TMDs at small x

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Medium energy QCD (12GeV-500GeV)

RHIC, Jlab, EIC

Outline:

- Joint TMD and small x evolution
- Spin dependent TMDs at small x

  1: unpolarized target

  2: transversely polarized target
- Summary

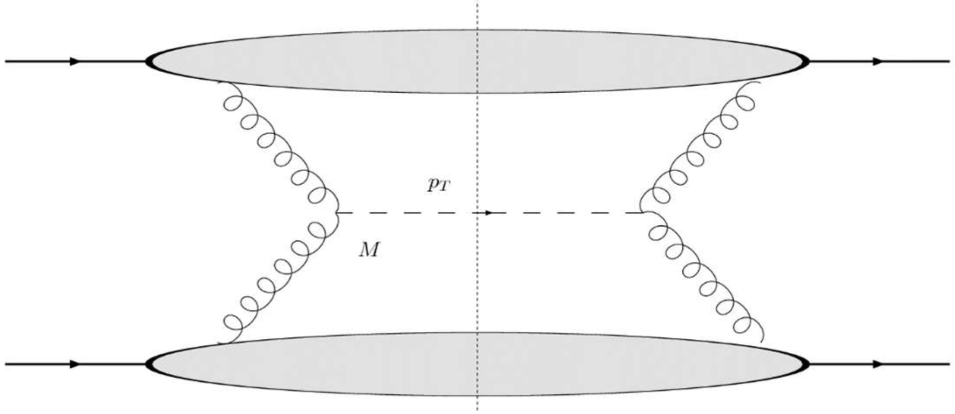
QCD evolution(resummation)

Desinged for summing large logarithms:

- DGLAP: $\ln \frac{Q^2}{\mu^2}$
- Collins-Soper: $\ln^2 \frac{Q^2}{p_T^2}$
- BFKL(MQ, BK,JIMWLK): $\ln \frac{S}{Q^2}$
- Threshold resummation: $\left(\frac{\ln(1-x)}{1-x}\right)_+$
- The infrared evolution equation $\ln^2 \frac{S}{Q^2}$
- …

To improve the convergence of perturbation series.

Gluon initiated Drell-Yan process



- $M^2 >> p_T^2$, TMD factorization, $\ln \frac{M^2}{p_T^2}$ resummed by Collins-Soper equation

1982-1983, Collins, Soper

- $S >> M^2$, Kt factorization, $\ln \frac{S}{M^2}$ resummed by BFKL equation

1991, Catani, Ciafaloni and Hautmann
1991, Collins and R. K. Ellis

The overlap region $S >> M^2 >> p_T^2$

An explicit NLO cross section calculation shows that both the large logarithm appear.

2013, Mueller, Xiao, Yuan

Such joint resummation has been also disscussed in other literatures.

2015, Balitsky and Tarasov; 2015 Marzani

Our starting point

$$xG(x,l_\perp,x\zeta) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3 P^+} e^{-ixP^+y^- + il_\perp\cdot y_\perp} \langle P|F_\mu^+(y^-,y_\perp)\mathcal{L}_{\tilde{n}}^\dagger(y^-,y_\perp)\mathcal{L}_{\tilde{n}}(0,0_\perp)F^{\mu+}(0)|P\rangle$$

Gloun---->gloun splitting kernel;

$$\mathcal{P}_{gg}(z) = 2C_A \frac{(z^2-z+1)^2}{z(1-z)}$$

When z(or x) --->0, large logarithm $\ln\frac{1}{x}$ summed by BFKL

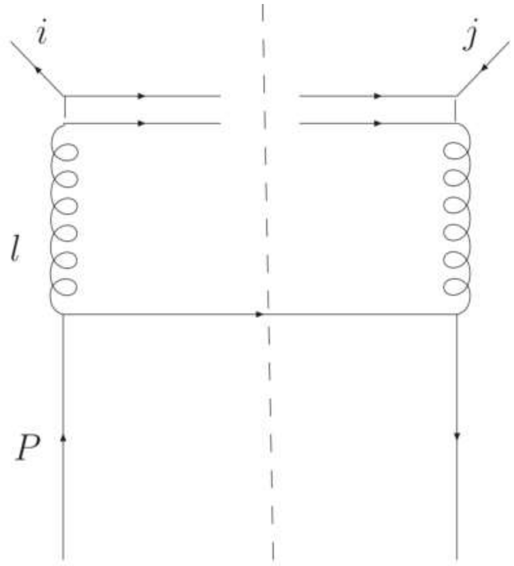
When z--->1, light cone divergence, introduce ζ to regularize

large logarithm $\ln\frac{x^2\varsigma^2}{k_T^2}$ summed by CS

$$\ln\frac{1}{x} \longrightarrow \ln\frac{S}{M^2} \qquad \ln\frac{x^2\varsigma^2}{k_T^2} \longrightarrow \ln\frac{M^2}{p_T^2}$$

small x gluon TMD at LO

In a simple quark model, at tree level:

