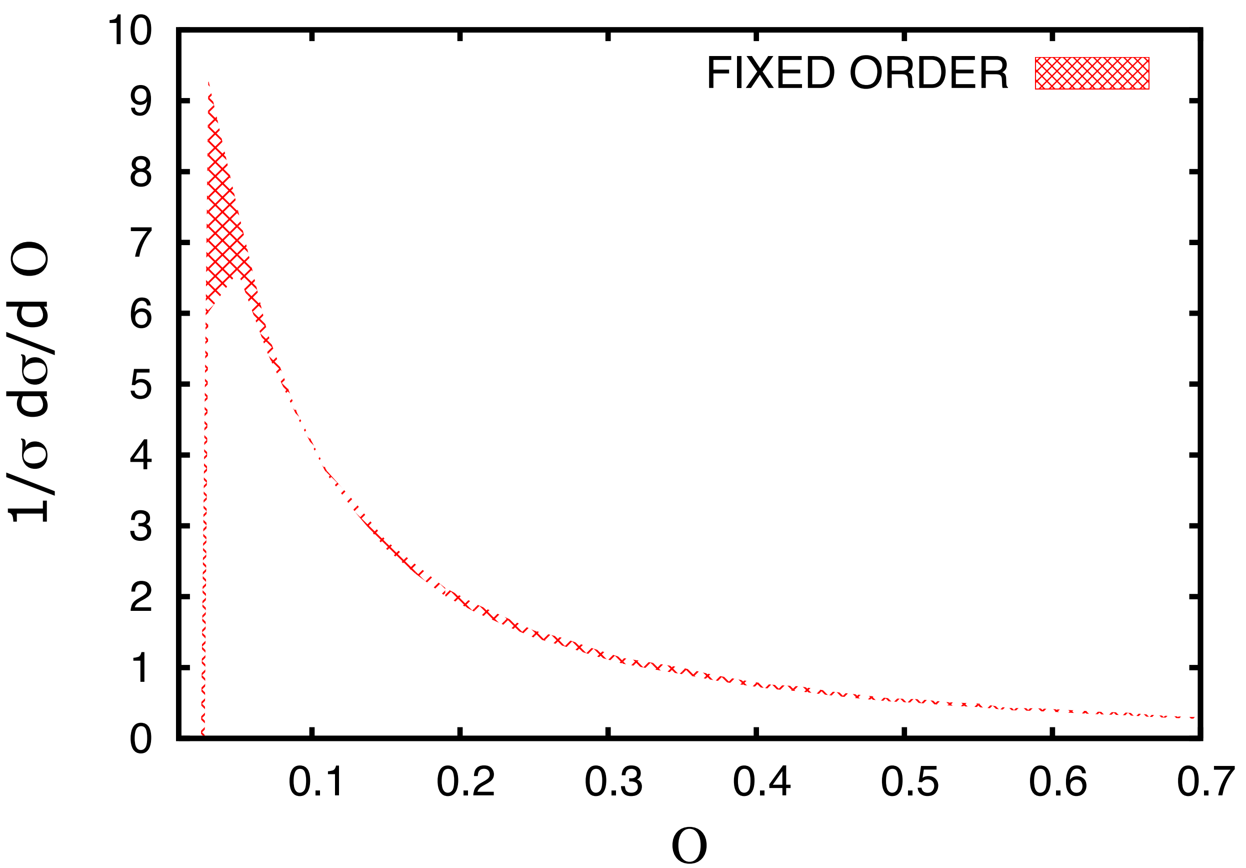


$$\sigma \sim 1 + \alpha_s + \alpha_s^2 + \alpha_s^3 + \dots$$

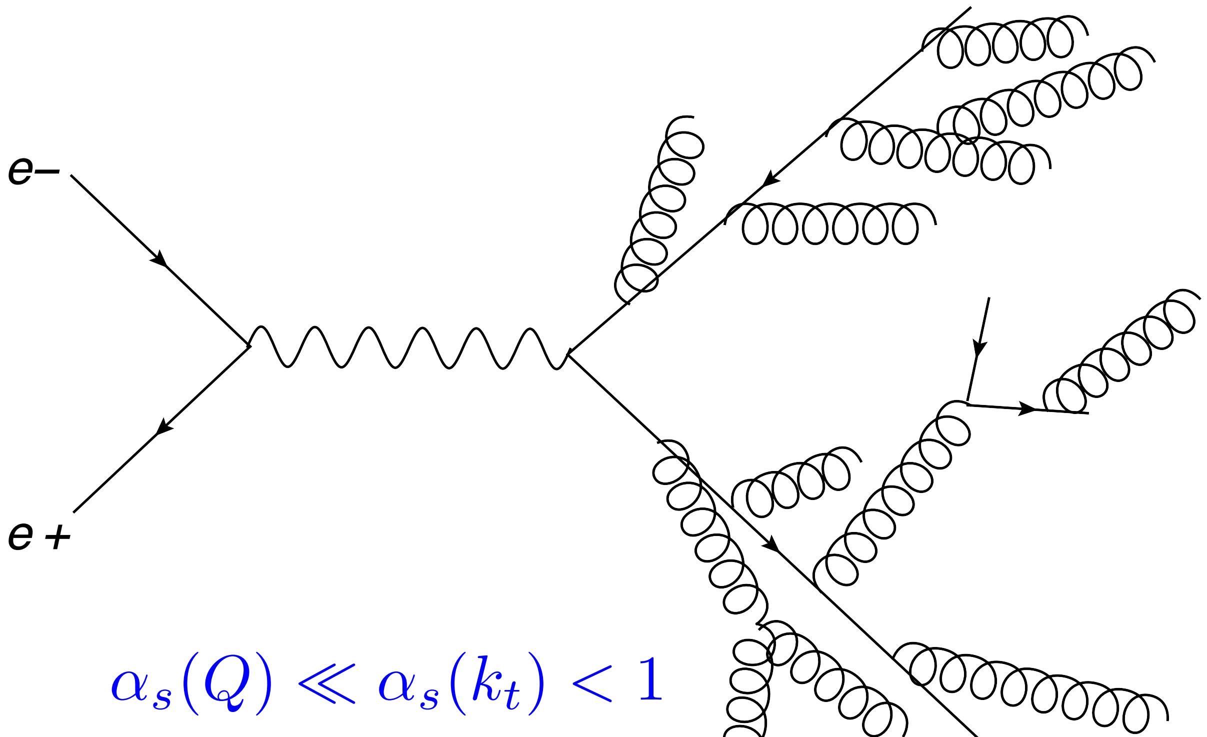
- Hard scattering with large momentum transfer into the hadronic system:
  - Theory description relies on perturbative expansion of observables in powers of the strong coupling constant
  - Coupling associated with extra radiation evaluated at typical "hardness" scales of the emissions (e.g. transverse momentum)

$$\alpha_s(k_t) \sim \alpha_s(Q) \ll 1$$

- First *few* terms of the series sufficient



Structure of a QCD event



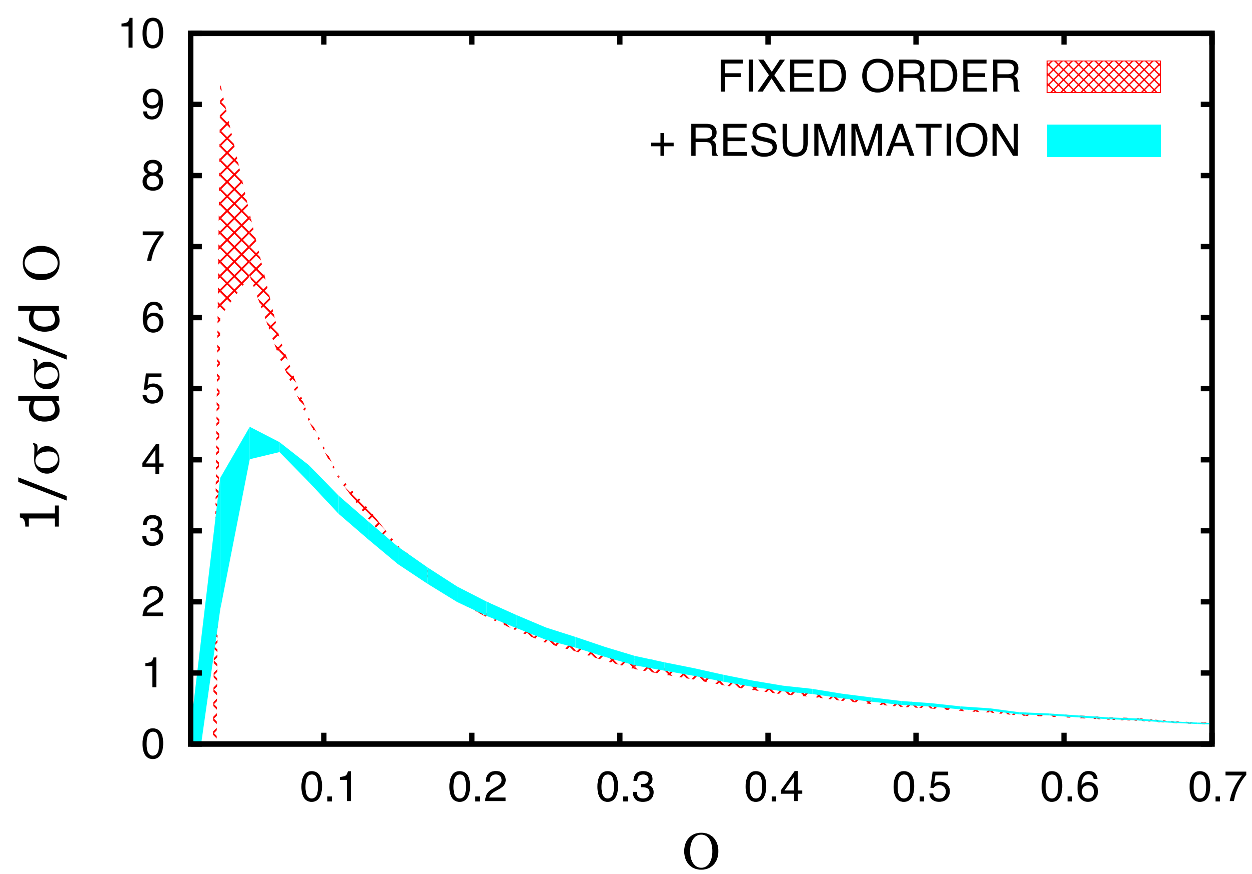
$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \sim \alpha_s \ln^2 O = \alpha_s L^2 \sim 1$$