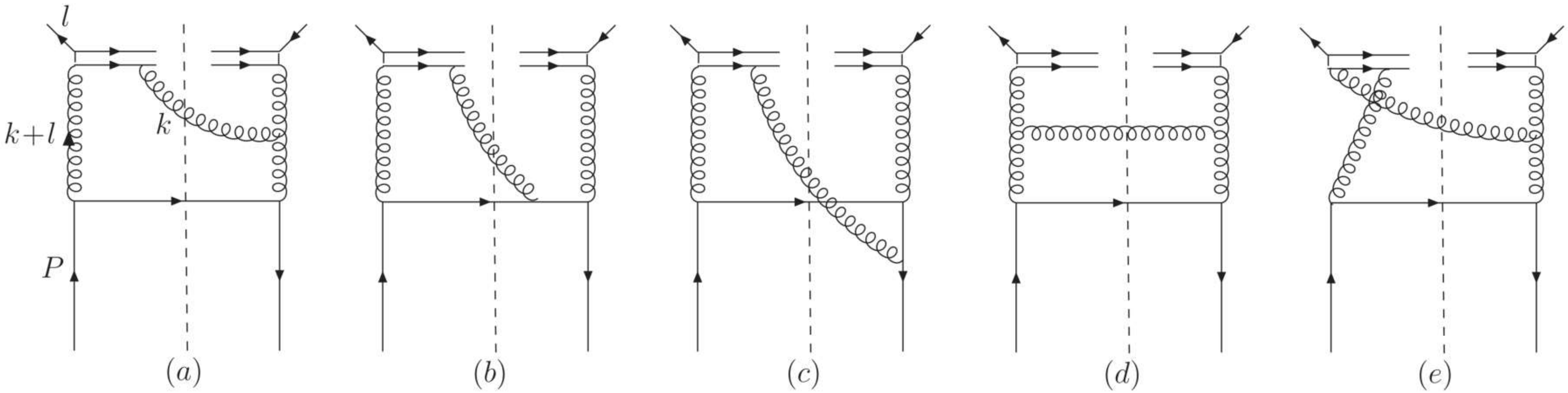


$$xG(x, l_\perp, x\zeta)_{LO}\big|_{x\to 0} = \frac{\alpha_s C_F}{\pi^2}\frac{1}{l_\perp^2}$$

- both large logarithms are absent at LO;
  how is it dressed by quantum corrections at NLO?

Real graphs

Sample diagrams:



- Fig.a is the only diagram contributing to both the CS and the BFKL evolution kernel

- Calculation formulated in the Ji-Ma-Yuan scheme.

$$\int_0^\infty \frac{dk^+}{k^+} = \int_{l^+}^\infty \frac{dk^+}{k^+} + \int_0^{l^+} \frac{dk^+}{k^+}$$