$$\alpha_s(Q) \ll \alpha_s(k_t) < 1$$

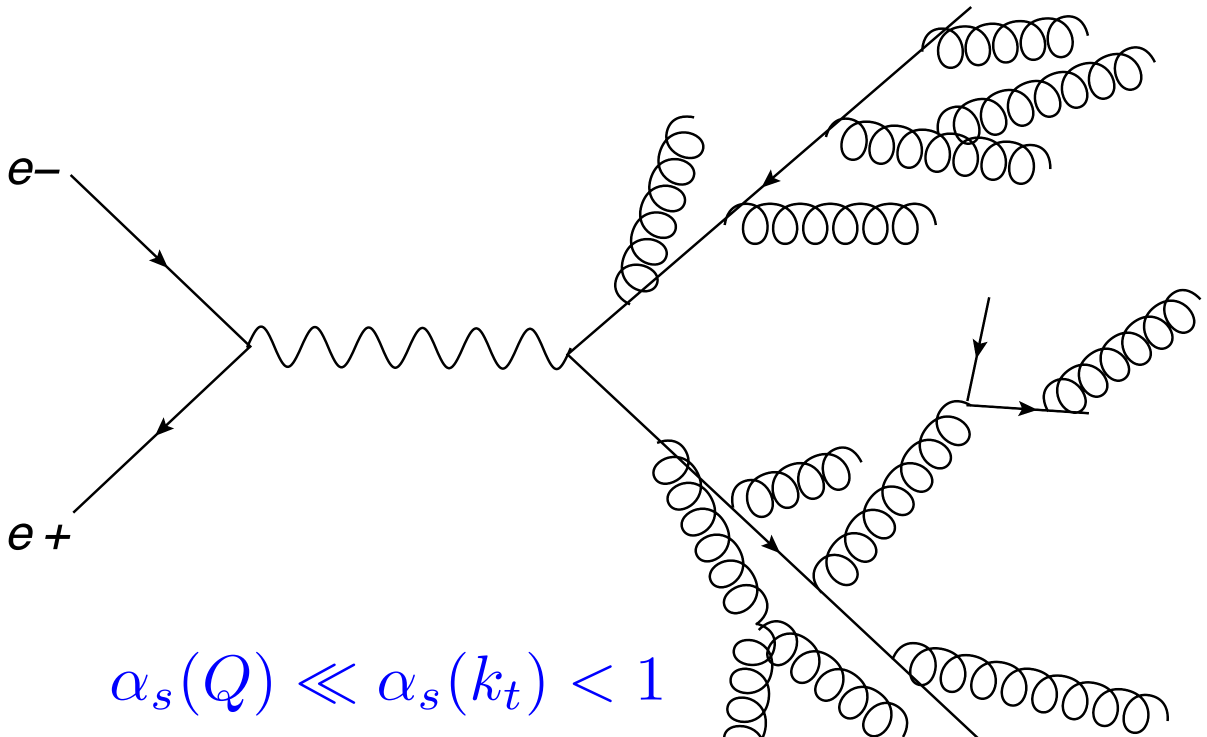
- As the coupling grows large, coloured particles are very likely to emit radiation with probability

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t)$$

- Inclusive Observables (e.g. total rates) are designed to be insensitive to such very soft and/or collinear (IRC) radiation
- However, the sensitivity to the IRC dynamics can become significant if one applies constraints on the radiation



Structure of a QCD event



$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \sim \alpha_s \ln^2 O = \alpha_s L^2 \sim 1$$

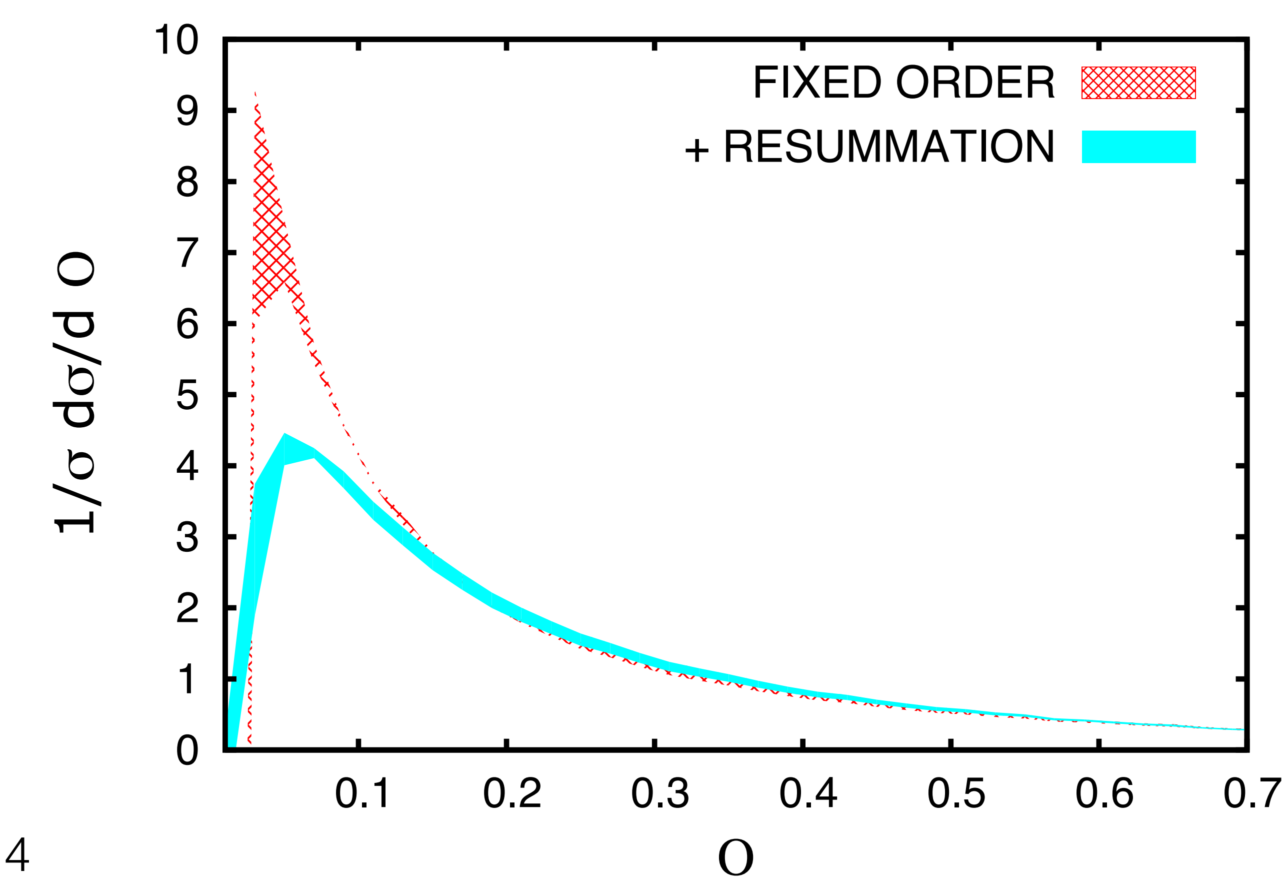
$$\sigma \sim 1 + \alpha_s L^2$$

$$\alpha_s(Q) \ll \alpha_s(k_t) < 1$$

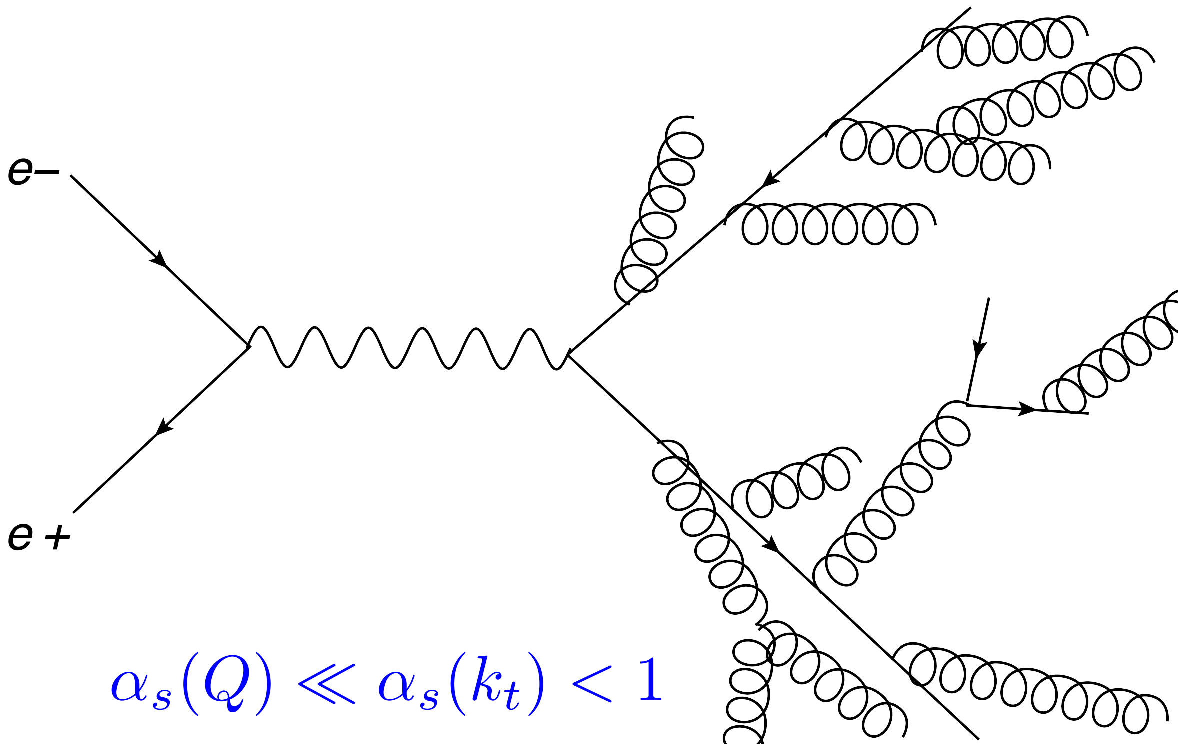
- As the coupling grows large, coloured particles are very likely to emit radiation with probability

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t)$$

- Inclusive Observables (e.g. total rates) are designed to be insensitive to such very soft and/or collinear (IRC) radiation
- However, the sensitivity to the IRC dynamics can become significant if one applies constraints on the radiation



Structure of a QCD event



$$\alpha_s(Q) \ll \alpha_s(k_t) < 1$$

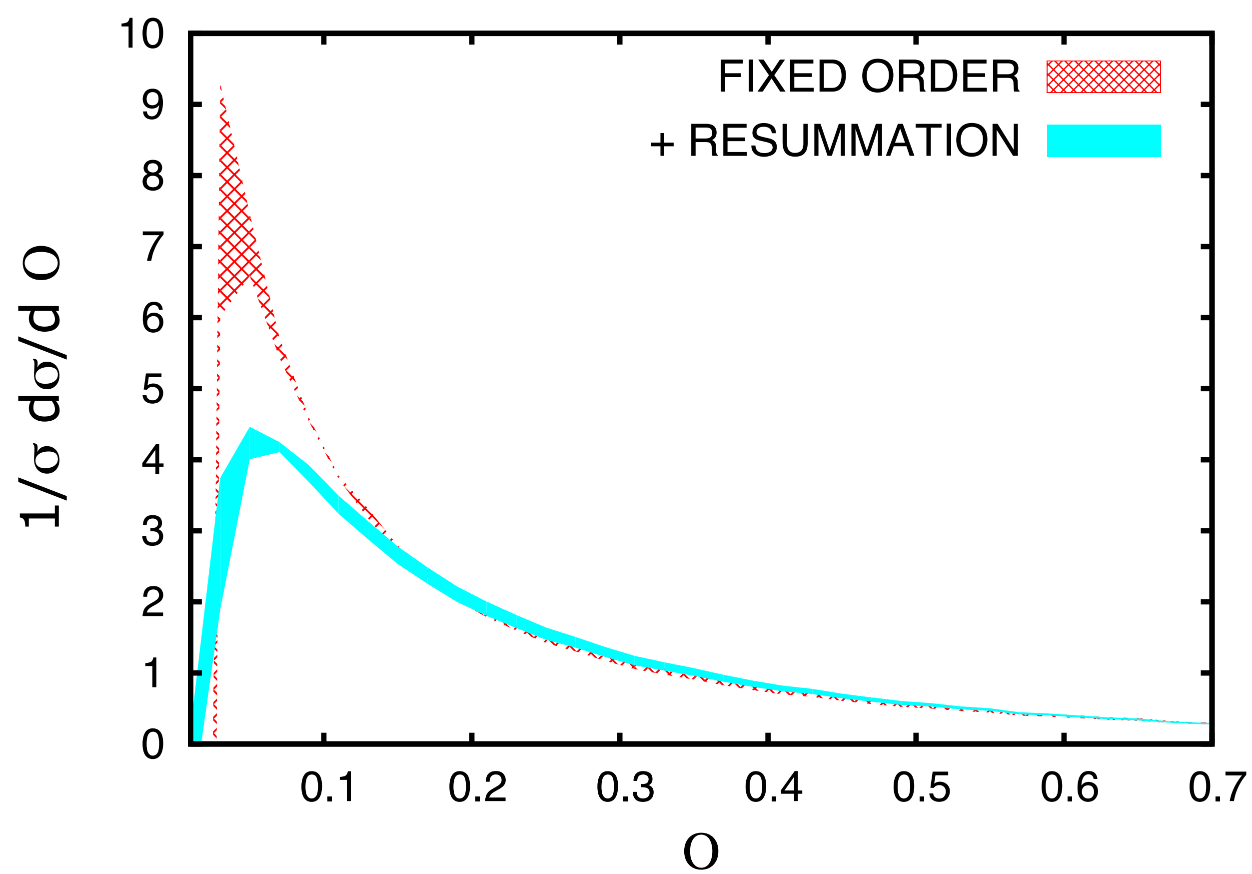
$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \sim \alpha_s \ln^2 O = \alpha_s L^2 \sim 1$$

$$\sigma \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4$$

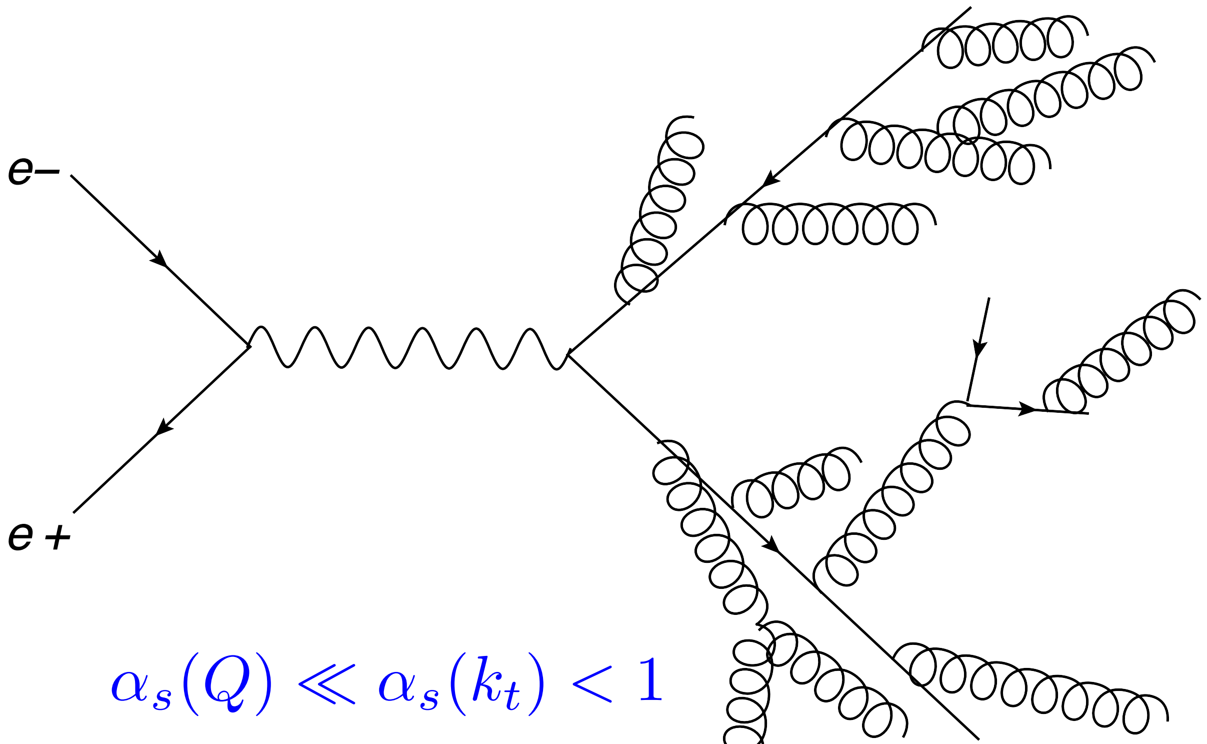
- As the coupling grows large, coloured particles are very likely to emit radiation with probability

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t)$$

- Inclusive Observables (e.g. total rates) are designed to be insensitive to such very soft and/or collinear (IRC) radiation
- However, the sensitivity to the IRC dynamics can become significant if one applies constraints on the radiation



Structure of a QCD event



$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \sim \alpha_s \ln^2 O = \alpha_s L^2 \sim 1$$