Virtual graphs

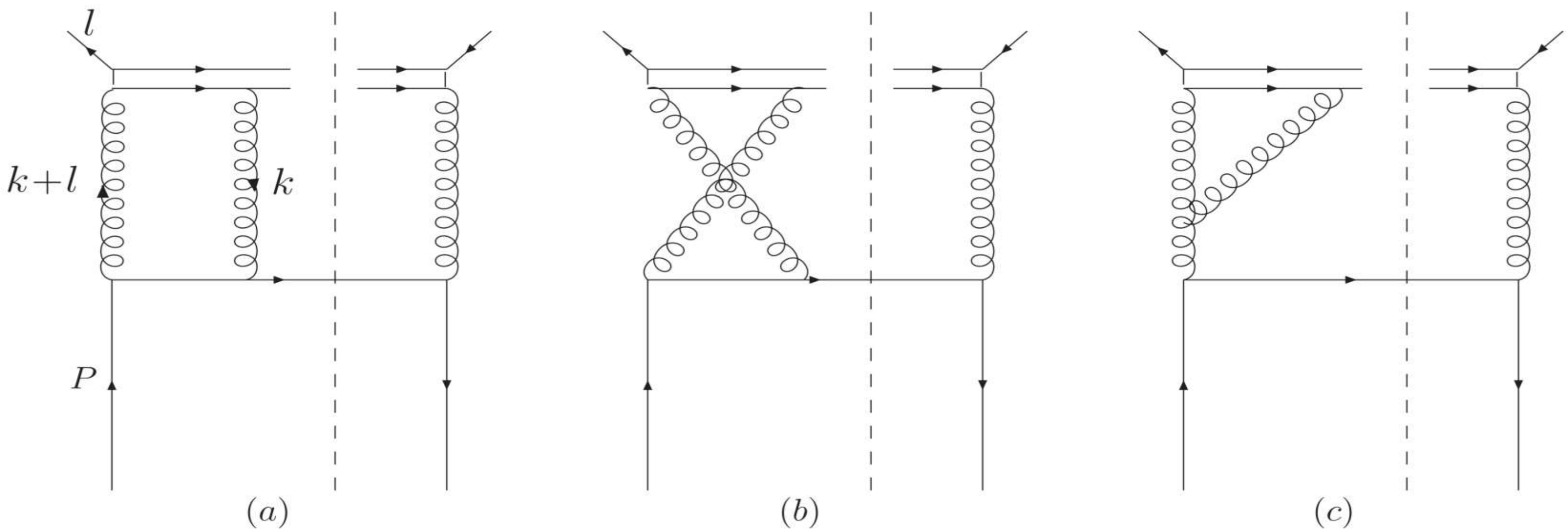


Fig.a & Fig.b ⟹ BFKL kernel

Fig.c ⟹ CS kernel

In the leading logarithm approximations,

Small x gluon TMD at NLO reads,

$$
\begin{aligned}
xG(x,l_\perp,x\zeta)_{NLO} &= xG(x,l_\perp,x\zeta)_{LO} \\
&+ \frac{\alpha_s N_c}{\pi^2}\ln\frac{1}{x}\int\frac{d^2k_\perp}{k_\perp^2}\left[xG_{LO}(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG_{LO}(x,l_\perp,x\zeta)\right] \\
&+ \frac{\alpha_s N_c}{2\pi}\left[\ln\frac{x^2\zeta^2}{l_\perp^2}-\frac{1}{2}\ln\frac{x^2\zeta^2}{\mu^2}-\left(\ln\frac{x^2\zeta^2}{l_\perp^2}\right)^2\right]xG_{LO}(x,l_\perp,x\zeta) \\
&+ \frac{\alpha_s N_c}{\pi^2}\int\frac{d^2k_\perp}{2\left[k_\perp^2+l_\perp^4/x^2\zeta^2\right]}\ln\frac{k_\perp^2(k_\perp^2+x^2\zeta^2)}{(k_\perp^2+l_\perp^2)^2}xG_{LO}(x,k_\perp+l_\perp,x\zeta)
\end{aligned}
$$

2016, ZJ

$$\ln\frac{1}{x} \Longrightarrow \ln\frac{S}{M^2} \qquad\qquad \ln\frac{x^2\varsigma^2}{k_T^2} \Longrightarrow \ln\frac{M^2}{p_T^2}$$

The resulting gluon TMD indeed simultaneously satisfies the both

BFKL equation:

$$\frac{\partial\left[xG(x,l_\perp,x\zeta)\right]}{\partial \ln(1/x)} = \frac{\alpha_s N_c}{\pi^2}\int\frac{d^2k_\perp}{k_\perp^2}\left\{xG(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG(x,l_\perp,x\zeta)\right\}$$

CS equation:

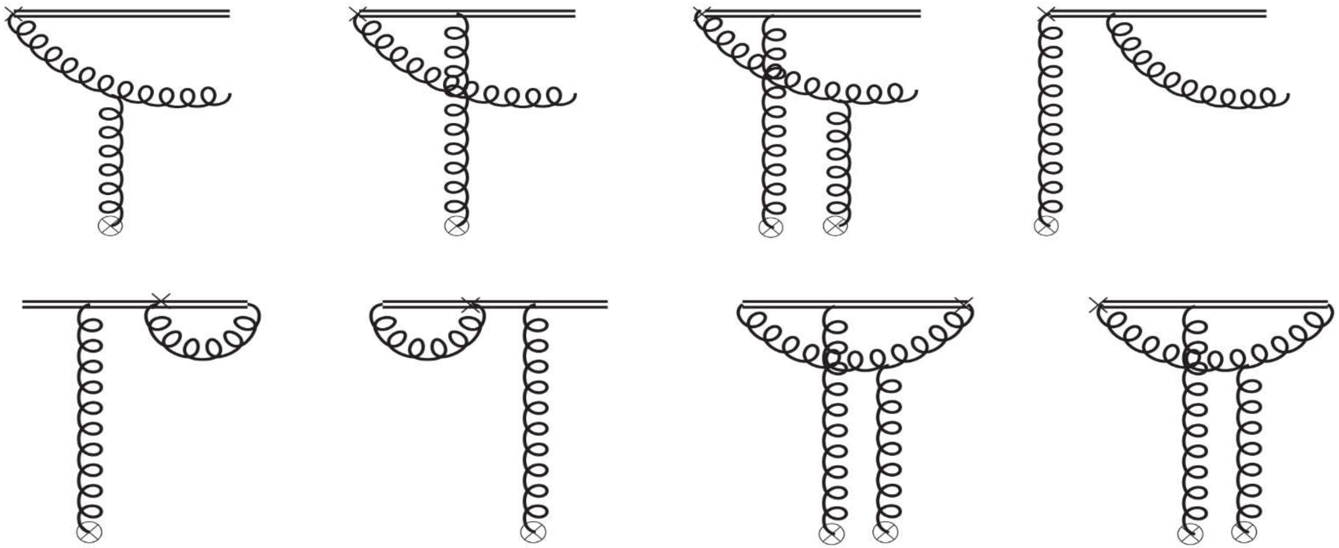
$$\frac{\partial\left[G(x,b_\perp,x\zeta)\right]}{\partial \ln\zeta} = -\frac{\alpha_s N_c}{\pi}\ln\left[\frac{x^2\zeta^2 b_\perp^2}{4}e^{2\gamma_E-\frac{1}{2}}\right]G(x,b_\perp,x\zeta)$$

More formal treatment

Computing small x gluon TMDs in CGC with Collins 2011 scheme

Small x TMDs in CGC at NLO

Sample diagrams



Xiao, Yuan and ZJ, 2017.

The basic nonperturbative ingredient:

$$U(x_\perp) = \langle \mathcal{P} e^{-ig \int_{-\infty}^{+\infty} dx^- A^+(x^-,\, x_\perp)} \rangle$$

Multiple gluon rescattering is treated in an equal way.

Collinear approach vs CGC I

- **TMDs in collinear approach**
  collinear divergence DGLAP
- **TMDs in CGC,**
  rapidity divergence BK or JIMWLK

$$\int_{l^+}^{\infty} \frac{dk^+}{k^+}$$

Collins-Soper light cone divergence appears in both collinear approach and CGC calculation

Match small x TMDs onto two point functions instead of PDFs.

$$\left\langle \mathrm{Tr} U(R_\perp + r_\perp) U^\dagger(R_\perp) \right\rangle$$

Collinear approach vs CGC II

$$\tilde{f}_g^{(sub.)}(x, r_\perp, \zeta_c{}') = e^{-S^g_{pert}(Q, r_\perp)} \sum_i C_{g/i}(\mu_r/\mu) \otimes f_i(x, \mu)$$

Sudakov factor — Hard coefficient — Colliner PDF

$$xG^{(1)}(x, k_\perp, \zeta_c) = -\frac{2}{\alpha_S} \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_s(Q)) e^{-S_{sud}(Q^2, r_\perp^2)} \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp)$$