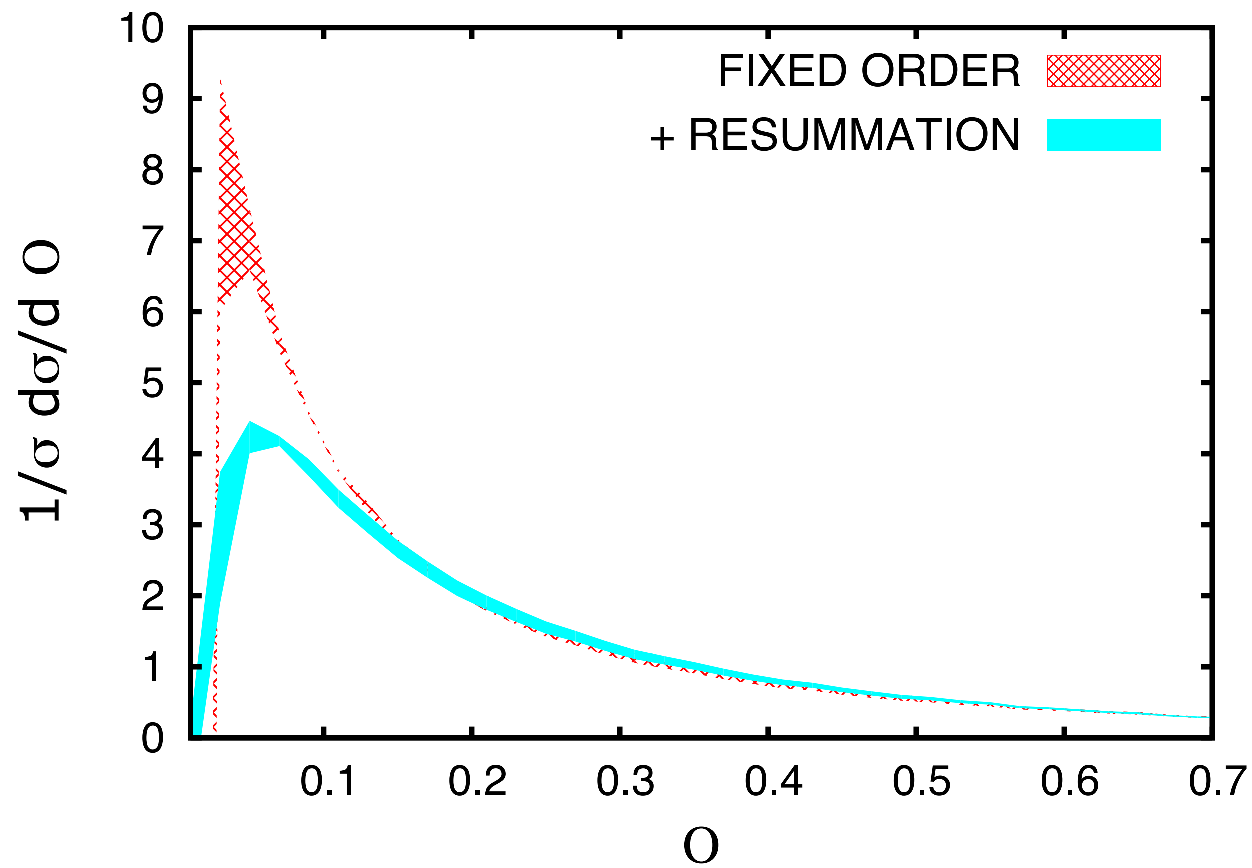
$$\sigma \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6$$

$$\alpha_s(Q) \ll \alpha_s(k_t) < 1$$

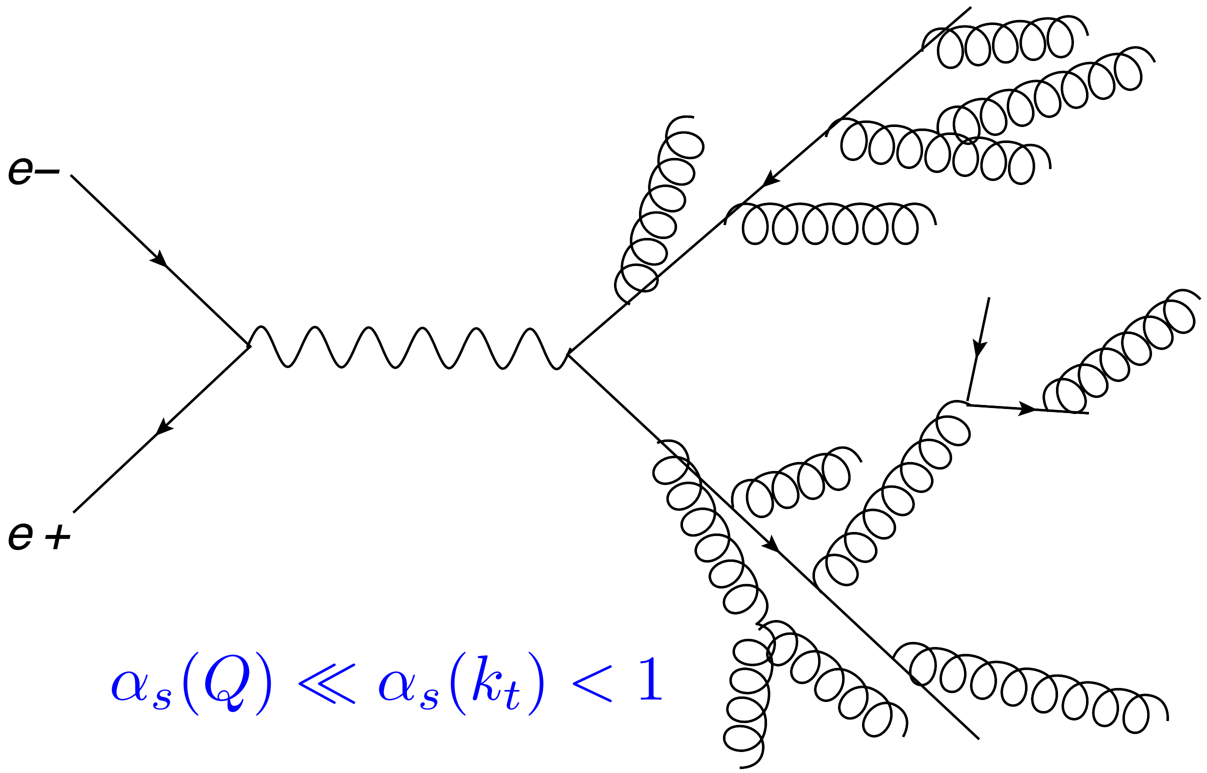
- As the coupling grows large, coloured particles are very likely to emit radiation with probability

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t)$$

- Inclusive Observables (e.g. total rates) are designed to be insensitive to such very soft and/or collinear (IRC) radiation
- However, the sensitivity to the IRC dynamics can become significant if one applies constraints on the radiation



Structure of a QCD event



$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \sim \alpha_s \ln^2 O = \alpha_s L^2 \sim 1$$

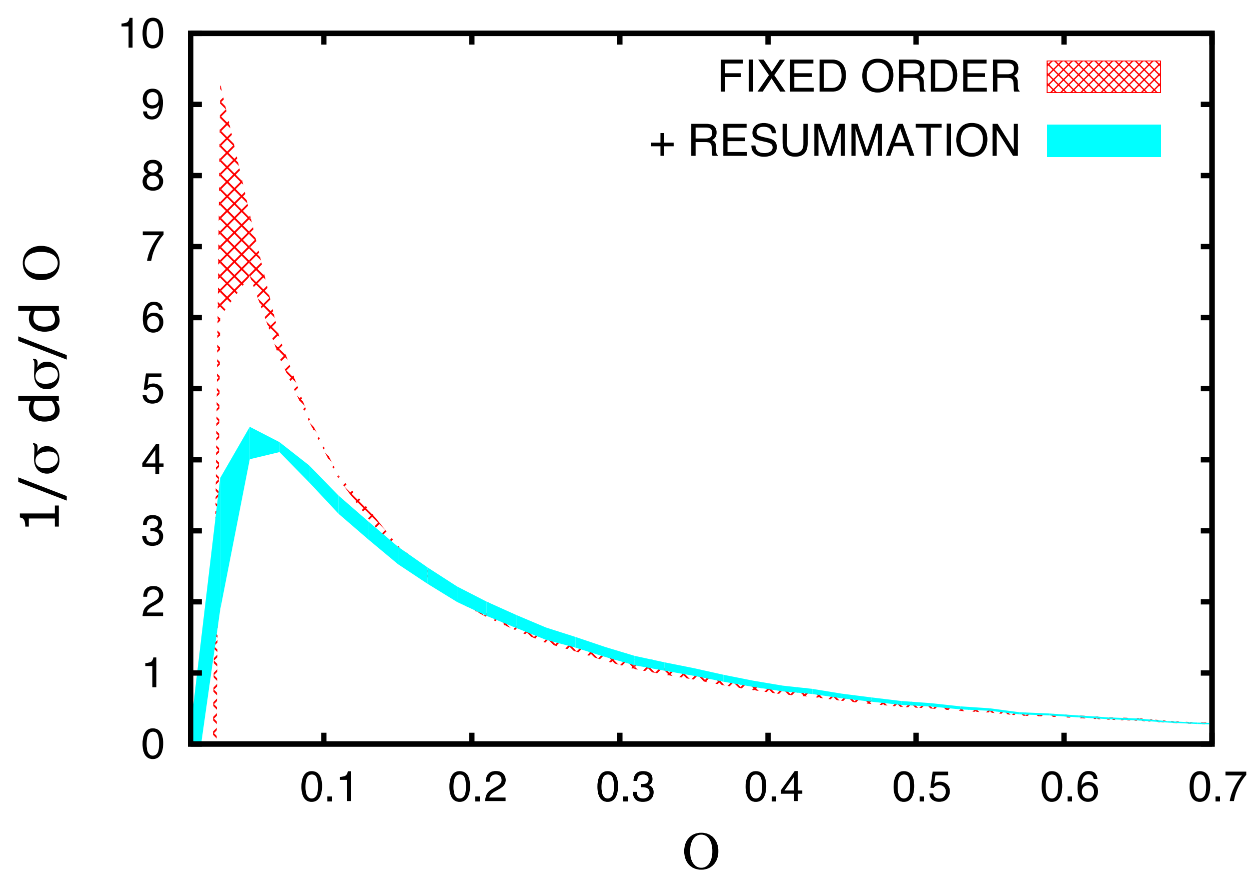
$$\sigma \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6 + \alpha_s^4 L^8 + \dots$$