Hard coefficient — Sudakov factor — Two point function

Two step evolution: $S \rightarrow M^2 \rightarrow k_T^2$

Spin dependent TMDs at small x

1: Inside an unpolarized target

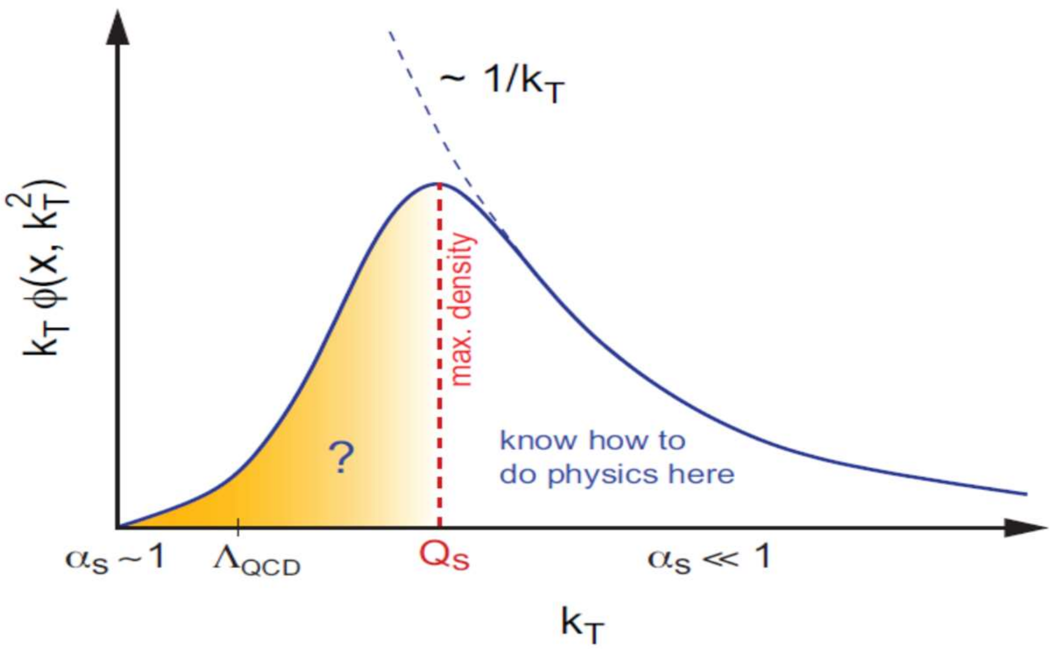
Linearly polarized gluon TMD

$$\int \frac{dr^- d^2r_\perp}{(2\pi)^3 P^+} e^{-ix_1 P^+ r^- + i\vec{k}_{1\perp}\cdot\vec{r}_\perp} \langle A|F^{+i}(r^- + y^-, r_\perp + y_\perp)\, L^\dagger\, L\, F^{+j}(y^-, y_\perp)|A\rangle$$

$$= \frac{\delta_\perp^{ij}}{2} x_1 G(x_1, k_{1\perp}) + \left(\hat{k}^i_{1\perp}\hat{k}^j_{1\perp} - \frac{1}{2}\delta_\perp^{ij}\right) x_1 h_1^{\perp g}(x_1, k_{1\perp}),$$

Mulders, Rodrigues, 2001

Unpolarized gluon TMD computed in the MV model

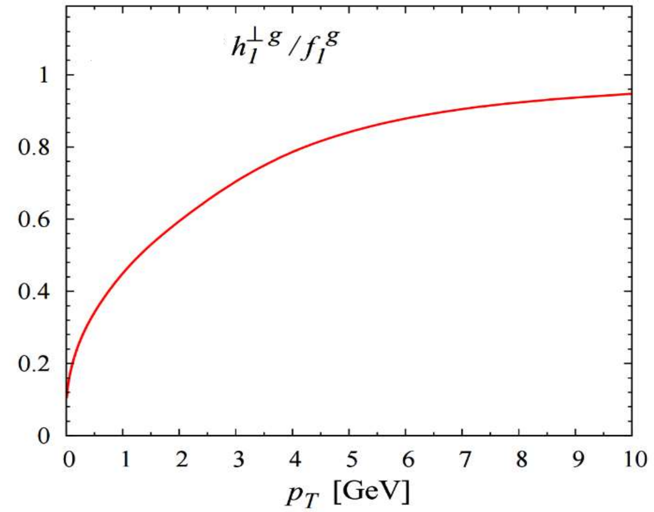


Kovchegov, 96
J. Marian, Kovner, Mclerran & Weigert, 97

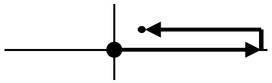
Gluon TMDs in the MV model

The linearly polarized gluon TMDs in the MV model,

Metz & ZJ, 2011

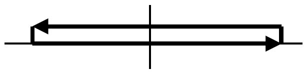


Weizsäcker-Williams(WW) distribution:



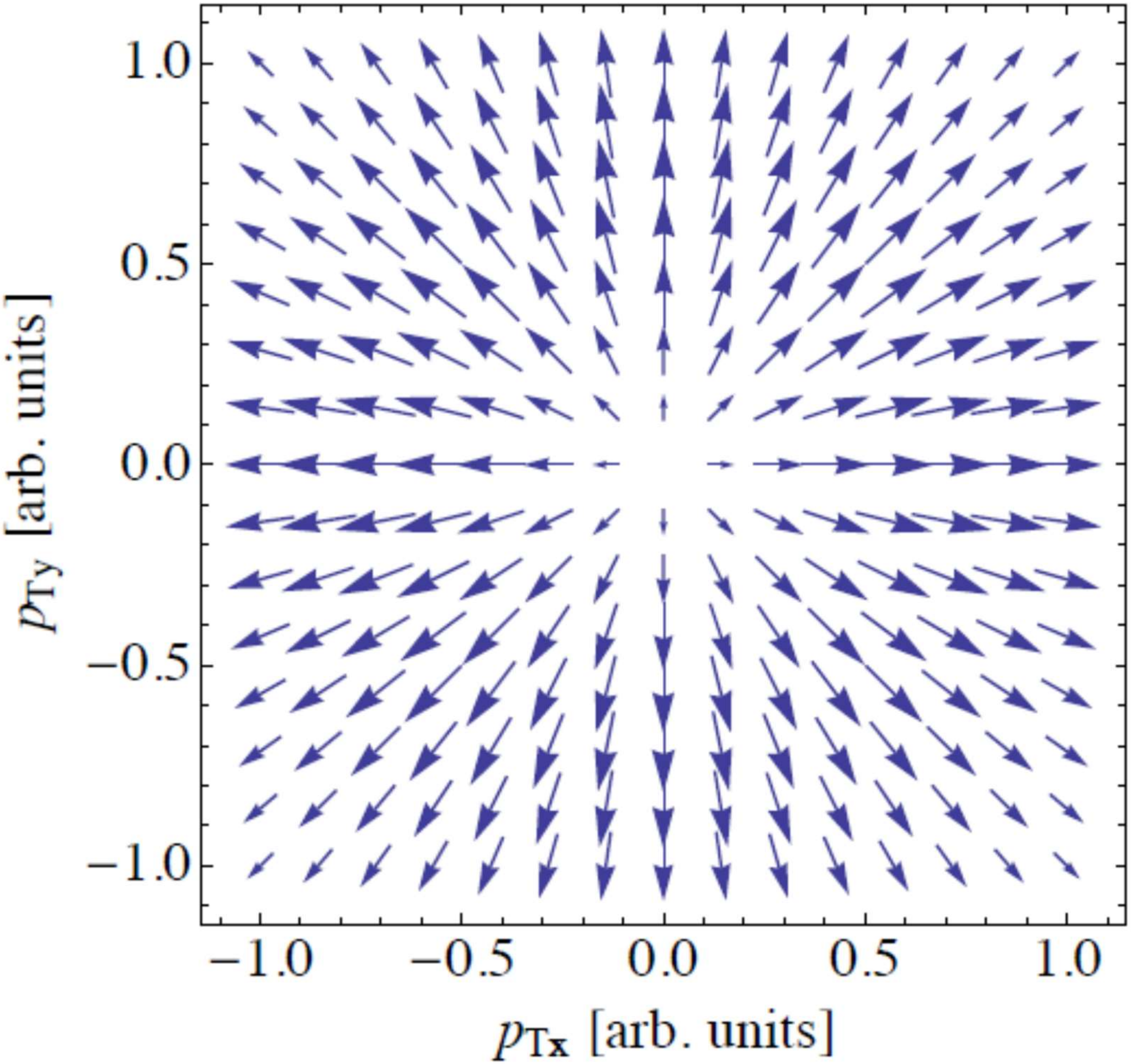
$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3} S_\perp \int d\xi_\perp \frac{K_2(k_\perp \xi_\perp)}{\frac{1}{4\mu_A}\xi_\perp Q_s^2}\left(1-e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right)$$

Dipole distribution:



$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}^g(x,k_\perp) = \frac{k_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-i\vec{k}_\perp\cdot\vec{\xi}_\perp} e^{-\frac{Q_{sq}^2\xi_\perp^2}{4}}$$

positivity bound saturated for any value of k_t



1.0
0.5
0.0
−0.5
−1.0
p_{Ty} [arb. units]
−1.0
−0.5
0.0
0.5
1.0
$p_{T\mathbf{x}}$ [arb. units]

TMD evolution effect

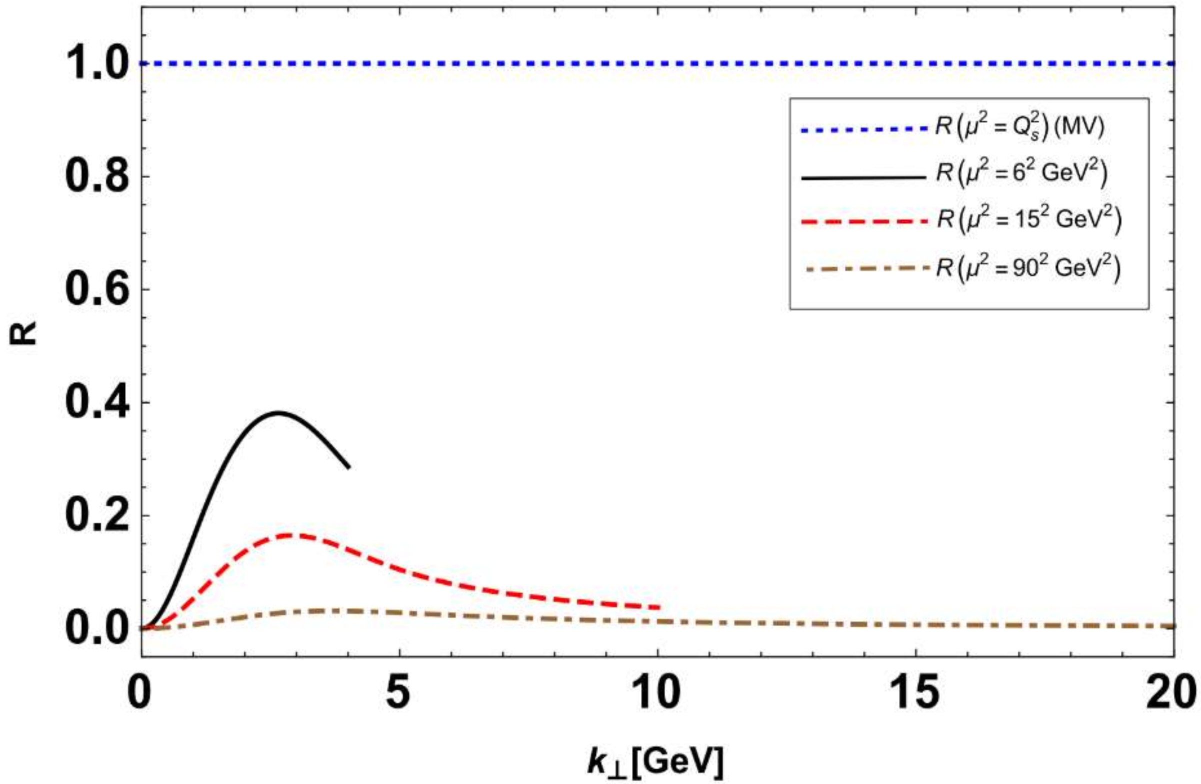


FIG. 2: The ratio $R = h_1^{\perp g}/f_1^g$ as function of $k_\perp$, at $x = 0.01$ for $\mu = 6$, 15 and 90 GeV.

D. Boer, P. Mulders, JZ, Y. J. Zhou, 2017

2: **Inside a transversely polarized target**

Three T-odd gluon TMDs

Identify 6 leading power gluon TMDs for a transversely polarized target (8 in total). Among them, 3 gluon TMDs are T-odd distributions.

$$\frac{1}{xP^+}\int\frac{dy^- d^2y_T}{(2\pi)^3}e^{ik\cdot y}\langle P,S_T|2\mathrm{Tr}\left[F^{\mu}_{+T}(0)UF^{\nu}_{+T}(y)U'\right]|P,S_T\rangle$$

$$= \delta_T^{\mu\nu} f_1^g + \left(\frac{2k_T^\mu k_T^\nu}{k_\perp^2} - \delta_T^{\mu\nu}\right) h_1^{\perp g} - \delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M} f_{1T}^{\perp g}$$

$$-i\epsilon_T^{\mu\nu}\frac{k_T\cdot S_T}{M} g_{1T}^g - \frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M} h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}} + \tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M} h_{1T}^g$$

Mulders, Rodrigues, 2001

Are the T-odd gluon TMDs relevant at small x?

T-odd gluon TMDs & the odderon

Starting point,

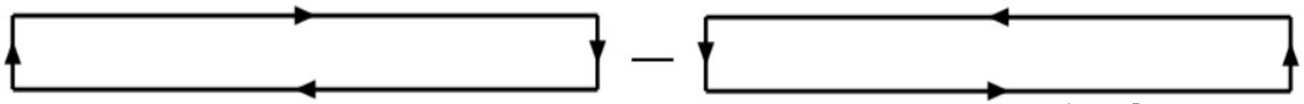
$$\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{1}{xP^+}\int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{ik\cdot y} \langle P, S_T | 2\mathrm{Tr}\left[F^{\mu}_{+T}(0) U^{[-]\dagger} F^{\nu}_{+T}(y) U^{[+]}\right] | P, S_T\rangle$$

Using time reversal invariance and parity symmetry, at small x one obtains,

$$\Gamma^{\mu\nu}_{\mathrm{T-odd}} = \frac{k_T^\mu k_T^\nu}{g^2 V x P^+}\int \frac{d^2 y_{1T} d^2 y_{2T}}{(2\pi)^3} e^{ik_T\cdot y_T} \langle P, S_T | \mathrm{Tr}\left[U^{[\Box]}(y_T) - U^{[\Box]\dagger}(y_T)\right] | P, S_T\rangle$$