$\alpha_s(Q) \ll \alpha_s(k_t) < 1$

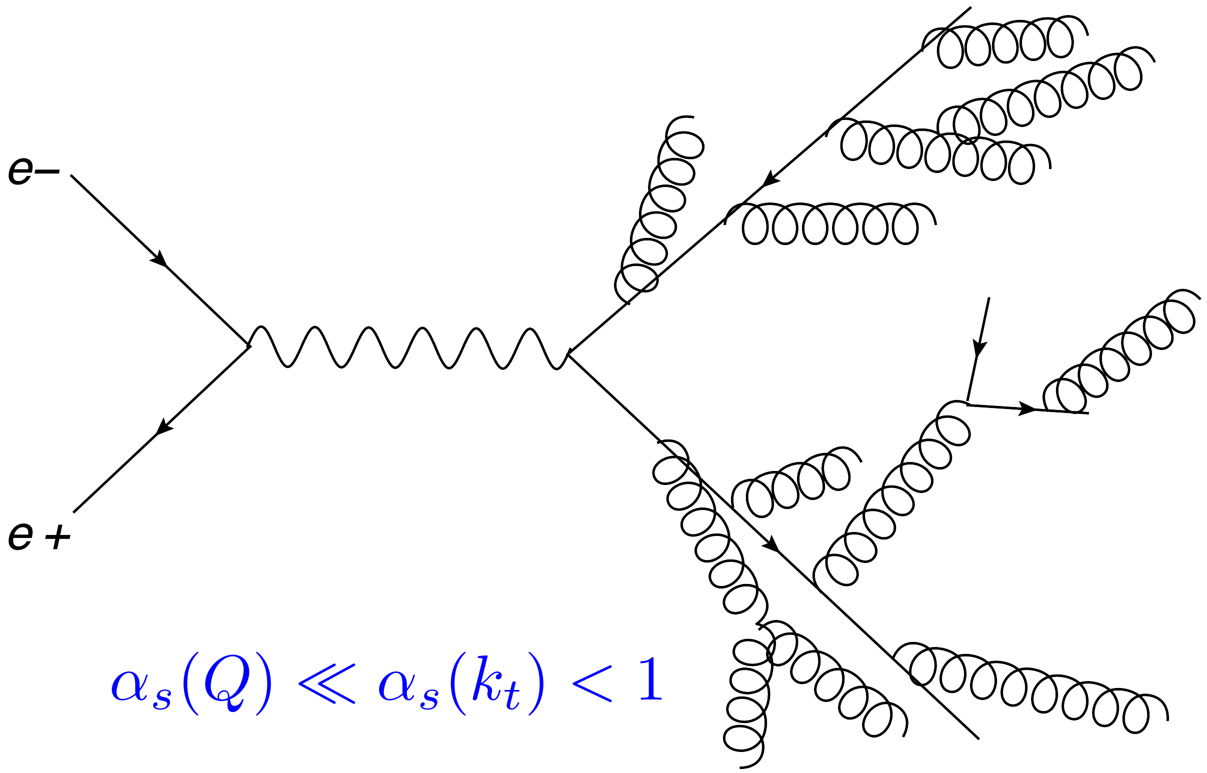
- As the coupling grows large, coloured particles are very likely to emit radiation with probability

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t)$$

- Inclusive Observables (e.g. total rates) are designed to be insensitive to such very soft and/or collinear (IRC) radiation
- However, the sensitivity to the IRC dynamics can become significant if one applies constraints on the radiation



Structure of a QCD event



$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s \sim \alpha_s \ln^2 O = \alpha_s L^2 \sim 1$$

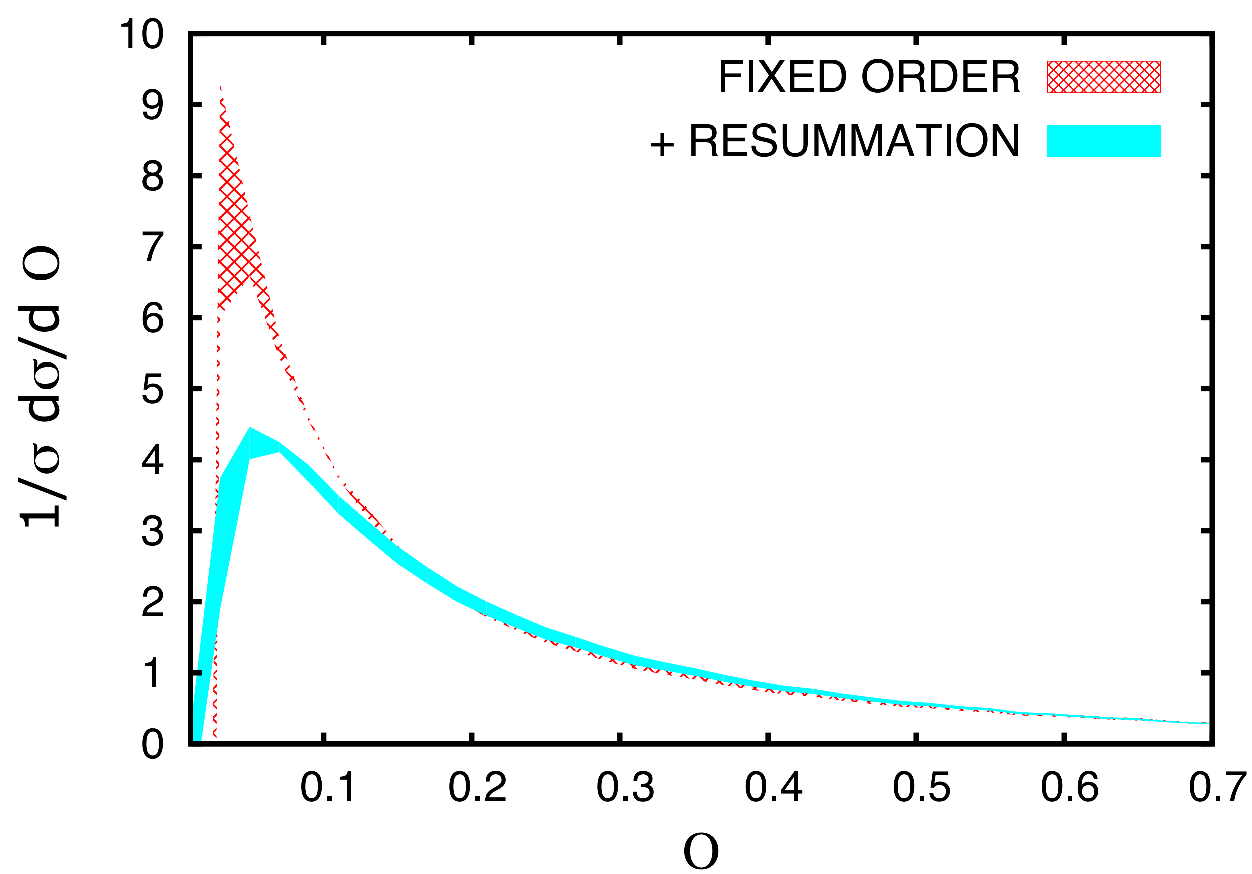
$$\sigma \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \alpha_s^3 L^6 + \alpha_s^4 L^8 + \ldots \overset{?}{\sim} e^{-\alpha_s L^2}$$

$\alpha_s(Q) \ll \alpha_s(k_t) < 1$

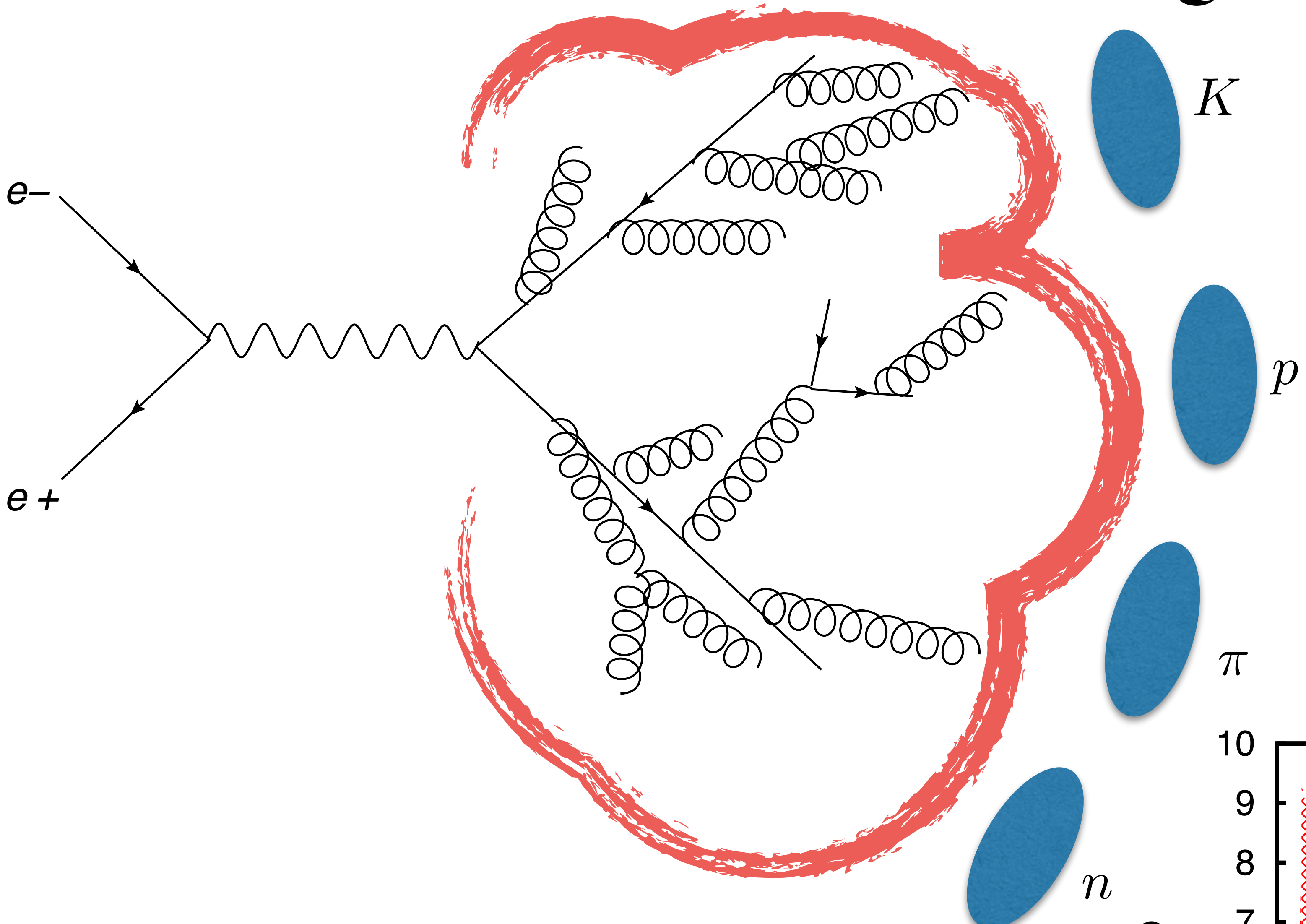
- As the coupling grows large, coloured particles are very likely to emit radiation with probability