Schematically,

$$\Gamma^{\mu\nu}_{T-odd} \propto \frac{1}{2} k_T^\mu k_T^\nu \{ \quad - \quad \}$$



$$\hat{D}(R_\perp, r_\perp) = \frac{1}{N_c}\mathrm{Tr}\left[U(R_\perp + \frac{r_\perp}{2}) U^\dagger(R_\perp - \frac{r_\perp}{2})\right]$$

Nothing but an odderon operator in CGC

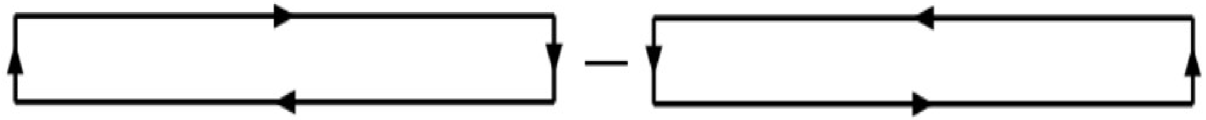
$$\hat{O}(R_\perp, r_\perp) = \frac{1}{2i}\left[\hat{D}(R_\perp, r_\perp) - \hat{D}(R_\perp, -r_\perp)\right]$$

Kovchegov, Szymanowski & Wallon 2004
Hatta, Iancu, Itakura & McLerran 2005

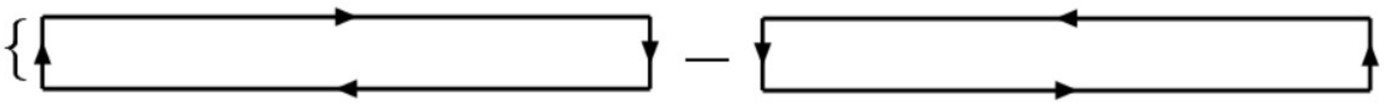
Spin dependent odderon

$$
\begin{aligned}
<\hat{O}(r_\perp)> &= -\frac{c_0\alpha_s^3\pi}{8M_pR_0^2}e^{-\frac{1}{4}r_\perp^2Q_s^2}r_\perp^2\epsilon_\perp^{ij}S_{\perp i}r_{\perp j}\int dx_q d^2z_\perp\sum_{u,d}\mathcal{E}(x_q,z_\perp^2)\\
&= -\frac{c_0\alpha_s^3\pi}{8M_pR_0^2}e^{-\frac{1}{4}r_\perp^2Q_s^2}r_\perp^2\epsilon_\perp^{ij}S_{\perp i}r_{\perp j}\left(\kappa_p^u+\kappa_p^d\right)
\end{aligned}
$$

In the momentum space:

$$\frac{\epsilon_T^{\alpha\beta}S_{T\alpha}k_{T\beta}}{M}O_{1T,x}^{\perp}(k_\perp^2)\propto\{$$  $$\}$$

ZJ 2013

$$\Gamma^{\mu\nu}_{T-odd}\propto\frac{1}{2}k_T^\mu k_T^\nu\{$$  $$\}$$

Two different paramertrizations

Equaling two parmetrizations:

$$\frac{k_T^\mu k_T^\nu N_c}{2\pi^2\alpha_s x}\frac{\epsilon_T^{\alpha\beta}S_{T\alpha}k_{T\beta}}{M}O^\perp_{1T,x}(k_\perp^2) = -\delta_T^{\mu\nu}\frac{\epsilon_{T\alpha\beta}k_T^\alpha S_T^\beta}{M}f_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}k_T^{\nu\}}}{k_\perp^2}\frac{k_T\cdot S_T}{M}h_{1T}^{\perp g} - \frac{\tilde{k}_T^{\{\mu}S_T^{\nu\}}+\tilde{S}_T^{\{\mu}k_T^{\nu\}}}{2M}h_{1T}^g$$

Simple algebra leads to

$$xf_{1T}^{\perp g} = xh_{1T}^g = xh_{1T}^{\perp g} = \frac{k_\perp^2 N_c}{4\pi^2\alpha_s}O^\perp_{1T,x}(k_\perp^2)$$

Boer, Echevarria, Mulders, ZJ; 2016

All of three dipole type T-odd gluon TMDs become identical at small x!

Summary

At small x:

- A joint resummation can be achieved
- Very rich polarization dependent phenomenology

It would be interesting to test these theoretical ideas at RHIC and the future EIC

Thank you for your attention.