$$\int \frac{dE}{E}\frac{d\theta}{\theta}\alpha_s(k_t)$$

- Inclusive Observables (e.g. total rates) are designed to be insensitive to such very soft and/or collinear (IRC) radiation
- However, the sensitivity to the IRC dynamics can become significant if one applies constraints on the radiation



Structure of a QCD event

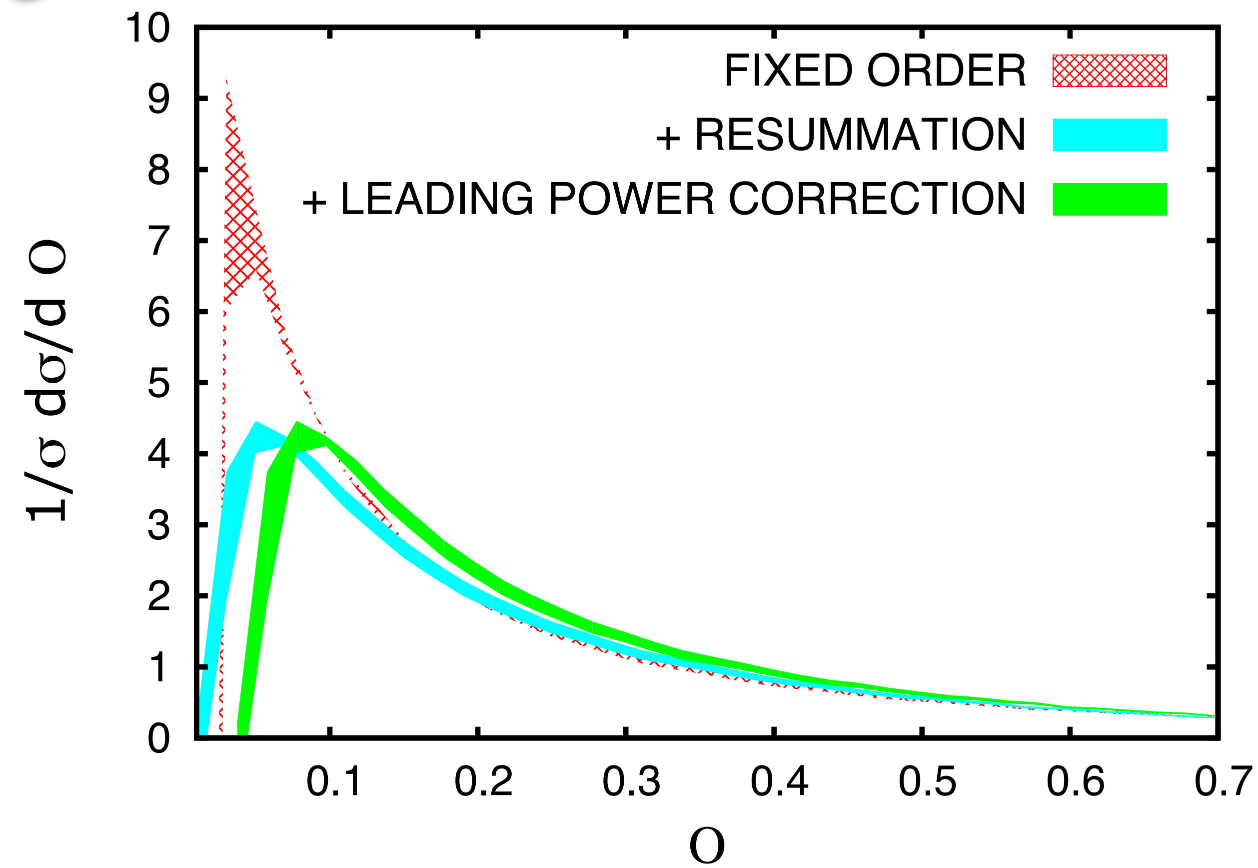


$$\sim \frac{1}{E_{\mathrm{cm}}}$$

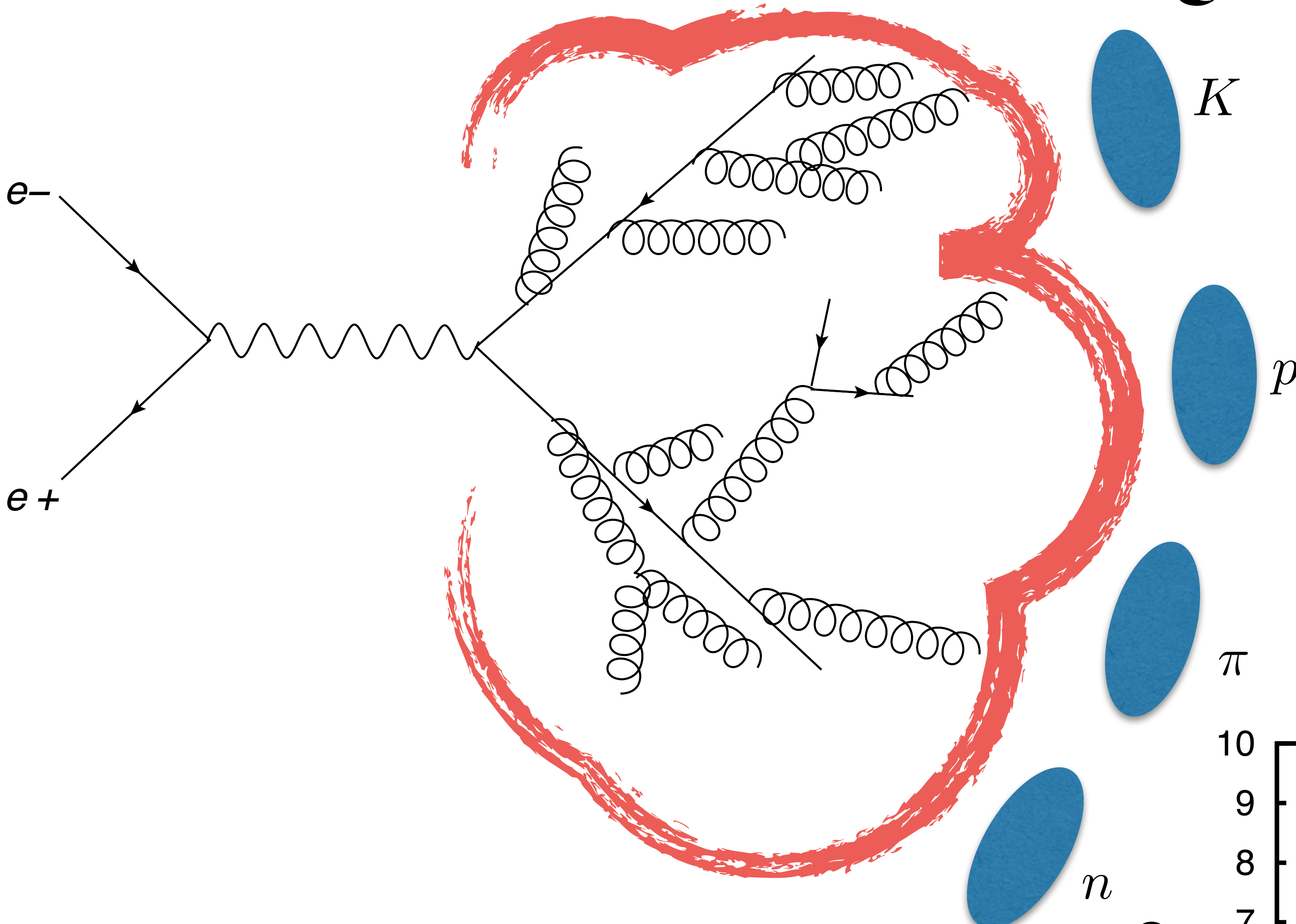
- At typical hadronic scales, hadronisation occurs

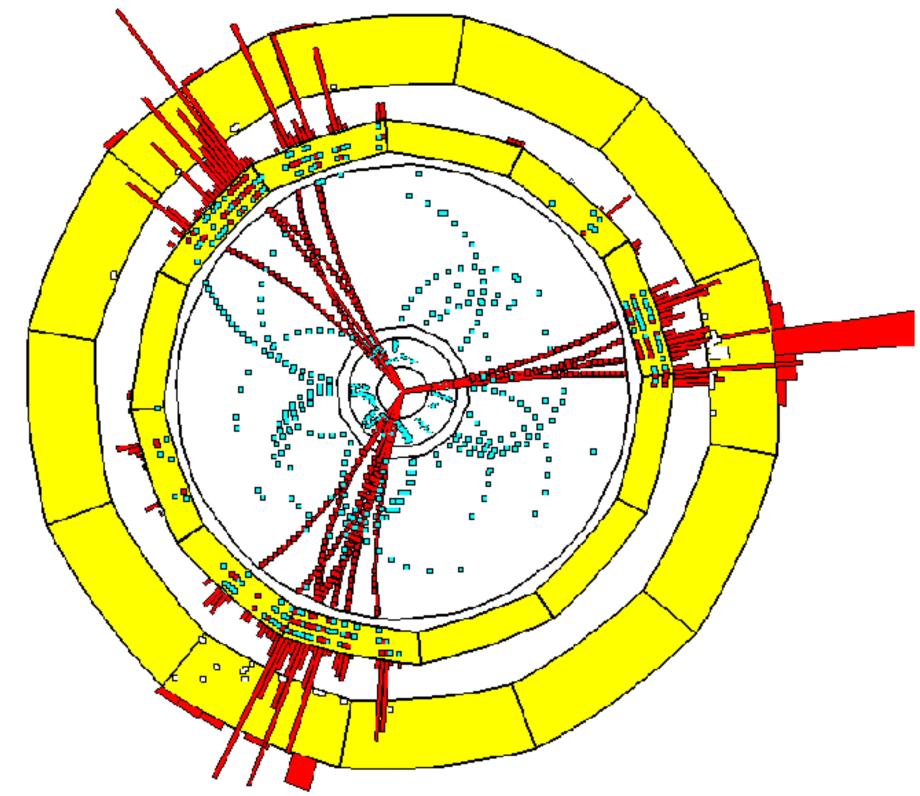
$$\alpha_s(k_t) \sim 1$$

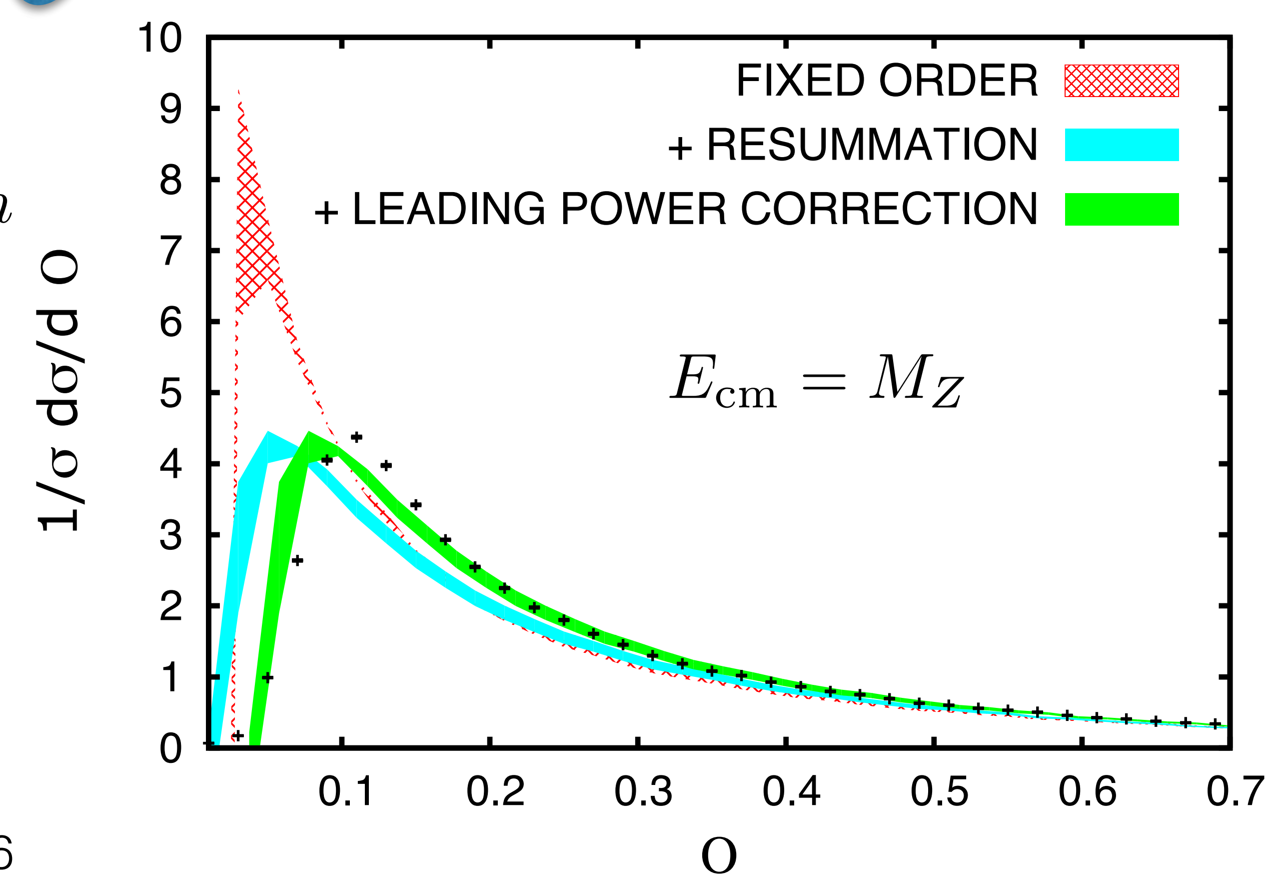
- Final state partons combine to form colourless hadrons: further kinematic reshuffle if cuts are applied



Structure of a QCD event







- At typical hadronic scales, hadronisation occurs

$$\alpha_s(k_t) \sim 1$$

- Final state partons combine to form colourless hadrons: further kinematic reshuffle if cuts are applied

Outline of the Lectures

- The course is divided into 3 lectures of 1.5h each
- Lecture I :
  - Introduction to resummation for final-state observables
  - Factorisation of the QCD matrix element
  - Classification of observables
- Lecture II :
  - Resummation for global observables in e^+e^- -> 2 jets
  - Branching formalism:
    - Analytic solution
    - Monte Carlo solution
- Lecture III :
  - Notions of Soft-Collinear Effective Theory
  - Factorisation for a global observable in e^+e^- -> 2 jets
  - Renormalisation-Group Evolution and Resummation

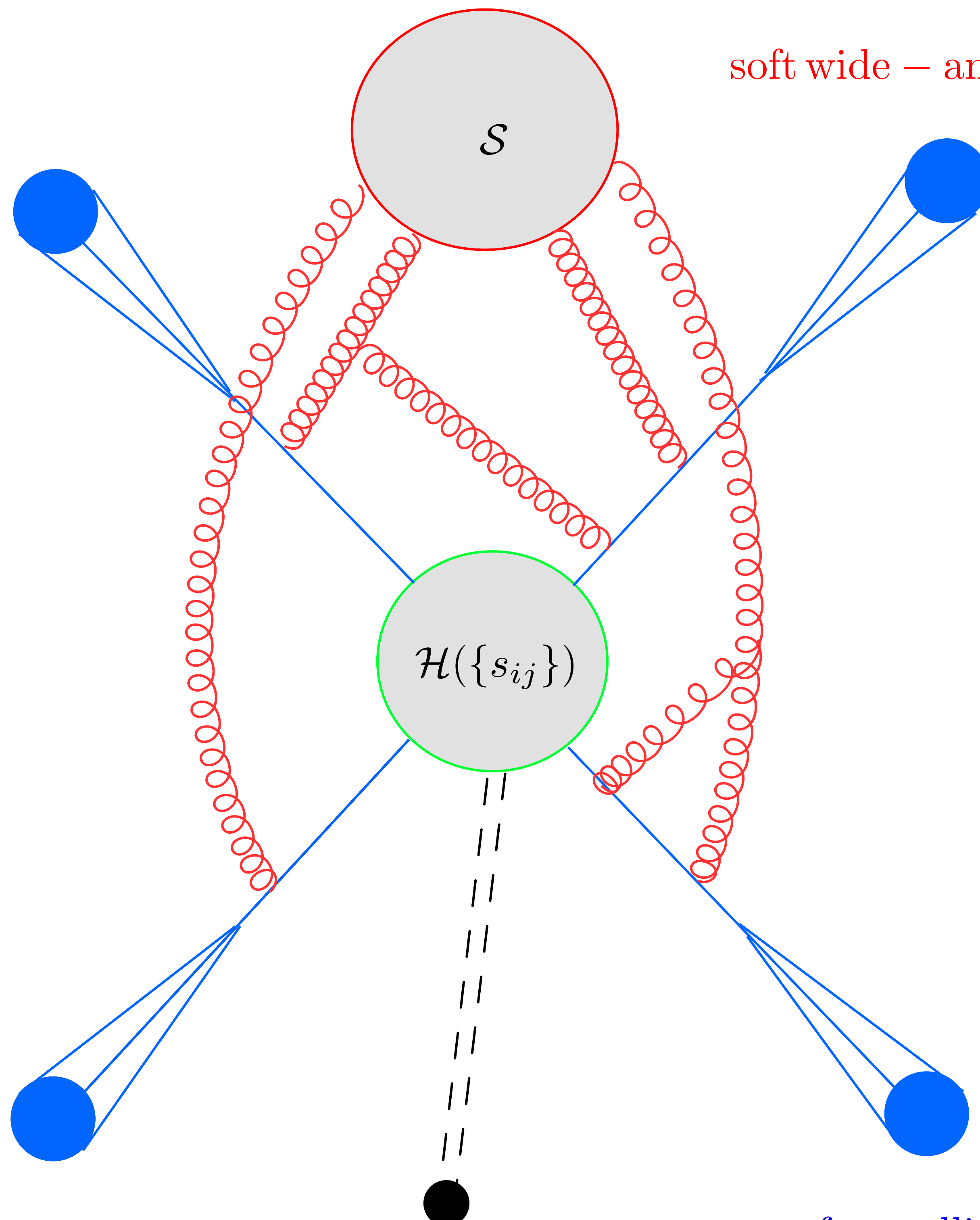
References (more during the lectures)

- Lecture I :
  - S. Catani's Academic Training Lectures: http://cds.cern.ch/record/377090/
  - R.K. Ellis, W.J. Stirling, and B.R. Webber: QCD and Collider Physics

- Lecture II :
  - S. Catani, L. Trentadue, G. Turnock, B.R. Webber: Nucl.Phys. B407 (1993) 3-42
  - A. Banfi, G. Salam, G. Zanderighi: hep-ph/0407286
  - A. Banfi, H. McAslan, P.F. Monni, G. Zanderighi: arXiv:1412.2126

- Lecture III :
  - T. Becher, A. Broggio, A. Ferroglia: arXiv:1410.1892
  - Iain Stewart's lectures on SCET: https://ocw.mit.edu/courses/physics/8-851-effective-field-theory-spring-2013/lecture-notes/MIT8_851S13_scetnotes.pdf
  - Bauer, Fleming, Lee, Sterman: arXiv:0801.4569

Factorisation of amplitudes in the IRC

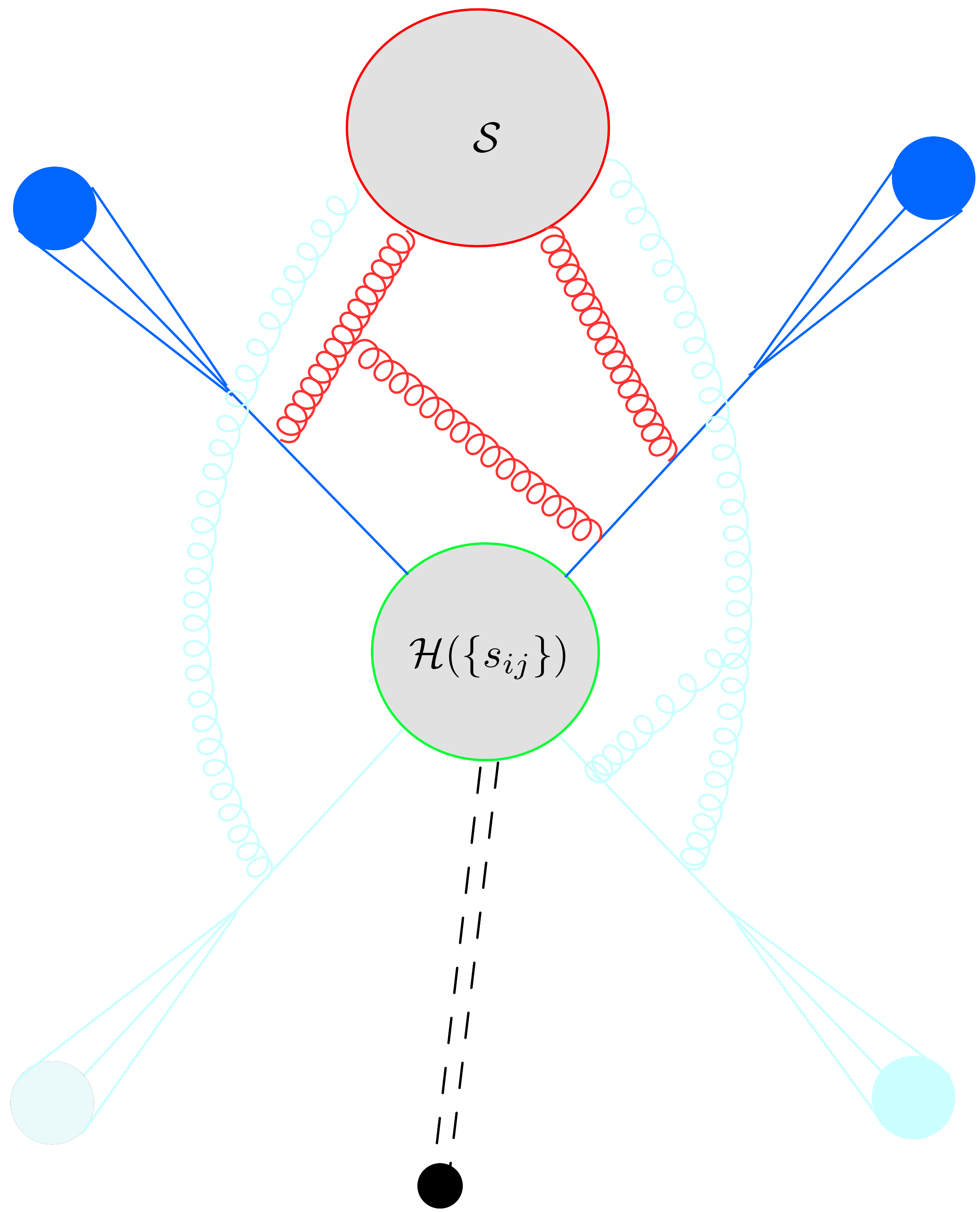


soft wide − angle : $\alpha_s^n L^m$ $(m \leq n)$

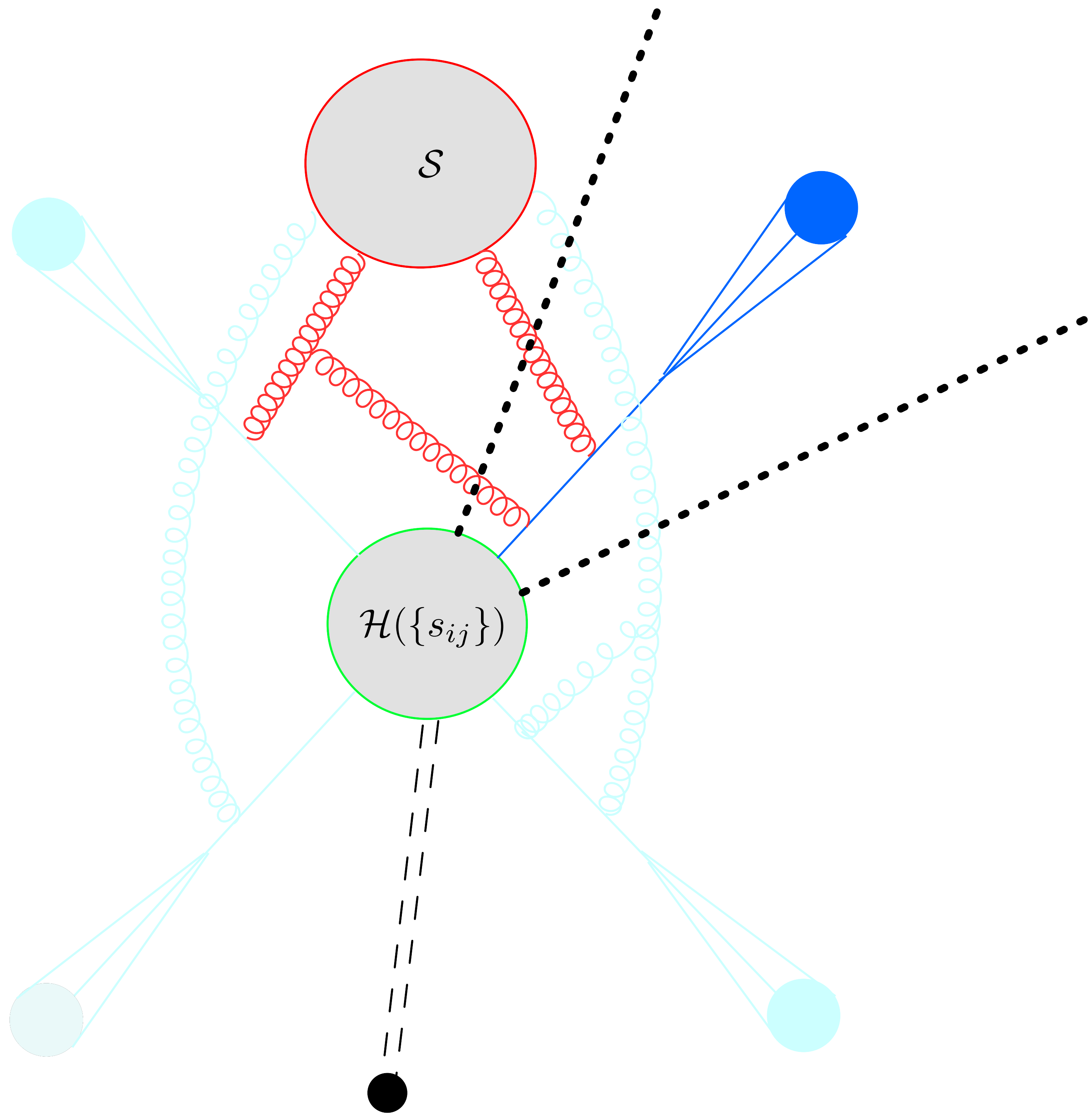
soft − collinear : $\alpha_s^n L^m$ $(m \leq 2n)$

hard − collinear : $\alpha_s^n L^m$ $(m \leq n)$

Two-emitter processes



Non-Global observables



Two-emitter processes (global case)